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this ability parameter

A STATISTICAL PROCEDURE FOR Assessing Test Dimensionality

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An important statistical problem in psychological test theory is the development of a sound method for determining whether a test which purports to measure the level of a certain ability is, in reality, significantly contaminated by the varying levels of one or more other abilities displayed by persons taking the test. For example, is a test of mathematical ability contaminated by varying levels of verbal ability displayed by persons taking the test or is a test of reading ability contaminated by varying levels of familiarity with middleclass American culture displayed by persons taking the test? Because of the large number of private and governmental organizations routinely using tests to screen people for the levels of various abilities, this problem of assessing the dimensionality of a test is of great importance.

The solution will be useful in settings other than psychological testing, since the problem is one of general interest and should, hence, be an important addition to the statistical methodology literature. Thus, it seems appropriate now to give a careful abstract statement of the problem, independent of its psychometric context.

Consider sampling units from a population and applying several treatments to each sampled unit. Suppose that the outcome of each unit-treatment combination is either success or failure. Suppose that associated with each unit is a parameter, θ (the ability parameter), which determines the likelihood of each treatment being successful for that unit. Assume that the dimensionality of θ is unknown (the precise mathematical definition of the dimensionality of θ will be given below). Thus, for each unit, dichotomous random variables $\{U_i\}$ are observed, where i is the treatment index. Let "treatment characteristic curves" $\{P_i(\cdot)\}$ be defined by

$$P_{i}(\underline{\theta}) = P[U_{i} = 1|\underline{\theta} = \underline{\theta}] = 1 - P[U_{i} = 0|\underline{\theta} = \underline{\theta}],$$
 [1]

the probability of treatment \underline{i} being successful, given that the sampled unit has ability θ . It is assumed that the process of random sampling units induces a probability distribution on the population of units with associated random variable $\underline{\theta}$.

Purpose

Described in this paper is an approach to the problem of finding a theoret-

ically sound and useful procedure for making inferences about the dimensionality of θ , that is, more precisely, the dimensionality of the distribution of 0. In order that this problem be well formulated mathematically, the dimensionality of θ needs to be defined precisely. The definition (Levine, 1981) that is used depends on the asymptotic behavior of "formula sequences." To define a linear formula sequence, a linear formula score must first be defined.

Definition of a Linear Formula Score

Given the outcomes $(U_1, U_2, ..., U_n)$ of <u>n</u> treatments resulting from a sampled unit, a linear formula score is a score of the form

$$\alpha_{n} = \sum_{i=1}^{n} a_{i}^{(n)} U_{i}$$
[2]

provided that

$$a_{i}^{(n)} \ge 0, \sum_{i=1}^{n} a_{i}^{(n)} = 1.$$
 [3]

Then, a formula sequence is a sequence of linear formula scores $(a_1, a_2, \ldots, a_n, \ldots)$ such that, referring to Equation 2,

$$a_{i}^{(n)} a_{i'}^{(n+1)} = a_{i}^{(n+1)} a_{i'}^{(n)}$$
 [4]

for all $i' \leq n$, $i \leq n$, and $n \geq 1$. The content of Equation 4 is that the contribution of a treatment, say, <u>i</u>, relative to another treatment, say, <u>i'</u>, is the same for all linear formula scores α_n for which $n \geq i$, $n \geq i'$. The prototype of a linear formula score and a formula sequence is the proportion-correct

$$\sum_{i=1}^{n} U_{i}/n$$
(5)

and

$$\{v_1, (v_1 + v_2)/2, \dots, \sum_{i=1}^n v_i/n, \dots\}$$
 [6]

respectively. Levine's (1981) definition can now be stated (below, Var [X|Y] denotes the variance of X, given Y):

A sequence of dichotomous random variables $\{U_1, U_2, ..., U_n, ...\}$ is <u>d</u> dimensional if there exist <u>d</u> formula sequences $\{h_1^{(n)}\}, \{h_2^{(n)}\}, ..., \{h_d^{(n)}\}$ such that for every formula sequence $\{h_1^{(n)}\}, \{h_2^{(n)}\}, \dots, \{h_d^{(n)}\}$

Var
$$[h^{(n)}|h_1^{(n)}, \ldots, h_d^{(n)}] \neq 0$$

as n + "; and, moreover, no smaller d works.

Note that it is the set of observables $\{U_1, U_2, ..., U_n, ...\}$ that is <u>d</u> dimensional. The ability Θ is not observable and is known only by inference. Nonetheless, let it be said that Θ is d dimensional, meaning that a <u>d</u>-dimensional random vector Θ and treatment characteristic curves $\{P_i(\cdot)\}$ (the conditional distributions of the U_i 's given Θ) can be constructed to specify the joint probability law of the <u>d</u>-dimensional U_i 's.

Assessment of Test Dimensionality

As stated above, the dimensionality problem is of particular importance in the field of psychological testing. In this case, the units are persons and the treatments are test items. The function $P_i(\cdot)$ is called the item characteristic curve for the ith item. The administration of a psychological test is modeled as a two-stage experiment, the first stage yielding J randomly sampled persons and the second stage consisting of the administration of I fixed test items (the test) to each sampled person. In this manner, dichotomous random variables $\{U_{ij}\}$; i = 1, 2, ..., I; j = 1, 2, ..., J are generated. The basic statistical assumptions made are as follows:

- 1. Experimental independence of persons. The appropriate assumptions are made concerning the joint distribution of the $\{U_{ij}\}$ that correspond to the psychometric assumption that persons are randomly sampled from a very large population and that sampled persons respond to items independently of one another.
- 2. Local independence of items. The appropriate probabilistic assumptions are made concerning the joint distribution of the $\{U_{ij}\}$ and Θ

that correspond to the psychometric assumption that for each person, his or her responses to different items are independent.

Consider again the example of the introductory paragraph, that of a "mathematics" test. It might be that while Θ is assumed to be a one-dimensional random variable measuring mathematical ability, in reality Θ is two dimensional with the first dimension being mathematical ability and the second dimension being verbal ability. In the case of psychological testing, the most important statistical problem concerning dimensionality is to test H : d = 1 vs. A : d > 1. Recently, this author has constructed a statistic to test this hypothesis and to be further used as an index that estimates the amount of regularity in the data attributable to the multidimensionality of Θ .

Illustration

It is rather easy to imagine applications in other fields. As an illustration, suppose that medical subjects (the units) undergo allergy sensitivity tests to various environmental substances (each such test is a treatment). Suppose that the result of each test is scored 1 or 0, depending on whether an al-

[7]

lergic reaction is observed or not. Let different values of the parameter θ be assigned to subjects according to each subject's sensitivity. Then, inferences about the dimensionality of θ become meaningful in attempting to develop a classification scheme for allergies.

Description of the Statistic

A description of the constructed statistic can now be given. In doing so, the psychological testing language of items, persons, and so forth, will be used.

- 1. The test being administered is split into two subtests of lengths M and n, respectively. Here, n should be considered as large and M as possibly not large. Let f_n denote the proportion correct on the second subtest of items M + 1, M + 2, ..., M + n.
- 2. [0,1) is partitioned into intervals

$$\bigcup_{k} A_{n}^{(k)} = [0,1)$$
 [8]

such that

$$\max_{k} \{ width (A_{n}^{(k)}) \} + 0 \text{ as } n + \infty .$$
 [9]

For example, let

$$A_{n}^{(k)} = \left[\frac{k-1}{\binom{n}{2}}, \frac{k}{\binom{n}{2}}\right] \qquad k = 1, 2, ..., [n^{\frac{n}{2}}] \qquad (10)$$

where [x] denotes the integer m such that m < x.

3. Persons are now grouped into categories according to the following rule: Assign a person to category (k,n) if for that person

$$f_n \in A_n^{(k)}$$
 $k = 1, 2, ..., K_n$. [11]

(Here, K_n denotes the number of categories.) Thus, persons are assigned to the same category if they all get about the same proportion correct. This categorization of persons is the only use made of the second subtest. Let $J_n^{(k)}$ denote the number of persons in category (k,n).

- 4. To construct the test statistic, take the ratio of two variance estimators, the denominator estimating a variance that is uninfluenced by the "amount" of multidimensionality present and the numerator estimating a variance that is inflated by the amount of multidimensionality present. The variance estimators are each based upon the first subtest, i.e., on Items 1, 2, ..., M.
- 5. Now, fix (k,n). That is, look at the persons in cell k of the

<u>nth</u> partition $\{A_n^{(1)}, A_n^{(2)}, \dots, A_n^{(K_n)}\}, K_n$ denoting the number of partition cells.

6. The denominator can now be constructed. Consider item \underline{n} (of the first subtest, hence, $1 \leq m \leq H$. Let

$$\hat{P}_{m}^{(k)} = \sum_{j=1}^{J} U_{mj}^{(k)} , \qquad [12]$$

where Uni indicates that correctness of the response of the ith person of cell k to item m. Let

$$\hat{\sigma}_{Pk}^{2} = \sum_{m=1}^{M} \hat{P}_{m}^{(k)} (1 - \hat{P}_{m}^{(k)}) / M^{2}, \qquad [13]$$

the denominator estimator of variance. Note that persons have been

summed over first, forming $\hat{p}^{(k)}_{m}$ and then items, forming $\hat{\sigma}_{Pk}^{2}$. 7. For the numerator, let $g_{j}^{(k)}$ be the proportion correct for person j on the first subtest, i.e.,

$$g_{j}^{(k)} = \sum_{m=1}^{M} U_{mj}/M.$$
 [14]

Let

$$\overline{g}^{(k)} = \sum_{j=1}^{n} g_{j}^{(k)} / J_{n}^{(k)}$$
[15]

and

$$\hat{\sigma}_{gk}^{2} = \sum_{j=1}^{n} (g_{j}^{(k)} - \overline{g}^{(k)}^{2}) / J_{n}^{(k)}, \qquad [16]$$

the numerator estimator of variance. Note that items have been summed over first, forming $g_4^{(k)}$ and then persons, forming $\hat{\sigma}_{gk}^2$. 8. For the estimator let

$$\mathbf{F}_{\mathbf{k}} = \hat{\sigma}_{\mathbf{g}\mathbf{k}}^2 / \hat{\sigma}_{\mathbf{P}\mathbf{k}}^2 .$$
 [17]

Thus, for each cell k, a statistic P_k is obtained. The $\{P_k\}$ are independent random variables.

The Asymptotic Distribution of $\{F_{L}\}$

In order to use the $\{F_k\}$ to make inferences about dimensionality, their asymptotic distribution is needed. To this end, the author has shown that for any K cells indexed by 1, 2, ..., K there exists $c_k > 0$ such that

$$\frac{K}{k=1} \frac{\frac{F_{k}-1}{c_{k}}}{\sqrt{K}}$$
[18]

is asymptotically normal with mean zero and variance one [notationally N(0,1)] when d = 1 and, moreover, estimators \hat{c}_k of c_k exist such that

$$\frac{K}{\sum_{k=1}^{r} \frac{F_{k}-1}{c_{k}}} / \sqrt{K}$$
[19]

is asymptotically N(0,1) when d = 1. Further, it has been shown that there exists a number C > 0 and numbers $A_{M,k} \ge CM$ such that $F_k + A_{M,k}$ in probability for $k = 1, 2, \ldots, K$ when d > 1. Hence, there exists a valid large sample level α procedure for testing H : d = 1 vs. A : d > 1.

It also follows that this procedure (even in the extreme case of K = 1) for an appropriate choice of M has asymptotic power one for any fixed alternative, i.e., any distribution of Θ for which d > 1. The procedure is to reject H if

$$\frac{K}{k} = \frac{k}{c_k} / \sqrt{K} > Z_{\alpha}, \qquad [20]$$

where Z_{α} is the 100 (1 - α) percentile of a standard normal distribution.

Discussion

There remain several important theoretical and practical questions that should be investigated. First, there are clearly several plausible ways of combining the F_k 's into a single test statistic and of obtaining the asymptotic distribution of this test statistic. Three such possibilities are

1. $z_k \frac{F_k - 1}{\hat{c}_k} / \sqrt{K}$

as was shown above;

^{2.}
$$z_k \left(I \left[\frac{r_k - 1}{\hat{c}_k} \rightarrow z_\alpha \right] - \alpha \right) / \sqrt{\kappa}$$

where I[A] denotes the indicator of the event A; and

3. A chi-square like statistic $[\Sigma_k(0_k - F_k)^2/E_k]$ based upon the number of k's such that

$$\frac{F_{k}-1}{\hat{c}_{k}} > Z_{\alpha} .$$

The author plans to investigate the asymptotic distributions of the second and third of these statistics as well.

Second, it is essential to carry out some carefully designed monte carlo studies to see for what range of test lengths and sample sizes of examinees the actual distribution of the F_k 's is well approximated by the asymptotic distribution of the F_k 's. This is essential because asymptotic distribution theory cannot by itself guarantee the accuracy of the approximation that it suggests.

Third, the meaningful and practical question is not whether d = 1 but, rather, whether taking d = 1 accounts for most of the explainable regularity in the data. Thus, what is called for is a reformulation of the hypothesis that d = 1 and possibly an estimation approach in order to estimate how much of the explainable regularity is accounted for by taking d = 1. This important practical concern needs to be dealt with by some combination of a theoretical analysis and a monte carlo study.

Fourth, some combination of a theoretical analysis and a monte carlo study is also needed so that some quantitative information is available about the power of the tests constructed from the F_{L} 's.

Fifth, the "regularity" conditions that were needed on the rate of growth of the $\{J_{n}^{(k)}\}$ (numbers of persons per cell) as $n \rightarrow \infty$ in order to establish the asymptotic normality of the F_{k} 's--and, hence, the asymptotic distribution of the statistics described above--can undoubtedly be improved upon. This would further strengthen the case for using the F_{k} 's in actual testing situations. Moreover, it is quite possible that the methods of proof used or the results obtained when abstracted from the present situation involving the F_{k} 's may add to the general body of knowledge in mathematical statistics.

Sixth, the procedures that are obtained from carrying out the above should be pilot tested on actual tests and populations.

Seventh, a thorough comparison between these procedures based on the F_k 's and on any other approaches (such as factor analytic) in the literature must be made.

Finally, procedures should also be developed for testing H : d = k vs. A : d > k for fixed $k \ge 2$. Although the derivation of the distribution of the F_k 's under the assumption d = 1 was surprisingly delicate, it seems clear that an analogous procedure for this hypothesis testing situation can be found and its properties studied.

The author plans to investigate these questions with the goal of producing a theoretically sound and practically important statistical approach to the problem of making inferences about the underlying dimensionality.

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