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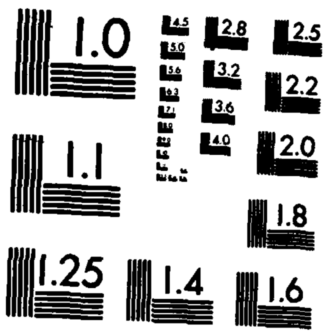
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DUAL OFFSET REFLECTOR ANTENNA SYSTEMS WITH ROTATIONALLY SYMMETRIC APERTURE DISTRIBUTIONS

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Carlyle J. Sletten (GTE)

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, NY 13441

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A detailed derivation is given of the basic design equation for a dual offset reflector antenna consisting of a paraboloid main reflector and a confocal/hyperboloid or ellipsoid subreflector. The basic design equation is a relation between the subreflector eccentricity; the relative orientations of the axes of the main reflector, subreflector, and feed; and the paraboloid focal length. If satisfied, the basic design equation guarantees that (to within the geometric optics approximation) a circularly symmetric and linearly polarized feed pattern will give rise to a circularly symmetric and linearly polarized main aperture distribution and far field pattern. The relation between the amplitude distribution of the feed pattern and main aperture is also derived.

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Block 11 (Contd)

Systems With Rotationally Symmetric Aperture Distributions

Preface

The authors wish to thank Dr. Luiz Costa da Silva of Catholic University, Rio de Janeiro, Brazil, for his proofs of some of the equations contained in this paper.

Contents

Illustrations

Dual Offset Reflector Antenna Systems With Rotationally Symmetric Aperture Distributions

1. INTRODUCTION

A rotationally symmetric paraboloid reflector fed by a rotationally symmetric feed aligned with the reflector axis and located at the paraboloid focus gives rise to a rotationally symmetric aperture distribution and far field pattern. If the feed is linearly polarized, then so is the aperture and far field. This positioning of the feed, however, partially blocks the reflected field. Offset reflectors can be used to reduce this blockage, but rotational symmetry and polarization purity are then lost to an extent proportional to the offset angle. The principal attraction of dual offset reflector systems, Cassegrainian and Gregorian, is that if they are suitably designed, it is possible not only to eliminate blockage but also to preserve rotational symmetry and polarization purity of the pattern. The geometries for such ideal dual reflector systems were first discovered by Japanese investigators[1, 2] and then generalized in an elegant way by Dragone.[3]

(Received for publication 12 July 1984)

1. Tanaka, H., and Mizusawa, M. (1975) Elimination of cross polarization in offset dual reflector antennas, Trans. IECE Japan, 58-B (No. 12).
2. Mizugutch, Y., Akagawa, M., and Yokoi, H. (1976) Offset dual reflector antenna, IEEE Sympos. Digest, University of Massachusetts, Amherst, AP-S:2-5.
3. Dragone, C. (1978) Offset multireflector antennas with perfect pattern symmetry and polarization discrimination, Bell System Tech. J., 57 (No. 7): 2663-2683.

The purpose of this report is to present a detailed derivation of the Japanese formula to design an offset dual reflector antenna system with a rotationally symmetric aperture distribution. Such a derivation has not appeared in an English language publication. The derivation given here amplifies at some length a personal communication of Mizugutch. In addition, we derive an expression for the aperture power distribution not given in the Japanese papers.

2. ANALYSIS

We consider a dual reflector system as shown in Figures 1A and 1B. The main reflector is an offset section of a paraboloid with focal length f. The subreflector is a section of either an ellipsoid (Gregorian system) or a hyperboloid (Cassegrainian system) with eccentricity e and interfocal distance 2c. The focal point F_o of the main reflector is also one of the foci of the parent subreflector surface, and the feed phase center is located at the other subreflector focal point, F_1.

Several coordinate systems are employed during the course of the analysis (see Figure 2). An x, y, z-system is established with origin at F_o, z-axis along the axis of the parent paraboloid in the plane of the paper and directed outward from the reflector, y-axis in the plane of the paper and directed downwards, and x-axis directed out from the plane of the paper. The feed pattern is described with reference to the spherical coordinates θ_o, ϕ_o, related in the usual way to the Cartesian x_o, y_o, z_o-system with origin F_1, z_o-axis directed from F_1 along the axis of the feed in the plane of the paper, and x_o-axis parallel to the x-axis. Also used is an x_1, y_1, z_1-system with origin at F_1, z_1-axis directed along the subreflector axis in the plane of the paper, and x_1-axis parallel to the x-axis; and an x', y', z'-system which is simply the x, y, z-system translated to the origin F_1.

The entire dual reflector system is assumed to be symmetric with respect to the x, y-plane. The orientation of the subreflector axis with respect to that of the main reflector is specified by the angle β through which the subreflector axis must be rotated around F_o to coincide with the z-axis. Counterclockwise rotation is taken to be positive. The orientation of the feed axis with respect to the subreflector axis is specified by the angle α through which the subreflector axis must be rotated around F_1 to coincide with the feed axis. Positive α is associated with a counterclockwise rotation direction.

The analysis employs geometric optics throughout. In the following, we will first trace through the dual reflector system a ray emanating from the feed phase center at F_1 in the direction specified by θ_o and ϕ_o, and express the image point P of the ray on the main reflector aperture as a function of θ_o, ϕ_o, and the

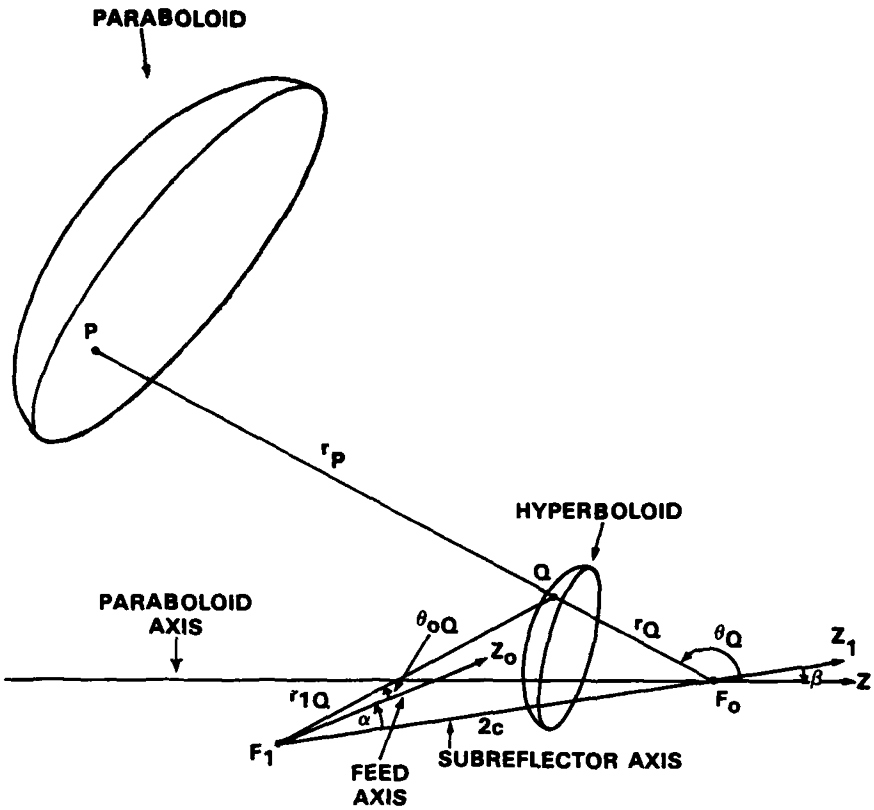

Figure 1A. Cassegrainian Reflector System Geometry

system parameters e, f, α, and β. We will then show that if the angles α and β, and the subreflector eccentricity, e, satisfy a certain condition (Eq. 24), then the images of the cones of rays, θ_o = constant, are concentric circles on the paraboloid aperture with center the image of the feed axis. Furthermore, we will show that if the feed pattern is rotationally symmetric (that is, dependent only on θ_o), then the paraboloid aperture power distribution is likewise rotationally symmetric.

Accordingly, we begin with a ray emanating from the feed phase center at F_1 in the direction specified by the angle θ_o between the feed axis (z_o-axis) and the ray, and by the angle ϕ_o between the x_o-axis and the projection of the ray on the x_o, y_o-plane. Let Q be the point of intersection of this ray with the subreflector surface. Then, in the x_o, y_o, z_o-system, the coordinates of Q are given by

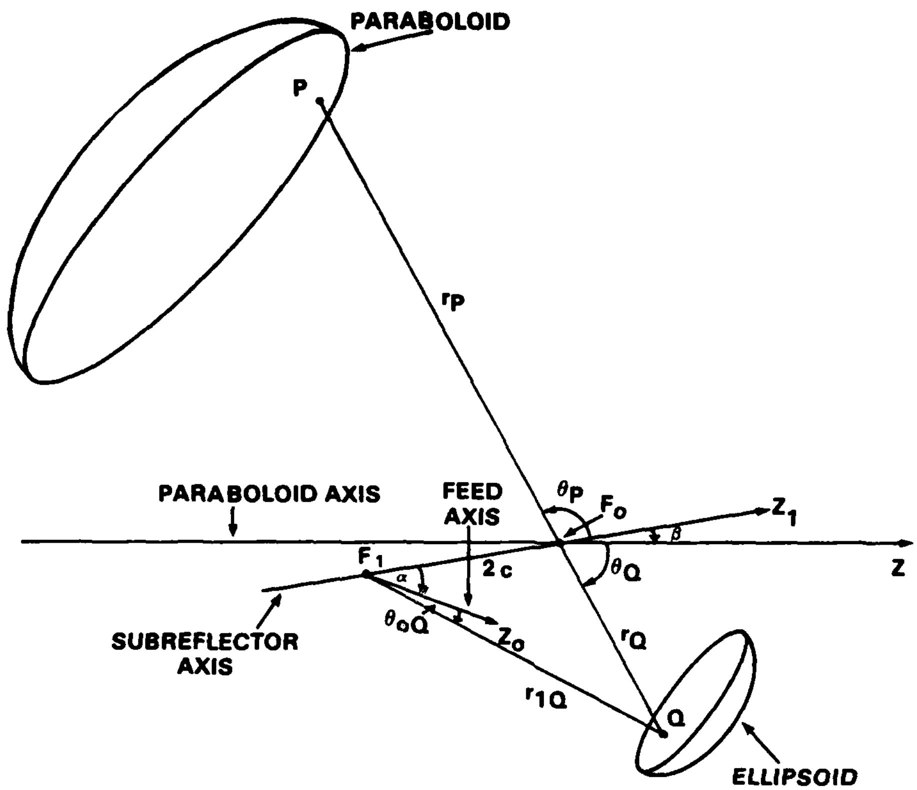

Figure 1B. Gregorian Reflector System Geometry

$$x_{oQ} = r_{1Q} \sin(\theta_o) \cos(\phi_o) \tag{1a}$$

$$y_{oQ} = r_{1Q} \sin(\theta_o) \sin(\phi_o) \tag{1b}$$

$$z_{oQ} = r_{1Q} \cos(\theta_o) \tag{1c}$$

The distance from F_1 to Q, which, instead of r_{oQ}, we have denoted r_{1Q} referring to the x_1, y_1, z_1-system, is given by the polar form[4] for either the hyperboloid or the ellipsoid,

$$r_{1Q} = \frac{(c/e)(1 - e^2)}{1 - e \cos(\theta_{1Q})} \tag{2}$$

4. Beyer, W.H., Ed. (1978) CRC Standard Mathematical Tables, 25th Edition, CRC Press, Inc., West Palm Beach, Fla.

with

$$\cos(\theta_{1Q}) = z_{1Q}/r_{1Q}$$

Since the x_1, y_1, z_1- and x_o, y_o, z_o-systems are related by a rotation of the y_1- and z_1-axis by an angle α about the x_1- (or x_o) axis (see Figure 2)

$$z_{1Q} = z_{oQ}\cos(\alpha) - y_{oQ}\sin(\alpha)$$

and hence, substituting from Eq. (1),

$$\cos(\theta_{1Q}) = \cos(\theta_o)\cos(\alpha) - \sin(\theta_o)\sin(\phi_o)\sin(\alpha) \tag{3}$$

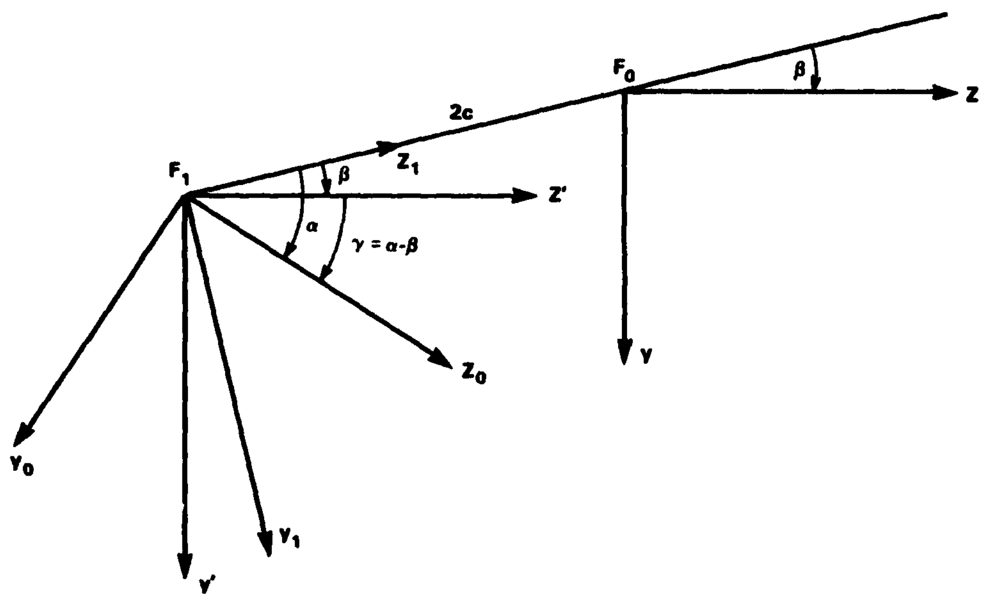

Figure 2. Coordinate Systems

To obtain the coordinates of Q in the x, y, z-system, we first obtain the coordinates of Q in the x', y', z'-system which is related to the x_o, y_o, z_o-system by a rotation of the y'- and z'-axes by the angle $\gamma = \alpha - \beta$ about the x'- (or x_o-) axis,

$$x'_Q = x_{oQ}$$

$$y'_Q = y_{oQ}\cos(\gamma) + z_{oQ}\sin(\gamma)$$

$$z'_Q = z_{oQ}\cos(\gamma) - y_{oQ}\sin(\gamma)$$

and then use the fact that the x, y, z- and x', y', z'-systems are related by a simple translation to obtain

$$x_Q = x'_Q$$

$$y_Q = y'_Q + 2c\sin(\beta)$$

$$z_Q = z'_Q - 2c\cos(\beta)$$

It follows that

$$x_Q = x_{oQ} \tag{4a}$$

$$y_Q = y_{oQ}\cos(\gamma) + z_{oQ}\sin(\gamma) + 2c\sin(\beta) \tag{4b}$$

$$z_Q = z_{oQ}\cos(\gamma) - y_{oQ}\sin(\gamma) - 2c\cos(\beta) \tag{4c}$$

If we let r_Q, θ_Q, ϕ_Q be the coordinates of Q in the spherical polar coordinate system based on the x, y, z-system, then

$$x_Q = r_Q\sin(\theta_Q)\cos(\phi_Q)$$

$$y_Q = r_Q\sin(\theta_Q)\sin(\phi_Q)$$

$$z_Q = r_Q\cos(\theta_Q)$$

or

$$\sin(\theta_Q)\cos(\phi_Q) = x_Q/r_Q \tag{5a}$$

$$\sin(\theta_Q)\sin(\phi_Q) = y_Q/r_Q \tag{5b}$$

$$\cos(\theta_Q) = z_Q/r_Q \tag{5c}$$

The distance, r_Q, from F_o to Q is related to the distance, r_{1Q}, from F_1 to Q by the equation

$$r_{1Q} - r_Q = 2c/e \tag{6}$$

for the hyperboloid, and by the equation

$$r_{1Q} + r_Q = 2c/e \tag{7}$$

for the ellipsoid. The distance 2c/e is the length of the transverse axis of the hyperboloid or the length of the major axis of the ellipsoid.

Now, let r_P θ_P, ϕ_P be the coordinates of P, the image on the main reflector of the point Q on the subreflector, in the spherical polar coordinate system based on the x, y, z-system. Then

$$x_P = r_P \sin(\theta_P) \cos(\phi_P)$$

$$y_P = r_P \sin(\theta_P) \sin(\phi_P)$$

$$z_P = (x_P^2 + y_P^2)/4f - f$$

For the hyperboloid

$$\theta_P = \theta_Q$$

$$\phi_P = \phi_Q$$

so that

$$x_P = r_P \sin(\theta_Q) \cos(\phi_Q) \tag{8a}$$

$$y_P = r_P \sin(\theta_Q) \sin(\phi_Q) \tag{8b}$$

with r_p given by the polar form of the paraboloid,

$$r_p = \frac{2f}{1 + \cos(\pi - \theta_Q)}$$

$$= \frac{2f}{1 - \cos(\theta_Q)} \tag{9}$$

For the ellipsoid, the ray passes through the focal point F_o so that

$$\theta_P = \pi - \theta_Q$$

$$\phi_P = \pi + \phi_Q$$

and

$$x_P = -r_P \sin(\theta_Q) \cos(\phi_Q) \tag{10a}$$

$$y_P = -r_P \sin(\theta_Q) \sin(\phi_Q) \tag{10b}$$

with

$$r_P = \frac{2f}{1 + \cos(\theta_Q)} \tag{11}$$

With Eqs. (8) and (9) for the hyperboloid case [or Eqs. (10) and (11) for the ellipsoid case] along with Eq. (5), Eq. (6) or Eq. (7), and Eqs. (4), (1), (2), and (3), we have thus expressed the image on the main reflector aperture of a ray emanating from the feed phase center in terms of the parameters c, e, and f, and trigonometric functions of the angles α, β, θ_o, and ϕ_o. Substituting and performing some algebraic and trigonometric manipulation then leads to the following equations for x_P and y_P for either the Cassegrainian or the Gregorian system:

$$x_P = -2f \frac{E \cos(\phi_o)}{A + B \sin(\phi_o)} \tag{12a}$$

$$y_P = -2f \frac{C + D \sin(\phi_o)}{A + B \sin(\phi_o)} \tag{12b}$$

where

$$A = u_1 + u_2 \cos(\theta_o)$$

$$B = u_3 \sin(\theta_o)$$

$$C = u_4 + u_5 \cos(\theta_o)$$

$$D = u_6 \sin(\theta_o)$$

$$E = (1 - e^2) \sin(\theta_o)$$

and

$$u_1 = \cos(\alpha - \beta) + e^2 \cos(\alpha + \beta) - 2e \cos(\alpha) \tag{13a}$$

$$u_2 = 1 + e^2 - 2e \cos(\beta) \tag{13b}$$

$$u_3 = 2e \sin(\alpha) - e^2 \sin(\alpha + \beta) - \sin(\alpha - \beta) \tag{13c}$$

$$u_4 = \sin(\alpha - \beta) - e^2 \sin(\alpha + \beta) \tag{13d}$$

$$u_5 = 2e \sin(\beta) \tag{13e}$$

$$u_6 = \cos(\alpha - \beta) - e^2 \cos(\alpha + \beta) \tag{13f}$$

Note that the interfocal distance, 2c, of the parent subreflector surface does not appear in this result. Thus, the image point on the paraboloid aperture depends only on the subreflector eccentricity, e; the paraboloid focal length, f; the angles, α and β, specifying the relative orientations of the axes of the feed, subreflector, and main reflector; and the ray direction. The dependence on the paraboloid focal length is that of a scale factor only.

We next show that the image on the paraboloid aperture of the circular cone of rays, θ_o = constant, is a circle. For squaring Eq. (12a)

$$x_P^2 \; [A + B \sin(\phi_o)]^2 = 4f^2E^2 \; [1 - \sin^2(\phi_o)]^2 \tag{14}$$

while from Eq. (12b)

$$\sin(\phi_o) = - \frac{Ay_P + 2fC}{By_P + 2fD} \tag{15}$$

Substituting Eq. (15) in Eq. (14) and completing the square, we obtain

$$\frac{(AD - BC)^2}{E^2(A^2 - B^2)} \quad x_P^2 \quad + \left(y_P \quad + \quad 2f \, \frac{AC - BD}{A^2 - B^2} \right)^2 = \frac{4f^2}{(A^2 - B^2)^2} (AD-BC)^2 \tag{16}$$

In Appendix A, it is shown that the following relations hold among the $\{ u_i \}$ defined in Eq. (13):

$$u_1^2 + u_3^2 = u_2^2 \tag{17a}$$

$$u_1 u_4 + u_3 u_6 = u_2 u_5 \tag{17b}$$

$$u_1 u_6 - u_3 u_4 = (1 - e^2) u_2 \tag{17c}$$

$$u_2 u_6 - u_3 u_5 = (1 - e^2) u_1 \tag{17d}$$

Using Eqs. (17c) and (17d), it is then straightforward to show that

$$AD - BC = (1 - e^2) \sin(\theta_o) \, [u_1 + u_2 \cos(\theta_o)]; \tag{18}$$

by using Eq. (17a), that

$$A^2 - B^2 = [u_1 + u_2 \cos(\theta_o)]^2; \tag{19}$$

and by using Eq. (17b), that

$$AC - BD = [u_1 + u_2 \cos(\theta_o)] \, [u_4 + u_5 \cos(\theta_o)]. \tag{20}$$

Substituting Eqs. (18), (19), and (20) in Eq. (16), we obtain the equation of the circle with center at $(0, y_c)$ and radius r_c,

$$x_P^2 + (y_P - y_c)^2 = r_c^2$$

where

$$y_c = -2f \frac{u_4 + u_5 \cos(\theta_o)}{u_1 + u_2 \cos(\theta_o)} \tag{21}$$

and

$$r_c = \left| \frac{2f(1 - e^2) \sin(\theta_o)}{u_1 + u_2 \cos(\theta_o)} \right| \tag{22}$$

It will be noticed from Eq. (21) that the center of the circle is a function of θ_o so that the circles corresponding to different values of θ_o are not in general concentric. However, differentiating Eq. (21) with respect to θ_o and equating the derivative to zero to make y_c independent of θ_o yields the equation

$$u_1 u_5 = u_2 u_4 \tag{23}$$

Substituting from Eq. (13) and performing some manipulation then gives the condition

$$\tan(\alpha) = \frac{(1 - e^2)\sin(\beta)}{(1 + e^2)\cos(\beta) - 2e} \tag{24}$$

which is equivalent to the equation

$$u_3 = 0 \tag{25}$$

or

$$2e \sin(\alpha) - e^2 \sin(\alpha + \beta) - \sin(\alpha - \beta) = 0 \tag{26}$$

Eq. (24) is the central result of this report. It gives the relation between the angle α (between the subreflector axis and the feed axis), the angle β (between the subreflector axis and the main reflector axis), and the subreflector eccentricity, e, that must be satisfied for the images of circular cones of rays from the feed phase center, θ_o = constant, to be concentric circles on the main reflector aperture.

Before proceeding to examine the transformation of power from the feed pattern to the main reflector aperture, it is worth noting some useful implications of Eq. (24) or Eq. (25). As shown in Appendix B, it is possible to express the angle β in terms of e and the angle α by the equation

$$\tan(\beta) = \frac{(1 - e^2)\sin(\alpha)}{(1 + e^2)\cos(\alpha) + 2e} \tag{27}$$

Eq. (27) is equivalent to the relation

$$2e \sin(\beta) + e^2 \sin(\alpha + \beta) - \sin(\alpha - \beta) = 0 \tag{28}$$

Using Eqs. (26) and (28), it is then simple to derive the formula (see Appendix C)

$$\tan(\alpha/2) = \frac{1+e}{1-e} \tan(\beta/2) \tag{29}$$

obtained by Dragone by a different method.[3] Conversely, Eqs. (26) and (28) can be derived by a single geometric argument starting with Dragone's method for determining the feed axis (see Appendix D).

Substituting Eq. (25) in Eq. (17a) gives

$$|u_1| = u_2$$

(note that u_2 is always positive), whereupon substituting Eq. (26) in Eq. (23) gives

$$|u_4| = u_5$$

(referring to Figure 1, β can always be taken to be positive if the feed is not to block the main reflector, so that u_5 is positive). Eq. (23) then also implies that u_1 and u_4 have the same sign. The possibility that u_1 and u_4 are both negative can be excluded by observing from Eq. (22) that then

$$r_c = \frac{2f}{u_2} \left|1 - e^2\right| \cot(\theta_o/2) \tag{30}$$

so that the feed pattern is inverted with the image of the central ray appearing at infinity. Hence, u_1 and u_4 are both positive and

$$\begin{aligned} u_1 &= u_2 \\ u_4 &= u_5 \end{aligned} \tag{31}$$

An additional equation results from substituting Eqs. (25) and (31) in Eq. (17c),

$$u_6 = 1 - e^2$$

or

$$\cos(\alpha - \beta) - e^2 \cos(\alpha + \beta) = 1 - e^2$$

The expression, Eq. (21), for the y-coordinate of the center of the circular image on the main reflector aperture of the circular cone of rays θ_o = constant, reduces to

$$y_c = -2f\frac{u_5}{u_2} = \frac{-4f\, e \sin(\beta)}{1 + e^2 - 2e\cos(\beta)}$$

independent of θ_o, while Eq. (22) for the radius of the image circle becomes

$$r_c = \frac{2f}{u_2}\,|1 - e^2|\,\tan(\theta_o/2) = \frac{2f\,|1 - e^2|\,\tan(\theta_o/2)}{1 + e^2 - 2e\cos(\beta)} \qquad (32)$$

Eqs. (12a) and (12b) for the x- and y-coordinates of the image point of the ray in the direction θ_o, ϕ_o become

$$x_P = -2f\,\frac{(1 - e^2)\,\sin(\theta_o)\,\cos(\phi_o)}{u_2\,[1 + \cos(\theta_o)]}$$

$$= -2f\,\frac{(1 - e^2)}{1 + e^2 - 2e\cos(\beta)}\,\tan(\theta_o/2)\cos(\phi_o) \qquad (33)$$

and

$$y_P = -2f\,\frac{u_5\,[1 + \cos(\theta_o)] + u_6\sin(\theta_o)\sin(\phi_o)}{u_2[1 + \cos(\theta_o)]}$$

$$= -2f\,\frac{u_5}{u_2} - 2f\,\frac{(1 - e^2)}{u_2}\,\tan(\theta_o/2)\sin(\phi_o)$$

$$= y_c - \frac{2f(1 - e^2)}{1 + e^2 - 2e\cos(\beta)}\,\tan(\theta_o/2)\,\sin(\phi_o) \qquad (34)$$

Referring to Eqs. (33), (34), and (32), we can also write

$$x_P = \pm\, r_c\,\cos(\phi_o)$$

$$y_P = y_c \pm r_c\sin(\phi_o)$$

where the plus and minus sign refers to the Cassegrainian and Gregorian system respectively. This means that for the Cassegrainian system the angle ϕ_o which defines the projection of a ray on the x_o, y_o-plane of the feed coordinate system, equals the azimuth angle ϕ_P of the image point on the main reflector aperture in the spherical coordinate system based on the x, y, z-system, while for the Gregorian system, $\phi_o = \phi_P \pm \pi$. This difference between the Cassegrainian and Gregorian system is, of course, attributable to the fact that in the Cassegrainian system, the rays do not pass through the focal point F_o, while for the Gregorian system, they do.

Turning now to the distribution of power in the paraboloid aperture, let $G(\theta_o, \phi_o)$ be the distribution of power on a unit sphere around the feed phase center. The power radiated through an element of area on the unit sphere is given by

$$G(\theta_o, \phi_o) \sin(\theta_o) d\theta_o d\phi_o$$

Letting $P(x, y)$ be the aperture power distribution, we then have

$$P(x, y)dxdy = G(\theta_o, \phi_o) \sin(\theta_o) d\theta_o d\phi_o$$

with

$$dxdy = \left| \frac{\partial(x, y)}{\partial(\theta_o, \phi_o)} \right| d\theta_o d\phi_o$$

so that

$$P(x, y) = \frac{G(\theta_o, \phi_o) \sin(\theta_o)}{\left| \frac{\partial(x, y)}{\partial(\theta_o, \phi_o)} \right|}$$

From Eqs. (33) and (34),

$$\frac{\partial(x, y)}{\partial(\theta_o, \phi_o)} = -F^2 \begin{vmatrix} 1/2 \sec^2(\theta_o/2) \cos(\phi_o) & -\tan(\theta_o/2) \sin(\phi_o) \\ 1/2 \sec^2(\theta_o/2) \sin(\phi_o) & \tan(\theta_o/2) \cos(\phi_o) \end{vmatrix}$$

$$= \frac{-F^2}{2} \tan(\theta_o/2) \sec^2(\theta_o/2)$$

with

$$F = \frac{2f(1 - e^2)}{u_2} = \frac{2f(1 - e^2)}{1 + e^2 - 2e \cos(\beta)} \tag{35}$$

Hence

$$P(x,y) = \frac{G(\theta_o, \phi_o)\sin(\theta_o)}{\frac{F^2}{2}\tan(\theta_o/2)\sec^2(\theta_o/2)}$$

$$= \frac{1}{F^2} G(\theta_o, \phi_o)\ [1 + \cos(\theta_o)]^2 \tag{36}$$

The feed pattern is thus transformed to the main reflector aperture distribution by multiplying by the factor

$$\frac{1}{F^2}[1 + \cos(\theta_o)]^2$$

which is independent of ϕ_o. Hence, if the feed pattern is rotationally symmetric, then so is the main reflector aperture power distribution.

It is worth noting that Eq. (36) can also be derived from the relation

$$P(x_P, y_P) = G(\theta_o, \phi_o)\ \frac{r_Q^2}{r_{1Q}^2\ r_P^2}$$

which expresses the fact that the power density decreases as a diverging spherical wave from F_1 to Q, increases as a converging spherical wave from Q to F_o, and decreases again as a diverging spherical wave from F_o to P. (This relation applies equally to Cassegrainian and Gregorian systems.) To show this, we use Eqs. (6) or (7), (9) or (11), and (5c) and (4c) to give

$$\pm\ \frac{r_Q}{r_{1Q}r_P} = \frac{c[1 - e\cos(\beta)]}{ef}\ \frac{1}{r_{1Q}} + \frac{\cos(\gamma)\cos(\theta_o) - \sin(\gamma)\sin(\theta_o)\sin(\phi_o) - 1}{2f}$$

whereupon, using Eqs. (2) and (3) we obtain

$$\pm\ \frac{r_Q}{r_{1Q}r_P} = \frac{1}{f}\left\{[1 - e\cos(\beta)]\ e\sin(\alpha) - \frac{\sin(\alpha-\beta)}{2}\right\}\sin(\theta_o)\ \sin(\phi_o)$$

$$+ \frac{1}{f}\left[\frac{1 - e\cos(\beta)}{1 - e^2} - \frac{1}{2}\right] + \frac{1}{f}\left\{\frac{\cos(\alpha-\beta)}{2} - \frac{[1 - e\cos(\beta)]e\cos(\alpha)}{1 - e^2}\right\}\cos(\theta_o) \tag{37}$$

The coefficient of $\sin(\theta_o)\ \sin(\phi_o)$ in Eq. (37) is equal to

$$\frac{1}{2f(1-e^2)}\ [2e\sin(\alpha) - \sin(\alpha - \beta) - e^2 \sin(\alpha + \beta)] = \frac{u_3}{2f(1-e^2)}$$

which is zero because of Eq. (25); the second term of the RHS of Eq. (37) equals

$$\frac{1 - 2e\cos(\beta) + e^2}{2f(1-e^2)} = \frac{u_2}{2f(1-e^2)}\ ;$$

while the coefficient of $\cos(\theta_o)$ is found to be

$$\frac{\cos(\alpha - \beta) - 2e\cos(\alpha) + e^2\cos(\alpha + \beta)}{2f(1-e^2)} = \frac{u_1}{2f(1-e^2)} = \frac{u_2}{2f(1-e^2)}$$

using Eq. (31). Thus,

$$\pm \frac{r_Q}{r_{1Q} r_P} = \frac{u_2}{2f(1-e^2)}\ [1 + \cos(\theta_o)]$$

$$= \frac{1 + e^2 - 2e\cos(\beta)}{2f(1-e^2)}\ [1 + \cos(\theta_o)]$$

$$= \frac{1 + \cos(\theta_o)}{F}$$

with F defined by Eq. (35), so that

$$\frac{r_Q^2}{r_{1Q}^2 r_P^2} = \frac{[1 + \cos(\theta_o)]^2}{F^2}$$

Although polarization will not be considered in detail here, it is important to note that the condition, Eq. (24), which guarantees that the images of circular cones of rays around the feed axis are concentric circles on the main reflector aperture and that a rotationally symmetric feed pattern produces a rotationally symmetric aperture power distribution, also guarantees that a feed with no cross polarization gives rise to an aperture field with no cross polarization. Cross polarization here is defined as in the third definition of Ludwig.[5] For a transmitted field polarized in the x_o direction at $\theta_o = 0$, this definition implies that the θ_o and ϕ_o components of the transmitted electric field satisfy the equation

$$E_{\theta_o} \sin(\phi_o) = - E_{\phi_o} \cos(\phi_o)$$

while for a transmitted field polarization in the y_o-direction,

$$E_{\theta_o} \cos(\phi_o) = E_{\phi_o} \sin(\phi_o)$$

The field of a Huygens source—that is, a combination of crossed electric and magnetic dipoles of equal strength—satisfies these equations. If Eq. (24) is satisfied, an x_o-polarized transmitted field gives rise to a paraboloid aperture field with $E_y = 0$, and a y_o-polarized field to an aperture field with $E_x = 0$. These results are theoretically established[2,3] and can be readily verified by computer calculation using the equation

$$\overline{E}_{refl} = 2 (\hat{n} \cdot \overline{E}_{inc}) \hat{n} - \overline{E}_{inc} \tag{38}$$

to handle the reflections at the subreflector and main reflector. In Eq. (38), $\overline{E}_{inc}$ and $\overline{E}_{refl}$ are the incident and reflected electric field vectors respectively, and $\hat{n}$ is the unit normal to the surface directed into the space from which the field is incident.

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Appendix A

Proof of Four Identities

In Appendix A, we prove four identities used in the main body of the report. Let u_i, $i = 1, 6$ be defined as in Eq. (13) by

$$u_1 = \cos(\alpha - \beta) + e^2 \cos(\alpha + \beta) - 2e \cos(\alpha) \tag{A1a}$$

$$u_2 = 1 + e^2 - 2e \cos(\beta) \tag{A1b}$$

$$u_3 = 2e \sin(\alpha) - e^2 \sin(\alpha + \beta) - \sin(\alpha - \beta) \tag{A1c}$$

$$u_4 = \sin(\alpha - \beta) - e^2 \sin(\alpha + \beta) \tag{A1d}$$

$$u_5 = 2e \sin(\beta) \tag{A1e}$$

$$u_6 = \cos(\alpha - \beta) - e^2 \cos(\alpha + \beta) \tag{A1f}$$

Then we will prove here that

$$u_1^2 + u_3^2 = u_2^2 \tag{A2}$$

$$u_1u_4 + u_3u_6 = u_2u_5 \tag{A3}$$

$$u_1u_6 - u_3u_4 = (1 - e^2)\, u_2 \tag{A4}$$

$$u_2u_6 - u_3u_5 = (1 - e^2)\, u_1 \tag{A5}$$

To prove Eq. (A2), substituting from Eq. (A1) in the LHS and using the relation for the cosine of the difference of angles gives

$$\begin{aligned} u_1^{\,2} + u_3^{\,2} &= 1 + e^4 + 4e^2 + 2e^2 \cos\,[(\alpha - \beta) - (\alpha + \beta)] - 4e \cos\,[\alpha - (\alpha - \beta)] \\ &\qquad - 4e^3 \cos\,[\alpha - (\alpha + \beta)] \\ &= (1 + e^2)^2 + 2e^2\,[1 + \cos\,(2\beta)] - 4e \cos\,(\beta)(1 + e^2) \\ &= (1 + e^2)^2 + 4e^2 \cos^2(\beta) - 4e \cos\,(\beta)(1 + e^2) \\ &= [(1 + e^2) - 2e \cos\,(\beta)]^2 \\ &= u_2^{\,2} \end{aligned}$$

To prove Eq. (A3), substituting from Eq. (A1) in the LHS and using the relation for the sine of the difference of two angles, we obtain

$$\begin{aligned} u_1u_4 + u_3u_6 &= 2e^2\,[\sin\,(\alpha - \beta) \cos\,(\alpha + \beta) - \sin\,(\alpha + \beta) \cos\,(\alpha - \beta)] \\ &\qquad + 2e \sin[\alpha - (\alpha - \beta)] + 2e^3 \sin\,[(\alpha + \beta) - \alpha] \\ &= 2e^2 \sin\,[(\alpha - \beta) - (\alpha + \beta)] + 2e \sin\,(\beta)(1 + e^2) \\ &= -4e^2 \sin\,(\beta) \cos\,(\beta) + 2e \sin\,(\beta)(1 + e^2) \\ &= [1 + e^2 - 2e \cos\,(\beta)]\, 2e \sin\,(\beta) \\ &= u_2u_5 \end{aligned}$$

To prove Eq. (A4), substituting in the LHS and using the relation for the cosine of the difference of two angles,

$$u_1u_6 - u_3u_4 = 1 - e^4 - 2e \cos [\alpha - (\alpha - \beta)] + 2e^3 \cos [\alpha - (\alpha - \beta)]$$

$$= 1 - e^4 - 2e \cos (\beta)(1 - e^2)$$

$$= (1 - e^2) [1 + e^2 - 2e \cos (\beta)]$$

$$= (1 - e^2) u_2$$

Finally, to prove Eq. (A5), substituting from Eq. (A1) in the LHS and using the relations for the cosine of the sum and the difference of two angles gives

$$u_2u_6 - u_3u_5 = \cos (\alpha - \beta) + e^2 [\cos (\alpha - \beta) - \cos (\alpha + \beta)]$$

$$- 2e \cos [\beta + (\alpha - \beta)] - e^4 \cos (\alpha + \beta)$$

$$+ 2e^3 \cos [\beta - (\alpha + \beta)] - 4e^2 \sin (\alpha) \sin (\beta)$$

$$= \cos (\alpha - \beta) + e^2 \cos (\alpha + \beta) - 2e \cos (\alpha)$$

$$- e^2 \cos (\alpha - \beta) - e^4 \cos (\alpha + \beta) + 2e^3 \cos (\alpha)$$

$$= (1 - e^2) [\cos (\alpha - \beta) + e^2 \cos (\alpha + \beta) - 2e \cos (\alpha)]$$

$$= (1 - e^2) u_1$$

Appendix B

Inversion of Eq. (24)

In Appendix B, we show that Eq. (24), or, equivalently, Eq. (26) can be used to obtain the angle β in terms of α and e. Substituting $\cos(\beta) = [1 - \sin^2(\beta)]^{1/2}$ in Eq. (26) gives a quadratic equation for $\sin(\beta)$ with the solutions

$$\sin(\beta) = \frac{(1-e^2)\sin(\alpha)\,[-2e\cos(\alpha) \pm (1+e^2)]}{(1-e^2)^2 + 4e^2\sin^2(\alpha)} \tag{B1}$$

The "+" sign must be taken for $\sin(\beta)$ to be positive as is assumed in the main body of the report. Similarly, substituting $\sin(\beta) = [1 - \cos^2(\beta)]^{1/2}$ in Eq. (26) gives a quadratic equation for $\cos(\beta)$ with the solution

$$\cos(\beta) = \frac{2e(1+e^2)\sin^2(\alpha) + (1-e^2)^2\cos(\alpha)}{(1-e^2)^2 + 4e^2\sin^2(\alpha)} \tag{B2}$$

corresponding to the "+" solution in Eq. (B1). The ratio of Eq. (B1) and Eq. (B2) then yields

$$\tan(\beta) = \frac{(1-e^2)\sin(\alpha)\,[1+e^2-2e\cos(\alpha)]}{[(1+e^2)\cos(\alpha)+2e]\,[1+e^2-2e\cos(\alpha)]}$$

$$= \frac{(1-e^2)\sin(\alpha)}{(1+e^2)\cos(\alpha)+2e}$$

Appendix C

Derivation of Eq. (29)

Using the half-angle formula

$$\tan\left(\frac{x}{2}\right) = \frac{\sin(x)}{1 + \cos(x)}$$

Eq. (29) is equivalent to

$$\sin(\alpha) - \sin(\beta) - e\sin(\alpha+\beta) - e\left[\sin(\alpha) + \sin(\beta)\right] + \sin(\alpha-\beta) = 0 \qquad \text{(C1)}$$

Adding Eqs. (26) and (28) gives

$$e\left[\sin(\alpha) + \sin(\beta)\right] - \sin(\alpha-\beta) = 0$$

while subtracting Eq. (28) from Eq. (26) yields

$$\sin(\alpha) - \sin(\beta) - e\sin(\alpha+\beta) = 0 \qquad \text{(C2)}$$

and so Eq. (C1) is satisfied.

Appendix D

Derivation of Eqs. (26) and (28) From Dragone's Construction of the Feed Axis

Dragone[3] has given a simple method of determining the orientation of the feed axis of a multi-confocal reflector system consisting of ellipsoids, hyperboloids, and paraboloids, so as to ensure circular symmetry and zero cross-polarization of the antenna far field. Applied to the dual reflector systems we consider in this report, the feed axis orientation is determined by the point of intersection, I, of the paraboloid axis with the parent subreflector surface (see Figures D1 and D2.) This construction guarantees that the ray from the feed phase center at F_1 in the direction of the feed axis is unchanged in direction after four successive reflections. the first from the subreflector, the second from the paraboloid, the third from infinity coinciding with the paraboloid axis (regarding the paraboloid as an ellipsoid with its second focus at infinity), and the fourth from the parent subreflector surface.

First, considering the Cassegrainian system and referring to Figure 3a,

$$r_1 = \frac{(c/e)(e^2-1)}{e\cos(\alpha)+1}$$

$$r_2 = \frac{(c/e)(e^2-1)}{e\cos(\beta)-1}$$

Applying the Law of Sines to the triangle F_oF_1I we have (remembering that α is taken to be negative for the Cassegrain system).

$$\frac{\sin(-\alpha)}{\left[\dfrac{\frac{c}{e}(e^2-1)}{e\cos(\beta)-1}\right]} = \frac{\sin[\pi-(-\alpha+\beta)]}{2e}$$

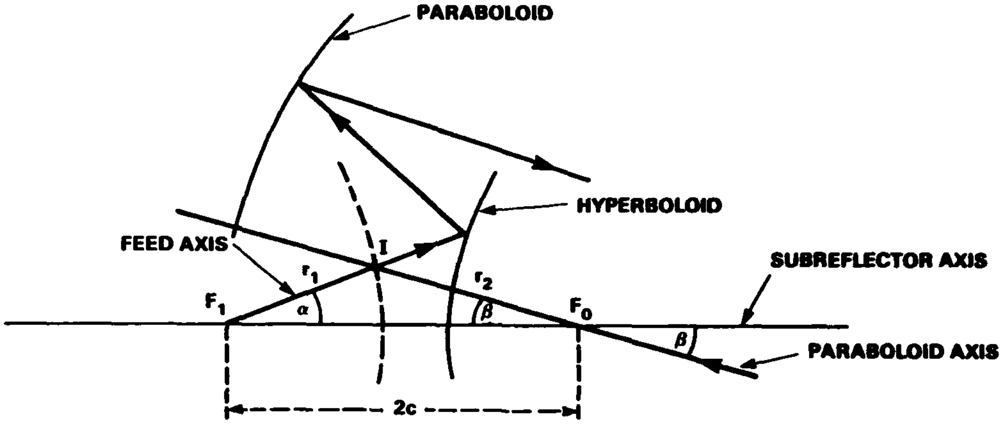

Figure Dl. Geometric Construction of Feed Axis for Cassegrainian System

whereupon we obtain

$$2e\sin(\alpha) - e^2\sin(\alpha+\beta) - \sin(\alpha-\beta) = 0 \tag{26}$$

A second Law of Sines relation for the same triangle gives

$$\frac{\sin(\beta)}{\left[\dfrac{\frac{c}{e}(e^2-1)}{e\cos(\alpha)+1}\right]} = \frac{\sin[\pi-(-\alpha+\beta)]}{2c}$$

and hence

$$2e\sin(\beta) + e^2\sin(\alpha+\beta) - \sin(\alpha-\beta) = 0 \tag{28}$$

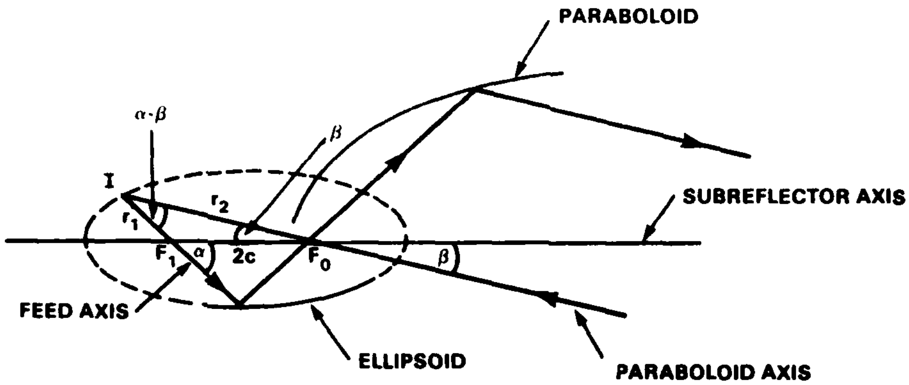

Figure D2. Geometric Construction of Feed Axis for Gregorian System

[The third Law of Sines relation gives the equation

$$\sin(\alpha) - \sin(\beta) = e \sin(\alpha + \beta)$$

Eq. (C2).]

For the Gregorian system, referring to Figure 3B

$$r_1 = \frac{(c/e)(1 - e^2)}{1 + e \cos(\alpha)} ,$$

$$r_2 = \frac{(c/e)(1 - e^2)}{1 - e \cos(\beta)}$$

Again applying the Law of Sines to triangle F_oF_1 I, we have

$$\frac{\sin(\pi - \alpha)}{\left[\dfrac{\frac{c}{e}(1 - e^2)}{1 - e\cos(\beta)}\right]} = \frac{\sin(\alpha - \beta)}{2c}$$

from which Eq. (26) is readily obtained, and

$$\frac{\sin(\beta)}{\left[\dfrac{\frac{c}{e}(1 - e^2)}{1 + e\cos(\alpha)}\right]} = \frac{\sin(\alpha - \beta)}{2c}$$

yielding Eq. (28). The third Law of Sines relationship again gives Eq. (C2).

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