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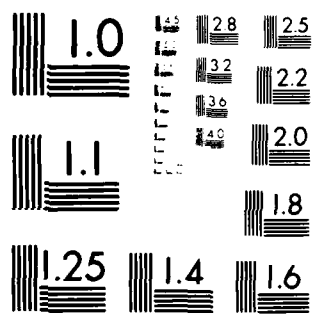
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SPREAD SPECTRUM, ACQUISITION AND TRACKING

Annual Technical Report RF 447105

March 1, 1983 - February 29, 1984

Grant AFOSR - 83-0102

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Bolling Air Force Base

Washington, D. C. 20322

Donald L. Schilling

H. J. Kayser Professor

Principal Investigator

COMMUNICATIONS SYSTEMS LABORATORY

DEPARTMENT OF ELECTRICAL ENGINEERING

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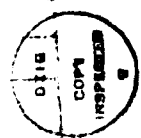
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examination instant. Using this technique a significant reduction in the acquisition time of a direct sequence spread spectrum signal is attained.

A New Double Threshold Acquisition Scheme Applied to the Fading Channel in Frequency Hopping Spread Spectrum. This technique employs the double threshold procedure used in Sec. 1.1. However, the design and results obtained are specific for frequency hopping in a fading channel such as the HF channel.

Table of Contents

	<u>Page</u>
1.0 Research Objectives	1
1.1 A New Rapid Acquisition Technique for Direct Sequence Spread Spectrum Communications	1
1.2 Double Threshold Acquisition Scheme Applied to the Fading Channel in Frequency Hopping Spread Spectrum	1
2.0 Status of the Research Effort	2
2.1 A New Rapid Acquisition Technique for Direct Sequence Spread Spectrum Communications	3
2.2 Double Threshold Acquisition Scheme Applied to the Fading Channel in Frequency Hopping Spread Spectrum	25
3.0 Paper Published	53

1.0 Research Objectives

1.1 A New Rapid Acquisition Technique for Direct Sequence Spread Spectrum Communications

The rapid acquisition technique described here can be used in direct sequence spread spectrum systems. The technique employs a double threshold which defines when a decision can be made. These thresholds change at each examination instant. Using this technique a significant reduction in the acquisition time of a direct sequence spread spectrum signal is obtained.

1.2 A New Double Threshold Acquisition Scheme Applied to the Fading Channel in Frequency Hopping Spread Spectrum

This technique employs the double threshold procedure used in Sec. 1.1. However, the design and results obtained are specific for frequency hopping in a fading channel such as the HF channel.

2.0 Status of the Research Effort

2.1

A NEW RAPID ACQUISITION TECHNIQUE FOR DIRECT
SEQUENCE SPREAD SPECTRUM COMMUNICATIONS

By

Sorin Davidovici¹
Laurence B. Milstein²
Donald L. Schilling³

This paper describes a rapid acquisition technique which can be used in direct sequence spread spectrum systems. It is sequential in nature and is based on a double threshold which defines when a decision can be made. The value of the thresholds change at every examination instant. The procedure makes use of a bound to the partial cross correlation function of PN sequences which insures that only the sequence length needs to be given as far as the PN sequence particulars are concerned. A decision is made only if the probability of error associated with a decision at that point is equal to or better than a prespecified value. Finally, it is shown that most of the computational difficulties associated with the bound as given by a recursive algorithm can be eliminated when using a low order polynomial approximation to the bound. However, either by using the bound or its polynomial approximation, a significant improvement in the acquisition time is obtained.

1. INTRODUCTION

The problem of acquisition of a spread spectrum signal consists of correctly estimating the phase of a received maximal-length shift register sequence (commonly referred to as a PN sequence) of arbitrary length $L = 2^N - 1$, where N refers to the number of delay stages in the generating shift register. The acquisition process consists of repeatedly comparing the received PN sequence $\sqrt{2P_s} g(t - jT_c)$ with a local generated PN sequence $g(t - iT_c)$ until the phases iT_c and jT_c are found to be equal. At that point,

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the locally generated PN sequence is said to be synchronized to the received PN sequence and the acquisition process is ended. Throughout this paper, the carrier phase and chip timing are assumed to be in perfect synchronism, and the only interference considered is additive white Gaussian noise. The in-phase/not-in-phase decision is based on a partial correlation process. As is well known [1], the partial correlation function of a PN sequence is not nearly as well behaved as the autocorrelation function, it being a function of the particular starting phases i and j , the specific feedback connections of the shift register, and the length of the correlation γT_c , where γ equals the number of chips (an integer) and T_c equals the chip duration. To circumvent all these difficulties, various bounds have been found ([2]-[4]) to the partial autocorrelation function, and one of these bounds is used in the analysis presented in the next section.

In particular, a sequential estimation technique is used in this paper and it is shown in Section 3 that the expected time to make a decision using the sequential procedure is upper bounded by the duration of time needed to make a decision using a fixed observation interval procedure operating with the same probability of error. A description of the technique as well as the supporting analysis are presented in the next section. Numerical results are presented in Section 3, and Section 4 presents the conclusions.

2. ANALYSIS

Reference [3] describes a method for generating a non-linear sequence which serves as an upper bound to the partial autocorrelation function of a PN sequence (via the shift-and-add property). This bound, unfortunately, is defined via an algorithm as opposed to an analytical equation. When the incoming sequence is correctly aligned with the reference sequence, the output of the correlator is proportional to γT_c (see, specifically, (3) below). Therefore, for simplicity of notation, the upper bound to the partial correlation when the incoming sequence and reference sequence are not in phase will be written as $\gamma T_c(1 - G(\gamma))$. This then results in $\gamma G(\gamma)$ as being the (normalized) difference between the in-phase and the not-in-phase correlation curves.

In addition to presenting precise results based upon the upper bound, approximate results will also be presented. These latter results arise from curve fitting the exact bound with a low order polynomial in γ . Specifically, it can be shown that

$$\gamma[1 - G(\gamma)] = \gamma[1 - \frac{\gamma}{L}]. \quad (1)$$

where L is the period (i.e., the length) of the PN sequence.

In Section 3, numerical results using the approximate bound will be compared to those obtained using the exact bound.

Referring now to Figure 1, the procedure works in the following manner: An integration time γ_1 (in integer number of chips) is established so that a given probability of incorrect decision P_e can be achieved with that integration time. Assuming that $\gamma_1 > N$, where N is the number of stages of the shift register which generates the spreading sequence, then if any integration time γ_2 such that $N < \gamma_2 < \gamma_1$ is chosen, P_e cannot necessarily be achieved. However, by allowing an uncertainty region such that if the test statistic falls in that uncertainty region then no decision is made, but rather the integration time is increased by, say, one chip, the test can be reapplied with the increased integration time to again see if P_e can be achieved. This process is repeated continually until an integration time of γ_1 is used, at which point P_e is guaranteed to be achieved. This will become more clear as the details of the procedure are described below.

The output of the correlator in Figure 1 is given by

$$V_o(\gamma T_c) = \int_0^{\gamma T_c} \left[\sqrt{2P_s} g(t - jT_c) + n_w(t) \right] g(t - iT_c) dt, \quad (2)$$

which consists of a signal term $s_o(\gamma T_c)$ and a noise term $n_o(\gamma T_c)$. The noise term can easily be shown to be Gaussian with zero mean and variance $\sigma_o^2(\gamma T_c) = \frac{\eta}{2} \gamma T_c$, where $\frac{\eta}{2}$ is the two-sided power spectral density of the input noise. The signal term $s_o(\gamma T_c)$ is given by

$$s_o(\gamma T_c) = \int_0^{\gamma T_c} \sqrt{2P_s} g(t - jT_c) \cdot g(t - iT_c) dt \begin{cases} = \sqrt{2P_s} \gamma T_c & \text{if } i = j \\ \leq \sqrt{2P_s} \gamma T_c (1 - G(\gamma)) & \text{if } i \neq j \end{cases} \quad (3)$$

where the lower term on the rhs arises from the bound in (1). Note that all the following equations which depend upon this bound are themselves bounds. For simplicity, however, they are written with equality signs.

The synchronization/no-synchronization decision is seen only to involve the choice of which signal term, $s_o(\gamma T_c)$, is present at the correlator's output. Due to the presence of noise, this decision can only be made with a specified accuracy. If, as shown in Figure 2, we wait for a time $T = \gamma_1 T_c$ before making a decision, then we can define a threshold such that if the correlator's output falls above the threshold, we decide synchronization is attained, and if it falls below the threshold, we decide the two PN sequences are not synchronized. The probability of an erroneous decision, given we are not looking at the correct phase position, is equal to

$$P_e = P[V_o(\gamma_1 T_c) > V_T] = P[n_o(\gamma_1 T_c) > \epsilon_1] \quad (4)$$

where ϵ_1 is shown in Figure 2 and V_T , the threshold, is given below [see (7)].

Note that both in Figure 2 and from Equation (4), for simplicity, the decision boundary has been located halfway between the two signals. Therefore,

$$\epsilon_1 = \frac{1}{2} [\sqrt{2P_s} \gamma_1 T_c - \sqrt{2P_s} \gamma_1 T_c (1 - G(\gamma_1))] = \frac{1}{2} \sqrt{2P_s} \gamma_1 T_c G(\gamma_1) \quad (5)$$

The probability of error of Equation (4) then becomes

$$P_e = P[n_0(\gamma_1 T_c) > \frac{1}{2} \sqrt{2P_s} \gamma_1 T_c G(\gamma_1)] = \frac{1}{2} \operatorname{erfc} \left\{ G(\gamma_1) \sqrt{\frac{E_{s1}}{2\eta}} \right\} \quad (6)$$

where $E_{s1} \triangleq \gamma_1 E_c$, E_c is the energy per chip and equals $P_s T_c$, and

$$\operatorname{erfc}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-y^2} dy.$$

Finally, $V_T(\gamma_1 T_c)$ is given by

$$V_T(\gamma_1 T_c) = \sqrt{2P_s} \gamma_1 T_c - \epsilon_1 = \sqrt{2P_s} \gamma_1 T_c \left[1 - \frac{1}{2} G(\gamma_1) \right] \quad (7)$$

The following discussion will show how the acquisition process can be improved by shortening the acquisition time.

The Decision Process prior to $\gamma_i = \gamma_1$: At times it may not be necessary to wait the full γ_1 chips to make a decision. The only requirement is that at any time a decision is made, the probability of error, P_e , be kept constant. If any attempt at making a decision after, say, γ_2 chips is made, then the threshold voltage must be set such that

$$P_e = P[n_0(\gamma_2 T_c) > \epsilon_2] = \frac{1}{2} \operatorname{erfc} \left[\frac{\epsilon_2}{\sqrt{2\sigma_0}(\gamma_2 T_c)} \right]. \quad (8)$$

Note that Equation (8) is identical to Equation (4), with the exception of the pertinent parameters (i.e., the decision mechanism is identical). But if the probability of error, P_e , is to be kept constant, then

$$P_e \Big|_{\text{at } t = \gamma_1 T_c} = P_e \Big|_{\text{at } t = \gamma_2 T_c} \quad (9)$$

or

$$\frac{1}{2} \operatorname{erfc} \left[\frac{\epsilon_1}{\sqrt{2\sigma_0(\gamma_1 T_c)}} \right] = \frac{1}{2} \operatorname{erfc} \left[\frac{\epsilon_2}{\sqrt{2\sigma_0(\gamma_2 T_c)}} \right]$$

This implies that the two arguments of the erfc function must be equal. Hence

$$\frac{\epsilon_1}{\sqrt{2\sigma_0(\gamma_1 T_c)}} = \frac{\epsilon_2}{\sqrt{2\sigma_0(\gamma_2 T_c)}} \quad (10)$$

or

$$\epsilon_2 = \epsilon_1 \frac{\sigma_0(\gamma_2 T_c)}{\sigma_0(\gamma_1 T_c)} = \frac{1}{2} \sqrt{2P_s} \sqrt{\frac{\gamma_2}{\gamma_1}} (\gamma_1 T_c) G(\gamma_1)$$

Equation (10) defines the parameter ϵ_2 such that at any time $T = \gamma_2 T_c$ a decision can be attempted. Figure 2 shows qualitatively the decision regions at any given $\gamma_2 < \gamma_1$.

If at time $T = \gamma_2 T_c$ the correlator's output voltage equals V_0 , the following decisions can be made with a given probability of error P_e :

- 1) Synchronization has been attained ($i = j$) if

$$V_0 > V_s = \sqrt{2P_s} \gamma_2 T_c (1 - G(\gamma_2)) + \epsilon_2$$

- 2) Synchronization has not been attained ($i \neq j$) if

$$V_0 < V_n = \sqrt{2P_s} \gamma_2 T_c - \epsilon_2$$

- 3) No decision can be made; continue the correlation if

$$V_n < V_0 < V_s$$

Thus, $V_n(\gamma_2)$ and $V_s(\gamma_2)$ become

$$V_n(\gamma_2) = \sqrt{2P_s} \gamma_2 T_c - \epsilon_2 = \sqrt{\frac{P_s}{2}} \gamma_1 T_c \left[2 \frac{\gamma_2}{\gamma_1} - G(\gamma_1) \sqrt{\frac{\gamma_2}{\gamma_1}} \right] \quad (11)$$

and

$$V_s(\gamma_2) = \sqrt{2P_s} T_c \gamma_2 (1 - G(\gamma_2)) + \epsilon_2 = \sqrt{\frac{P_s}{2}} \gamma_1 T_c \left[2 \frac{\gamma_2}{\gamma_1} (1 - G(\gamma_2)) + G(\gamma_1) \sqrt{\frac{\gamma_2}{\gamma_1}} \right] \quad (12)$$

respectively.

From Figure 2, the probability of no decision can be seen to equal

$$P(\text{no decision}) \triangleq P_{nd} = P[V_n < s_0 + n_0 < V_s] \quad (13)$$

With V_n and V_s given in Equations (11) and (12), respectively, the probability of no decision at time $\gamma_i T_c$ becomes

$$P_{nd}(\gamma_i) = \frac{1}{2} \left[\operatorname{erf} \left[\sqrt{\frac{E_{s1}}{2n}} \left(G(\gamma_1) - 2 \sqrt{\frac{\gamma_i}{\gamma_1}} G(\gamma_i) \right) \right] - \operatorname{erf} \left[\sqrt{\frac{E_{s1}}{2n}} G(\gamma_1) \right] \right] \quad (14)$$

Of more interest than (14) is the conditional probability of no decision at the j^{th} step, given that no decision had been made at the $j-1^{\text{st}}$ step. For ease of notation, let $i \triangleq j-1$, and denote by f_{s_i} and f_{n_j} the value of the in-sync and not-in-sync curve of Figure 2, respectively, at the point γ_j . Denoting the probability referred to above as P_{ij} , we have

$$P_{ij} = P \left\{ v_{nj} < f_{sj} + n_j < v_{sj} \mid v_{ni} < f_{si} + n_i < v_{si} \right\} \quad (15)$$

where

$$v_{nj} \triangleq V_n(\gamma_j) \quad (16)$$

$$v_{sj} \triangleq v_s(\gamma_j) \quad (17)$$

and

$$n_j \triangleq \int_0^{\gamma_j T_c} n_w(t) g(t - kT_c) dt, \quad (18)$$

where k corresponds to the specific phase position of the spreading sequence that is currently being examined.

The noise output at time t_j is given by

$$n_j = n_i + \Delta n$$

where Δn is the noise contribution in the interval Δt (e.g., $\Delta t = T_c$), and the threshold voltages are given by

$$v_{nj} = f_{sj} - \epsilon_j$$

and

$$v_{sj} = f_{nj} + \epsilon_j$$

Therefore, (15) can be rewritten as

$$\begin{aligned} P_{ij} &= P [-\epsilon_j - n_i < \Delta n < (f_{nj} - f_{sj}) + \epsilon_j - n_i \mid v_{ni} < f_{si} + n_i < v_{si}] \\ &= P \left\{ \frac{-\epsilon_j - n_i < \Delta n < (f_{nj} - f_{sj}) + \epsilon_j - n_i, -\epsilon_i < n_i < (f_{ni} - f_{si}) + \epsilon_i}{P \{-\epsilon_i < n_i < (f_{ni} - f_{si}) + \epsilon_i\}} \right\} \frac{P_n}{P_d} \end{aligned} \quad (19)$$

If, for simplicity, $D_j \triangleq G(\gamma_j)$, then the numerator of (19) can be shown to reduce to

$$\begin{aligned}
P_n = & \int \frac{\gamma_j}{\gamma_j} - \left[D_i + \frac{1}{2} \sqrt{\frac{\gamma_1}{\gamma_i}} D_1 \right] \sqrt{\frac{1}{\pi}} \sqrt{\frac{2E_{s_j}}{n}} \sqrt{\frac{\gamma_j}{\gamma_i}} e^{-\frac{2E_{s_j}}{n} \frac{\gamma_j}{\gamma_i} x^2} \\
& - \frac{1}{2} \frac{\gamma_1}{\gamma_j} \sqrt{\frac{\gamma_i}{\gamma_1}} D_1 \\
& \cdot \left[\operatorname{erf} \left[\sqrt{\frac{2E_c}{n}} \gamma_j \left[-D_j - x + \frac{1}{2} \sqrt{\frac{\gamma_1}{\gamma_j}} D_1 \right] \right] \right. \\
& \left. - \operatorname{erf} \left[\sqrt{\frac{2E_c}{n}} \gamma_j \left[-\frac{1}{2} \sqrt{\frac{\gamma_1}{\gamma_j}} D_1 - x \right] \right] \right] dx \quad (20)
\end{aligned}$$

Similarly, the denominator of (19) can be simplified to

$$P_d = \operatorname{erf} \left[\sqrt{\frac{2E_s}{n}} \left[-D_i + \frac{1}{2} \sqrt{\frac{\gamma_1}{\gamma_i}} D_1 \right] \right] - \operatorname{erf} \left[-\sqrt{\frac{E_{s_1}}{2n}} D_1 \right] \quad (21)$$

Let $\{E_j^{(1)}\}$ be the event {making a decision for the first time at γ_j }.
Then

$$P \{E_j^{(1)}\} = (1 - P_{j-1,j}) P_{j-2,j-1} \cdots P_{N,N+1} P_{n1}(\gamma_N) \quad (22)$$

where P_{ij} is given by (19) and $P_{nd}(\gamma_i)$ is given by (14). Implicit in (22) is that the first time a decision is attempted is $\gamma_i = N$.

Finally, it is of interest to determine the average time needed to make a decision. To evaluate this quantity, the Markov chain model described in [5] can be used. The transition diagram is shown in Figure 3, where states $N+1, N+2, \dots, (\gamma_1-1)$ indicate both a possible path to state D, the decision state (absorbing state), and also a possible path to the next higher state. On the other hand, when state γ_1 is entered, the next state has to be D.

The canonical form of the transition matrix (see [5]) for this scheme is given by

$$\bar{P} = \begin{array}{c|cccccccc} & D & N & N+1 & N+2 & & & (\gamma_1-1) & \gamma_1 \\ \hline D & 1 & 0 & 0 & 0 & - & - & - & 0 \\ N & 1 - P_{N,N+1} & 0 & P_{N,N+1} & 0 & 0 & - & - & 0 \\ N+1 & 1 - P_{N+1,N+2} & 0 & 0 & P_{N+1,N+2} & - & - & - & 0 \\ N+2 & 1 - P_{N+2,N+3} & 0 & 0 & 0 & P_{N+2,N+3} & - & - & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_1-1 & 1 - P_{\gamma_1-1,\gamma_1} & 0 & 0 & 0 & - & - & - & 0 \\ \gamma_1 & 1 & 0 & 0 & 0 & - & - & - & 0 \end{array} \quad (23)$$

$$\bar{N} \triangleq \begin{bmatrix} \bar{S} & \bar{Q} \\ -\bar{R} & -\bar{Q} \end{bmatrix}$$

where the partitions $\bar{S}$, $\bar{Q}$, $\bar{R}$, and $\bar{Q}$ are defined in [5]. If we define the matrix $\bar{N} \triangleq [\bar{I} - \bar{Q}]^{-1}$, where $\bar{I}$ is an identity matrix, it is shown in [5] that

$$\bar{\tau} = \bar{N} \bar{\zeta} \quad (24)$$

where $\bar{\tau}$ is a column vector whose j^{th} element is the mean time to a decision given the process started in state j , and $\bar{\zeta}$ is a column vector whose elements are the duration of time needed to make each transition.

For this particular acquisition scheme,

$$\bar{\zeta} = \begin{bmatrix} N T_c \\ T_c \\ T_c \\ \vdots \\ T_c \\ T_c \end{bmatrix}, \quad (25)$$

Also, it is only the first entry τ_1 of the vector $\bar{\tau}$ that is of interest, since we are interested in the average time to a decision given that we start in state p_1 . From (23), (24), (25), and the definition of $\bar{N}$, we can show that

$$\tau_1 = (N + P_{N,N+1} + P_{N,N+1}P_{N+1,N+2} + \dots + P_{N,N+1}P_{N+1,N+2}\dots \quad (26)$$

$$\dots P_{\gamma_1-1,\gamma_1}) T_c$$

$$= \left[N + \sum_{j=N}^{\gamma_1-1} \sum_{i=N}^j P_{i,i+1} T_c \right] \quad (27)$$

Notice that τ_1 is upper bounded by $\gamma_1 T_c$, which would be the value of the integration time if a fixed interval was used to make the decision.

3. NUMERICAL RESULTS

In this section, the key equations describing the performance of the system will be evaluated numerically to illustrate how the acquisition scheme behaves. The parameters are chosen arbitrarily to be indicative of realistic values on one hand, but to not result in excessive computation time on the other hand. In particular, there are six figures presented, each one containing two curves graphed on a common set of axes. These curves represent the probability of making a decision within γ chips. Results are presented for the exact bound as well as for the approximation to the exact bound. The curves plot the probability of a decision along the ordinate; the number of chips within which the decision is made is the abscissa. For each curve we have set the length of the PN register to be 10 so that the period of the sequence is $L = 1023$. The curves are then plotted for a chip energy-to-noise density ratio of 0 dB to -20 dB and a probability of error of either 0.1 or 0.01. However, using Eq. (22), any other set of parameters may be employed.

It can be seen, for example from Figure 6, that if the ratio of energy-per-chip-to-noise density E_c/η is -20 dB and the probability of error is .1, then for a probability of decision of 0.99, the two-threshold sequential procedure will result in detection within 140 chips, compared to a single threshold fixed dwell time scheme which requires an interval of 560 chips (with, of course, a probability of decision of 1.0). If the probability of error is decreased to 0.01, it is seen from Figure 9 that the two-level technique will result in detection within 520 chips whereas the single threshold fixed dwell scheme requires 820 chips.

If the approximation curves shown in Figures 4 through 9 are compared to the corresponding exact curves, it will be seen that at low values of E_c/n and/or higher values of P_e , the approximation yields fairly accurate results. On the other hand, for lower values of P_e and/or higher values of E_c/n , the approximation is rather loose. However, the simplicity of evaluating the approximation together with the insight one can obtain by having a closed-form expression (as indeed is the case with the approximation) make it a useful tool in this type of design.

Finally, it is seen from some of the figures (e.g., Figure 4) that the upper curve, which corresponds to the exact upper bound, has a very irregular shape. This is due to jumps in the upper bound itself and to the discrete nature of the computation (that is, due to the fact that each additional increment of integration time is of length one chip).

4. CONCLUSION

In this paper, we have analyzed a rapid acquisition technique for DS spread spectrum communications. The technique employed was sequential estimation, and used two thresholds at each step in the procedure to determine whether or not synchronization was to be declared; the probability of error at each step in the process was always held constant.

Numerical results were presented which illustrated the performance of this approach. We have determined the probability of making a decision within γ chips and have proven that the average time to making a decision using this technique is always upper bounded by the time needed to make a decision using a fixed interval acquisition procedure.

Specifically, Figures 4-9 illustrated the actual savings in acquisition time which may be expected for a sequence of length 1023, a chip-to-noise density ratio of 0 dB to -20 dB, and an acceptable P_e of either 0.1 or 0.01. These results showed that savings of 40% to 75% in the acquisition time are entirely feasible if the double threshold scheme is used instead of a single threshold procedure.

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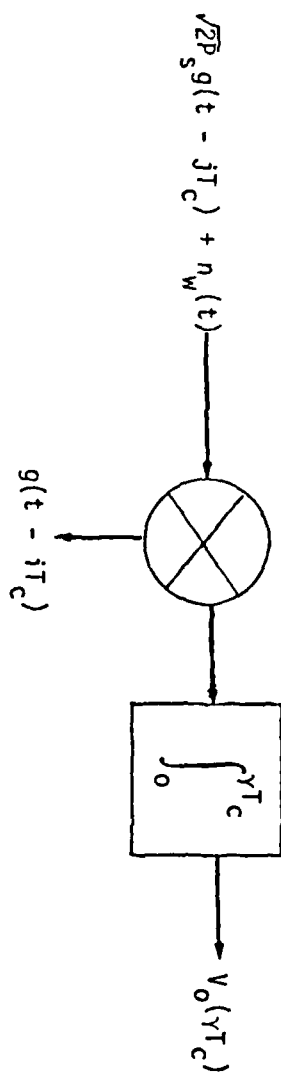


Figure 1. Block Diagram

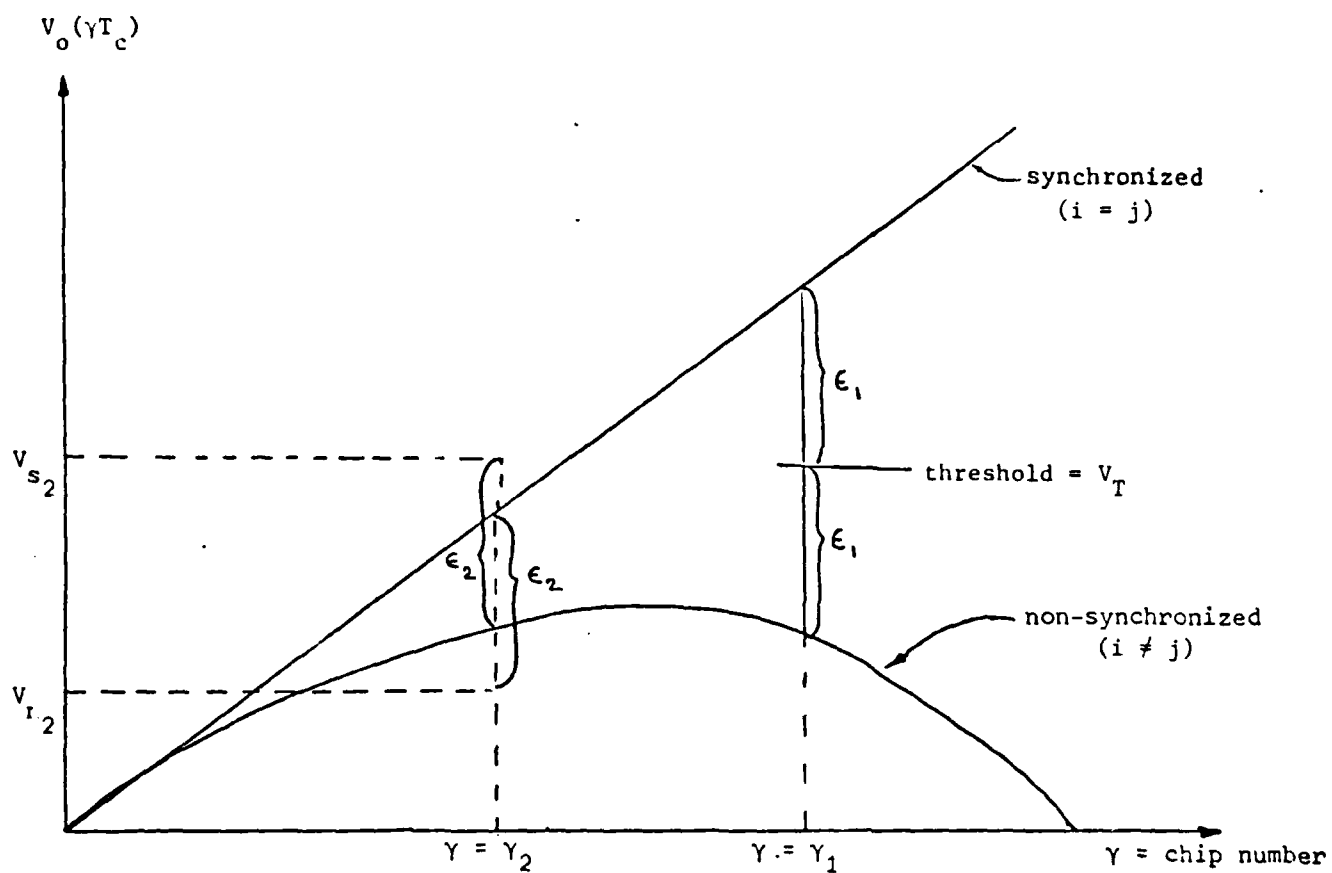


Figure 2. Decision Regions

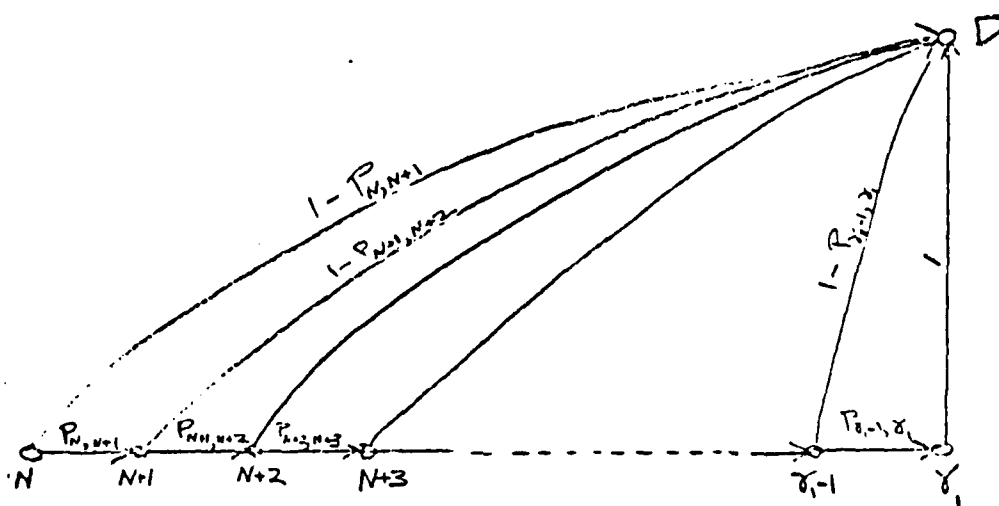
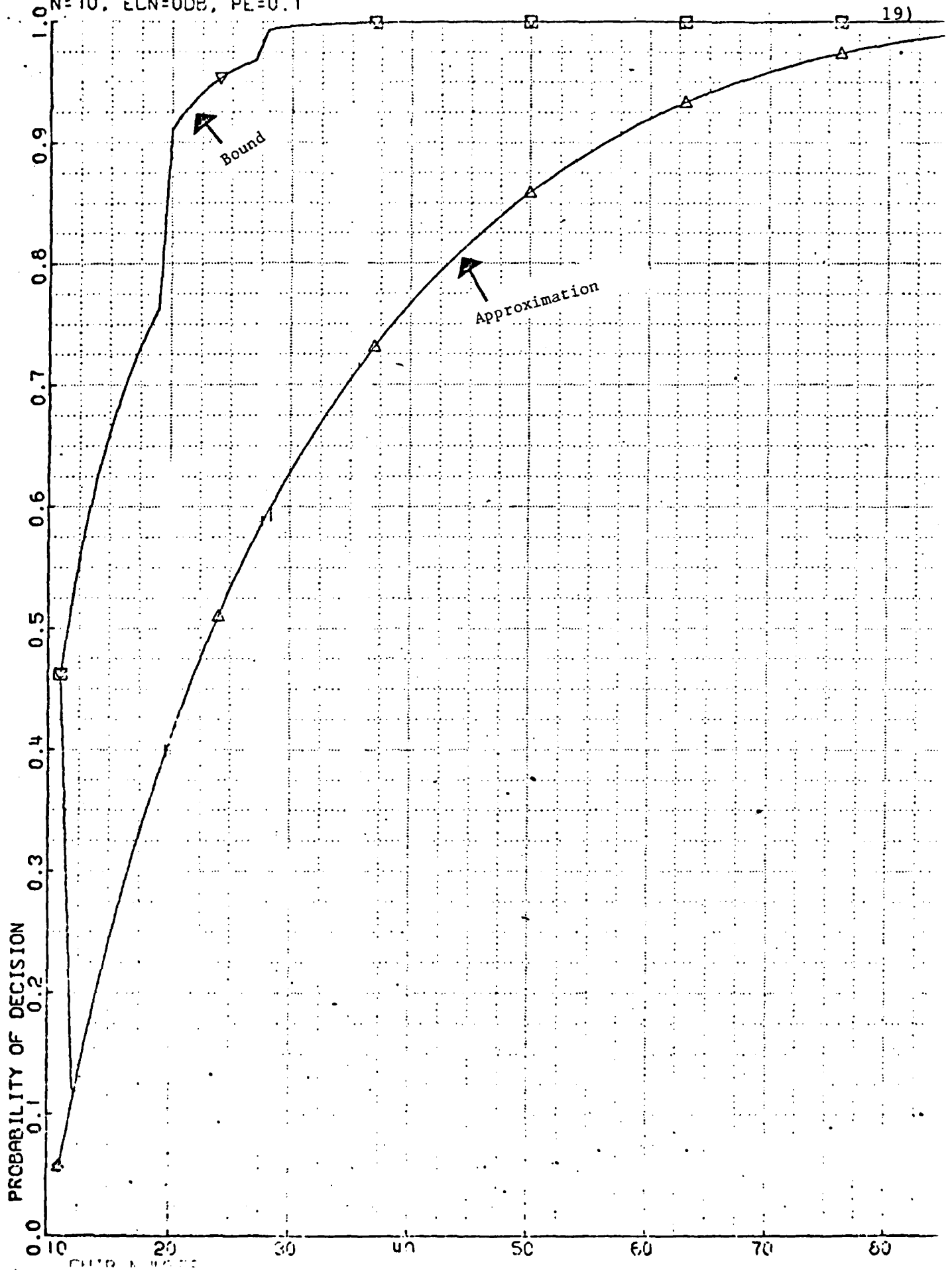


Figure 3. State Diagram

N=10, ECN=0DB, PE=0.1



N=10, ECN=-1025, PE=0.1

20)

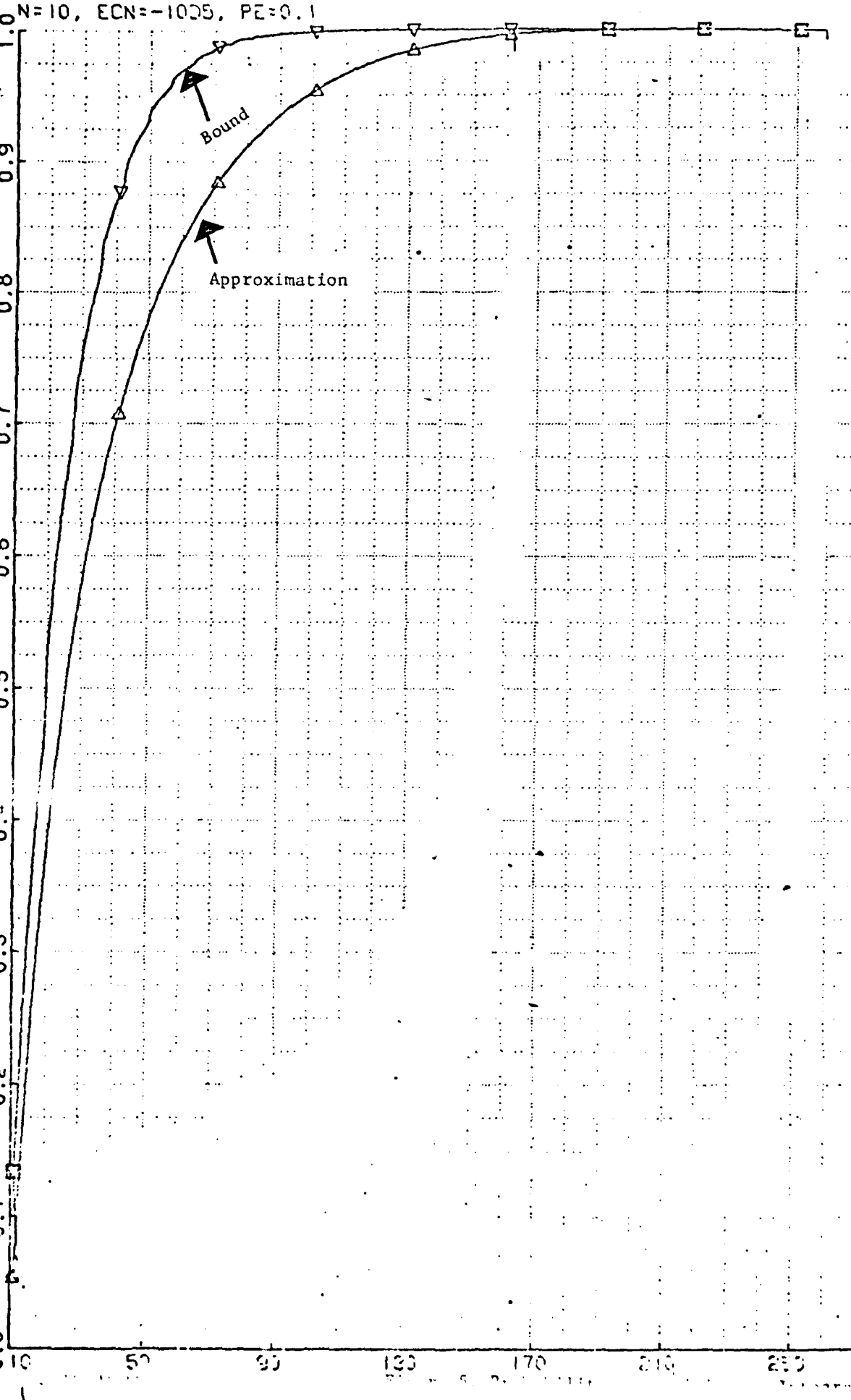
PROBABILITY OF DECISION

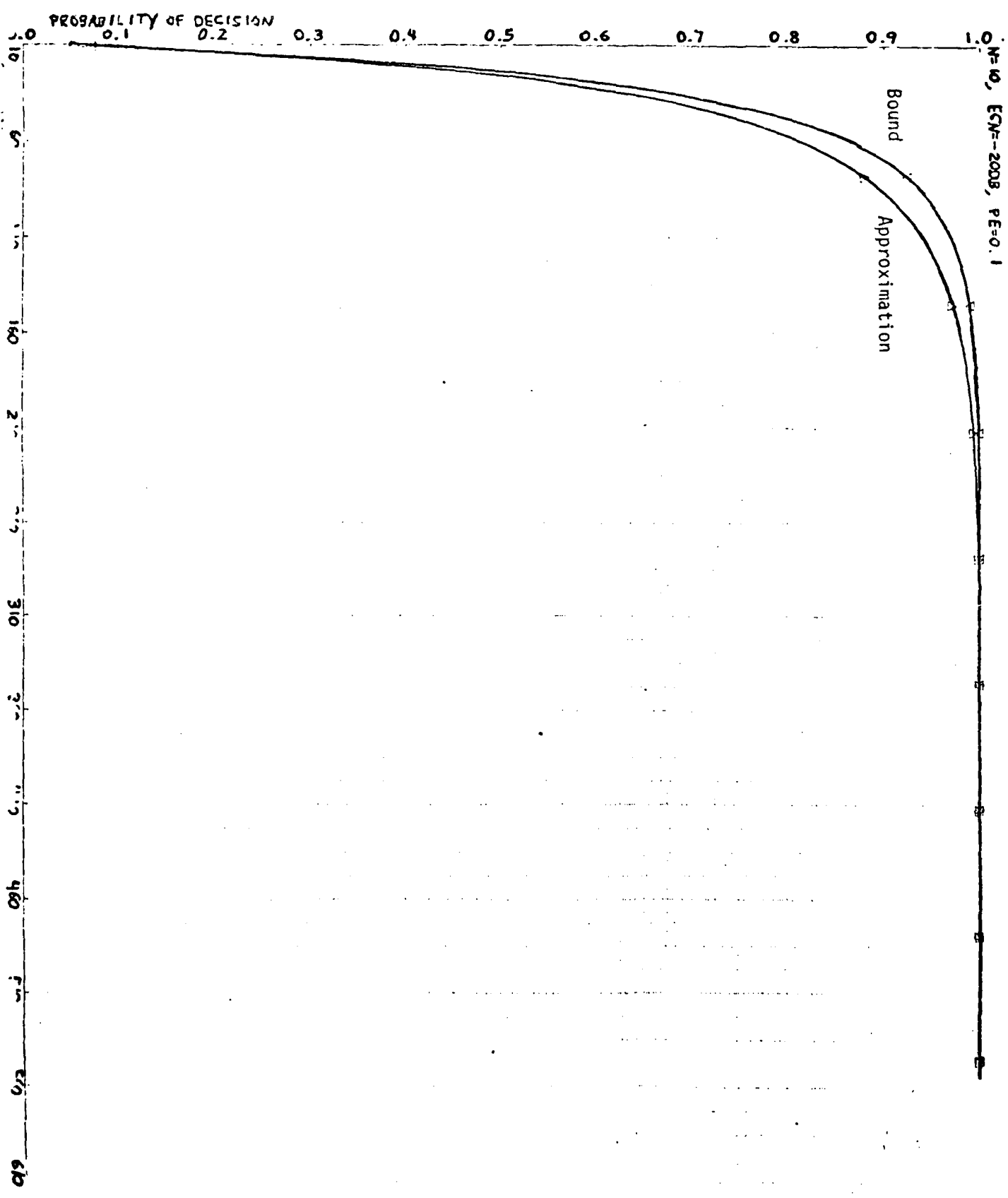
Bound

Approximation

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

10 50 90 130 170 210 250 290
Observation Time





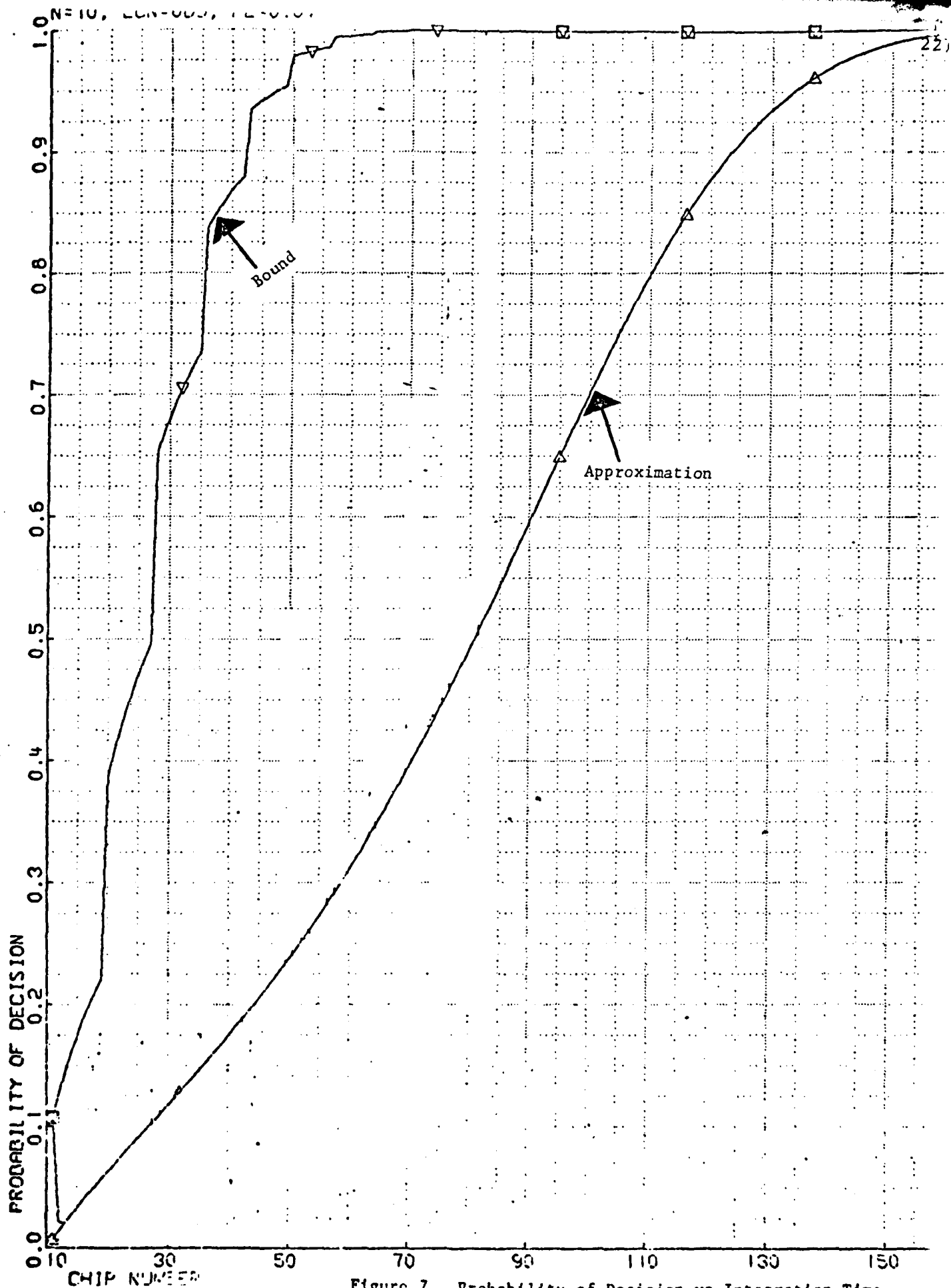


Figure 7. Probability of Decision vs Integration Time

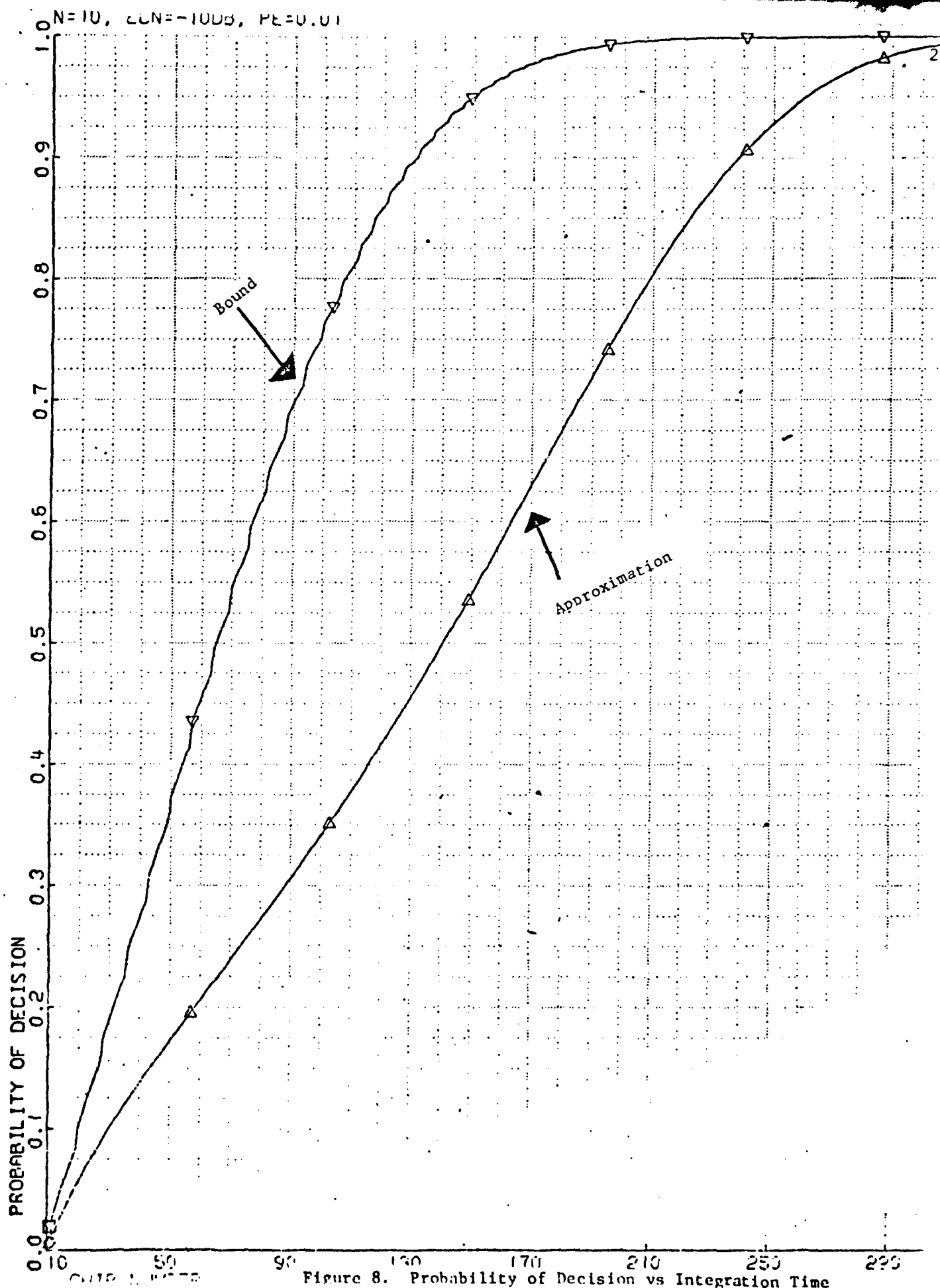
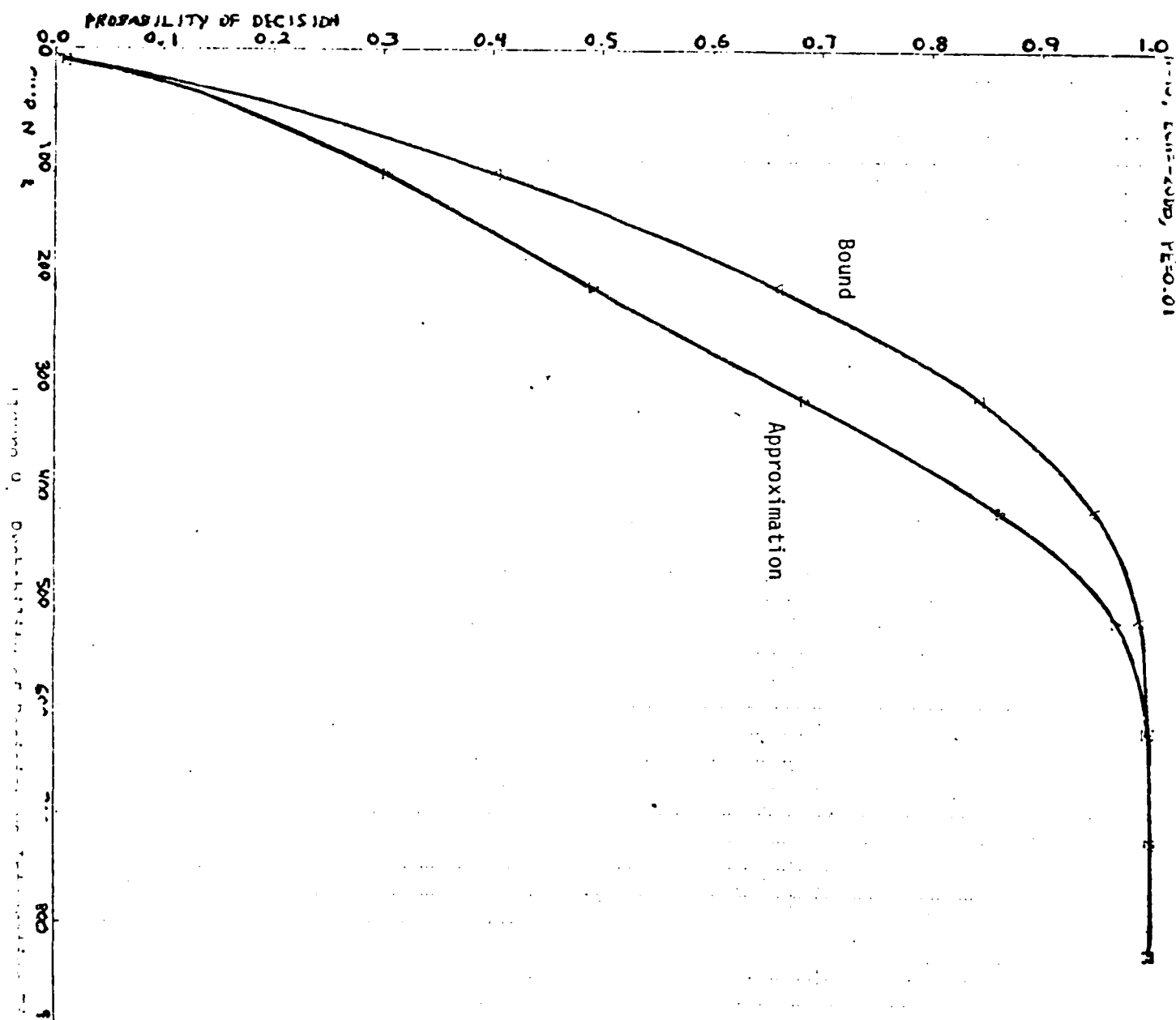


Figure 8. Probability of Decision vs Integration Time



2.2 DOUBLE THRESHOLD ACQUISITION SCHEME APPLIED TO THE
FADING CHANNEL IN FREQUENCY HOPPING SPREAD SPECTRUM

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ABSTRACT

In this paper a double threshold acquisition scheme is applied to the fading channel in frequency hopping spread spectrum in order to minimize the mean acquisition time. The model of the fading channel is presented. The acquisition strategy is described. The equations for the two thresholds are derived. A computer simulation program is used to determine the mean and the variance of the acquisition time. The results are analyzed and compared to the nonfading channel and also compared to the single threshold scheme in the fading channel.

DOUBLE THRESHOLD ACQUISITION SCHEME APPLIED TO THE
FADING CHANNEL IN FREQUENCY HOPPING SPREAD SPECTRUM

1. INTRODUCTION

In FH communication system the transmitting carrier frequency is controlled by a pseudonoise sequence generated by linear feedback shift registers. The state of the shift registers controls a frequency synthesizer which provides the transmitting carrier frequencies. In the receiver there must be such a PN sequence replica which controls the local carrier. The two PN sequence should be synchronized such that the local carrier frequency synthesizer provides exactly the same hopping frequency and hops at the same rate as the transmitter in order to dehop the signal and recover the data from demodulation such as FSK. Before the system is synchronized no data can be sent. Therefore, the mean acquisition time is greatly concerned. The purpose of this paper is to minimize the mean acquisition time. Based on the work that we have done to the nonfading channel [reference 1] a double threshold acquisition scheme is also applied to the fading channel. A computer simulation program is used to determine the mean acquisition time and the variance with respect to the other parameters. In this paper the acquisition strategy is described. The faded signal has been studied. The probability density function of the signal

envelope square is given. The equation of the two thresholds are derived. The computer simulation is done statistically for each of the specific parameter. The results are analyzed and compared to the nonfading channel and also compared to the single threshold scheme in the fading channel.

2. ACQUISITION STRATEGY

The tracking loop works only when the alignment between the transmitter and the receiver is greater than half of the hop duration; therefore, the acquisition technique includes two stages, pre-acq and true-acq, to guarantee this requirement.

Acquisition strategy of FH signal is the following:

Refer to Fig. 1 and Fig. 2.

The incoming signal having frequency ω_i and phase θ is embedded in noise. The local frequency synthesizer is multiplied by the incoming signal and the product passes through a BPF which eliminates the sum frequency components and presents the difference component $\omega_i - \omega_j$ and the noise term to the integrator. ω_i and ω_j are selected such that the output of the integrator

$$\sqrt{P_s} \int_0^{rT_c} \cos(\omega_i - \omega_j)t \, dt = 0, \quad \text{for } \omega_i \neq \omega_j \quad (2-1)$$

$$\text{ie, } (f_i - f_j)T_c = \text{integer} \quad (2-2)$$

$$\text{and } \sqrt{P_s} \int_0^{rT_c} \cos(\omega_i - \omega_j)t \, dt = rT_c \sqrt{P_s} \quad (2-3)$$

for $\omega_i = \omega_j$

The noise has a mean value of 0 and a variance of $\eta/2$. After integration the noise term present at the output of the integrator has a mean value of 0 and the variance σ^2 is proportional to $\eta r T_c/2$.

The signal has two components, in phase and quadrature. We then take the envelope (or envelope square). The envelope square is compared with two thresholds THA (lower threshold) and THB (upper threshold). Refer to Fig 3, at time t_1 the output of the integrator V_0 is less than THA, we say that the incoming signal is not synchronized with the local frequency synthesizer. The comparator outputs a signal which controls the local PN generator to shift to the next state, thus the frequency synthesizer also hops to the next frequency f_1 . at time t_2 V_0 is greater than THB, we say that pre-acquisition has occurred. The frequency synthesizer and PN generator remain in the old state until the incoming signal hops to the next frequency f_2 . At time t_3 V_0 again drops below THA, the comparator declares that true acquisition has occurred, the PN generator shifts to next state and the frequency synthesizer hops to next frequency f_2 . In this case the receiver is at most one chip (rt_c) later than the transmitter; that is, the alignment between the incoming signal and the local frequency synthesizer is longer than half of the hop duration. At this moment the tracking loop starts to work which brings the local hopper completely in phase with the incoming signal and thus, fine acquisition is fulfilled.

If V_0 is between THA and THB , no decision is made and more samples must be taken. At the end of the second rT_c another comparison is made....etc. This procedure is repeated until a decision is made.

In our simulation $rT_c = T_b$ (one bit duration).

3. PROBABILITY DENSITY FUNCTION OF THE SIGNAL ENVELOPE IN THE FADING CHANNEL AND THE TWO THRESHOLDS

The mean value of the coefficients of the fading channel is zero so we say that the fading channel without specular component is Rayleigh channel.*

In general, if X and Y are Gaussian random variable with variance σ^2 .

$$Z = \sqrt{X^2 + Y^2} \quad (3-1)$$

$$f(z) = \frac{z}{\sigma^2} e^{-z^2/2\sigma^2} U(z) \quad \text{---Rayleigh Density} \quad (3-2)$$

$U(z)$ = unit step function

For convenience we consider the envelope squared. We define

$W = z^2$, W = envelope square

The probability density function of W is:

$$f(W) = \frac{1}{2\sigma^2} e^{-W/2\sigma^2} U(W) \quad (3-3)$$

*: See Appendix A

The probability that W is between α and β is:

$$P(\alpha \leq W < \beta) = \int_{\alpha}^{\beta} f(W) dW = e^{-\alpha/2\sigma^2} - e^{-\beta/2\sigma^2} \quad (3-4)$$

Before synchronization we have noise only. The variance equals σ_N^2 .** When the system is going to synchronize, we have signal and noise. The variance equals σ_{SN}^2 .** Refer to Fig.4, when the system is not synchronized, for each comparison we make an error if the noise is greater than THB.

$$\begin{aligned} P_{FA} &= P(\text{noise} \geq \text{THB}) = \int_{\text{THB}}^{\infty} f(W) dW \\ &= e^{-\text{THB}/2\sigma_N^2} \end{aligned} \quad (3-5)$$

When the system is synchronized, we do not acknowledge the synchronization if the signal + noise is less than THA.

$$\begin{aligned} P_D &= P(\text{signal} + \text{noise} \leq \text{THA}) = \int_0^{\text{THA}} f(W) dW \\ &= 1 - e^{-\text{THA}/2\sigma_{SN}^2} \end{aligned} \quad (3-6)$$

From eqs (3-5) and (3-6) we have

$$\text{THA} = -2\sigma_{SN}^2 \ln(1 - P_D) \quad (3-7)$$

$$\text{THB} = -2\sigma_N^2 \ln P_{FA} \quad (3-8)$$

Fig. 4 shows the probability density function of

Appendix B

envelope square, P_{FA} and P_D . Fig.5 and Fig.6 give THA and THB versus number of bits being integrated for the different signal to noise ratio E_b/η and the different P_D .

In order to minimize the probability of false acquisition P_{FA} will be less than or equals to 0.0001. P_D affects the mean acquisition time significantly. This can happen in two ways:

Refer to Fig 4,

a) With a small P_D most of the noise will be above THA and we always cannot make a decision which increases the mean acquisition time .

b) If P_D is large the signal + noise will also lie below THA and it will be falsely dismissed.

Both of the above situation will increase the mean acquisition time.

4. COMPUTER SIMULATION RESULTS --- MEAN ACQUISITION TIME AND VARIANCE VERSUS E_b/η FOR DIFFERENT P_D .

The normalized mean acquisition time versus E_b/η for the different P_D are shown in Fig.7 and Fig.8. Fig.7 is for 30 hops/S and Fig.8 is for 100 hops/S. From these figures we observe that the mean acquisition time doesn't depend on the hop rate and mostly depends on the energy.

Comparing with the nonfading channel [reference 1] we observe that when $E_b/\eta=3\text{db}$, fading increases the mean acquisition time 22% only because comparing to the noise fading is not important. When $E_b/\eta=30\text{db}$, fading doesn't

affect the mean acquisition time at all. The mean acquisition time is close to the minimum possible value which equals $(L/2)*1.1$.***

L = code length, in our simulation $L=511$

$(L/2)*1.1=(511/2)*1.1=281$ bits

When E_b/η equals 15db, P_D equals 0.3 and 0.5, (points A and B in Fig.7) and also refer to Fig.6 we observe that THA and THB intersect earlier than or at the vicinity of one bit. Considering the acquisition strategy, referring to Fig.2, we would find that acquisition only depends on THB. In that case the double threshold scheme becomes single threshold scheme and the mean acquisition time goes high. Point C ($P_D=0.1$) is still the case of double threshold scheme. Hence, this is the improvement of the double threshold scheme comparing to the single threshold scheme. Again at points D and E double threshold scheme becomes single threshold scheme and we may reduce the P_D to, say, 0.02 to find another double threshold scheme. Point F is the improvement.

The variances of the acquisition time are listed in Table 1 and Table 2.

***: See Appendix C

APPENDIX A
MODEL OF THE FADING CHANNEL

The fading channel is represented by two tapped delay lines, one representing the in phase component and another representing the quadrature component. To represent a HF channel the taps are spaced about 1ms apart. There are either 3 or 4 taps employed. The signal samples from those taps are multiplied by independent Gaussian noise processed as shown in Fig.9. The output signal from the fading channel is then represented by two equations.

$$FP = \sum_{j=1, TDS, \frac{NSC}{2}} DSP(j) * CP(j) - DSQ(j) * CQ(j) \quad (A-1)$$

$$FQ = \sum_{j=1, TDS, \frac{NSC}{2}} DSQ(j) * CP(j) + DSP(j) * CQ(j) \quad (A-2)$$

In equation (A-1) and (A-2)

FP: Fading in phase sample

FQ: Fading quadrature sample

TDS: Total number of delayed sample, in phase or quadrature

$$TDS = ((NTAPS-1) * \frac{NSC}{2}) + 1 \quad (A-3)$$

NTAPS: Total number of taps, in phase or quadrature

$$NTAPS = CHIP RATE * CHANNEL SPACING * SPREAD TIME * 2 \quad (A-4)$$

NSC: Number of samples/chip

DSP(J): Individual in phase delayed sample

DSQ(J): Individual quadrature delayed sample

CP(J): Individual in phase coefficient

CQ(J): Individual quadrature coefficient

RP: Transmitting in phase signal sample

RQ: Transmitting quadrature signal sample

CP and CQ are random variables which represent the characteristic of the fading channel. CP and CQ equals a Gaussian random number multiplied by its standard deviation and then passes through a low pass filter whose cutoff frequency equals the fading rate. The data passing through the low pass filter will lose energy and therefore its variance is adjusted accordingly. The adjustment is proportional to the sampling frequency and inversely proportional to the fading rate. The constant of proportionality is about 1/3. The variance of CP and CQ equals the total normalized power divided by the number of taps and multiplied by the factor of the adjustment.

$$\text{Variance before filtering} = \frac{1}{2 * NTAPS} * \frac{\text{sampling rate}}{\text{fading rate}} * \frac{1}{3} \quad (\text{A-5})$$

$$\text{Variance after filtering} = \frac{1}{2 * NTAPS} \quad (\text{A-6})$$

In the case of FSK, as shown in Fig.10, RQ=0, Eqs (A-1) and (A-2) become Eqs (A-7) and (A-8).

$$FP = \sum_{j=1, TDS, \frac{NSC}{2}} DSP(j) * CP(j) \quad (\text{A-7})$$

$$FQ = \sum_{j=1, TDS, \frac{NSC}{2}} DSP(j) * CQ(j) \quad (\text{A-8})$$

APPENDIX B

SIGNAL IN THE FADING CHANNEL AND THE VARIANCE OF THE
RECEIVED SIGNAL AT THE OUTPUT OF THE INTEGRATOR

The fading channel is represented by two delay lines. When the transmitter hops to a new frequency, the signal is passed through a new fading channel. Consider the in phase component only. (The quadrature component is the same). Referring to Fig.10, for a 2-tap model we see that the first to the fifth fading samples are:

$$\begin{aligned} FP(1) &= \sqrt{2} * CP(1) \\ &\vdots \\ FP(5) &= \sqrt{2} * CP(1) \\ &= \sqrt{2} * \sqrt{\frac{1}{2NTAPS}} = \sqrt{\frac{1}{2}} = 0.707 \end{aligned} \quad (B-1)$$

The sixth to tenth fading samples are:

$$FP(6) = \sum_{j=1}^2 \sqrt{2} CP(j) \quad (B-2)$$

Because CP's are Gaussian random variables with a variance of $1/2NTAPS$ and a frequency bandwidth of $1 H_z$, the fading rate, the summation among the different taps is calculated according to the power. Hence,

$$\begin{aligned} FP(6) &= \sqrt{2} \sum_{j=1}^2 CP(j) \\ &= \sqrt{2} * \sqrt{2} * \sqrt{\frac{1}{2NTAPS}} = 1 \end{aligned} \quad (B-3)$$

For FP(11) to FP(100), since all the delay lines are filled with signal, the magnitude of FP will be 1.

At the time of the 101th sample, the transmitter hops to the next frequency and the delay lines are gradually filled by new values. Nevertheless, some delay lines are still filled with the old frequency signal, however, the magnitude of the old frequency signal decreases gradually according to the same rule.

For the 101th to 105th samples

$$FP(101) = \sqrt{2} \text{ CP}(2) = \sqrt{2} * \sqrt{\frac{1}{2NTAPS}} = 0.707 \quad (B-4)$$

For a 4-tap model, NTAPS=4, similarly,

FP(1) to FP(5)=0.5

FP(6) to FP(10)=0.707

FP(11) to FP(15)=0.866

FP(16) to FP(100)=1

FP(101) to FP(105)=0.866

FP(106) to FP(110)=0.707

FP(111) to FP(115)=0.5

The signal variation with time in the fading channel is shown in Fig.11 to Fig.13 for the different hopping rates.

When the system is not synchronized, the received detected output consists of noise only. When the receiver hops to the transmitting frequency, there are ten possible situations if we have ten bits in one hop. The receiver can

only hop at the end of the bit time. If the transmitter and the receiver hop simultaneously, there is no signal in the channel; otherwise, some samples are already in the fading channel. Now we consider the situation that both the transmitter and the receiver hop simultaneously. At the end of the first bit the receiver receives ten samples. The first five samples are correlated because they have the same CP(1). Due to the low fading rate these five samples add almost linearly. (The precision is up to the third digits). Sample 6 to sample 10 also add linearly. But sample 1 to sample 5 and sample 6 to sample 10 only add partly linearly because they have the common coefficient CP(1) but samples 1 to 5 don't have CP(2). For simplicity we suppose all the samples add linearly. Of course, this is an approximation. But the influence of such approximation will be partly cancelled if the signal is already in the fading channel when the receiver hops to the transmitting frequency. Therefore, for a two-tap model at the end of the first bit the standard deviation of the received signal at the output of the integrator is:

$$\begin{aligned}\sigma_{s1} &= \left[\sqrt{P_s} \sqrt{\frac{1}{2NTAPS}} * \frac{NSC}{2} + \sqrt{P_s} \sqrt{\frac{2}{2NTAPS}} * \frac{NSC}{2} \right], \quad P_s=2 \\ &= \frac{NSC}{2} \sqrt{\frac{1}{NTAPS}} [1 + \sqrt{2}] \quad (B-5)\end{aligned}$$

From bit 2 to bit 10, because all the delay lines are

filled with signal; hence,

$$\sigma_{s2} = \sigma_{s1} + \text{NSC}$$

$$\sigma_{s3} = \sigma_{s2} + \text{NSC}$$

⋮

$$\sigma_{s10} = \sigma_{s9} + \text{NSC} \quad (\text{B-6})$$

for bit 11, the signal in the delay lines gradually move out; hence,

$$\sigma_{s11} = \sigma_{s10} + \frac{\text{NSC}}{2} \sqrt{\frac{1}{\text{NTAPS}}} (\sqrt{1} + 0) \quad (\text{B-7})$$

For a four-tap model:

$$\sigma_{s1} = \frac{\text{NSC}}{2} \sqrt{\frac{1}{\text{NTAPS}}} [1 + \sqrt{2}] \quad (\text{B-8})$$

$$\sigma_{s2} = \frac{\text{NSC}}{2} \sqrt{\frac{1}{\text{NTAPS}}} [\sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4}] \quad (\text{B-9})$$

$$\sigma_{s3} = \sigma_{s2} + \text{NSC}$$

⋮

$$\sigma_{s10} = \sigma_{s9} + \text{NSC} \quad (\text{B-10})$$

$$\sigma_{s11} = \sigma_{s10} + \frac{\text{NSC}}{2} \sqrt{\frac{1}{\text{NTAPS}}} [\sqrt{3} + \sqrt{2} + \sqrt{1} + 0] \quad (\text{B-11})$$

$$\sigma_{s12} = \sigma_{s11} + \frac{\text{NSC}}{2} \sqrt{\frac{1}{\text{NTAPS}}} [\sqrt{1} + 0] \quad (\text{B-12})$$

As for the noise:

At the end of the M^{th} bit, the standard deviation of the noise is:

$$\sigma_{NM} = \sqrt{M} * NSC * NCB * 10^{-\frac{WNL}{20}} \quad (B-13)$$

WNL: $E_b /$ in db

NCB: number of chips/bit, which equals 1.

At the end of the M^{th} bit, the standard deviation of the signal plus noise is:

$$\sigma_{SNM} = \sqrt{(\sigma_{SM})^2 + (\sigma_{NM})^2} \quad (B-14)$$

APPENDIX C

THE MINIMUM POSSIBLE ACQUISITION TIME

If there is no noise the minimum possible acquisition time equals $(L/2)*1.1$

L = code length

The average difference between the transmitter and the receiver is half of the code length $L/2$. The factor 1.1 is due to the hopping rate of the transmitter during acquisition. If there is no noise so that the receiver can make a decision at the end of each bit, it needs $L/2$ bit time to catch up the transmitter. But in this time interval the transmitter also hops $L/2/10$ times. (in our case there are 10 bits in one hop.) The receiver still have to catch up. The time needed is $L/2/10$ bits. Therefore, the total time = $(L/2)+(L/2/10)=(L/2)*1.1$ bits.

Table 1

Variance of Acquisition Time (Experiment Result)Hop rate = 30 hops/S, 2-tap Model

E_b/n (db)	P_D	Variance	Normalized Standard Deviation
30	0.1	0.418418	0.62966
	0.02	0.418418	0.62966
	0.01	0.303081	0.55344
	0.005	0.275799	0.55099
	0.002	0.435575	0.56009
25	0.1	0.455856	0.61307
	0.02	0.337374	0.50216
	0.01	0.412955	0.55545
	0.005	0.921933	0.62320
	0.002	2.070953	0.51520
20	0.1	0.524299	0.65931
	0.02	0.778812	0.69792
	0.01	1.200690	0.63962
15	0.5	6.588859	0.93680
	0.3	5.536296	0.97243
	0.1	1.629105	0.82622
	0.02	6.980358	0.95249

Table 1 (Cont'd.)

E_D/η (db)	P_D	Variance	Normalized Standard Deviation
12	0.5	6.665808	0.89386
	0.3	2.210138	0.76763
	0.1	0.840163	0.56669
9	0.5	9.171375	0.89559
	0.3	4.471619	0.83897
	0.1	7.753166	0.93249
6	0.5	16.20247	0.77684
	0.3	13.74216	0.94278
	0.1	18.79460	0.84126
3	0.5	36.24931	0.79713
	0.3	48.56720	1.15495
	0.1	146.13722	0.97152

Table 2

Variance of Acquisition Time (Experiment Result)

Hop rate = 100 hops/S, 4-tap Model

E_D/η (db)	P_D	Variance	Normalized Standard Deviation
30	0.1	0.056443	0.68907

Table (Cont'd.)

E_b/η (db)	P_D	ance	Normalized Standard Deviation
30	0.02	0.028083	0.55752
	0.01	0.039172	0.60551
	0.005	0.039606	0.51557
25	0.1	0.022601	0.56278
	0.02	0.033616	0.52921
	0.01	0.069021	0.59486
20	0.1	0.045693	0.62705
	0.02	0.106365	0.64079
15	0.5	0.308951	0.86610
	0.3	0.289192	1.00024
	0.1	0.146960	0.80117
12	0.5	0.358209	0.99068
	0.3	0.144358	0.77637
	0.1	0.201937	0.69824
9	0.5	0.811281	0.92320
	0.3	0.485079	0.94808
	0.1	0.388513	0.55792
6	0.5	1.514624	0.87441
	0.3	1.176961	1.00146
	0.1	2.291038	0.84144
3	0.5	3.302562	0.90081
	0.3	2.015820	0.88359

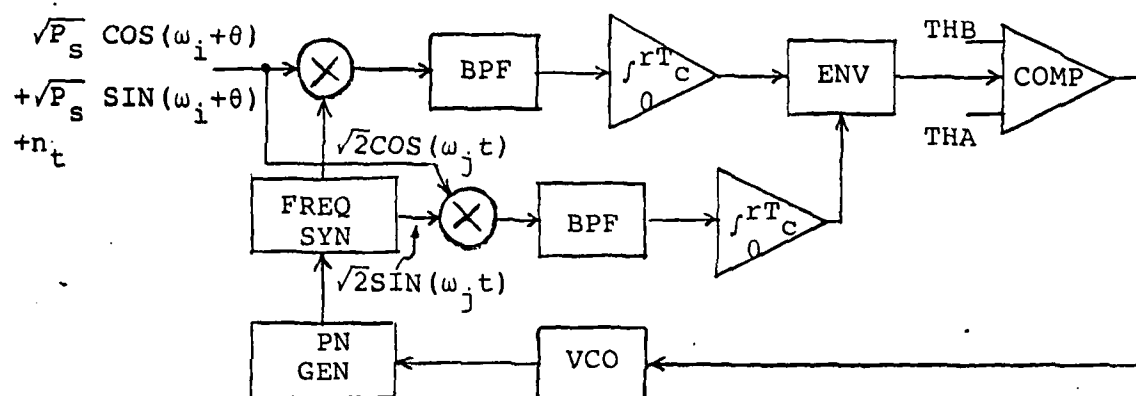


Fig. 1 The receiver in FH SS

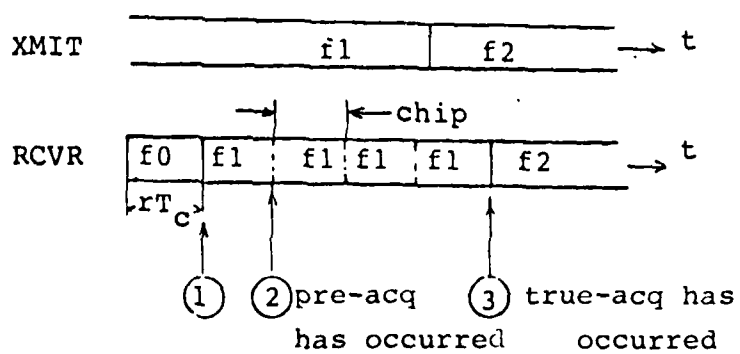


Fig. 3 Hopping frequency slot

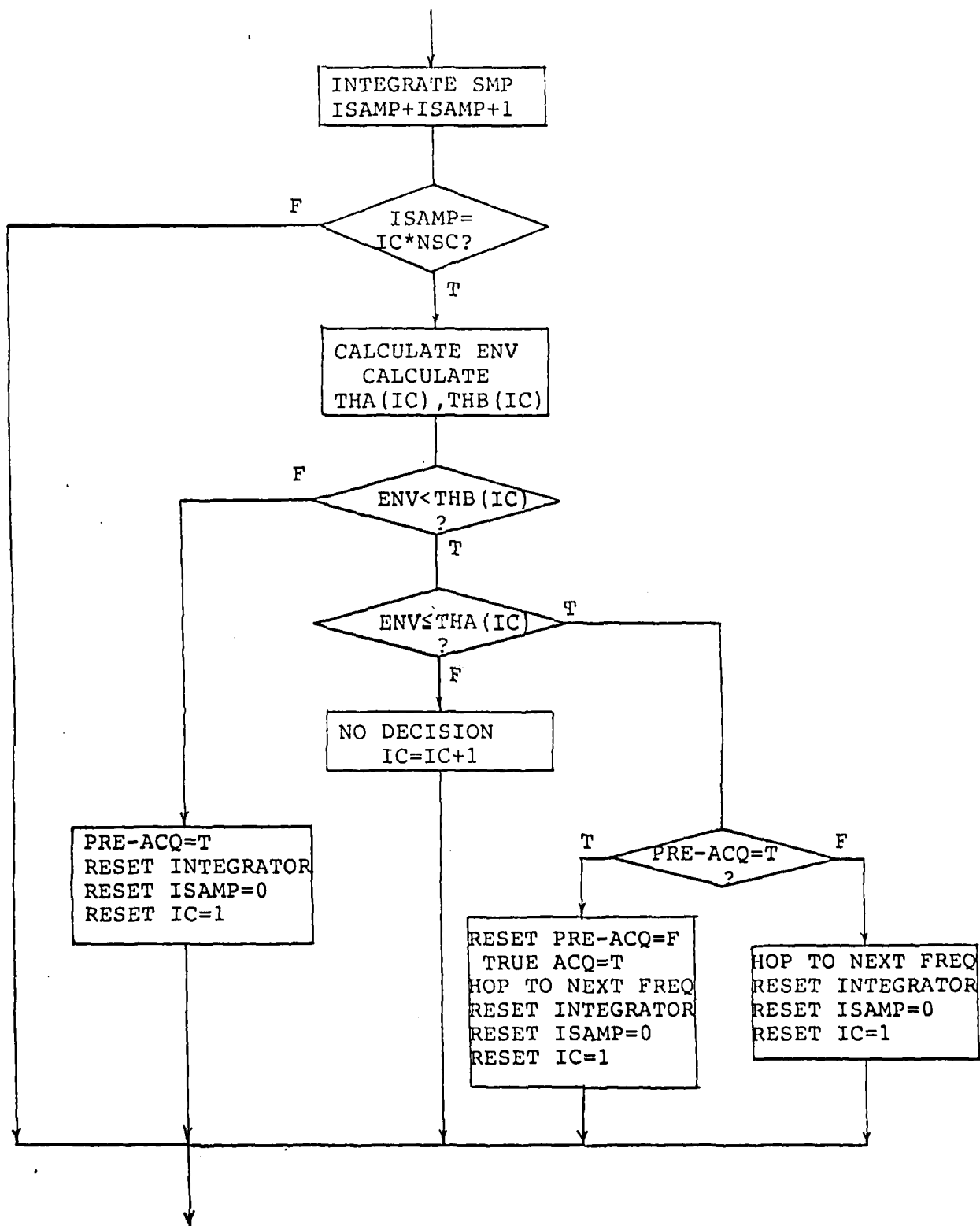


Fig. 2 ACQ strategy for fading channel
(300 bits/S, 30 hops/S, spread time=1ms)

Probability density function
of envelope square

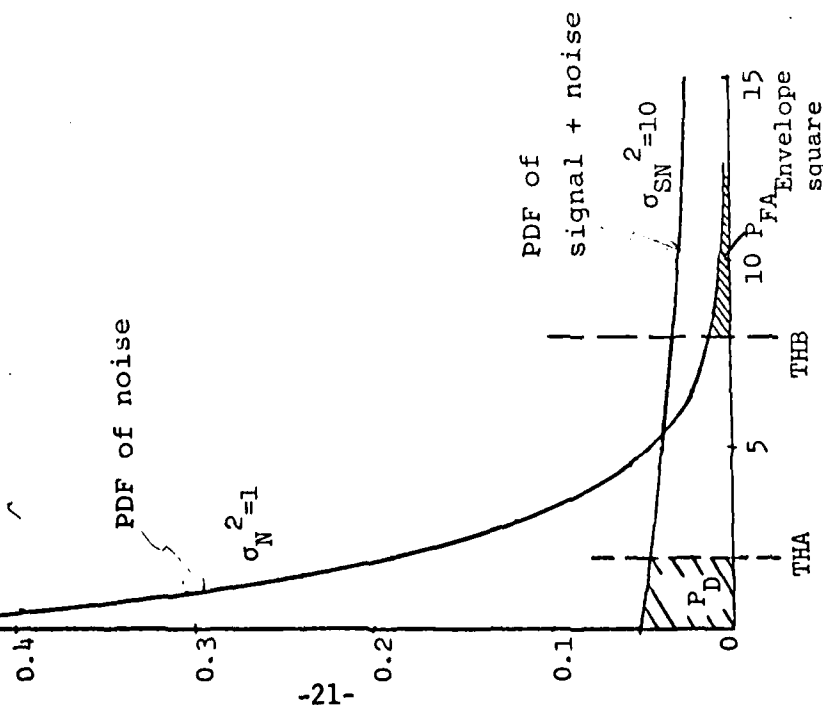


Fig. 4 Prob density function of envelope square, P_{FA} and P_D

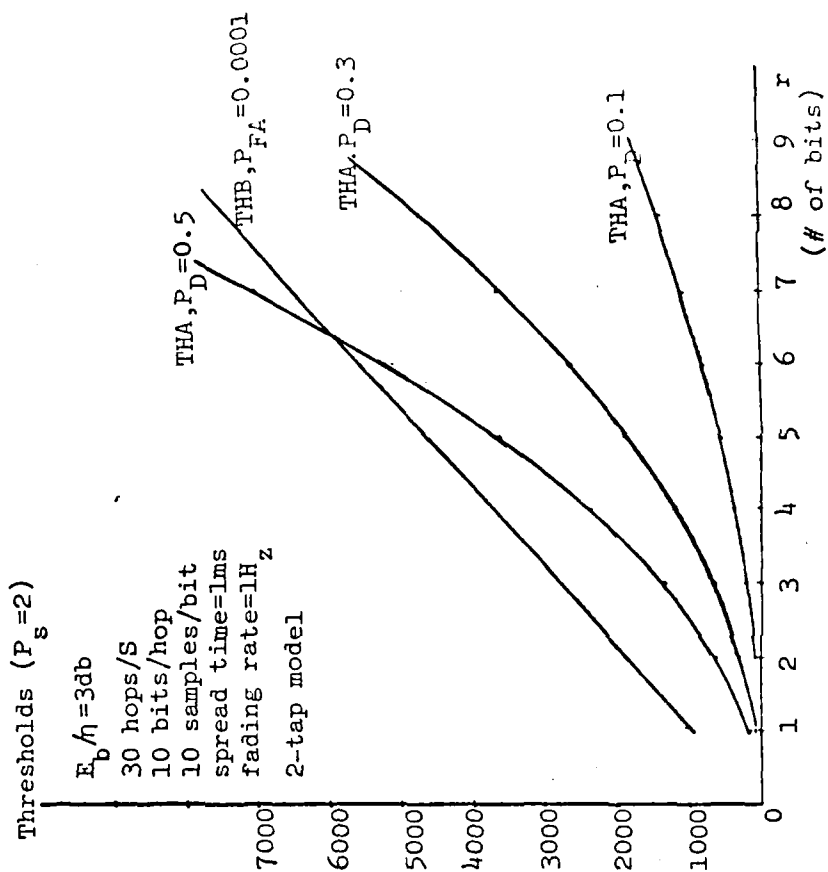


Fig. 5 Thresholds versus NO. of bits to be integrated

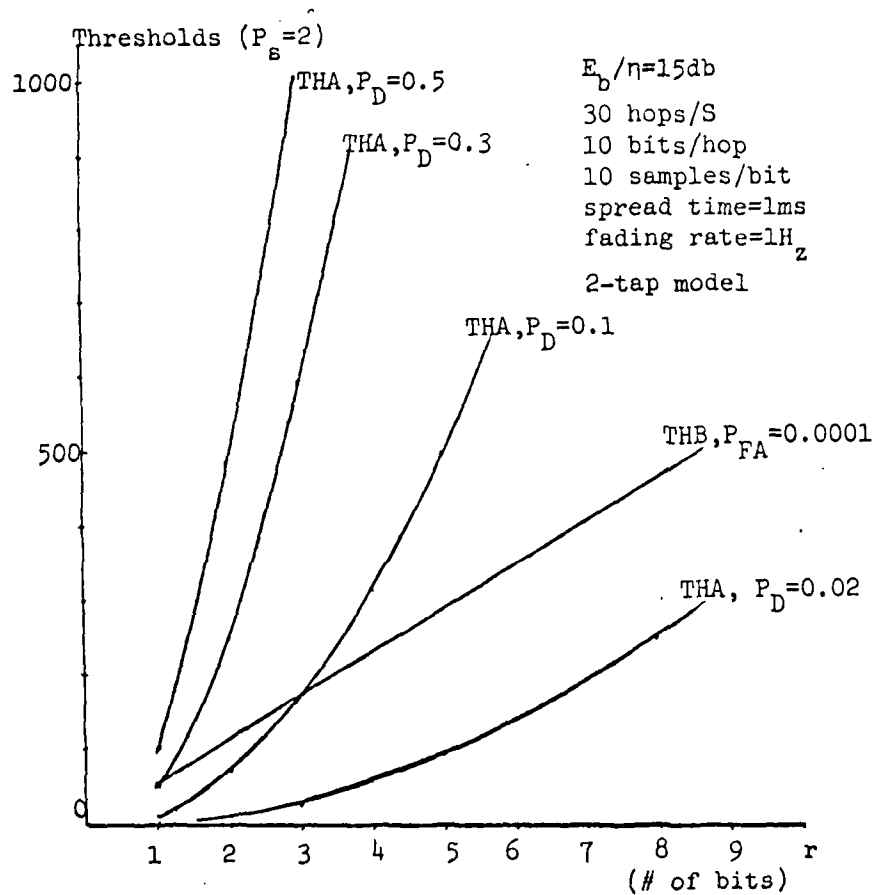
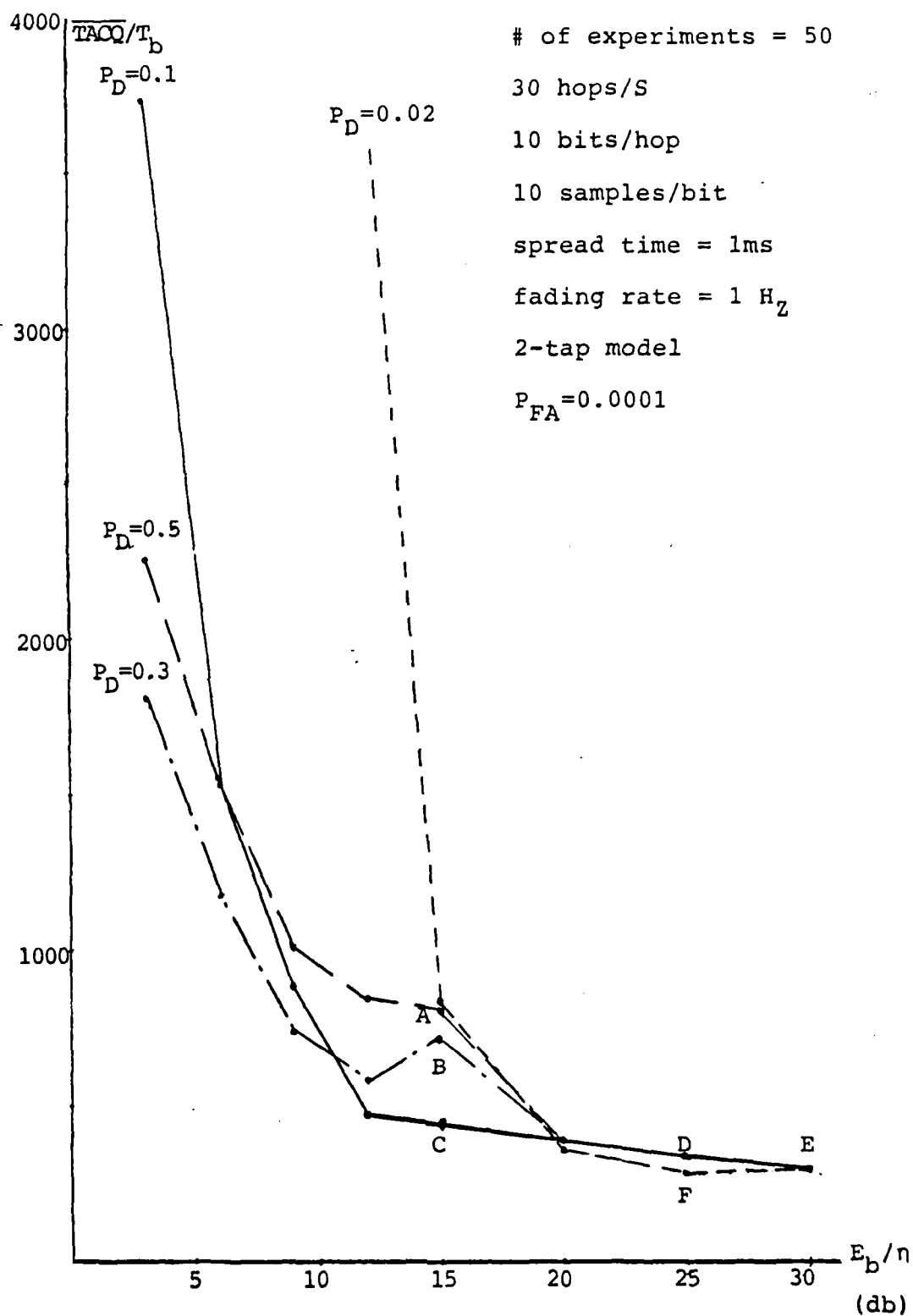


Fig. 6 Thresholds versus NO. of bits to be integrated

Fig. 7 Mean acquisition time versus E_b/η

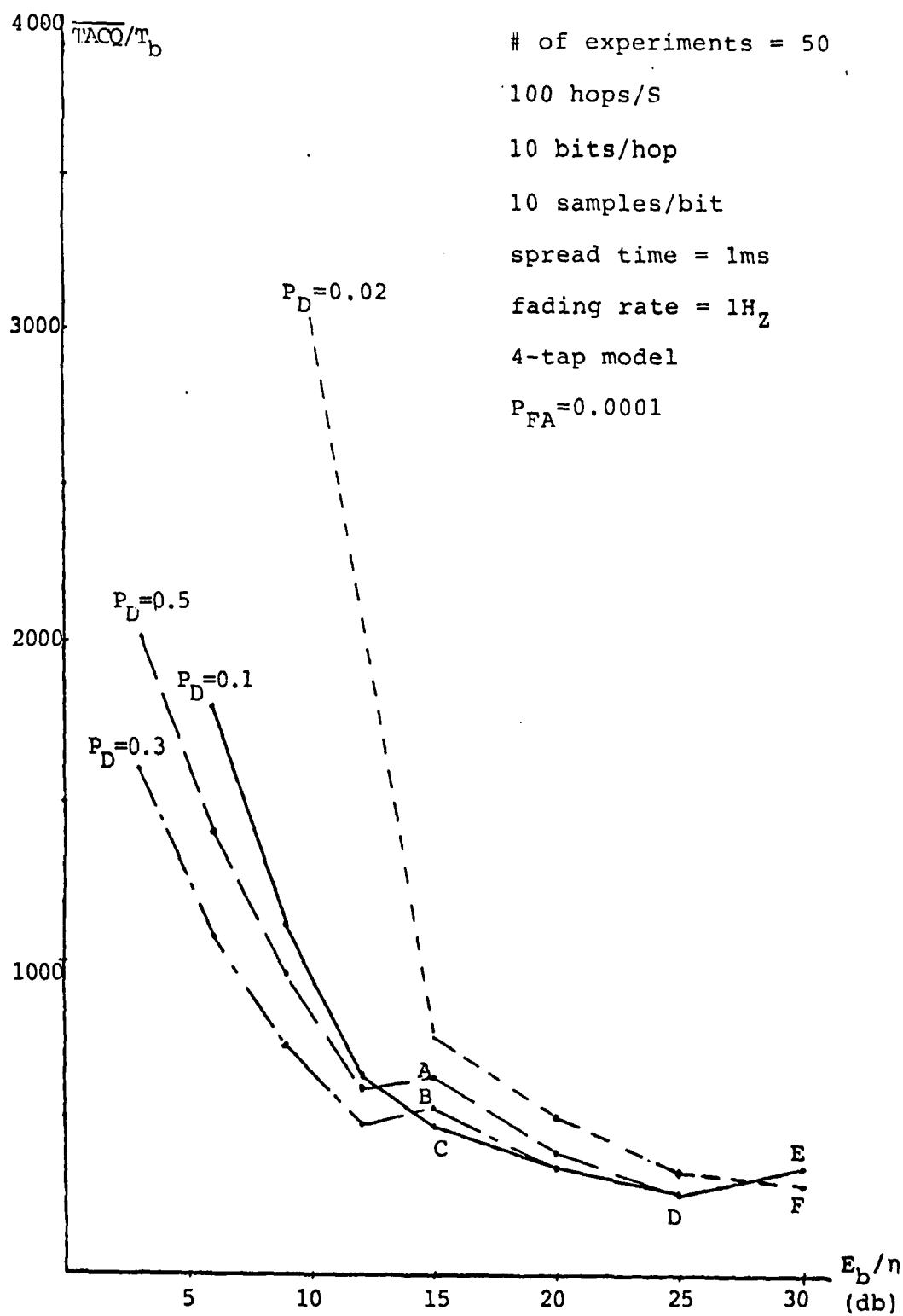


Fig. 8 Mean acquisition time versus E_b/n

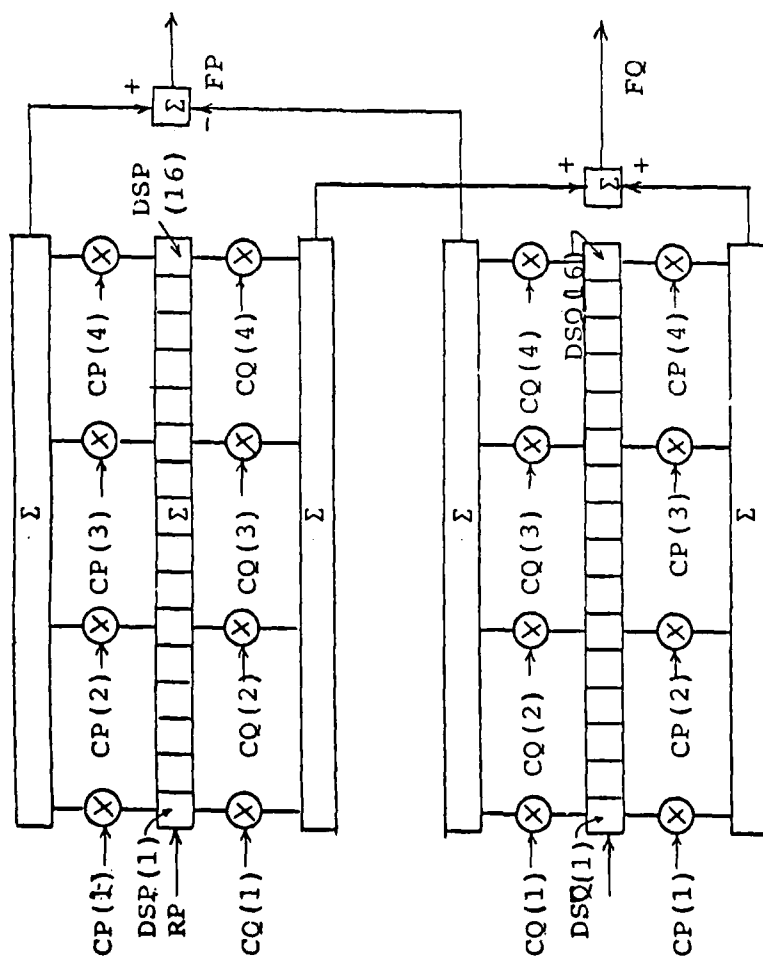


Fig. 9 Model of fading channel

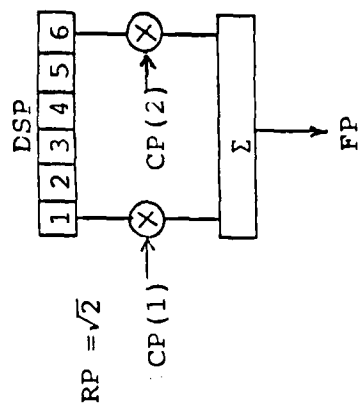


Fig. 10 Fading channel using FSK

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