AD-A140 527 MULTI-LENGTH SCALE TURBULENCE MODELS(U) BOEING MILITARY AIRPLANE CO SEATTLE WA S F BIRCH AUG 83 AFOSR-TR-84-0249 F49620-82-C-0032 1/1

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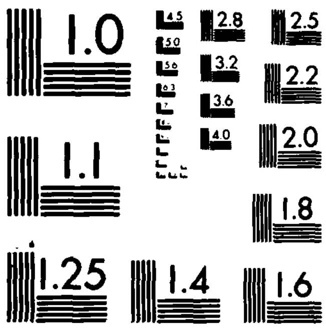

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Multi-length Scale Turbulence Models

By

Stanley F. Birch
August 1983

Boeing Military Airplane Co.
P. O. Box 3999
Seattle , Washington 98124

Final Report AFOSR Contract No. F49620-82-C-0032

REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER AFOSR-TR- 84-0249	2. GOVT ACCESSION NO. AD-A140 527	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MULTI-LENGTH SCALE TURBULENCE MODELS		5. TYPE OF REPORT & PERIOD COVERED FINAL 15 Apr 82 - 18 Jul 83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) STANLEY F. BIRCH		8. CONTRACT OR GRANT NUMBER(s) F49620-82-C-0032
9. PERFORMING ORGANIZATION NAME AND ADDRESS BOEING MILITARY AIRPLANE COMPANY P.O. BOX 3999 SEATTLE, WASHINGTON 98124		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2307/K2
11. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA BOLLING AFB, DC 20332		12. REPORT DATE August 1983
		13. NUMBER OF PAGES 49
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

TURBULENCE MODELS
LENGTH SCALE EQUATIONS
JETS
MIXING LAYERS
INITIAL CONDITIONS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Development of a new two-length scale turbulence model is described. This work was undertaken because the performance of current models for certain flows appears to be limited by an inadequate treatment of the turbulence length scale. One flow for which current turbulence models are not adequate is the initial developing region of a plane mixing layer. Available mixing layer data is briefly reviewed. An improved ability to analyze this flow is required for improved prediction of the near field of a jet for a wide range of applications including those for STOL aircraft applications. The work was performed in the

context of developing an improved turbulence model for general application to complex three-dimensional jets.

The new model is based on the physical observation that the turbulence shear stress and the turbulence energy production are associated primarily with the large scale eddies, while most of the turbulence energy is dissipated by small scale eddies near the high frequency end of the turbulence energy spectrum. Therefore, except for flows close to equilibrium, separate turbulence length scales are required to characterize the large and small scale motions.

Both a one-dimensional and a two-dimensional turbulence model were developed around this concept. The addition of a second length scale is equivalent to adding an elliptic term to the model, and an elliptic procedure was developed in a relaxation scheme for solving the set of equations. Many of the flows for which the new model is needed are already elliptic, because of reverse flows or elliptic pressure effects, and use of an elliptic turbulence model will not add to the cost of calculating these flows. It is felt that the importance of elliptic effects has probably been underestimated in most previous turbulence model studies.

From the results obtained, the overall behavior of the two-length scale model appears very encouraging. Results shown provide interesting insights into the behavior of the mixing layer over the initial development region. While a considerable effort was made to obtain an optimum set of model constants, the major effort was concentrated on a study of the initial development region of a mixing layer and successfully demonstrating that the addition of a second length scale equation extends the range of application of current turbulence models. A number of issues that require more detailed study remain, including an extensive evaluation and refinement of the model constants for a wider range of flows.

Modeling problems tend to be interrelated in the sense that a modification that leads to an improvement for one flow may yield worse predictions for another flow. Currently different constants are necessary for predictions of planar and axisymmetric jets. This problem needs resolution to obtain an understanding of the physical reasons for planar vs axisymmetric modeling differences. A balanced judgement can then be formed on how best to proceed in the development of an improved turbulence model for complex three-dimensional flows.

ACKNOWLEDGEMENT

As with most research efforts, the present work draws heavily on previous work. Much of the detailed review and analysis of mixing layer data discussed in this report was performed as part of the preparations for the 1980-81 AFOSR-HTTM Stanford Conference on Complex Turbulent Flows. The review and the subsequent work described in this report has benefited greatly from inputs from numerous individuals, in particular P. Bradshaw, A. K. M. F. Hussain and S. J. Kline, I am also greatful to I. E. Strom and J. L. Julum for their help in solving programming problems that arose during the present work.

This work was supported by contract no. F49620-82-C-0032 from the Air Force Office of Scientific Research. The contract was managed by James D. Wilson, whose encouragement and interest in the work are greatly appreciated.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1.0 SUMMARY AND INTRODUCTION

1.1 SUMMARY

Development of a new two-length scale turbulence model is described. This work was undertaken because the performance of current models for certain flows appears to be limited by an inadequate treatment of the turbulence length scale. One flow for which current turbulence models are not adequate is the initial developing region of a plane mixing layer. Available mixing layer data is briefly reviewed. An improved ability to analyze this flow is required for improved prediction of the near field of a jet for a wide range of applications including those for STOL aircraft applications. The work was performed in the context of developing an improved turbulence model for general application to complex three-dimensional jets.

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Both a one-dimensional and a two-dimensional turbulence model were developed around this concept. The addition of a second length scale is equivalent to adding an elliptic term to the model, and an elliptic procedure was developed in a relaxation scheme for solving the set of equations. Many of the flows for which the new model is needed are already elliptic, because of reverse flows or elliptic pressure effects, and use of an elliptic turbulence model will not add to the cost of calculating these flows. It is felt that the importance of elliptic effects has probably been underestimated in most previous turbulence model studies.

From the results obtained, the overall behavior of the two-length scale model appears very encouraging. Results shown provide interesting insights into the behavior of the mixing layer over the initial development region. While a considerable effort was made to obtain an optimum set of model constants, the major effort was concentrated on a study of the initial development region of a mixing layer and successfully demonstrating that the addition of a second length scale equation extends the range of application of current turbulence models. A number of issues that require more detailed study remain, including an extensive evaluation and refinement of the model constants for a wider range of flows.

Modeling problems tend to be interrelated in the sense that a modification that leads to an improvement for one flow may yield worse predictions for another flow. Currently different constants are necessary for predictions of planar and axisymmetric jets. This problem needs resolution to obtain an understanding of the physical reasons for planar vs axisymmetric modeling differences. A balanced judgement can then be formed on how best to proceed in the development of an improved turbulence model for complex three-dimensional flows.

1.2 GENERAL DISCUSSION

It is now generally accepted[1] that the Navier-Stokes equations are the appropriate equations for the description of turbulent flow. In this sense, the physics of turbulence is completely understood. If there existed a fast and economical method for solving these equations, interest in turbulent research as a scientific discipline would quickly disappear. The great difficulty in solving the Navier-Stokes equations for most flows of interest, however, has all but eliminated any near term prospect of using these equations directly for the solution of practical problems.

The problem that we are faced with then is in finding a simpler set of equations that provide an approximation for the behavior of real flows and that are simple enough to be tractable using present computers. There is, of course, no guarantee that any such set of equations exist. Nevertheless there have been major advances in our ability to calculate the behavior of viscous flows over the last ten or fifteen years and there is no obvious reason why this progress should not continue. The extent of this progress is well illustrated by a comparison of the results of the 1968 Stanford meeting with the results obtained at the recent 1980-81 Stanford meeting.

In spite of the success of current models in calculating the behavior of complex turbulent flows, their performance for certain flows appears to be limited by an inadequate treatment of the turbulence length scale. All current turbulence models -- or at least all those that have been developed enough to be useful for practical calculations -- use a single turbulence length scale. This implies a universal turbulence energy spectrum shape and is probably not physically reasonable for even a limited range of flows.

One flow for which current turbulence models are not adequate is the initial developing region of a plane mixing layer. This flow can be regarded as an idealization of the near field of an axisymmetric jet. Since a free mixing layer entrains mass, it acts as a sink and thus influences the local external flow. The magnitude of these effects depends on the initial conditions and the geometry of the flow, but can, under certain circumstances, be quite important. For example, many STOL aircraft in design or development use the strong coupling between the jet engine exhaust and the inviscid outer flow to achieve enhanced landing and take-off performance, or to improve maneuvering capability.[2,3] This coupling changes the circulation on the wing and, hence, the aerodynamic characteristics of the aircraft. Since adequate methods for predicting these strongly coupled jet/aerodynamic surface interactions do not exist at present, design of these aircraft is achieved through expensive parametric wind tunnel testing. In addition, many of these tests must be run at full scale because of the problems associated with attempts to scale-up model scale test data. The understanding and prediction of the near field mixing region is thus of very practical as well as scientific interest. The work described in this report is a study of a new two length scale turbulence model with primary emphasis on the developing region of a plane mixing layer. The plane mixing layer was selected for detailed study, in part because it is a relatively simple, well documented flow for which current single length scale models are clearly inadequate; but primarily because an improved ability to analyze this flow is required for improved prediction of the near field of a jet for a wide range of applications including those for STOL aircraft.

The work described in this report is an attempt to improve the prediction of the relaxation region of a mixing layer by the addition of a second turbulence length scale. This work was performed in the context of developing an improved turbulence model for general application to complex three-dimensional jets. Before discussing work on the new model the available mixing layer data is briefly reviewed.

2.0 MIXING LAYER

2.1 FULLY DEVELOPED REGION

The radius of an axisymmetric jet is generally large enough compared with the width of the mixing layer so that lateral curvature effects can be ignored for a distance of about two diameters downstream of the nozzle exit. The mixing layer in the near field of an axisymmetric jet is often idealized as a planar layer between two semi-infinite streams. In practice, there is little difference between the development of at least the mean velocity profiles of the two flows until near the end of the potential region. Therefore, in the present review both flows will be discussed together.

It is convenient to divide the mixing layer into an initial developing region and a fully developed region (Figure 1). The developing region is a transition region between an initial wall boundary layer flow and the fully developed flow farther downstream. At lower Reynolds numbers, when the wall boundary layer is laminar, the developing region will also involve a transition from laminar to turbulent flow. The flow in the fully developed region is self-similar and spreads linearly with downstream distance.

Although the mixing layer is geometrically simple, it has caused considerable problems experimentally. So much so, in fact, that until recently the variation in the reported data was such that there was no general agreement that a unique asymptotic flow, independent of initial conditions, existed.

Some of this experimental scatter can be attributed to experimental error or to confusion arising from the use of different definitions for the width of the mixing layer; but the major problem was an underestimate of the persistence of the effects of initial conditions and the experimental difficulty of achieving a fully developed flow, particularly for a mixing layer developing from a turbulent wall boundary layer.

Bradshaw(4) studied the influence of initial conditions earlier and particularly noted that tripping the initial wall boundary layer would extend

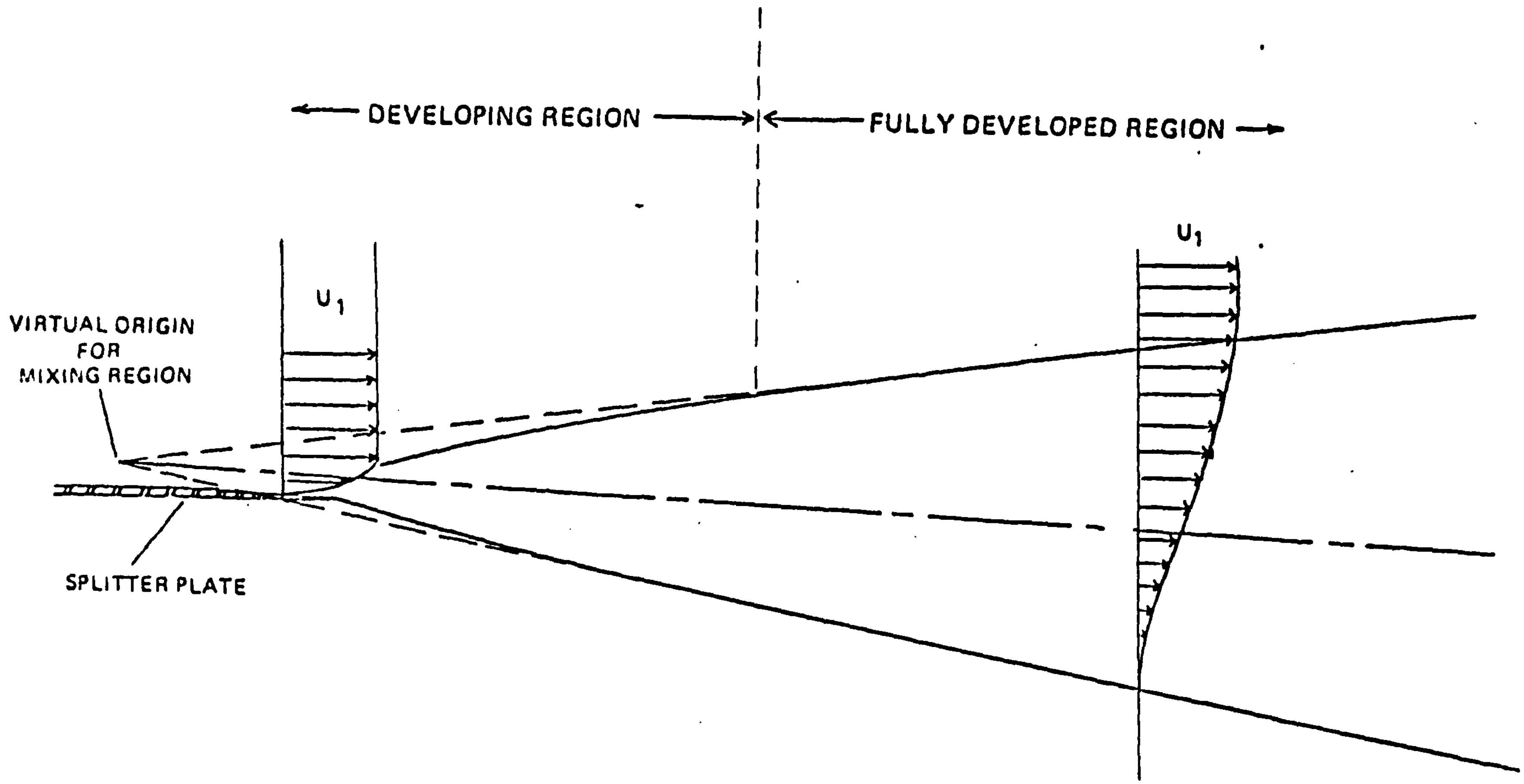

FIGURE 1. FREE MIXING LAYER

the length of the developing region; but the importance of these results was not fully appreciated until much later. The nature of the problem is probably best illustrated by a plot of the reported spreading rate of nominally fully developed mixing layers as a function of a Reynolds number based on downstream distance. This is shown in Figure 2. The tabulated data are given in Table 1. The definition of the width of the mixing layer is the distance between the point at which the velocity squared is 0.1 and 0.9 times the freestream value.

Note that although the plotted spreading rates differ by more than 50%, there is little evidence of genuine data scatter. The flows developing from laminar wall boundary layers appear to reach a constant spreading rate for values of Re_x greater than about $7x10^5$ -- the value originally suggested by Bradshaw. The effect of tripping the boundary layer is also well illustrated in this plot. When the wall boundary layer is turbulent at the separation point, the mixing layer does not appear to become fully developed until a value of Re_x of greater than $2x10^6$.

For most of these experimental studies, the initial conditions are not sufficiently well documented to make a direct comparison of the various data sets very meaningful. There are, however, four sets of data (references 5, 19,37) for which the initial conditions are well documented. All four flows develop from fully developed, turbulent, wall boundary layers. These data sets, normalized by the momentum thicknesses of the boundary layers at their separation points, are shown in Figure 3. The solid line is an estimate of the asymptotic mixing rate slope. The lack of scatter in this data is even more surprising when one realizes that two of the flows are axisymmetric and two are planar. These flows do not become fully developed for a distance of at least 1500 Θ, where Θ is the momentum thickness of the boundary layer at its separation point.

Apart altogether from the fact that most jets encountered in practical applications develop from turbulent wall boundary layers, jets developing from laminar wall boundary layers are not, in general, suitable for evaluating numerical calculations. The problem is that the thin laminar shear layer just downstream of the separation point is very sensitive to external disturbances,

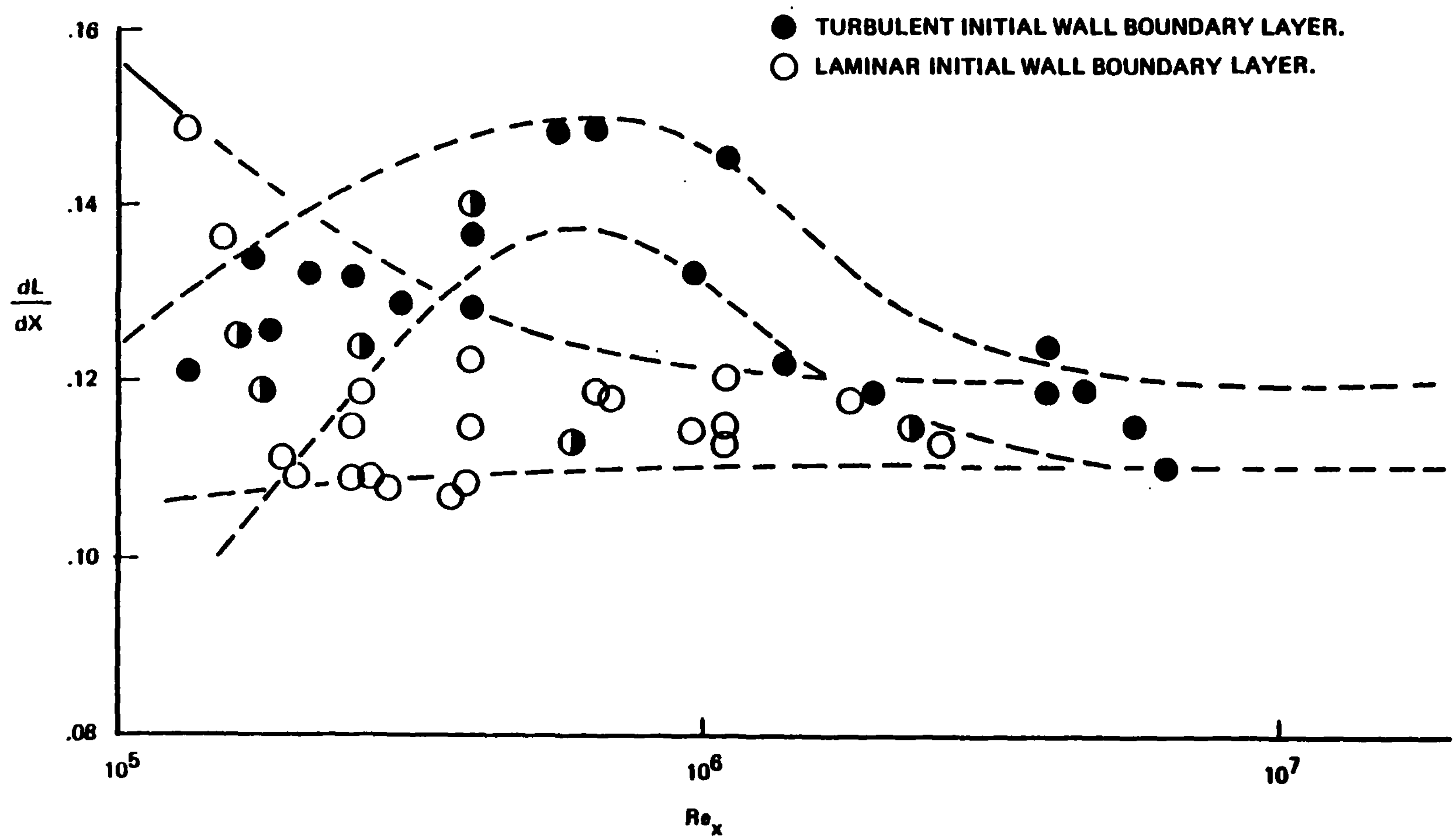

Figure 2. Spreading Rate of Single Stream Mixing Layers

INCOMPRESSIBLE SINGLE STREAM MIXING LAYERS

INVESTIGATOR	INITIAL CONDTIONS	$Re_x \times 10^{-6}$	$\frac{dL}{dX}$	REFERENCE
TOLLMIEN	?	2.3	.115	13
CORDES	?	0.6	.113	14
REICHARDT	?	2.8	.098	15
LIEPMANN & LAUFER	LAMINAR	1.1	.113	16
MAYDEW & REED	TURBULENT	4.6	.119	17, 18
		5.6	.115	
		6.3	.110	
BRADSHAW	LAMINAR	0.7	.118	4
GARTSHORE	TURBULENT	1.4	.121	19
ILIZAROVA	TURBULENT	?	.110	20
MILLS	?	0.175	.119	21
	?	0.26	.124	
SUNYACH & MATHIEU	LAMINAR (?)	0.16	.125	22
WYGNANSKI & FIEDLER	TURBULENT	0.47	.148	23
PATEL	TURBULENT(?)	1.8	.118	24
SPENCER	LAMINAR	2.6	.113	25, 26
BATT, KUBOTA & LAUFER	LAMINAR	0.66	.119	27
	TURBULENT	"	.148	
JOHNSON	LAMINAR	0.13	.148	28
CASTRO	LAMINAR	1.1	.115	29, 30
FIEDLER	LAMINAR	0.4	.122	31
CHAMPAGNE, PAO & WYGNANSKI	TURBULENT (?)	0.4	.138	32
BIRCH	TURBULENT	4.0	.119	33
	"	2.0	.119	
	"	4.0	.124	
OSTER, WYGNANSKI &	LAMINAR	1.1	.120	34
FIEDLER	TURBULENT	1.1	.145	
HUSSAIN & ZEDAN	NOMINALLY	0.37	.107	35, 36
	LAMINAR	0.25	.109	
	↓	0.27	.109	
	↓	0.19	.110	
	↓	0.25	.115	
	↓	0.40	.115	
HUSSAIN & ZEDAN	TURBULENT	0.18	.126	
		0.25	.132	
		0.40	.136	
		0.31	.128	
		0.40	.128	
HUSSAIN & ZEDAN	[illegible]LLY LAMINAR,	0.39	.108	
	[illegible] & 8 ARE	0.29	.108	
	[illegible]S HIGHLY	0.20	.109	
	ST[illegible]	0.26	.119	
		0.132	.121	
		0.21	.132	
		0.17	.134	
		0.15	.136	
HUSAIN & HUSSAIN	LAMINAR	0.97	.115	37
	TURBULENT	"	.132	

TABLE 1. INCOMPRESSIBLE SINGLE STREAM MIXING LAYERS

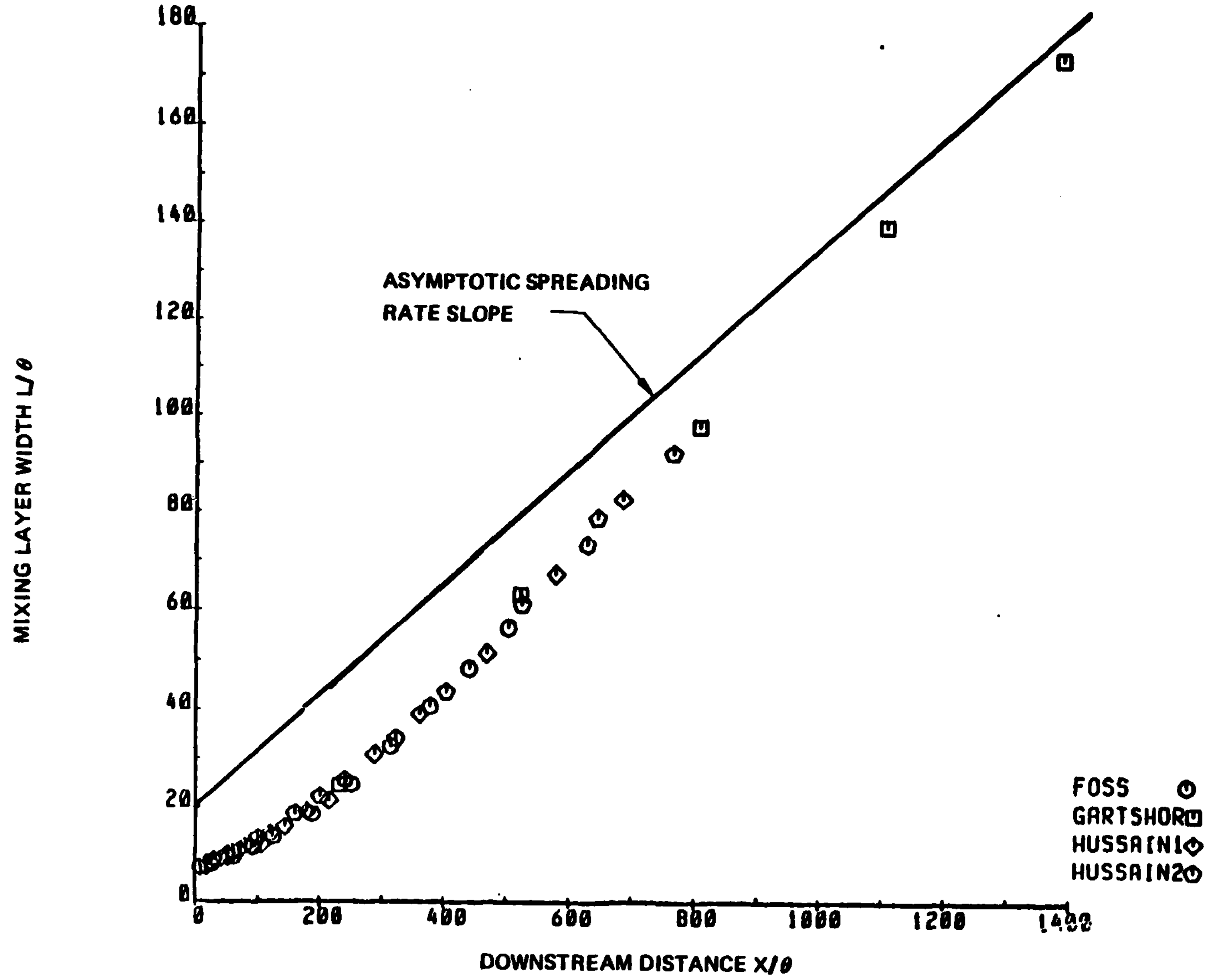

Figure 3. Spreading of a Planar Mixing Layer Developing from a Fully Developed Turbulent Wall Boundary Layer

including freestream turbulence, noise and mechanical vibrations. Since it is difficult to completely document or control these sources of disturbances, the experimental data for such experimental studies tend to be apparatus dependent. For a detailed discussion of some of these problems see references 6 and 7.

2.2 DEVELOPING REGION

A plot of the local spreading rate, obtained by directly differentiating the data in Figure 3 is shown in Figure 4. Again note the surprisingly small scatter in the data. The first point to be made about the developing region is its length. For a fully turbulent initial wall boundary layer, a single stream mixing layer does not appear to reach its asymptotic state for a distance of about 1500 O downstream of the separation point. For more strongly perturbed flows, this distance can be considerably longer. The initial spreading rate for this flow is approximately half its fully developed value. The spreading rate increases steadily with downstream distance, overshoots its fully developed value, and at 700 O is about 20% larger than its asymptotic value. From then on, it slowly relaxes back to equilibrium. The maximum local spreading rate is, incidentally, almost exactly what one would infer from the data of Figure 2.

The second point of interest is the tendency of the turbulence to overshoot during the relaxation process, although the precise behavior will, of course, depend on details of the flow. For example, for a two stream mixing layer[8] with a velocity ratio of 0.3, the turbulence overshoots when the initial boundary layer is laminar; but for a turbulent initial boundary layer it appears to approach its asymptotic state monotonically. With strong excitation over the initial region, the shear stress will undershoot during the relaxation process. If the excitation is strong enough, the shear stress may collapse completely, leading to a region of almost no growth.[9] The important point is that the mixing layer behaves like an underdamped second order system and it can only be meaningfully discussed from this point of view. A discussion, for example, of whether a given perturbation increased or decreased the mixing rate is therefore not very useful. The fully developed

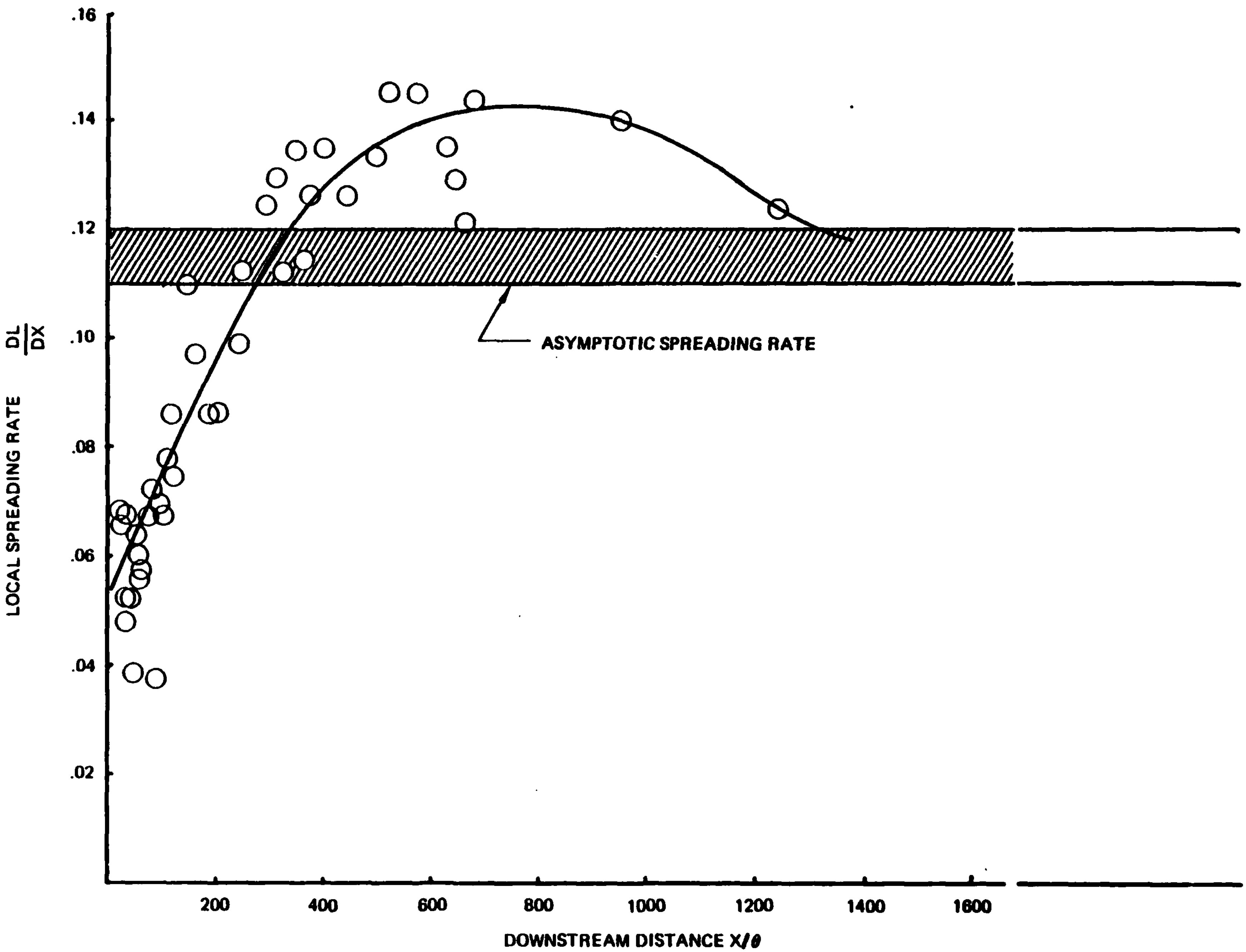

Figure 4. Local Spreading Rate of a Planar Mixing Layer Developing from a Fully Developed Turbulent Wall Boundary Layer

flow will always remain unchanged, while the flow in the relaxation region will generally contain regions that have spreading rates that are both higher and lower than the fully developed value, irrespective of the perturbation.

An interesting consequence of this behavior is that, although there may be large variations in the local spreading rate, the spreading rate averaged over the whole region is often close to the fully developed value. For example, Foss[5] studied the development of a planar mixing layer developing from both laminar and fully developed turbulent wall boundary layers in the same experimental apparatus. The results showed that although the local spreading rate was different over the whole region, depending on whether the flow was initially laminar or turbulent, the total width of the layer at the last station was almost the same for both flows. Similar results for an axisymmetric mixing layer are reported by Hussain.[37]

3.0 MULTI LENGTH SCALE MODELS

3.1 THEORETICAL BASIS

The theoretical basis of the approach used here for the development of the new two length scale turbulence model is described in detail in references 10-12. In brief, it is based on the idea that turbulent flow tends toward a dynamic equilibrium characterized by a critical Reynolds number,

$$\frac{UL}{\nu^*_T} \tag{1}$$

where U and L are characteristic velocity and length scales for the mean flow and ν_T^* is the equilibrium effective turbulent viscosity.

If the equilibrium viscosity is written as ν_T^*, then it seems reasonable to expect that the local eddy viscosity can be written in terms of a series expansion about this value. In one dimension this is simply

$$\nu_T^* = \nu_T + F_1 \frac{d\nu_T}{dx} + F_2 \frac{d^2\nu_T}{dx^2} + \ldots \tag{2}$$

The simplest approximation, assuming that $\nu_T = \nu_T^*$, is equivalent to the use of a local equilibrium model. The next simplest approximation is

$$\nu_T^* = \nu_T + F_1 \frac{d\nu_T}{dx} \tag{3}$$

or

$$F_1 \frac{d\nu_T}{dx} = \nu_T^* - \nu_T \tag{4}$$

If one assumes that the turbulence viscosity is a funcion of the turbulence kinetic energy 'k' and a turbulence length scale 'ℓ',

$$\nu_T = \text{const.}\ k^{1/2}\ell \tag{5}$$

then because

$$\frac{1}{\text{const.}} \frac{D\nu_T}{Dt} = \frac{m-2n}{2m} \frac{\ell}{k^{1/2}} \frac{Dk}{Dt} + k^{(1/2-n)} \frac{\ell^{(1-m)}}{m} \frac{D(k^n \ell^m)}{Dt} \tag{6}$$

equation (4) can be regarded as a contraction of a model based on the solution of differential equations for 'k' and a length scale containing quantity $k^n \ell^m$. The two equation models can in turn be regarded as a simplification of a complete Reynolds stress model.

3.2 ONE DIMENSIONAL MODEL

From equation (2), we would expect that a second order approximation would lead to a turbulence model with an improved range of application. In terms of Eq. (2), this means models of the form

$$\nu_T^* = \nu_T + F_1 \frac{d\nu_T}{dx} + F_2 \frac{d^2\nu_T}{dx^2} \tag{7}$$

Since there are advantages in formulating a turbulence model from conservation type equations, we will do this starting with the first order model equation (4), written in a slightly different form.

$$\bar{u} \frac{d\nu_T}{dx} = A\nu_T \frac{\partial u}{\partial y} - B \frac{\nu_T^2}{L^2} \tag{8}$$

where A and B are constants, u is the average convection velocity and L is a local length scale.

A major problem with this model is the presence of a local length scale 'L' in the dissipation term. This suggests adding a second differential equation for this length scale or for the dissipation term, D, itself. A simple version of the resulting model is

$$\bar{u}\,\frac{d\nu_T}{dx} = A\nu_T\,\frac{\partial u}{\partial y} - D \tag{9}$$

$$\bar{u}\,\frac{dD}{dx} = BD\,\overline{\frac{\partial u}{\partial y}} - C\frac{D^2}{\nu_T} \tag{10}$$

where A, B, and C are constants. Note that differentiating Eq. (9) with respect to "x" and substituting for dD/dx from Eq. (10) yields an equation of the same form as Eq. (7) containing a term of the form $F_2\ \partial^2\nu_T/\partial x^2$.

These equations can be written in nondimensional form by replacing u by $(u_1+u_2)/2$ and $\partial u/\partial y$ by $(u_1-u_2)/L$ to give

$$\frac{1}{\nu_T}\,\frac{d\nu_T}{dx} = 2A\lambda\ (1 - \frac{D}{P}) \tag{11}$$

$$\frac{1}{D}\,\frac{dD}{dx} = 2B\lambda\ (1 - \frac{D}{P}) \tag{12}$$

where A, B, and C are again constants, $\lambda = \frac{u_1-u_2}{u_1+u_2}$ and $P = \nu_T\,\frac{u_1-u_2}{L}$

A two-dimensional version of the model can obviously be derived by replacing Eqs. (9) and (10) by their two-dimensional equivalents. Although one of these more complex versions of the model will be needed for some practical applications, the present discussion will be confined to the one-dimensional model given here.

3.2.1 Physical Implications

Before discussing the performance of the second order model derived above in detail, the physical implications of adding an equation for the dissipation will be briefly considered. Adding an equation for the dissipation 'D' in combination with an equation for the viscosity 'ν_T' is equivalent to adding a length scale equation, since a length scale can be inferred from the ratio $(\nu_T^2/D)^{1/2}$. But since 'ν_T' already contains a length scale, which has been identified with the energy containing large eddy structure, the change to a second order model involves the addition of a second length scale. This second length scale is used to characterize the dissipation process. At high Reynolds number, most of the energy is concentrated near the low frequency end of the turbulence energy spectrum while most of the dissipation takes place at the high frequency end of the spectrum. The two length scales used in the present turbulence model, therefore, constitute a simple two parameter description of the turbulence energy spectrum.

In first order models, the length scale which appears in the dissipation term is assumed to be proportional to either a local length scale or to a length scale which characterizes the turbulence energy. Of these two assumptions the second is probably most valid, but even this implies a universal shape for the turbulence energy spectrum, which cannot in general be true. In the present model, since the turbulence energy and the dissipation are characterized by different length scales, this assumption is not required.

The effect of adding a second length scale equation will now be studied by comparing the predictions of the second order model derived here with a range of experimental data.

3.2.2 Comparison with Data

Before it is possible to evaluate the performance of the present turbulence model, the values of the constants A, B, and C, which appear in Equations (9) and (10) must be specified. The values used here are

$A = 0.16$

$B = 0.17$

$C = 0.16$

A comparison between the predictions of the resulting model and experimental data for the spreading rates of three free shear flows are given in Table 2.

Table 2

	SHEAR LAYER dL/dx	2-D JET dL/dx	RADIAL JET dL/dx
2nd Order Model	0.12	.110	.102
Experimental Data	0.115	.105	.110

This shows that the agreement between experimental and prediction, using this model, is good for all three flows; probably within the uncertainty in the data themselves. Compared with the results presented in, for example, reference (12) it is clear that the present turbulence model gives as good or better overall agreement with the experimental data than any other model.

Although these results are encouraging, the most dramatic improvement in prediction ability occurs for strongly perturbed flows. This will be illustrated by considering the initial developing region of a free shear layer. This flow is shown diagramatically in Figure 1. The far field region is the region where the shear layer has become fully developed and is spreading at a constant rate. This is the region already considered above. The region under consideration here is the developing region where the flow changes from a wall boundary layer to a fully developed free shear layer. To the author's knowledge, no previous turbulence model will naturally predict this region of the flow, even qualitatively, correctly.

The flow in this developing region is known to be sensitive, not only to the initial boundary layer, but also to external excitation, such as noise, mechanical vibration, free stream turbulence, etc. This is particularly true when the boundary layer is initially laminar. Since it is not possible to accurately simulate the initial wall boundary layer with a one dimensional model, it is only possible to demonstrate that the present model will predict the correct general behavior of the flow in this region.

Two specific situations will be considered. In one it will be assumed that the wall boundary layer is initially laminar and in the other it will be assumed that the boundary layer is initially turbulent. For the laminar case the calculation is started at a position that corresponds to the region just downstream of transition, where the shear stress has started to increase but where the turbulent dissipation is still small. This is simulated using a value for the turbulence viscosity ν_T which is approximately one quarter of the value it would have if the shear layer were fully developed. The dissipation is assumed to be small but finite. The turbulent initial boundary layer is simulated using the same initial value for the viscosity. In this case, however, the dissipation is selected so that the production and the dissipation are approximtely equal. To show that the prediction is not particularly sensitive to the value of the dissipation selected, a second run was made in which the initial dissipation was increased by a factor of three. This resulted in an initial decrease in the shear stress level for a short distance downstream. The subsequent development of the flow, however, remained the same.

These results are shown in Figure 5. In this Figure, the shear stress shown is the peak shear stress at a given 'x' station. It is normalized by the value it would have in a fully developed shear layer. The Reynolds number Re_x is defined as

$$Re_x = ux/\nu \tag{13}$$

There are two important points to note for the case where the initial boundary layer is laminar. The first is that the shear stress initially rises very fast, overshooting its fully developed value, and then slowly relaxes back farther downstream. The second point to note is that the Reynolds number based on this total distance is approximtely 7×10^5; the value found experimentally by Bradshaw.[4] When the boundary layer is initially turbulent, the shear stress rises more slowly. It again overshoots its equilibrium value, but not by as much as when the boundary layer was laminar. The important difference here is that the length of the development region has approximately doubled. This is again in good agreement with Bradshaw's results. It is also consistent with the experimental data discussed in section 2.

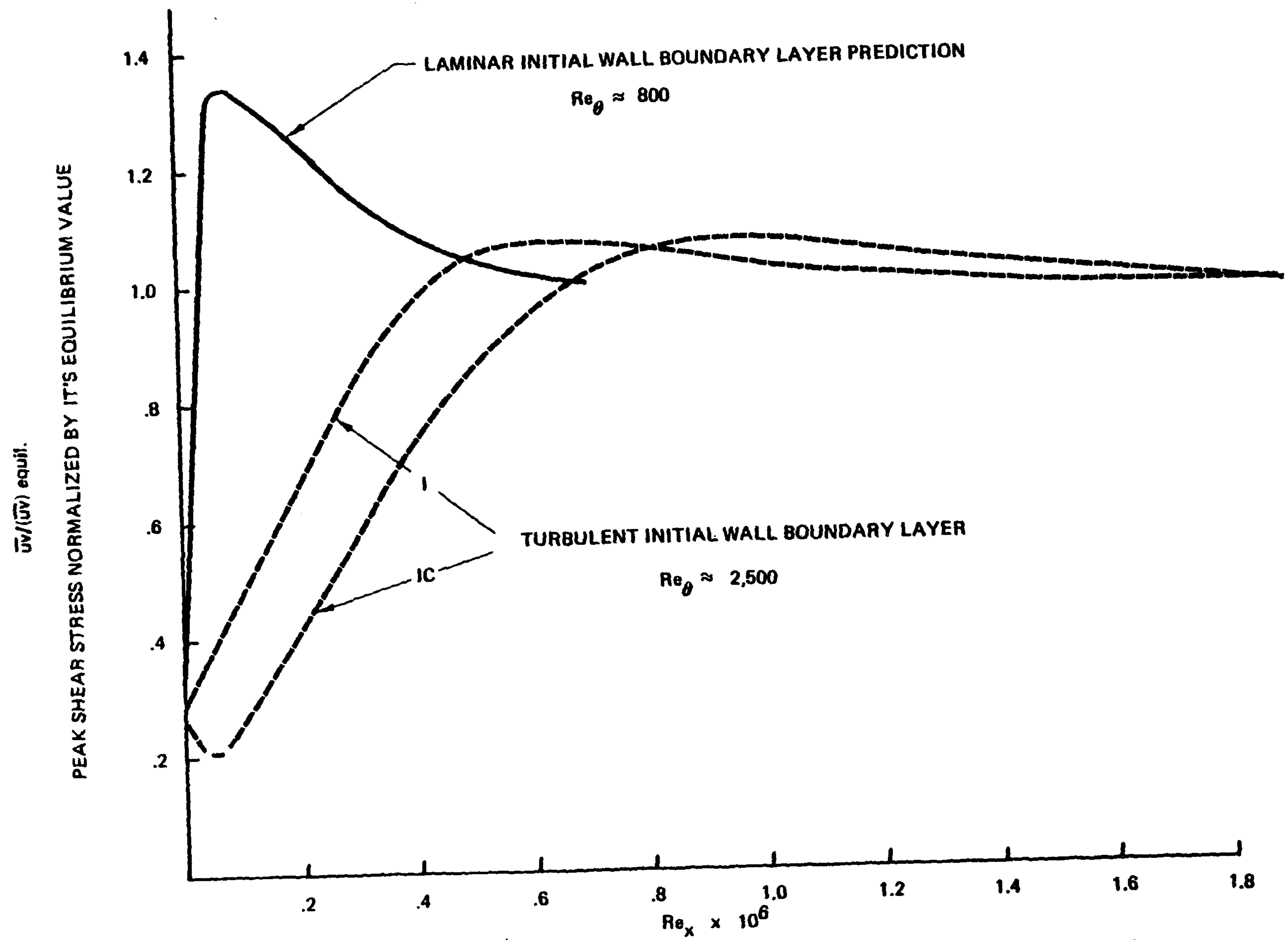

Figure 5. Numerical Calculation of the Developing Region of a Mixing Layer Using One-Dimensional, Two-Length Scale Model

3.3 TWO DIMENSIONAL MODEL

In order to make a new two-dimensional model as compatible as possible with existing models, it was decided to develop the two length scale model as an extension of an existing model. To minimize the complexity at this stage of development, it was decided to start with a two equation model, although eventually it may be desirable to extend the model to a full Reynolds stress model, or at least an algebraic stress model.

Since there appears to be very little basic difference among the various two-equation models it was decided to start with a standard version of the k, kℓ model. The selection of the kℓ equation for a length scale equation was based simply on the belief that it would be less confusing to use an equation for kℓ to calculate the integral length scales than, for example, an equation for the dissipation rate 'ε'.

The particular version of the model that was used here is that developed by Rodi and described in detail in reference 11, and is:

$$\rho\overline{uv} = \mu_t \frac{\partial u}{\partial y}, \qquad \mu_t = C_\mu \rho k^{1/2} \ell \tag{14}$$

$$\rho u \frac{\partial k}{\partial x} + \rho v \frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + \mu_t\left(\frac{\partial u}{\partial y}\right)^2 - C_D \rho \frac{k^{3/2}}{\ell} \tag{15}$$

$$\rho u \frac{\partial (k\ell)}{\partial x} + \rho v \frac{\partial (k\ell)}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu_t}{\sigma_{k1}}\frac{\partial (k\ell)}{\partial y}\right) + C_B \mu_t \ell \left(\frac{\partial u}{\partial y}\right)^2 - C_s \rho k^{3/2} \tag{16}$$

$$C_\mu = 1.0 \qquad C_B = 0.98$$
$$C_D = 0.09 \qquad C_S = 0.058 \tag{17}$$

The basis for using an integral length scale in the dissipation term is the argument that the dissipation scales act merely as an energy sink and that the dissipation rate is determined primarily by the rate at which energy is fed from the large scale energy containing eddies. Therefore, the appropriate length scale for the dissipation term is the same integral length scale that characterizes the large scale energy containing eddies. Although this argument is probably correct in a Lagrangian sense, it does not necessarily follow that the same length scale can be used to model the turbulence energy production and dissipation rates in an Eulerian formulation of the equations.

The reason for this is the finite life of the large eddy structure. The turbulence energy production is necessarily associated with the initial growth phase of the large eddies while the dissipation (or the transfer of energy from the large scales to the small scale dissipation range) is necessarily associated with the decay phase.

For mixing layers and wakes, the width of the mixing region approximately doubles during the life of a typical large eddy. The problem is compounded by the results of reference (38), which suggest that most of the dissipation takes place in a short time near the end of the life of the large eddies. Therefore, because of the finite life of these eddies, the production and dissipation processes associated with a given group of eddies are separated both in space and time. At a given location, the eddies responsible for the turbulence energy production will, in general, differ from those responsible for the dissipation. Therefore, unless the flow is only changing slowly with downstream distance, the length scales characterizing the production and dissipation processes at a given location in the flow will in general be different. Note that this is a somewhat different physical argument than that used by Hanjalic and Launder in reference (39).

In the present model a single additional equation is added for the turbulence energy dissipation rate 'ε'. This equation is:

$$\rho u \frac{\partial \varepsilon}{\partial x} + \rho v \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + C_3 \rho \varepsilon \frac{k^{1/2}}{\ell} - C_4 \rho \frac{\varepsilon^2}{k} \qquad (18)$$

In equilibrium when the rate of production of 'ε' is equal to its rate of destruction this reduces to the conventional expression for 'ε'.

$$C_3 \rho \varepsilon \frac{k^{1/2}}{\ell} = C_4 \rho \frac{\varepsilon^2}{k} \tag{19}$$

$$\varepsilon = \frac{C_3}{C_4} \frac{k^{3/2}}{\ell} \tag{20}$$

For the two length scale model 'ε' is substituted directly into the turbulence energy equation, equation (15) for the dissipation term. For the '$k\ell$' equation, equation (16), the destruction term is rewritten as:

$$C_s \rho k^{3/2} = \text{const.}\ \rho \varepsilon \ell \tag{21}$$

3.3.1 Numerical Method

In the present work, use was made of a modified version of a general two-dimensional parabolic program. This program is part of a series of programs that includes a three-dimensional parabolic program and a three-dimensional wall boundary layer program, together with a number of supporting utility programs. These programs share a common structure and common input and output procedures. A detailed discussion of the three-dimensional parabolic program can be found in references 40 and 41.

The describing equations are written in cylindrical coordinates in the following form:

Continuity

$$\frac{\partial}{\partial x} [\rho r u (r_o - r_I)] + \frac{\partial}{\partial \eta} [\rho r v - \rho r u (\frac{dr_I}{dx} + \eta \frac{d}{dx}(r_o - r_I))] = 0 \tag{22}$$

Momentum

$$\frac{\partial}{\partial x}[\rho r u^2 (r_0 - r_I)] + \frac{\partial}{\partial \eta}[\rho r u v - \rho r u^2 (\frac{dr_I}{dx} + \eta \frac{d}{dx}(r_0 - r_I))]$$

$$= \frac{1}{(r_0 - r_I)} \frac{\partial}{\partial \eta}(\Gamma\ r \frac{\partial u}{\partial \eta}) - \frac{dP}{dx} \tag{23}$$

where Γ is the effective viscosity, r_I and r_0 are the inner and outer radii of the computational domain and

$$\eta = \frac{r - r_I}{r_0 - r_I} \tag{24}$$

Provision is also made for the solution of up to five additional equations written in the general form

$$\frac{\partial}{\partial x}[\rho u r (r_0 - r_I)\phi] + \frac{\partial}{\partial \eta}[\{\rho r v - \rho r u (\frac{dr_I}{dx} + \eta \frac{d}{dx}(r_0 - r_I))\}\phi]$$

$$= \frac{1}{(r_0 - r_I)} \frac{\partial}{\partial \eta}(\Gamma_\phi\ r \frac{\partial \phi}{\partial \eta}) + S_\phi \tag{25}$$

where S_ϕ is the appropriate source term for the general variable ϕ.

The density ρ is obtained from the ideal gas law

$$\rho = P/RT \tag{26}$$

where R is the gas constant and T is the static temperature. The static temperature can be specified as an input or obtained from the solution of an energy equation.

The finite difference equations are formed by integrating the equations over small control volumes that surround the grid nodes. First order dffferencing is used for the convection terms and a hybrid central/upwind differencing scheme is used for the diffusion terms. The resulting equations are solved sequentially using a standard tri-diagonal solver and the solution is iterated between planes to convergence.

3.3.2 Relaxation Procedure

Initial attempts to run the 2-D multi-length scale model led to instability problems on some runs. As is obvious from the previous discussion, the addition of a second length scale is equivalent to adding an elliptic term to the model, although the individual equations are still parabolic. Experience with the 1-D model suggested that the elliptic effect would be small enough to allow the use of a marching solution procedure and, at least if the initial perturbation was not too large, this was probably correct. The problem, however, was that the turbulence model equations are dominated by the source terms and small changes in the adjustable constants can lead to large differences in the behavior of the model. The tendency of the equation set to develop instabilities made the selection of an optimum set of constants extremely difficult, since it was often difficult to determine whether the behavior with a given set of constants was due to an inappropriate selection of constants or to stability problems. After a considerable effort to develop a stable marching procedure, it was eventually decided to develop an elliptic procedure.

The reluctance to use an elliptic procedure initially was due to the fact that it would be more expensive to run than a marching procedure and that this would tend to hamper model development. Although this is true, the disadvantage is not as bad as it might at first seem. First, the use of an elliptic procedure is not necessarily that much more expensive than a marching procedure. Second, many of the flows for which the additional length scale will be needed are already elliptic, either because of reverse flow or because of elliptic pressure effects, so that the use of an elliptic turbulence model will not necessarily add anything to the cost of calculating such flows. Even for many of the simpler self-similar flows such as jets, the normal stress gradients are not really negligible, about 10% of the shear stress gradients, so the argument that they can be ignored is at best questionable and is in reality due more to the mathematical simplification that results than to the fact that the terms are genuinely negligible.

The sensitivity of the model equations to changes in the source terms, initially led to problems in developing an efficient iteration procedure. Simply calculating the dissipation on one sweep and using it on the next sweep did not work even when the iteration procedure was strongly underrelaxed. The iteration procedure eventually developed turned out to converge very fast. On the first sweep the ratio

$$\frac{\varepsilon}{k_o^{3/2}/\ell_o} \tag{27}$$

was calculated and stored. On the second sweep the dissipation term in the energy equation was multipled by the ratio calculated during the first sweep and a new ratio based on the most recent values of the variable was calculated and stored. The dissipation term was therefore

$$\frac{\varepsilon}{k_{i-1}^{3/2}/\ell_{i-1}} \quad \frac{k_i^{3/2}}{\ell_i} \tag{28}$$

where the subscript i refers to the latest iteration.

3.4 RESULTS AND DISCUSSION

In order to obtain some idea of the sensitivity of the standard model to changes in the dissipation term, a calculation for a simple mixing layer was run with the dissipation term set to zero in both equations. The results obtained are shown in Figure 6. It was known from previous work that a reduction in the dissipation term in the turbulence energy equation would lead to a reduction in the shear stress; but it was not anticipated that a reduction in the dissipation term in both equations would also lead to a reduction in the shear stress.

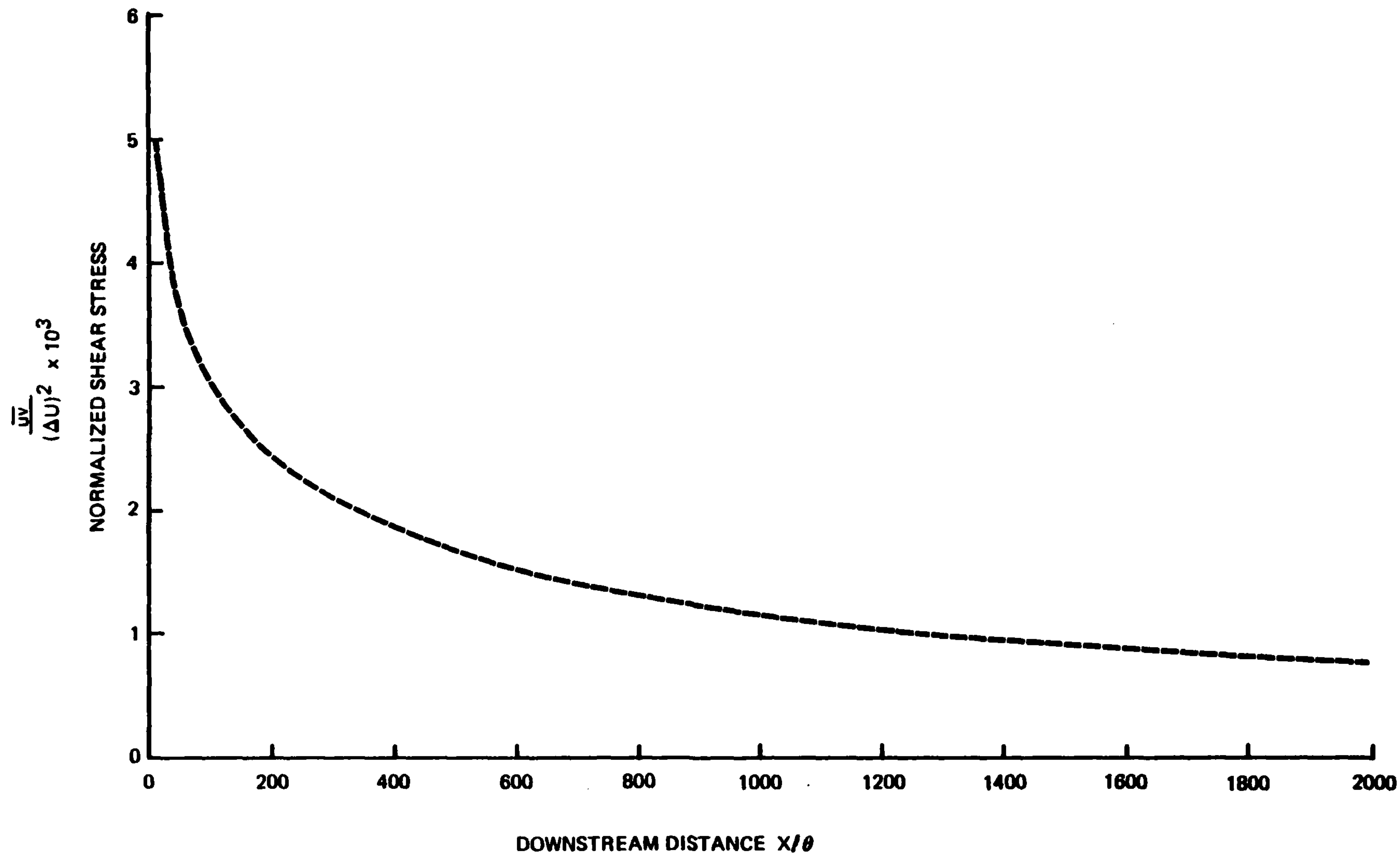

Figure 6. Numerical Calculation of Mixing Layer Wing Standard K, Kl Model with Dissipation Terms Set at Zero

Both equations are source term dominated, so that when the dissipation terms are set to zero both k and $k\ell$ must increase. Since the effective viscosity is assumed to be proportional to the product of $k^{1/2}$ and ℓ, one would expect the effective viscosity also to increase. This does not happen because the length scale "ℓ" used in the definition for the effective viscosity and in the source term for the equation for "$k\ell$" is obtained from the relation

$$\ell = \frac{(k\ell)}{k} \tag{29}$$

What happens when the source terms are set to zero is that "k" increases much faster than "$k\ell$" with the result that "ℓ" and the effective viscosity both decrease. Calculations with a standard version of the k, ε model yielded essentially the same result.

This behavior is obviously not physically meaningful; but it does provide an excellent example of why the modeling of a particular term can only be meaningfully discussed in the context of the model as a whole. It must, after all, be remembered that we are dealing with a coupled set of partial differential equations. What appears to happen in this case is that in a standard two-equation model the production and dissipation terms in both equations are expressed in terms of the single turbulence velocity scale and a single turbulence length scale. The coupling that results from this effectively constrains the ratio τ/k so that it remains approximately constant. When an attempt is made to specify the dissipation independently, however, this constraint is removed and the model starts to exhibit a physically unrealistic behavior.

The simplest solution to this problem is to insert the relation

$$\tau = \text{const. } k \tag{30}$$

directly into the production term in the "$k\ell$" equations to give

$$C_3 \mu_T \ell \left(\frac{\partial u}{\partial y}\right)^2 \times \text{const} \frac{k}{\tau} = \text{const } C_3 k \ell \frac{\partial u}{\partial y} \tag{31}$$

One would not expect this modification to have a major influence on the behavior of the basic two equation model and a limited number of calculations supported this view; but it does dramatically change the behavior of the model when a second length scale is introduced. When the dissipation terms are set to zero in the modified two equation model, a mixing layer calculation yields a self-similar solution and approximately a doubling of the shear stress and the spreading rate.

The resulting turbulence energy, length scale and dissipation equations that constitute the new two-length scale turbulence model, written in cartesian coordinates, are as follows:

$$\rho u\frac{\partial k}{\partial x} + \rho v\frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + \mu_t\left(\frac{\partial u}{\partial y}\right)^2 - \rho\varepsilon \tag{32}$$

$$\rho u\frac{\partial (k\ell)}{\partial x} + \rho v\frac{\partial (k\ell)}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu_t}{\sigma_{k\ell}}\frac{\partial (k\ell)}{\partial y}\right) + C_1\rho(k\ell)\frac{\partial u}{\partial y} - C_2\rho\varepsilon\ell \tag{33}$$

$$\rho u\frac{\partial \varepsilon}{\partial x} + \rho v\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + C_3\rho\varepsilon\frac{k^{1/2}}{\ell} - C_4\rho\frac{\varepsilon^2}{k} \tag{34}$$

$$\mu_t = C_\mu\rho k^{1/2}\ell \tag{35}$$

The constants used for the calculations presented in this report are:

$$C_\mu = 1.0, \quad C_1 = .355 \quad C_2 = .645$$
$$C_3 = .045, \quad C_4 = .5 \tag{36}$$

From the results obtained so far, the overall behavior of the two-length scale model appears to be very close to what one would expect from the results obtained using the simple one-dimensional model. Although a considerable effort was made to obtain an optimum set of constants, the major effort was concentrated on a study of the initial developing region of a mixing layer and it is probable that the constants used for the results presented here are not optimum for a wide range of flows.

Before discussing these results in detail, it is useful to briefly discuss the behavior of the modified two-equation model with particular emphasis on the initial developing region of a planar mixing layer. The calculation was run, starting from a fully developed turbulent wall boundary layer using the data specified as starting conditions for CASE 0311 in the 1980/81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows.[42] The results shown in Figure 7 were obtained using the standard k, $k\ell$ turbulence with the source term in the $k\ell$ equation modified as described by equation (31). The same calculation was performed using the k,ε model and the results are shown in Figure 8. The only significant difference between the two calculations is that the constants in the k, $k\ell$ model have been adjusted to match the asymptotic spreading rate of the fully developed mixing layer and, so, is in slightly better agreement with the experimental data. Otherwise, both calculations are very similar.

A detailed examination of the numerical calculations provides some interesting insights into the behavior of the mixing layer over the initial developing region. In the wall boundary layer upstream of the separation point, the turbulence energy production and dissipation rates are approximately in balance in the steep gradient region close to the wall. Downstream of the separation point, when the influence of the wall is removed, the dissipation starts to decrease. The turbulence energy and length scale increase; but since this is accompanied by a decrease in the mean velocity gradient, there is no sudden increase in either the shear stress or the spreading rate of the mean velocity profile. The subsequent relaxation rate appears to be determined simply by the turbulence time scale. The development of the mixing layer in this region is, therefore, dominated by the small scale turbulence and the overall behavior of the flow appears to be predictable from fairly simple considerations, in spite of the fact that the detailed behavior of the turbulence may be very complex and still be poorly understood.

Farther downstream both models predict that the spreading rate approaches its asymptotic value monotonically; there is no evidence of the overshoot observed experimentally. One consequence of this is that the length of the developing

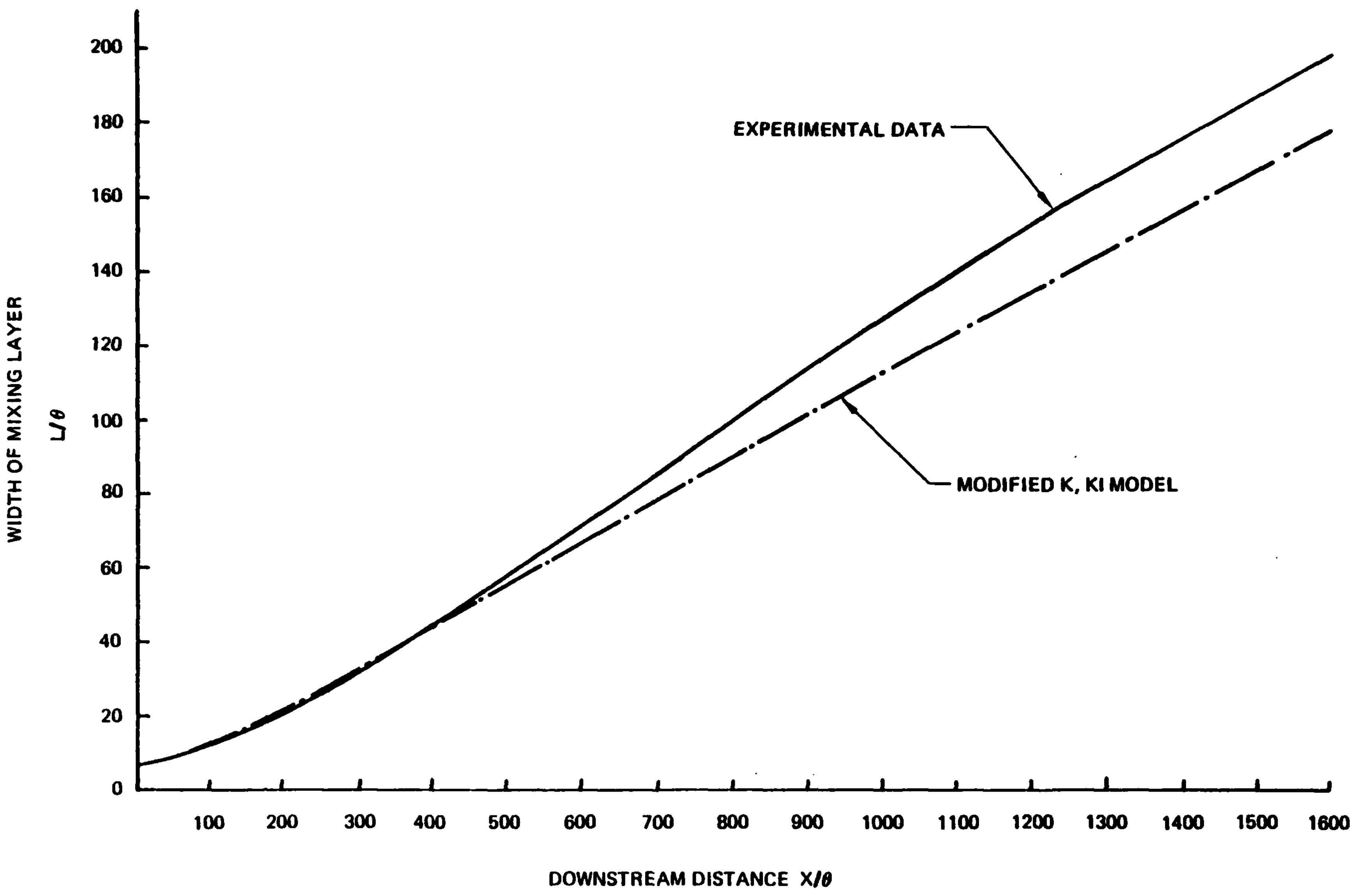

Figure 7. Comparison of Numerical Calculations with Modified K, KI Model with Experimental Data for Mixing Layer

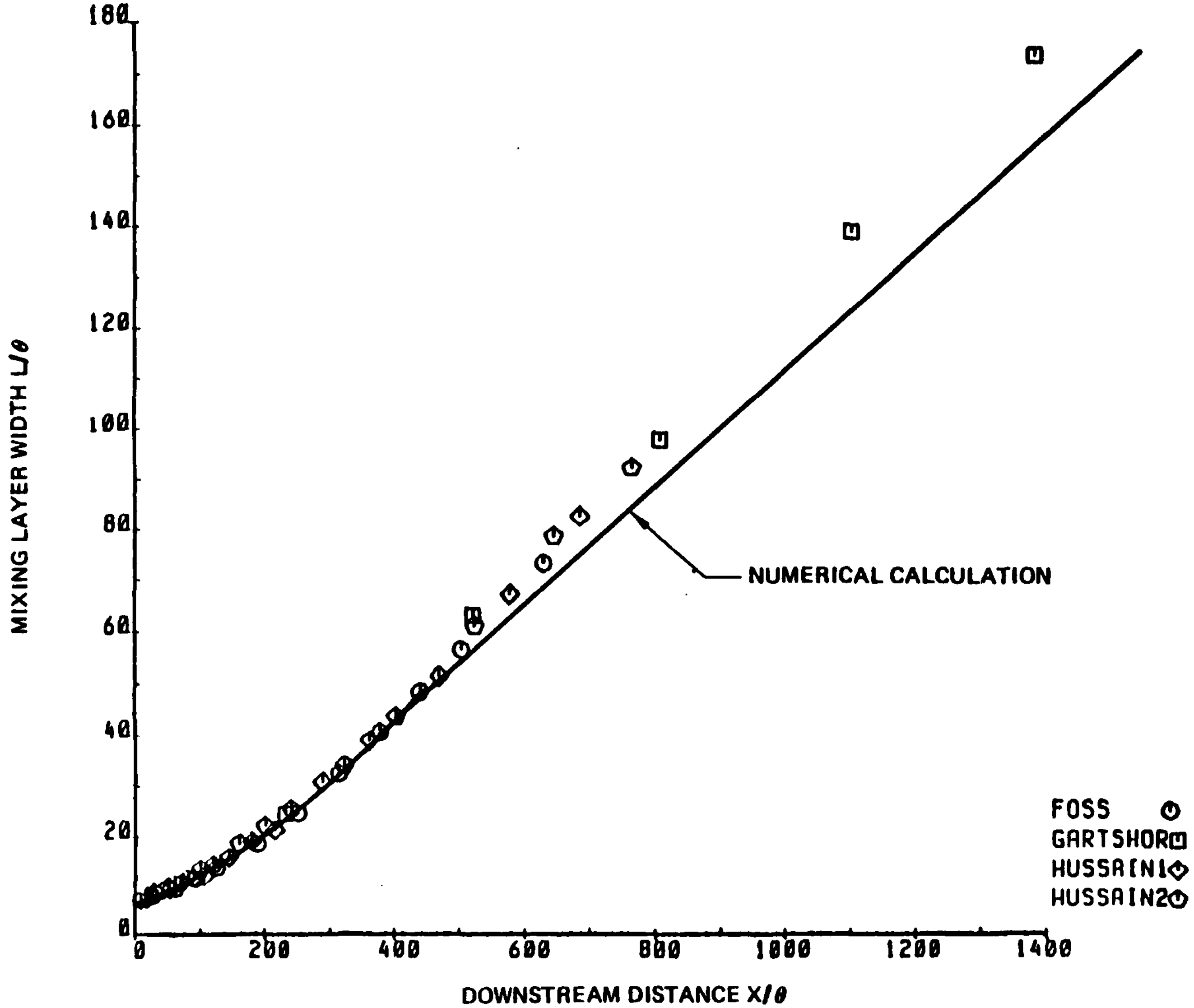

Figure 8. Comparison of the Calculated Spreading Rate of a Planar Mixing Layer with the Experimental Data from Figure 3.

region is under predicted by about a factor of four. Even in the fully developed region of the flow, where the calculated and measured spreading rates are in agreement, the width of the mixing layer is under predicted.

A calculation of the same flow using the new two-length scale model is shown in Figure 9 and a plot of the calculated normalized shear stress is shown in Figure 10. It is clear that use of the new model leads to a considerable improvement in the agreement between the calculations and experimental data.

The first point to note is that the overall qualitative behavior of the relaxation process is well predicted. In particular, the length of the relaxation region is in good agreement with experimental data and it was found to be fairly insensitive to the values of the adjustable constants. This again seems to be an indication that the time scale of the relaxation process is determined simply by the time scale of the turbulence and that it tends to emerge naturally once the qualitative behavior of the model is correct.

The available experimental shear stress data is not reliable enough for a direct evaluation of the calculations; but it appears that the calculated overshoot in the shear stress and mixing rate is somewhat less than that observed experimentally. The magnitude of the predicted overshoot is sensitive to the values of the selected constants; but it is believed that further refinements of the constants is unlikely to significantly improve the overall agreement. This is due in part to the fact that the one-dimensional model studied earlier also seemed to under predict the overshoot and in part to the fact that attempts to obtain a larger overshoot increased the initial growth rate of the shear stress. This led to a shear stress peak that was too close to the separation point and no real overall improvement in the agreement between the calculations and experimental data.

One possible cause of this discrepancy, that needs to be investigated, is that the small extra strain rate terms that are generally ignored in thin shear layer calculations may be important in the relaxation region. This is most likely for the region of the flow just downstream of the separation point where conditions are changing rapidly.

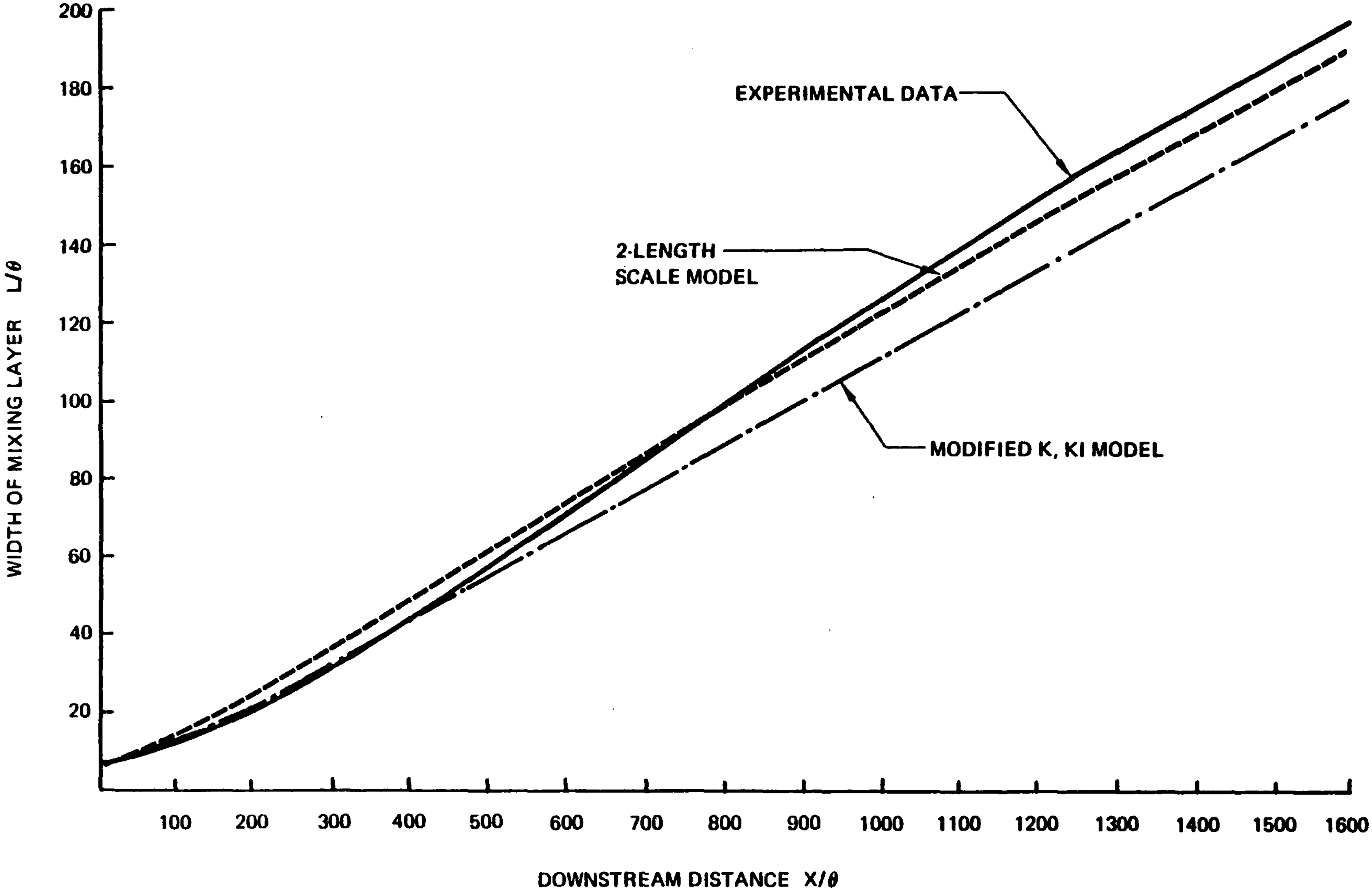

Figure 9. Comparison of Numerical Calculations with Experimental Data for Mixing Layer

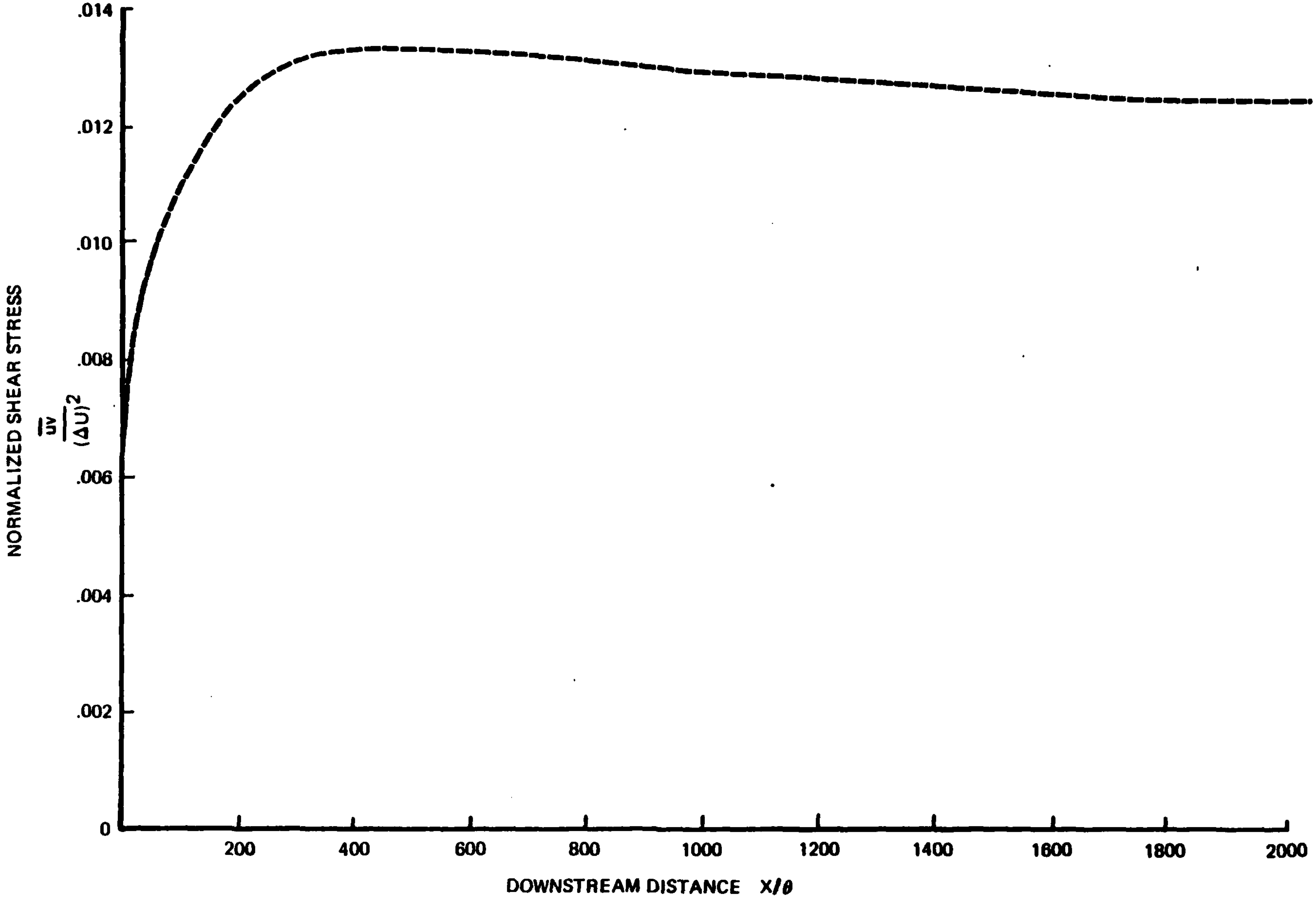

Figure 10. Mixing Layer Developing from Turbulent Wall Boundary Layer (New Model)

In spite of this it is believed that the discrepancy is most likely to be due to the use of constant coefficients. It seems likely, even from simple physical considerations, that at least some of the coefficients should depend on the shape of the turbulence energy spectrum. One simple way of incorporating such a dependence would be to make one or more of the constants a function of the ratio of the turbulence energy production to its dissipation rate. Such a study, however, was not possible in the time available for the present work.

Although the present study was successful in demonstrating that the addition of a second length scale equation extends the range of application of current turbulence models, there remain a number of issues that require more detailed study. In addition, the development of the new model to the point where it could be used reliably for practical predictions would require an extensive evaluation and refinement of the model for a wide range of flows. It is believed that such an effort would be premature at this time.

The reason for this is that the problem considered here is just one of the problems encountered in attempts to predict the behavior of complex three-dimensional jets. Another serious problem is the necessity of using different constants for planar and axisymmetric jets. Since modeling problems tend to be interrelated in the sense that a modification that leads to an improvement for one flow may well yield worse predictions for another flow, it would be desirable at this time to at least obtain an understanding of the physical reason or reasons for the planar/axisymmetric jet problem before embarking on an extensive development program for a new turbulence model. With this information in hand, it would then be possible to form a balanced judgment of how best to proceed in the development of an improved turbulence model.

4.0 CONCLUSIONS

This report presents results of a study of a new two-length scale turbulence model. The addition of a second length scale is intended to extend the range of application of turbulence models to strongly perturbed flows that are far from equilibrium. The model is based on the physical observation that the turbulence shear stress and the turbulence energy production are associated primarily with the large scale eddies, while most of the turbulence energy is dissipated by small scale eddies near the high frequency end of the turbulence energy spectrum. Therefore, except for flows close to equilibrium, separate turbulence length scales are required to characterize the large and small scale motions.

Most of the work described in this report deals with a detailed study of the developing region of a plane mixing layer. Nevertheless, it is believed that the new model is generally applicable to a wide range of flows, although some further refinement of the turbulence model constants is probably required. The new model correctly predicts an overshoot in the developing region of a mixing layer although the magnitude of the overshoot is slightly less than that observed experimentally. With a single length scale equation model, there is no overshoot at all.

Two important conclusions of the present study, which are only indirectly related to the new model, are that the form of the production term in most current length scale equations appears to be incorrect and that the importance of elliptic effects has probably been underestimated in most previous turbulence model studies.

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