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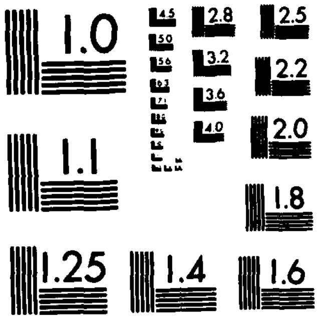

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THE DERIVATION OF THE MAPPING EQUATIONS AND DISTORTION FORMULAE FOR THE SATELLITE TRACKING MAP PROJECTIONS

by

Gregory Cote Arnold

Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

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in

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Rigorous derivations of the Satellite Tracking conic and cylindrical projections are presented. The first fundamental quantities of the projection surface parameters as functions of spherical earth parameters are developed. Using these quantities, newly derived in this paper, general equations for the distortion in length, area, and azimuth are developed. Examples of the graticule and distortion values are given for the Landsat orbit.

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1. INTRODUCTION

The growing use of remotely sensed earth imagery has prompted interest in a new type of map projection, one in which some relationship between satellite position and the earth is preserved during the transformation, rather than the preservation of some feature or property of the earth itself. A good example is the Space Oblique Mercator described by Snyder (1978) on which conformality is preserved along a satellite groundtrack.

The Satellite Tracking projections have the property of portraying all groundtracks of a particular satellite as straight lines. The advantage of this is that, once the graticule is constructed, knowing the location on the sphere of any subpoint allows the entire revolution to be drawn.

1.1 Background

In his article "Map Projections for Satellite Tracking", Snyder (1981) introduces the cylindrical and conic Satellite Tracking projections. The article briefly discusses the concept of satellite apparent longitude from which the relationships between the revolving satellite and the rotating earth are derived, and includes the development of the mapping equations and examples of the graticules of both projections.

Although the mapping equations are developed, the complete relationship between coordinates on the sphere and coordinates on the projection surface is not. To define this relationship, and subsequently derive general formulae for expressing various types of distortion on the projection surface, the fundamental quantities of each projection must be known. These are not determined by Snyder.

1.2 Objective

The objective of this thesis is the derivation of general distortion formulae for the Satellite Tracking map projections. This is accomplished by first deriving the fundamental quantities of the projections which relate the change in ϕ,λ on the sphere to the change in the projection parameters on the plane. From quantities, general equations for length, area, and angular distortion are derived. The equations used in the computation of the fundamental quantities and distortion formulae are given in Pearson (1977).

1.3 Scope

After a brief explanation of terms and concepts as given in Richardus and Adler (1972), a complete rederivation of the two projections is presented. The derivations are included here for two reasons: first, each step is completely explained, which should make them easier to follow than the original; and second, the resulting mapping equations are the basis for the determination of the distortion formulae.

In the chapter on distortion analysis, the fundamental quantities and distortion equations are derived. Finally, numerical examples using the Landsat orbit are given.

Changes in the definition of some variables will affect the appearance of several equations. Specifically, motion of the satellite along its orbit will be calculated from the ascending node, the customary point of reference, rather than the descending node. Also, Snyder refers to a non-rotating earth which he later allows to rotate to complete his description of satellite motion. Here, the non-rotating earth concept is replaced by referencing the satellite to an astronomical coordinate system.

Three conditions are assumed in the derivation of the mapping equations

1. The earth is spherical and of uniform mass,
2. the orbit is circular, i.e., satellite velocity is constant,
3. the rotation axes of the selected astronomical and earth-fixed coordinate systems are coincident.

Plotting the groundtrack is equivalent to orbit determination where the above conditions serve as the math model of the orbit. Recognizing the limitations of such a crude model, extrapolation on any groundtrack should be restricted to the revolution containing the known satellite subpoint.

1.4 Satellite Apparent Longitude

Two types of longitude are used in the derivation of the mapping equations: first, earth-fixed, or geodetic, longitude (λ), and second, satellite apparent longitude (λ'), defined in section 3.1.3. The meridians on the projection represent geodetic longitude. However, in order to compute the parallel spacing necessary to allow straight line groundtracks, the motion of the satellite must be taken into consideration. Therefore, the derivation of the mapping equations for ϕ will be based on λ' instead of λ.

2. MAP PROJECTION CONCEPTS

2.1 The Projections

The transformations are applied to positions on a spherical earth of radius R, defined in the latitude, longitude (ϕ,λ) coordinate system. The two projection surfaces, the cylinder and the cone, are oriented relative to the sphere in the normal aspect, i.e., their axes coincide with the polar axis of the earth. The plane coordinate systems are chosen such that the parametric lines of that system correspond with the projection of the parametric lines of the sphere. On the cylinder, where the graticule is composed of perpendicular straight lines, the Cartesian (x,y) system is used. On the cone, where meridians are straight lines converging at its apex, and parallels are arcs of concentric circles, the (ρ,θ) system is used. The line of contact between the projection surface and the sphere is a parallel of latitude along which scale is true. A secant cone or cylinder will have two standard parallels, and a tangent surface will have one.

2.2 Distortion On Map Projections

Regardless of the type of transformation, all of the relationships existing on the sphere cannot be duplicated on the plane. Any projection will contain distortions in distance, direction, size or shape. The Satellite Tracking projections sacrifice some desirable properties in order that satellite groundtracks are preserved as straight lines.

2.2.1 Length distortion

Length, or scale, distortion is a measure of the difference in len of a line on the projection surface compared with the length of th same line on a sphere which has been reduced to map scale. It is usually expressed as a scale factor, the ratio of the two distance

$$\text{Scale factor} = \frac{\text{distance on the plane}}{\text{distance on the sphere}}$$

The scale factor along a standard parallel or meridian, where scal true, is 1. A scale factor greater than 1 indicates an increase i length on the projection; a value less than 1 indicates a decrease in length.

On most projections, the scale factor is dependent on the azimuth line. Formulae to be developed here are for distortion along the meridians (Mm) and along the parallels (Mp).

Conformality is the property which maintains the shapes of differe tially small areas on a projection. This means that, although the magnitude of the scale distortion changes across the map, at any p

$$Mm = Mp.$$

This condition will be used in the derivation of the mapping equat

2.2.2 Area distortion

Area distortion (Da), also expressed as a ratio, compares an area the map with the same area on the earth at map scale.

$$Da = \frac{\text{area on the plane}}{\text{area on the sphere}}$$

Because the parametric curves of the two plane coordinate systems are orthogonal, the area bounded by two differentially close pairs of curves is the product of the lengths of the lines separating them.

$$Da = Mm\ Mp$$

2.2.3 Angular distortion

Angular distortion is measured by comparing an azimuth on the sphere to its projected azimuth on the plane. For the Satellite Tracking projections, it is a function of azimuth and latitude. Angular distortion will be zero, i.e., sphere azimuth equals plane azimuth, wherever the condition of conformality is satisfied; otherwise, some distortion will be present.

At each point on the sphere there exists a pair of perpendicular lines that will remain perpendicular during the transformation; along these lines, angular distortion is zero.

3. THE DEVELOPMENT OF THE MAPPING EQUATIONS

3.1 Elements Of The Satellite Orbit

Unlike projections which deal only with parameters of the earth-fixed coordinate system, the Satellite Tracking projections must take into account the motions of satellite revolution and earth rotation. This is because, under the above assumptions, the satellite orbital plane is stationary with respect to the stars, with its earth orientation constantly changing due to earth rotation. Because the transformation between these two systems is a rotation about the polar axis, only longitude is affected.

To develop the projections, satellite motion along its orbit must be defined in terms of earth-fixed coordinates (ϕ,λ). This is done by first defining the orbit in astronomical coordinates (Φ,Λ), and then applying the transformation to (ϕ,λ). The elements of the orbit are shown in Figure 1.

3.1.1 Distance along the satellite track as a function of ϕ

Applying the sine law to triangle ACD in Figure 1 gives the relationship between the angular distance traveled by the satellite (α) as a function of ϕ and the inclination of the orbital plane (ι).

$$\sin\alpha = \frac{\sin\Phi}{\sin\iota} \tag{1}$$

Under the conditions imposed for the derivation, astronomical latitude equals geodetic latitude. Therefore, $\Phi = \phi$, and equation (1) may be

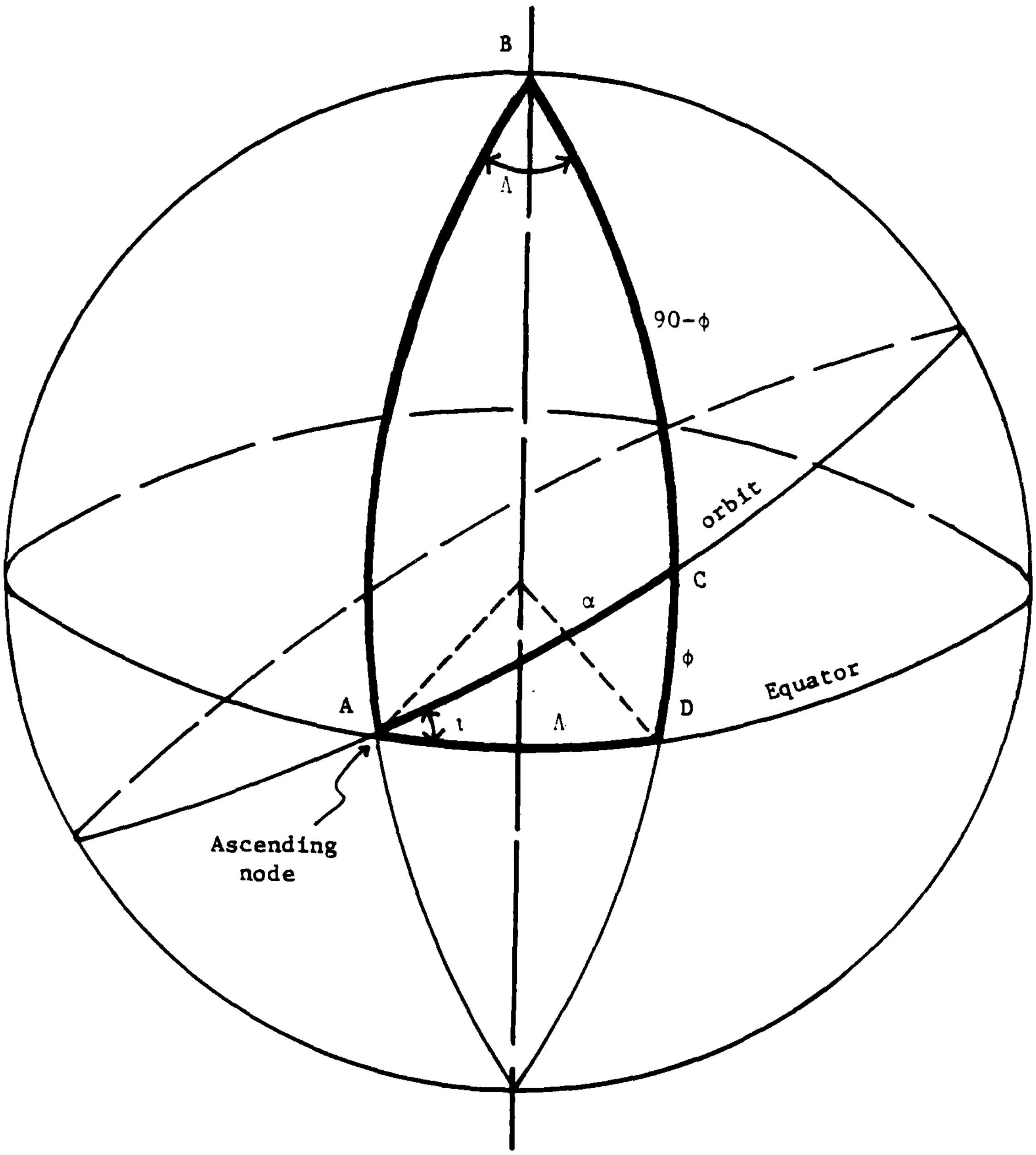

Figure 1. Elements of the satellite orbit.

rewritten as

$$\sin\alpha = \frac{\sin\phi}{\sin\iota} \tag{2}$$

3.1.2 Distance along the satellite track as a function of Λ

Applying the sine law to triangle ABC, and the cosine law to triangle ACD in Figure 1 provide the following relationships. In both cases, Λ is measured from the ascending node.

$$\sin\alpha = \frac{\cos\Phi \sin\Lambda}{\cos\iota} \tag{3}$$

$$\cos\alpha = \cos\Phi \cos\Lambda \tag{4}$$

Dividing equation (3) by (4) eliminates Φ and expresses α as a function of Λ only.

$$\tan\alpha = \frac{\tan\Lambda}{\cos\iota} \tag{5}$$

3.1.3 Satellite apparent longitude

The geodetic longitude of a point beneath the satellite orbit is equal to the astronomical longitude plus the combined effects of satellite and earth motion on earth-fixed longitude ($\Delta\lambda$).

$$\lambda' = \Lambda + \Delta\lambda \tag{6}$$

where the primed λ indicates longitude of the satellite, known as satellite apparent longitude. The angular distance along the orbit covered over time t is

$$\alpha = \frac{2\pi t}{p} \tag{7}$$

where p is the satellite period of revolution. During the same interval, the earth-fixed longitude of the satellite subpoint changes by

$$\Delta\lambda = \frac{-2\pi t}{P} \tag{8}$$

where P is the earth period of rotation. $\Delta\lambda$ is negative because the earth rotates in the positive λ direction, therefore, the longitude of the point is always decreasing. Equate (7) and (8) to obtain

$$\Delta\lambda = -\frac{p}{P}\alpha \tag{9}$$

Substitute equation (9) into (6) to obtain the final form of satellite apparent longitude

$$\lambda' = \Lambda - \frac{p}{P}\alpha \tag{10}$$

3.1.4 The relationship between λ' and ϕ

Groundtracks on these projections will be constrained as straight lines by spacing the parallels as a linear function of the satellite apparent longitude as computed in equation (10). To do this, $d\lambda'/d\phi$ is required. Differentiating equations (10), (2), and (5)

$$\frac{d\lambda'}{d\phi} = \frac{d\Lambda}{d\phi} - \frac{p}{P}\frac{d\alpha}{d\phi} \tag{11}$$

$$\frac{d\alpha}{d\phi} = \frac{\cos\phi}{\sin\iota\,\cos\alpha} \tag{12}$$

$$\frac{d\Lambda}{d\phi} = \frac{\sec^2\alpha\,\cos\iota}{\sec^2\Lambda}\frac{d\alpha}{d\phi} \tag{13}$$

Substituting (12) and (13) into (11)

$$\frac{d\lambda'}{d\phi} = \left(\frac{\sec^2\alpha \cos\iota}{\sec^2\Lambda} - \frac{p}{P}\right)\frac{\cos\phi}{\sin\iota \cos\alpha}$$

$$\frac{d\lambda'}{d\phi} = \left(\frac{\cos\iota}{\cos^2\alpha\ (1 + \tan^2\Lambda)} - \frac{p}{P}\right)\frac{\cos\phi}{\sin\iota \cos\alpha}$$

Reducing terms in the last equation:

$$\begin{aligned}
\cos^2\alpha\ (1 + \tan^2\Lambda) &= \cos^2\alpha + \cos^2\alpha \tan^2\Lambda \\
&= \cos^2\alpha + \cos^2\alpha \tan^2\alpha \cos^2\iota && \text{from (5)} \\
&= 1 - \sin^2\alpha \sin^2\iota \\
&= 1 - \frac{\sin^2\phi}{\sin^2\iota}\sin^2\iota && \text{from (2)} \\
&= \cos^2\phi
\end{aligned}$$

$$\begin{aligned}
\sin\iota \cos\alpha &= \sin\iota\left(1 - \frac{\sin^2\phi}{\sin^2\iota}\right)^{\frac{1}{2}} && \text{from (2)} \\
&= (\sin^2\iota - \cos^2\phi)^{\frac{1}{2}} \\
&= (\cos^2\phi - \cos^2\iota)^{\frac{1}{2}}
\end{aligned}$$

Continuing the derivation:

$$\frac{d\lambda'}{d\phi} = \left(\frac{\cos\iota}{\cos^2\phi} - \frac{p}{P}\right)\frac{\cos\phi}{(\cos^2\phi - \cos^2\iota)^{\frac{1}{2}}}$$

$$\frac{d\lambda'}{d\phi} = \frac{\cos\iota - (p/P)\cos^2\phi}{\cos\phi\ (\cos^2\phi - \cos^2\iota)^{\frac{1}{2}}} \qquad (14)$$

Given $d\lambda'$ and $d\phi$ on the sphere, the azimuth (A) of a great circle at latitude ϕ is computed as:

$$\tan A = \frac{d\lambda'}{d\phi}\cos\phi \qquad (15)$$

Substituting the expression for $d\lambda'/d\phi$ from (14) gives the equation expressing groundtrack azimuth on the sphere as a function of the orbit parameters.

$$\tan A = \frac{\cos\iota - (p/P)\cos^2\phi}{(\cos^2\phi - \cos^2\iota)^{1/2}} \tag{16}$$

3.2 The Mapping Equations For The Cylindrical Projection

Formulae for length distortion on the projection surface along the meridians (Mm) and along the parallels (Mp) are defined as:

$$Mm = \frac{1}{R}\frac{dy}{d\phi} \tag{17a}$$

$$Mp = \frac{1}{R\cos\phi}\frac{dx}{d\lambda} \tag{17b}$$

3.2.1 <u>The mapping equation for x</u>

To derive the mapping equation for x, define the scale to be true in the x direction at ϕ_1, the conformal latitude.

$$Mp = 1$$

$$dx = R\cos\phi_1\, d\lambda$$

Integrating with the constraint that $x=0$ when $\lambda=0$ gives the mapping equation for x.

$$x = R\lambda\cos\phi_1 \tag{18}$$

3.2.2 <u>The mapping equation for y</u>

To derive the parallel spacing, impose the condition of conformality at ϕ_1.

$$Mm = Mp$$

$$dy = \frac{dx}{\cos\phi_1 \left(\frac{d\lambda}{d\phi}\right)_{\phi_1}} \quad (19)$$

Integrate equation (19) and substitute from (18) replacing λ with λ' to make y a linear function of the satellite apparent longitude. $d\lambda'/d\phi$ is a constant since it is evaluated at ϕ_1.

$$y = \frac{R\lambda'\cos\phi_1}{\cos\phi_1 \left(\frac{d\lambda'}{d\phi}\right)_{\phi_1}} \quad (20)$$

Because conformality is defined at ϕ_1, the groundtrack azimuth at ϕ_1 on the map (dx/dy) should equal the azimuth on the globe (A). Rearranging equation (19), again using λ' in place of λ yields

$$\frac{dx}{dy} = \left(\frac{d\lambda'}{d\phi}\right)_{\phi_1} \cos\phi_1$$

This agrees with the value of tanA in (15), therefore

$$\left(\frac{d\lambda'}{d\phi}\right)_{\phi_1} = \frac{\tan A_1}{\cos\phi_1}$$

Substituting into equation (20) gives the mapping equation for y.

$$y = \frac{R\lambda'\cos\phi_1}{\tan A_1} \quad (21)$$

3.3 The Mapping Equations For The Conic Projection

3.3.1 The mapping equation for θ

Figure 2 shows the elements of the projection to the plane in the (ρ,θ)

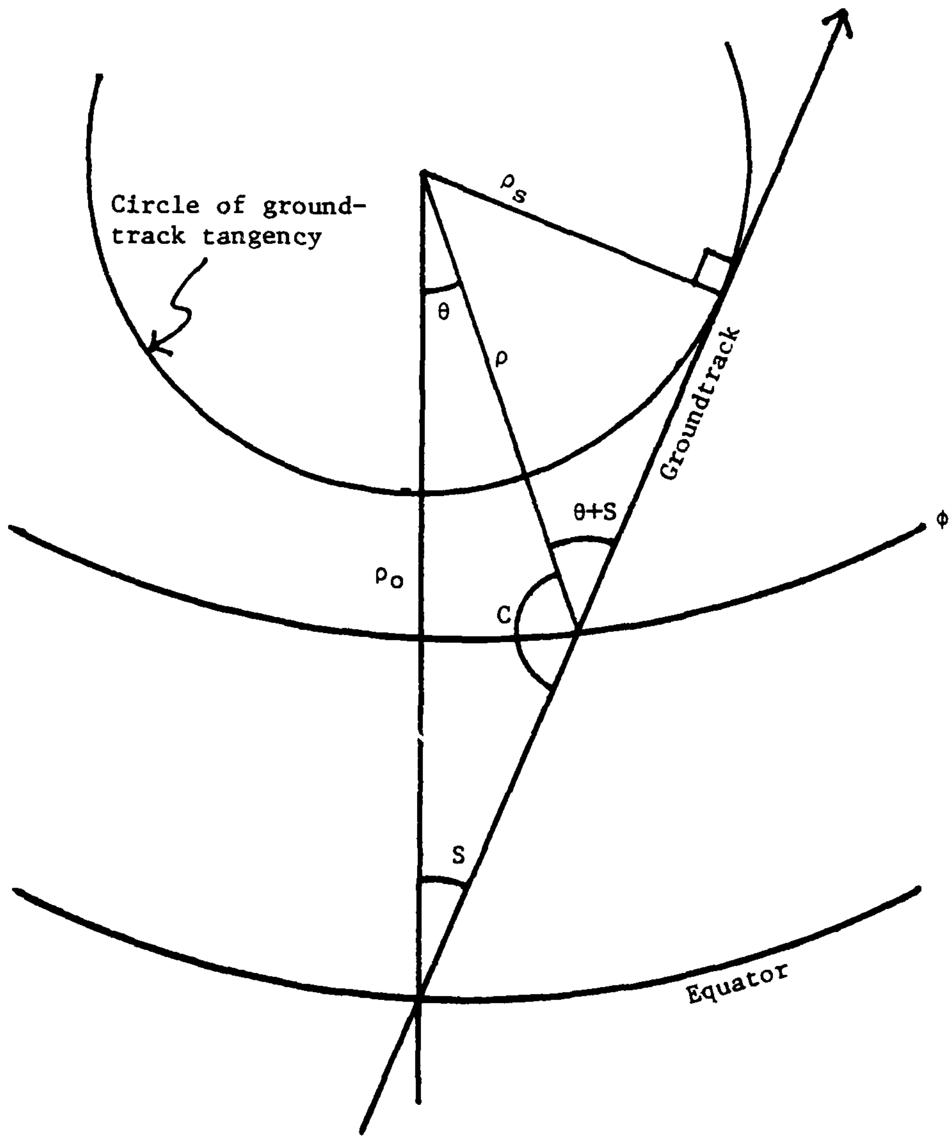

Figure 2. Elements of the conic projection to the plane.

coordinate system. As with conventional conics in the normal aspect, the mapping equation for θ is

$$\theta = n\lambda \tag{22a}$$

In the derivation of ρ, the satellite apparent longitude is substituted for earth-fixed longitude.

$$\theta = n\lambda' \tag{22b}$$

where

n is the constant of the cone, and

θ is the angular separation of the meridians on the projection surface.

3.3.2 The mapping equation for ρ

From the geometry of the plane triangle in Figure 2

$$\text{angle } C = 180° - (\theta + S)$$

where S is the azimuth of the groundtrack at the equator on the projection surface. Applying the sine law gives a relationship between ρ and ρ_o, the radius from the apex of the cone to the equator.

$$\rho = \frac{\rho_o \sin S}{\sin(\theta + S)} \tag{23}$$

To obtain the mapping equation for ρ, evaluate the length ρ_o at a conformal latitude and substitute into equation (23).

Formulae for length distortion in the (ρ, θ) system are

$$Mm = -\frac{1}{R}\frac{d\rho}{d\phi} \tag{24a}$$

$$Mp = \frac{\rho}{R\cos\phi}\frac{d\theta}{d\lambda} \qquad (24b)$$

The minus sign in Mm indicates that ρ decreases as ϕ increases.

Differentiating equation (22b) with respect to ρ and λ' yields

$$\frac{d\theta}{d\lambda'} = n \qquad (25)$$

$$\frac{d\theta}{d\phi} = n\,\frac{d\lambda'}{d\phi}$$

Substituting the value of $d\lambda'/d\phi$ from equation (15)

$$\frac{d\theta}{d\phi} = \frac{n\ \tan A}{\cos\phi} \qquad (26)$$

Imposing conformality at ϕ_1 produces an expression for the length ρ_1.

$$Mm = Mp$$

$$\rho_1 = -\cos\phi_1\left(\frac{d\lambda'}{d\theta}\right)\left(\frac{d\rho}{d\phi}\right)_{\phi_1}$$

Substituting ρ from equation (23) and $d\lambda'/d\theta$ from (25) yields

$$\rho_1 = -\cos\phi_1\,\frac{1}{n}\,\frac{d}{d\phi}\left(\frac{\rho_o\ \sin S}{\sin(\theta + S)}\right)_{\phi_1}$$

$$\rho_1 = \frac{\cos\phi_1\ \rho_o\ \sin S\ \cos(\theta_1 + S)}{n\ \sin^2(\theta_1 + S)}\,\frac{d}{d\phi}(\theta + S)_{\phi_1}$$

Substituting $d\theta/d\phi$ from equation (26)

$$\rho_1 = \frac{\rho_o\ \sin S\ \tan A_1}{\sin(\theta_1 + S)\ \tan(\theta_1 + S)}$$

Comparing this with equation (23)

$$\tan A_1 = \tan(\theta_1 + S)$$

$$A_1 = \theta_1 + S \tag{27}$$

Substituting equation (27) into (23) evaluated at ϕ_1

$$\rho_1 = \frac{\rho_o \sin S}{\sin A_1} \tag{28}$$

Imposing true scale at ϕ_1 produces a second expression for ρ_1.

$$Mp = 1$$

$$\rho_1 = R\cos\phi_1 \frac{d\lambda'}{d\theta}$$

$$\rho_1 = \frac{R\cos\phi_1}{n} \tag{29}$$

Substitute equation (29) into (28) to eliminate ρ_1 and obtain the equation for ρ_o.

$$\rho_o = \frac{R\cos\phi_1 \sin A_1}{n \sin S} \tag{30}$$

Substitute equation (30) into (23) to eliminate ρ_o and get the mapping equation for ρ.

$$\rho = \frac{R\cos\phi_1 \sin A_1}{n \sin(\theta + S)} \tag{31}$$

3.3.3 <u>The constant of the cone for a projection with two conformal latitudes</u>

Equation (27) gives the value of θ at the conformal latitudes ϕ_1 and ϕ_2.

$$\theta_1 = A_1 - S$$

$$\theta_2 = A_2 - S$$

Substituting into equation (22b)

$$\theta_1 = n\lambda_1' = A_1 - S$$

$$\theta_2 = n\lambda_2' = A_2 - S$$

Subtracting:

$$n\lambda_2' - n\lambda_1' = (A_2 - S) - (A_1 - S)$$

$$n = \frac{A_2 - A_1}{\lambda_2' - \lambda_1'} \qquad (32)$$

3.3.4 <u>The constant of the cone for a projection with one conformal latitude</u>

As ϕ_2 approaches ϕ_1, equation (32) becomes

$$n = \frac{dA}{d\lambda'} = \left(\frac{dA}{d\phi}\right)\left(\frac{d\phi}{d\lambda'}\right)_{\phi_1}$$

Differentiating equation (16) gives the expression for $dA/d\phi$.

$$\sec^2 A \frac{dA}{d\phi} = \{(\cos^2\phi - \cos^2\iota)^{\frac{1}{2}} (2(p/P)\sin\phi \cos\phi)$$
$$+ (\cos\iota - (p/P)(\cos^2\phi)(\tfrac{1}{2}(\cos^2\phi - \cos^2\iota)^{-\frac{1}{2}}(-2\sin\phi \cos\phi)\}$$
$$\div (\cos^2\phi - \cos^2\iota)$$

$$\frac{dA}{d\phi} = \frac{\sin\phi \cos\phi \{(p/P)(\cos^2\phi - 2\cos^2\iota) + \cos\iota\}}{(\cos^2\phi - \cos^2\iota)^{3/2} (1 + \tan^2 A)}$$

Substituting this and the value of $d\phi/d\lambda'$ from (15) into the equation for n evaluated at ϕ_1 yields

$$n = \frac{\sin\phi_1 \cos\phi_1 \{(p/P) \cos^2\phi_1 - 2\cos^2\iota) + \cos\iota\}}{(\cos^2\phi_1 - \cos^2\iota)^{3/2} (1 + \tan^2 A_1)} \frac{\cos\phi_1}{\tan A_1}$$

$$n = \frac{\sin\phi_1 \cos^2\phi_1 \{(p/P) \cos^2\phi_1 - 2\cos^2\iota) + \cos\iota\}}{(\cos^2\phi_1 - \cos^2\iota)^{3/2} (\tan A_1 + \tan^3 A_1)}$$

Expanding the denominator using the expression for tanA in (16)

$$(\cos^2\phi_1 - \cos^2\iota)^{3/2} \left(\frac{\cos\iota - (p/P) \cos^2\phi_1}{(\cos^2\phi_1 - \cos^2\iota)^{1/2}} + \frac{(\cos\iota - (p/P) \cos^2\phi_1)^3}{(\cos^2\phi_1 - \cos^2\iota)^{3/2}} \right)$$

Thus

$$n = \frac{\sin\phi_1 \cos^2\phi_1 \{(p/P) (\cos^2\phi_1 - 2\cos^2\iota) + \cos\iota\}}{(\cos^2\phi_1 - \cos^2\iota)(\cos\iota - (p/P)\cos^2\phi_1)+(\cos\iota - (p/P)\cos^2\phi_1)^3} \quad (\text{:}$$

The equation for n appearing in Snyder (1981) contains additional expansions of the denominator. See Appendix A for a continuation of the derivation.

3.3.5 The constant of the cone when the conformal latitude is the tracking limit

The tracking limit, the highest latitude reached by the satellite equal to the inclination of the orbit. To determine the value of replace ϕ_1 with ι in equation (33).

$$n = \frac{\sin\iota \cos^2\iota (\cos\iota - (p/P) \cos^2\iota)}{(\cos\iota - (p/P) \cos^2\iota)^3}$$

$$n = \frac{\sin\iota}{(1 - (p/P) \cos\iota)^2} \quad ($$

3.3.6 The circle of groundtrack tangency

If the projection is to be constructed manually, the plotting of groundtracks is made easier by computing the circle of tangency. is a circle of radius ρ_s to which all groundtracks are tangent.

From the geometry of the plane triangle in Figure 2

$$\rho_S = \rho_O \sin S$$

Substituting the value of ρ_O from equation (30)

$$\rho_S = \frac{R\cos\phi_1 \sin A_1}{n} \tag{35}$$

The direction of the groundtracks is obtained from the sign of either ρ_S in equation (35) or S in equation (27). A negative sign indicates a west azimuth when the satellite is travelling northward, or east azimuth when it's going southward.

Equation (35) is needed to compute ρ_S because equation (31) breaks down when ϕ equals the tracking limit. Because parallel spacing was derived based on the satellite groundtrack, radii poleward of the tracking limit are undefined, i.e., the sine of α computed in equation (2) is greater than unity.

4. DISTORTION ANALYSIS

The Satellite Tracking projections were developed by defining the relationship between parameters on the projection surface and parameters on the sphere at specific locations, i.e., conformality and true scale at one or two latitudes. Now that the mapping equations have been developed, general equations describing the distortion properties of the projections can be computed from fundamental quantities developed using the following transformation matrix.

$$\begin{bmatrix} E \\ F \\ G \end{bmatrix} = \begin{bmatrix} \frac{du^2}{d\phi^2} & 2\frac{du}{d\phi}\frac{dv}{d\phi} & \frac{dv^2}{d\phi^2} \\ \frac{du}{d\phi}\frac{du}{d\lambda} & \frac{du}{d\phi}\frac{dv}{d\lambda} + \frac{du}{d\lambda}\frac{dv}{d\phi} & \frac{dv}{d\lambda}\frac{dv}{d\phi} \\ \frac{du^2}{d\lambda^2} & 2\frac{du}{d\lambda}\frac{dv}{d\lambda} & \frac{dv^2}{d\lambda^2} \end{bmatrix} \begin{bmatrix} E' \\ F' \\ G' \end{bmatrix}$$

where

u,v are the parametric quantities on the projection surface, either (x,y) or (ρ,θ),

ϕ,λ are the parameters on the sphere,

E', F', G' are the fundamental quantities of u,v with respect to the projection surface,

E, F, G are the fundamental quantities of u,v with respect to the sphere.

Also used in the distortion analysis are e, f, and g, the fundamental quantities of ϕ,λ with respect to the sphere.

The following fundamental quantities for the sphere and plane (projection surface) are known through the equations defining differential length

ϕ,λ on the sphere: $ds^2 = R^2(d\phi)^2 + R^2\cos^2\phi\ (d\lambda)^2$

$$e = R^2 \qquad f = 0 \qquad g = R^2\cos^2\phi$$

x,y on the plane: $dS^2 = (dx)^2 + (dy)^2$

$$E' = 1 \qquad F' = 0 \qquad G' = 1$$

ρ,θ on the plane: $dS^2 = (d\rho)^2 + \rho^2(d\theta)^2$

$$E' = 1 \qquad F' = 0 \qquad G' = \rho^2$$

4.1 Cylindrical Projection Distortion Equations

4.1.1 <u>The first fundamental quantities</u>

Recalling equations (15), (18), and (21), and evaluating their partials

$$\frac{d\lambda'}{d\phi} = \frac{\tan A}{\cos\phi} \tag{15}$$

$$x = R\lambda\cos\phi_1 \tag{18}$$

$$y = \frac{R\lambda'\cos\phi_1}{\tan A_1} \tag{21}$$

$$\frac{dx}{d\phi} = 0$$

$$\frac{dx}{d\lambda} = R\cos\phi_1$$

$$\frac{dy}{d\phi} = \frac{R\cos\phi_1}{\tan A_1}\frac{d\lambda'}{d\phi} = \frac{R\cos\phi_1}{\tan A_1}\frac{\tan A}{\cos\phi}$$

$$\frac{dy}{d\lambda} = 0$$

Substitute the known values into the transformation matrix.

$$\begin{bmatrix} E \\ F \\ G \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dfrac{R^2\cos^2\phi_1 \tan^2 A}{\tan^2 A_1 \cos^2\phi} \\ 0 & \dfrac{R^2\cos^2\phi_1 \tan A}{\tan A_1 \cos\phi} & 0 \\ R^2\cos^2\phi_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$E = \frac{R^2\cos^2\phi_1 \tan^2 A}{\tan^2 A_1 \cos^2\phi} \qquad (36a)$$

$$F = 0 \qquad (36b)$$

$$G = R^2\cos^2\phi_1 \qquad (36c)$$

These are the fundamental quantities of the cylindrical projection.

4.1.2 Scale distortion in the meridians and parallels

$$Mm = \left(\frac{E}{e}\right)^{1/2}$$

$$= \left(\frac{\dfrac{R^2\cos^2\phi_1 \tan^2 A}{\tan^2 A_1 \cos^2\phi}}{R^2}\right)^{1/2}$$

$$Mm = \frac{\cos\phi_1 \tan A}{\tan A_1 \cos\phi} \qquad (37)$$

$$Mp = \left(\frac{G}{g}\right)^{1/2}$$

$$= \left(\frac{R^2\cos^2\phi_1}{R^2\cos^2\phi}\right)^{1/2}$$

$$Mp = \frac{\cos\phi_1}{\cos\phi} \qquad (38)$$

4.1.3 Area distortion

$$Da = \left(\frac{EG - F^2}{eg - f^2}\right)^{1/2}$$

$$= \left(\frac{\dfrac{R^4\cos^4\phi_1 \tan^2 A}{\tan^2 A_1 \cos^2\phi}}{R^4\cos^2\phi}\right)^{1/2}$$

$$Da = \frac{\cos^2\phi_1 \tan A}{\tan A_1 \cos^2\phi} \qquad (39)$$

4.1.4 Angular distortion

A derivation of the ω to Ω equation is given in Appendix B.

$$\cos\Omega = \frac{\sqrt{E}\cos\phi \cot\omega}{(E\cos^2\phi \cot^2\omega + G)^{1/2}}$$

$$= \frac{\dfrac{R\cos\phi_1 \tan A \cos\phi \cot\omega}{\tan A_1 \cos\phi}}{\left(\dfrac{R^2\cos^2\phi_1 \tan^2 A \cos^2\phi \cot^2\omega}{\tan^2 A_1 \cos^2\phi} + R^2\cos^2\phi_1\right)^{1/2}}$$

$$\cos\Omega = \frac{\tan A \cot\omega}{(\tan^2 A \cot^2\omega + \tan^2 A_1)^{1/2}} \qquad (40)$$

4.2 Conic Projection Distortion Equations

4.2.1 The first fundamental quantities

Recalling equations (15), (22a), (22b), and (31) and evaluating their partials:

$$\frac{d\lambda'}{d\phi} = \frac{\tan A}{\cos\phi} \qquad (15)$$

$$\theta = n\lambda \qquad (22a)$$

$$\theta = n\lambda' \qquad (22b)$$

$$\rho = \frac{R\cos\phi_1 \sin A_1}{n \sin(\theta + S)} \tag{31}$$

$$\frac{d\theta}{d\phi} = 0$$

$$\frac{d\theta}{d\lambda} = n$$

$$\frac{d\rho}{d\phi} = \frac{R\cos\phi_1 \sin A_1}{n} \frac{d}{d\phi}\left(\frac{1}{\sin(\theta + S)}\right)$$

$$= \frac{-R\cos\phi_1 \sin A_1 \cos(\theta + S)}{n \sin^2(\theta + S)} \frac{d}{d\phi}(\theta + S)$$

Because the mapping equation for ρ was developed using satellite apparent longitude, the value for θ in $d\rho/d\phi$ is gotten from equation (22b). Substituting from (22b) and (15) yields

$$\frac{d}{d\phi}(\theta + S) = \frac{d}{d\phi}(n\lambda') + \frac{dS}{d\phi} = n \frac{d\lambda'}{d\phi} = \frac{n \tan A}{\cos\phi}$$

$$\frac{d\rho}{d\phi} = \frac{-R\cos\phi_1 \sin A_1 \tan A}{\cos\phi \sin(\theta + S) \tan(\theta + S)}$$

$$\frac{d\rho}{d\lambda} = 0$$

Substituting the partials into the transformation matrix

$$\begin{bmatrix} E \\ F \\ G \end{bmatrix} = \begin{bmatrix} \frac{R^2\cos^2\phi_1 \sin^2 A_1 \tan^2 A}{\cos^2\phi \sin^2(\theta + S) \tan^2(\theta + S)} & 0 & 0 \\ 0 & \frac{-n\, R\cos\phi_1 \sin A_1 \tan A}{\cos\phi \sin(\theta + S) \tan(\theta + S)} & 0 \\ 0 & 0 & n^2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \rho^2 \end{bmatrix}$$

$$E = \frac{R^2\cos^2\phi_1 \sin^2 A_1 \tan^2 A}{\cos^2\phi \sin^2(\theta + S) \tan^2(\theta + S)} \tag{41a}$$

$$F = 0 \tag{41b}$$

$$G = n^2\rho^2 \tag{41c}$$

Since ρ is known as a function of ϕ, substitute equation (31) into (41c) to obtain the equation for G.

$$G = \frac{R^2\cos^2\phi_1 \sin^2A_1}{\sin^2(\theta + S)} \tag{41d}$$

These are the fundamental quantities of the conic projection.

4.2.2 Scale distortion in the meridians and parallels

$$Mm = \left(\frac{E}{e}\right)^{\frac{1}{2}}$$

$$= \left(\frac{\dfrac{R^2\cos^2\phi_1 \sin^2A_1 \tan^2A}{\cos^2\phi \sin^2(\theta + S) \tan^2(\theta + S)}}{R^2}\right)^{\frac{1}{2}}$$

$$Mm = \frac{\cos\phi_1 \sin A_1 \tan A}{\cos\phi \sin(\theta + S) \tan(\theta + S)} \tag{42}$$

$$Mp = \left(\frac{G}{g}\right)^{\frac{1}{2}}$$

$$= \left(\frac{\dfrac{R^2\cos^2\phi_1 \sin^2A_1}{\sin^2(\theta + S)}}{R^2\cos^2\phi}\right)^{\frac{1}{2}}$$

$$Mp = \frac{\cos\phi_1 \sin A_1}{\cos\phi \sin(\theta + S)} \tag{43}$$

4.2.3 Area distortion

$$Da = \left(\frac{EG - F^2}{eg - f^2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{\dfrac{R^4\cos^4\phi_1 \sin^4 A_1 \tan^2 A}{\cos^2\phi \sin^4(\theta + S) \tan^2(\theta + S)}}{R^4\cos^2\phi} \right)^{\frac{1}{2}}$$

$$Da = \frac{\cos^2\phi_1 \sin^2 A_1 \tan A}{\cos^2\phi \sin^2(\theta + S) \tan(\theta + S)} \qquad (44)$$

4.2.4 Angular distortion

$$\cos\Omega = \frac{\sqrt{E} \cos\phi \cot\omega}{(E \cos^2\phi \cot^2\omega + G)^{\frac{1}{2}}}$$

$$= \frac{\dfrac{R\cos\phi_1 \sin A_1 \tan A \cos\phi \cot\omega}{\cos\phi \sin(\theta + S) \tan(\theta + S)}}{\left(\dfrac{R^2\cos^2\phi_1 \sin^2 A_1 \tan^2 A \cos^2\phi \cot^2\omega}{\cos^2\phi \sin^2(\theta + S) \tan^2(\theta + S)} + \dfrac{R^2\cos^2\phi_1 \sin^2 A_1}{\sin^2(\theta + S)} \right)^{\frac{1}{2}}}$$

$$\cos\Omega = \frac{\tan A \cot\omega}{(\tan^2 A \cot^2\omega + \tan^2(\theta + S))^{\frac{1}{2}}} \qquad (45)$$

5. RESULTS AND ANALYSIS

Inspecting the distortion equations for both projections reveals that they do satisfy the conditions of true scale and conformality imposed during the derivation of the mapping equations.

$$Mm_{\phi_1} = Mp_{\phi_1}$$

$$Mm_{\phi_2} = Mp_{\phi_2}$$

$$Mm_{\phi_1} = 1$$

$$Mp_{\phi_1} = 1$$

$$\Omega_{\phi_1} = \omega_{\phi_1}$$

$$\Omega_{\phi_2} = \omega_{\phi_2}$$

As a further check, $Da = Mm\ Mp$.

The following analysis is illustrated with plots of the various distortions for the Landsat orbit. Tables containing the numerical distortion values appear in Appendix C.

The parameters used in the calculations are

satellite revolution period = 103.267 minutes,

earth rotation period = 1440.0 minutes,

satellite inclination = $99\overset{\circ}{.}092$.

5.1 The Cylindrical Projection

The meridian scale distortion, Figure 3, is a function of the secant of the latitude. However, because the spacing of the parallels is based on

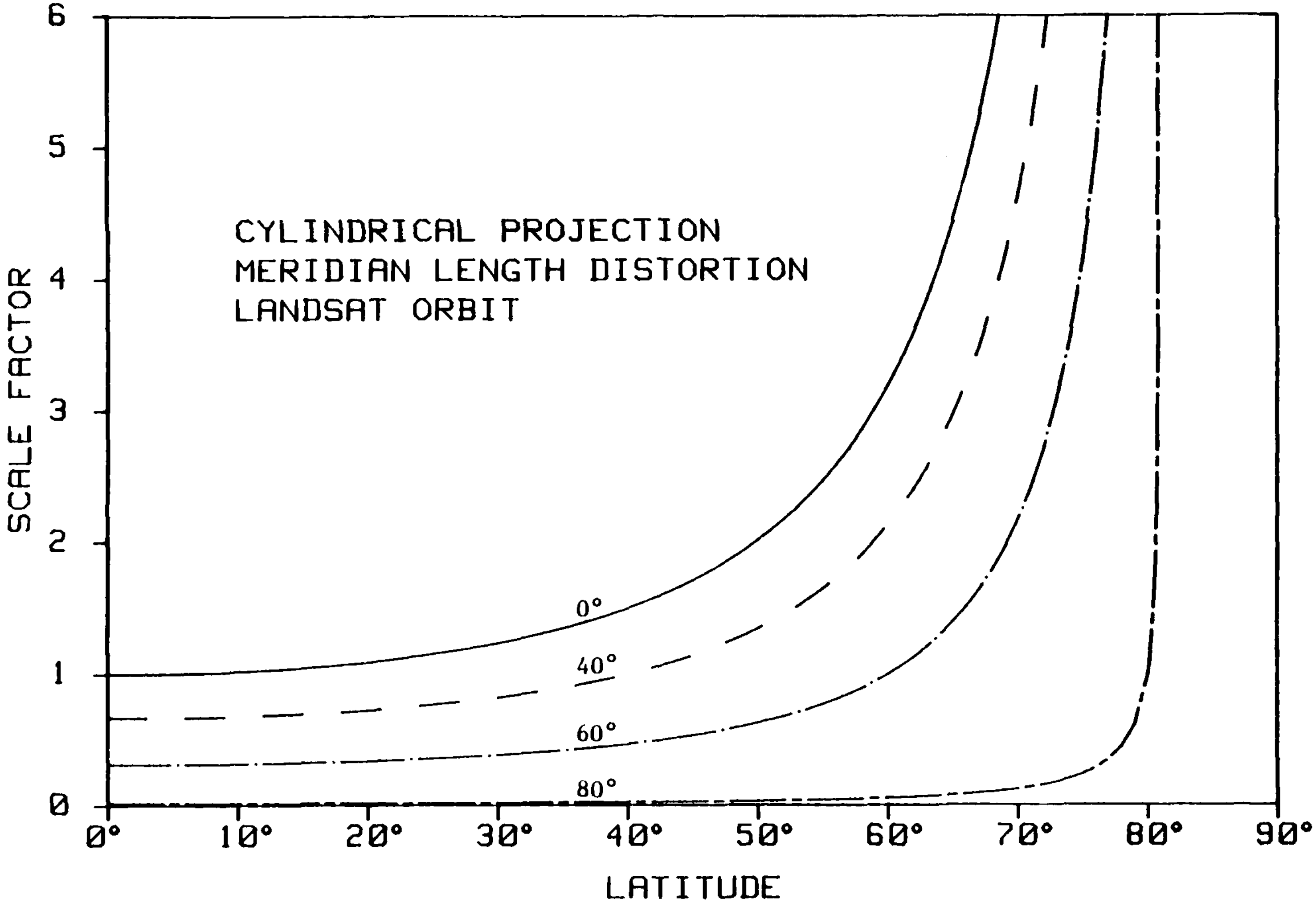

Figure 3. Meridian length distortion at standard parallels of 0°, 40°, 60°, and 80°.

the satellite orbit, the effect of the orbit appears in the tanA term. Since the azimuth of the groundtrack on the sphere increases with latitude, the large range of Mm reflects the increased parallel spacing required to introduce sufficient angular distortion to keep the projected azimuth straight.

The ϕ_1 subscripted terms in all of the distortion equations allow the scale factor to equal unity at the standard parallel.

The parallel scale distortion, Figure 4, is a function only of the secant of the latitude. This indicates that latitude length is independent of its projected distance from the equator because the meridians are cast as equidistant vertical lines. Area distortion, the product of Mm and Mp, is shown in Figure 5.

Figures 6, 7, and 8 show the effect of angular distortion at standard parallels of 20°, 45°, and 70°. Conformality insures that the azimuth on the map equals the azimuth on the sphere at the standard parallel. Because azimuth is a constant on the projection, but increases with latitude on the sphere, constraining the groundtracks as straight lines causes map azimuth to be greater than sphere azimuth below the standard parallel and less than sphere azimuth above it. This effect can also be seen in the scale distortions: Mm is less than Mp below ϕ_1 and greater than Mp above ϕ_1 due to the relative sizes of A and A_1 in equation (37).

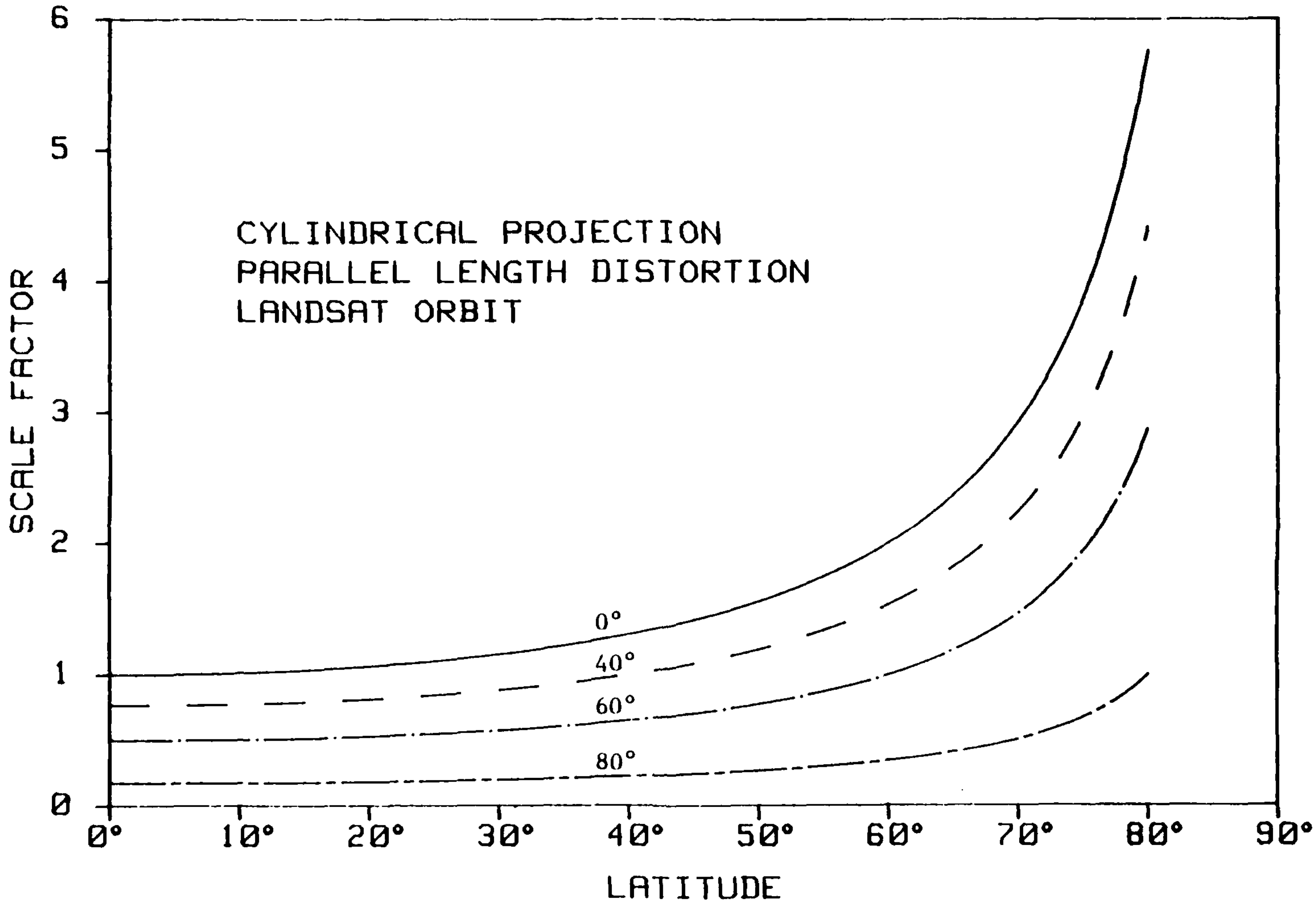

Figure 4. Parallel length distortion at standard parallels of 0°, 40°, 60°, and 80°.

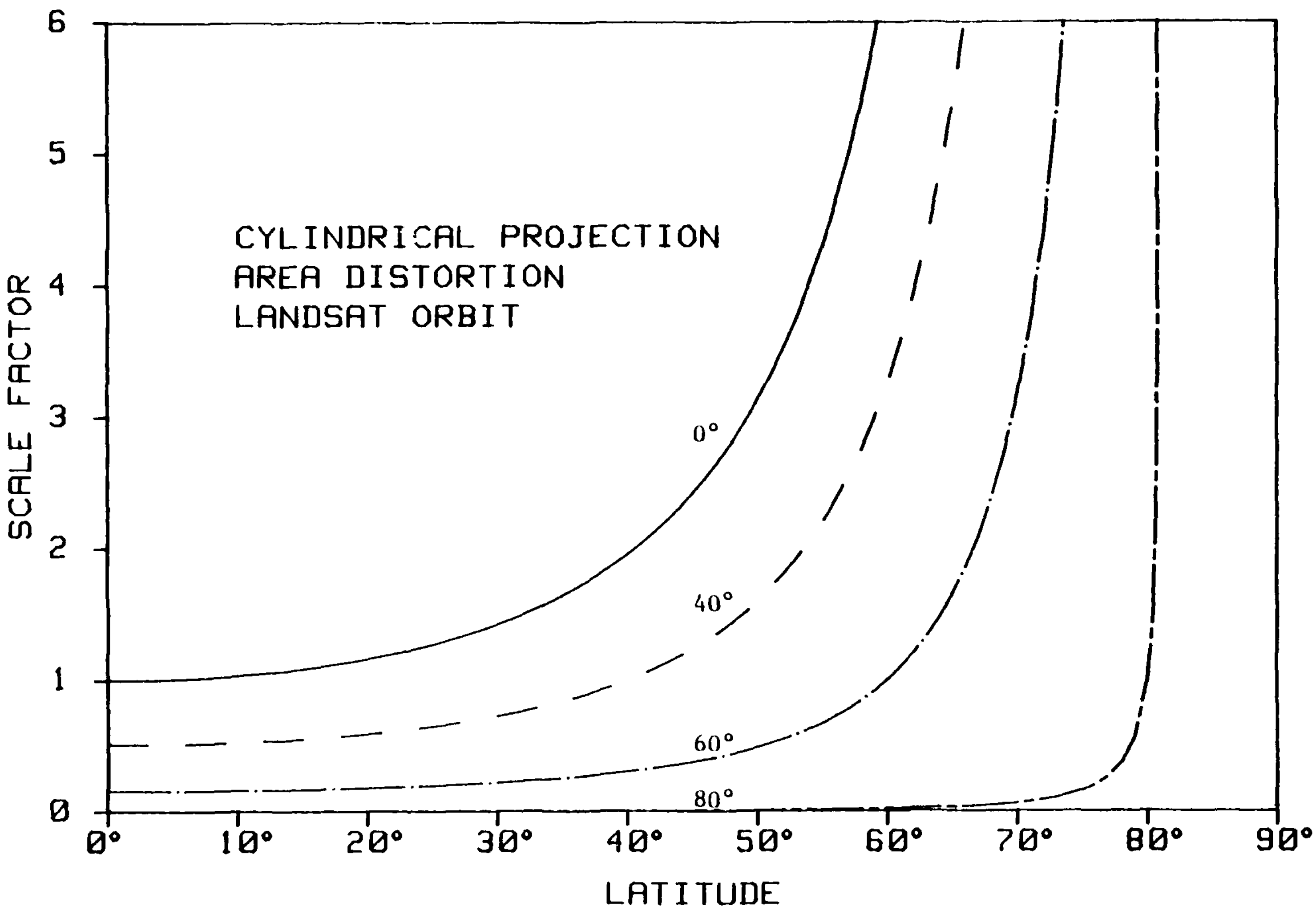

Figure 5. Area distortion at standard parallels of 0°, 40°, 60°, and 80°.

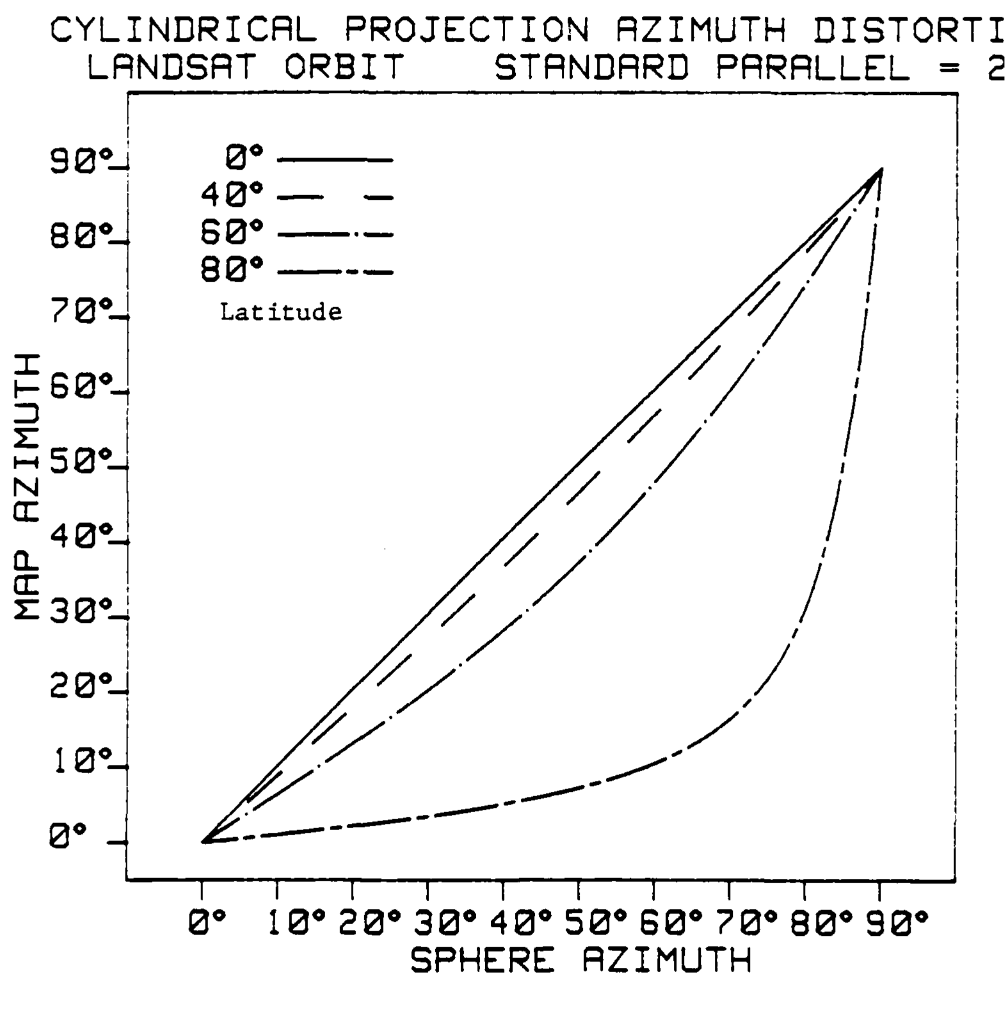

Figure 6. Cylindrical projection azimuth distortion.
Standard parallel = 20 degrees.

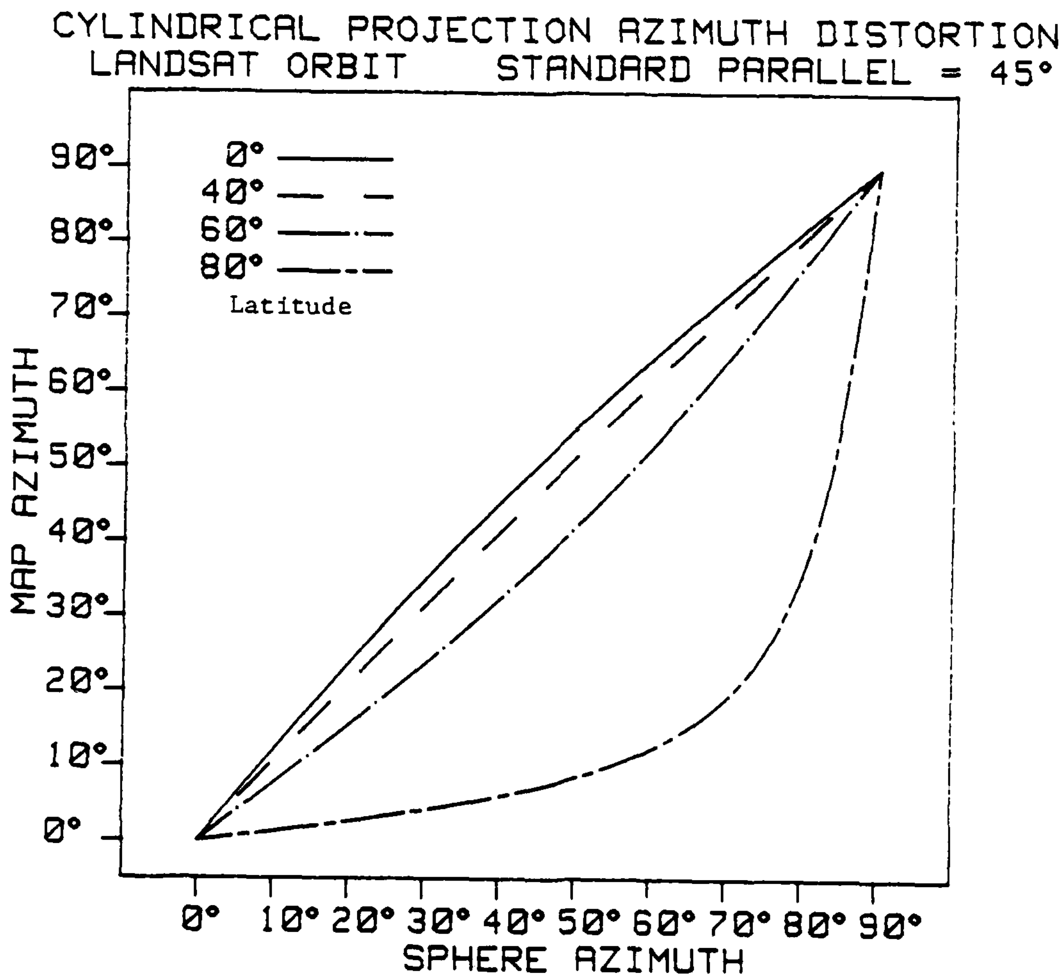

Figure 7. Cylindrical projection azimuth distortion.
Standard parallel = 45 degrees.

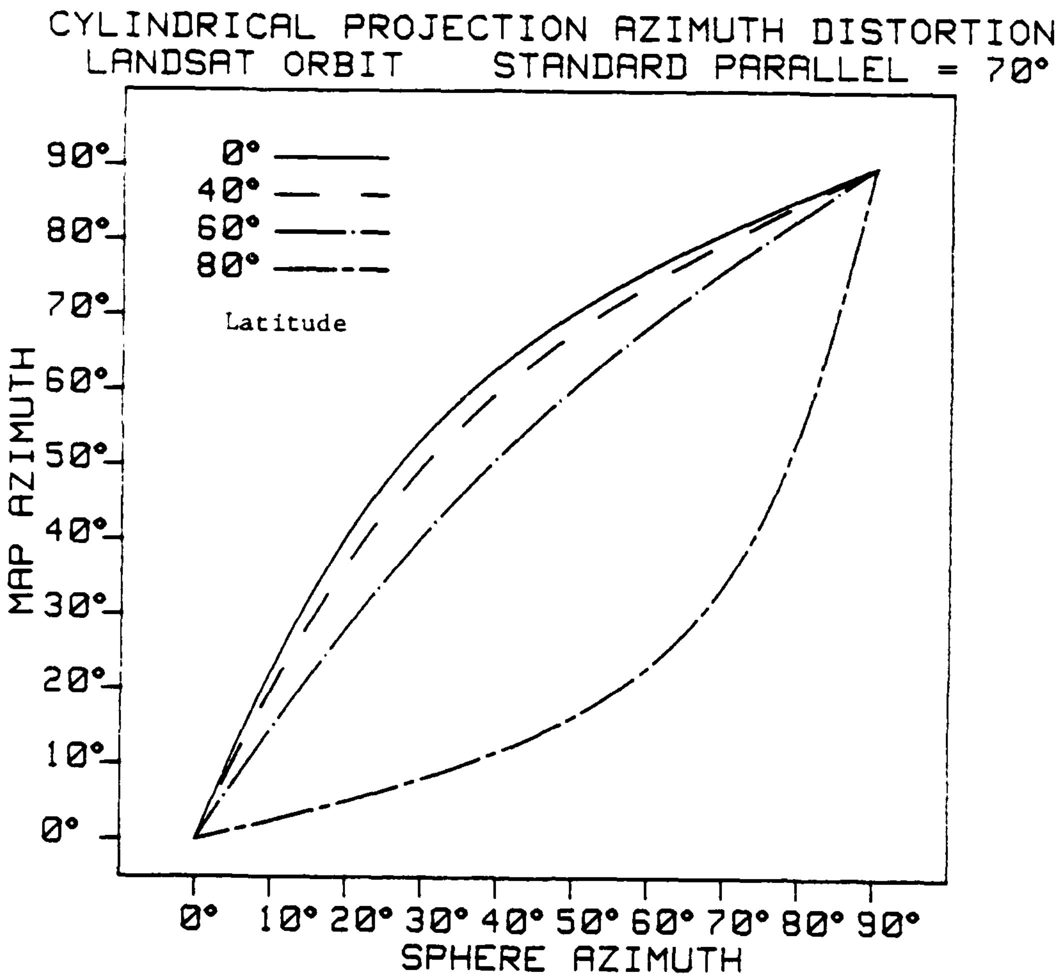

Figure 8. Cylindrical projection azimuth distortion.
Standard parallel = 70 degrees.

5.2 The Conic Projection

The equations for scale distortion are similar to those for the cylindrical projection; but here, the azimuth of the groundtrack changes on the projection as well as on the sphere. This is reflected in the addition of the $\sin A_1/\sin(\theta + S)$ term in both equations, which relates the azimuth at the standard parallel, A_1, to its general value of $(\theta + S)$. Plots of the meridian and parallel scale distortions for the tangent conic projection are shown in Figures 9 and 10; area distortion is shown in Figure 11.

Inspecting the tables of the scale distortion values in Appendix C reveals that, for the tangent cone, Mm is greater than Mp everywhere except at the standard parallel where they are equal. For this to be true, comparing equations (42) and (43), the azimuth on the sphere, A, must always be greater than or equal to the azimuth on the plane, $(\theta + S)$. This is confirmed in the angular distortion plots in Figures 12, 13, and 14.

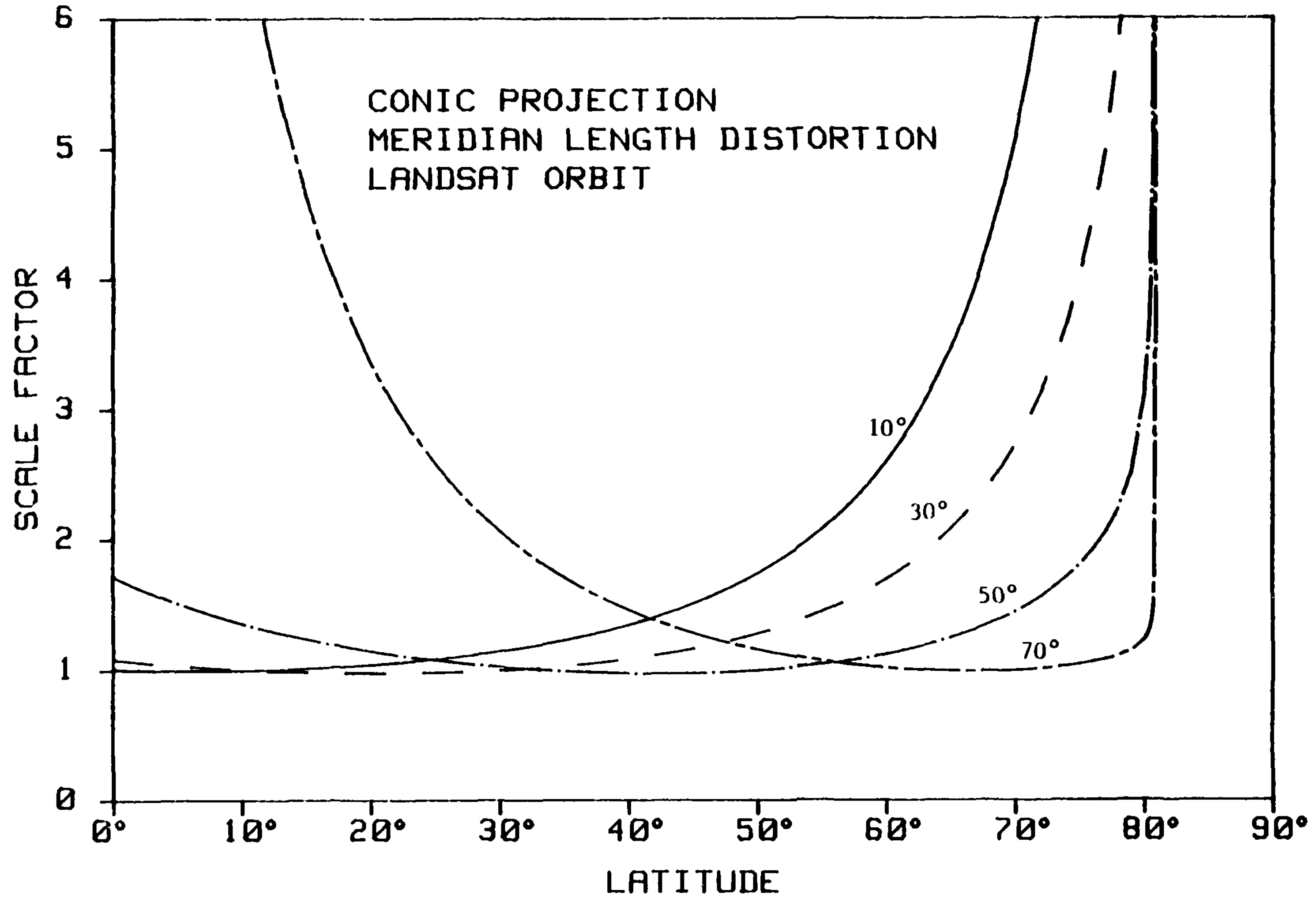

Figure 9. Meridian length distortion at standard parallels of 10°, 30°, 50°, and 70°.

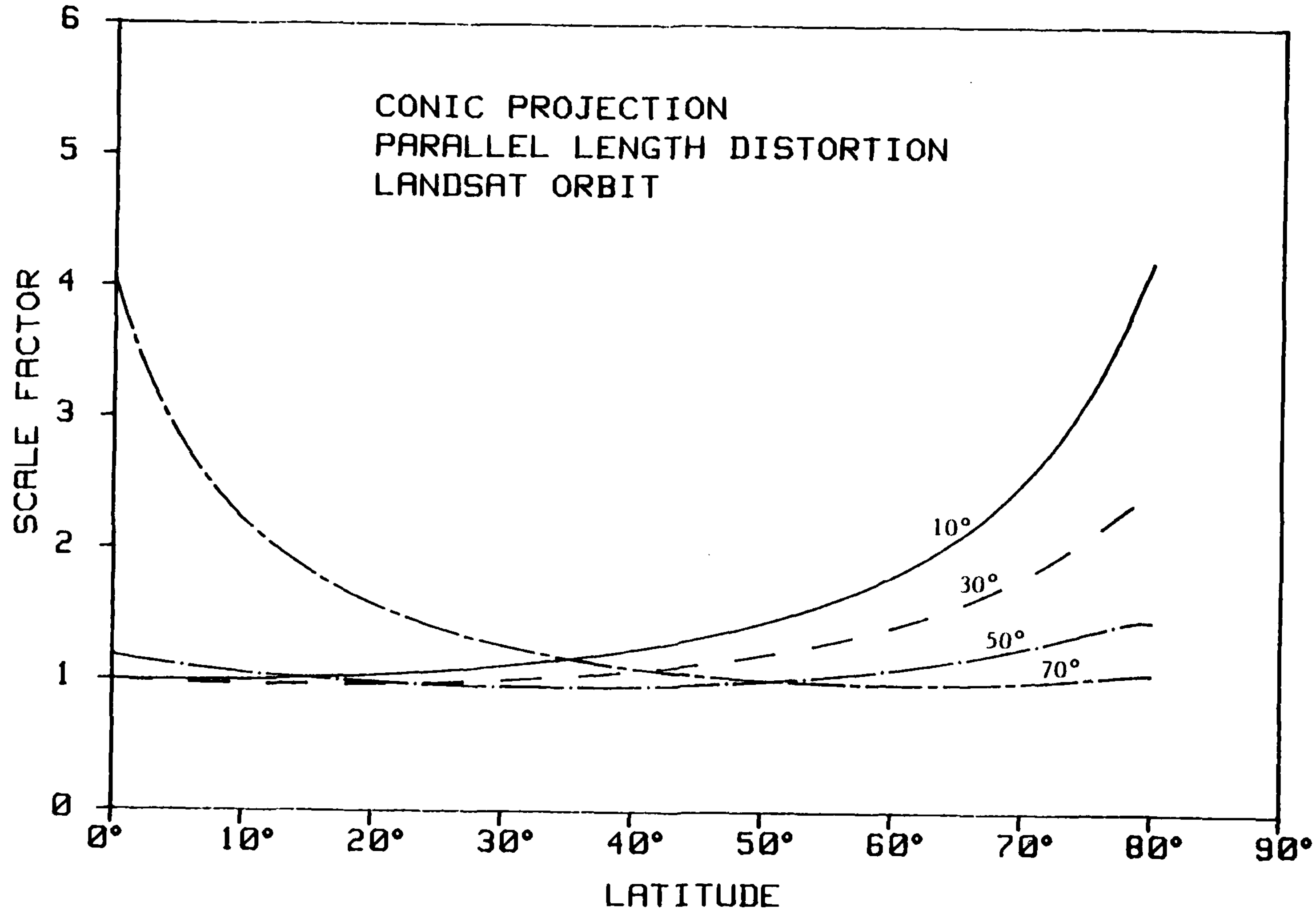

Figure 10. Parallel length distortion at standard parallels of 10°, 30°, 50°, and 70°.

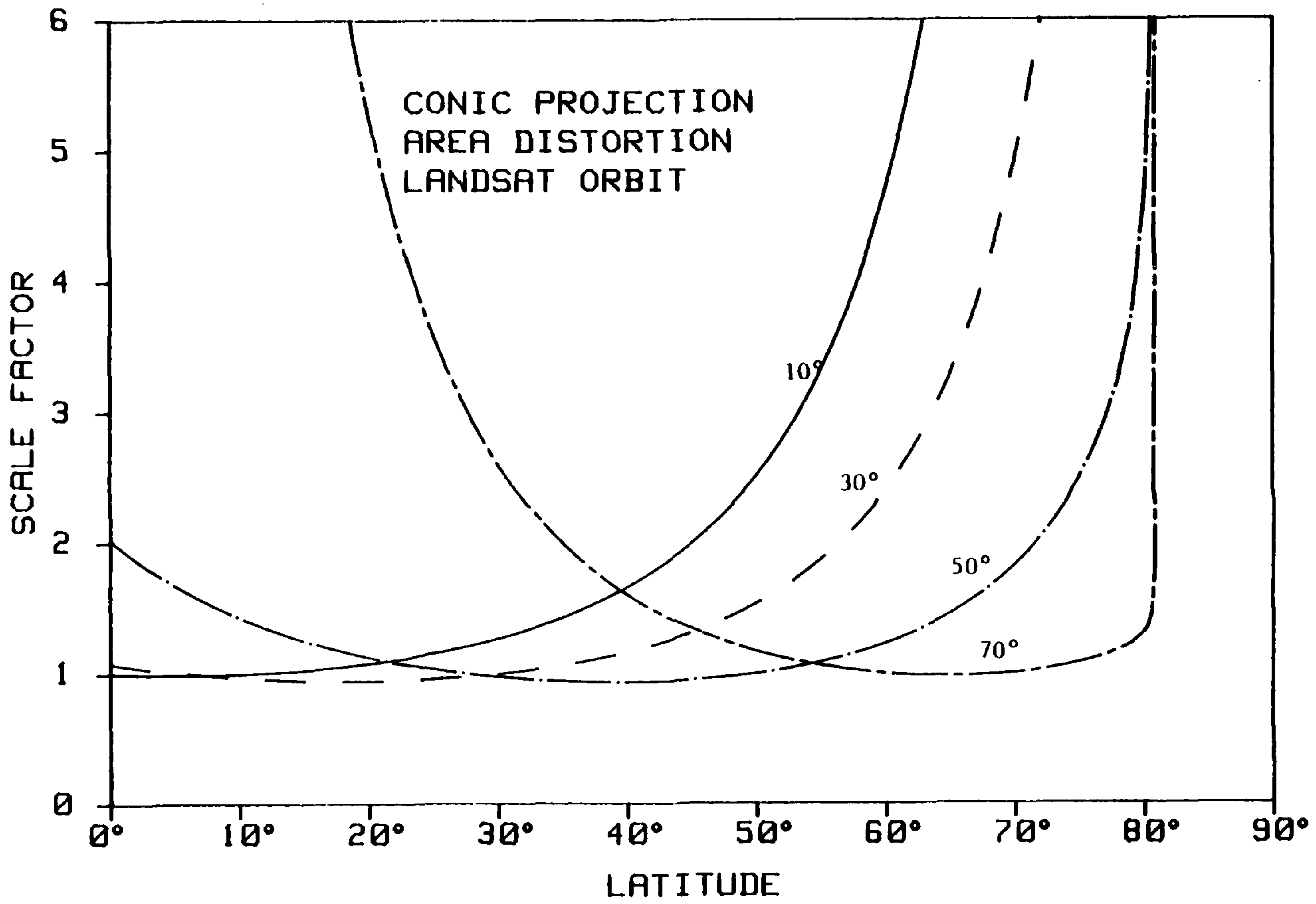

Figure 11. Area distortion at standard parallels of 10°, 30°, 50°, and 70°.

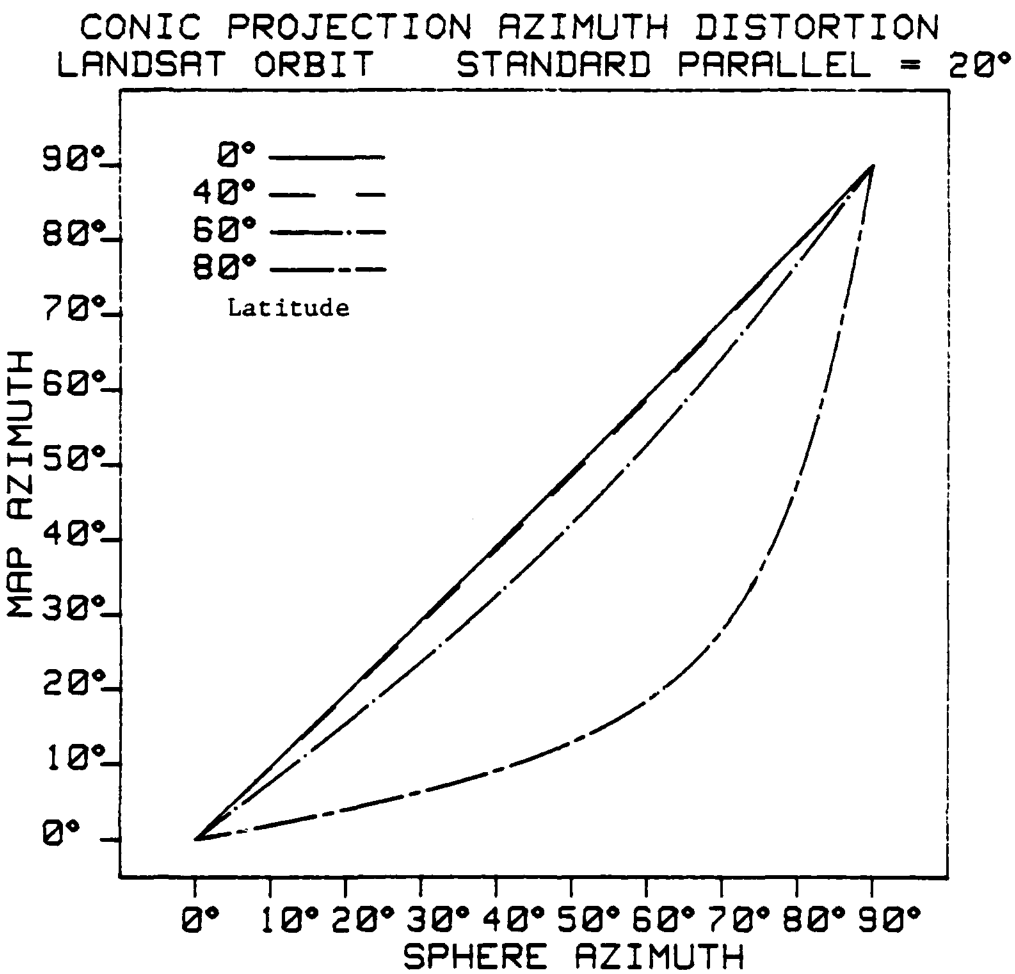

Figure 12. Conic projection azimuth distortion.
Standard parallel = 20 degrees.

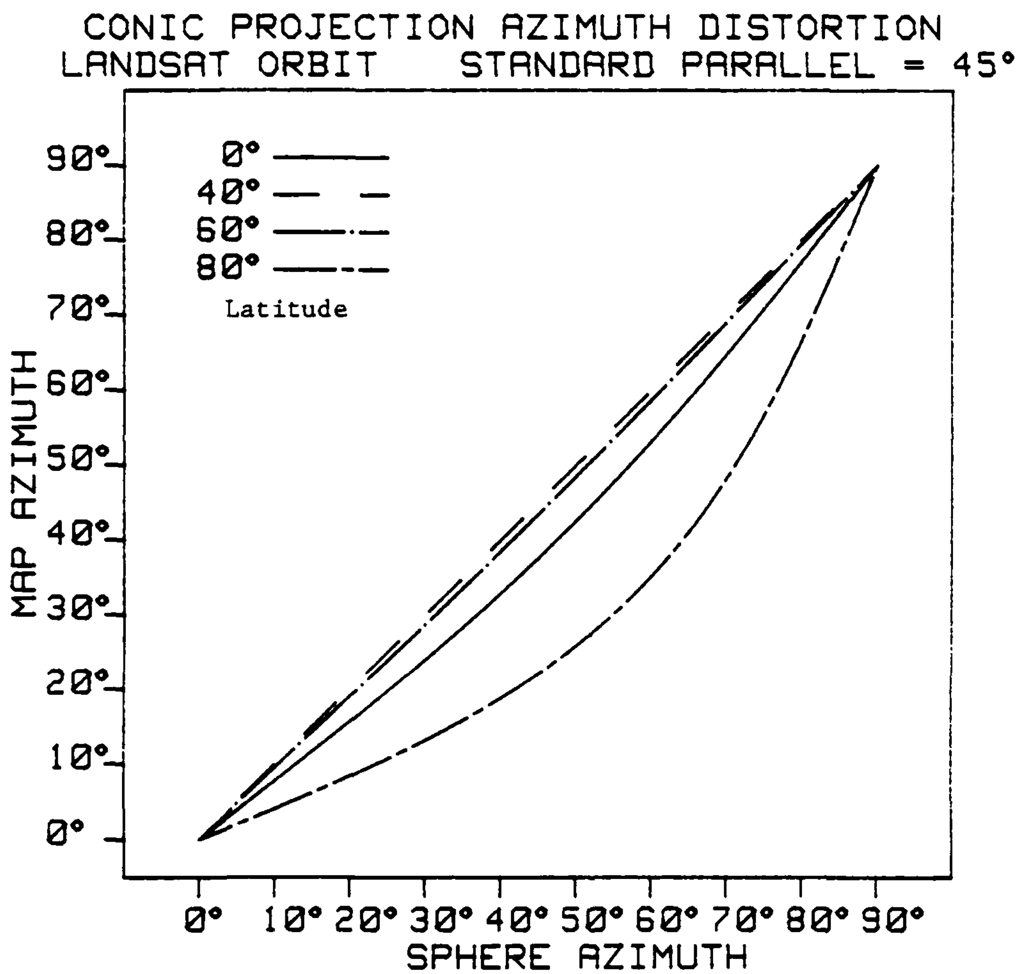

Figure 13. Conic projection azimuth distortion.
Standard parallel = 45 degrees.

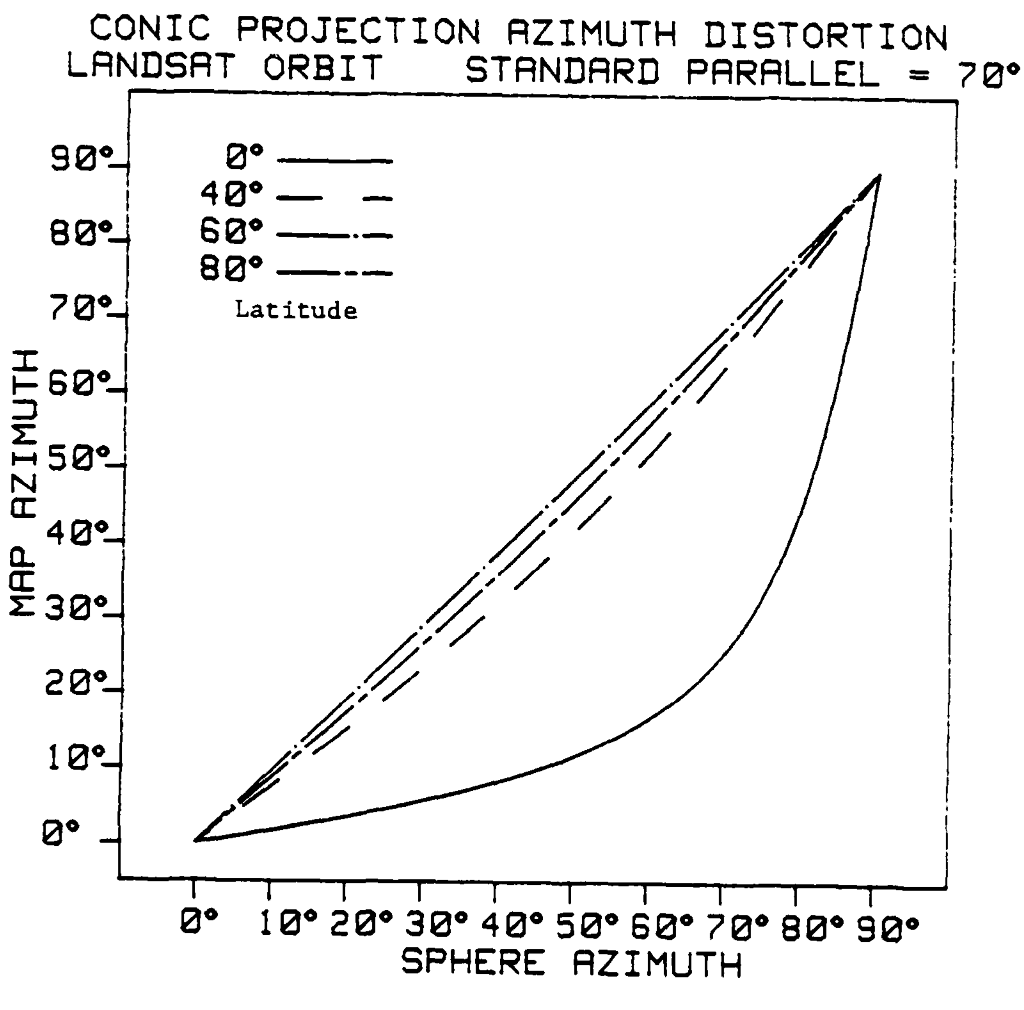

Figure 14. Conic projection azimuth distortion.
Standard parallel = 70 degrees.

6. CONCLUSIONS

The purpose of this thesis is to derive general distortion formulae for the Satellite Tracking map projections. The following conclusions can be drawn:

1. Equations for length, area, and azimuth distortion for the Satellite Tracking projections were derived.
2. Limitations in the use of these projections are now quantitatively defined, e.g., the size, shape, and orientation of a Landsat scene.
3. The cylindrical projection is preferred when the entire earth is to be shown because it allows true scale to be defined both above and below the equator.
4. The conic projection is preferred for depicting small areas because conformality can be defined at any two parallels within the area of interest.

Recommendations for further study:

These projections are valid only under the assumption of a spherical earth and circular satellite orbit.

1. What errors in the plotted groundtrack are introduced as a result of these assumptions.
2. Can the projections be modified to accept a non-circular orbit.
3. Can the transformation be made from an ellipsoidal earth.

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Appendix A. A continuation of the derivation of the constant of the cone for a projection with one conformal latitude.

Recalling equation (33):

$$n = \frac{\sin\phi_1 \cos^2\phi_1 \{(p/P)(\cos^2\phi_1 - 2\cos^2\iota) + \cos\iota\}}{(\cos^2\phi_1 - \cos^2\iota)(\cos\iota - (p/P)\cos^2\phi_1) + (\cos\iota - (p/P)\cos^2\phi_1)^3}$$

Expanding the denominator:

$$(\cos^2\phi_1 - \cos^2\iota)(\cos\iota - (p/P)\cos^2\phi_1) +$$

$$(\cos^3\iota - 3(p/P)\cos^2\phi_1 \cos^2\iota + 3(p/P)^2 \cos^4\phi_1 \cos\iota - (p/P)^3 \cos^6\phi_1)$$

$$= -2(p/P)\cos^2\phi_1 \cos^2\iota + 3(p/P)^2\cos^4\phi_1 \cos\iota -$$

$$(p/P)^3 \cos^6\phi_1 + \cos^2\phi_1 \cos\iota - (p/P)\cos^4\phi_1$$

$$= \cos^2\phi_1\{-2(p/P)\cos^2\iota + 3(p/P)^2 \cos^2\phi_1 \cos\iota -$$

$$(p/P)^3 \cos^4\phi_1 + \cos\iota - (p/P)\cos^2\phi_1\}$$

$$= \cos^2\phi_1 (-(p/P)\cos^2\phi_1 \{(p/P)^2 \cos^2\phi_1 - 2(p/P)\cos\iota + 1\} +$$

$$\cos\iota \{(p/P)^2 \cos^2\phi_1 - 2(p/P)\cos\iota + 1\})$$

$$= \cos^2\phi_1 ((\cos\iota - (p/P)\cos^2\phi_1)\{(p/P)^2 \cos^2\phi_1 - 2(p/P)\cos\iota + 1\})$$

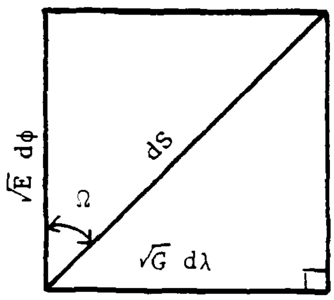
$$= \cos^2\phi_1 ((\cos\iota - (p/P)\cos^2\phi_1)\{(p/P) ((p/P)\cos^2\phi_1 - 2\cos\iota) + 1\})$$

Substituting this back into equation (33) yields the final form of n.

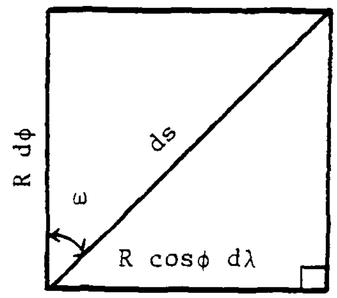
$$n = \frac{\sin\phi_1 \{(p/P) (\cos^2\phi_1 - 2\cos^2\iota) + \cos\iota\}}{(\cos\iota - (p/P)\cos^2\phi_1)\{(p/P)((p/P)\cos^2\phi_1 - 2\cos\iota) + 1\}}$$

This form is useful to test the condition where $\phi_1 = 90°$ and the projection becomes azimuthal. This could not be done using equation (33) because of the $\cos^2\phi_1$ term in the numerator.

Appendix B. The Derivation of the Angular Distortion Equation



Projection surface



Spherical earth

On the projection surface:

$$\cos\Omega = \sqrt{E}\,\frac{d\phi}{dS}$$

$$= \frac{\sqrt{E}\,d\phi}{\sqrt{E\,(d\phi)^2 + G\,(d\lambda)^2}}$$

$$\cos\Omega = \frac{\sqrt{E}}{\sqrt{E\left(\frac{d\phi}{d\lambda}\right)^2 + G}}\,\frac{d\phi}{d\lambda}$$

On the sphere:

$$ds\,\cos\omega = R\,d\phi$$

$$ds\,\sin\omega = R\,\cos\phi\,d\lambda$$

$$\frac{d\phi}{d\lambda} = \frac{R\,\cos\phi\,\cos\omega}{R\,\sin\omega}$$

$$\frac{d\phi}{d\lambda} = \cos\phi\,\cot\omega$$

Substituting:

$$\cos\Omega = \frac{\sqrt{E}\,\cos\phi\,\cot\omega}{\sqrt{E\,\cos^2\phi\,\cot^2\omega + G}}$$

Appendix C. Distortion Tables.

Table 1. Cylindrical projection length and area distortion for the Landsat orbit.

φ \ Standard Parallel	0°	40°	60°	80°
	Meridian Length Distortion			
0°	1.0	.66762	.31363	.01816
10	1.02179	.68217	.32047	.01855
20	1.09298	.72969	.34279	.01985
30	1.23456	.82421	.38720	.02242
40	1.49787	1.0	.46978	.02720
50	2.01389	1.34451	.63162	.03657
60	3.18846	2.12866	1.0	.05790
70	6.89443	4.60283	2.16231	.12519
80	55.07144	36.76658	17.27213	1.0
80.908	∞	∞	∞	∞
	Parallel Length Distortion			
0	1.0	.76604	.5	.17365
10	1.01543	.77786	.50771	.17633
20	1.06418	.81521	.53209	.18479
30	1.15470	.88455	.57735	.20051
40	1.30541	1.0	.65270	.22668
50	1.55572	1.19175	.77786	.27015
60	2.0	1.53209	1.0	.34730
70	2.92380	2.23976	1.46190	.50771
80	5.75877	4.41147	2.87939	1.0
80.908	6.32830	4.84776	3.16415	1.09890
	Area Distortion			
0	1.0	.51142	.15682	.00315
10	1.03756	.53063	.16270	.00327
20	1.16312	.59485	.18240	.00367
30	1.42555	.72906	.22355	.00449
40	1.95533	1.0	.30663	.00617
50	3.13306	1.60232	.49131	.00988
60	6.37691	3.26130	1.0	.02011
70	20.15798	10.30926	3.16109	.06356
80	317.14380	162.19480	49.73312	1.0
80.908	∞	∞	∞	∞

Table 2. Conic projection length and area distortion for the Landsat orbit.

Standard Parallel	10°	30°	50°	70°
ϕ	Meridian Length Distortion			
0°	1.00313	1.07991	1.71646	23.26322
10	1.0	1.00398	1.35754	6.92079
20	1.04275	.97729	1.14552	3.37126
30	1.14572	1.0	1.02572	2.06490
40	1.34649	1.08700	.97692	1.46129
50	1.73949	1.28047	1.0	1.15710
60	2.60194	1.69556	1.12369	1.01679
70	5.08699	2.73190	1.44871	1.0
80	29.00155	9.41190	3.09745	1.22864
80.908	∞	∞	∞	∞
	Parallel Length Distortion			
0	.99671	.99621	1.17680	4.05740
10	1.0	.96599	1.05324	2.22532
20	1.03510	.96693	.98251	1.57647
30	1.10816	1.0	.95174	1.26294
40	1.23355	1.07194	.95670	1.09489
50	1.44188	1.19859	1.0	1.00824
60	1.80333	1.41446	1.09269	.97861
70	2.51103	1.79895	1.25919	1.0
80	4.20804	2.42253	1.47016	1.06676
80.908	4.23325	2.24930	1.35775	1.04704
	Area Distortion			
0	.99983	1.07582	2.01993	94.38810
10	1.0	.96984	1.42982	15.40095
20	1.07935	.94497	1.12548	5.31469
30	1.26964	1.0	.97622	2.60784
40	1.66097	1.16520	.93462	1.59995
50	2.50814	1.53477	1.0	1.16663
60	4.69216	2.39830	1.22784	.99504
70	12.77360	4.91455	1.82420	1.0
80	122.03977	22.80060	4.55374	1.31066
80.908	∞	∞	∞	∞

Table 3. Cylindrical projection azimuth distortion for the Landsat orbit.

Latitude	0°	40°	60°	80°
Sphere Azimuth	Map Azimuth			
	Standard Parallel = 20°			
0°	0.0°	0.0°	0.0°	0.0°
10	10.26495	8.96897	6.48077	1.08436
20	20.49676	18.04498	13.19640	2.23744
30	30.66688	27.32916	20.40258	3.54643
40	40.75502	36.90925	28.39457	5.14695
50	50.75153	46.84937	37.51585	7.29022
60	60.65803	57.17734	48.13385	10.53262
70	70.48670	67.87198	60.53498	16.43235
80	80.25838	78.85585	74.69298	31.33252
90	90.0	90.0	90.0	90.0
	Standard Parallel = 45°			
0	0.0	0.0	0.0	0.0
10	12.03481	10.52545	7.61675	1.27646
20	23.75263	20.98287	15.43133	2.63343
30	34.91723	31.31478	23.64675	4.17284
40	45.41324	41.48222	32.47151	6.05278
50	55.23919	51.46883	42.10802	8.56439
60	64.47477	61.28085	52.71885	12.34601
70	73.24646	70.94427	64.36273	19.14672
80	81.70266	80.50024	76.91131	35.62808
90	90.0	90.0	90.0	90.0
	Standard Parallel = 70°			
0	0.0	0.0	0.0	0.0
10	22.57733	19.91899	14.61773	2.48838
20	40.63871	36.79633	28.29625	5.12597
30	53.70224	49.87562	40.49658	8.09850
40	63.18850	59.89055	51.14122	11.68437
50	70.41251	67.78976	60.43412	16.36862
60	76.24248	74.30808	68.67779	23.11683
70	81.22568	79.95673	76.17587	34.10376
80	85.72366	85.09605	83.20187	54.41874
90	90.0	90.0	90.0	90.0

Table 4. Conic projection azimuth distortion for the Landsat orbit.

Latitude	0°	40°	60°	80°
	Map Azimuth			
Sphere Azimuth	Standard Parallel = 20°			
0°	0°.0	0°.0	0°.0	0°.0
10	9.71601	9.55806	7.64103	1.95248
20	19.46493	19.16619	15.47873	4.02519
30	29.27632	28.86987	23.71462	6.36914
40	39.17314	38.70529	32.55515	9.21472
50	49.16891	48.69490	42.19983	12.97502
60	59.26561	58.84354	52.80775	18.51405
70	69.45275	69.13623	64.43463	27.97665
80	79.70805	79.53849	76.95195	47.63448
90	90.0	90.0	90.0	90.0
	Standard Parallel = 45°			
0	0.0	0.0	0.0	0.0
10	7.70744	9.95383	9.44635	4.08611
20	15.60828	19.91319	18.95443	8.38833
30	23.89994	29.88297	28.58072	13.16523
40	32.78316	39.86682	38.37095	18.77562
50	42.44963	49.86671	48.35445	25.77252
60	53.04910	59.88269	58.53894	35.05824
70	64.62944	69.91288	68.90689	48.06408
80	77.06188	79.95362	79.41530	66.48014
90	90.0	90.0	90.0	90.0
	Standard Parallel = 70°			
0	0.0	0.0	0.0	0.0
10	1.76150	7.52606	9.63174	8.70415
20	3.63232	15.25417	19.30566	17.53739
30	5.75013	23.39270	29.05984	26.62384
40	8.32610	32.15784	38.92427	36.07508
50	11.74211	41.76285	48.91710	45.97808
60	16.80910	52.38369	59.04168	56.37768
70	25.60347	64.09087	69.28501	67.25653
80	44.68725	76.75732	79.61826	78.52028
90	90.0	90.0	90.0	90.0

Appendix D. FORTRAN program and sample output.

This program computes values for constructing the graticules of the Satellite Tracking projections. The type of the projection is determined by the values in the assignment statements in the "initialize variables" section. For example, if PHI1 equals zero, or PHI1 equals -PHI2, a cylindrical projection is produced; otherwise the result is a conic.

Output is included for cylindrical and tangent conic projections with a standard parallel of 30°. Plots of the graticules are shown in Figures 15 and 16.

```
C   SATELLITE TRACKING PROJECTIONS
C   PROJECTIONS PORTRAYING SATELLITE GROUNDTRACKS AS
C   STRAIGHT LINES.  THE TRANSFORMATION IS FROM THE SPHERE.
C   REFERENCE: JOHN SNYDER, 'MAP PROJECTIONS FOR SATELLITE TRACKING',
C   PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING,VOL 47, NO 2, FEB 81
C
C   VARIABLES
C
C   DELTA   - DEGREE INCREMENT FOR CREATING THE GRATICULE.
C   PHI1    - LOWER CONFORMAL LATITUDE = LATITUDE OF TRUE SCALE.
C   PHI2    - UPPER CONFORMAL LATITUDE.
C   R       - EARTH RADIUS AT MAP SCALE.
C   PE      - EARTH ROTATION PERIOD (MINUTES).
C   PS      - SATELLITE REVOLUTION PERIOD (MINUTES).
C   SATI    - INCLINATION OF THE SATELLITE ORBIT.
C   AZ(PHI)- AZIMUTH OF THE GROUNDTRACK AT LATITUDE PHI ON THE SPHERE.
C   TL      - TRACKING LIMIT = MAXIMUM SATELLITE LATITUDE.
C   N       - CONSTANT OF THE CONE.
C   SZERO   - AZIMUTH OF THE GROUNDTRACK AT THE EQUATOR ON THE MAP.
C   RHONUM - THE NUMERATOR OF RHO.
C   RHOMIN - RADIUS OF THE CIRCLE OF GROUNDTRACK TANGENCY.
C             WILL BE NEGATIVE WHEN SATI IS GREATER THAN 90.
C   RHOZRO - RADIUS FROM THE APEX OF THE CONE TO THE EQUATOR.
C   RHOTL   - RADIUS OF THE TRACKING LIMIT.
C   ALPHA   - ARC LENGTH MEASURED ALONG THE ORBIT.
C   ASTROL - ASTRONOMICAL LONGITUDE OF A POINT ON THE ORBIT.
C   L       - SATELLITE APPARENT EARTH FIXED LONGITUDE.
C   LTL     - SATELLITE APPARENT LONGITUDE AT THE TRACKING LIMIT.
C             THIS MUST BE COMPUTED SEPARATELY BECAUSE THE TANGENT
C             COMPUTED IN ASTROL(X) BREAKS DOWN WHEN SATI = PHI.
```

```
C    SIGN   = THE SIGN OF COS(SATI) IN ASTROL(X).  USED IN LTL.
C    RADCON = RADIANS PER DEGREE.
C    DEGRAD = DEGREE TO RADIAN CONVERSION FACTOR.
C    RADDEG = RADIAN TO DEGREE CONVERSION FACTOR.
C
C
C    AUTHOR:  GREGORY C. ARNOLD
C    DATE:    MARCH, 1984
C
C   ****************************************************************
C
      IMPLICIT REAL*8 (A-H,L-Z)
C
C... STATEMENT FUNCTIONS...
C
      ALPHA(X)  = DASIN( DSIN(X) / DSIN(DEGRAD(SATI)) )
      ASTROL(X) = DATAN( DTAN(ALPHA(X)) * DCOS(DEGRAD(SATI)) )
      L(X)      = ASTROL(X) - ALPHA(X) * PS / PE
      AZ(X)     = DATAN( (DCOS(DEGRAD(SATI)) - DCOS(X)**2 * PS / PE )
     .            / (DSQRT(DCOS(X)**2 - DCOS(DEGRAD(SATI))**2)) )
      THETA(X)  = N * L(X)
      RHO(X)    = RHONUM / (N * DSIN(THETA(X) + SZERO))
      DEGRAD(X) = X * RADCON
      RADDEG(X) = X / RADCON
C
C...INITIALIZE VARIABLES...
C
```

```
      RADCON = 3.14159265358980D0 / 180.D0
      DELTA  = 10.D0
      SIGN   = 1.D0
      R      = 1.D0
      PE     = 1440.D0
      PS     = 103.267D0
      SATI   = 99.092D0
      TL     = 180.D0 - SATI
      PHI1   = DEGRAD(30.D0)
      PHI2   = DEGRAD(-30.D0)
      IF(SATI.GT.90)  SIGN = -1.D0
      LTL = DEGRAD(SIGN * 90.D0 - 90.D0 * PS / PE)
C
C...TEST FOR THE CYLINDRICAL OR CONIC PROJECTION...
C
      IF(PHI1.NE.0.AND.PHI1.NE.-PHI2) THEN
C
C...CONIC PROJECTION...
C
C...COMPUTE N BASED ON 1 OR 2 STANDARD PARALLELS...
C
         IF(PHI1.EQ.PHI2) THEN
            T = DCOS(DEGRAD(SATI)) - PS/PE * DCOS(PHI1)**2
            N = (DSIN(PHI1) * DCOS(PHI1)**2 *(PS/PE *(DCOS(PHI1)**2 -
     .          2 * DCOS(DEGRAD(SATI))**2) + DCOS(DEGRAD(SATI))))
     .          / ((DCOS(PHI1)**2 - DCOS(DEGRAD(SATI))**2) * T + T**3)
         ELSE
            N = (AZ(PHI2) - AZ(PHI1)) / (L(PHI2) - L(PHI1))
         END IF
```

```
C
C...COMPUTE PROJECTION CONSTANTS...
C
         SZERO = AZ(PHI1) - THETA(PHI1)
         RHONUM = R * DCOS(PHI1) * DSIN(AZ(PHI1))
         RHOMIN = RHONUM / N
         RHOTL = RHONUM / (N * DSIN( N * LTL + SZERO))
         RHOZRO = RHO(0,D0)
         WRITE(6,600) N,RHOMIN,RADDEG(SZERO),RADDEG(PHI1),RADDEG(PHI2)
C
C...COMPUTE AND WRITE VALUES OF RHO AND THETA...
C
         WRITE(6,610)
         DO 10 I = 0,TL,DELTA
            WRITE(6,650) FLOAT(I), RHO(DEGRAD(I))
   10    CONTINUE
         WRITE(6,650) TL, RHOTL
C
         WRITE(6,620)
         DO 20 J = 0,90,DELTA
            WRITE(6,650) FLOAT(J), RADDEG(N * DEGRAD(J))
   20    CONTINUE
C
      ELSE
C
C...CYLINDRICAL PROJECTION...
C
C...COMPUTE VALUES OF X AND Y...
C
```

```
      WRITE(6,640) RADDEG(PHI1)
      DO 30 I = 0,TL,DELTA
         WRITE(6,650) FLOAT(I),
     *         R * L(DEGRAD(I)) * DCOS(PHI1) / DTAN(AZ(PHI1))
   30 CONTINUE
      WRITE(6,650) TL, R * LTL * DCOS(PHI1) / DTAN(AZ(PHI1))
C
      WRITE(6,660)
      DO 40 J = 0,90,DELTA
         WRITE(6,650) FLOAT(J), R * DEGRAD(J) * DCOS(PHI1)
   40 CONTINUE
C
    END IF
    STOP
C
  600 FORMAT('1','CONSTANT OF THE CONE =',F10.5/
     *       '0','RADIUS OF CIRCLE OF GROUNDTRACK TANGENCY =',F10.5/
     *       '0','EQUATOR GROUNDTRACK AZIMUTH ON THE MAP =',F10.5/
     *       '0','STANDARD PARALLEL 1 =',F10.2/
     *       '0','STANDARD PARALLEL 2 =',F10.2)
  610 FORMAT(///' ','    LATITUDE       RHO')
  620 FORMAT(///' ','   LONGITUDE      THETA')
  630 FORMAT(' ')
  640 FORMAT('1','CYLINDRICAL PROJECTION'/
     *       ' ','STANDARD PARALLEL =',F5.2//
     *       ' ','    LATITUDE        Y')
  650 FORMAT(' ',5X,F5.2,F12.5)
  660 FORMAT(///' ','   LONGITUDE        X')
C
    END
```

CYLINDRICAL PROJECTION
STANDARD PARALLEL =30.00

LATITUDE	Y
.00	.00000
10.00	.14239
20.00	.29121
30.00	.45470
40.00	.64591
50.00	.88979
60.00	1.24489
70.00	1.89918
80.00	4.33417
80.91	5.86098

LONGITUDE	X
.00	.00000
10.00	.15115
20.00	.30230
30.00	.45345
40.00	.60460
50.00	.75575
60.00	.90690
70.00	1.05805
80.00	1.20920
90.00	1.36035

CONSTANT OF THE CONE = .24794

RADIUS OF CIRCLE OF GROUNDTRACK TANGENCY = -.84314

EQUATOR GROUNDTRACK AZIMUTH ON THE MAP = -12.11332

STANDARD PARALLEL 1 = 30.00

STANDARD PARALLEL 2 = 30.00

LATITUDE	RHO
.00	4.01791
10.00	3.83683
20.00	3.66461
30.00	3.49284
40.00	3.31185
50.00	3.10733
60.00	2.85239
70.00	2.48152
80.00	1.69663
80.91	1.43346

LONGITUDE	THETA
.00	.00000
10.00	2.47943
20.00	4.95886
30.00	7.43829
40.00	9.91772
50.00	12.39715
60.00	14.87657
70.00	17.35600
80.00	19.83543
90.00	22.31486

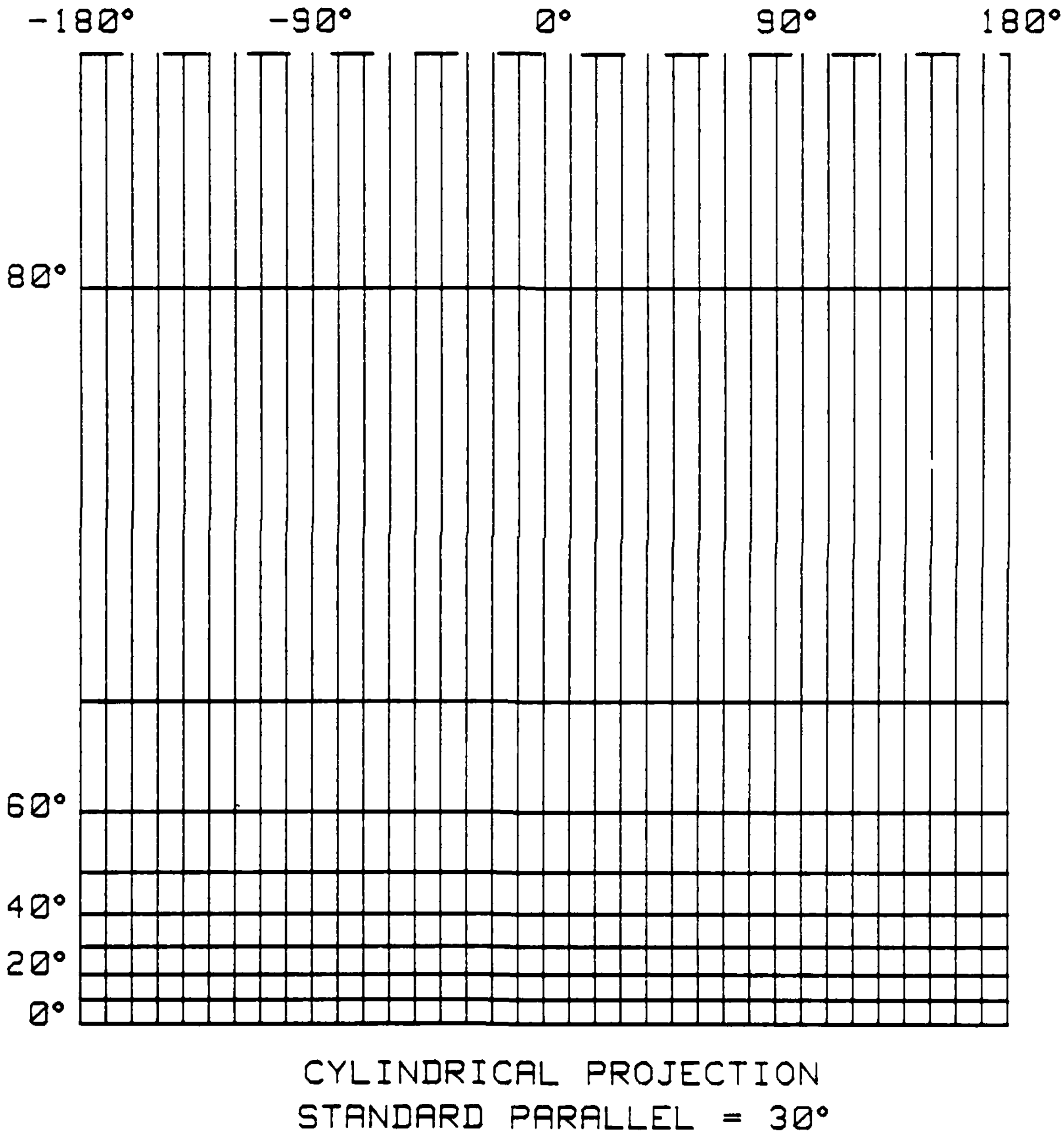
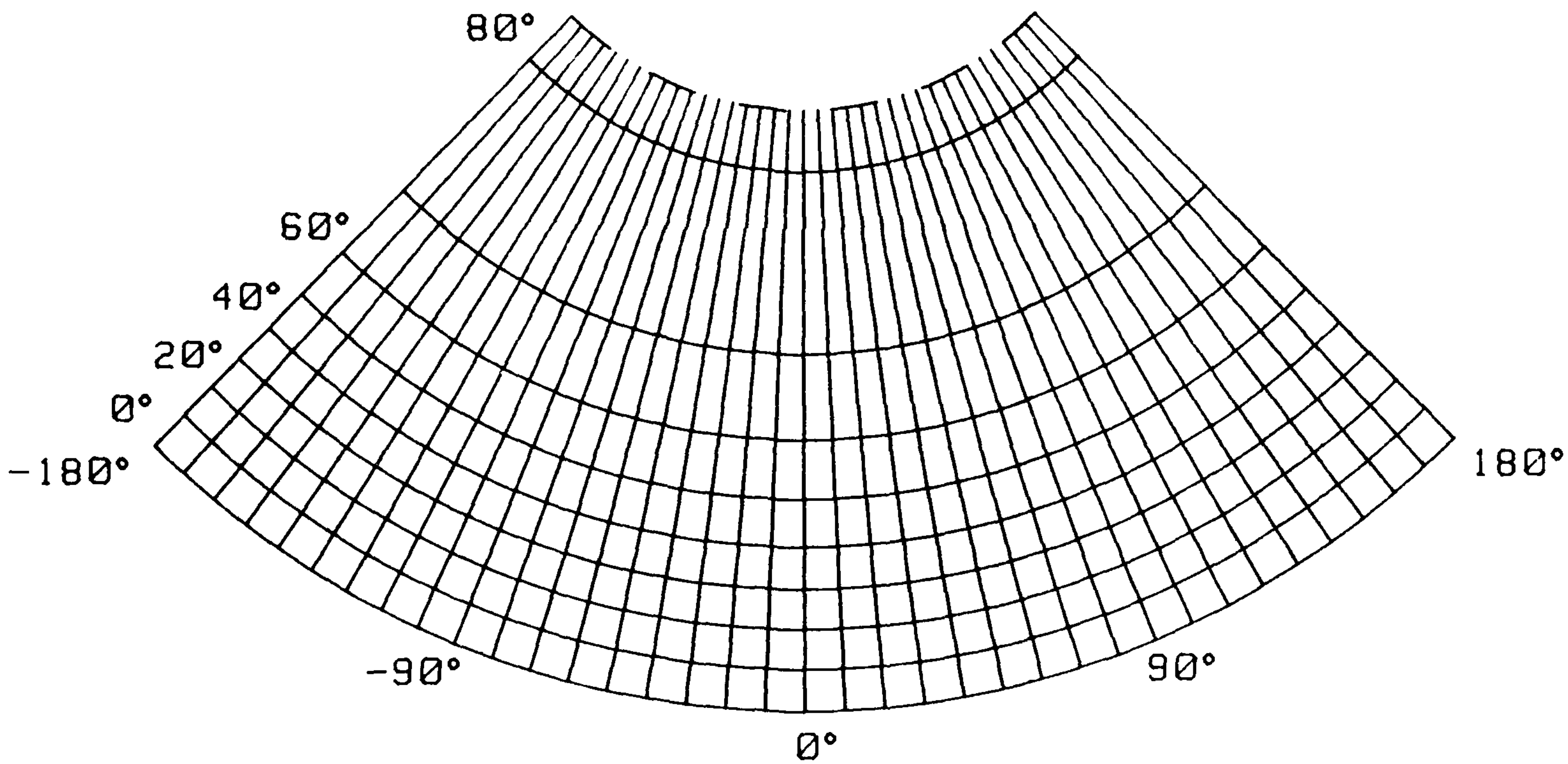


Figure 15. Cylindrical projection graticule.



CONIC PROJECTION STANDARD PARALLEL = 30°

VITA

Gregory Arnold was born in Reading, Pa, August 8, 1950. He received a B.A. in Geology from Lehigh University in 1972 and a B.S. in Cartography from George Washington University in 1982.

He served in the U.S. Army from 1972 to 1975. Since then he's worked as a Cartographer for NOAA and the Geological Survey. He is now a Physical Scientist working for the Defense Mapping Agency.

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