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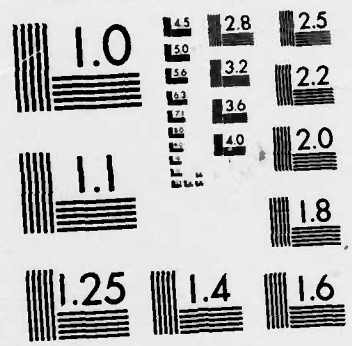

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NSWC TR 82-552

AD-A134938

AN ACTIVE FILTER PRIMER, MOD 1

BY ARTHUR D. DELAGRANGE

UNDERWATER SYSTEMS DEPARTMENT

1 FEBRUARY 1983

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In the past few years active filters have become very popular. This report explains why, and explains what active filters can (and can't) do. It gives the basics of active filter design, both theory and practice. It can be used as a handbook to build working active filters of the most common types. This report is an update of the original issued in 1979.

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FOREWORD

This report gives the basics of active filter design, both theory and practice. It is an update and expansion of a previous report issued in 1979. It will be of interest to persons working in the fields of analog filter design or signal processing.

Approved by:

W. C. Jones

W. C. JONES, Acting Head
Surface ASW Division

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INTRODUCTION

In the past ten years or so, active filters have become quite popular. Probably the biggest single cause is the availability of good monolithic integrated-circuit (IC) operational amplifiers (op-amps). Other factors are improvements in resistors and capacitors (both discrete and monolithic) some advances in the basic theory, new methods for achieving certain functions, availability of computers for analysis and simulation, and simply the need for more exotic filters as part of more sophisticated systems.

To some extent the term "active filter" has become a magic "buzzword", and some misconceptions about them have become widespread. The primary advantage of modern active filters is that they are the most practical solution to a large number of common filter design problems. Most of the theory is not new at all, some of it having been around for more than half a century. The applications are new. In theory, active filters can't do much that couldn't be done with passive filters and simple amplifiers. The difference is in practicality. Active filters cannot do everything imaginable; there are definite limits. Often on a given project, the filter considerations are left until last and the filter designer handed an impossible job. There is no such thing as a "best" type of active filter. There are many tradeoffs and the designer must realistically decide what properties he must have and what he can do without.

Active filter design theory could fill a large book, and indeed a number of books have sprung up recently. Some are very good and some are not so good. Table 1 gives the author's preferences. Number one was used as the basis for the original report.[1] Number two is another good, easy to understand book. Numbers three and four are very thorough references on active filters and op-amps respectively. Some of the material in this report was previously published in numbers five and six. This report is really a companion to number five. Number seven is a classic reference on filter theory, but done entirely for passive circuits. Number eight is a newer book which can be used to replace number seven. Number nine is one of the better simple "cookbooks". Number ten is a thorough book covering the most recent techniques.

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[1]Delagrange, A. D., An Active Filter Primer, NSWC TR 79-137, 1 Mar 1979.

TABLE 1. RECOMMENDED SOURCES

1. Arthur B. Williams, Active Filter Design, Artech House, 1975.
2. Hilburn and Johnson, Manual of Active Filter Design, McGraw-Hill 1973.
3. Blinchikoff and Zverev, Filtering in Time and Frequency Domain, Wiley, 1976.
4. Tobey, Graeme and Huelsman, Operational Amplifiers, Design and Applications, McGraw-Hill, 1971.
5. A. D. Delagrange, "An Operational Amplifier Primer," NOLTR 72-166.
6. A. D. Delagrange, "A Useful Filter Family," NSWC WOL TR 75-170.
7. E. A. Guillemin, Synthesis of Passive Networks, Wiley, 1957.
8. Harry Y-F. Lam, Analog and Digital Filters; Design and Realization, Prentice-Hall, 1979.
9. Don Lancaster, Active Filter Cookbook, Sams, 1975.
10. Huelsmand and Allen, Introduction to the Theory and Design of Active Filters, McGraw-Hill, 1980.

Most of the books attempt to simplify the design to the point where an engineer with no filter experience can build a working filter. That will be done in this report, too; however, the less one knows of the theory, the more likely it is that one will run into problems. Therefore, this report will begin with a review of the theory and then proceed to actual designs. The novice reader should not be discouraged if he has trouble with some of the early sections dealing with theory, as later examples will make some points more clear. (On the other hand, the experienced designer will be bored by some of the background material.) Many of the statements will be generalizations or approximations, but all are accurate enough for most engineering design. The filter circuits shown have all been tested and work. The curves shown in Appendix A are all obtained from actual circuits; they are not theoretical or computer-simulation curves.

There are several reasonable assumptions implicit in these or any other filter designs. The filters must be driven from a low impedance; a simple op-amp voltage follower can be added at the input if necessary. A reasonable load is assumed; since all circuits here are arranged to have an op-amp at the output, this means, simply, a load that can be driven by an op-amp. The frequency range must be such that the op-amps perform pretty much like ideal op-amps. Note that an op-amp with a gain-bandwidth of 1 MHz is useless at 1 MHz; the useful range would typically be 10 KHz. And of course the allowable op-amp voltage swing must not be exceeded.

This report is limited to analog, linear, frequency-domain filters. Although there are many types that fall outside this scope, they comprise a small percentage of actual use. Note that although digital filters are active devices, they are normally considered a separate category and not included in the term "active filters."

This report is hopefully an improvement over the previous edition. It has become more sophisticated, including more options and some new, advanced material. The basic approach is unchanged, though, and it should be only slightly more difficult for beginners. Whereas the original included only two, three and four-pole versions of the basic filter types, this version presents a better, unified approach where component values are listed in a table and go up to nine-poles. This report gives the basic filters in a more desirable form where capacitance values are all equal and only resistors need be selected; also the need for three-pole sections is eliminated. Elliptic filters are usually mentioned only as an idea; here practical methods for building them are presented. A general method of converting any passive prototype, including bandpass and bandstop, to an active version is given. The original report should be retained, as occasionally the old alternate filter forms might be desired.

ADVANTAGES AND DISADVANTAGES

Table 2 gives a summary of the characteristics of active filters as compared to passive and digital. Passive filters were the forerunners of active filters, and much of the basic theory carries over. (Crystal and mechanical filters can be reasonably well thrown in with passive filters here.) Digital filters are sort of the next step beyond analog active filters, although a direct comparison is not completely fair. For example, a digital filter usually requires an analog prefilter anyway. Many digital filters are derived directly from counterparts, and the theory carries over with modification, but many others perform functions one would never attempt with analog circuitry. To confuse the issue farther, there is a class of filters that uses both analog and digital techniques. These will only be mentioned in passing.

A word or two of justification for the entries of Table 2: Inductors are, of course, the most serious handicap of passive filters. The requirement of a power supply is no longer a significant handicap; but note that active filters require better regulation while digital usually require more power. Passive filters have virtually no high-frequency limit, but become bulky below 10 KHz and virtually unacceptable below 1 KHz. Active filters suit the mid-range from 1 Hz to 1 KHz, but can be used with some care to 10 KHz and in limited applications as high as 1 MHz. Digital filters have no inherent low-frequency limit because the clock can always be slowed down, but are limited by sample frequency and are generally used below 1 KHz, with limited application to 10 KHz or 100 KHz. Passive filters usually must be impedance-matched on both input and output. Active filters normally have high enough input impedance and low enough output impedance that impedance is not a problem. In digital filters, impedance is not directly relevant; instead one must observe loading rules and compatibility between logic types. Some types of passive filters can have many sections; 23-pole crystal filters are common. Active filter design becomes difficult beyond 10 poles. Digital filters may be expanded nearly indefinitely by adding more hardware (or software) if possible. Similarly, with dynamic range, passive has no inherent limit beyond practical considerations; active is limited by power supply on the high end and semiconductor noise on the low end; digital can be expanded, but note that more bits means more expense and slower operation. Passive tends to be expensive because of inductors, and digital expensive because of the number of parts; however, large-scale integration (LSI) will soon provide exceptions to the latter. Passive filters often show large discrepancies between calculated and actual performance, while active ones tend to be good, especially if tolerance errors are accounted for; and digital is, of course, exact if quantization is included in the analysis.

TABLE 2. COMPARISON OF FILTER METHODS

Property \ Method	Passive	Active/Analog	Digital
Uses Inductors?	Yes	No	No
Requires Power Supply?	No	Yes	Yes
High-Frequency Capability	Good	Fair	Poor
Low-Frequency Capability	Poor	Fair	Good
Input Impedance	Matched	High	Irrelevant
Output Impedance	Matched	Low	Irrelevant
Number of Sections	Many	Few	Expandable
Dynamic Range	Good	Fair	Expandable
Expense	Medium	Low	High
Match to Calculations	Poor	Good	Exact

BASIC DEFINITIONS

The reader should be familar with the basic definitions and facts of this section. Otherwise, difficulties are almost inevitable.

Decibels (dB) is defined as 20 times the logarithm (base ten) of a voltage ratio. In this report, it is always the ratio of the output voltage, to the input voltage equivalent to gain. Attenuation is the opposite of gain, the reciprocal of the voltage ratio or simply the negative number of decibels. Table 3 gives approximate numbers for the most common conversions. By memorizing these few numbers, most other combinations can quickly be estimated mentally. For example, a gain of 100 is 10 squared, so the conversion is 40 dB. An attenuation of 1/5 is an attenuation of 1/10 plus a gain of 2 of -14 dB; a gain of 35 dB is 40 dB - 6 dB + 1 dB, or (100/2)(1.1) = 55; and so forth.

An octave is a factor of two, normally applied to frequency; a decade is a factor of ten. Thus, a function that is directly proportional to frequency (or the reciprocal), such as a simple single-pole filter, has a slope of 6 dB/octave, equivalent to 20 dB/decade.

TABLE 3. APPROXIMATE DECIBEL CONVERSION

Number of Decibels	Equivalent Gain	Equivalent Attenuation
0	1	1
1	1.1	0.9
3	1.4	0.7
6	2	0.5
10	3	0.3
20	10	0.1

The radian frequency ω is equal to $2\pi f$ where f is in Hertz (cycles per second). The factor 2π appears often in filter theory. $\omega=1/\tau$ where τ is the time constant of a simple RC; $f=1/T$ where T is the time period of the corresponding sine wave. So again, there is a factor of 2π between τ and T; unfortunately, the component values work out in terms of ω or τ, but filter time-domain responses are most meaningful in terms of f or T. For phase, one complete cycle is equivalent to either 360 degrees ($^{\circ}$) or 2π radians (rads) and again the factor of 2π occurs between cycles and radians. Phase response is sometimes plotted in terms of group delay, which is the derivative of phase shift with respect to frequency ($d\phi/df$), so a linear phase shift curve corresponds to constant (flat) group delay.

The Laplace transform variable S (also called generalized frequency) is equal to $\sigma + j\omega$, where $j = \sqrt{-1}$. The real part, σ, has to do with damping or exponential decay, and is associated with transient behaviour. The imaginary part, ω, is the same previously mentioned and has to do with periodic signals; in particular to obtain the response of a filter to a sine wave of arbitrary frequency (frequency response) simply take the filter transfer function and substitute $j\omega$ in place of S, implying that any transients have died out and the filter is operating in a steady-state condition.

The usual filter functions are shown schematically in Figure 1. A low-pass filter retains the low frequencies while rejecting the higher frequencies as completely as possible. A high-pass is the exact opposite, rejecting the lower frequencies. A bandpass retains some band of frequencies while rejecting both highs and lows. A band-reject is the opposite of bandpass, retaining all but a specific band. An all-pass is constant over all frequencies. Here the amplitude is left undisturbed; the properties to be altered are phase and time response, and further graphs are necessary.

Figure 2 shows samples of the graphs of filter performance used in this report. The amplitude response versus frequency, also simply called the frequency response, is usually the plot of interest. Either axis may be either linear (lin) or logarithmic (log); always be careful to note which is used in any given graph, as the apparent shape changes considerably when the scale is changed. The best method is log/log, but unfortunately, oscilloscopes are linear on both axes, and oscillators and spectrum analyzers often do not have a log sweep capability. The plots in this report (see Appendix A) are log/lin.

The amplitude response (see Figure 2.A; low-pass is shown) has three regions: Response in the passband is as flat (constant) as possible; the graph is usually normalized to unity gain (0dB) although the filter itself may not be. It may have droop toward the edge of the band and/or ripple throughout the band; the latter is usually of constant amplitude, i.e., the peaks and valleys are all a given deviation from the gain at DC. Response in the stopband will eventually approach a straight line on a log/log (or lin/lin) plot. The transition band is simply the region between the two, desired to

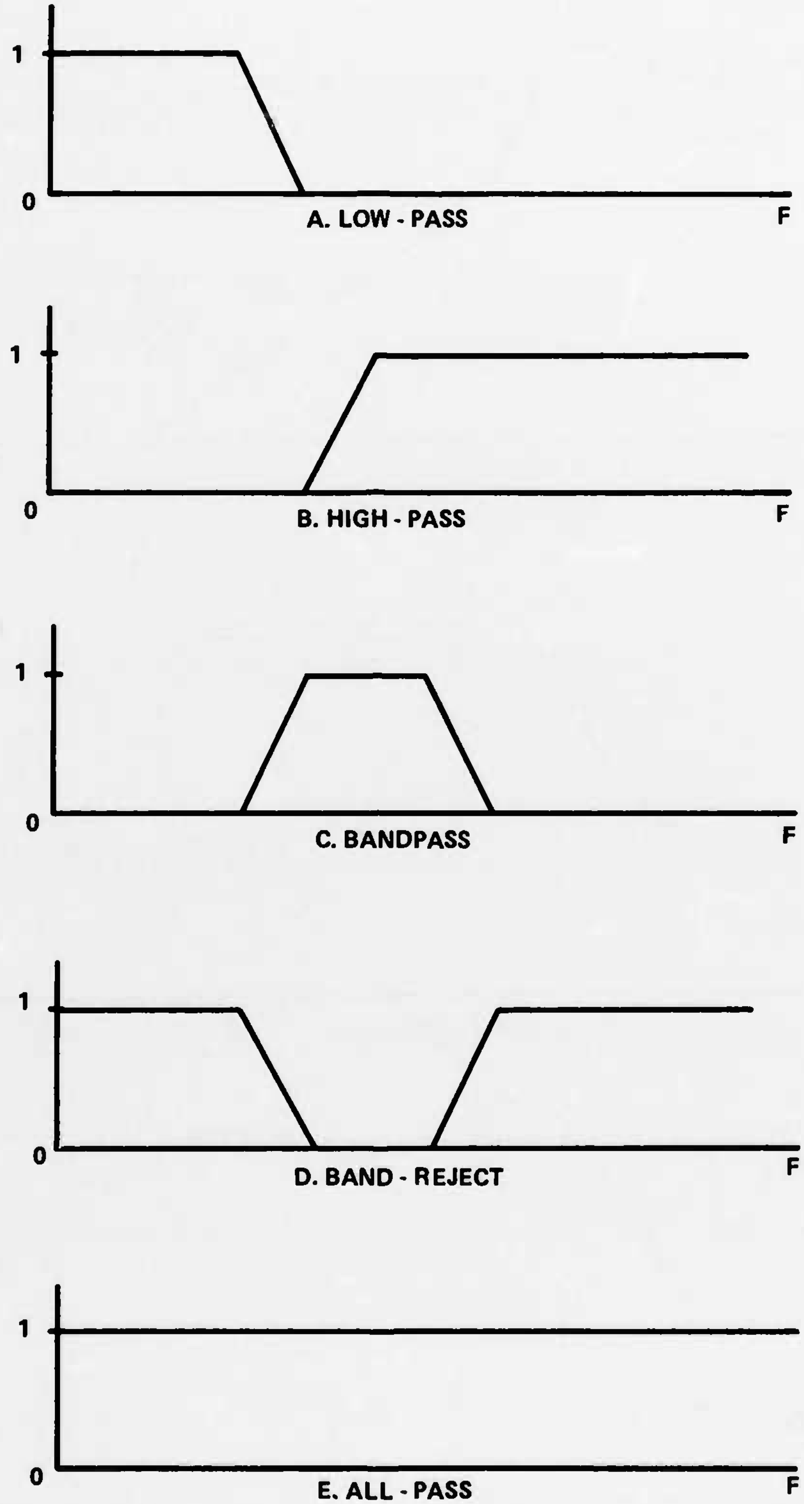


FIGURE 1. COMMON FILTER FUNCTIONS

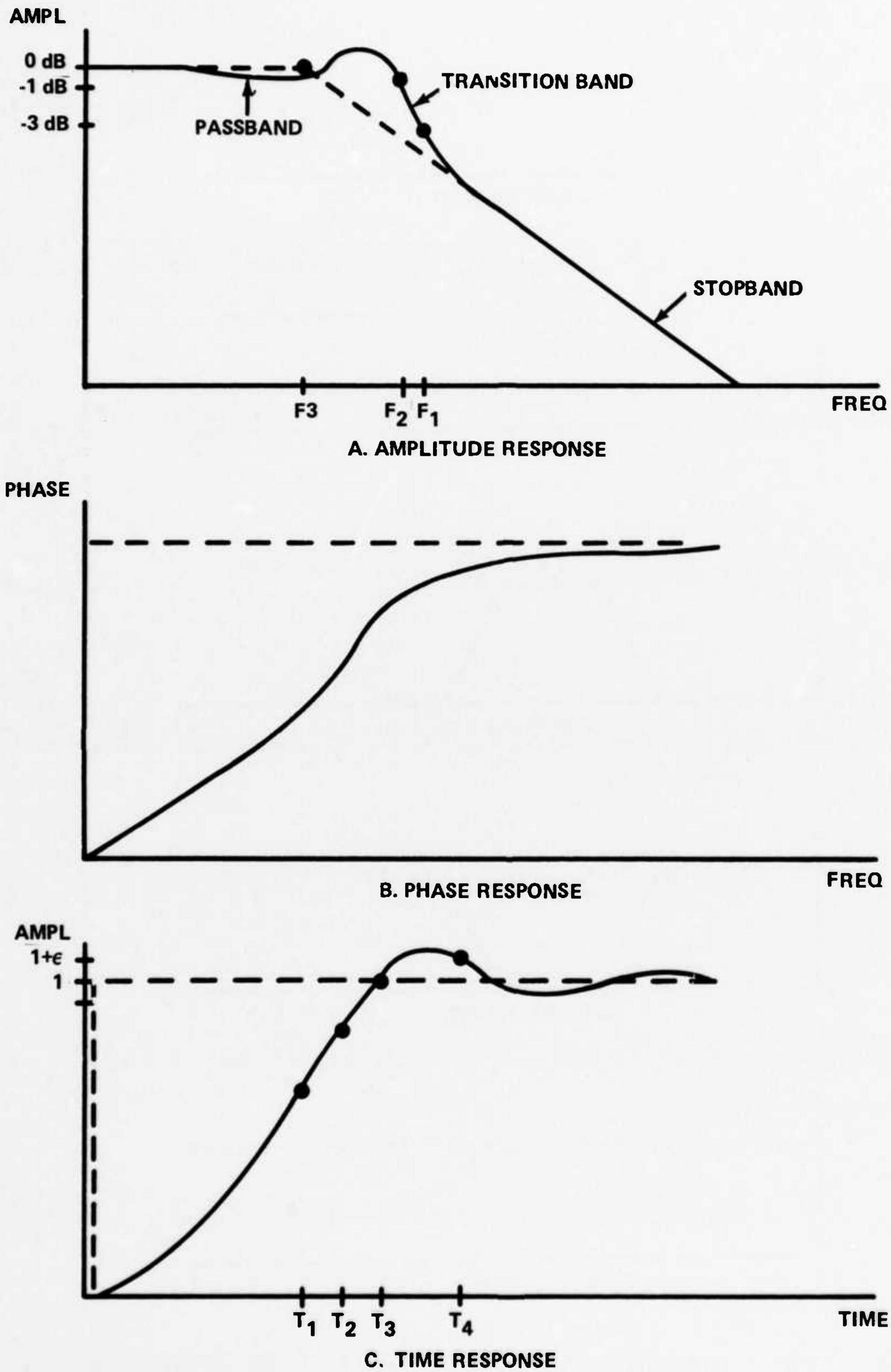


FIGURE 2. GENERALIZED FILTER RESPONSE

be as steep as possible.

The filter cutoff frequency, or simply filter frequency, is the edge of the passband; it may be defined in at least five different ways, which may or may not give the same point, and the user should understand which is used for any given plot. By far the most common is the 3 dB down point (f_1). However, if the filter is a type that has ripple, the cutoff frequency may be defined as the point the curve leaves the ripple band (f_2), which is usually less than 3dB. It may be defined at the intersection of the linear extensions of the curve from the response at low and high frequency, (f_3), as both will eventually become straight lines on a log/log plot. It may be some convenient number that falls out of the mathematical analysis (f_4 not shown). Lastly, it may be defined by phase, even for an amplitude filter; for example, the frequency at which the phase shift is 45° times the number of sections (f_5 not shown). This report in various places uses f_1, f_2, and f_3; f_4 and f_5 are sometimes the same as the one used. Designs are normalized to a cutoff frequency of 1 rad/sec.

Usually ignored, but of increasing interest especially in feedback, correlation or direction-determining systems, is the phase response (see Figure 2B). The vertical may be plotted in either degrees or radians, differing by only a scale factor; in phase lag or phase lead, differing by only a minus sign; but in any case, linear. The horizontal may be either linear or log; linear is best, but the graph then may not correspond to the amplitude response. On a log frequency scale the curve will flatten out to a constant at both high and low ends; however, phase response is usually not of interest in the stopband. Phase lag usually increases with increasing frequency. As noted, instead of phase, the plot may be for group delay, which would be the derivative of the curve shown. Actually group delay is more often the quantity of interest, but phasemeters exist and delay-meters do not. Appendix A uses phase.

The third quantity of interest is the time domain response, also called transient response or step response. Mathematically, the function of primary interest is the impulse response, the time response of the filter to a pulse of infinite amplitude and infinitesimal width. Since the Fourier transform of an impulse is a flat frequency spectrum and a flat phase spectrum, the Fourier transform of the filter impulse response gives the amplitude and phase response; hence the impulse response completely defines the filter. Unfortunately, an ideal impulse cannot be generated in the real world, so instead the step response is usually used. A unit step is the integral of a unit impulse, so the filter response is the integral of the impulse response; also step response often has direct physical significance.

For a unity gain filter, the output of a low-pass filter will eventually rise to the input value. It will always have a finite slope, may exhibit a noticeable delay and/or overshoot/ringing.

Filter rise time is often of interest, and may be defined in a number of ways. It may be the point on the curve (1-1/e), e=2.718...(T_1), the 90% point (T_2), the time it crosses unity (T_3), or the time it enters and stays within some arbitrary ($1\pm\varepsilon$) error band (T_4). Rise times are listed in some references, but not here; however, they may be obtained approximately from the curves of Appendix A. Step response is generally most meaningfully normalized to 2π seconds (sec) for a filter normalized to radian frequency of unity; that is done here. Scale is, by definition, lin/lin.

SPECIFYING A FILTER

A filter problem is usually specified as an amplitude response curve. For instance, an ideal low-pass filter would have rectangular characteristics (Figure 3A); the passband perfectly flat, the transition band infinitely steep, and the reject band having complete rejection everywhere. By Murphy's Law, such a filter cannot be built, at least not with a finite number of parts. The first step toward a more realistic filter is to add a finite slope (refer to Figure 3A); however, it turns out that this is still unrealizable because the corner is infinitely sharp. If the corner is rounded, the characteristic may be achievable, but there is no indication yet of how to go about it. (Note that the filter is not completely specified without adding phase response, but we usually don't care.) Therefore, filter transfer functions are usually specified by polynomials, or more precisely by a ratio of polynomials, such as:

$$H(S) = \frac{S + 2}{S^3 + 3S^2 + 4S + 2}$$

Each term after the first in either the numerator or the denominator creates one bend in the amplitude response curve, which also corresponds to one reactive component in a circuit. Thus a ratio of finite polynomials implies a finite number of parts. There are techniques for deriving a filter design from this, but the polynomial does not yet imply a realizable design, as the filter may be unstable or some components with negative values might be required. There are also mathematical tests for determining which polynomials are realizable. Even so, the correspondence between the polynomials and the shape of the curve is not obvious, but requires plotting by hand or by computer. To make the frequency response plot, recall that S must be replaced by $j\omega$; the polynomial becomes:

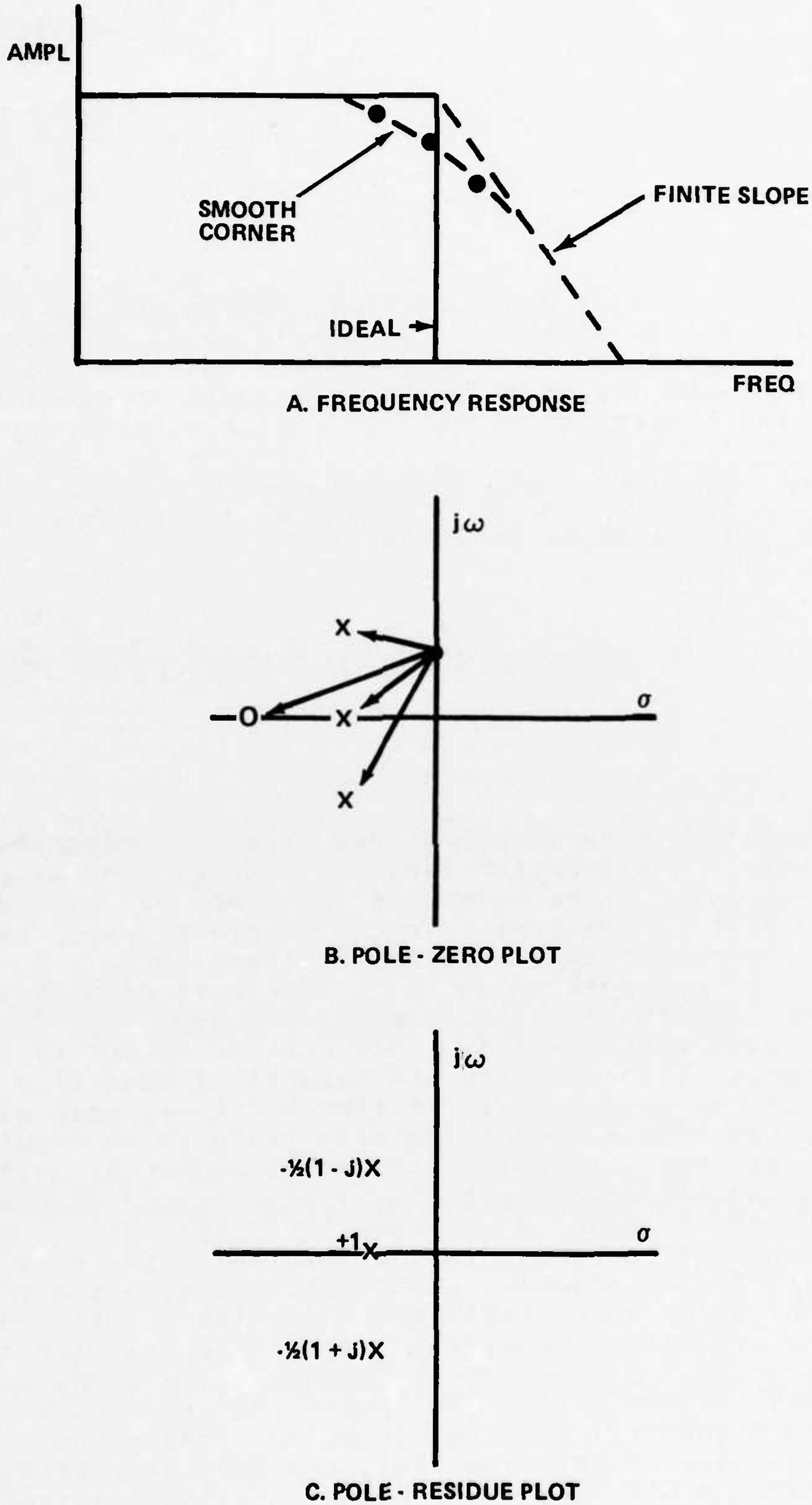


FIGURE 3. GRAPHICAL METHODS OF FILTER SPECIFICATION

$$\frac{(j\omega) + 2}{(j\omega)^3 + 3(j\omega)^2 + 4(j\omega) + 2}$$

This can be reduced to two terms, one real (having no j's) and the other imaginary (multiplied by j). If amplitude is of interest, the magnitude is found by taking the square root of the sum of the squares of the real and imaginary parts; if phase is required, it is found by taking the arctangent of the ratio of the imaginary part to the real part.

The next step is to factor both numerator and denominator into a product of terms. The example above becomes

$$H(S) = \frac{(S + 2)}{(S + 1)(S + 1 + j)(S + 1 - j)}$$

The numerator factors are termed zeros and the denominator factors are termed poles; the transfer function becomes zero at the zeros and infinte at the poles. The poles and zeros may be plotted on a two-dimensional plot of real part versus imaginary part, called the pole-zero plot, S-plane or complex plane (Figure 3B). A zero is indicated by a "O" and pole by an "X". (The word complex should be used only to mean "having both real and imaginary parts" and not "complicated"). Both axes are lin. This plot turns out to be useful for several reasons. The poles are the natural frequencies of the filter; they give the modes of oscillation and decay that will occur if the filter is hit with a bunch of energy (such as an impulse) and then left to its own devices. The poles cannot be in the right half-plane, or the natural response would be an indefinitely increasing function instead of eventually dying out. A practical filter cannot have poles directly on the imaginary ($j\omega$) axis either, as this implies a sinusoidal response that continues forever; in fact filters having poles very close to the imaginary axis may have problems. In a transfer function, there is no restriction on the placement of the zeros; as long as the response is zero, we don't really care what caused it. The poles and zeros must be placed symmetrically about the real (σ) axis (complex conjugates) so that when the factors are multiplied out, the imaginary parts cancel; otherwise the design would require components having imaginary values (imaginary components?). Since a sinewave at a given frequency represents a point on the imaginary axis, the magnitude of the filter response at that frequency will be the product of the magnitudes of the vectors from that point to each of the zeros divided by the product of the magnitudes of the vectors from that point to each of the poles; the phase angle is the sum of the vector angles to the poles minus the sum of the vector angles to the zeros (See Figure 3B). As the frequency passes near a pole, the response will become larger because one denominator factor will become small, vice-versa for a zero, and so forth. Thus, the frequency response can be mentally estimated to some degree, and the effect of moving one of the poles or zeros can be observed. Frequency may be positive or negative; response will be the same due to the symmetry of the plot; really only half of each need be plotted. The pole-zero plot completely defines the filter except for gain factor.

An alternate method which is sometimes useful is to separate the polynomial into a sum of terms instead of a product; a partial fraction expansion. The polynomial becomes:

$$H(S) = \frac{1}{S+1} - \frac{1}{2}\,\frac{(1-j)}{(S+1-j)} - \frac{1}{2}\,\frac{(1+j)}{(S+1+J)}$$

Again this may be plotted in the complex plane, and is called a pole-residue plot (Figure 3C). The zeros are not shown; instead each pole has associated with it a residue, which is the value of the function at that point if the "infinity" is eliminated by removing the denominator factor. Residues must be complex conjugates so the polynomial will be real when multiplied out. Again filter response may be visualized by vectors, but this now requires adding vectorially while correcting for residues, which is difficult. The pole-residue plot completely describes the filter, although the gain factor may be ignored. Note that Figures 3B and 3C correspond to each other but not to 3A.

Of course, what is actually done in 99% of the design problems is to use a standard filter type that is already well-known and well-documented. These types usually exhibit some special property in frequency, phase or transient response or circuit realization, are named by that property or the man responsible for the design, and often exhibit noticeable patterns in the pole plot. Most texts, including this one, tabulate these filters; but using the tabulations requires another element of theory which is described in the next section.

TRANSFORMATIONS

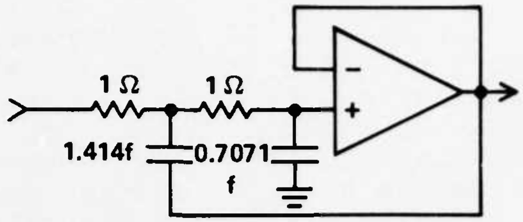
Because of the large number of variables (characteristic, function, number of sections, frequency, impedance, etc.) involved in filter design, it is impossible to provide a complete catalog of ready-to-use circuits. That would fill a library, so some generalization is necessary. What is done is to give a filter in "normalized" or "prototype" form, typically having a cutoff frequency of 1 rad/sec and an impedance level of 1 ohm. The prototype would never be used directly, but a filter having the desired frequency and impedance may be obtained from the prototype by simple transformations. In some texts, the prototype is always given as a low-pass, and an additional transformation is necessary if high-pass is desired; but both are given in this report. An example of a normalized 1 rad/sec, 1 ohm design is shown in Figure 4A.

If all capacitor values (and inductors if any) are reduced by a factor K_F, it is fairly evident that the filter will act the same way as before, only K_F times faster; in other words, the frequency characteristic will have the same shape, but all frequencies will be K_F times higher. This is termed "frequency scaling". Figure 4B shows the same filter scaled up to 10,000 rad/sec (about 1.6 KHz). Already the capacitor values look more reasonable.

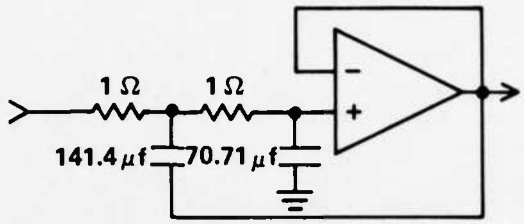
If all resistors (and inductors if any) are multiplied by a factor K_I and all capacitors are divided by the same factor K_I, it can be shown fairly easily that the voltage transfer ratio will be unchanged; all currents will be K_I times less, but this does not affect any voltage ratio. This is termed "impedance scaling". Figure 4C shows the circuit of Figure 4B scaled up by a factor of 10,000 in impedance. For our purposes, performance is identical for the two. The component values are now entirely reasonable; this is a working filter. These two transformations are all the reader need know to use this report. However, the following additional ones are useful at times, and indeed some were used to generate some of the circuits given.

To change a low-pass function to a high-pass function, S is replaced by 1/S everywhere in the polynomial. It can be shown that the new prototype circuit may be obtained by replacing each capacitor with an inductor (and vice-versa if necessary); this is not directly useful because the point of using active filters in the first place was to avoid inductors. Further transformations may or may not be possible to eliminate the inductors. Alternately, in some circuits the resistors and capacitors may be simply interchanged; Figure 4D is the high-pass version of Figure 4A. The component values are inverted because the impedance of a capacitor is inversely proportional to its value; they appear to interchange only because the capacitor values in Figure 4A happen to be reciprocals for this example. Note: Component values are given to four significant figures to prevent possible roundoff errors when making transformations. The usual components for filter design are ±1% tolerance (three significant figures) with ±5% or ±10% (two significant figures) allowable in some applications.

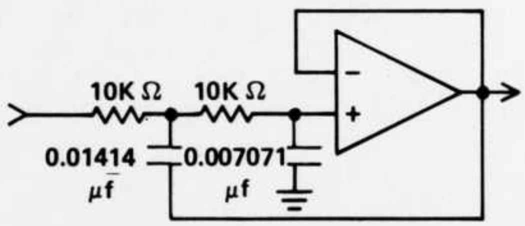
A low-pass may be transformed to a bandpass by replacing S by S+1/S in the polynomial, which replaces each capacitor with a capacitor in parallel with an inductor. This is less useful to us,



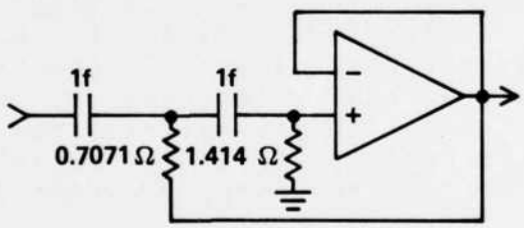
A. NORMALIZED LOW-PASS FILTER



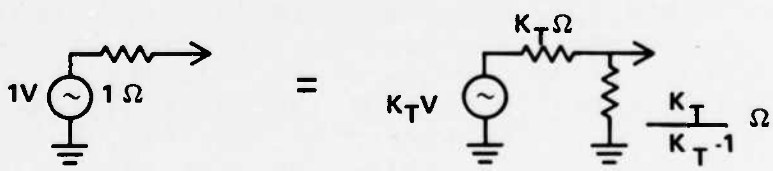
B. FREQUENCY SCALING



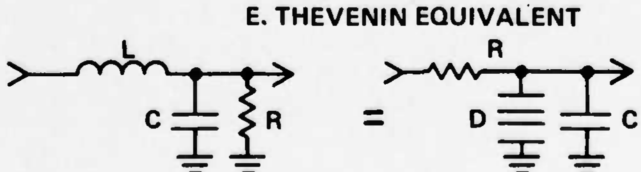
C. IMPEDANCE SCALING



D. HIGH - PASS TRANSFORMATION



E. THEVENIN EQUIVALENT



F. SUPER - CAPACITOR TRANSFORMATION

FIGURE 4. CIRCUIT TRANSFORMATIONS

and there is no equivalent RC transformation. Similarly, a band-reject is generated by replacing S by 1/(S+1/S), equivalent to replacing each capacitor with a capacitor in series with an inductor.

Thevenin's theorem says that any network of sources and resistors may be replaced by a single voltage source in series with a single resistor. An example is shown in Figure 4E; the two circuits act identically at the output for any K_T. Norton's theorem is similar but uses a current source in parallel with a resistor.

Sometimes an inductor may be replaced with a synthetic inductor, a circuit made using op-amps, resistors, and capacitors which looks like an inductor at its terminals. It turns out this works best for cases where one terminal is grounded. If the inductor is "floating" (neither end grounded), a transformation using "super-capacitors," also referred to as D-elements or FDNR's (Frequency-Dependent Negative Resistors)[2] may be possible. Here each inductor is replaced with a resistor, each resistor with a capacitor, and each capacitor with a super-capacitor, an active circuit having an impedance of 1/DS . Each impedance in the filter is simply multiplied by 1/S and the voltage transfer ratio remains unchanged. An example is shown in Figure 4F.

RESONATORS

Usually the easiest way to realize a filter characteristic is to use a combination of standard "building block" circuits. Caution: There is often confusion between filter characteristics and filter circuits. The two are virtually independent; a given basic circuit design might be used to realize several different characteristics by simply changing component values; conversely, a given filter characteristic might be realized using a number of different circuit topologies.

The pole-zero and pole-residue plots indicate that any filter may be built by assembling a number of individual single real axis poles and zeros and pairs of complex-conjugate poles and zeros (referred to simply as pole-pairs and zero-pairs). Caution again: A filter may be specified by the number of sections, but a section may have either one pole or a pole-pair. To make matters worse, a pole-pair is sometimes referred to simply as a pole, particularly in bandpass circuits. Filters are often specified by the number of poles and zeros, but the zeros are sometimes ignored. The number of poles may be also referred to as the "order" of the filter, but this becomes less clear when zeros are added.

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[2] Bruton and Treleavan, "Active Filter Design Using Generalized Impedance Converters," EDN Magazine, 5 Feb 1973.

Single real-axis poles are realized by simple RC circuits without feedback. Pole-pairs are obtained with circuits termed resonators, second-order active feedback circuits which may also be described in terms of the feedback theory parameters natural frequency and damping ratio. In general, zeros are difficult to generate; few filters use them. Zeros always come with associated poles that must be accounted for, as any circuit that does something must have a natural frequency.

Figure 5A shows the most common circuit for generating a complex pole-pair, attributed to Sallen and Key. (They used an emitter-follower, but an op-amp works better except at high frequency; this report will use op-amps exclusively.) The low-pass version is shown; the equivalent high-pass version has the resistors and capacitors interchanged. Either circuit looks somewhat like an oscillator; indeed a resonator may be thought of as an oscillator that can't quite maintain oscillation but exhibits a ringing which dies out.

The universal active filter (Figure 5B), also in slightly different forms known as the state-variable or bi-quad, is so called because it simultaneously provides low-pass, high-pass and bandpass outputs. With the addition of a summing amp it can also provide a band-reject or all-pass output. It is indeed an oscillator circuit with negative feedback added to damp the oscillation. It is somewhat complicated but is becoming more popular now that a quad of op-amps in a single package is available, sometimes with matched resistors for this specific application.

As indicated earlier a passive LC circuit may be adapted by synthesizing the inductor. The dotted line in Figure 5C indictes that it actually contains not an inductor but an active-RC circuit. Figure 5C then provides a high-pass pole-pair. For low-pass the super-capacitor transformation must be used (Figure 5D); again the dotted line indicates not a simple two-terminal component (which is not possible), but an entire active RC circuit.

There are methods of making resonators using gyrators, GIC's (Generalized Impedance Converters), NIC's (Negative Immittance Converters), etc. Often a circuit using these that works in theory will not work properly when built because of the actual limitations of these devices. The theory is also more complicated; none are included here.

GAIN AND IMPEDANCE VARIATIONS

Since a frequency-domain filter by its very purpose removes part of the energy in the signal, the output is often considerably lower in amplitude than the input, so some gain may be desired. All filters given here are arranged to have an op amp at the output, and the op-amp can also provide gain.[3] Figure 6A shows the 2-pole

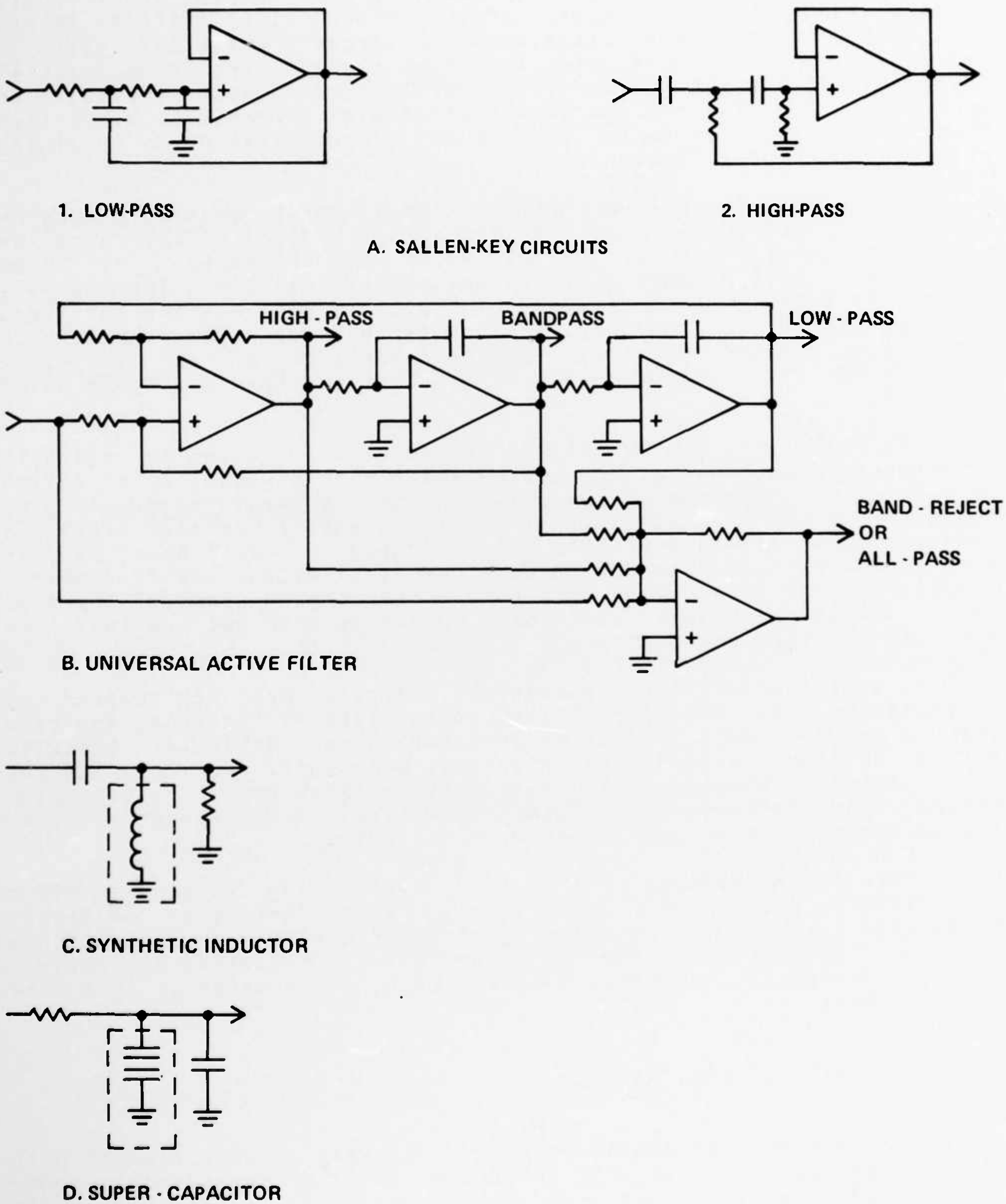
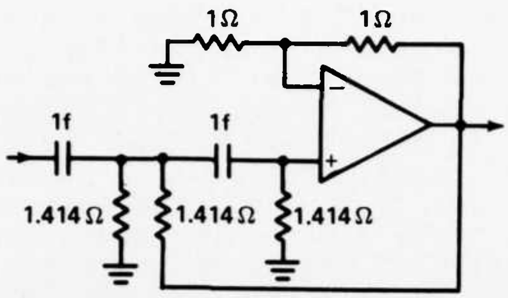
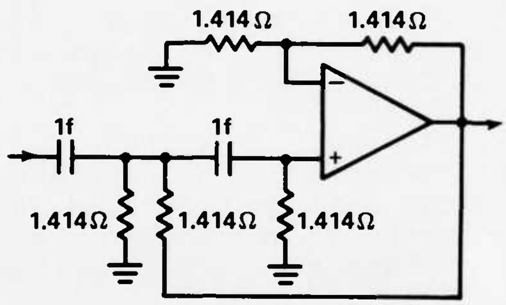


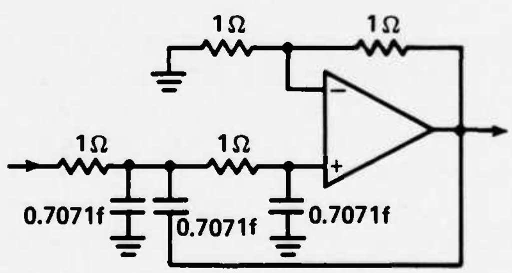
FIGURE 5. RESONATOR CIRCUITS



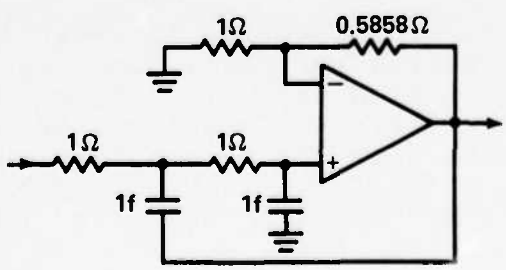
a. HIGH-PASS, GAIN OF 2



b. HIGH-PASS, GAIN OF 2, RESISTORS EQUAL



c. LOW-PASS, GAIN OF 2



d. LOW-PASS, RESISTORS AND CAPACITORS EQUAL

FIGURE 6. GAIN AND IMPEDANCE VARIATIONS

high-pass of Figure 4D modified to have a gain of two (6dB). The buffer is modified to be a gain-of-two amplifier, and the feedback resistor is changed to an attenuator having the same impedance but a gain of one-half to cancel the op-amp gain (Thevenin equivalent). This particular example is attractive because the resistors in the RC network happen to be all equal, whereas they would not be in the unity gain version. In fact, since the impedance of the negative feedback network is arbitrary and the resistors are equal, a trivial change gives the circuit of Figure 6B where now all five resistors are equal. Figure 6C is the low-pass version having a gain of two. Note that the negative feedback network need not be transformed to a capacitive divider; indeed that would not work as the op-amp requires a small DC input current. The feedback capacitor is now a capacitive divider, which places a capacitive load on the op-amp; most op-amps will tolerate this but some types will not. Again, all capacitors are equal and all resistors are equal.

Another way of obtaining equal capacitance values is to let the amplifier gain be the variable. The circuit of 6D has the same characteristic as 6C, but the gain must be 1.58 for the capacitors to be equal. Unfortunately, there is no simple transformation to obtain this circuit from one of the others; some calculations must be done.

INDEPENDENT RC SECTION FILTERS

The simplest possible filter is a number of independent single RC sections (Figure 7). This really isn't even an active filter, but we might as well start from scratch. Figure 7A shows a two-pole low-pass; Figure 7B shows a two-pole high-pass. Higher orders are not shown because they are obtained simply by adding more sections. (Of course a single pole may also be used; the filter characteristic is then meaningless as there are no additional degrees of freedom and all types degenerate to the same form for a single-pole filter.) All sections normally have the same frequency and hence these are sometimes called synchronous filters; a slight improvement in the amplitude response may be obtained by shifting the poles somewhat, but this isn't really worth the effort.

The pole-zero plot for the low-pass is shown in Figure 7C. As stated, all poles are on the real axis, usually at the same point. The poles and zeros for the high-pass are obtained by reflecting the locations through the unit circle. Thus the pole-zero plot for the synchronous high-pass would have the <u>same</u> poles but also an equal number of zeros at the origin (zero on both axes). The low-pass may be said to have its zeros at infinity, but this is inconvenient to draw on the plot and is usually ignored.

[3] Delagrange, A. D., "Op-Amp in Active Filter Can Also Provide Gain," EDN Magazine, 5 Feb 1973.

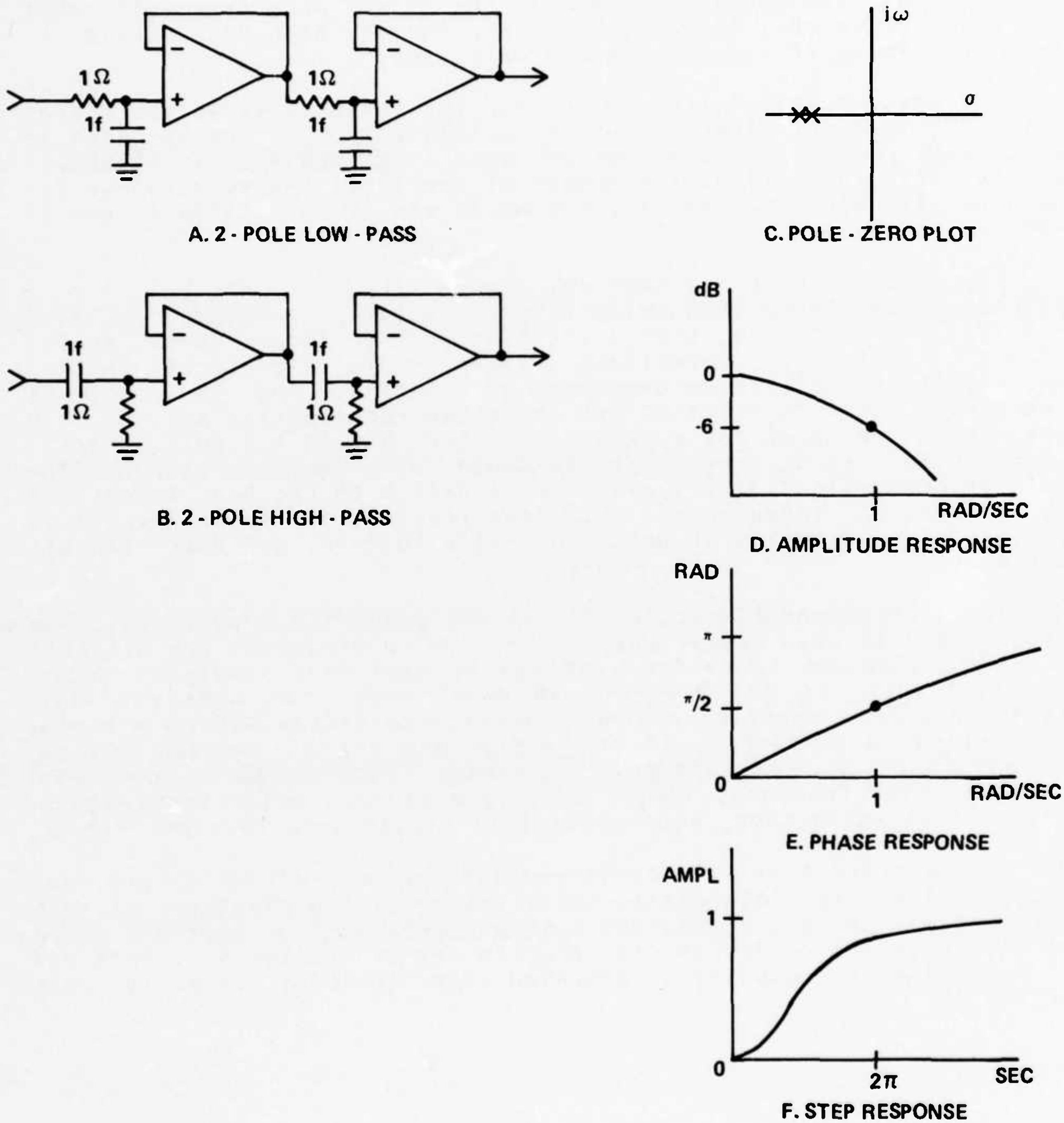


FIGURE 7. INDEPENDENT RC SECTION FILTERS

The amplitude (frequency) response is shown in Figure 7D. It exhibits just a gradual bend downward. A steeper slope may be obtained by adding more sections, but each section adds another 3dB of droop at the bandedge. Here is one of the many trade-offs that will be encountered. Amplitude response for the high-pass would be the mirror image if a log/log scale were used.

The phase response (Figure 7E) for the low-pass is an arctangent curve. It is fairly linear near the origin only. At the bandedge it is $\pi/4$ (45°) times the number of sections. Eventually it flattens out to $\pi/2$ (90°) times the number of sections. Phase response for the high-pass would be similar, but would end at a multiple of 2π ($=0^\circ$) at infinity.

The step response is shown in Figure 7F. In general for a low-pass filter there is a delay before the filter responds, followed by a fairly linear rise, then a settling to the final value (unity for a completely normalized filter). The identical section independent RC exhibits no overshoot or ringing, and is sometimes used where this is critical but the other requirements are not. In fact, it can be shown for a two-pole filter that if the poles are not on the real axis, the step response must have overshoot. The familiar Krohn-Hite* 3200 series[4] has a switch on the back to convert the filter to independent RC. Step response for the high-pass is less useful; it begins at unity and falls to zero, and does exhibit overshoot.

The independent RC section filter is really the crudest possible filter, and is used mostly where filtering requirements are minimal. It is the simplest to design, and may be expanded, simply by adding more sections. It is, however, an exact model for some real-life situations. For example, a long transmission system having a number of identical amplifiers. If the amplifiers are AC coupled each may be thought of as a single-pole high-pass filter. Also, each will have some high frequency limit; usually past some point the response falls off at 6dB/octave, equivalent to a single-pole low-pass filter.

The sketches that will accompany each set of circuits are not exact. They are intended to point out particular features of each type, and may not all correspond to the same number of sections. For exact curves the reader should refer to one of the texts or Appendix A. Responses are usually indicated for low-pass only in this section.

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*Registered Trademark.

[4]"Solid-State Variable Filter Model 3200, Operating and Maintenance Manual," Krohn-Hite Corp., Cambridge, Massachusetts.

BUTTERWORTH FILTERS

The most popular type of filter is the Butterworth, or maximally flat. The latter name indicates that this type is as flat as possible at zero frequency. The name may be misleading, for if the requirement is that the filter response be as flat as possible all the way to the cutoff frequency, this is not necessarily the best type.

The prototype circuits for Butterworth high-pass and low-pass are shown in Figure 8. Note that the same basic circuit also serves for Chebyshev and Bessel (and others); only component values will change. Component values are given in Table 4; frequency and impedance scaling are then applied. The circuits are not unity-gain as in the previous report; in fact, the gain depends on the characteristic chosen and the number of poles used. This form is used here because it greatly reduces the paperwork. Also, the two capacitors in each stage are equal; in fact, impedance scaling can be done separately for each stage in such a way as to make all the capcitors not only equal but a standard value (i.e., factor of ten). The gain can easily be corrected back to unity if desired. Each gain setting network can be scaled to a different convenient impedance value, or each could be a potentiometer (pot), which is often good enough as only the resistance ratio need be stable.

Noise bandwidth is the width of an equivalent ideal rectangular-characteristic filter which would pass equal noise power for white (flat frequency spectrum) noise in. For the ideal filter noise power out would be simply proportional to bandwidth, but for a realizable filter some power is lost in passband droop and some is gained in the imperfect transition and reject bands, so equivalent bandwidth may be either greater or less than unity. One must also be careful with even-order filters with ripple; if filter gain is set to unity at DC, the peaks will be above unity, increasing apparent noise bandwidth. Noise bandwidth is meaningless for high-pass.

Characteristics and responses are shown in Figure 9. As indicated, for Butterworth the poles lie equally spaced on a circle (a unit circle for the normalized filter) except that the mirror image poles in the right-half plane are absent. The poles for the high-pass are the same for Butterworth.

The amplitude response is very flat at the low frequency end, but begins to bend downward approaching the edge of the passband and then is down 3dB at the cutoff frequency. As more poles are added, the response remains 3dB down at that point, but the cutoff becomes steeper and the passband stays flat closer to the bandedge, approaching the ideal rectangular shape.

The phase response is fairly linear at the lower frequencies, but bends upward near the bandedge. The vertical axis is not labeled because the total amount of phase shift is directly proportional to

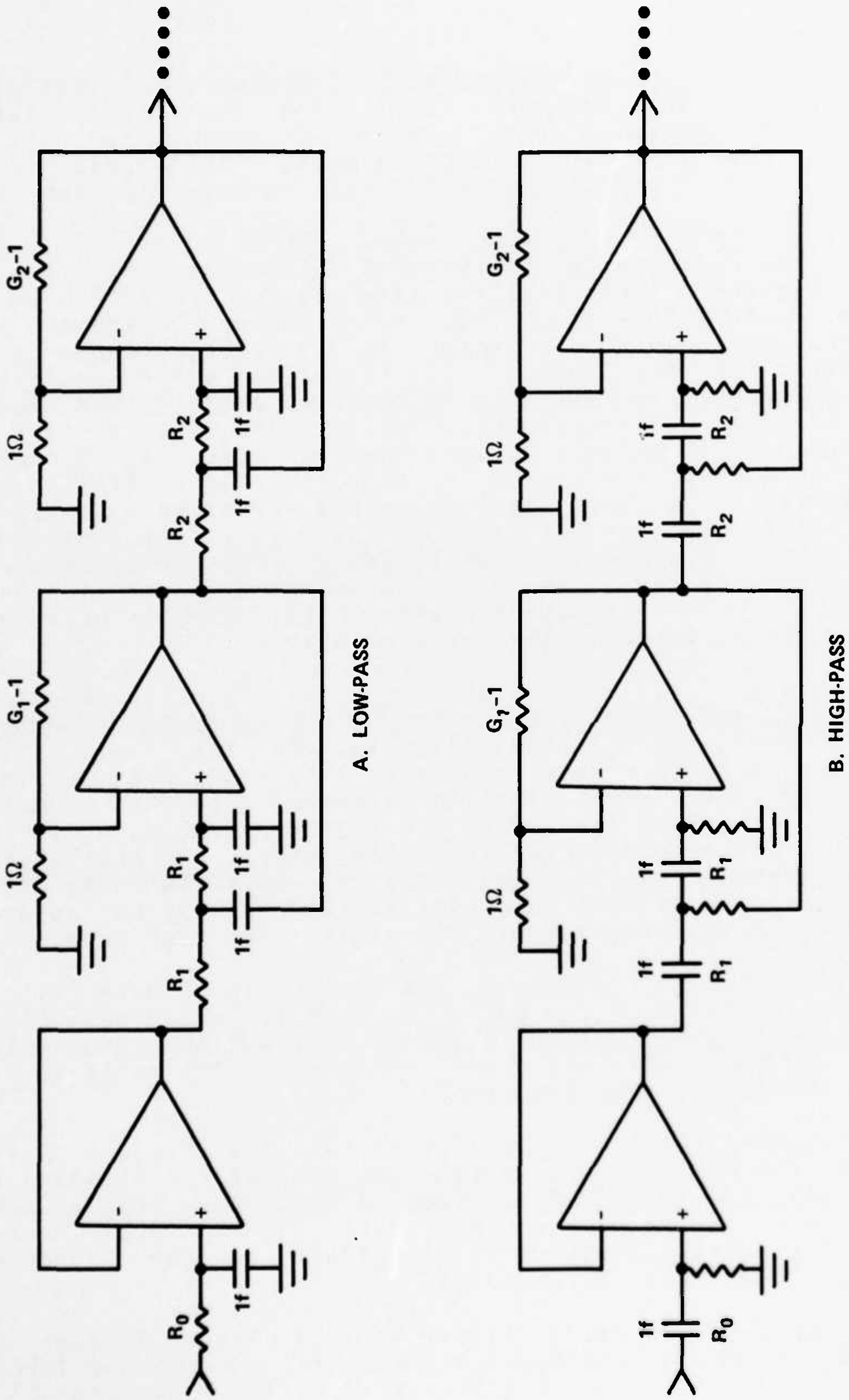

FIGURE 8. PROTOTYPE CIRCUITS (BUTTERWORTH, CHEBYSHEV, BESSEL)

TABLE 4. BUTTERWORTH VALUES

Order	G_1	G_2	G_3	G_4	NOISE BANDWIDTH
1					1.571
2	1.5858				1.110
3	2.0000				1.047
4	1.1522	2.2346			1.026
5	1.3820	2.3820			1.017
6	1.0681	1.5858	2.4824		1.012
7	1.1981	1.7530	2.5550		1.008
8	1.0385	1.3371	1.8889	2.6098	1.006
9	1.1206	1.4679	2.0000	2.6527	1.005

Note: All resistors 1.0000 except gain-setting

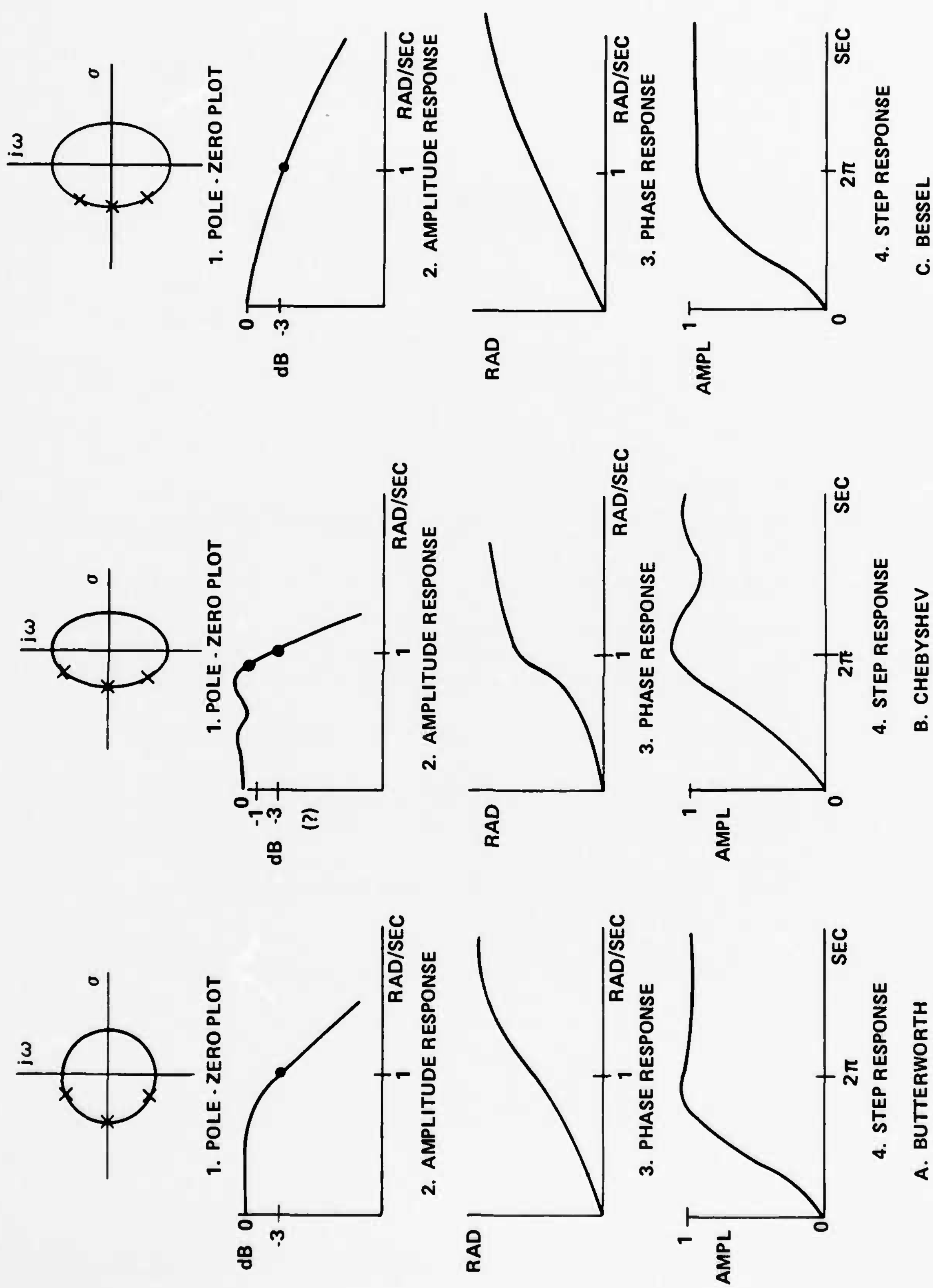

FIGURE 9. FILTER RESPONSES (BUTTERWORTH, CHEBYSHEV, BESSEL)

the number of poles used. The step response shows slight overshoot.

The Butterworth then is a good general-purpose filter, with reasonable amplitude response, fair phase response, and reasonable transient response. It is the most often used type. The Krohn-Hite* Model 3200 series has a 4-pole Butterworth characteristic.

A filter having slightly improved transition band steepness is the Legendre filter; in fact the Legendre has the maximum possible steepness without having a rise in the passband, i.e., still be monotonically decreasing. However, it is seldom used and will not be included here.

CHEBYSHEV FILTERS

The Chebyshev (also spelled Tshebycheff or various other ways) filter, also called the equiripple, achieves improved steepness in the transition band at the expense of having ripple in the passband. It is derived in such a way that all peaks and valleys have the same magnitude; hence the second name. Often a filter passband requirement will be that the response stay within some fixed limits; the Chebyshev fits this application well. For this reason the cutoff frequency is best specified as the point the curve passes through the ripple level on its way down, hence the question mark on Figure 9B2. In this report it is specified at the ripple point. Note also that the ripple may be specified as either peak or peak-to-peak; here it is the latter.

The poles are located on an ellipse (see Figure 9.B.1); the height/width ratio depends on the amount of ripple specified. The phase response has a rather sharp bend near the bandedge. The transient response has considerable overshoot and ringing. Note that for the normalized filter, one cycle of the ringing takes about 2π seconds.

Since the ripple must be specified, another variable is added, making the Chebyshev more difficult to catalog. Tables (5-8) are provided corresponding to several different values of ripple. Note that the frequency-determining resistors are not equal to unity, and values must be inverted for high-pass. Since the bandedge here is given at the ripple point which is not necessarily 3dB, the relative bandwidth at the 3dB point is listed. Note that although there is little difference for the sharper filters usually used, there is considerable difference for the low-order low-ripple cases. Noise bandwidth is correspondingly high for the latter cases; for comparison to other filter types one should first divide the noise bandwidth value by the 3dB-bandwidth value.

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*Registered Trademark

TABLE 5. CHEBYSHEV 0.1dB RIPPLE VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	3dB BANDWIDTH	NOISE BANDWIDTH
1	0.1526									6.6039	10.292
2		1.6968	0.5493							1.9432	2.144
3	1.0316	2.2542	0.7693							1.3890	1.442
4		1.3839	1.2670	2.5419	0.8671					1.2131	1.233
5	1.8556	1.9065	1.2540	2.6953	0.9148					1.1347	1.142
6		1.3318	1.9486	2.2490	1.1983	2.7842	0.9410			1.0929	1.094
7	2.6541	1.8185	1.7402	2.4586	1.1522	2.8396	0.9568			1.0680	1.065
8		1.3142	2.6206	2.1547	1.5500	2.5923	1.1188	2.8763	0.9670	1.0519	1.047
9	3.4428	1.7834	2.2286	2.3691	1.4177	2.6820	1.0948	2.9018	0.9739	1.0410	1.034
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$		

TABLE 6. CHEBYSHEV 0.5dB RIPPLE VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	3dB BANDWIDTH	NOISE BANDWIDTH
1	0.3493									2.8629	4.498
2		1.8422	0.8121							1.3897	1.489
3	1.6014	2.4139	0.9356							1.1675	1.167
4		1.5818	1.6750	2.6599	0.9697					1.0931	1.066
5	2.7600	2.1510	1.4483	2.7800	0.9826					1.0593	1.021
6		1.5372	2.5238	2.4476	1.3019	2.8465	0.9887			1.0410	0.997
7	3.9037	2.0839	1.9847	2.6117	1.2155	2.8869	0.9920			1.0310	0.983
8		1.5221	3.3700	2.3792	1.6698	2.7115	1.1614	2.9138	0.9941	1.0230	0.974
9	5.0402	2.0570	2.5291	2.5481	1.4867	2.7767	1.1255	2.9314	0.9954	1.0182	0.967
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$		

TABLE 7. CHEBYSHEV 1dB RIPPLE VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	3dB BANDWIDTH	NOISE BANDWIDTH
1	0.5088									1.9654	3.088
2		1.9545	0.9524							1.2176	1.253
3	2.0236	2.5044	1.0029							1.0949	1.041
4		1.7254	1.8919	2.7190	1.0068					1.0530	0.973
5	3.4543	2.2851	1.5262	2.8200	1.0059					1.0338	0.943
6		1.6857	2.8317	2.5450	1.3390	2.8751	1.0047			1.0234	0.927
7	4.8682	2.2290	2.0831	2.6831	1.2371	2.9082	1.0037			1.0172	0.918
8		1.6720	3.7726	2.4889	1.7128	2.7656	1.1756	2.9298	1.0029	1.0132	0.911
9	6.2763	2.2064	2.6503	2.6314	1.5100	2.8191	1.1356	2.9445	1.0024	1.0104	0.907
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$		

TABLE 8. CHEBYSHEV 3db RIPPLE VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	NOISE BANDWIDTH
1	0.9976									1.571
2		2.2335	1.1885							0.864
3	3.3487	2.6740	1.0916							0.774
4		1.9718	2.5002	2.8208	1.0523					0.744
5	5.6329	2.5322	1.6286	2.8866	1.0336					0.731
6		2.0425	3.3557	2.7108	1.3843	2.9218	1.0234			0.724
7	7.9061	2.4957	2.2127	2.8009	1.2626	2.9427	1.0172			0.720
8		2.0326	4.4591	2.6754	1.7651	2.8535	1.1922	2.9563	1.0132	0.717
9	10.1756	2.4810	2.8101	2.7685	1.5379	2.8872	1.1473	2.9655	1.0104	0.715
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$	

There is also a characteristic called the inverse Chebyshev which has the ripple in the stopband instead of the passband. This is seldom used and will not be included here. Chebyshev minimizes the least-square error in the passband; a generalization called "least-squares" may be made to incorporate a weighting function to emphasize portions of the passband having more importance than others.

The Chebyshev, then, is used where better transition band steepness is required and passband ripple can be tolerated. Larger ripple means steeper slope, so we have another tradeoff. Note, however, that past the transition band the slope in the reject band is the same as for the Butterworth. This is true in general since the ultimate slope is determined only by the number of poles and zeros. Any two filters having an equal number of poles (and equal number of zeros if used) will ultimately have the same slope. For Chebyshev a wider spread of component values occurs than for the Butterworth. This can cause a problem because high and low valued components may be made out of different materials (even within a given type designation) and hence may not track with temperature; however, it is minimized in this design by making the capacitors equal, as resistors are better in this respect.

BESSEL FILTERS

The Bessel, or Thompson or maximally - linear-phase filter is derived by the technique that is used for the Butterworth except the phase characteristic rather than amplitude is made as linear as possible at zero frequency.

The poles lie approximately on an ellipse again (see Figure 4) but relatively close to the real axis. The amplitude response falls off very gradually; the cutoff frequency is usually taken to be the 3dB down point for any number of poles (it may instead be specified in terms of phase or time delay), but unlike the Butterworth for a large number of poles, the shape approaches not a rectangular characteristic but a Gaussian "bell" shape. Here the 3dB point is used (see Table 9).

The phase characteristic is very linear to the 3dB point and beyond. The step response achieves a fairly sharp rise with little overshoot (less than 1%). (Some texts incorrectly say no overshoot.) In fact, it can be shown that the sharpest possible rise without overshoot is for the Gaussian frequency response. This shape is interesting because it transforms into an impulse response which is also Gaussian. However, this extends to infinity in both directions, so an exact Gaussian is not possible without an infinite time delay.

The Bessel characteristic is used only where phase linearity or transient response is of driving importance and the attenuation characteristic is secondary. The high-pass versions are of lesser

TABLE 9. BESSEL VALUES

LOW-PASS	R_0	G_1	R_1	G_2	R_2	G_3	R_3	G_4	R_4	NOISE BANDWIDTH
1	1.0000									1.571
2		1.2679	0.7852							1.152
3	0.7536	1.5529	0.6885							1.073
4		1.0842	0.7046	1.7585	0.6284					1.047
5	0.6636	1.2255	0.6406	1.9089	0.5679					1.038
6		1.0405	0.6227	1.3639	0.5913	2.0228	0.5244			1.039
7	0.5934	1.1216	0.5823	1.4867	0.5484	2.1121	0.4877			1.042
8		1.0238	0.5606	1.2130	0.5441	1.5932	0.5104	2.1842	0.4555	1.046
9	0.5384	1.0758	0.5321	1.3032	0.5131	1.6852	0.4804	2.4278	0.4304	1.049
HIGH-PASS	$1/R_0$	G_1	$1/R_1$	G_2	$1/R_2$	G_3	$1/R_3$	G_4	$1/R_4$	

interest, as the linear phase characteristic does not transform in a meaningful way and the step response does have significant overshoot.

There is a class of filters called transitional filters which are a compromise between Butterworth and Bessel. These combine the advantages of both, or the disadvantages of both, depending on how you look at it. These are seldom used and will not be included here. One company[5] markets a type they term Besselworth* which appears to be a phase-corrected Butterworth, having a Butterworth amplitude characteristic but good phase linearity. Equiripple (Chebyshev) or least-squares techniques may also be applied to phase linearity. The filter type having the fastest realizable rise with no overshoot is the prolate, an obscure type seldom used. An approximation to the Gaussian filter may be made by expressing the Gaussian as a power series and truncating at a finite number of terms, and is referred to as a Gaussian filter.

ELLIPTIC FILTERS

An extension of the Chebyshev approach is elliptic filters (Figure 10), also called Cauer or double-equiripple, which achieve even better transition-band steepness at the expense of ripple in both passband and stopband. The elliptic is essentially a Chebyshev modified by adding zeros in the stopband. Strictly speaking the number of zeros equals the number of poles, but in practice filters having an arbitrary number of zeros are included in this category. The poles are again all on an ellipse, although not quite the same as for the Chebyshev, and the zeros are all on the imaginary axis. The cutoff frequency may be specified in the same two ways as the Chebyshev, and the amount of passband ripple must again be specified. Additionally the height of the stopband peaks, called the stopband rejection, must be specified, making this type doubly difficult to catalog. (The valleys are all zero amplitude, which is minus infinity on a log scale.) The response at very high frequencies falls off relatively slowly, or not at all if the number of zeros exactly equals the number of poles.

The phase response in the passband is similar to the Chebyshev; it eventually reverses and falls back down due to the zeros. A filter requirement may specify that the passband response stay within certain limits and the stopband response reach a certain rejection at a specified frequency and then stay below that limit; the elliptic filter suits such a specification well. This is often encountered with sampled (digital) systems such as spectrum analyzers in which

[5]"Linear Phase, Maximally Flat Gain, Low Pass Filter ," EIT Corp., no date.

*Registered Trademark.

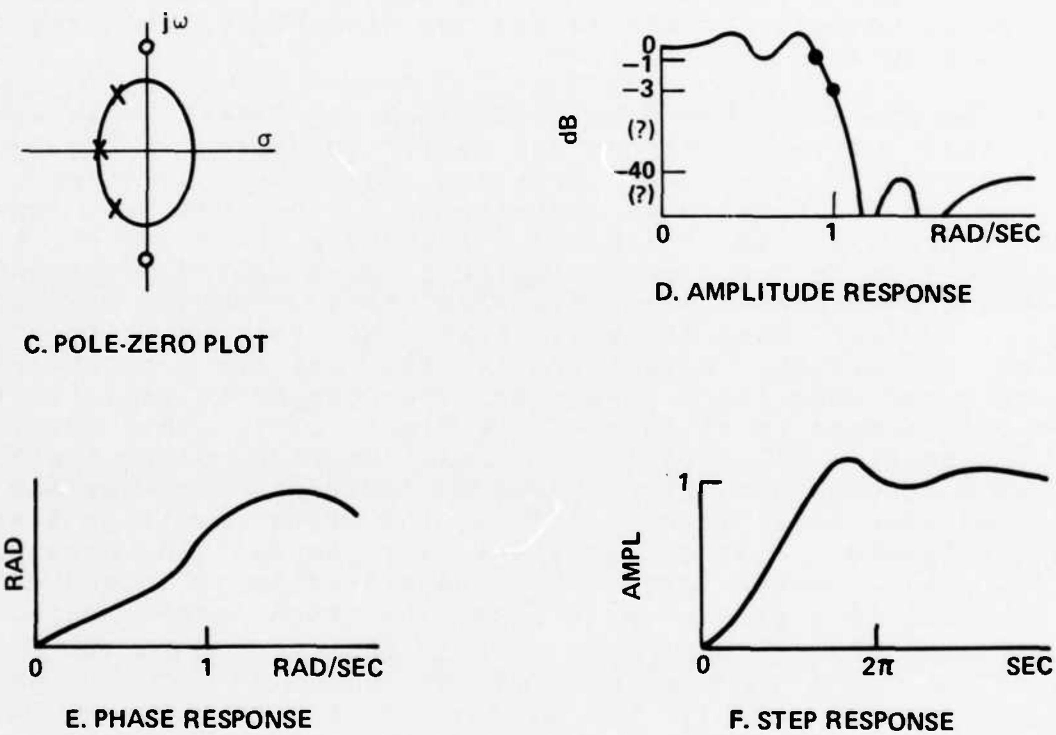

FIGURE 10. ELLIPTIC FILTER RESPONSES

the higher frequencies would be "aliased" or folded back into the passband. The Rockland* Model 753A Elliptic Filter 6 advertises 115dB/octave, but this is a misleading specification for this type of filter, as the response is not down 115dB at one octave.

Circuits for elliptic filters are difficult not only to catalog, due to the number of variables, but to design; the problem here is the creation of the required zeros. Each zero-pair comes with a pole-pair, which must be made to correspond to one of the pole-pairs required. Early active circuits, employing twin-tees which require redundant components, were clumsy. Recent texts point out that elliptic filters may be constructed using universal active filters as the resonator elements, but a conversion must be made between the poles and zeros normally cataloged and the element values required; also redundant op-amps are required.

A unified approach[7] has been devised which takes advantage of the fact that elliptic filters are easier to design in the passive domain. Furthermore, computer programs have been written[8], and element values extensively catalogued.[9] The passive low-pass prototype is shown in Figure 11A. The best way to convert to an active circuit is to use super-capacitors, also called D-elements or FDNR's (Frequency-Dependent-Negative- Resistors), which are active elements that have the property that the voltage is the <u>double</u> integral of the current. Capacitors in the passive prototype are changed to super-capacitors, resistors are changed to capacitors and inductors are changed to resistors (see Figure 11B). This gets rid of the inductors while giving a configuration where the active elements are grounded. Each element impedance in the passive prototype has been multiplied by 1/S, so the overall voltage transfer <u>ratio</u> is unchanged. Extra resistors are needed to provide DC continuity, which gets lost in the transformation. To convert the passive low-pass to a passive high-pass, inductors become capacitors and vice versa (Figure 11C). The high-pass is converted to an active version simply by synthesizing the inductors, since they are all grounded (Figure 11D); the active circuit within the dotted lines appears at its input as a grounded inductor. Circuit behavior in the saturated state is undefined; the extra resistors here prevent

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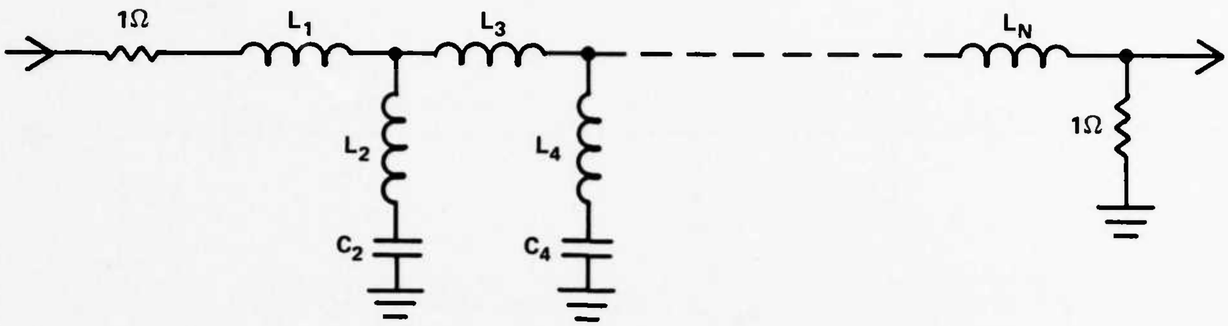
*Registered Trademark

[6] "Dual Section High-Pass/Low-Pass Brickwall Filter, Model 753A," Wavetek Rockland, Inc. Apr 1982.

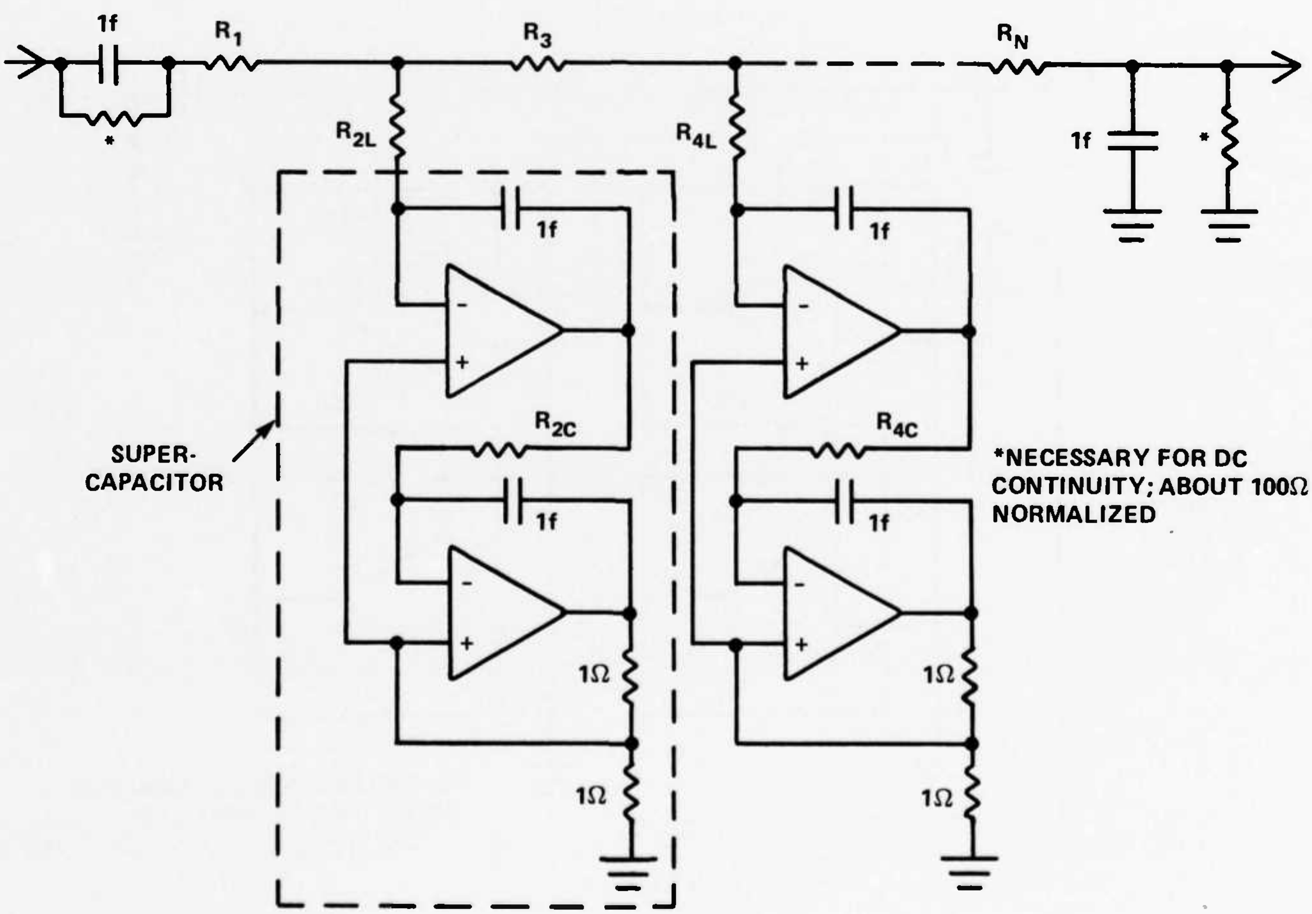
[7] Delagrange, D. A., "Design Active Elliptic Filters With A 4-Function Calculator," <u>EDN Magazine</u>, 3 March 1982.

[8] Baezlopez, David Jose, "Sensitivity and Synthesis of Elliptic Function," Ph.D. Dissertation, U. of Arizona, Tucson, 1978.

[9] Zverev, Anatol I., <u>Handbook of Filter Synthesis</u> (New York: John Wiley & Sons Inc., 1967).

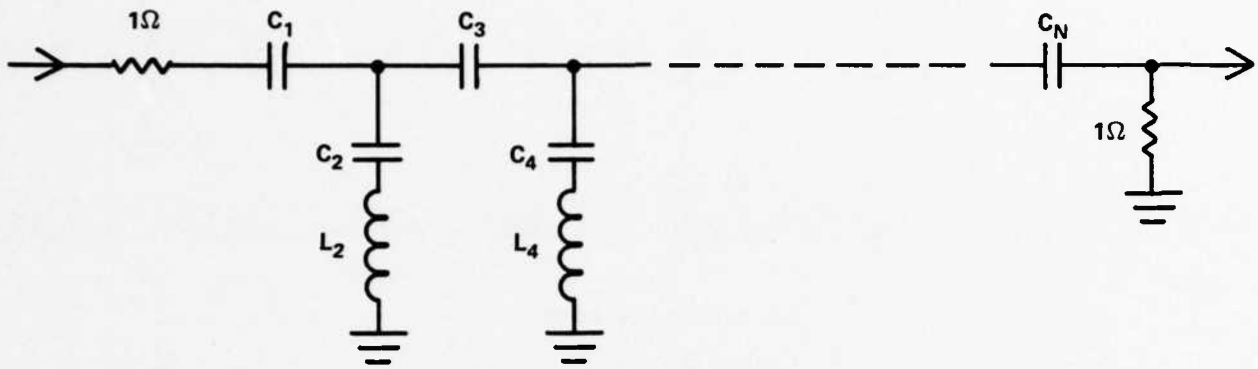


A. PASSIVE LOW-PASS

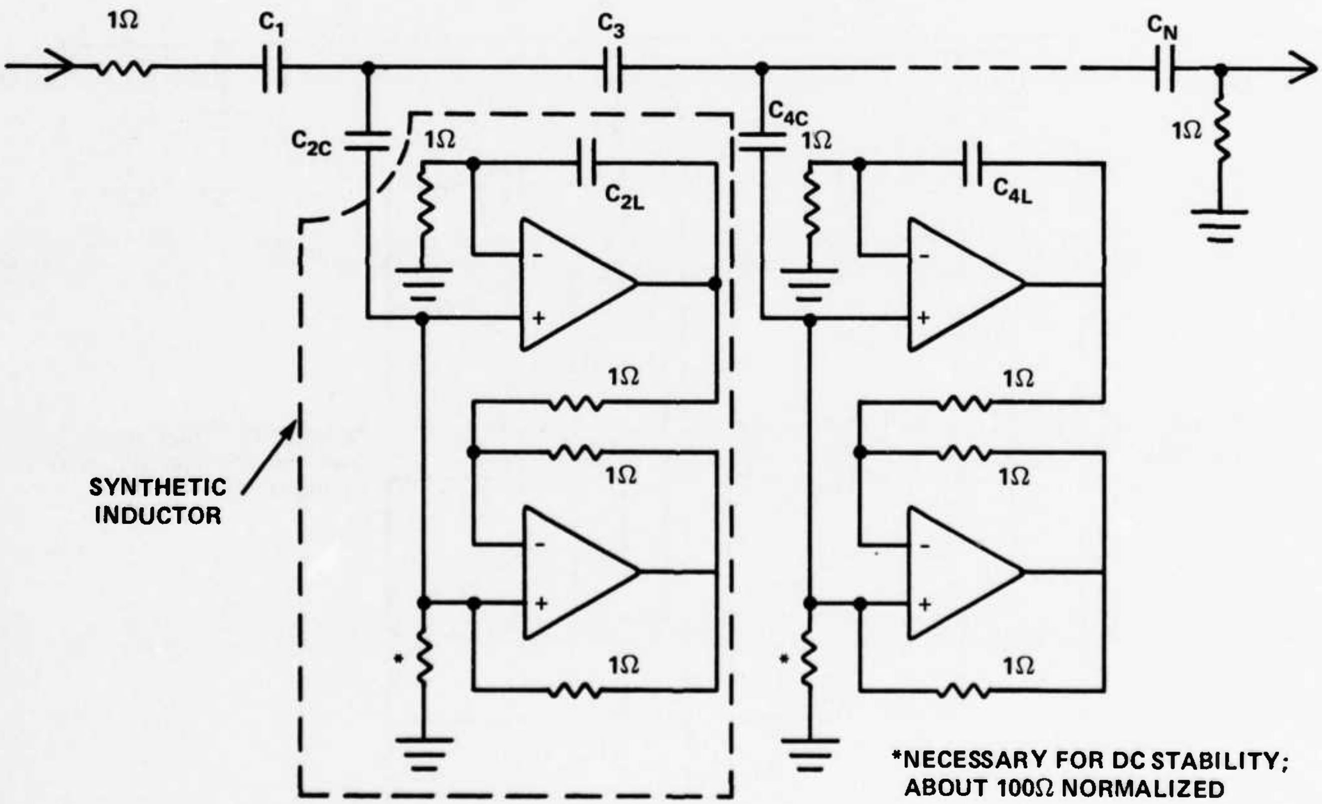


B. ACTIVE LOW-PASS

FIGURE 11. ELLIPTIC FILTER CIRCUITS



C. PASSIVE HIGH-PASS



D. ACTIVE HIGH-PASS

FIGURE 11. ELLIPTIC FILTER CIRCUITS (CONT.)

circuit latch-up.

Element values have been assigned so the same table can apply to all four cases. Tables 10-15 give a sampling of designs, taken from Ref. 9. Note that among steepness of the transition band, passband ripple, and stopband attenuation, any one may be traded off for any other. The active low-pass has been arranged so all capacitors are equal; this cannot be done for the others.

CONSTANT K-FILTERS

One of the oldest filter types is the constant-K ladder (Figure 12); the simplest form has all sections identical as shown. See reference 10 for a more detailed explanation. These are derived by an approximation technique, not analytical, and are not optimized with respect to any particular variable. The original circuits were passive (12A and 12B). The dotted lines indicate that more sections can be added; hence the descriptor "n-pole".

The pole-plot is difficult to calculate and is not shown. The amplitude characteristic does have ripple; the amount cannot be controlled and depends on the number of sections used. The attenuation slope can be quite steep simply because many poles can be easily added. The cutoff frequency is approximate and is calculated from the component values. It is exact for the 3-pole (reference 10 mistakenly says 5); higher orders have more attenuation at the cutoff frequency. The phase characteristic is fairly linear most of the way across the passband, but bends sharply upward at the bandedge. The step response exhibits considerable overshoot and ringing; however, it is a fair approximation to a delay line, having a rise time noticeably shorter than the delay time.

The active circuits (Figures 12C and 12D) are similar to the elliptic filters. These designs have been juggled so most of the capacitors and resistors are equal; since the values on the ends of the ladder are just different by a factor of two they may be obtained using the same components and paralleling. These filters are a general-purpose compromise; they are useful in a large number of applications which simply require a filter with reasonable performance for all parameters but no stringent requirements on any one.

[10]Delagrange, A. D., A Useful Filter Family, NSWC/WOL TR 85-170, 20 October 1975.

TABLE 10. ELLIPTIC 0.28 DB RIPPLE 3-POLE 2-ZERO VALUES

			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	8.8	19.9	32.5	55.2
PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
L_1	R_1	$1/C_1$	$1/C_1$	0.7944	1.0813	1.2382	1.3260
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	1.4111	0.4251	0.1469	0.0246
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.4575	0.8099	1.0067	1.1172
L_3	R_3	$1/C_3$	$1/C_3$	0.7944	1.0813	1.2382	1.3260

TABLE 11. ELLIPTIC 0.28 DB RIPPLE 5-POLE 4-ZERO VALUES

			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	29.5	48.9	70.0	107.9
PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
L_1	R_1	$1/C_1$	$1/C_1$	1.1677	1.3332	1.4084	1.4475
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	0.3872	0.1511	0.0567	0.0098
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.9858	1.1701	1.2534	1.2972
L_3	R_3	$1/C_3$	$1/C_3$	1.5108	1.9056	2.1283	2.2553
L_4	R_{4L}	$1/C_4$	$1/C_{4C}$	1.3083	0.4257	0.1519	0.0258
C_4	R_{4C}	$1/L_4$	$1/C_{4L}$	0.5458	0.9370	1.1553	1.2792
L_5	R_5	$1/C_5$	$1/C_5$	0.7337	1.1225	1.3216	1.4318

TABLE 12. ELLIPTIC 0.28 DB RIPPLE 7-POLE 6-ZERO VALUES

			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	50.9	78.0	107.5	160.6
PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
L_1	R_1	$1/C_1$	$1/C_1$	1.3232	1.4204	1.4623	1.4836
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	0.2000	0.0795	0.0300	0.0052
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	1.1584	1.2666	1.3137	1.3378
L_3	R_3	$1/C_3$	$1/C_3$	1.6183	2.0239	2.2400	2.3612
L_4	R_{4L}	$1/C_4$	$1/C_{4C}$	1.0436	0.3814	0.1411	0.0243
C_4	R_{4C}	$1/L_4$	$1/C_{4L}$	0.7013	1.0861	1.3012	1.4241
L_5	R_5	$1/C_5$	$1/C_5$	1.3663	1.8941	2.1859	2.3514
L_6	R_{6L}	$1/C_6$	$1/C_{6C}$	0.7233	0.2673	0.0985	0.0169
C_6	R_{6C}	$1/L_6$	$1/C_{6L}$	0.7893	1.0886	1.2413	1.3248
L_7	R_7	$1/C_7$	$1/C_7$	0.9741	1.2588	1.3974	1.4720

TABLE 13. ELLIPTIC 1.25 DB RIPPLE 3-POLE 2-ZERO VALUES

			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	15.3	26.8	39.4	62.2
PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
L_1	R_1	$1/C_1$	$1/C_1$	1.4922	1.8694	2.0701	2.1819
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	1.4198	0.4822	0.1732	0.0294
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.4547	0.7140	0.8537	0.9317
L_3	R_3	$1/C_3$	$1/C_3$	1.4922	1.8694	2.0701	2.1819

TABLE 14. ELLIPTIC 1.25 DB RIPPLE 5-POLE 4-ZERO VALUES

PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
		STOPBAND/PASSBAND RATIO		1.15	1.52	2.28	5.24
		STOPBAND ATTENUATION		36.5	55.9	76.9	114.9
L_1	R_1	$1/C_1$	$1/C_1$	1.9444	2.1590	2.2572	2.3086
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	0.4805	0.1898	0.0715	0.0124
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.7945	0.9318	0.9949	1.0280
L_3	R_3	$1/C_3$	$1/C_3$	2.1125	2.6821	2.9919	3.1664
L_4	R_{4L}	$1/C_4$	$1/C_{4C}$	1.4882	0.5206	0.1898	0.0325
C_4	R_{4C}	$1/L_4$	$1/C_{4L}$	0.4798	0.7662	0.9250	1.0152
L_5	R_5	$1/C_5$	$1/C_5$	1.4228	1.8974	2.1482	2.2887

TABLE 15. ELLIPTIC 1.25db RIPPLE 7-POLE 6-ZERO VALUES

			STOPBAND/PASSBAND RATIO	1.15	1.52	2.28	5.24
			STOPBAND ATTENUATION	57.8	85.0	114.4	167.6
PASSIVE LOW-PASS	ACTIVE LOW-PASS	PASSIVE HIGH-PASS	ACTIVE HIGH-PASS				
L_1	R_1	$1/C_1$	$1/C_1$	2.1357	2.2625	2.3175	2.3457
L_2	R_{2L}	$1/C_2$	$1/C_{2C}$	0.2535	0.1012	0.0382	0.0066
C_2	R_{2C}	$1/L_2$	$1/C_{2L}$	0.9141	0.9954	1.0310	1.0493
L_3	R_3	$1/C_3$	$1/C_3$	2.2594	2.8064	3.0977	3.2612
L_4	R_{4L}	$1/C_4$	$1/C_{4C}$	1.3297	0.4952	0.1842	0.0318
C_4	R_{4C}	$1/L_4$	$1/C_{4L}$	0.5504	0.8366	0.9966	1.0880
L_5	R_5	$1/C_5$	$1/C_5$	1.9028	2.6284	3.0243	3.2479
L_6	R_{6L}	$1/C_6$	$1/C_{6C}$	0.8835	0.3359	0.1250	0.0216
C_6	R_{6C}	$1/L_6$	$1/C_{6L}$	0.6462	0.8665	0.9786	1.0399
L_7	R_7	$1/C_7$	$1/C_7$	1.7004	2.0580	2.2350	2.3309

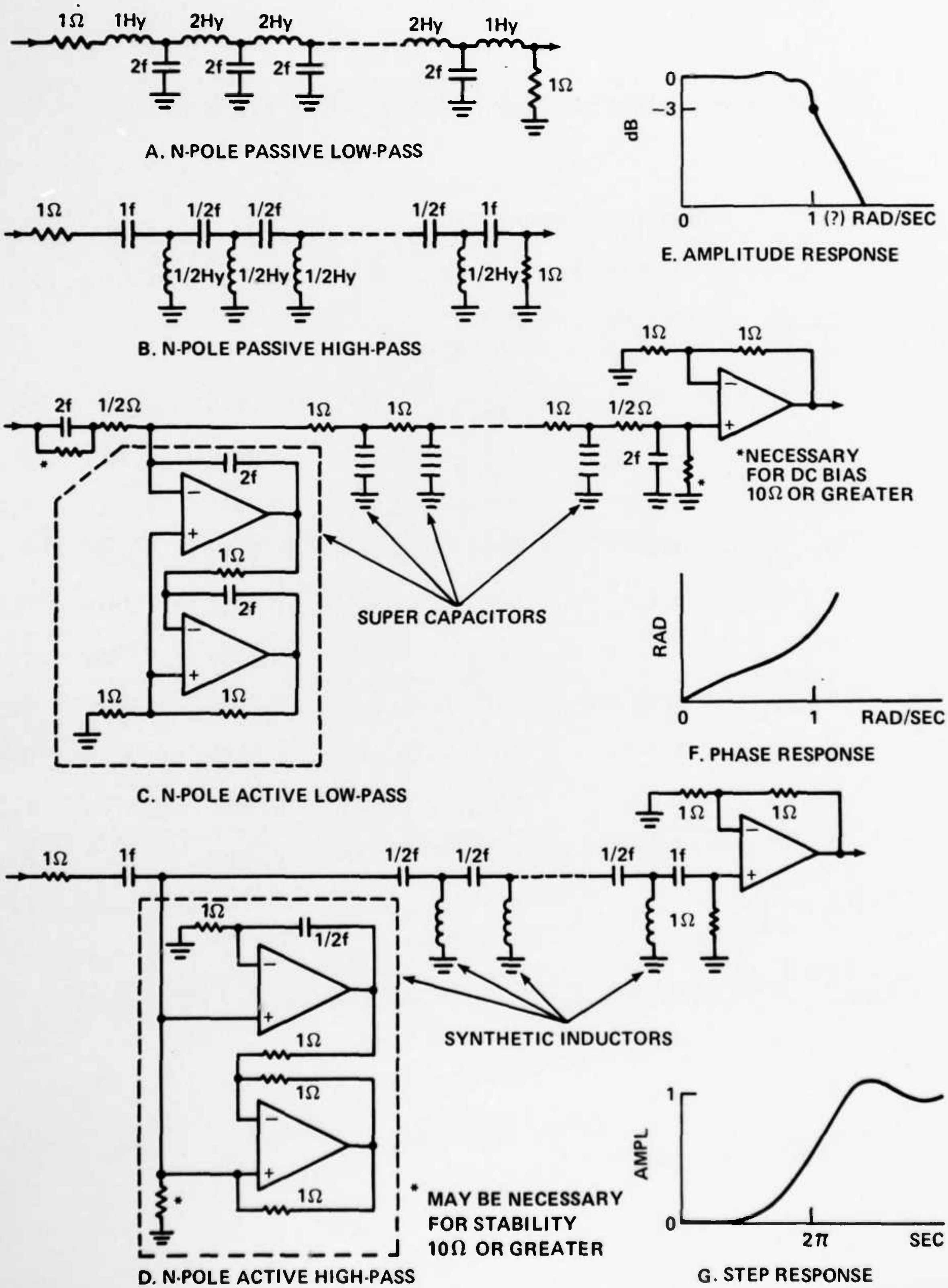


FIGURE 12. CONSTANT-K FILTERS

LERNER FILTERS

In the filters covered thus far, there has been an implied trade-off between squareness of amplitude response and linearity of phase. This occurs because they were all "minimum phase" a term which has not been explained yet. Thinking of the pole-zero plot, consider a zero in the left-half plane and its mirror image in the right-half plane. The effect on amplitude response would be the same for either one, as a vector from any point on the imaginary axis will be same length to either one. However, the phase angle is different; in particular, it is obvious that the angle to the zero in the left-half plane is always less (phase lag is measured counter-clockwise from the negative real axis for positive frequencies for poles; opposite for zeros.) A filter having no zeros in the right-half plane is said to be minimum phase. (Remember that poles in the right-half plane are not allowed.) In a minimum-phase filter, the amplitude and phase response are directly related by an equation called the Hilbert transform; improving one characteristic unfortunately always makes the other worse.

For years no one pursued the problem much further, but in 1963 Lerner pointed out that having minimum-phase is not a requirement in most systems, and that if right-half plane zeros are used, good performance may be achieved in both amplitude and phase response.[11] Furthermore, he devised a method for generating a good filter. Lerner's work has been nearly ignored, but has recently been updated;[12] the results will be summarized in this section.

Lerner derived his filters as passive filters; Figure 13A shows his basic design. Figure 13B shows a conversion to an active filter by means of super-capacitors. (The device on the right is a differential summer.) Lerner's method utilizes the pole-residue plot (Figure 13C), not the pole-zero plot. The prototype filter is a bandpass, not the usual low-pass. Basically, the poles are spaced parallel to the imaginary axis, with the sign alternating. The spacing is twice the distance from the axis. At each end of the string are "corrector" or "termination" poles having half the spacing but the same distance from the axis, and half the residue (amplitude).

The pole-residue plot converts directly to the passive filter (see 13A). Each pole-pair is created by an LC; two sets are driven from opposite sides of a differential transformer to give the alternating signs. (This creates the zeros which are required,

[11] Lerner, Robert M., "Bandpass Filters With Linear Phase," <u>Proceedings Of the IEEE</u>, Mar 1964.

[12] Delagrange, A. D., "Design Lerner Filters Using Op-Amps," <u>Electronic Design Magazine</u>, 15 Feb 1979.

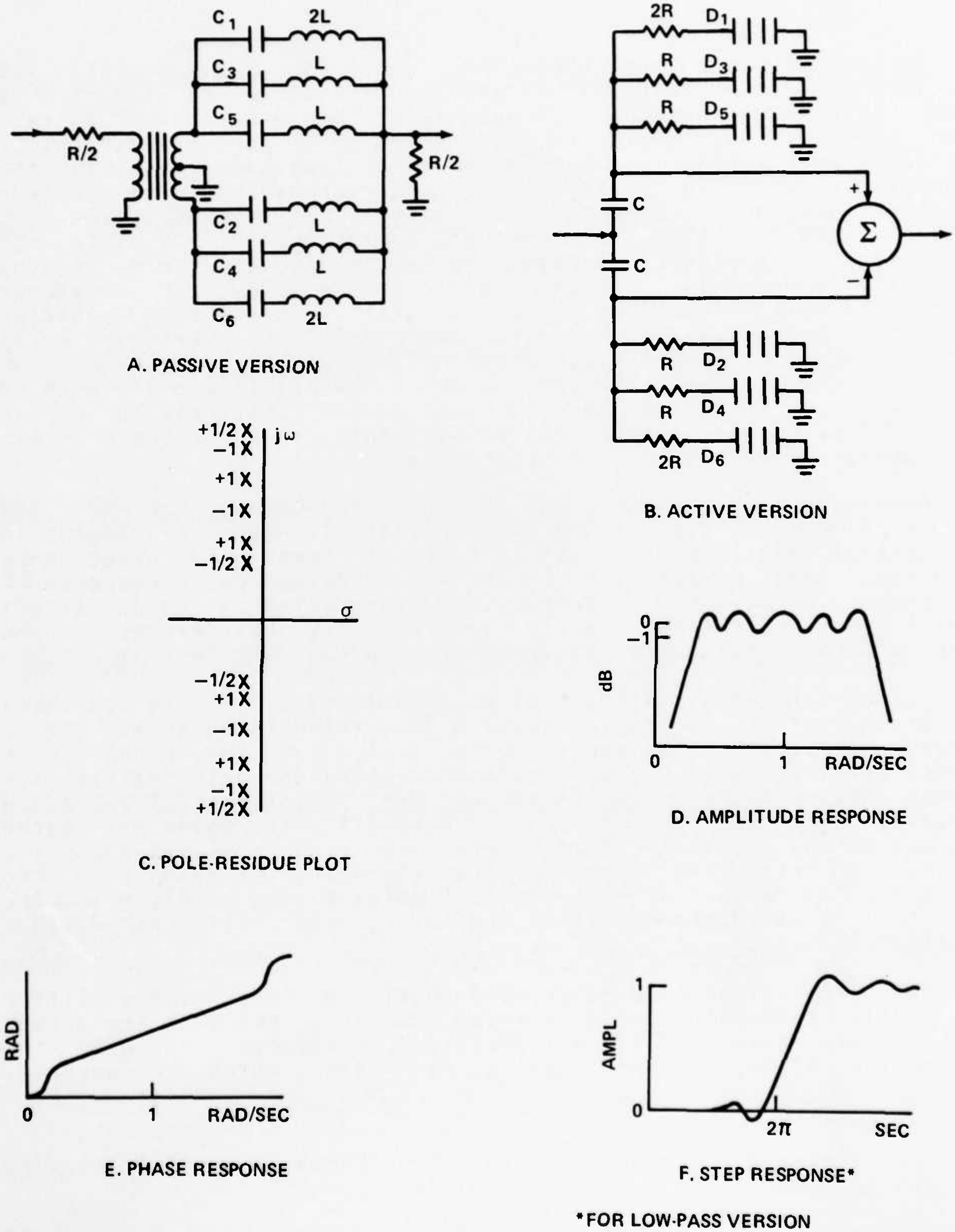


FIGURE 13. LERNER FILTER METHOD

although they are not explicitly shown in the pole-residue plot.) The residues are determined by the inductors, which are hence all equal except for the two on the end. The capacitors determine the frequency of the resonators. The filter is terminated on each end with half the usual matching impedance. The circuit may be transformed to an active circuit in several ways; suffice it to say the one shown (13B) is the best found by this author.

The Lerner filter has ripple in both amplitude and phase, although the amount is not directly controllable, as decreasing one would increase the other. Theoretically the amplitude ripple is within 1dB and the phase ripple is within 5°; in practice errors due to component value tolerances are usually predominant over the theoretical limitations. The amplitude response (13D) typically falls quite steeply at the edges because ripple is allowed and a fairly large number of poles are normally used. The phase response is quite linear clear past the bandedge, and then does weird things (irrelevant anyway). Step response of the low-pass is much like a delay line due to the good phase linearity, showing a clear delay, quick rise and considerable ringing. Note the "pre-shoot." The step response of the bandpass is not particularly meaningful since both high and low frequencies are missing, and is not shown.

An actual circuit for a 12-pole bandpass filter of approximately octave bandwidth is shown in Figure 14. The double op-amp circuits are super-capacitors; the three op-amps on the right are a standard differential amplifier (summer). The values have been juggled so all capacitors are the same; resonator frequencies are determined by resistances, which are varied in pairs to minimize the range of values required. Again the super-capacitor transformation removes the bias path for the op-amps so dribble resistors must be added. The cutoff frequency points are taken as the corrector (termination) resonator frequencies which may be read directly from the reciprocals of the resistor values in the end resonators; these of course represent peaks in the response so the bandwidth at 1dB or 3dB down will be slightly wider.

For the low-pass the poles continue down to and include the real axis with equal spacing (not shown, see reference 12); hence the lower termination becomes a single real pole. In the circuit (Figure 15) this corresponds to eliminating the super-capacitor from the lowest frequency resonator. Note that the cutoff frequency is not normalized to unity; instead the lowest resonator frequency is normalized to unity, which means the cutoff frequency is determined by the corrector resonator.

The transformation to high-pass is a little more obscure. The upper corrector pole-pair of the bandpass is replaced with a single real pole of cutoff frequency equivalent to the next pole-pair were it <u>equally</u> spaced (not shown). In the circuit (Figure 16) this corresponds to eliminating the resistor only in the highest frequency resonator; the frequency-determining resistors in the super-capacitor acquire a square-root over the expected value. Since

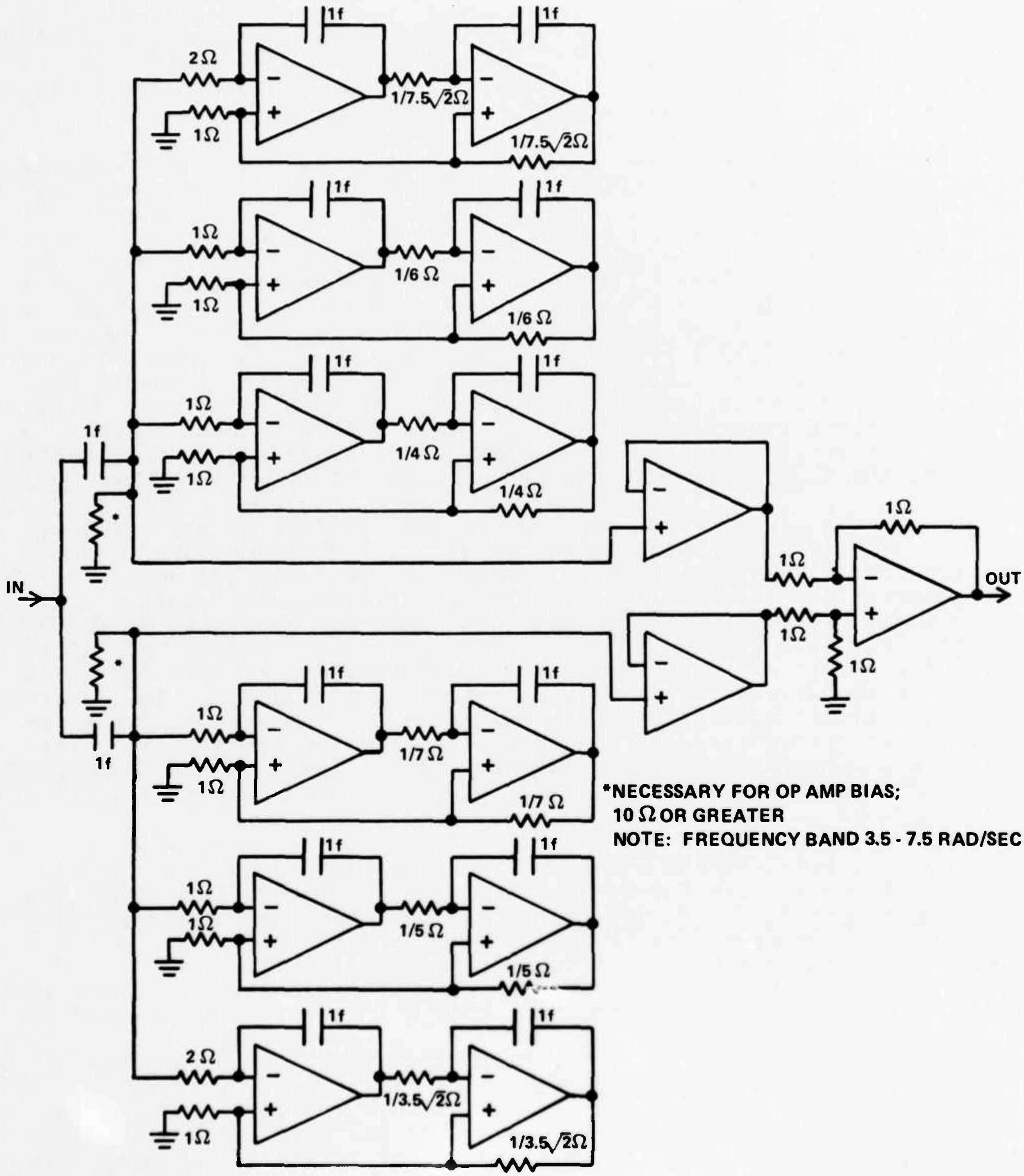


FIGURE 14. 12-POLE BANDPASS LERNER FILTER

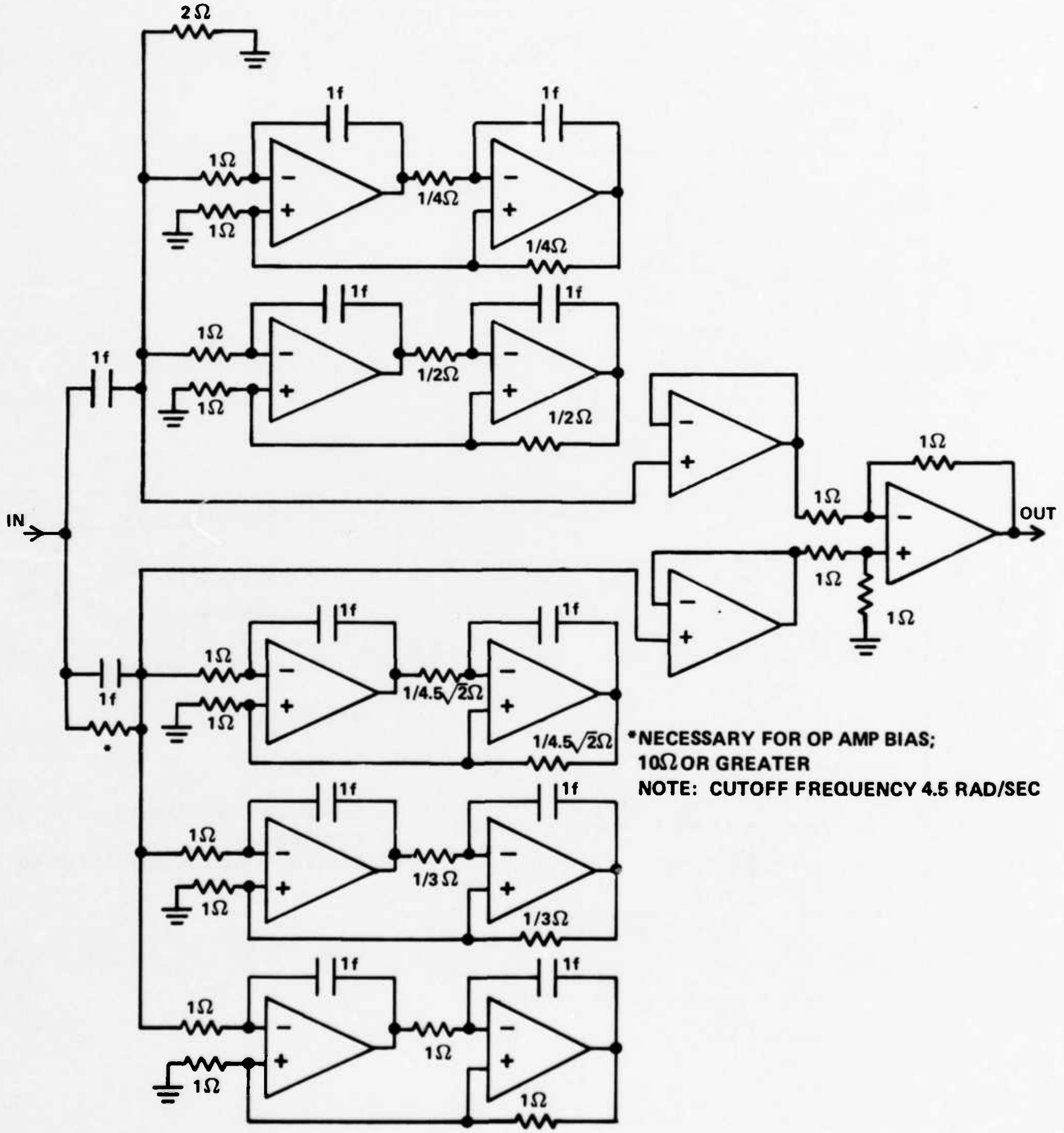


FIGURE 15. 11-POLE LOW-PASS LERNER FILTER

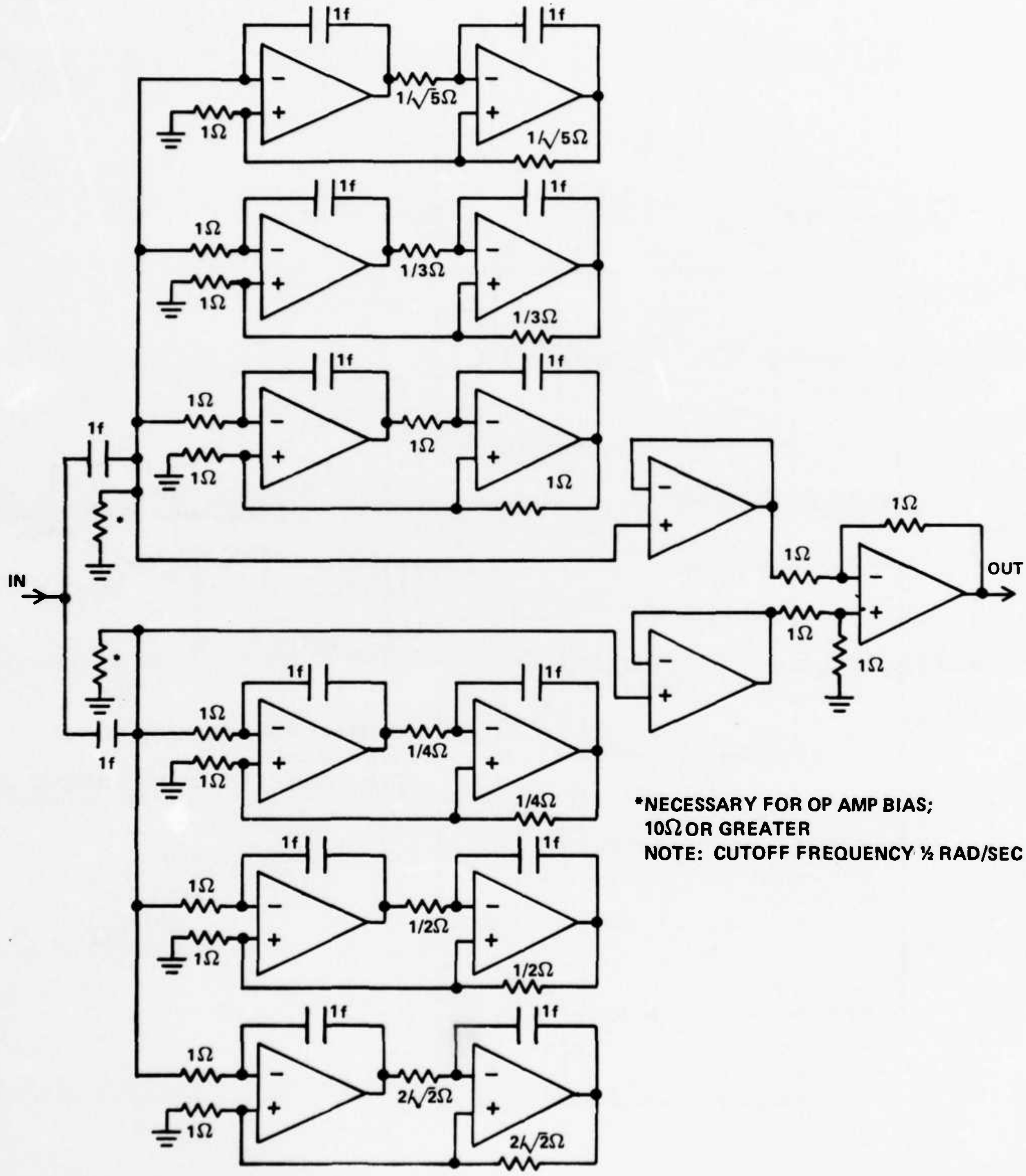


FIGURE 16. 11-POLE HIGH-PASS LERNER FILTER

the Lerner characteristic is derived on a linear frequency scale, the high-pass cutoff on a log scale may appear either very sharp or very gradual depending entirely on where the poles are placed. Note also that linear phase shift cannot be continued indefinitely without an infinite number of stages; phase shift of the high-pass version flattens out to a constant above the frequency of the termination pole.

Lerner filters are to be used when both sharp amplitude cutoff and linear phase response are required. They are practical only when using a fairly large number of poles because they are basically an ideal repetitive design with non-ideal terminations, which makes the filters rather large. However, the design is fairly simple, and the large number of poles insures a sharp cutoff. Stopband attenuation is limited, however, by the accuracy of the differential amplifier, typically about 60dB. For more detail see reference 12.

BANDPASS AND BANDSTOP FILTERS

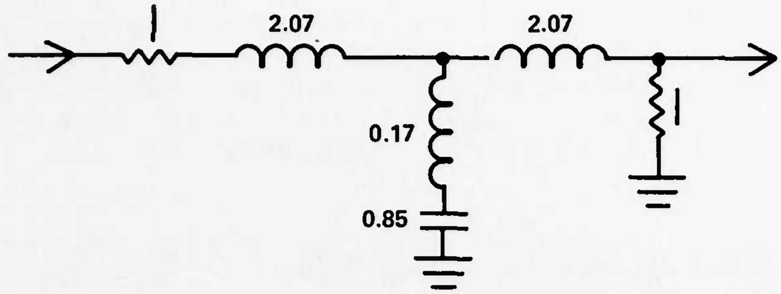
Thus far, except for the Lerner filters, only low-pass and high-pass filters have been discussed. Many texts include the bandpass version of each filter type, but the usual transformations produce circuits requiring inductors. Typically half of the inductors and half of the capacitors are not grounded, so neither the synthetic inductors or the super capacitors used earlier will work.

The most common bandpass transformation[13] is to make the substitution

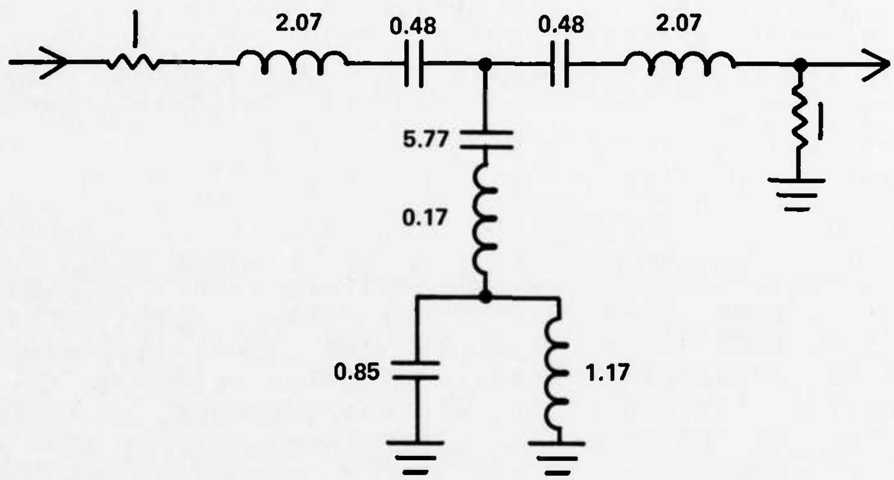
$$S \rightarrow \frac{S^2 + \omega_0^2}{S}$$

in the filter function. This moves the low-pass function (including its mirror image) up to a center frequency of ω_0 (on a geometric basis) and compresses it by a factor of two, so the bandwidth is essentially unchanged. In the circuit this adds a parallel inductor to each capcitor and adds a series capacitor to each inductor (see Figs 17A and 17B), with each resonator having a frequency of ω_0. The conversion to bandstop (also called band-reject) (Figure 17C) is complementary; here each new capacitor or inductor has a value equal to the reciprocal of the value of the original inductor or capacitor respectively (same impedance magnitude), and the original inductor or capacitor is then chosen to resonate at the desired center frequency. The example shown is a 1.25dB, 39dB stopband

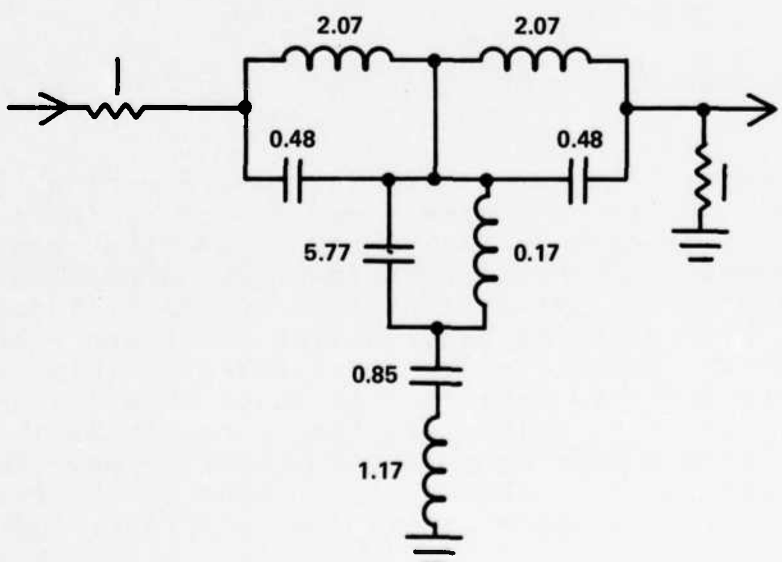
[13] Guillemin, E. A., Synthesis of Passive Networks (New York: John Wiley & Sons, Inc., 1957).



A. PASSIVE LOW-PASS PROTOTYPE



B. PASSIVE BANDPASS PROTOTYPE



C. PASSIVE BANDSTOP PROTOTYPE

FIGURE 17. BANDPASS AND BANDSTOP TRANSFORMATIONS

elliptic design taken from Reference 9. Center frequency was chosen to be 1 rad/sec, which gives about an octave band.

There are exotic techniques for getting around the problem of floating inductors such as GIC embedding[14] but these are fairly involved and applicable only to certain forms. It is easier to extend the synthetic inductor design already given to make it floating; Figure 18 gives such a circuit.[15] It is equivalent to a 1-Henry inductor; larger/smaller values are obtained by scaling the capacitor in direct proportion. The dotted resistors may or may not be necessary for stability. Substitution of this circuit for each inductor of Figure 17B and appropriately scaled for 10KΩ, 1.0Krad/sec (160Hz) gives the circuit of Figure 19; the bandstop for the special case of octave-band consists of the same circuit sections simply rearranged.

Figure 20 shows the bandpass circuit performance; the solid line is almost "textbook". The dashed line represents raising the center frequency to 1.6KHz by reducing the capacitors by a factor of ten; the stability resistors had to be reduced considerably and performance is noticeably degraded. Figure 21 shows the performance of the bandstop version. The dotted line represents the first attempt. The resonator frequencies were mismatched so the high impedance of the shunt parallel resonator was not cancelled by the two series resonators as it should be (see Figure 17C), allowing signal leak-through at the center frequency; slight tweaking gave the solid wave.

These techniques are required for bandwidths on the order of an octave. There are two special cases, which fortunately are easier to handle: If a wide bandpass (much greater than one octave) is required, the signal may be passed through a low-pass and a high-pass in tandem, first removing the high frequencies and then the lows. If a narrow bandpass is required (the common problem of picking out one frequency from all others), a special class of circuits called narrowband or tuned circuits exists. Indeed, any resonator circuit theoretically can be made into a tuned circuit, but there are a number of practical problems. Tuned circuits near resonance exhibit large voltage and/or current swings; this is not a serious problem with passive circuits, but an active circuit may go into limiting and the calculated linear characteristic then becomes meaningless.

The pole-zero plot of a bandpass tuned circuit is shown in Figure 22C. The poles are much closer to the imaginary axis than the real axis; there is a single zero at the origin. Obviously the amplitude is large when the frequency is in the vicinity of one of the poles. One of the practical problems is that for some circuits a variation in component values or addition of stray capacitance or inductance may move the poles into the right half-plane, meaning the circuit will oscillate. The amplitude response is a single peak at

[14] "Active Filters," Amperex Corp.., Report No. S-166, 1977.

[15] Delagrange, A. D., "Make Passive Filters Active With A Floating Synthetic Inductor," EDN Magazine, Vol. 28, 13, 23 June 1983.

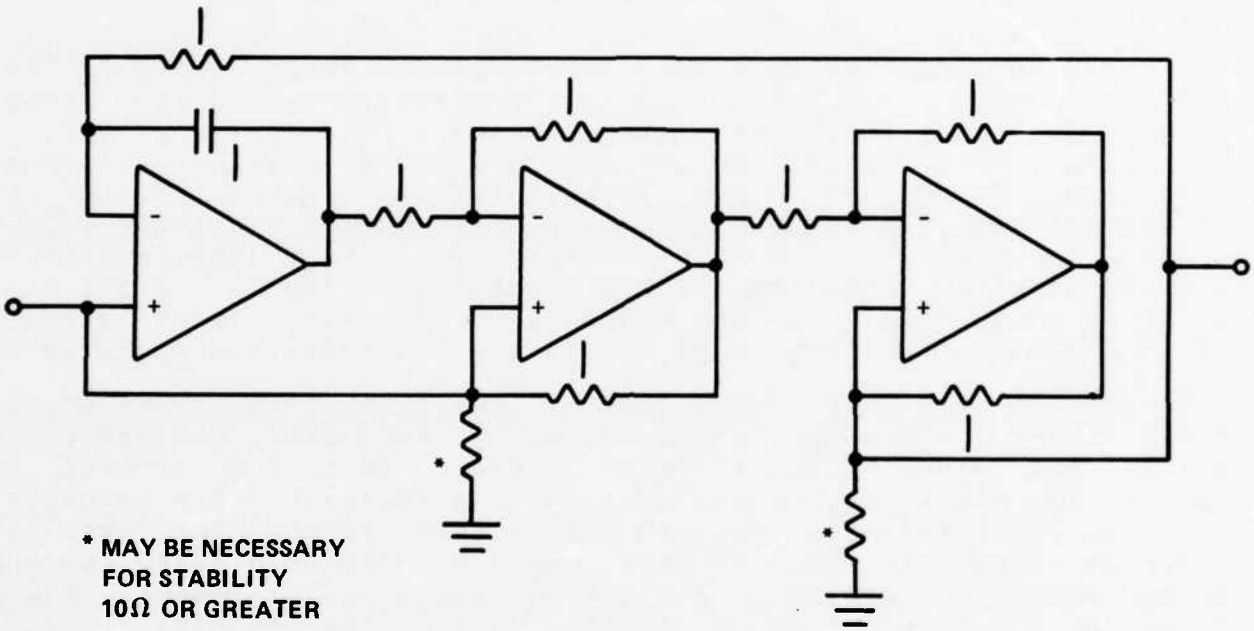
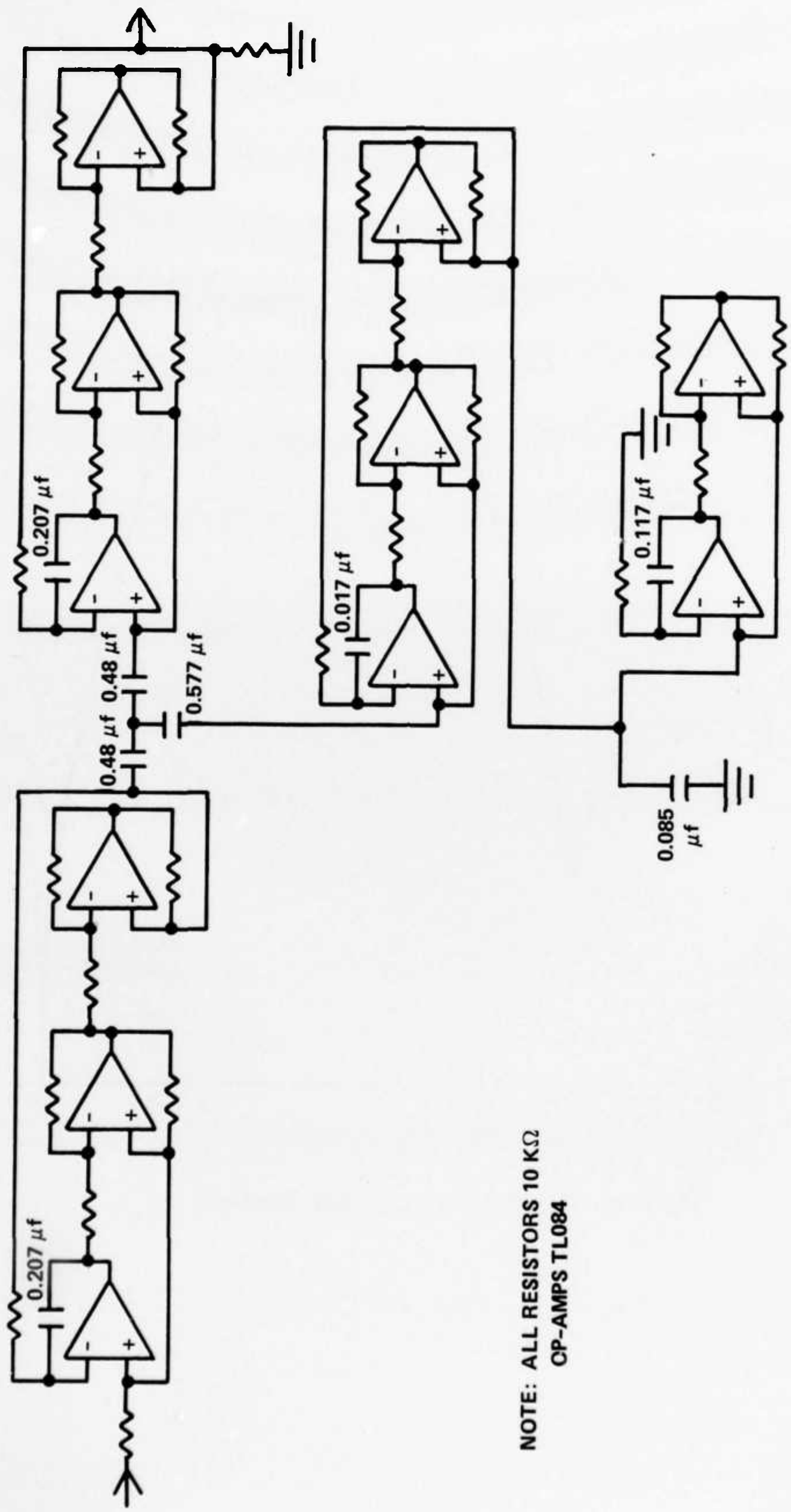


FIGURE 18. FLOATING SYNTHETIC INDUCTOR



NOTE: ALL RESISTORS 10 KΩ
CP–AMPS TL084

FIGURE 19. ACTIVE BANDPASS CIRCUIT

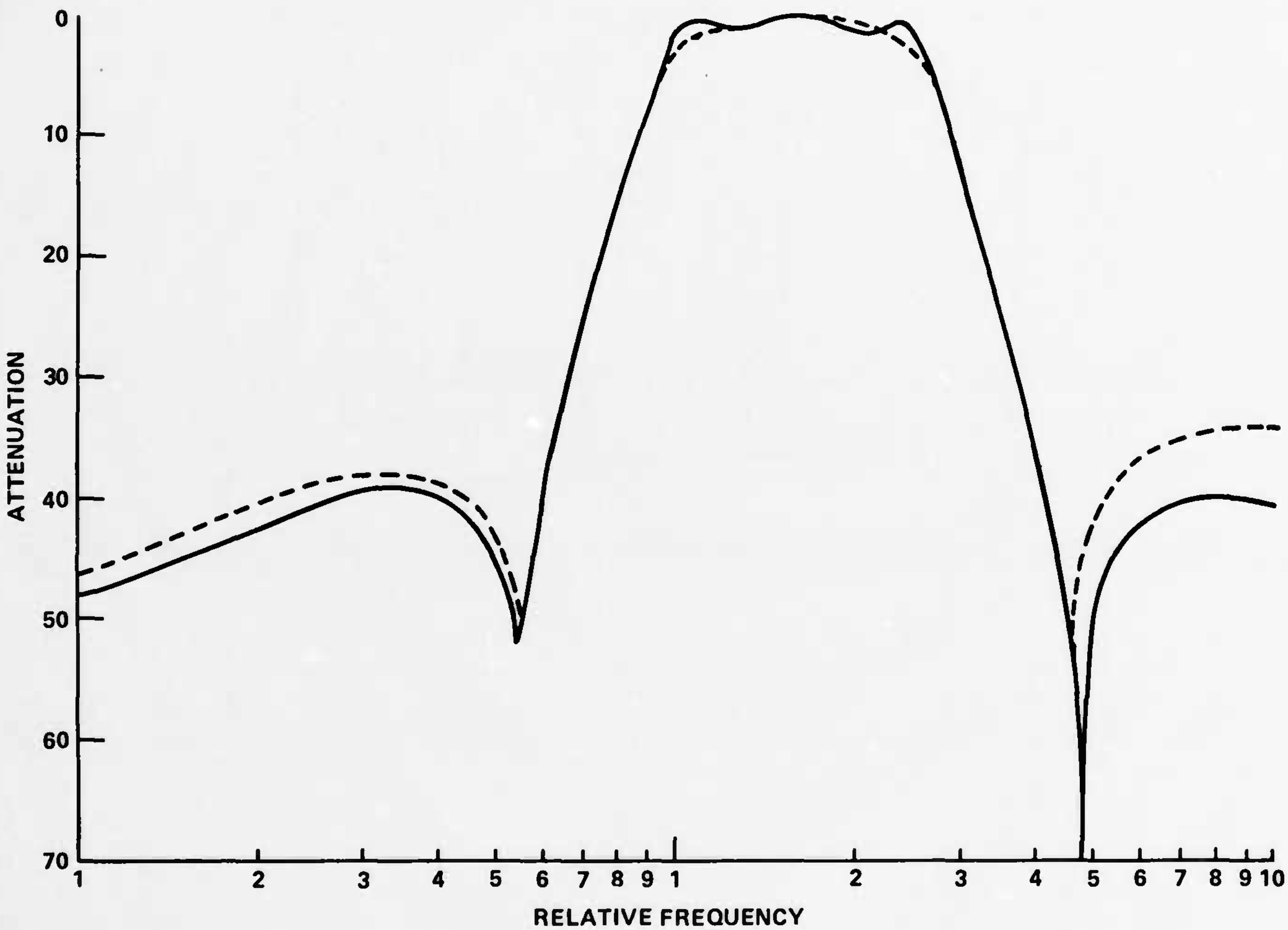


FIGURE 20. BANDPASS FILTER PERFORMANCE

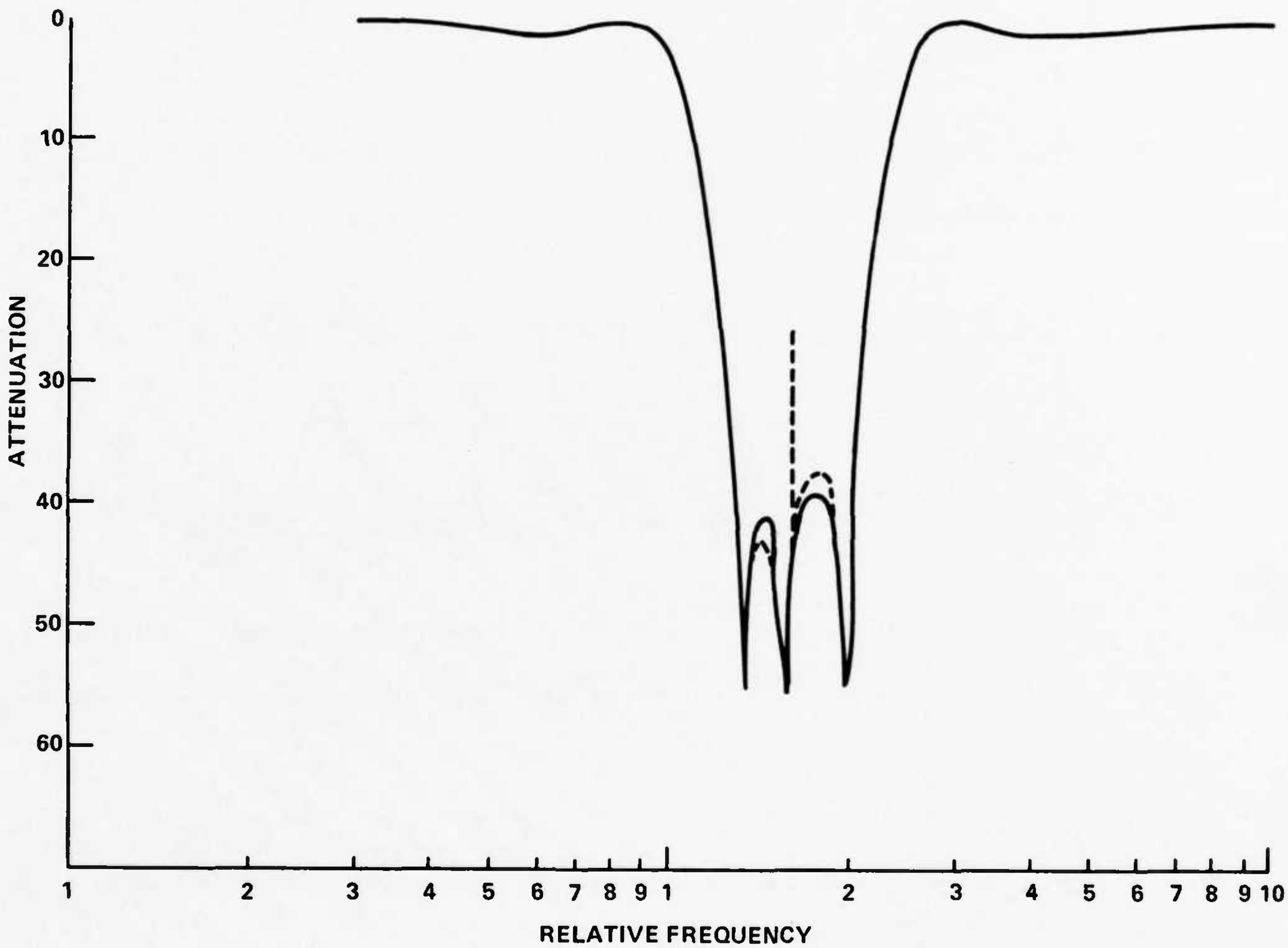


FIGURE 21. BANDSTOP FILTER PERFORMANCE

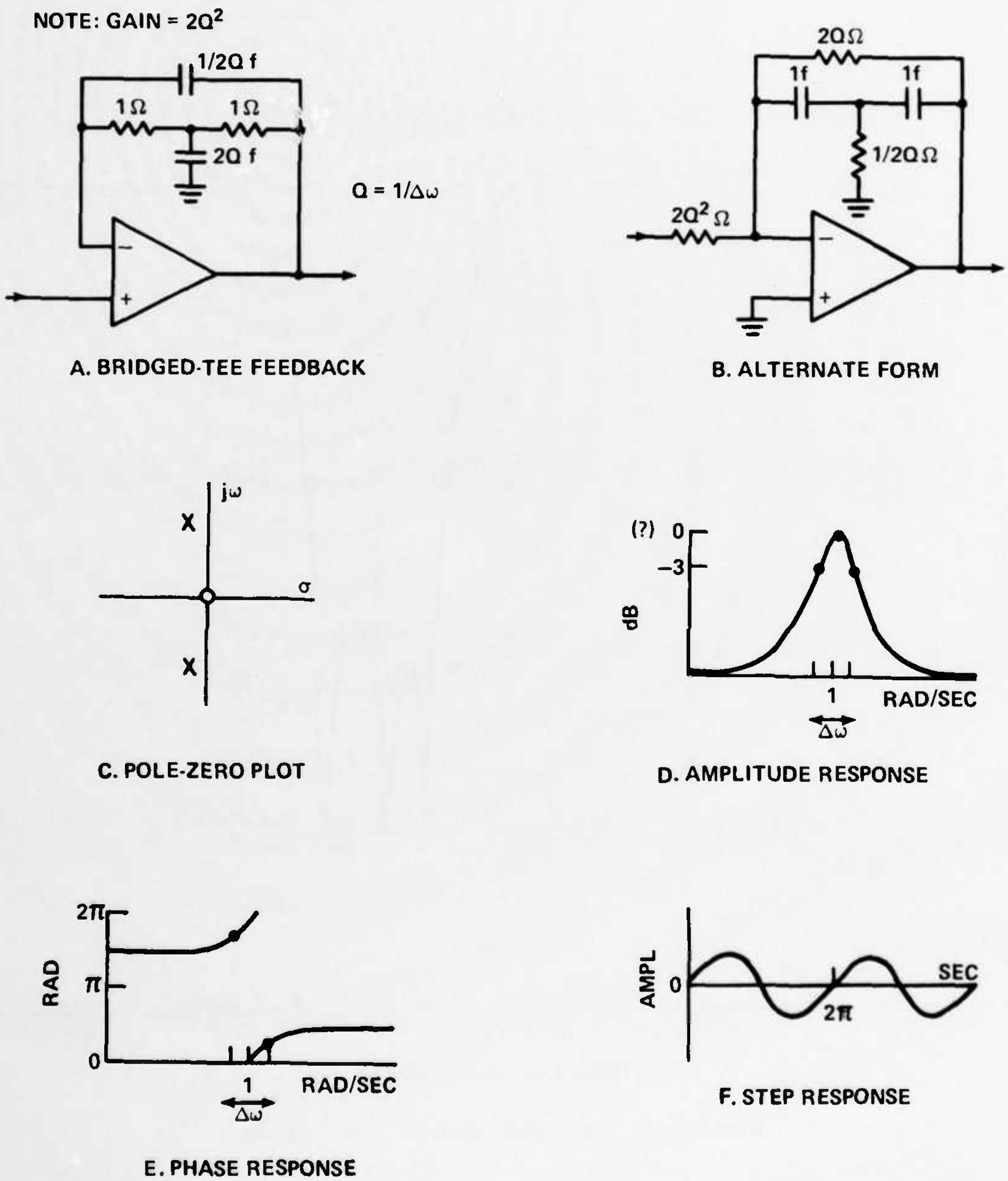


FIGURE 22. NARROWBAND FILTERS

the resonant frequency ω_0, or unity for the normalized case. The response is by definition down 3dB at the bandedges; however, these circuits are usually not inherently unity gain, hence the question mark in Figure 22D. In fact, the gain is usually high, causing problems. The ratio of the center frequency to the bandwidth $\Delta\omega$ is defined as the "Quality factor" or simple "Q."

The phase response is a constant $3\pi/2$ at low-frequency (equivalent to $-\pi/2$, but irrelevant anyway as the amplitude is near zero), increasing by $\pi/4$ at the bandedge, through zero at the center frequency, another $\pi/4$ at the other bandedge, and to constant $+\pi/2$ at high frequency (irrelevant again). The response of a narrowband filter to any input whatsoever is essentially a sine wave at its center frequency. The step response is an exponentially decaying sine wave, with each peak smaller than the previous; the vertical axis is not labeled because the amplitude depends on the filter bandwidth.

Many different circuits are given in the texts for narrowband filters; all seem to have drawbacks, especially for high Q. The circuit shown in Figure 22A is one of the most common. The Q is determined by the ratio of the capacitors; it is thus limited by available capacitance values, but is quite stable. The gain is very high for high Q, but an attenuator can easily be added at the input. The center freuquency can be adjusted somewhat by adding a potentiometer to one or both of the resistors, with minimal effect on Q.

Two modifications to this circuit are possible, as shown in Figure 22B. The capacitors and resistors in the bridged-tee may be interchanged. This makes the capacitors equal, but the op-amp sees a capacitive load at medium frequencies and a low-impedance resistive load at high frequencies, either of which may make the op-amp unhappy. The circuit may also be put in inverting form, as shown; the high gain is compensated to unity by the large input resistor. Here the center frequency and Q may both be adjusted by pots, but the adjustments are not independent.

Narrowband filters can also be built using the state variable filter or a passive design with a synthesized inductor. In either case, both center frequency and Q can be adjusted by pots, but adjustability often implies lack of stability.

The band-reject problem is similar to the bandpass, except worse. If a wideband is desired, separate low-pass and high-pass filters may be used and the outputs summed. The narrowband-reject filter is termed a notch (Figure 23); the goal here is to reject one frequency while retaining all others. The pole-zero plot is a pair of zeros on the imaginary axis with a pair of poles close by; all vectors will be nearly equal except in the vicinity of the zeros. The amplitude response is unity except for a dip at the center frequency. Bandwidth and Q are usually defined in terms of the 3dB-down frequencies, but occasionally in terms of the points 3dB up from the bottom of the notch, which may not be a true zero.

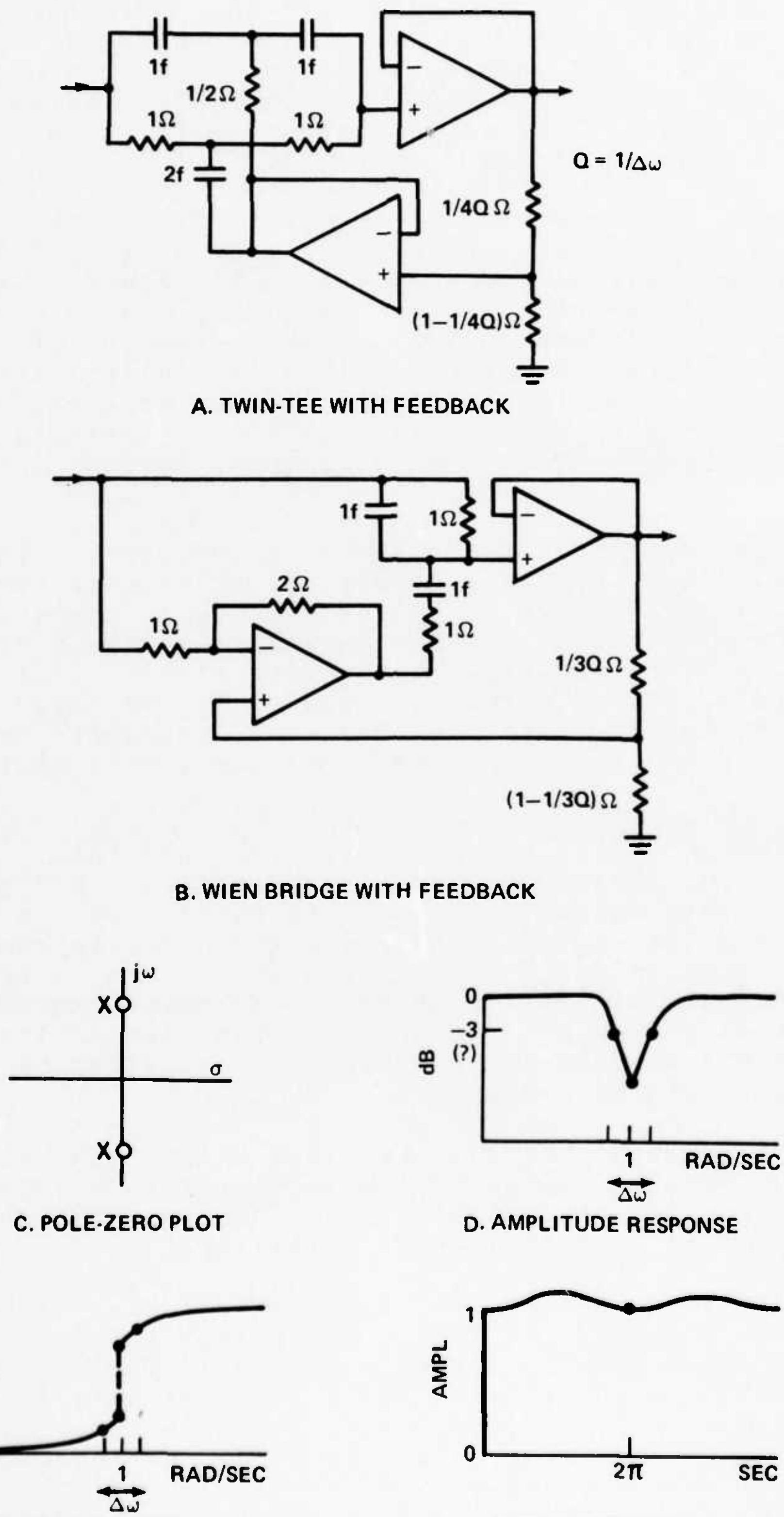


FIGURE 23. NOTCH FILTERS

Phase response is near zero at low frequency and near 2π at high frequencies, which is the same. It passes through $\pi/4$ phase shift at the bandedges and jumps from $+\pi/2$ to $3\pi/2$ (same as $-\pi/2$) at the center frequency. The step response is basically a step, as both high and low frequencies are passed, but it does exhibit a decaying ringing because the circuit is, in a sense, a tuned circuit.

Notch circuits have all the problems of the narrowband, and have the additional problem that finite-frequency zeros must be realized. The standard circuit is the twin-tee with feedback shown in Figure 23A. The frequency is determined by RC's of equal value (if components are paralleled); components must be matched for good performance. Q is adjustable; note that the resistors affecting Q add up to unity, i.e., they may be a pot. Frequency is essentially non-adjustable, as a triple or quadruple pot with good matching would be required.

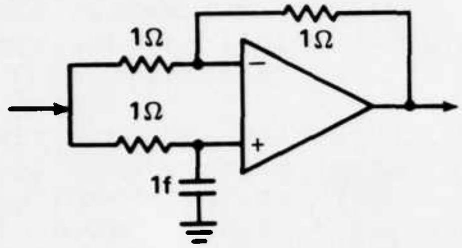
Recently it has been pointed out that a similar circuit can be built using a modified Wien bridge.[16] There are several mistakes in the article, but the circuit does work. The circuit (23B) is quite similar to 23A, but has the advantage that it uses only two equal capacitors and resistors. Contrary to the article, they must be matched. Q is again adjustable. Frequency could conceivably be adjusted with a double pot.

Notch filters may also be built using the state-variable filter or synthesized inductors. The most obvious synthesized inductor circuit turns out to have a maximum input amplitude limit proportional to 1/Q, making it virtually useless in high-Q applications.

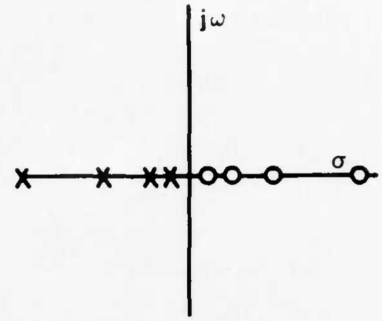
ALL-PASS FILTERS

All-pass filters, not surprisingly, are filters that pass all frequencies. The object here is not to alter the amplitude response but to impart a desired phase shift (or time delay), or often to correct for an undesired phase shift that has already occurred elsewhere. There are several types of all-pass filters, as indicated in the pole-zero plots of Figures 24B and 24C. Usually for each pole there is a mirror-image zero so the vector magnitudes cancel and the gain is unity everywhere. However, this need not be true; the amplitude may not be exactly flat but only an equiripple approximation. The poles and zeros may be spaced in simple pairs along the real axis (24B) or in conjugate pairs along the imaginary axis (24C). They may be spaced logarithmically (24B), usually the case for constant phase; or linearly (23C), usually the case for

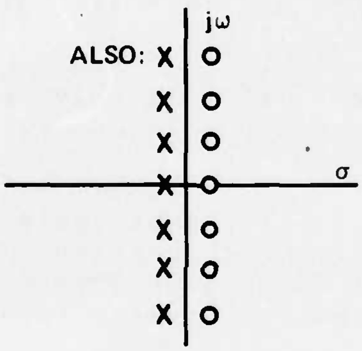
[16] D. Fellot, "When Bridge and Op-Amp Select Notch Filter's Bandwidth," Electronics Magazine, 7 Dec 1978.



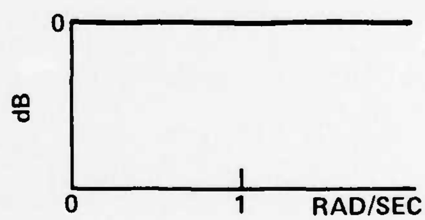
A. ONE-POLE, ONE ZERO CIRCUIT



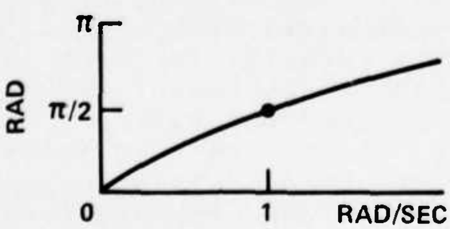
B. POLE-ZERO PLOT



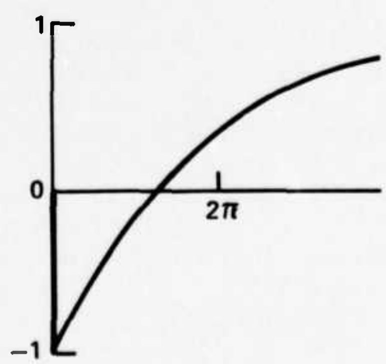
C. ALTERNATE POLE-ZERO PLOT



D. AMPLITUDE RESPONSE



E. PHASE RESPONSE



F. STEP RESPONSE

FIGURE 24. ALL-PASS FILTERS

constant delay.

The circuit of Figure 24A gives a single real pole and matching zero. The amplitude response (24D) is perfectly flat. The phase response is an arctangent curve, going from 0° at zero frequency through 90° at the "breakpoint" frequency to 180° at high frequency. The step response is amusing; since the high frequency response is of negative sign, it steps in the "wrong" direction and then recovers exponentially to the "correct" value. The frequency may be adjusted by adding a pot without affecting the gain. Since the DC gain is set by resistors, a trimming pot must be added at the junction if exactly unity gain is required; this does not affect the frequency.

There are circuits available which give a pair of poles and matching zeros; the state-variable filter may be used. However, these are more difficult to adjust as there are four variables, requiring four pots. The Lerner filter may be made into an all-pass by taking the bandpass and applying both the low-pass and high-pass modifications; however this gives the equiripple approximation and not true unity gain.

All-pass networks are sometimes used to provide linear phase shift (delay lines); in most cases, though, a linear-phase low-pass can be used. More often they are used to provide constant phase shift, normally 90°. It is difficult to design a single network to maintain a constant phase shift; often it is acceptable to substitute a pair of networks whose overall phase shift is arbitrary but which maintain a fixed 90° phase difference between their outputs, which is easier. (See Reference 17 for an example.) In any case, the design usually consists of cascading a number of sections, each having a single pole or pole-pair.

COMPARISON OF FILTER TYPES

Table 16 gives a direct comparison of filter types (here meaning primarily the characteristic but also to a lesser extent the circuit realization) with respect to the various filter parameters. These descriptors are generalizations, and perhaps to some extent the opinion of the author. Although there are a number of different parameters, in any given application they will fortunately not all be important. This report has gone through the filters by type (column); now we will summarize by comparing them by each parameter (row).

Passband droop is worst in the Independent RC and Bessel. It is variable in the Chebyshev and Elliptic because the amount of ripple

[17] Delagrange, A. D., An Improved Translating Filter Design, NSWC/WOL TR 77-172, 30 Nov 1977.

TABLE 16. COMPARISON OF FILTER TYPES

FILTER TYPE / PROPERTY	INDEPENDENT RC	BUTTERWORTH (MAXIMALLY FLAT)	CHEBYSHEV (TSCHEBYCHEFF) (EQUIRIPPLE)	ELLIPTIC (CAVER) (DOUBLE EQUIRIPPLE)	BESSEL (MAXIMALLY LINEAR PHASE)	CONSTANT - K IDENTICAL SECTIONS	LERNER
PASSBAND DROOP	POOR	FAIR	FAIR TO EXCELLENT	FAIR TO EXCELLENT	POOR	FAIR	GOOD
PASSBAND RIPPLE?	NO	NO	YES	YES	NO	YES	YES
TRANSITION STEEPNESS	POOR	FAIR	GOOD TO FAIR	EXCELLENT	POOR	GOOD	GOOD
STOPBAND MONOTONIC?	YES	YES	YES	NO	YES	YES	YES
ULTIMATE ATTENUATION	GOOD	GOOD	GOOD	POOR	GOOD	EXCELLENT	FAIR
ALL-POLE?	YES	YES	YES	NO	YES	YES	NO
NOISE BANDWIDTH	POOR	FAIR	FAIR	POOR TO GOOD	POOR	EXCELLENT	FAIR
MINIMUM-PHASE?	YES	YES	YES	YES	YES	YES	NO
PHASE LINEARITY	POOR	FAIR	POOR	POOR	GOOD	FAIR	EXCELLENT
TRANSIENT RESPONSE	GOOD	FAIR	POOR	POOR	GOOD	POOR	FAIR
COMPONENT SENSITIVITY	GOOD	FAIR	POOR	POOR	GOOD	GOOD	FAIR
EASILY EXPANDABLE?	YES	NO	NO	NO	NO	YES	SORT OF
COMPLEXITY/COST	EXCELLENT	GOOD	GOOD	POOR	GOOD	FAIR	FAIR

is controllable. Note, however that decreased ripple amplitude implies worse transition band steepness; a 0.01 dB ripple Chebyshev is essentially a Butterworth.

Transistion band steepness correlates with passband droop, since the bands are adjacent. Again the Independent RC and Bessel are worst. The Chebyshev has a trade off with the amount of ripple. The Elliptic is the best available because of the stopband zeros.

Response in the reject band is normally monotonic (continually decreasing), but the Elliptic is an exception. Note that if the number of zeros equals the number of poles the reject band response is eventually flat and does not drop off any further. Ultimate attenuation is tied to montonicity. Thus the Elliptic is poor. The Lerner eventually reaches a "floor" because of the accuracy of the difference amplifier. The Constant-K is best simply because a large number of sections can be added. Whether the design is all-pole (no zeros) directly affects monotonicity and ultimate attenuation.

Noise bandwidth is often important in signal processing systems where Signal-to-Noise-Ratio (SNR) must be calculated. This parameter is affected by transition band steepness, monotonicity, and ultimate atttenuation. Again the Independent RC and Bessel are poor. The elliptic can range from poor to good depending on the parameters chosen. The constant-K is generally best, again because a large number of sections may be used and it is monotonic.

Whether a filter is minimum phase is determined by the presence or absense of zeros in the right half-plane. Most are, but the Lerner is an exception. This is of no consequence in most systems. In a minimum-phase filter if either the amplitude or phase is good, the other must be poor, as can be seen by comparing the phase linearity with the transition steepness. However, it is possible for both to be poor, as the Independent RC shows. The only filter type that does well in both is the Lerner, which of course, is not minimum-phase. It has the best linearity for all frequencies passed with any significant amplitude; but if the phase response is of interest primarily at the lower part of the band, the Bessel is preferable. Transient response correlates closely with phase linearity. Recall that the Bessel exhibits quick rise with minimal overshoot.

Component sensitivity is defined as:

$$S_C^{\omega} = \frac{\partial \omega}{\partial C} \frac{C}{\omega} \simeq \frac{\Delta\omega/\omega}{\Delta C/C}$$

That is, the sensitivity of some filter parameter (for example the center frequency ω) with respect to some circuit value (say a capacitance C) is the calculated partial derivative of the parameter with respect to the value, normalized by their ratio. For practical purposes (also the way it is measured experimentally), the sensitivity is the percent change that occurs in the parameter when the value is changed one percent. For passive components the ratio should be on the order of unity; in some undesirable circuits it can be tens or hundreds, meaning the design will be very sensitive to component tolerance. For active components the ratio should be near zero, as parameters such as op-amp gain may be expected to vary widely; fortunately, this is usually easy to achieve. The actual calculation from circuit equations can be very tedious; reference 18 is a sensitivity analysis of a simple bandpass filter which takes 135 pages. The descriptors of sensitivity in Table 16 are a general judgement by the author based on working with the filters. The simpler filters tend to be better with respect to sensitivity.

As indicated in the next line of the table, filters are not usually expandable by adding sections without recalculations. Exceptions are the Independent RC, which doesn't make a very good filter for any number of sections, and the Constant-K, where the cutoff frequency varies only slightly. More sections can be added to the Lerner fairly easily, but the cutoff frequency changes directly with each new section added.

Complexity is again a general estimate covering number of parts, number of different values and circuit design; cost tends to go right along with complexity. Here the Independent RC finally stands out. The more exotic filters are usually more complicated, and the Elliptic is the worst in this category.

COMMUTATING FILTERS

Several special types of filters deserve at least passing mention. They have all been around as curiosities for some time, but some have recently become practical due to advanced integrated circuit techniques. They belong to a class which may be termed quasi-linear; to a first approximation the circuits behave as linear filters, but the internal circuit is actually nonlinear, which produces some side effects.

The most common is the commutating filter, (Figure 25), usually shown as a narrow bandpass (Figure 25A). A bank of low-pass filters, shown here as simple RC's are isolated by input and output switches which connect one at a time into the circuit. The switches "commutate" through the bank at a repetition rate f_0. If the input signal is a sine wave at f_0, each capacitor acquires a DC

[18] Martin, Stephen, <u>Minimizing the Worst-Case Drift of an Active Bandpass Filter</u>, NSWC/WOL TR 78-1, 11 April 1978.

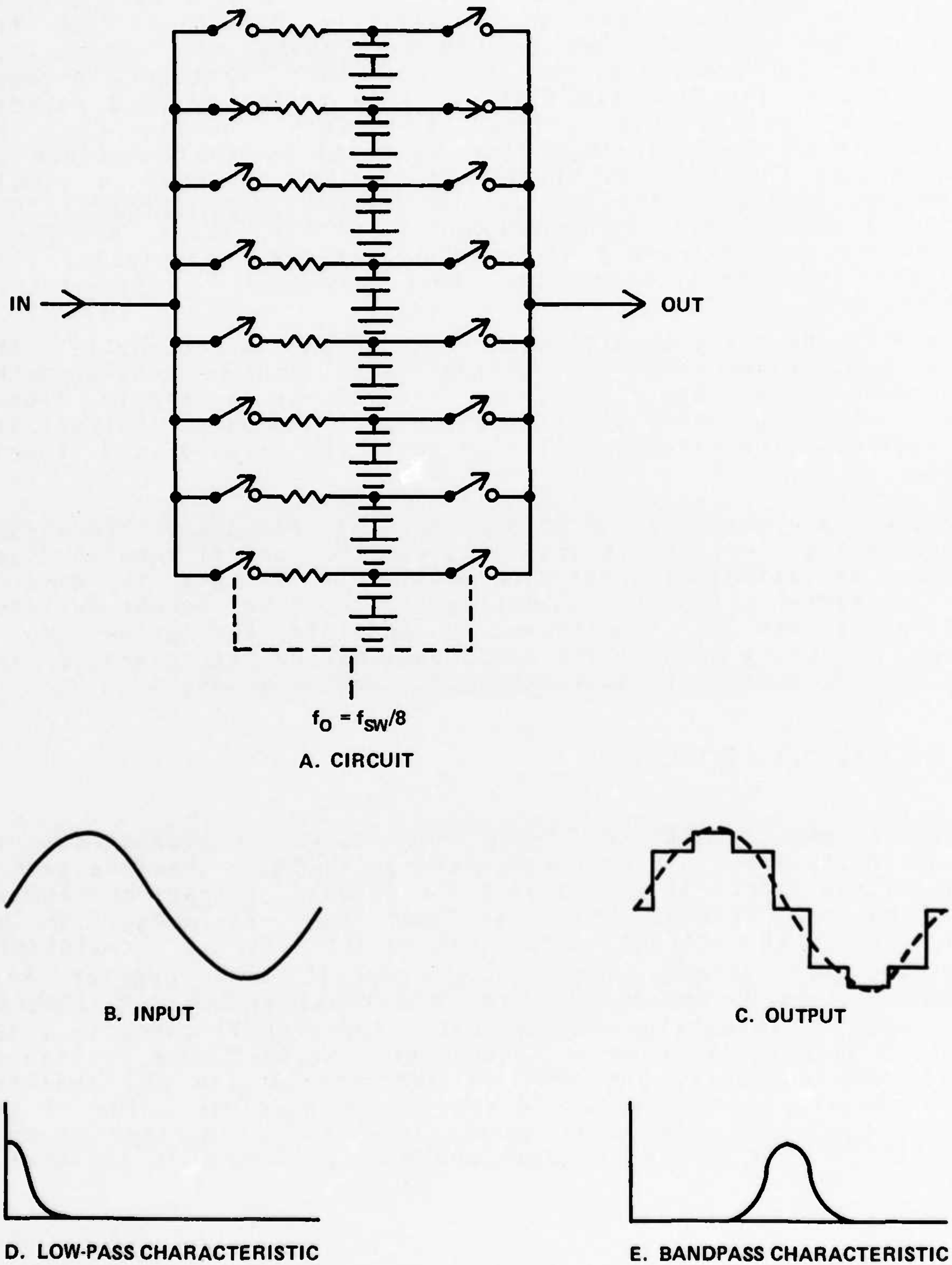


FIGURE 25. COMMUTATING FILTER

value equal to the average of that particular segment of the sine wave, and the output is a "stepped" version of the sine wave (Figure 25C). If the input frequency is slightly different from the commutating frequency, the "DC" values will change slowly, in fact at the difference frequency. If the difference frequency exceeds the frequency of the low-pass filters, they cannot respond rapidly enough, and the output level drops. It is fairly obvious that the half-bandwidth of the overall filter is equal to the bandwidth of the low-passes; the low-pass characteristic (Figure 25D) is simply moved up and reflected about a center frequency f_0 (Figure 25E). The center frequency will be determined by an oscillator, and hence can be made either extremely accurate or externally variable. The filter shape is determined by the low-passes, and can be closely controlled.

There is obviously quantization noise in the output, but if the sections are matched it is at the eighth and higher harmonics of the center frequency, and can usually be removed by a simple linear filter. Switching noise is similar. Input and output buffers are often required. The circuit will also pass the second and fourth harmonics.

Switches are normally FET gates. A simpler version requiring only one set of switches is possible, but the general version shown here allows variations such as using multi-pole filters to give a squarer characteristic, or commutating the input and output switches at a different rate to simultaneously modulate the signal to a different frequency band.[19] The components may be rearranged to give a notch, or the output simply subtracted from the input.

SWITCHED-CAPACITOR FILTERS

Consider the circuit of Figure 26A. When the capacitor is connected to the input, it acquires a charge $V_{in}C_{sw}$. When the switch is changed, the charge is dumped into the output. <u>Average</u> current is charge flow per second, which is then $I_{out} = \overline{V_{in}C_{sw}f_{sw}}$. On the <u>average</u>, then, the circuit is equivalent to a resistance $V_{in}/I_{out} = 1/C_{sw}F_{sw}$ (Figure 26B). The circuit must operate into a ground, but an op-amp can provide a virtual ground; it must be averaged, but an integrator can do that. The overall circuit, which works quite nicely, is shown in Figure 26C. We need some resistance to limit peak currents, but that is inherent in the FET switches that are normally used anyway. Although the absolute value of the capacitors will vary, the time constant of the integrator depends only on their ratio, which can be closely controlled with integrated circuit techniques.

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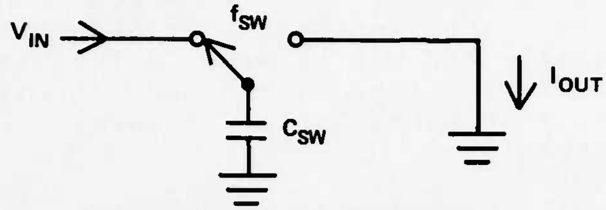
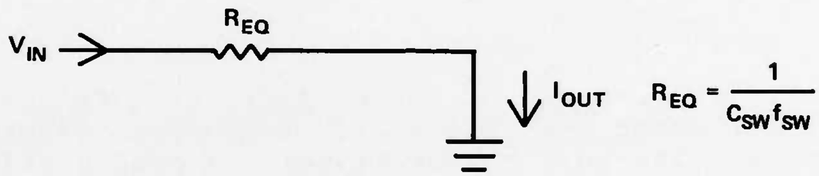
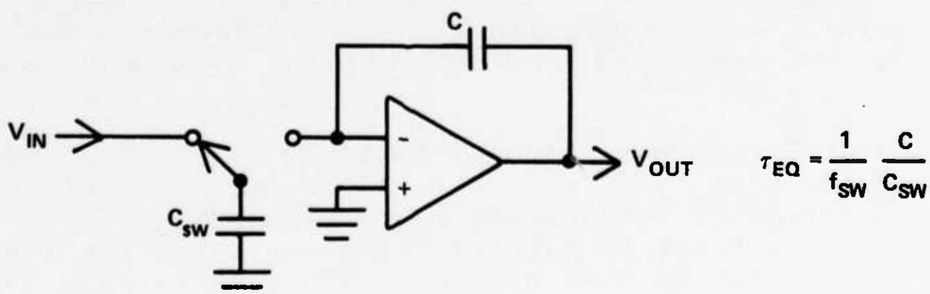
[19] Leehey, Jonathan, "Frequency Sampling and Translation Using Commutating Filters," M. S. Thesis, Massachusetts Institute of Technology, 1979.



A. CIRCUIT



B. EQUIVALENT CIRCUIT



C. SWITCHED-CAPACITOR INTEGRATOR

FIGURE 26. SWITCHED-CAPACITOR FILTER METHOD

We can take two of these integrators and an ordinary inverter and make a universal active filter. The frequency of the overall resonator will be controlled by the switching frequency; in fact, it will be directly proportional. Thus we have a resonator where the frequency is controlled externally and may be made either extremely stable or variable. Furthermore, the clock may control several stages, giving a multipole filter with variable frequency, which is difficult to do with linear techniques.

There will be noise present, principally at the clock frequency. This is typically well above the resonant frequency and can be eliminated by a simple linear filter, at least for the low-pass case. It may be a problem for the high-pass case; sometimes cancellation techniques can be used, since the offending frequency is known and is readily available.

TRANSVERSAL FILTERS

Another class of filters can be built, different in the sense that they are constructed basically from time response properties rather than frequency. The most common is built around a delay line (Figure 27). The different stages are summed with different weights and may have either sign. A pulse input will produce an output seven periods long (in this simple example) whose amplitude in each one is the weighting. Mathematically, the frequency response of the circuit is the Fourier transform of this pulse response. Circuit-wise it should be apparent the circuit will respond to some frequencies more than others. For example, let the weights be of equal magnitude but alternating sign. If the input frequency is half the shift frequency, then the "contents" of the stages will have alternating sign and all stage outputs will reinforce; but for an input frequency much lower than the shift frequency the contents will be nearly the same, and with alternating sign will average to nearly zero.

This type of filter has become feasible with the advent of charge-coupled devices (CCD's), the first really practical analog delay line. Transversal filters are especially attractive for two special cases: The first is matched filtering where the transmitter response is the reverse in time of the receiver, achieved simply be reversing the tap weights. The other is linear phase. If the weights are arranged symmetrically about the center tap, then that tap represents the delay and each pair around it represent relative phase shifts that are equal in magnitude but opposite in sign; hence the phase errors relative to the pure delay cancel.

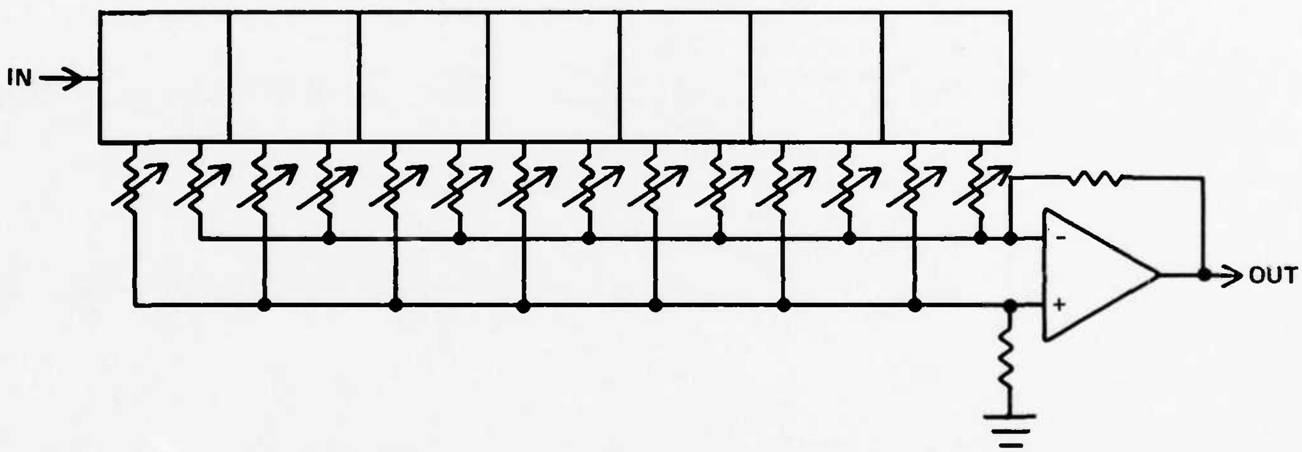


FIGURE 27. TRANSVERSAL FILTER

APPENDIX A

FILTER RESPONSE DATA

The filter designs given in this updated report have become too numerous to catalog all the responses; so only a sampling will be given. Circuits were built from the prototypes and tables given in this report, and the actual responses recorded on oscilloscopes.

The same three curves sketched for each filter type in the body of the report are used here. Amplitude response, here denoted by the common term "frequency response," was done with a voltmeter having an auxiliary DC voltage output with the frequency swept linearly by a VCO (Voltage-Controlled-Oscillator), driven by a triangular wave. Scale is linear with the filter frequency at center (unless noted) and the left-hand edge is zero; thus the right-hand edge is an octave above the filter frequency. Vertical scale is 10dB/large division (log) for all types; this allows direct comparison between them, but utilizes little of the scale for some types. Phase was done similarly with a phasemeter; vertical scale is 100°/large division. Horizontal scale corresponds to that of the frequency response, but note that the two oscilloscopes do not have the same size graticule; also, the traces will not line up identically with their respective graticules due to DC drift. The step response is done simply by driving the filter with a low-frequency square wave. The step occurs at the left-hand edge; the center of the sweep corresponds to one complete cycle at the filter frequency (unless noted.) The step goes from two large divisions below the centerline to two above; filters are normalized to unity gain. The circuits were constructed mostly using 1% capacitors, 5% resistors; sometimes tweaking was necessary to prevent oscillation or serious errors, mostly with the higher-order filters. FET-input op-amps (which are about an order of magnitude faster than 741's) were used; filter frequency was usually around 1KHz. The curves at times will show noticeable deviations from the ideal; if exact curves are required the mathematical function must be plotted, which is done in some of the references, usually by computer. Some of the more pertinent aspects of some of the photographs will now be discussed.

The simple RC low-pass (A1) offers only about 6dB rejection an octave above cutoff, a minimal filter. The phase shift appears linear only because it is so low; a straight edge will confirm this. The step response is a simple exponential, about 1/6 of the period due to the 2π factor. The 1-pole RC high-pass (A2) appears to have a sharper cutoff only because a linear frequency scale is used; any high-pass must go to zero ($-\infty$dB) at zero frequency. (The VCO is set to go not quite to zero; otherwise the voltmeter and phasemeter give wild gyrations.) Conversely the high-pass cannot reach 0dB on the picture as the low-pass does. If a log frequency scale were used, the low-pass and high-pass would be mirror images in most cases. The stairstep glitch is due to a lag in the voltmeter as it changes ranges. The phase also would be a mirror image if a log frequency scale were used, except it is negative (phase lead). The step response is also a simple exponential, but falling, as a high-pass shows the initial step but must return to zero.

The 4-pole equal-RC (A3) shows four times as much attenuation at an octave, but also at the cutoff. Note that at +180° the phasemeter shifts to -180°. The step response shows no overshoot. The phase response of the 4-pole high-pass shows a glitch at one-third the cutoff frequency because at that point the third harmonic in the VCO is interferring. Note that the phasemeter gives up when the amplitude gets too low. These effects will appear in other pictures. The no-overshoot property does not carry over to the high-pass.

For the Butterworth curves (A5 through A12) note that the attenuation at the cutoff frequency remains 3dB but the attenuation at an octave improves with increasing order. On the other hand, ringing in the step response becomes worse. The frequency response in the higher orders show slight peaking; this is in error and is probably due to "Q-enhancement" in the higher-frequency sections. The lower end of the 8-pole high-pass does not fall off as it should, but exhibits "stairstepping." This is caused by oscillator harmonics; the third harmonic does not get attenuated until it falls below one-third the cutoff in frequency, etc. This curve idicates the third harmonic level is about -50dB and the fifth is about -55dB.

The Chebyshev curves (A13 through A28) show how the cutoff slope may be made considerably steeper by allowing as little as 1dB ripple in the passband. However, the ringing in the step response is worse than Butterworth. For 3dB ripple, adjustment becomes critical. For the 8-pole filters, the 3dB version does not have noticeably better attenuation than the 1dB; conversely, misadjustment in the 1dB versions is also evident as the 1dB limit is exceeded in some of the pictures. Filters with less than 1dB ripple were not even attempted.

The Bessel filters (A29 through A32) show excellent phase linearity clear to the bandedge and beyond. The attenuation at an octave, though, is pitiful. The step response overshoot is barely detectable. The phase linearity does not carry over to the high-pass versions, so they are not shown.

The elliptics offer the steepest transition band. The 5-pole 4-zero low-pass (A33) hits the stopband about 20% past the cutoff frequency. The zeros are clearly evident. The 5-pole 4-zero high-pass is chosen to have deeper rejection, so its transition slope is not as steep. The 7-pole 6-zero low-pass appears comparable only because of the distortion of the linear scale; it is really steeper than the 5-pole 4-zero high-pass. Phase shift and step response are similar to Chebyshev filters of the same steepness.

The constant-k filters (A36 and A37) are reasonably well-behaved. A good amplitude characteristic is obtained with no tweaking. The low-pass makes a good delay line; the phase response is nearly linear over most of the passband, and the step response exhibits a delay comparable to the rise time. The high-pass is interesting in that the ringing is not a constant frequency, but slows down as it dies out.

The good phase linearity of the Lerner filters (A35 through A40) is evident. That of the high-pass is less than ideal, but it is the best shown yet for a high pass. Step response is meaningful only for the low-pass. Again we have a good delay line; observe the "pre-shoot". The sweeps here have been lengthened to show the details of the wiggles. Cutoff frequency is not normalized.

The narrowband response (A41) was obtained from the equal-capacitor version. The phase response rises sharply by 180° as the peak is traversed. (The trash at the lower end of the trace is due to the phasemeter jumping between -180° and +180° on the low-amplitude, low-phase signal.) The response of a narrowband filter to any input is of course a sine wave at the center frequency. The height of each peak can be seen to be diminishing. The initial amplitude is not normalized, as it depends on Q. The notch filter was constructed using the Wein-bridge circuit. The depth of the notch depends on the accuracy of the components. The phase shift jumps 180° going through the notch. It might be surprising that the step response rings at the center frequency, since that is the frequency at which the filter transmits least. One way to explain this is to observe that the ringing is upside down; what we are seeing is all frequencies (the step) except (minus) the center frequency.

The amplitude response of the all-pass (A43) is decidedly uninteresting. The phase is the arctangent curve, exactly double that of the simple RC low-pass. The step response is interesting in that it initially steps in the wrong direction, due to the inversion at high frequencies, then recovers exponentially to the final value. The vertical scale is compressed slightly to get the whole trace on the picture. Figure A44 shows the performance of a phase-difference network, two 2-pole 2-zero networks whose outputs are 90° apart across the frequency range 200Hz-1400hz. (Center scale is 1000Hz). Phase shift is shown for each output, and the difference between them which is nearly constant 90°. The step response is for one side, and shows two spikes resulting from using two stages.

FREQUENCY

PHASE

STEP

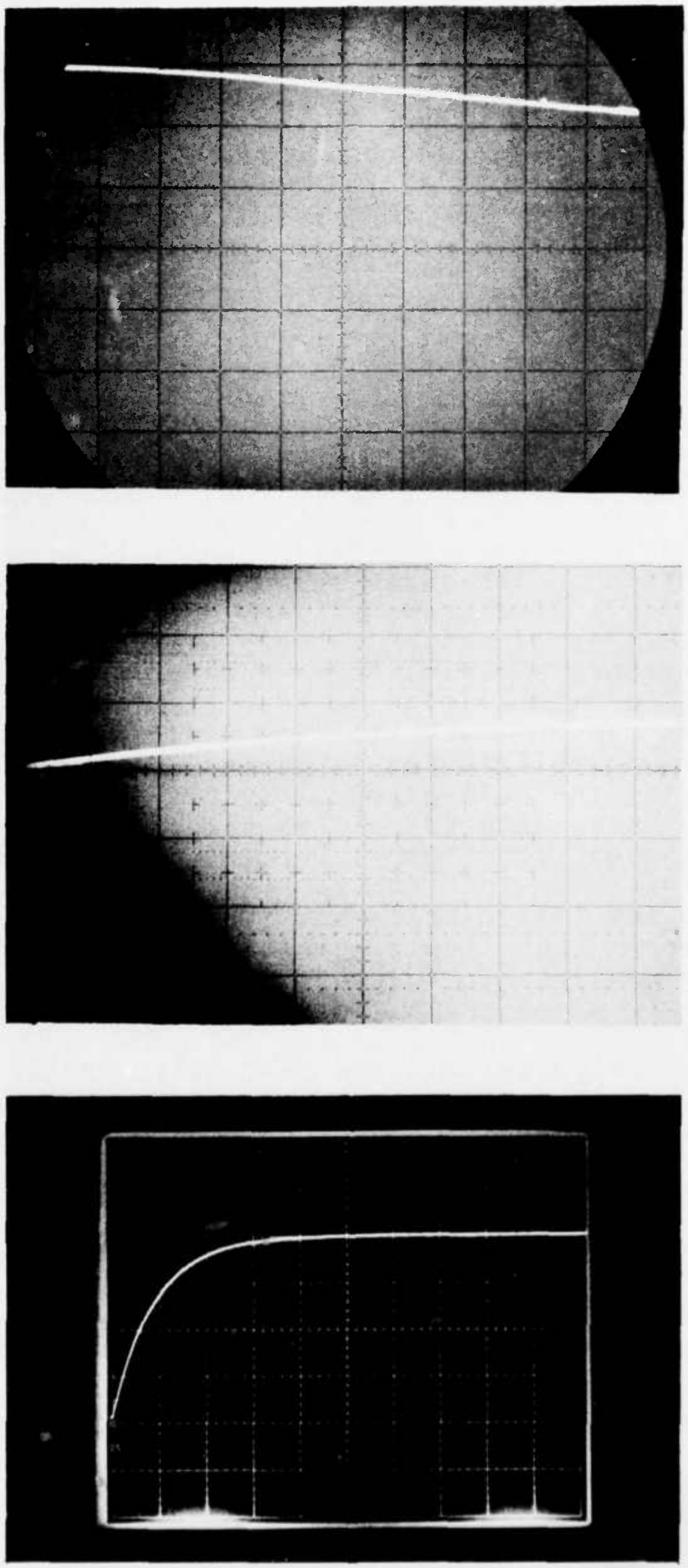


FIGURE A1. 1-POLE RC LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

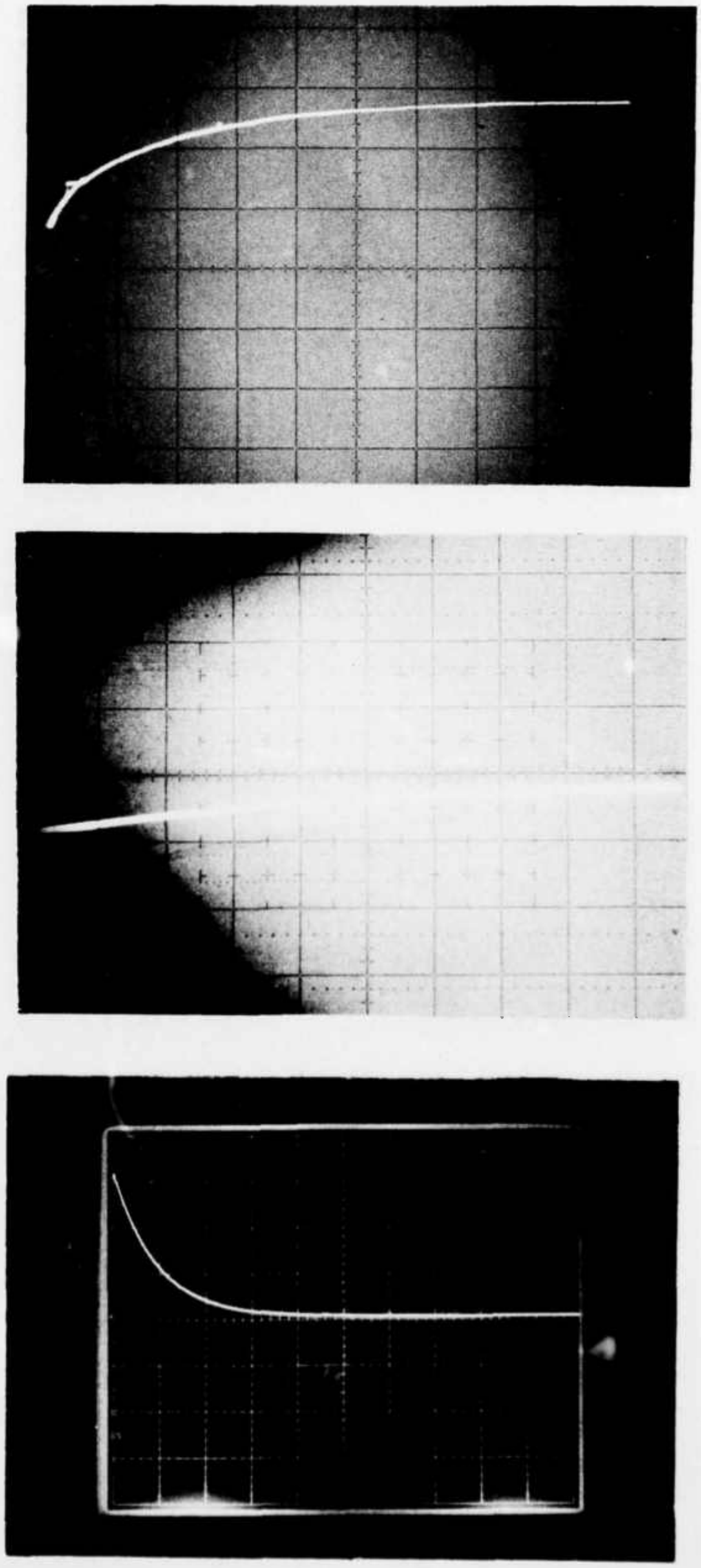


FIGURE A-2. 1-POLE RC HIGH-PASS RESPONSE

FREQUENCY

PHASE

STEP

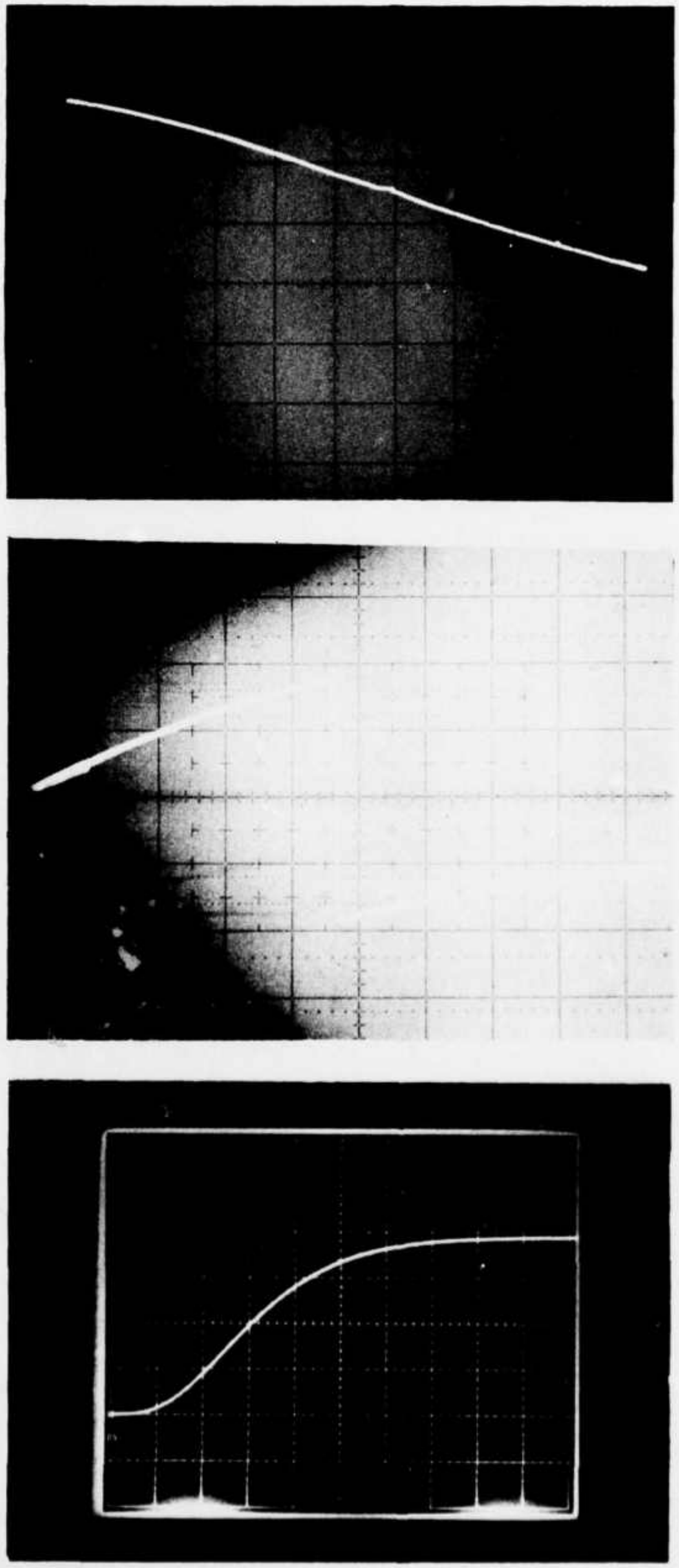
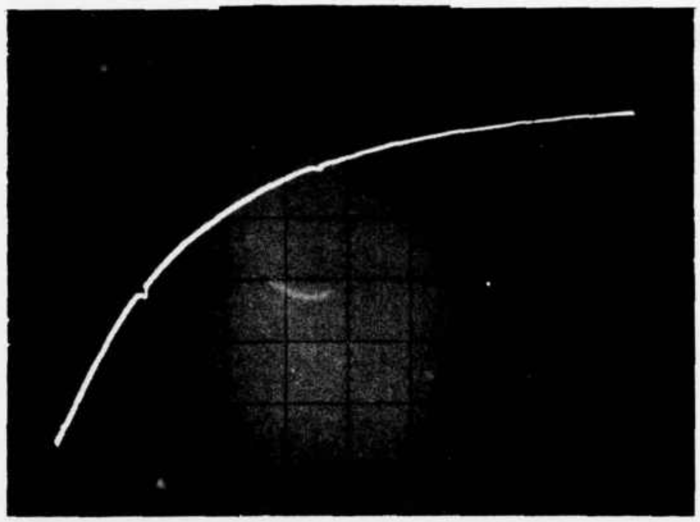
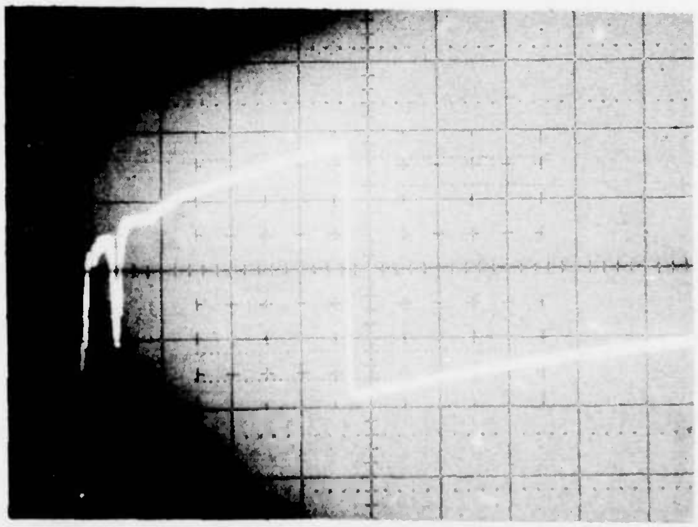


FIGURE A3. 4-POLE EQUAL-RC LOW-PASS FILTER RESPONSE

FREQUENCY



PHASE



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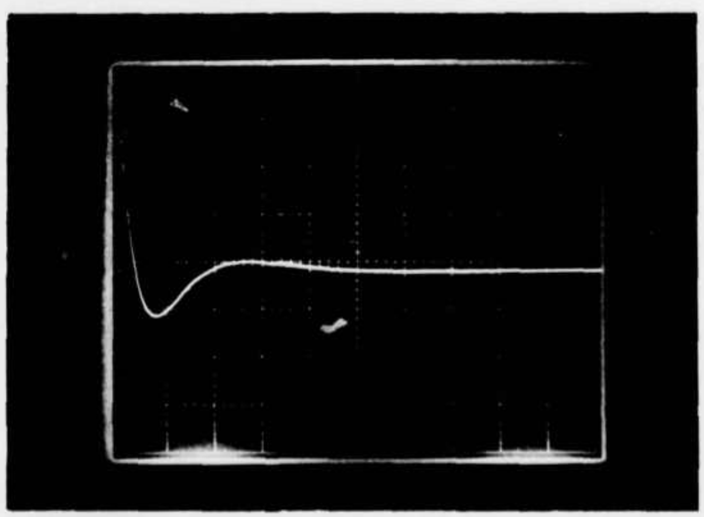
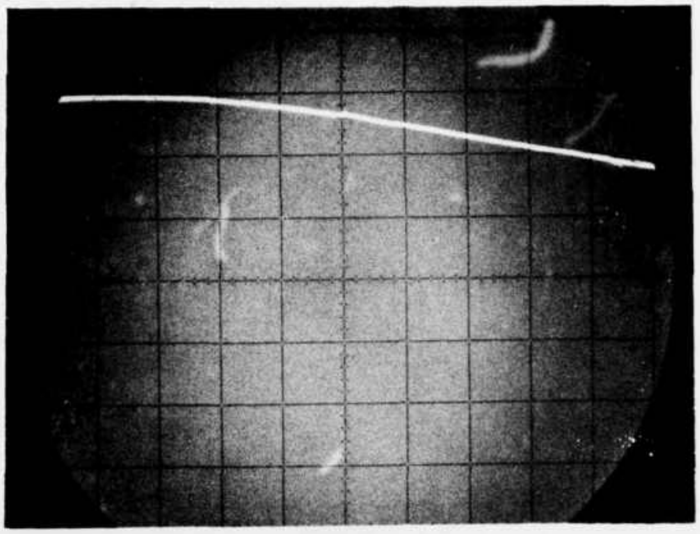
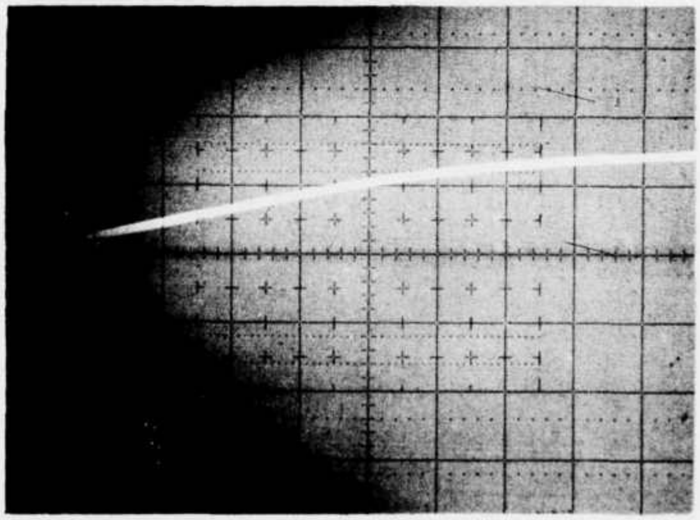


FIGURE A-4. 4-POLE EQUAL-RC HIGH-PASS RESPONSE

FREQUENCY



PHASE



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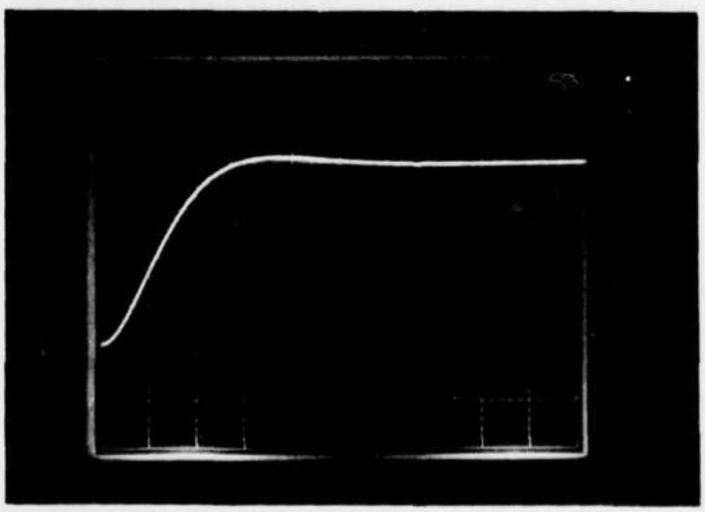


FIGURE A5. 2-POLE BUTTERWORTH LOW PASS RESPONSE

FREQUENCY

PHASE

STEP

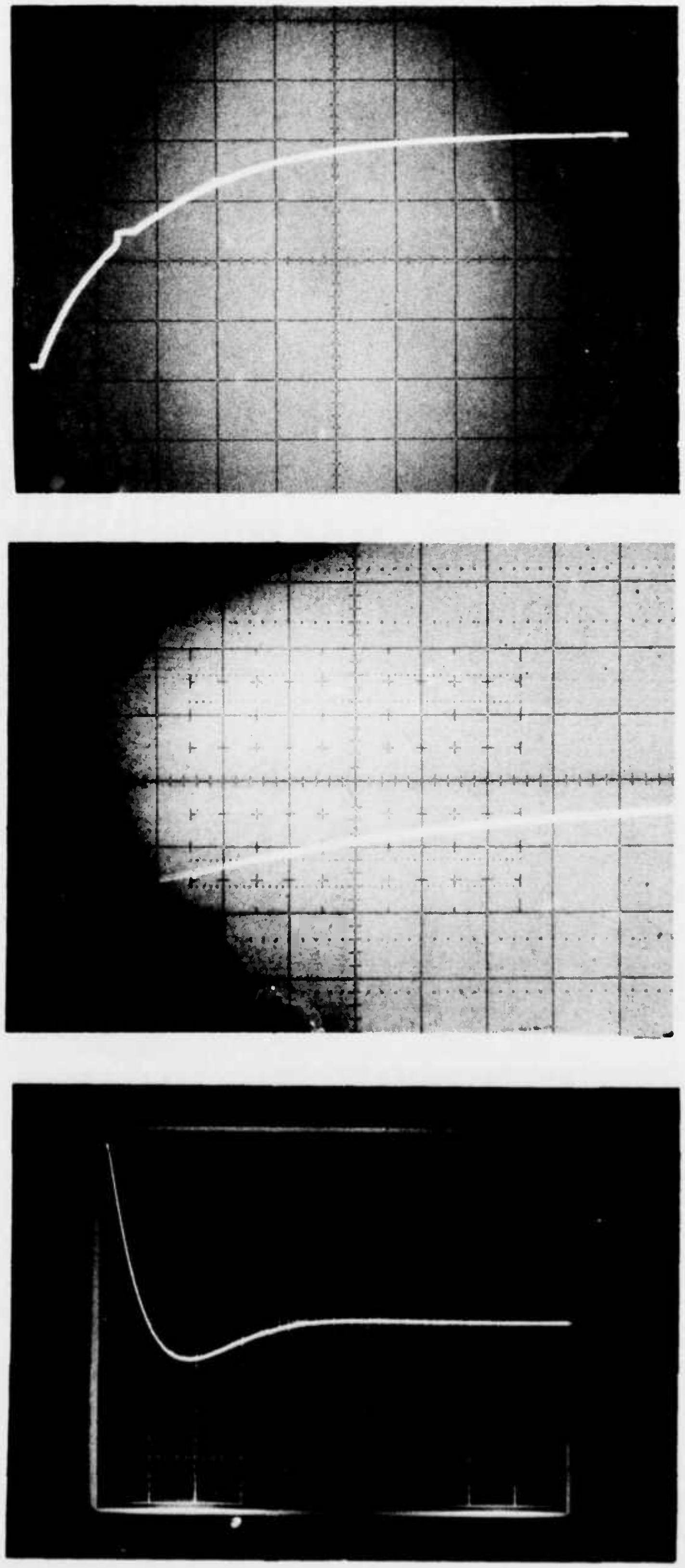
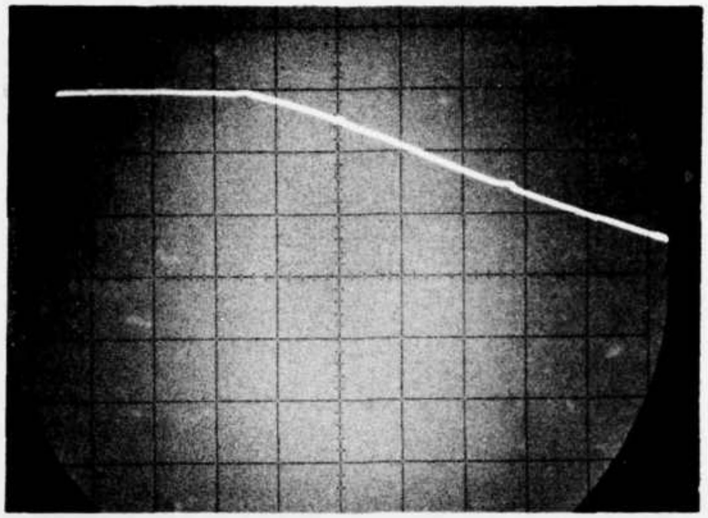


FIGURE A6. 2-POLE BUTTERWORTH HIGH-PASS RESPONSE

FREQUENCY



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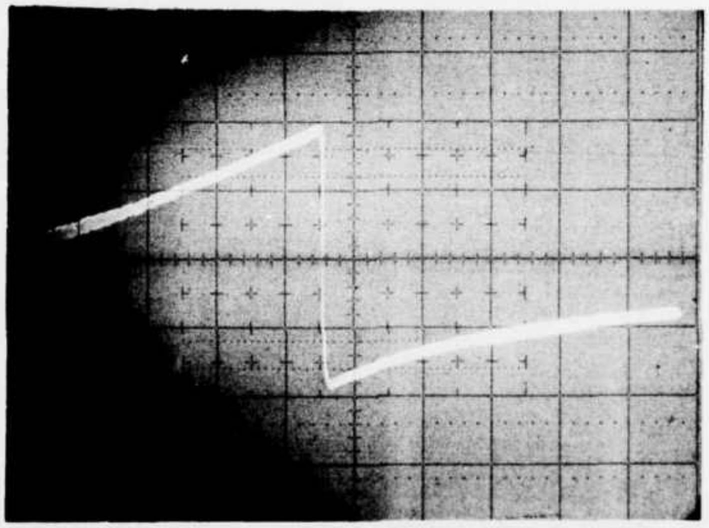


STEP

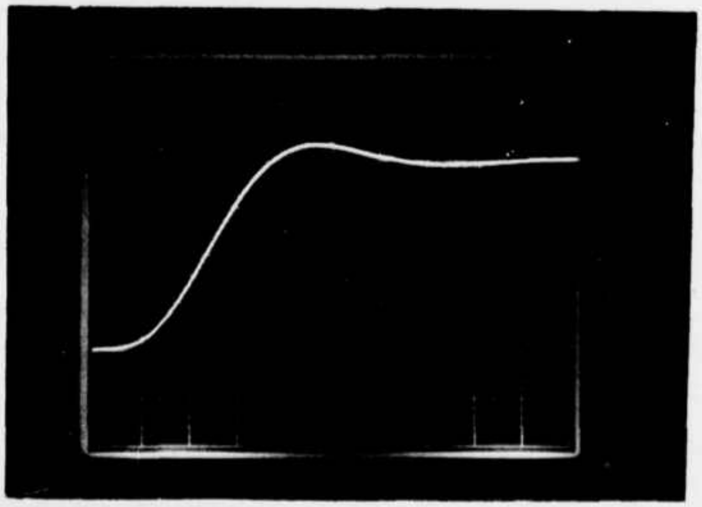
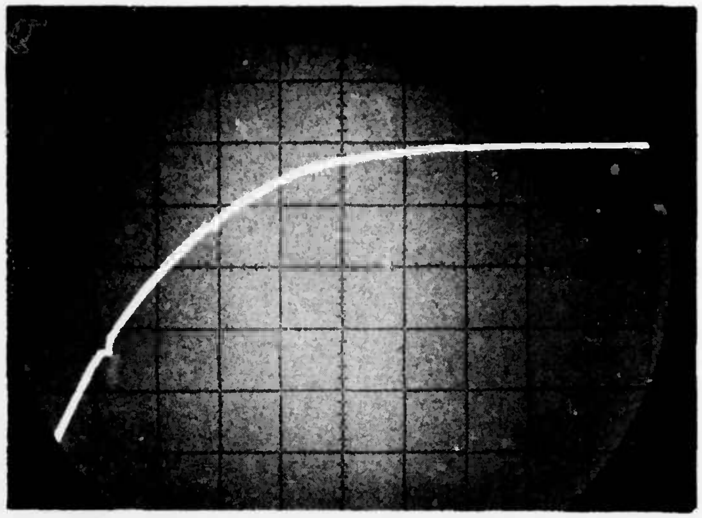
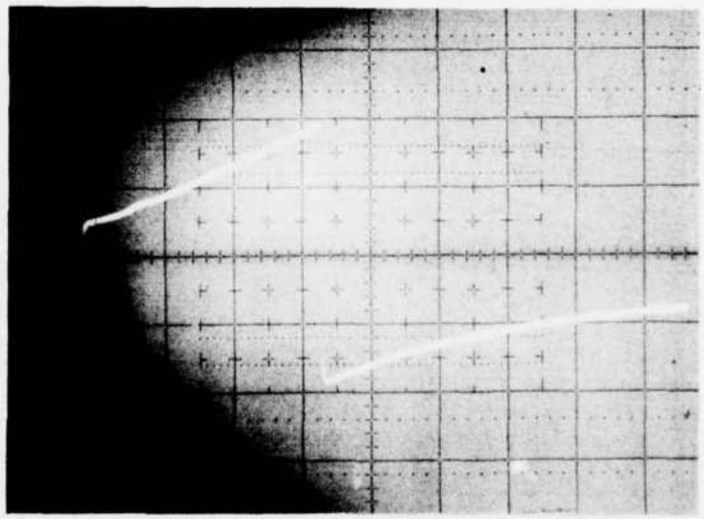


FIGURE A7: 4-POLE BUTTERWORTH LOW-PASS RESPONSE

FREQUENCY



PHASE



STEP

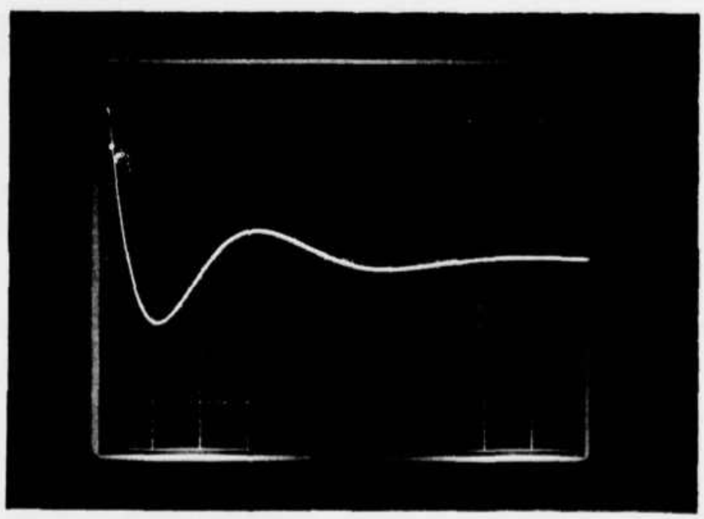
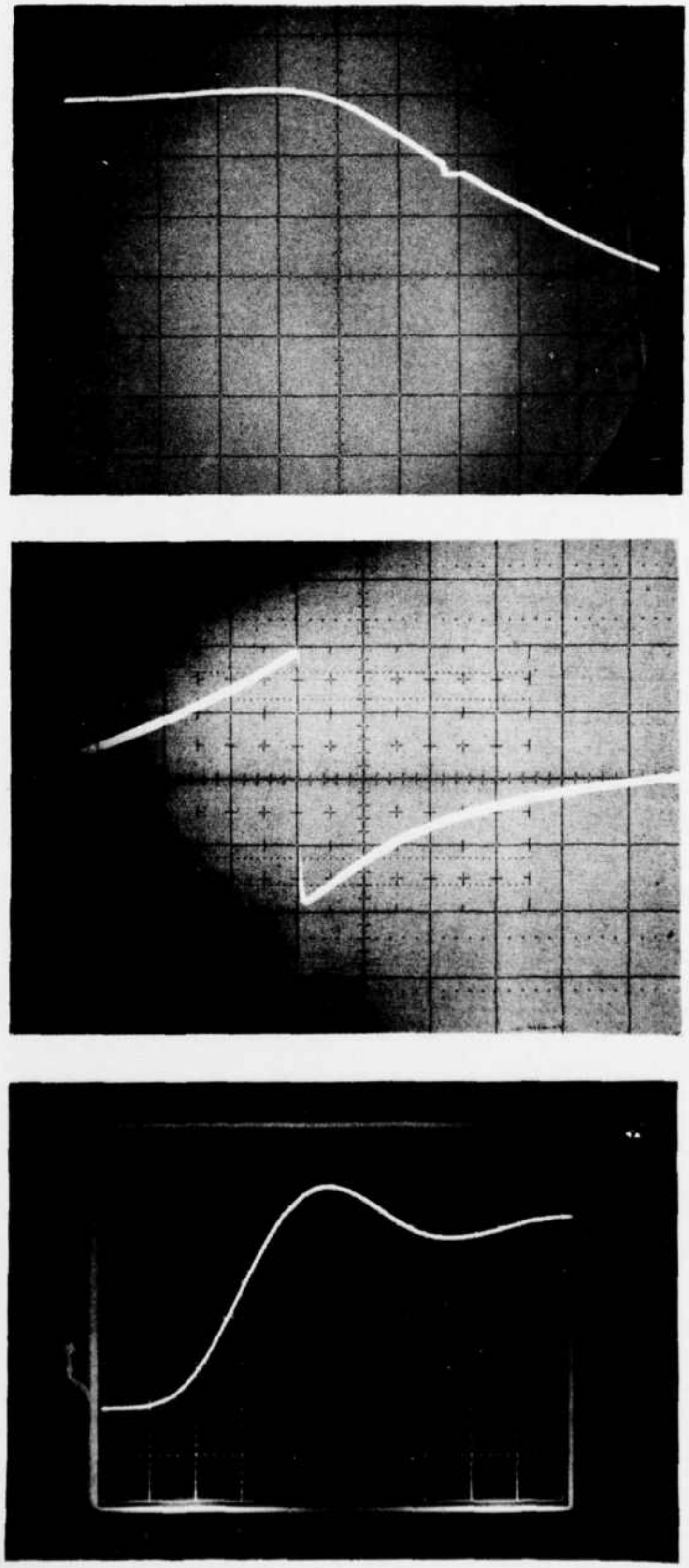


FIGURE A8. 4-POLE BUTTERWORTH HIGH-PASS RESPONSE



FREQUENCY

PHASE

STEP

FIGURE A9. 6-POLE BUTTERWORTH LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

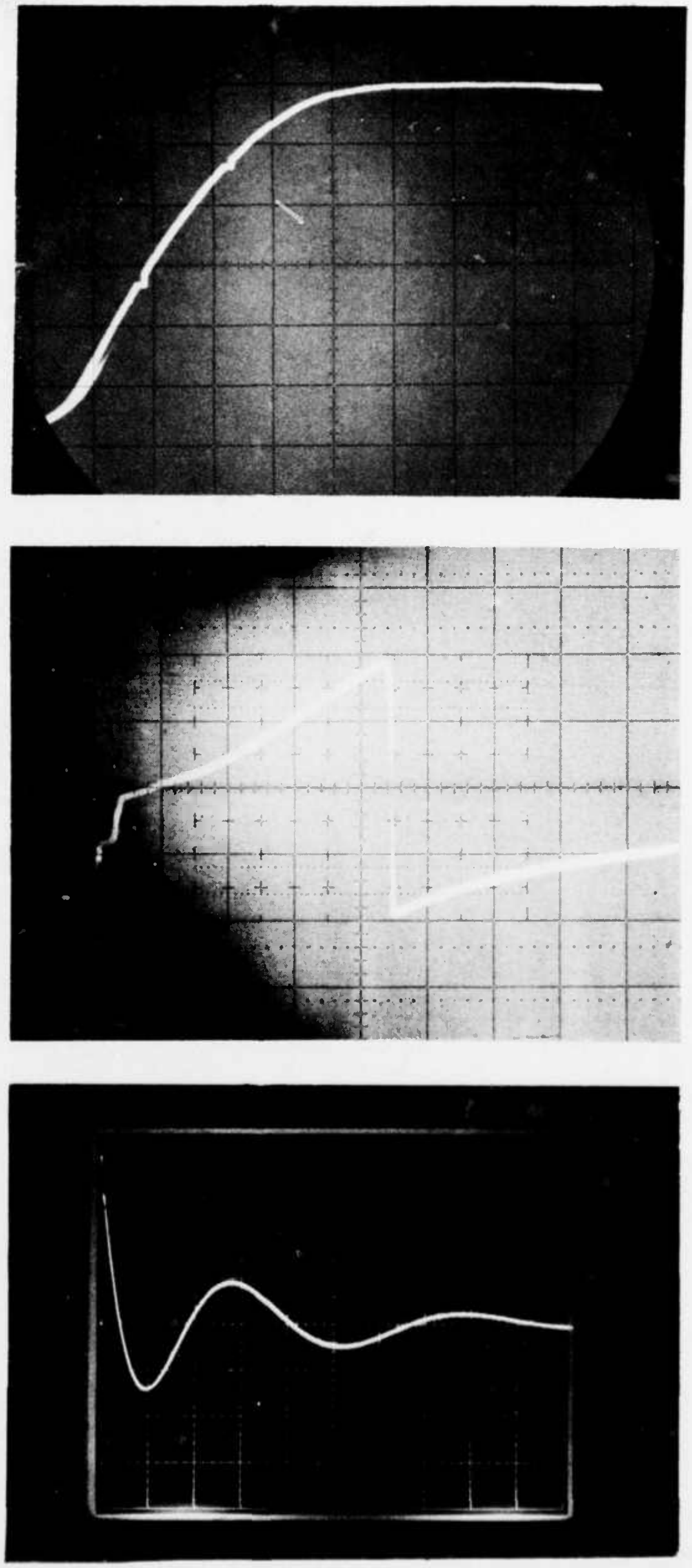
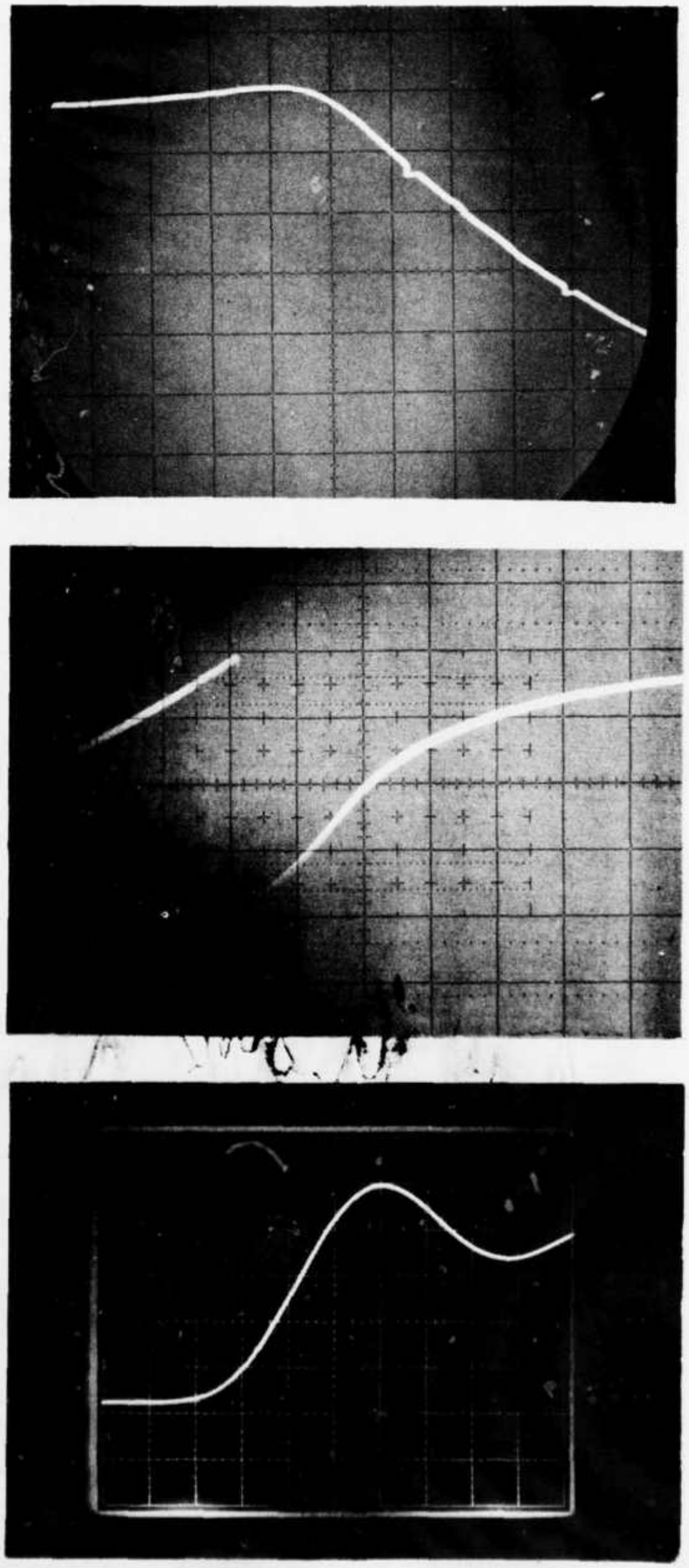


FIGURE A10. 6-POLE BUTTERWORTH HIGH-PASS RESPONSE



FREQUENCY

PHASE

STEP

FIGURE A11. 8 POLE BUTTERWORTH LOW PASS RESPONSE

AD-A134 838 AN ACTIVE FILTER PRIMER MOD 1(U) NAVAL SURFACE WEAPONS CENTER SILVER SPRING MD A D DELAGRANGE 01 FEB 83 NSWC/TR-82-552 2/2

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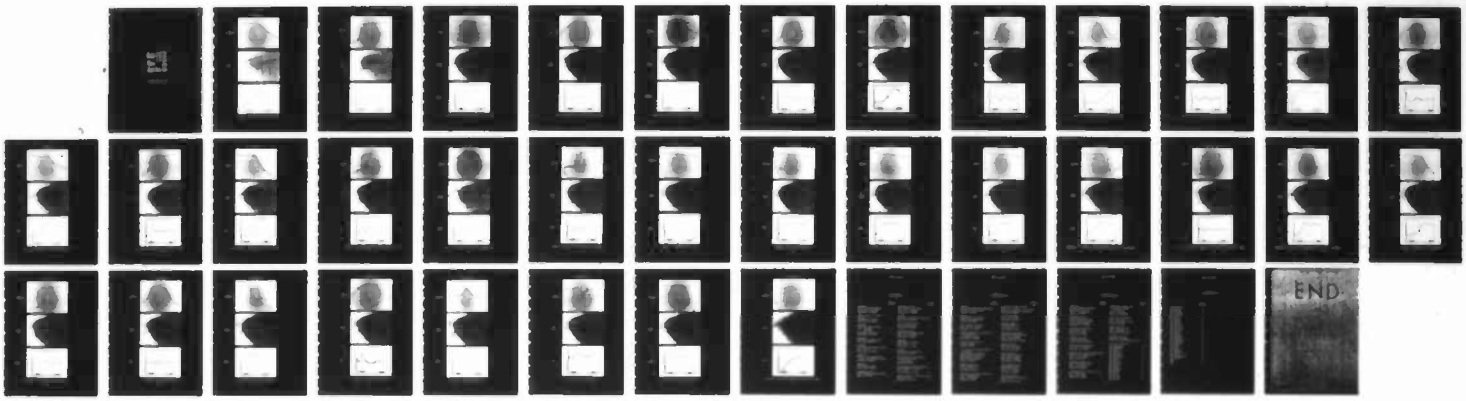

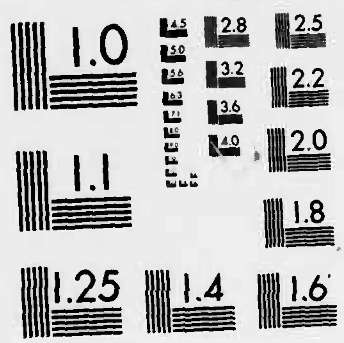

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NATIONAL BUREAU OF STANDARDS-1963-A

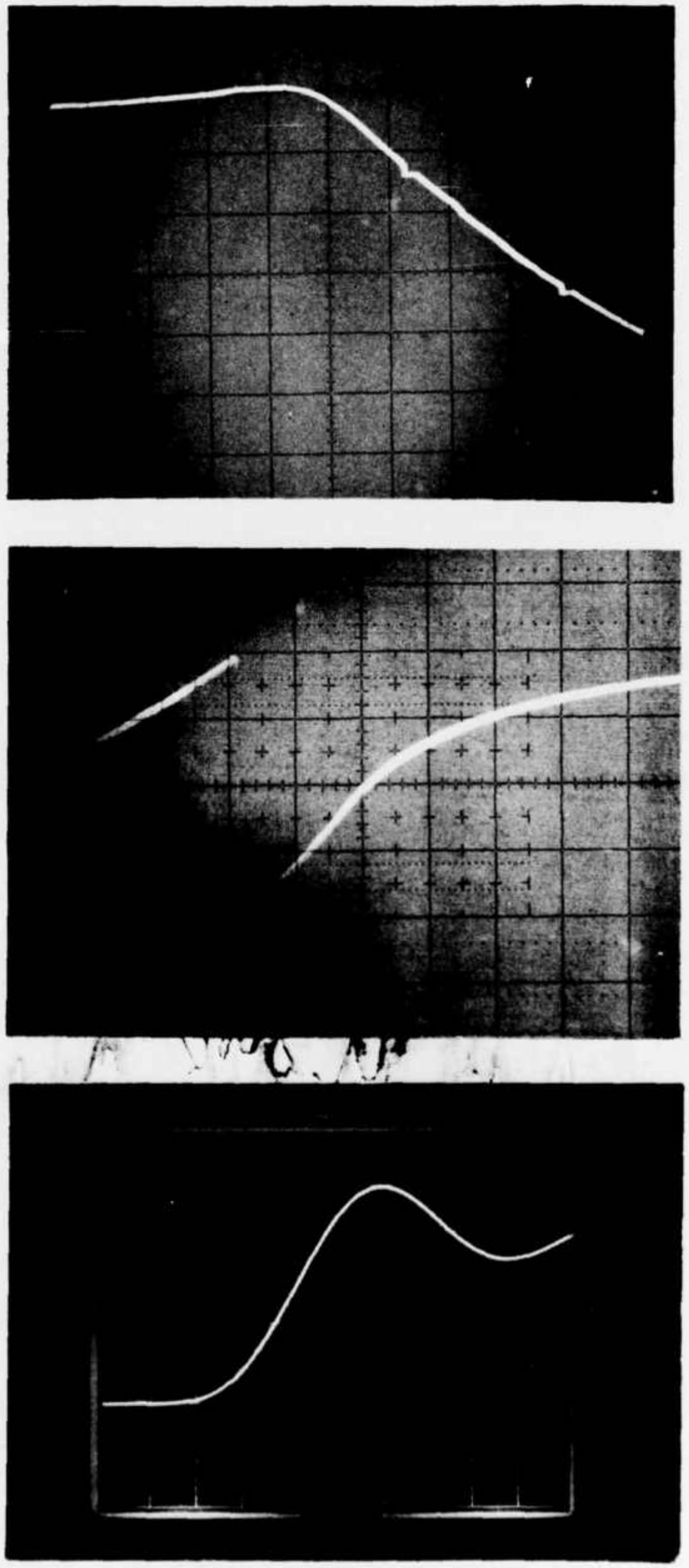
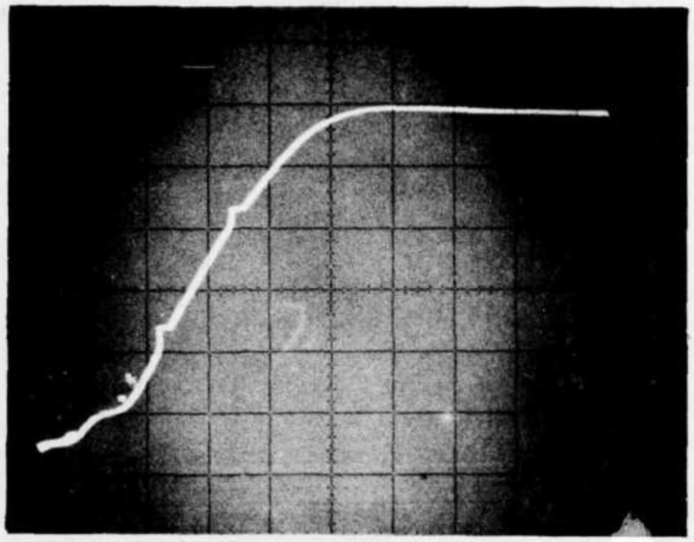


FIGURE A11. 8-POLE BUTTERWORTH LOW-PASS RESPONSE

FREQUENCY



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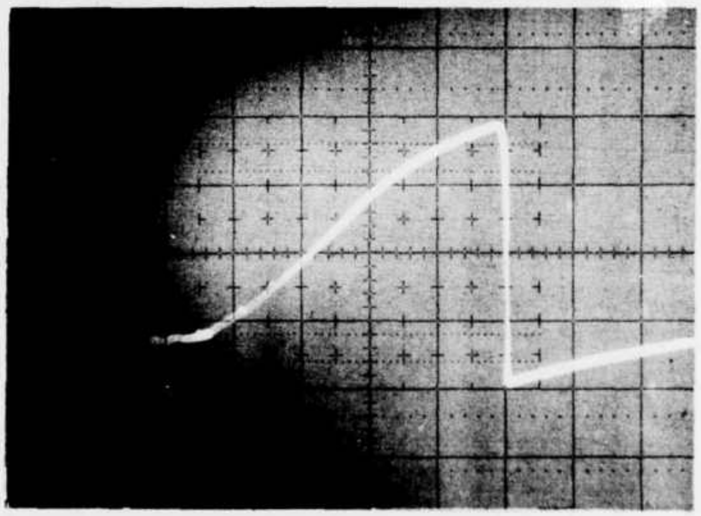

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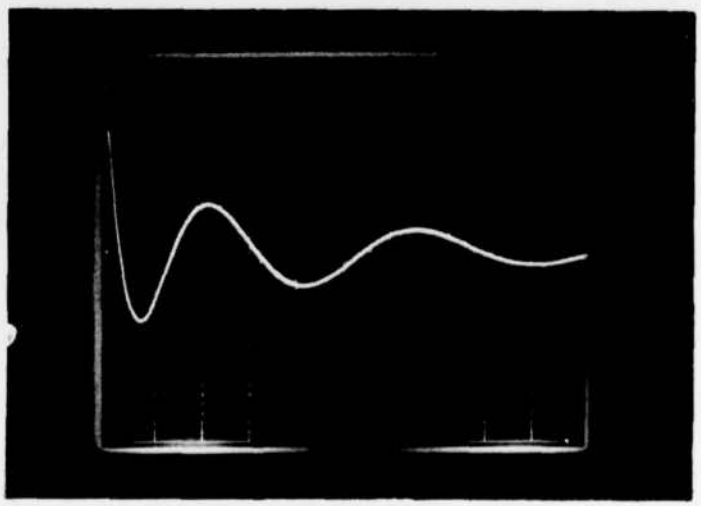


FIGURE A-12. 8-POLE BUTTERWORTH HIGH-PASS RESPONSE

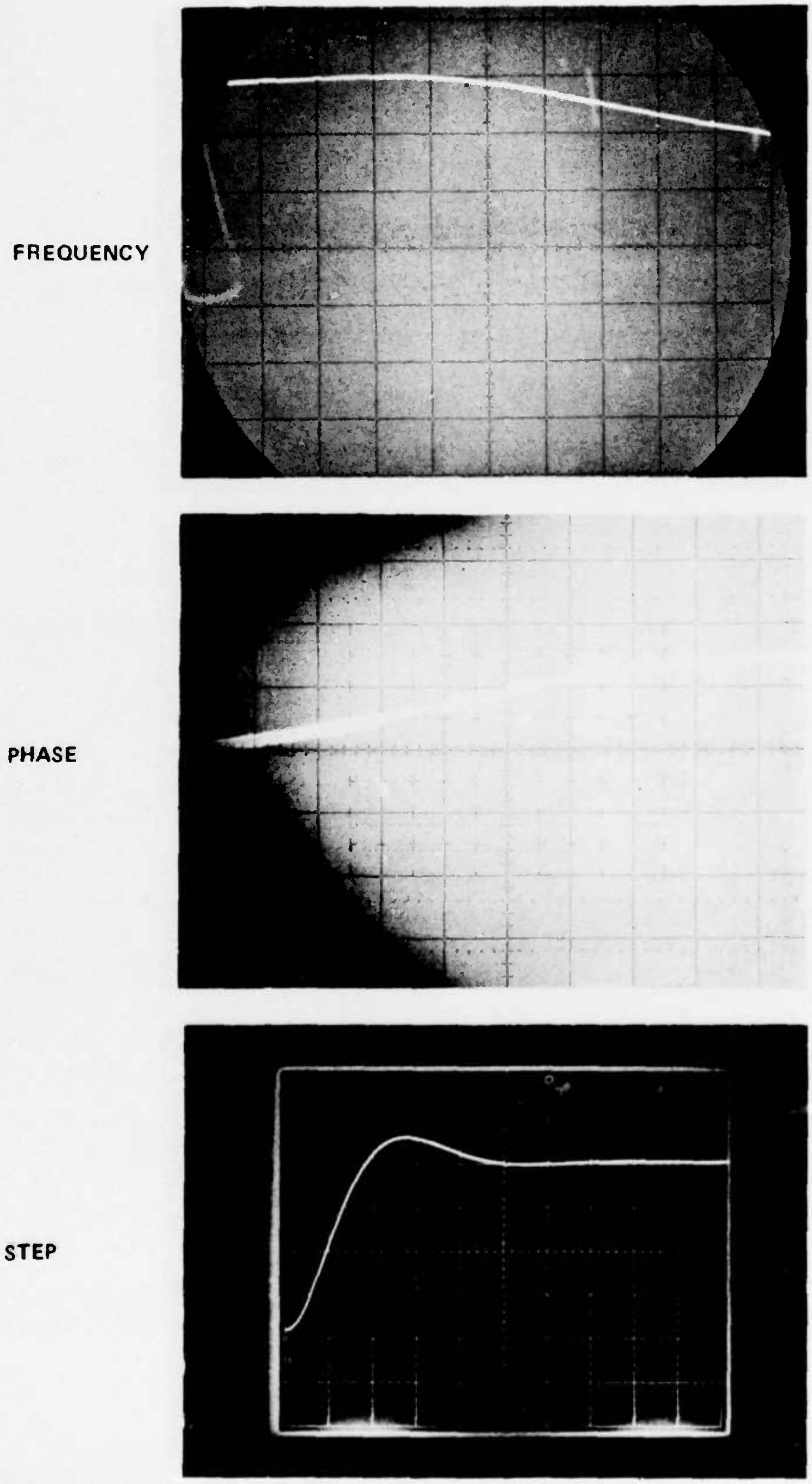
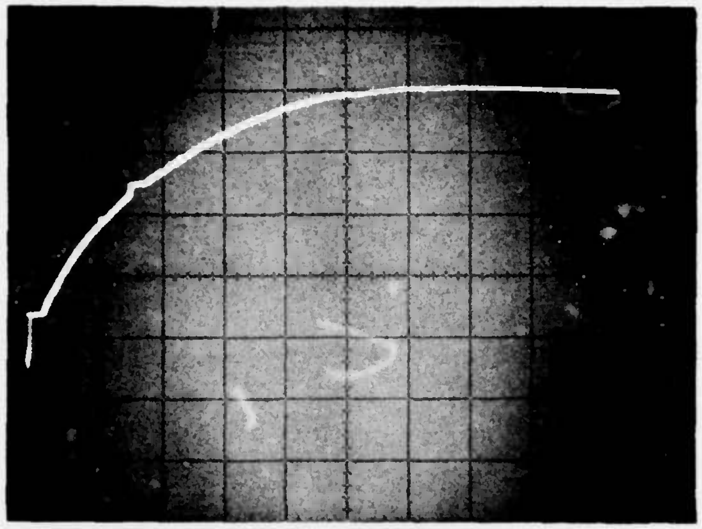


FIGURE A-13. 2-POLE 1 DB-RIPPLE CHEBYSHEV LOW-PASS RESPONSE

FREQUENCY



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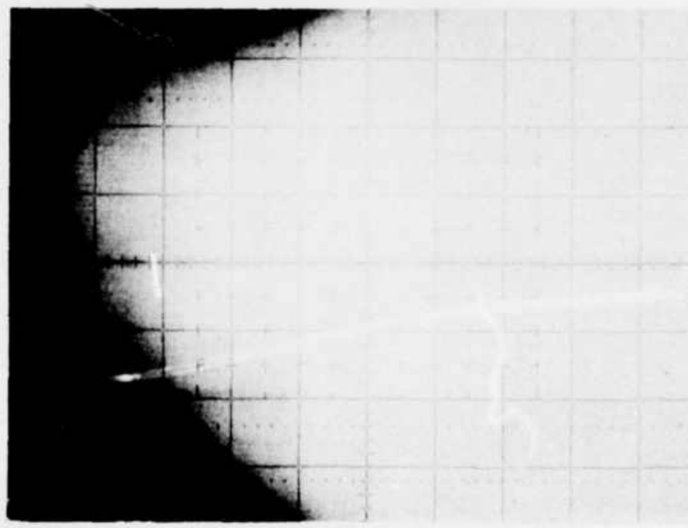

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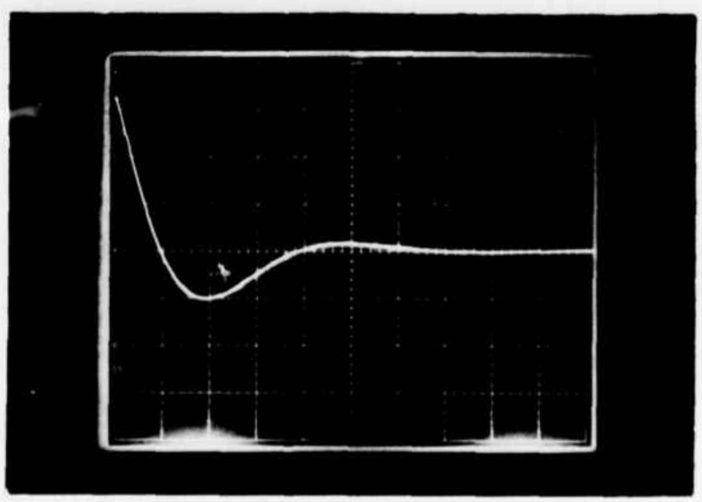


FIGURE A-14. 2-POLE 1 DB-RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

FREQUENCY

PHASE

STEP

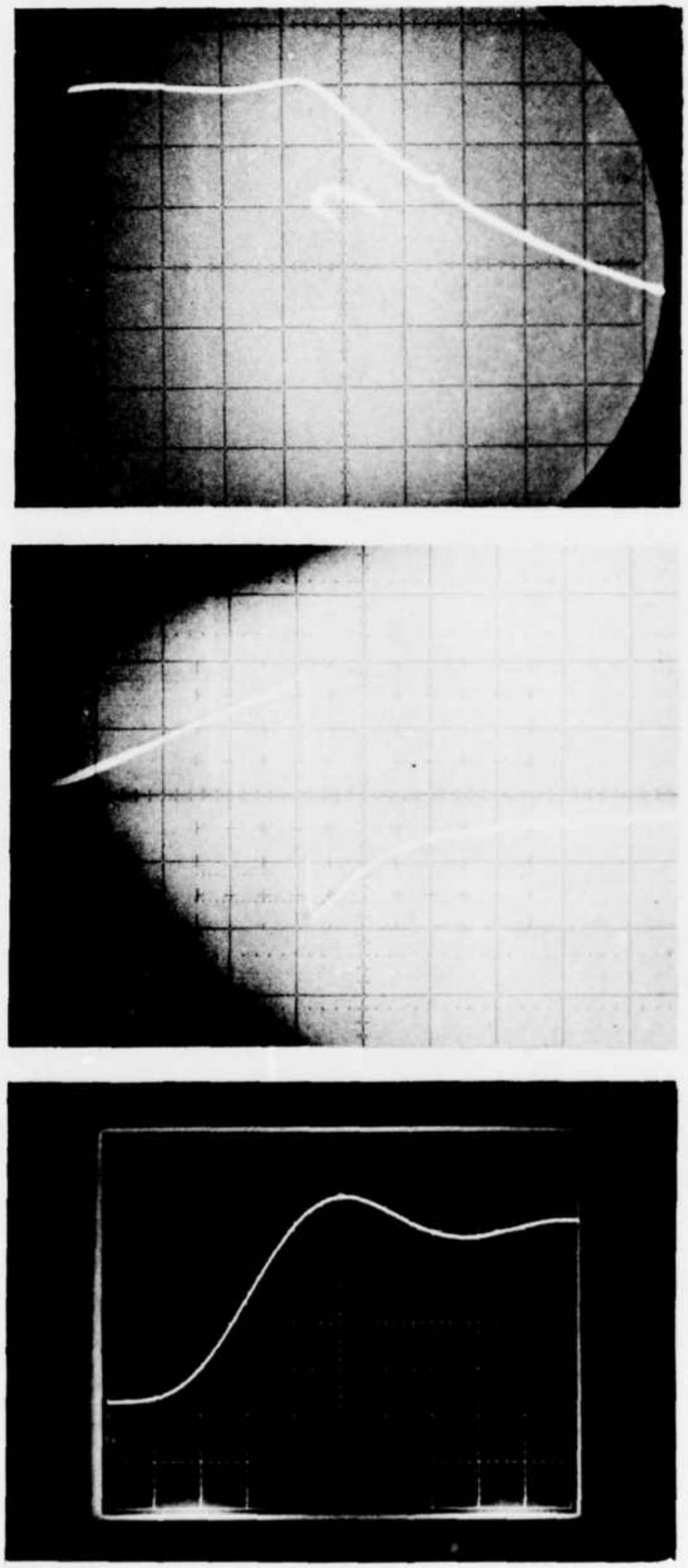


FIGURE A-15. 4-POLE 1 DB-RIPPLE CHEBYSHEV LOW-PASS RESPONSE

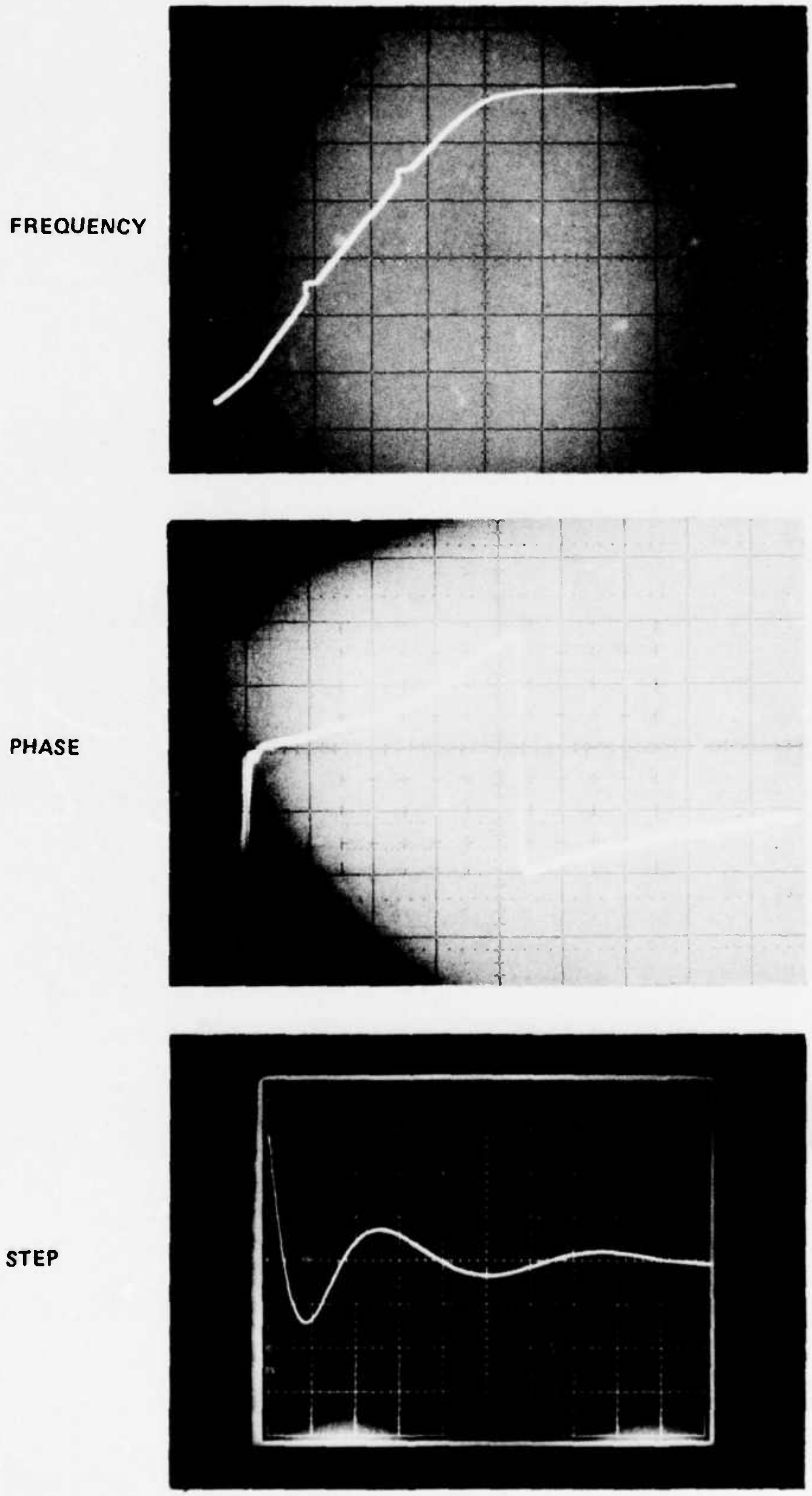


FIGURE A-16. 4-POLE 1 DB-RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

FREQUENCY

PHASE

STEP

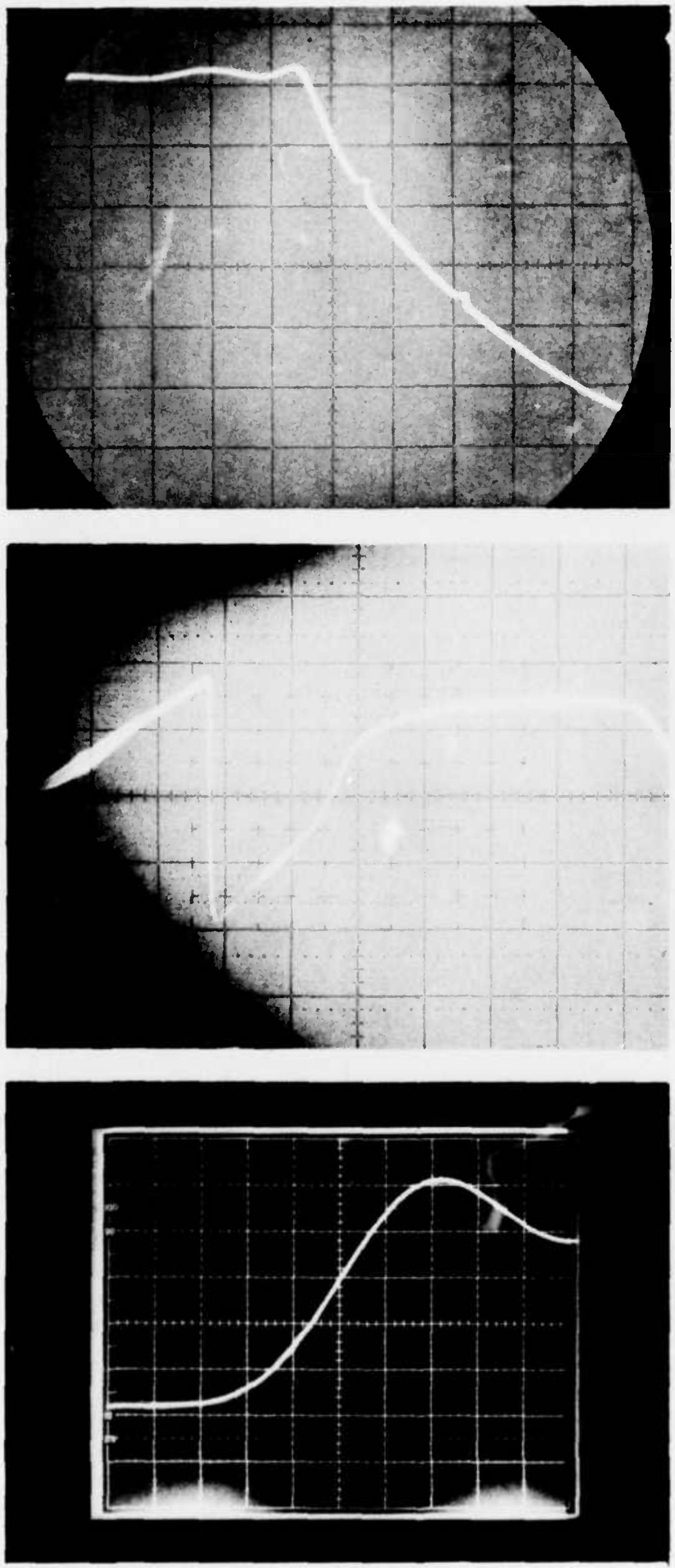


FIGURE A-17. 6-POLE 1 DB-RIPPLE CHEBYSHEV LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

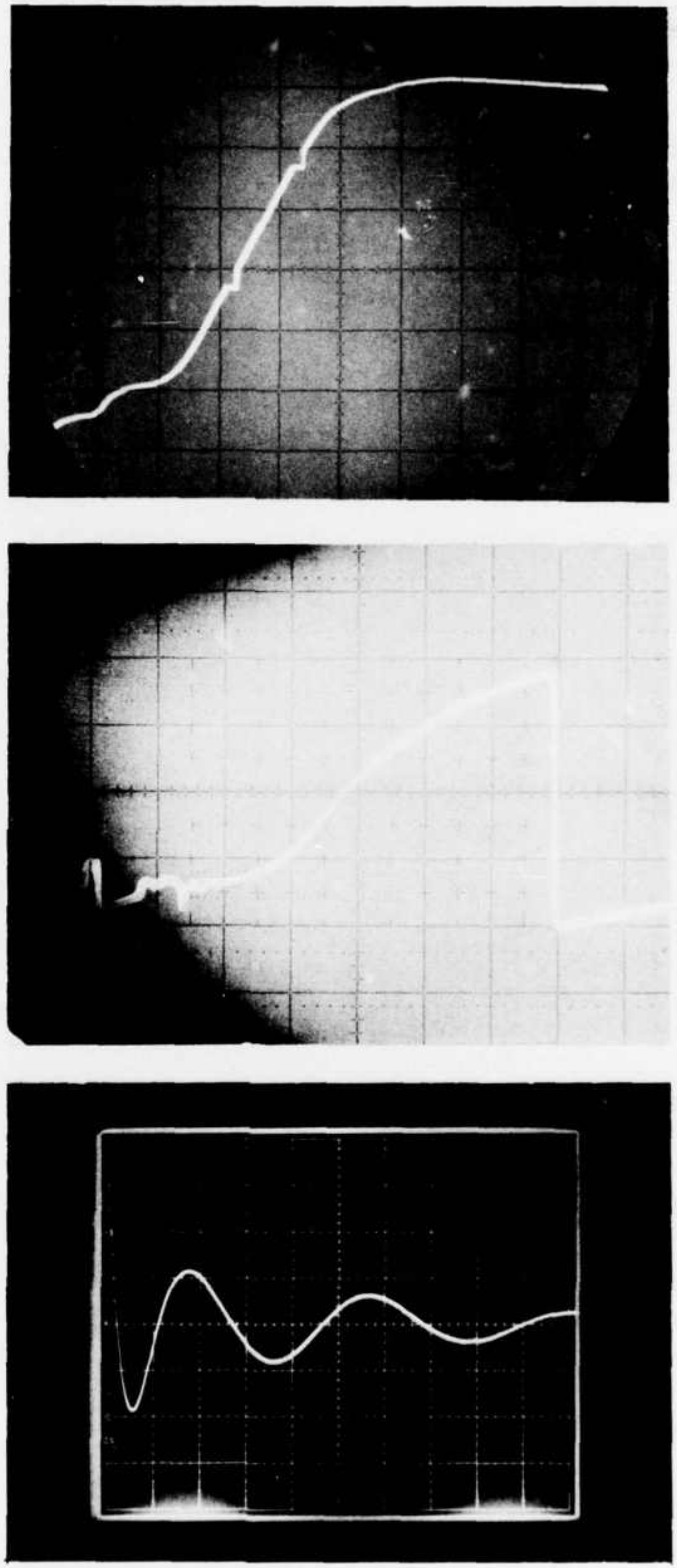
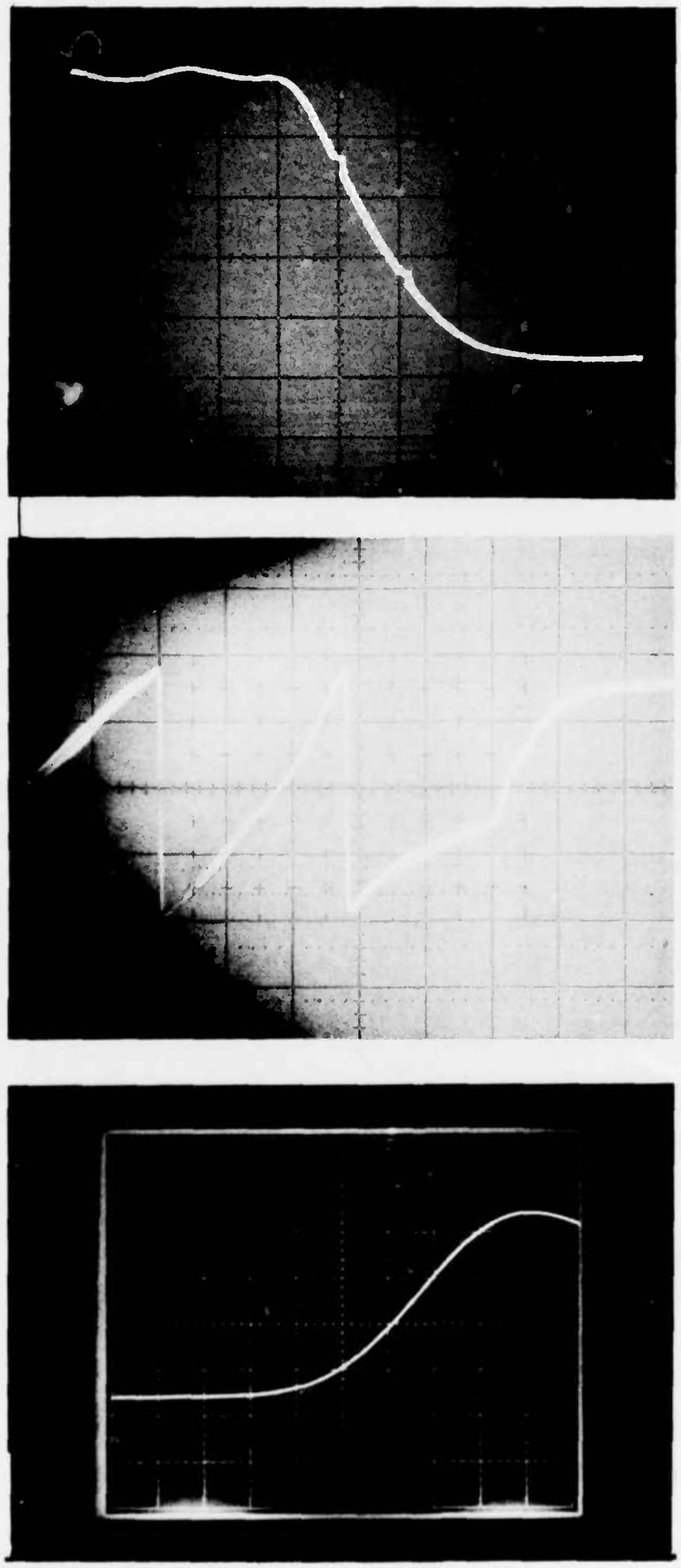


FIGURE A-18. 6-POLE 1 DB-RIPPLE CHEBYSHEV HIGH-PASS RESPONSE



FREQUENCY

PHASE

STEP

FIGURE A-19. 8-POLE 1 DB-RIPPLE CHEBYSHEV LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

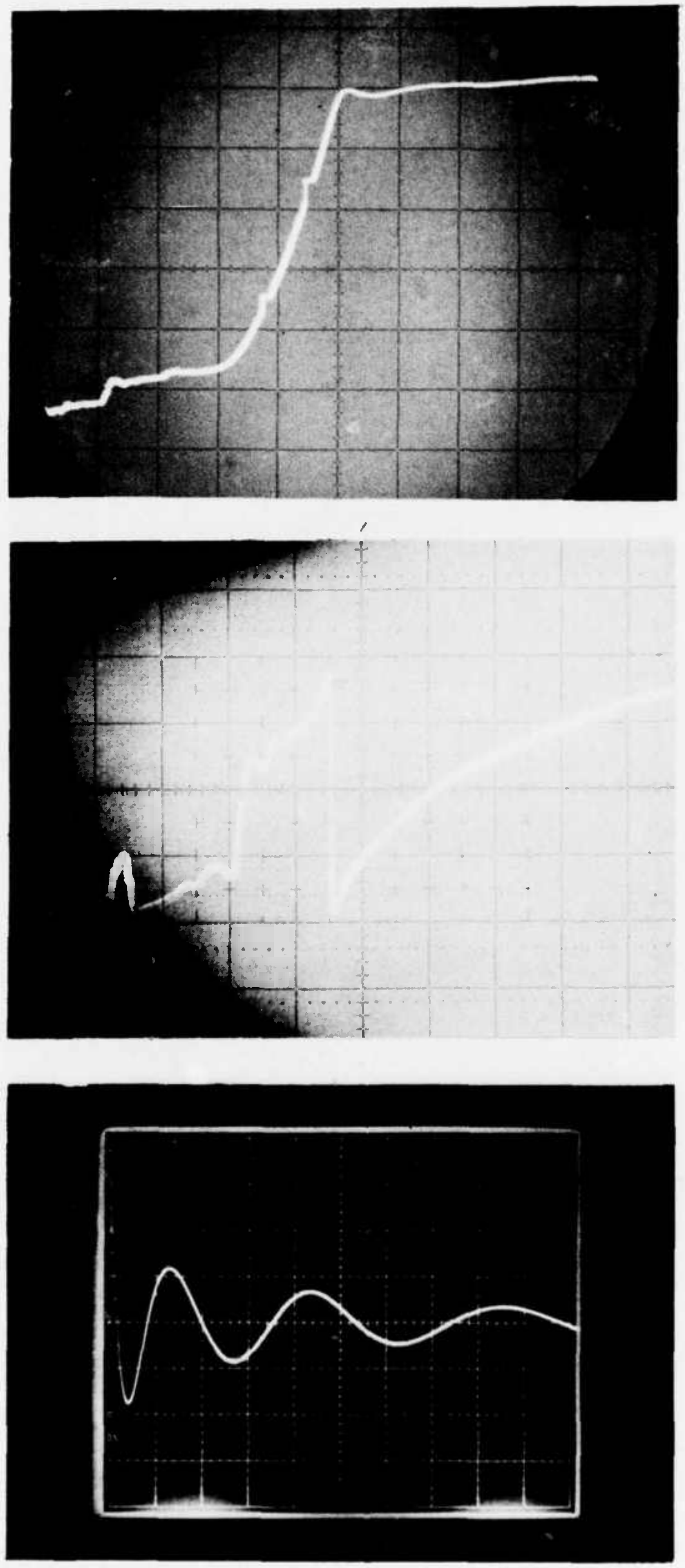
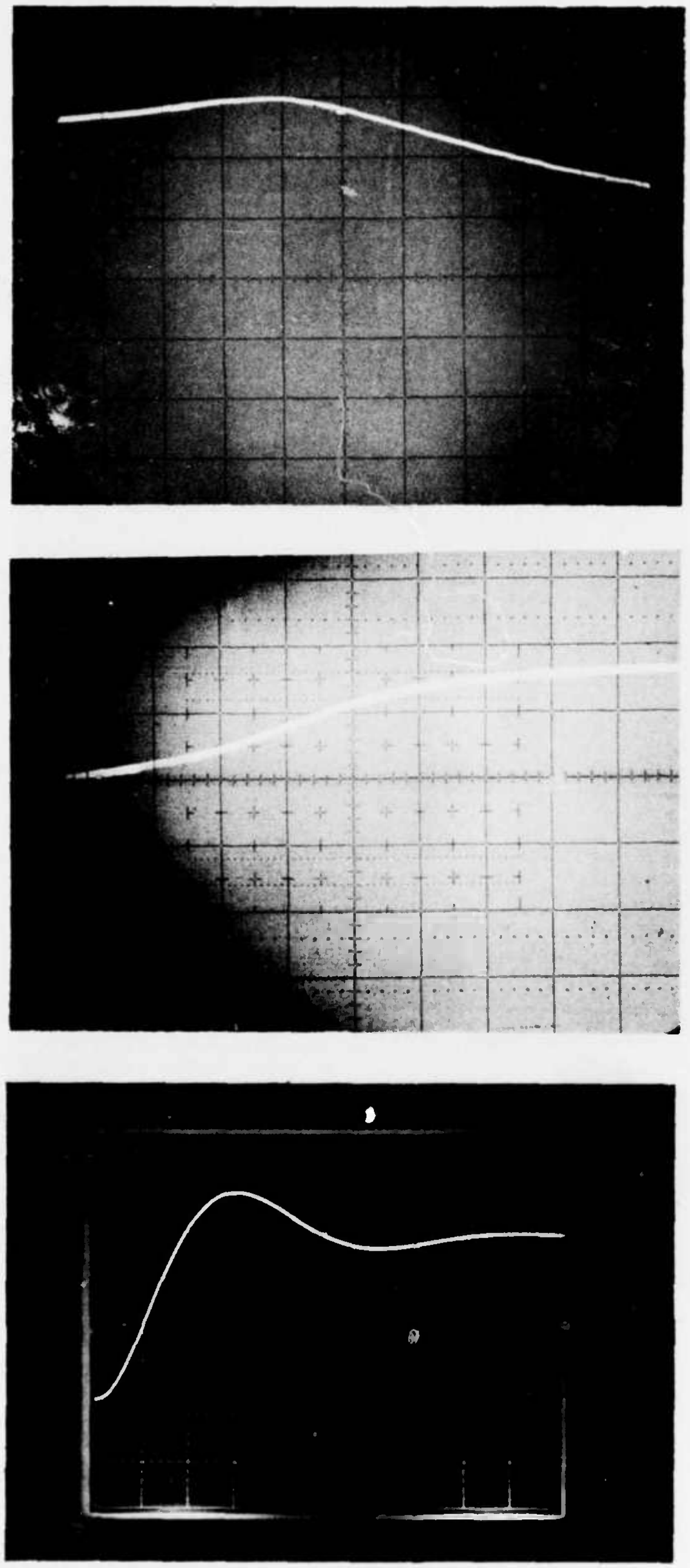


FIGURE A-20. 8-POLE 1 DB-RIPPLE CHEBYSHEV HIGH-PASS RESPONSE



FREQUENCY

PHASE

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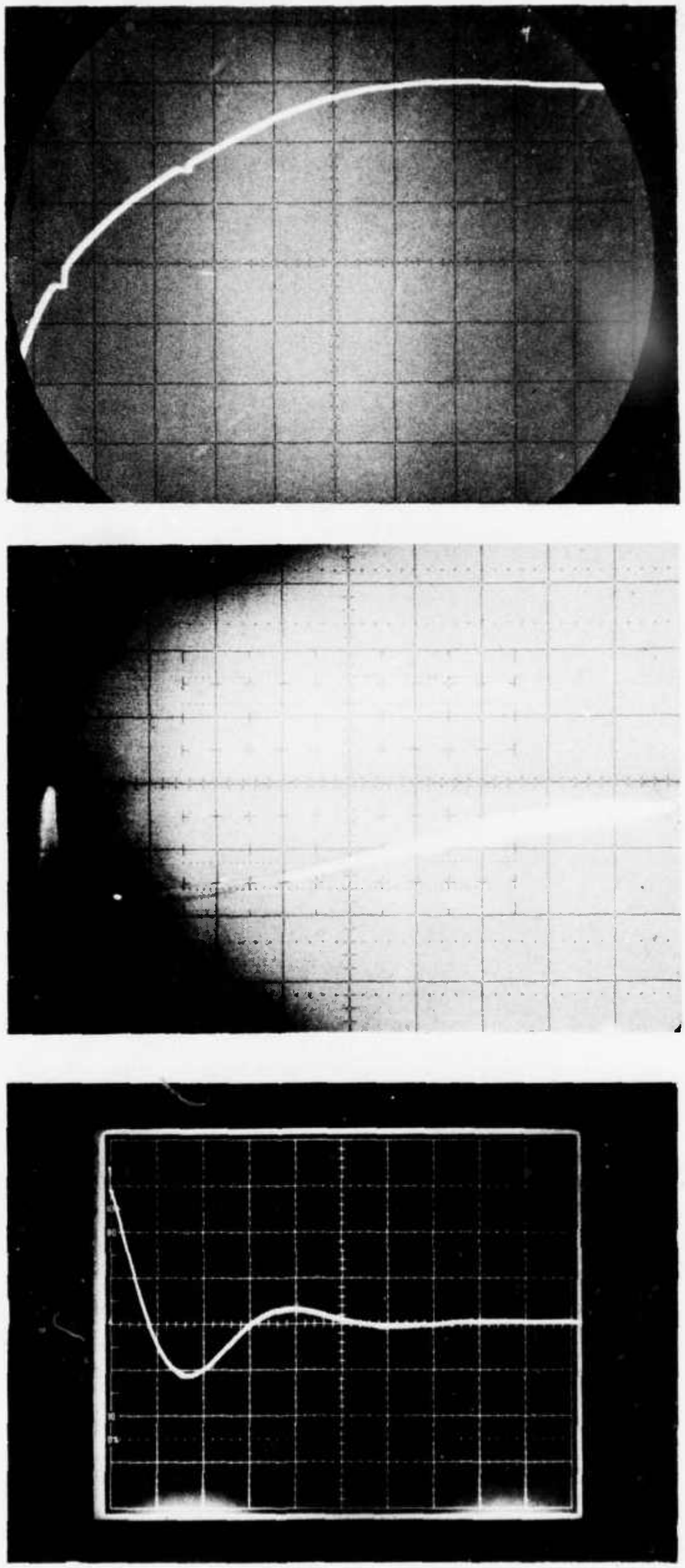
FIGURE A-21. 2-POLE 3 DB-RIPPLE CHEBYSHEV LOW-PASS RESPONSE



FREQUENCY

PHASE

STEP

FIGURE A-22. 2-POLE 3 DB-RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

FREQUENCY

PHASE

STEP

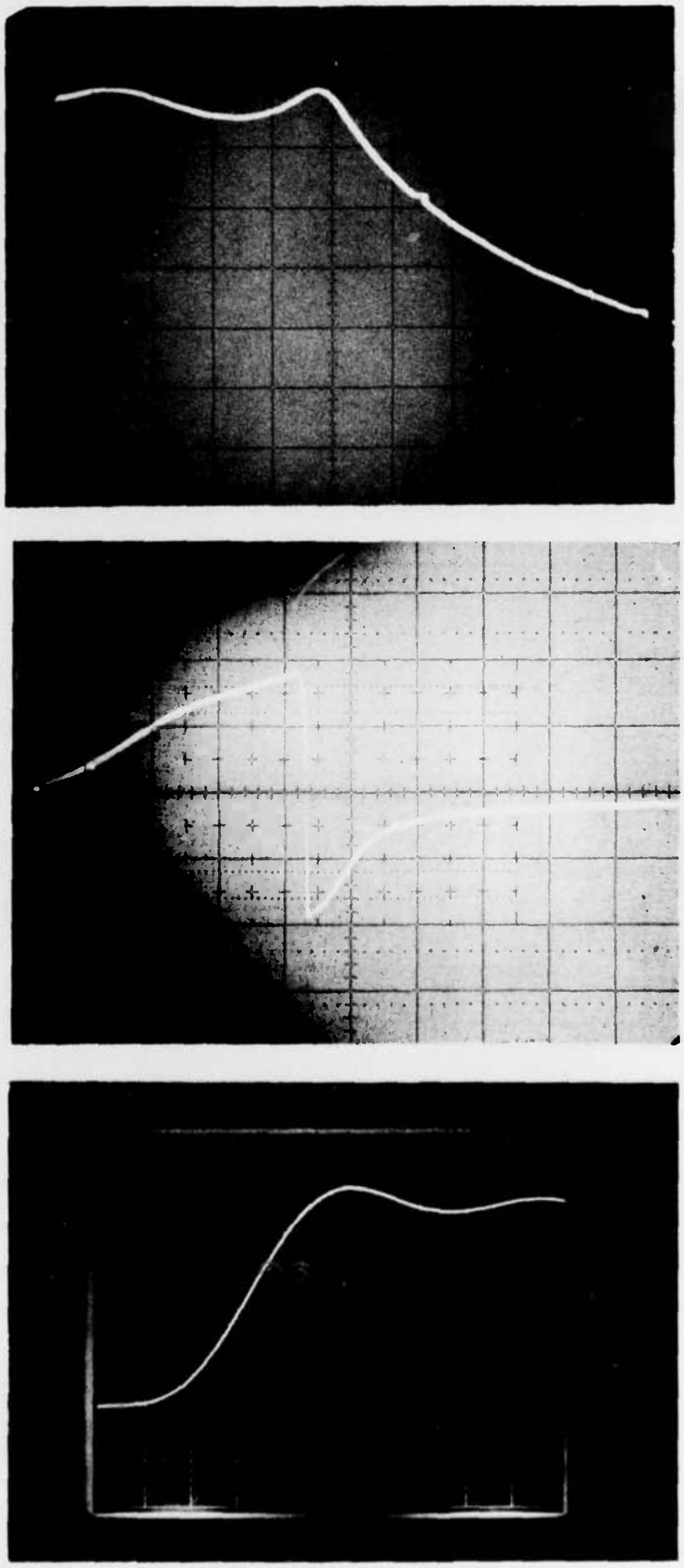


FIGURE A-23. 4-POLE 3 DB-RIPPLE CHEBYSHEV LOW PASS RESPONSE

FREQUENCY

PHASE

STEP

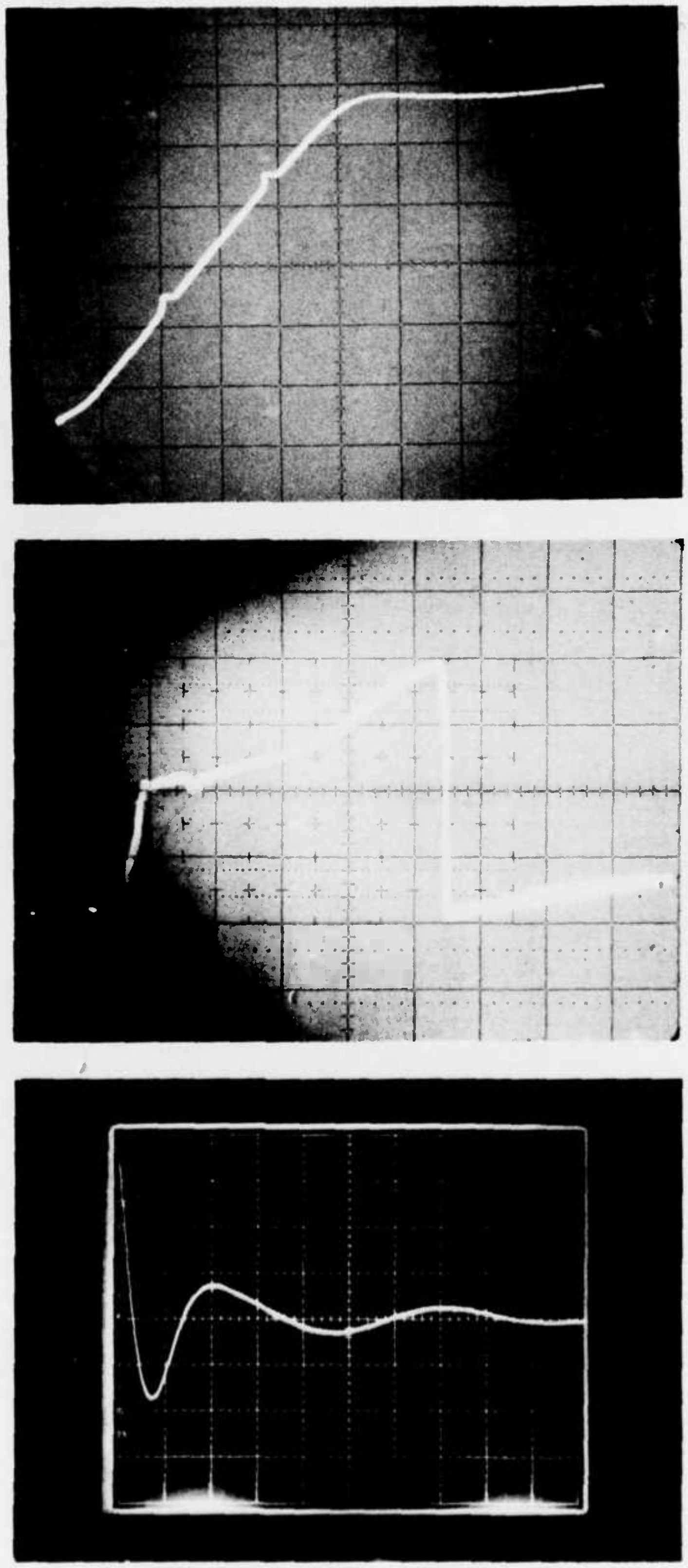
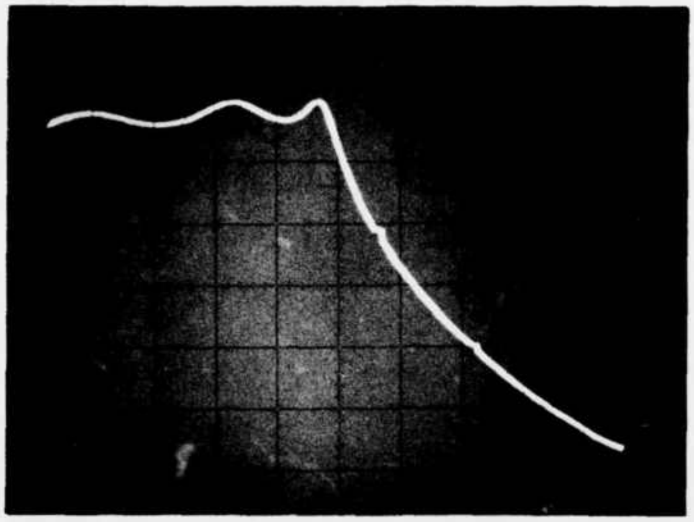


FIGURE A-24. 4-POLE 3 DB-RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

FREQUENCY



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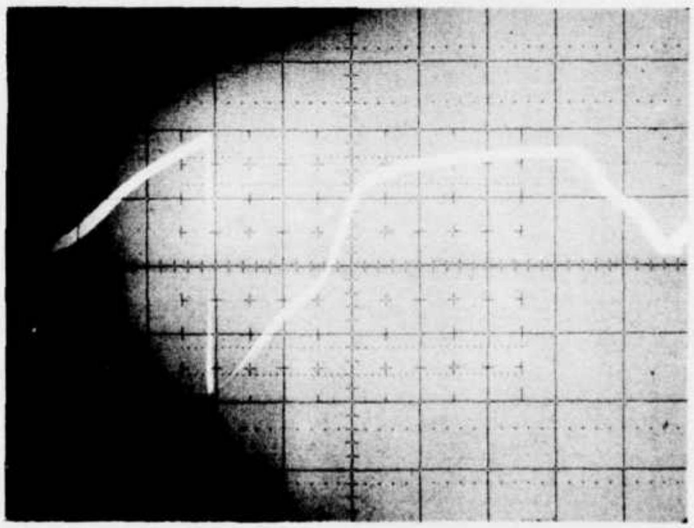

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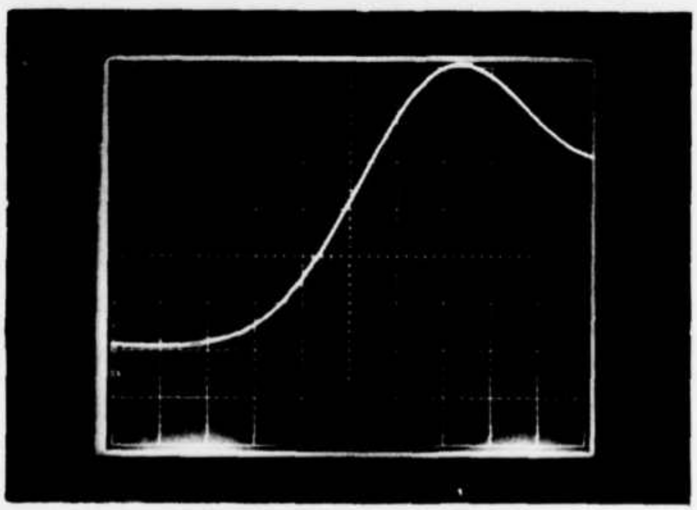
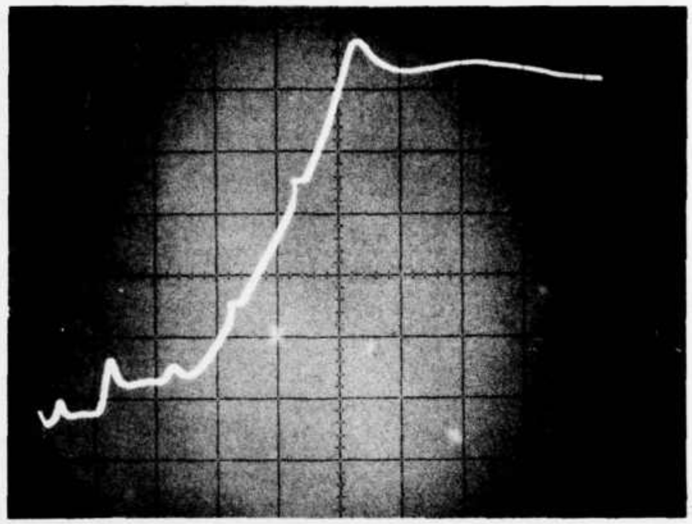


FIGURE A-25. 6-POLE 3 DB-RIPPLE CHEBYSHEV LOW-PASS RESPONSE

FREQUENCY



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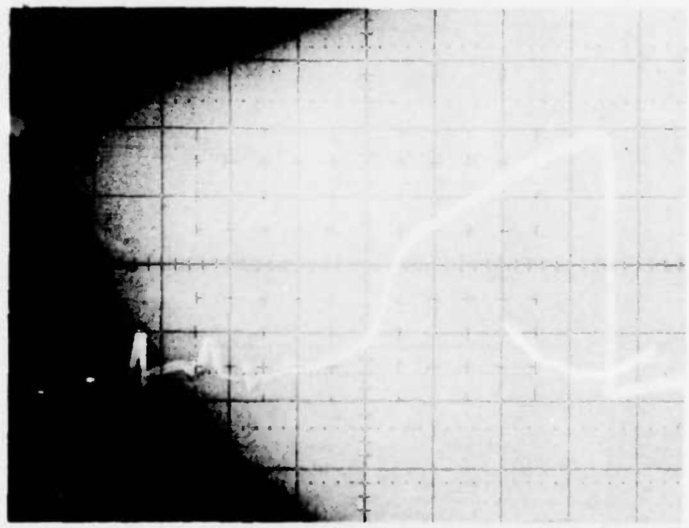

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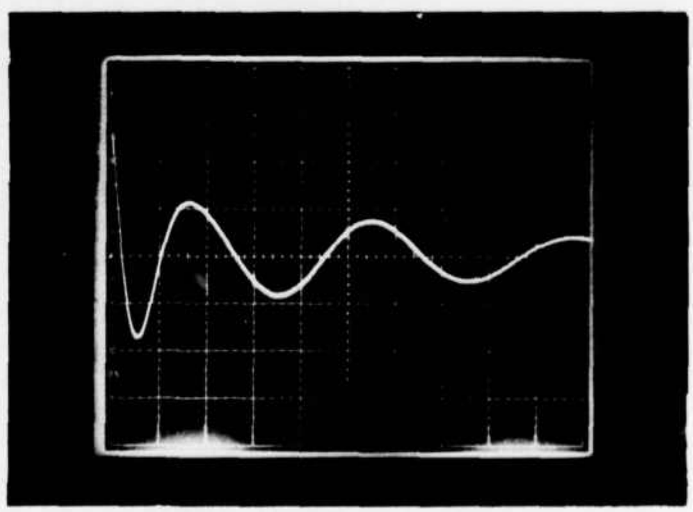
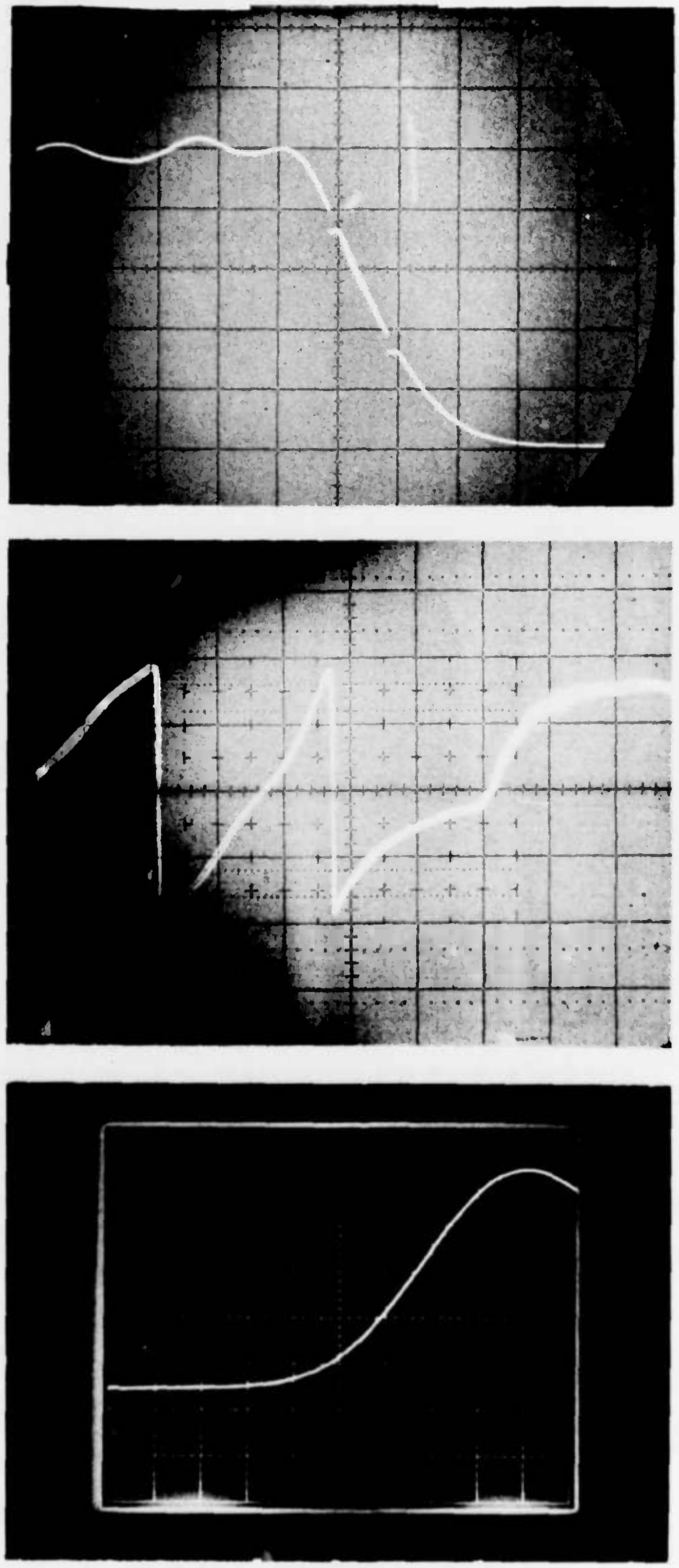


FIGURE A-26. 6-POLE 3 DB-RIPPLE CHEBYSHEV HIGH-PASS RESPONSE



FREQUENCY

PHASE

STEP

FIGURE A-27. 8-POLE 3 DB-RIPPLE CHEBYSHEV LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

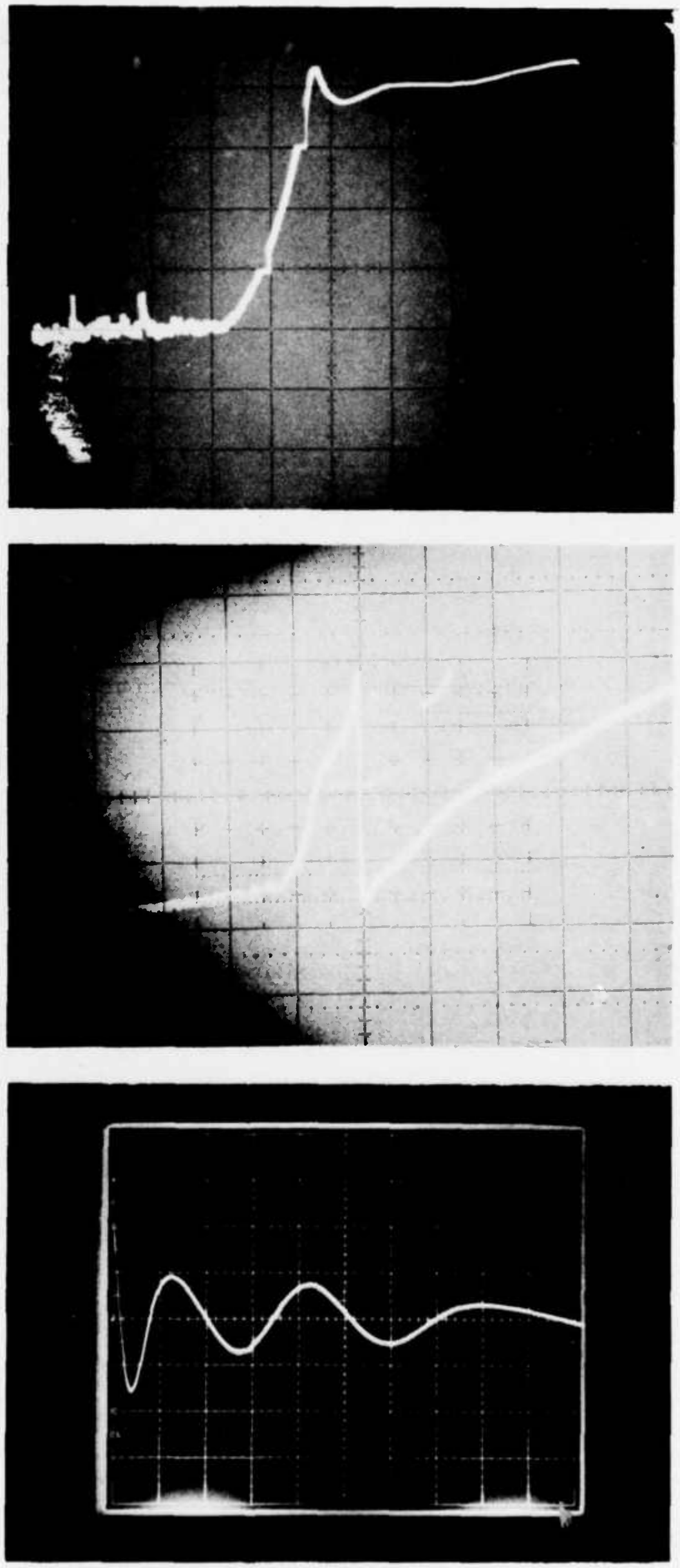
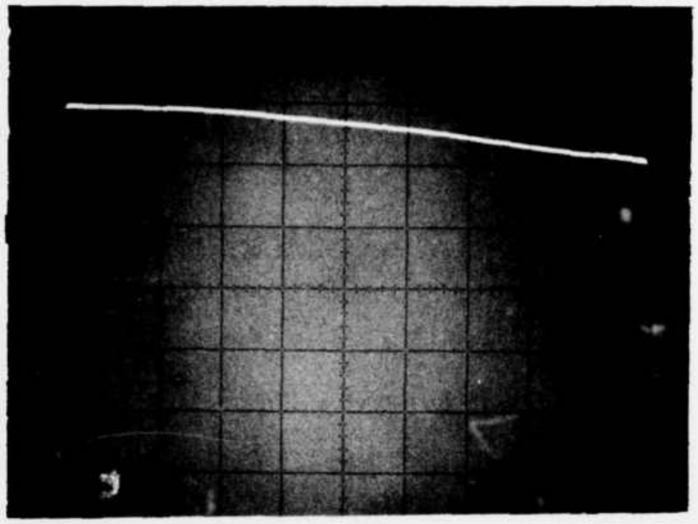
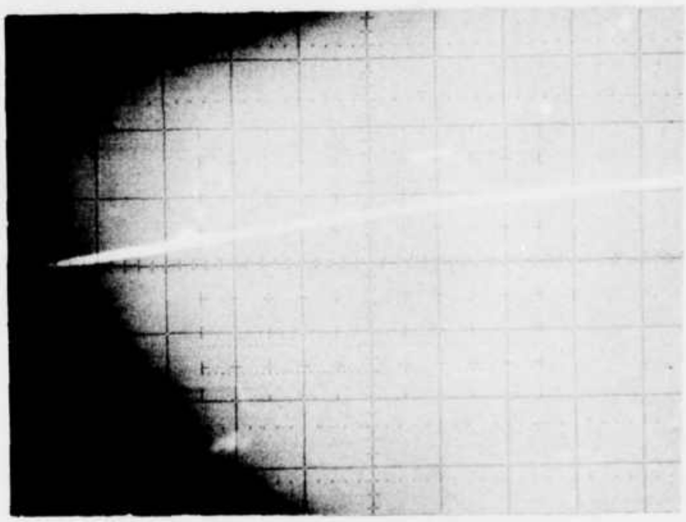


FIGURE A-28. 8-POLE 3 DB-RIPPLE CHEBYSHEV HIGH-PASS RESPONSE

FREQUENCY



PHASE



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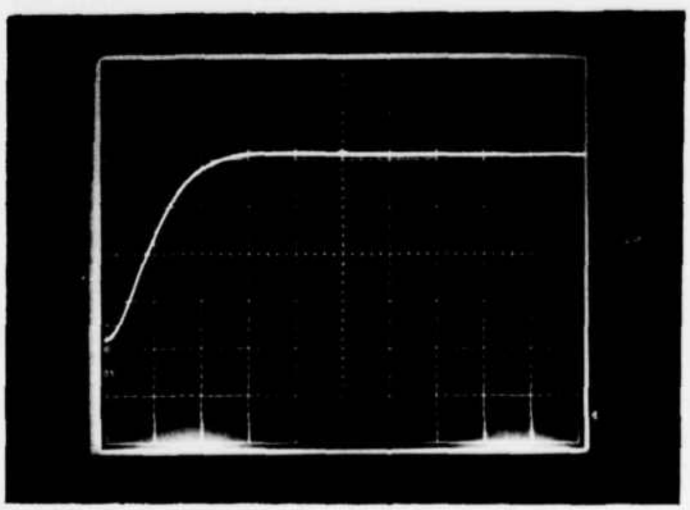
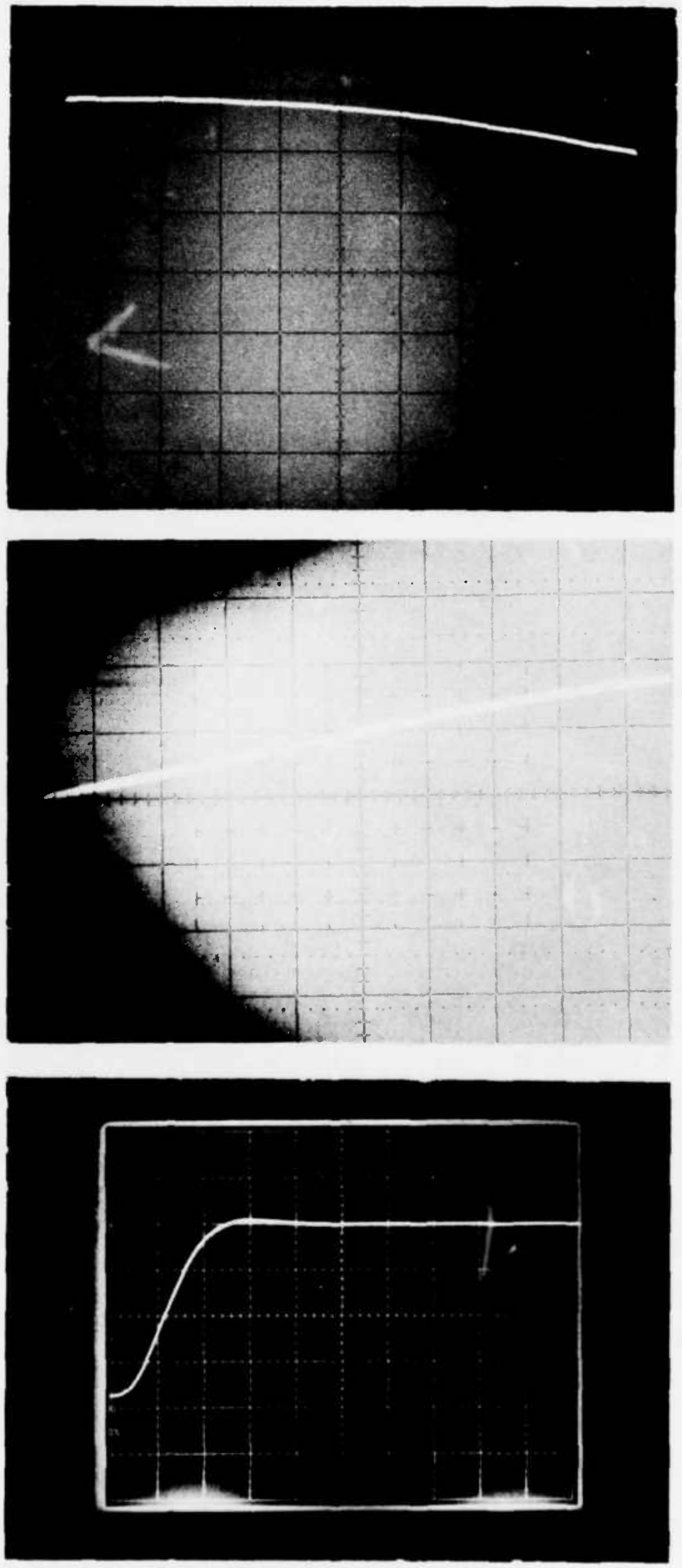


FIGURE A-29. 2-POLE BESSEL LOW-PASS RESPONSE



FREQUENCY

PHASE

STEP

FIGURE A-30. 4-POLE BESSEL LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

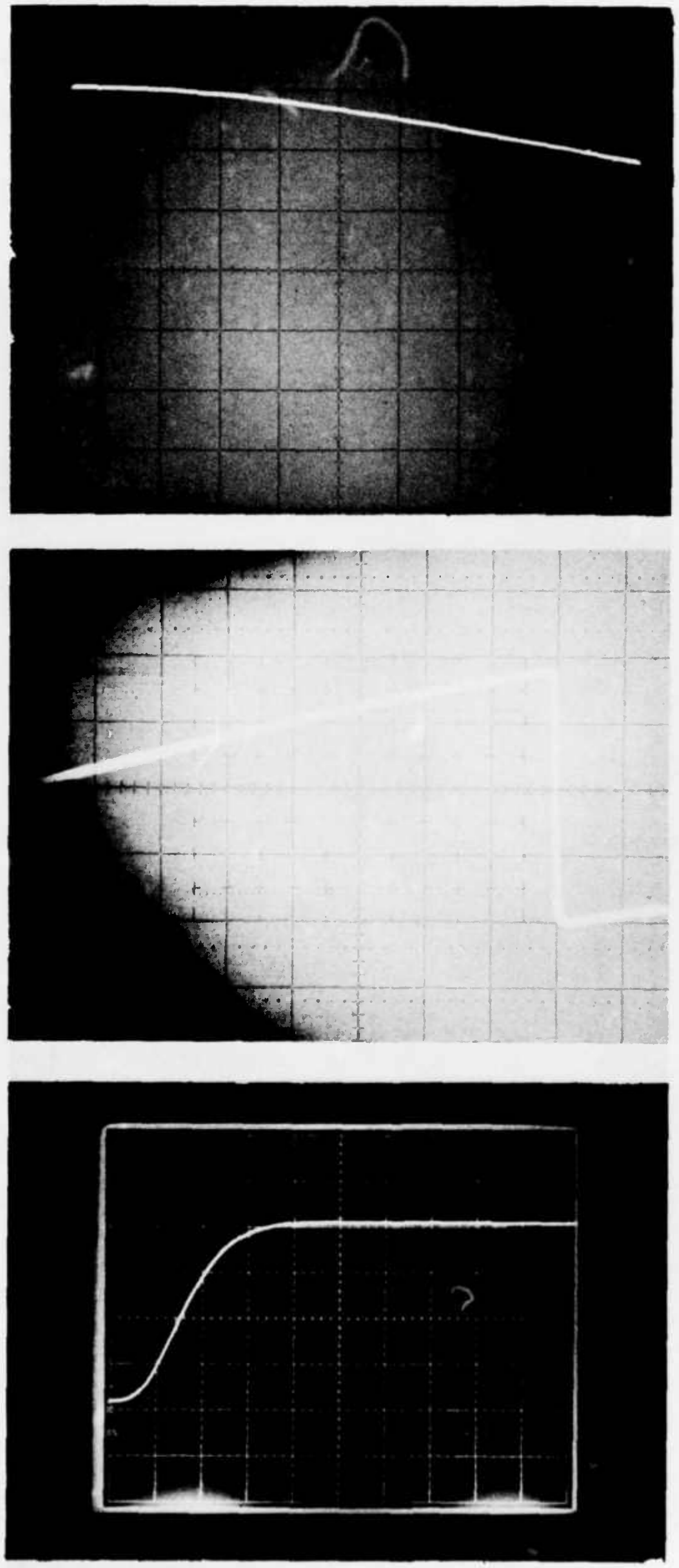


FIGURE A-31. 6-POLE BESSEL LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

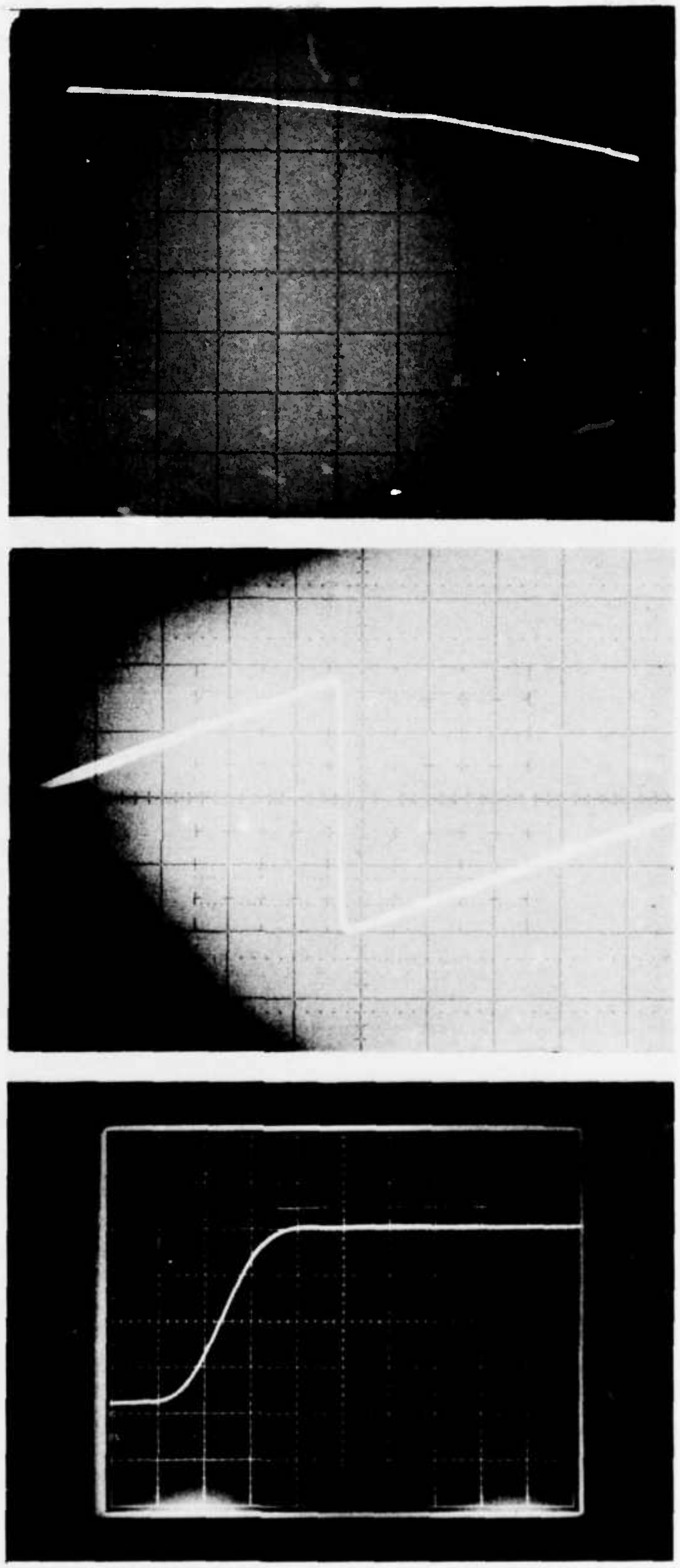
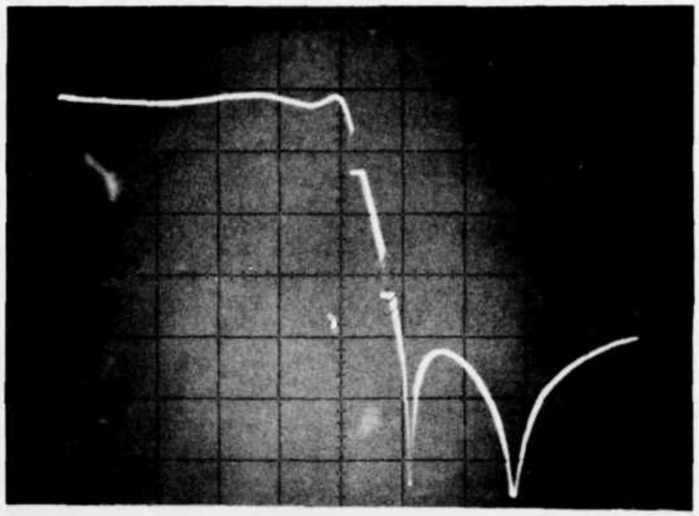
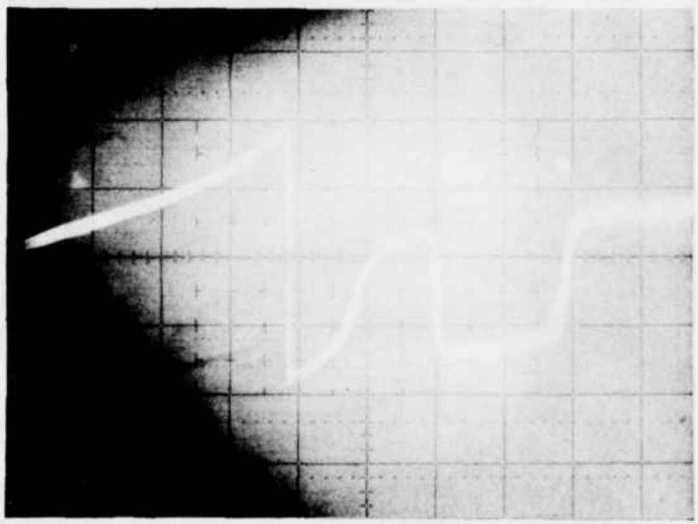


FIGURE A-32. 8-POLE BESSEL LOW-PASS RESPONSE

FREQUENCY



PHASE



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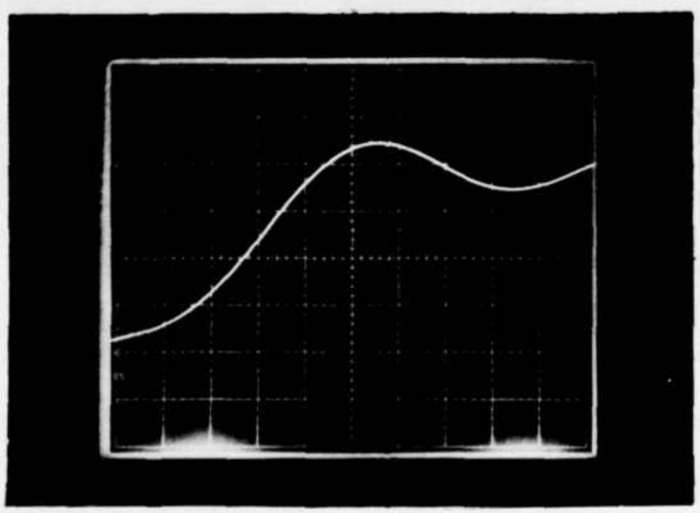

FIGURE A-33. 5-POLE 4-ZERO 1.25 DB-RIPPLE 37 DB-REJECTION ELLIPTIC LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

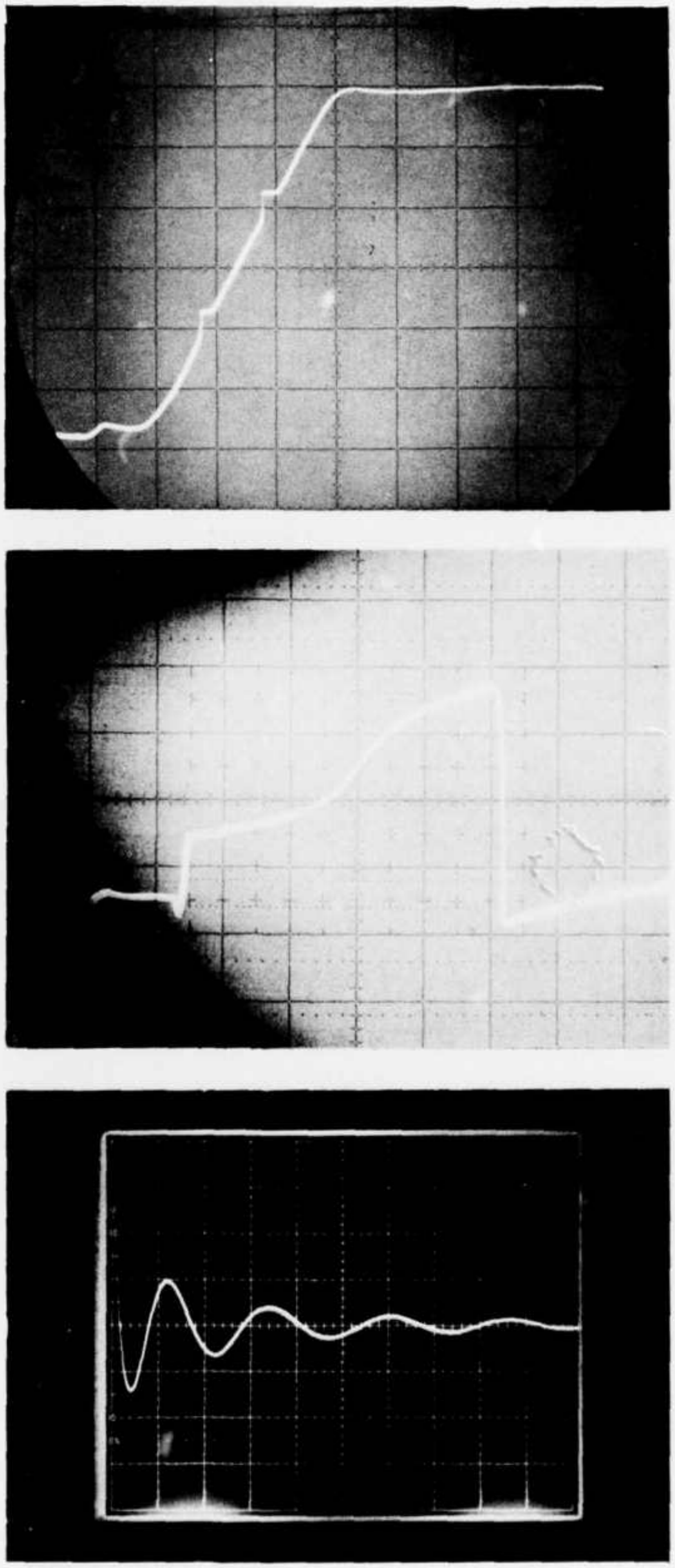

FIGURE A-34. 5-POLE 4-ZERO 1 DB-RIPPLE 69 DB-REJECTION ELLIPTIC HIGH-PASS RESPONSE

FREQUENCY

PHASE

STEP

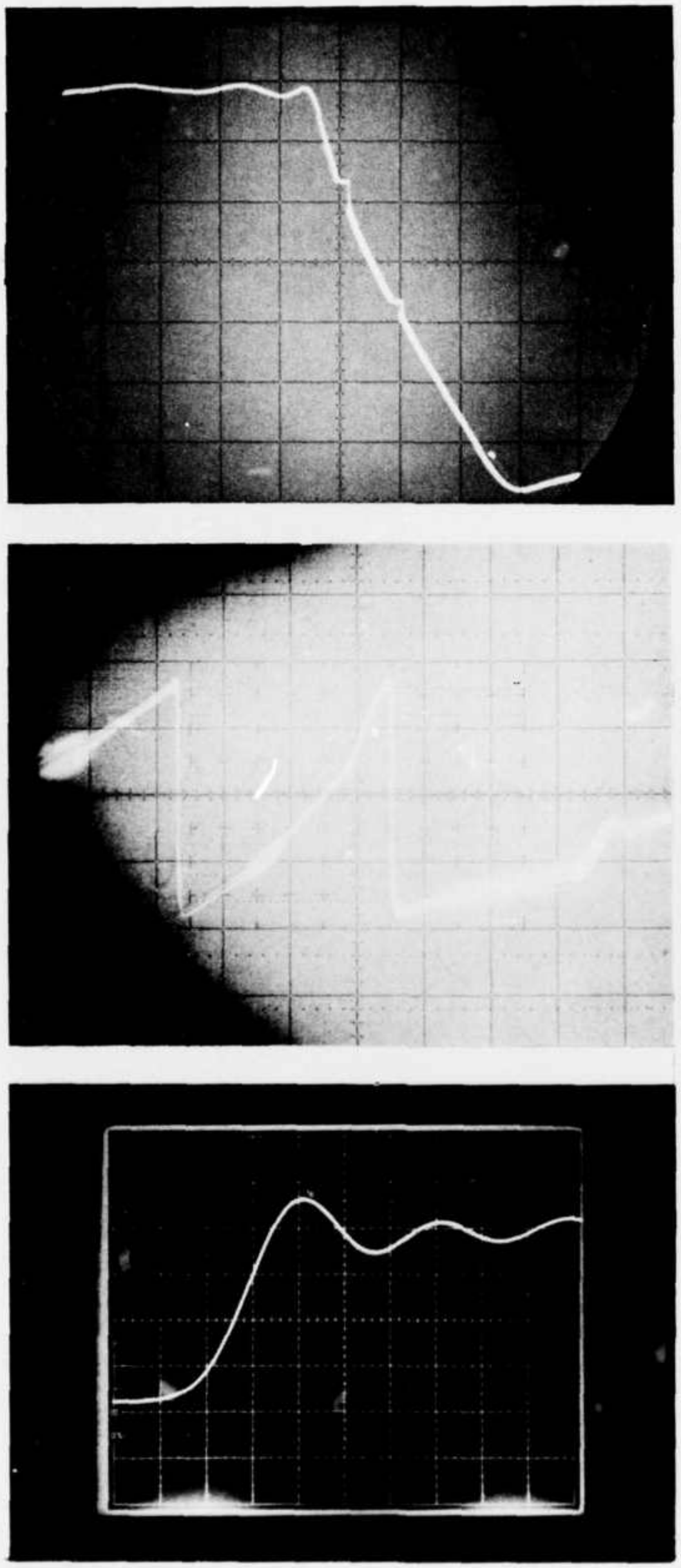

FIGURE A-35. 7-POLE 6-ZERO 1 DB-RIPPLE 104 DB-REJECTION ELLIPTIC LOW-PASS RESPONSE

FREQUENCY

PHASE

STEP

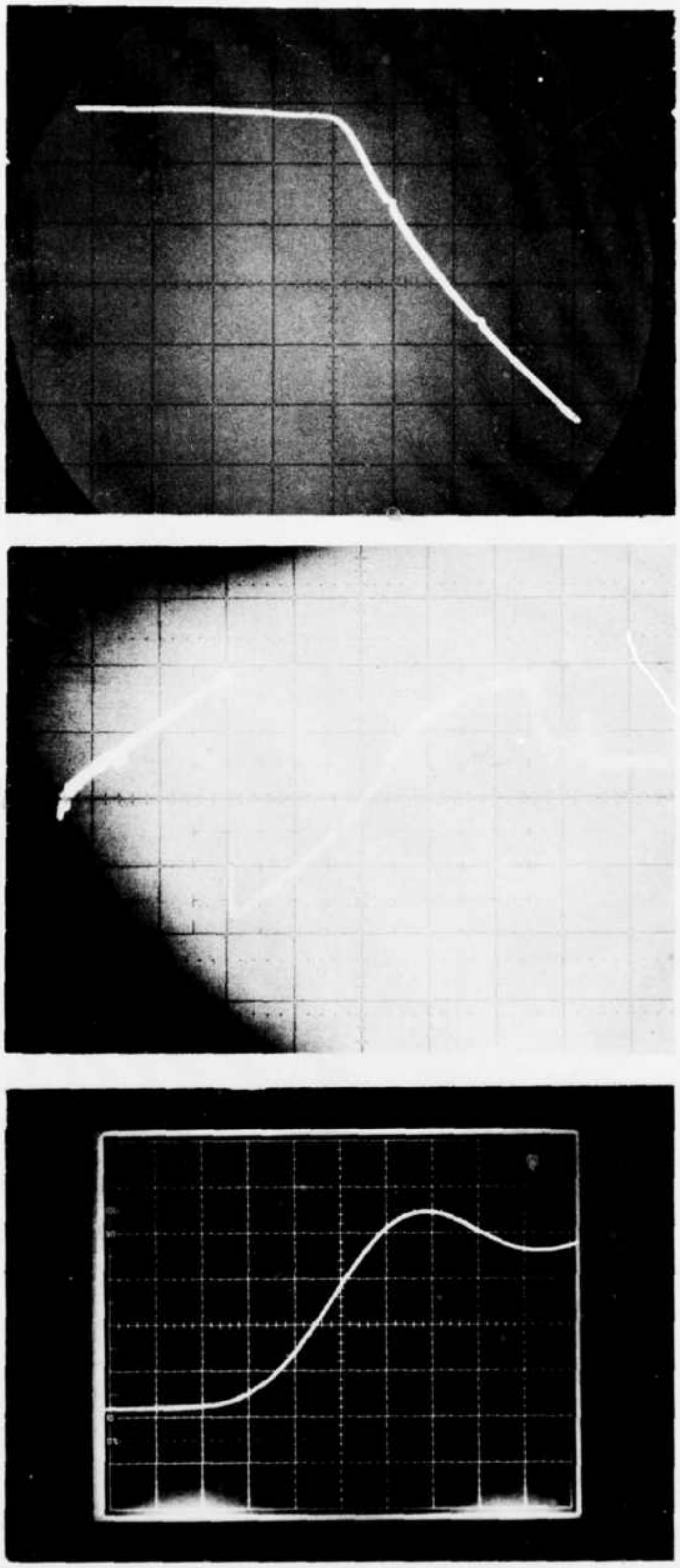

FIGURE A-36. 7-POLE CONSTANT-K LOW-PASS RESPONSE

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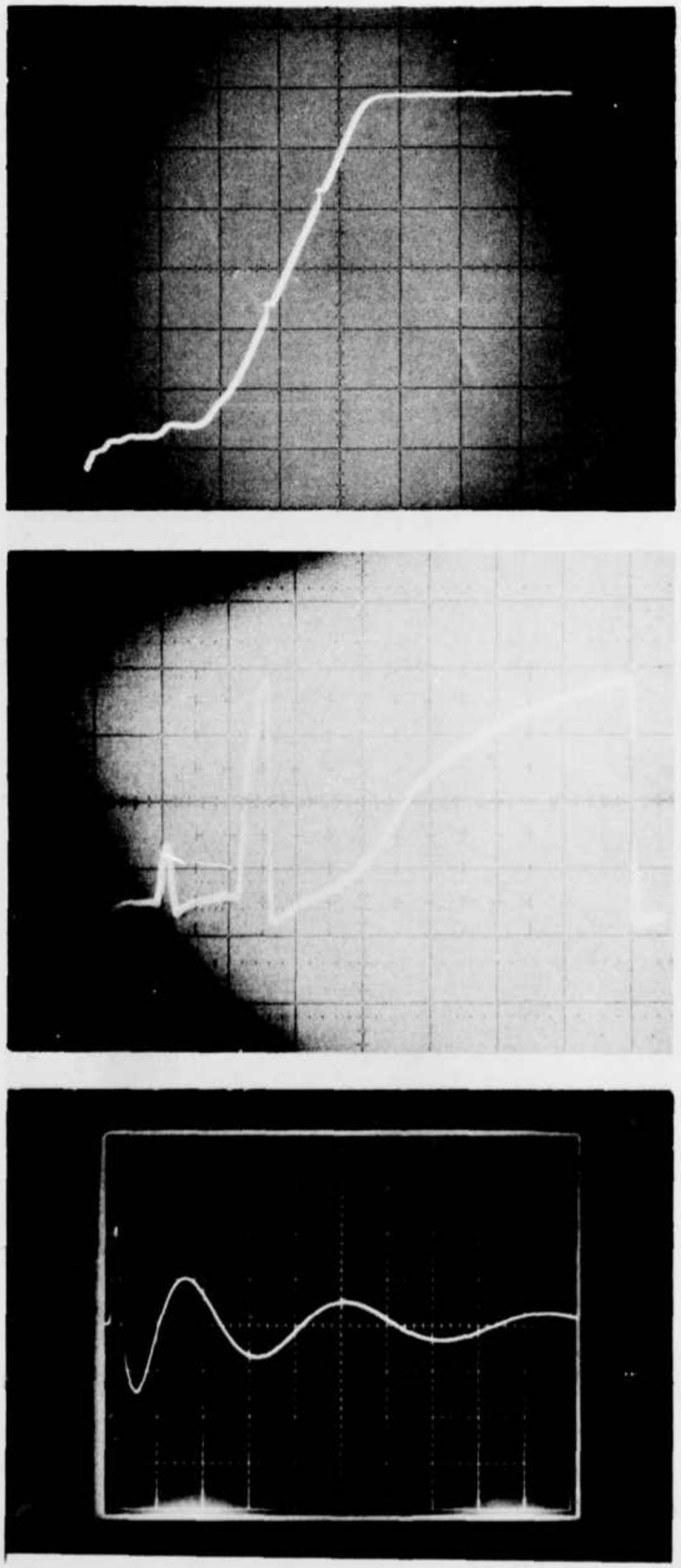
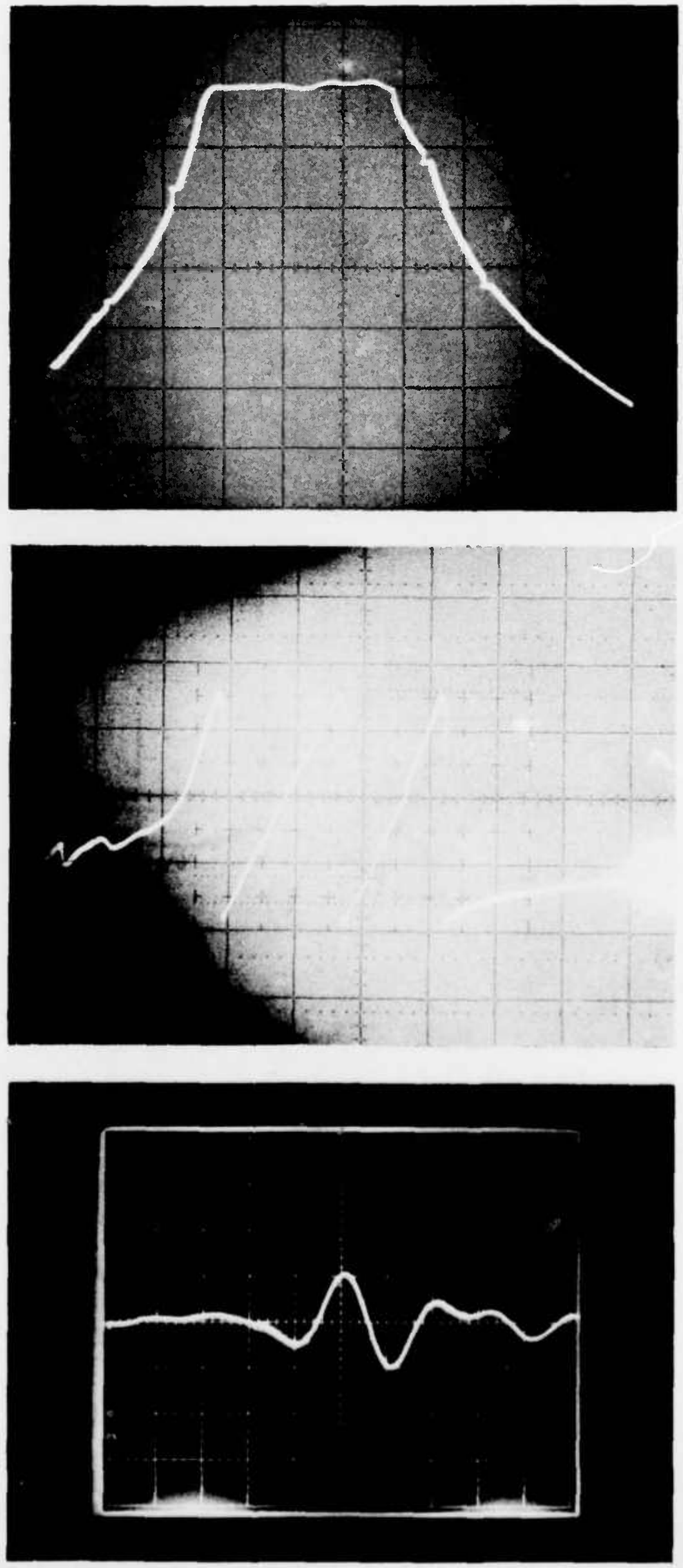


FIGURE A-37. 7-POLE CONSTANT-K HIGH-PASS RESPONSE



FREQUENCY

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FIGURE A-38. 12-POLE LERNER BANDPASS RESPONSE

FREQUENCY

PHASE

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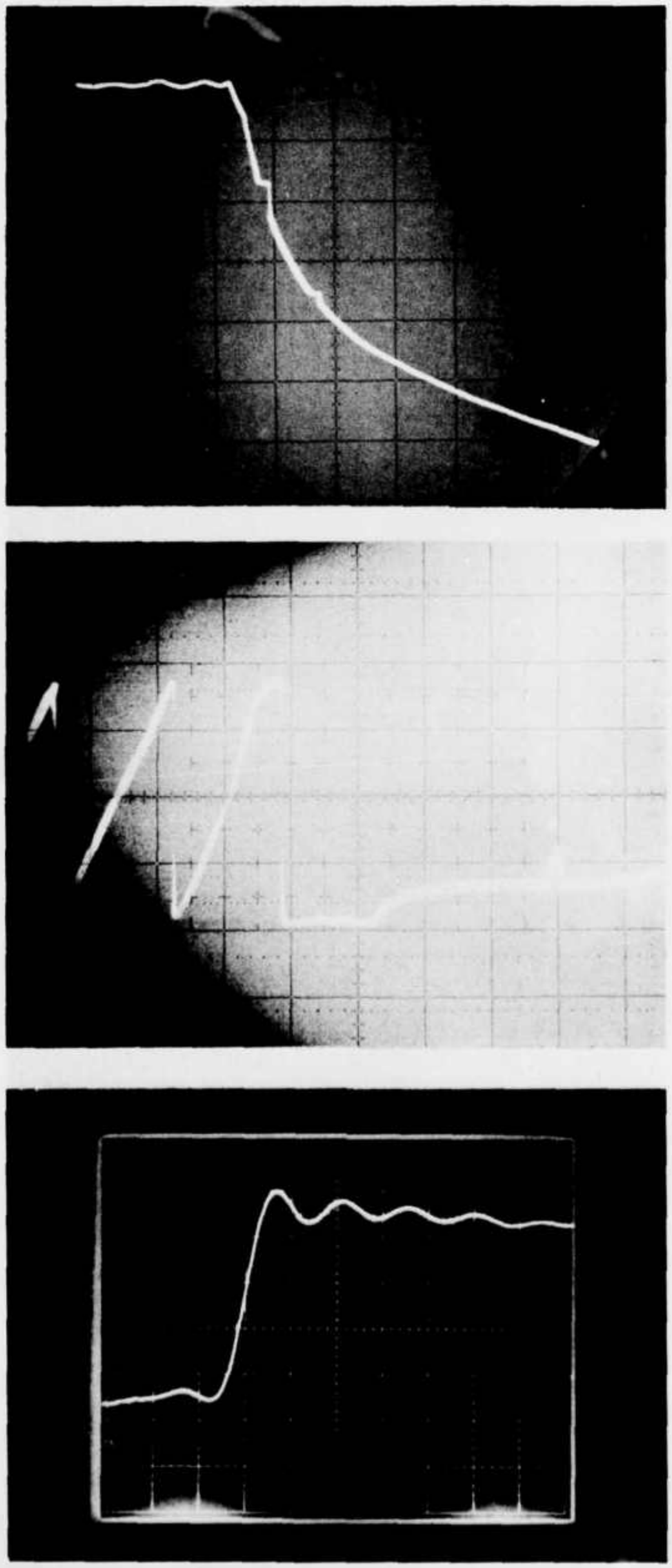
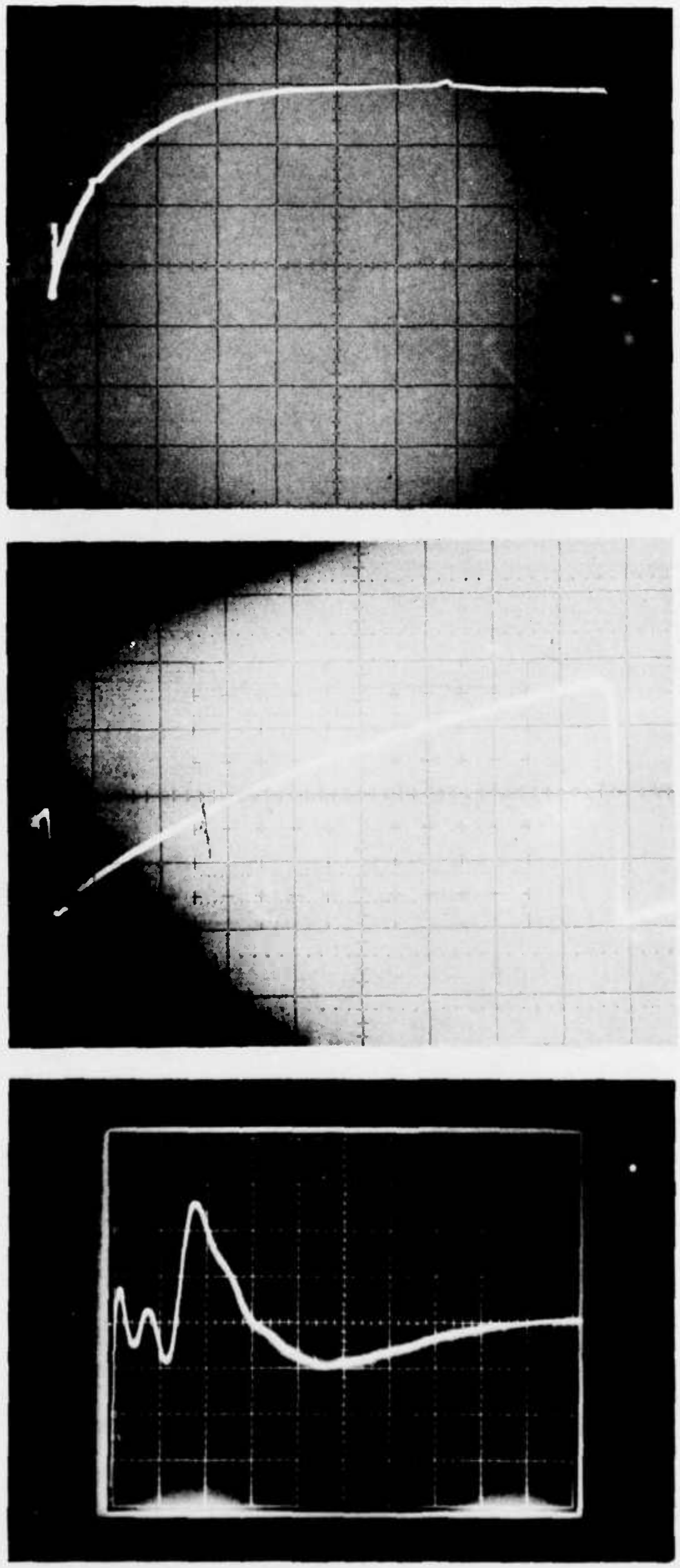


FIGURE A-39. 11-POLE LERNER LOW-PASS RESPONSE



FREQUENCY

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FIGURE A-40. 11-POLE LERNER HIGH-PASS RESPONSE

FREQUENCY

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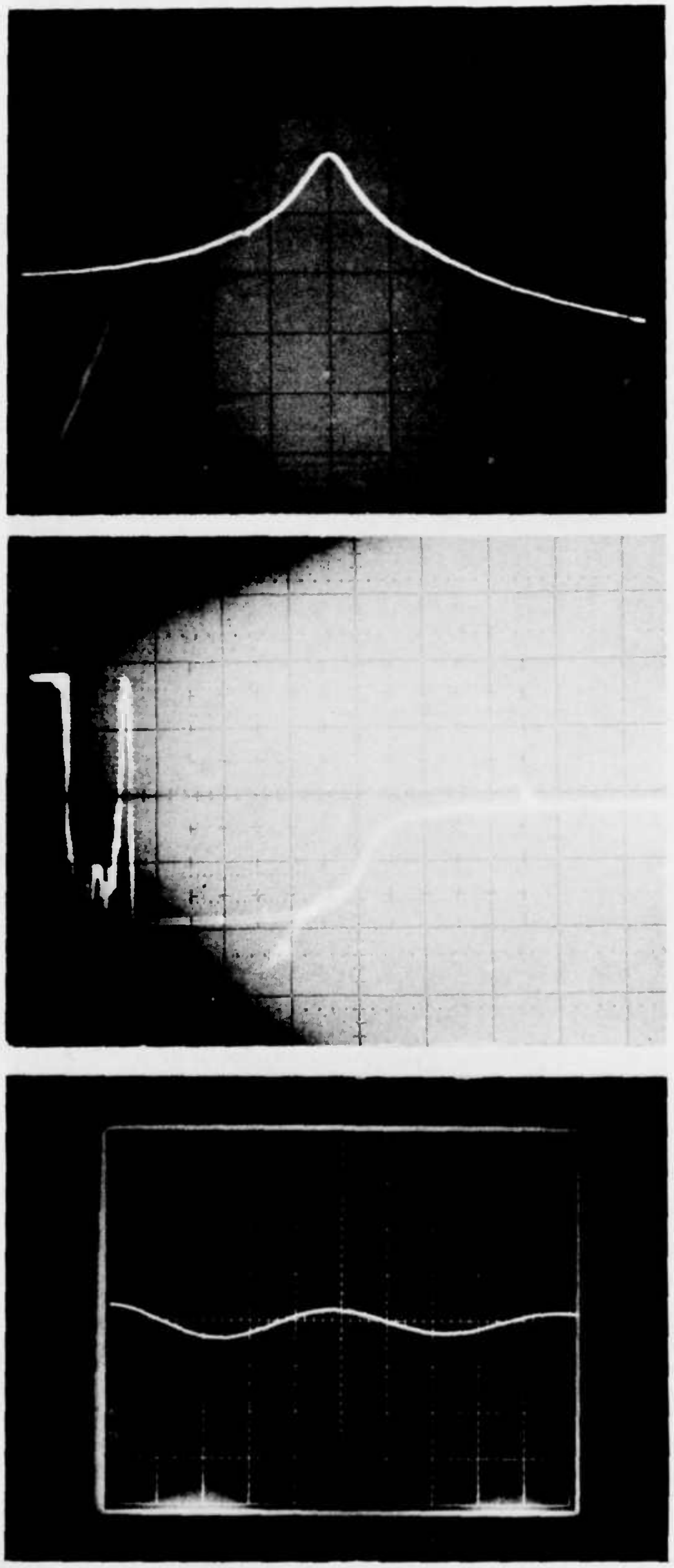

FIGURE A-41. Q-OF-10 NARROWBAND RESPONSE

FREQUENCY

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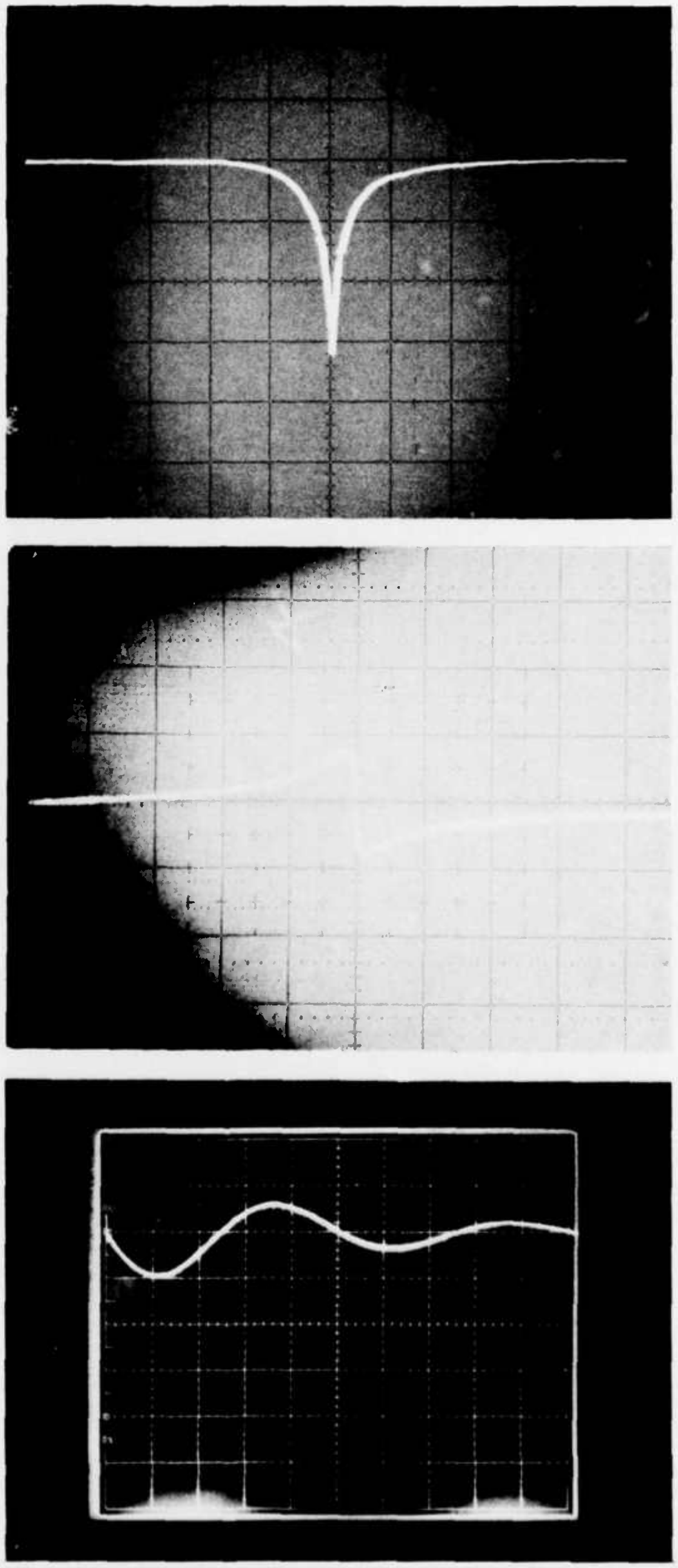

FIGURE A-42. Q-OF-3 NOTCH RESPONSE

FREQUENCY

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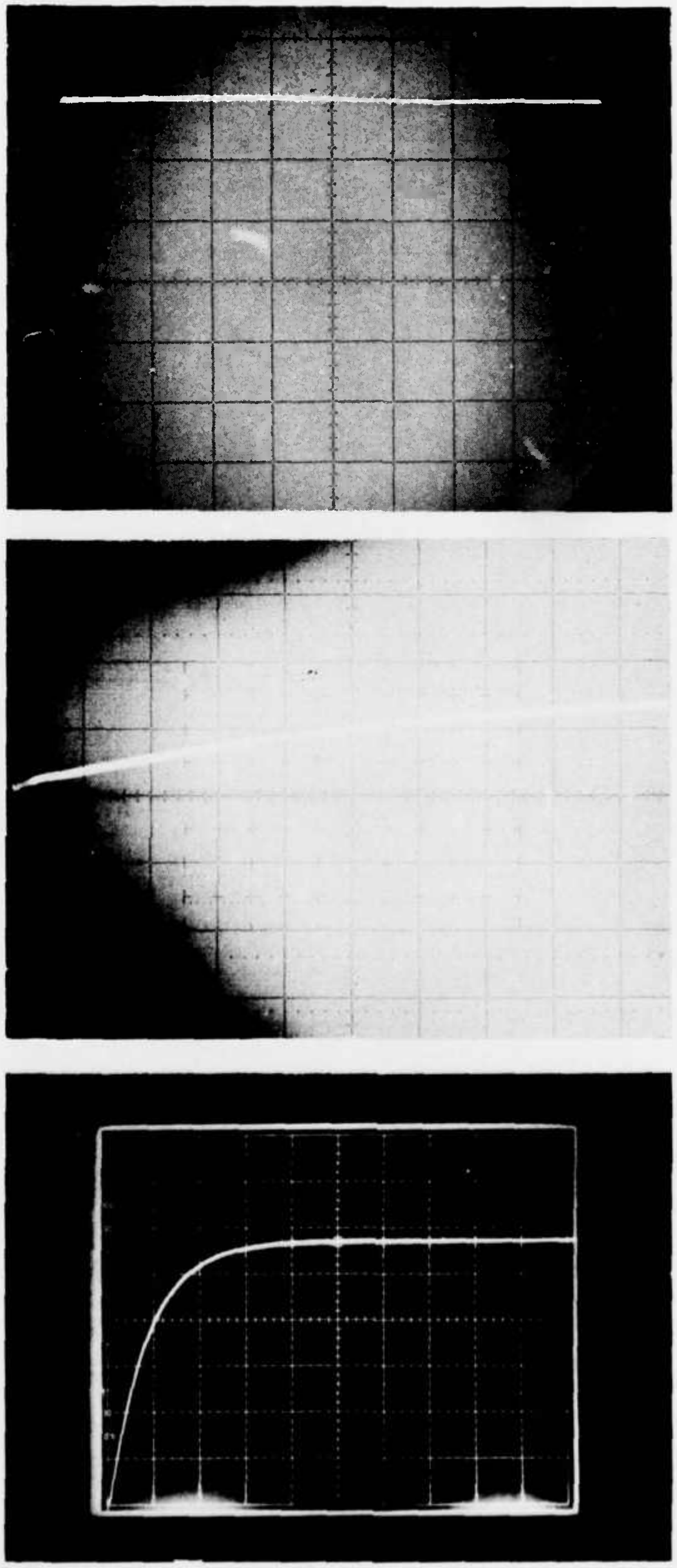

FIGURE A-43. 1-POLE 1-ZERO ALL-PASS RESPONSE

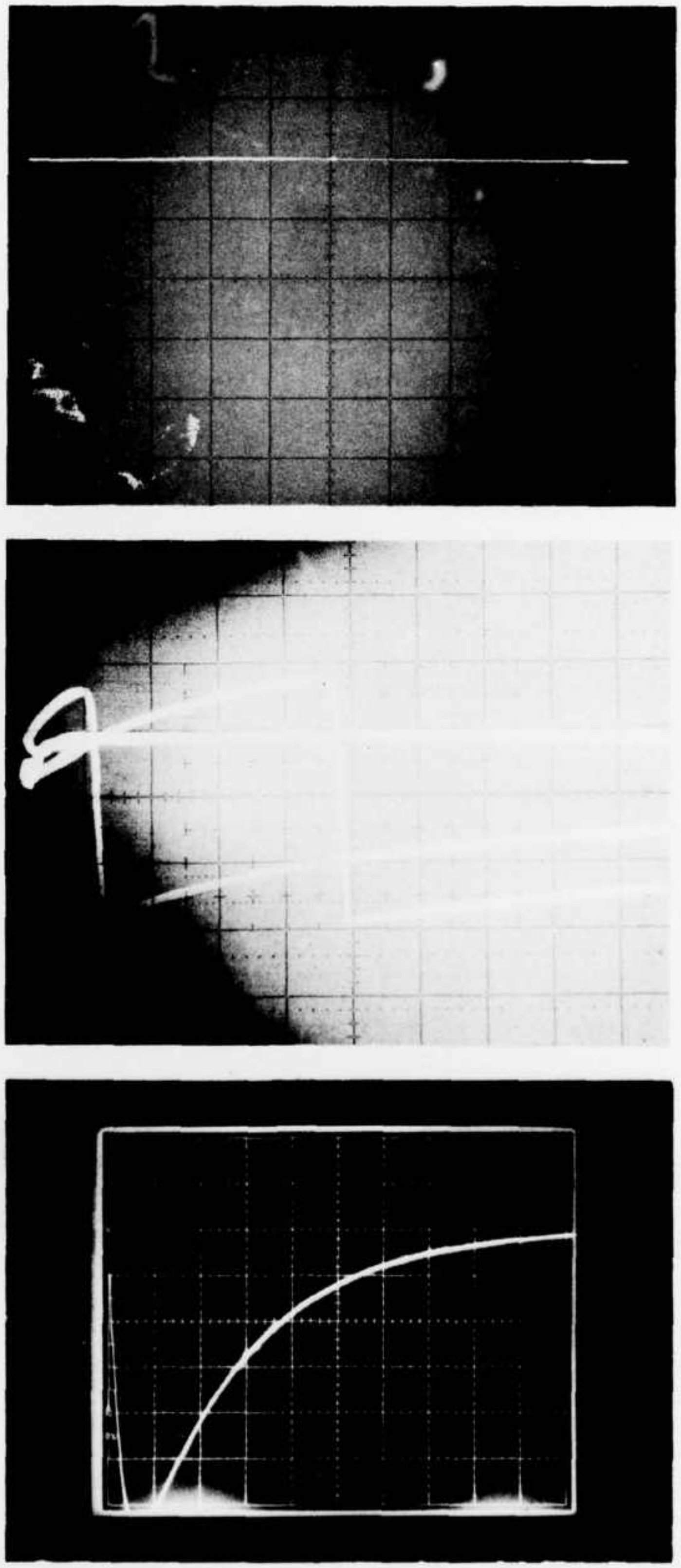

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FIGURE A-44. 2-POLE 2-ZERO 90° PHASE-DIFFERENCE NETWORK RESPONSE

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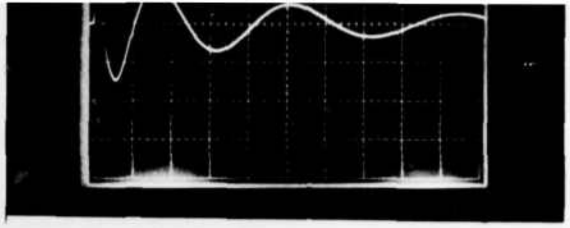

FIGURE A-37. 7-POLE CONSTANT-K HIGH-PASS RESPONSE

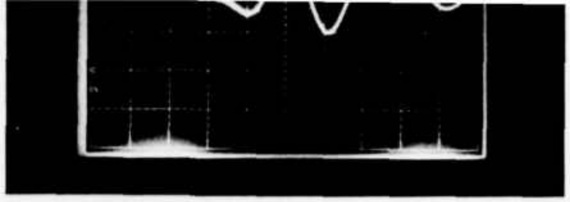

FIGURE A-38. 12-POLE LERNER BANDPASS RESPONSE

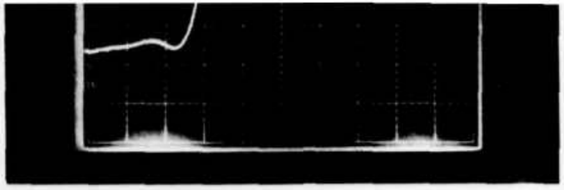

FIGURE A-39. 11-POLE LERNER LOW-PASS RESPONSE

FIGURE A-40. 11-POLE LERNER HIGH-PASS RESPONSE

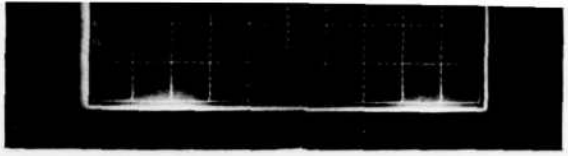

FIGURE A-41. Q-OF-10 NARROWBAND RESPONSE

FIGURE A-42. Q-OF-3 NOTCH RESPONSE

FIGURE A-43. 1-POLE 1-ZERO ALL-PASS RESPONSE

FIGURE A-44. 2-POLE 2-ZERO 90° PHASE-DIFFERENCE NETWORK RESPONSE