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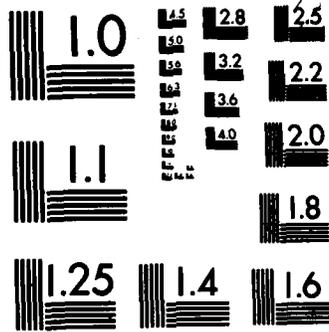
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# SOME PROBLEMS IN THE DESIGN OF THE TUBE WIND TUNNEL\*

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## ABSTRACT

The problem of the boundary layer growth in the charge tube is discussed. An analytical solution is derived and has been reduced into an algebraical expression. The result contains various factors which affect boundary layer growth. The calculation is simple and the results agree with experiments.

Based on F. L. Shope's model, analytical expressions for the test section starting process are derived. Various factors which affect the starting process are analyzed. The precision of calculation is adequate for design purposes. The calculating procedure is much simpler than F. L. Shope's.

## NOTATIONS

a speed of sound  
d diameter of tube  
L length  
M Mach number  
p pressure  
r radius  
T temperature  
t time  
u axial component of velocity  
V velocity of expansion wave or volume  
x axial coordinate  
y radial coordinate  
 $C_f$  coefficient of surface friction

Re Reynolds' number  
 $\gamma$  adiabatic index for gases  
 $\tau_v$  characteristic time of opening of valve

A area or coefficient

### Subscripts

3 parameter of central flow in gas storage tube  
4 state of stored gas  
e outward flow parameter of boundary layer  
t parameter of the test section  
w wall parameter  
p storage chamber parameter  
pe exhaust parameter of storage chamber

\* received in October 1981.

$\delta$	thickness of boundary layer	$f$	parameter of regulating plate
$\delta^*$	displacement thickness of boundary layer	$s$	state after starting time
$\theta$	momentum thickness of boundary layer	$e_0$	parameter at the stationary point of the flow
$\delta_f$	width of opening of the regulating plate		
$\rho$	density		

The tube wind tunnel has a simple structure and it is relatively easy to increase the pressure of the stored gas. As the ratio of the pressure at the stationary point of the flow in the test section to that of the stored gas is determined by the ratio of the area of the stored gas tube to that of the nozzle throat, one can increase the Reynolds' number of the flow in the test section by raising the pressure of the stored gas. Hence, sufficiently high Reynolds' numbers can be obtained with small equipment. The Reynolds' number can be further increased by combining this method with low temperature techniques.

Some special problems are associated with design of tube wind tunnels to be used as high-Reynolds' number transonic test equipment. We will discuss two of these problems in what follows:

1. Boundary layer growth in the gas storage tube

In the tube wind tunnel, a one-dimensional stable constant flow is created during the process of propagation of unstable non-constant expansion waves. Owing to the viscosity of the flow, a boundary layer is created along the tube wall, its thickness increasing with time. This boundary layer growth affects the effective flow duration and quality of flow in the tunnel.

This problem has been discussed by E. Becker [1], J. C. Sivells [2] and H. Ludwig [3]. All of them used complicated numerical methods. J. C. Sivells had to modify his results on the basis of experimental results. Our analysis consists of directly integrating the integral equation of the momentum of the unstable non-constant

boundary layer and reducing the analytical solution thus obtained to an algebraic expression. The axially symmetric integral equation of momentum takes the following form for cylindrical flows:

$$\frac{1}{u_e} \frac{\partial}{\partial t} (\delta^* - \delta_e) - \frac{\partial \theta}{\partial x} = \frac{1}{2} C_f \quad (1)$$

where

$$\left. \begin{aligned} \theta &= \int_0^{\delta^*} \left(1 - \frac{y}{r_0}\right) \frac{\rho u}{\rho_e u_e} \left(1 - \frac{u}{u_e}\right) dy \\ \delta^* &= \int_0^{\delta^*} \left(1 - \frac{y}{r_0}\right) \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy \\ \delta_e &= \int_0^{\delta^*} \left(1 - \frac{y}{r_0}\right) \left(1 - \frac{\rho}{\rho_e}\right) dy \end{aligned} \right\} \quad (2)$$

Let

$$\xi = Vt - x \quad (3)$$

Here,  $V$  is the velocity of the unstable non-constant expansion waves. Its relation with the speed of sound  $a_u$  in the stored gas state has already been derived by E. Becker:

$$\frac{V}{a_u} = 1 - \frac{\frac{1}{3}(\gamma + 1)M_3}{1 + \frac{(\gamma + 1)}{2}M_3} \quad (4)$$

$M_3$  is the Mach number corresponding to  $u_e$ .

Assume that  $\frac{\delta^* - \delta_e}{\theta}$  is independent of time, and we have

$$\left(1 + \frac{V}{u_e} \frac{\delta^* - \delta_e}{\theta}\right) \frac{d\theta}{d\xi} = \frac{1}{2} C_f \quad (5)$$

We use equation (5) to analyze the following factors:

(1) Assuming that  $C_f$  can be expressed as a power expansion in  $\theta$ , and that the velocity distribution can be expressed as a power expansion in  $\frac{y}{\delta^*}$ , we can derive from  $n \geq 7$ , and  $\delta/r_0 \ll 1$  the relation

$$\delta_e/\delta^* \ll 10/7$$

Here,  $n$  is the inverse of the index of the expansion of the velocity section.

(2) The effect of  $C_f$  is obvious. Different expressions have been given for  $C_f$  for different Reynolds' numbers [4]. Comparing

the formulas given by Blasius and Karman, the result obtained from one is more than twice that obtained from the other, for  $Re > 10^7$ .

(3) The effect of the width of the expansion waves is on the velocity  $V$  of the effective expansion waves. It can be seen from equation (4) that this effect can be neglected only for small  $M_3$ .

(4) The effect of the velocity section is expressed through  $n$ . Taking a two-dimensional slab as an example, we have approximately

$$\frac{\theta}{\delta} = \frac{n}{(n+1)(n+2)},$$

$n$  varies as  $Re$  [5].

In the process of solving equation (5), we have been able to take all of the above four factors into account. Calculations show that when  $n = 7-9$ , the error produced by the assumption that  $(\delta^* - \delta_e)/\theta$  is independent of time is approximately 4%, which is acceptable.

We take the following expression for the coefficient of friction. This is a combination of Karman's and Frankle, F. Voishel, V's (sic) equations.

$$C_f = \frac{(0.242)^2}{(\lg Re_0 + 1.1696)(\lg Re_0 + 0.3010)} \left(1 + \frac{\gamma-1}{2} M_3^2\right)^{-0.45} \quad (6)$$

Inserting this in equation (5), and integrating, we obtain, after rearrangement,

$$\delta = \frac{0.0293(Vt - x)}{\frac{\theta}{\delta} + \frac{V}{u_e} \frac{\delta^* - \delta_e}{\delta}} [\lg(2Re_0)]^{-2} \left(1 + \frac{\gamma-1}{2} M_3^2\right)^{-0.45} \quad (7)$$

After  $\frac{\theta}{\delta}$ ,  $\frac{\delta^*}{\delta}$  and  $\frac{\delta_e}{\delta}$  are calculated, the above equation can be written as

$$C = B(\delta/r_0) - A(\delta/r_0)^2 \quad (8)$$

where

$$\left. \begin{aligned} A &= \frac{n}{(2n+2)(2n+3)} \left(1 + \frac{2}{2n+1} \frac{T_c}{T_w}\right) + \frac{V}{u_e} \frac{n}{(2n+1)(2n+2)} \left(1 + \frac{1}{n} \frac{T_c}{T_w}\right) \\ B &= \frac{n}{(n+2)(n+3)} \left(1 + \frac{2}{n+1} \frac{T_c}{T_w}\right) + \frac{V}{u_e} \frac{n}{(n+1)(n+2)} \left(1 + \frac{2}{n} \frac{T_c}{T_w}\right) \\ C &= \frac{0.0293(Vz-x)}{r_0 [\lg(2Re_0)]^2 \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{0.427}} \end{aligned} \right\} \quad (9)$$

When we are calculating for the thickness of the boundary layer at the entrance of the nozzle,  $x = 0$ . If we let  $\frac{T_c}{T_w} = 1$ , then

$$\left. \begin{aligned} A &= \frac{n}{(2n+1)(2n+2)} + \frac{V}{u_e} \frac{1}{2(2n+1)} \\ B &= \frac{n}{(n+1)(n+2)} + \frac{V}{u_e} \frac{1}{n+1} \\ C &= \frac{0.0293Vz}{r_0 [\lg(2Re_0)]^2 \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{0.427}} \end{aligned} \right\} \quad (10)$$

The procedure for the calculation is very simple: given  $r_0$ ,  $M_3$ ,  $Re_d$  and  $a_4$ . Assume values for  $\frac{\delta}{r_0}$  and find  $Re_\delta$  and  $Re_\theta$ . From  $M_3$  find  $\frac{V}{u_e}$  and  $V$ . From  $n$  and  $\frac{V}{u_e}$ , find A and B. After C is found,  $\frac{\delta}{r_0}$  can be readily obtained. Figures 1 and 2 give the calculated results as compared with experimental results. The agreement is very good. Our computation has a fairly high accuracy, at least, for  $\frac{\delta}{r_0} < 0.75$

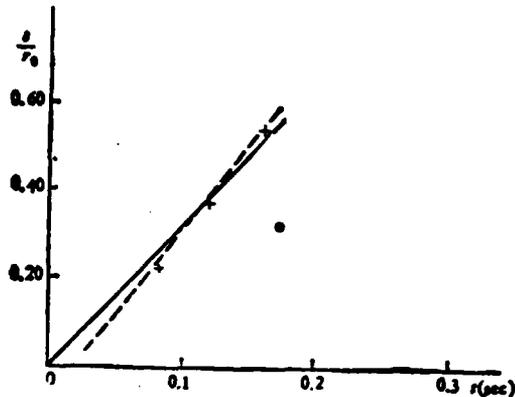


Figure 1. Comparison of calculated results with experimental results of AEDC  
 + experimental values for  $M_3=0.265$   $Re=3.0 \times 10^7$   
 ----J. C. Sivells calculation  
 ——our calculation

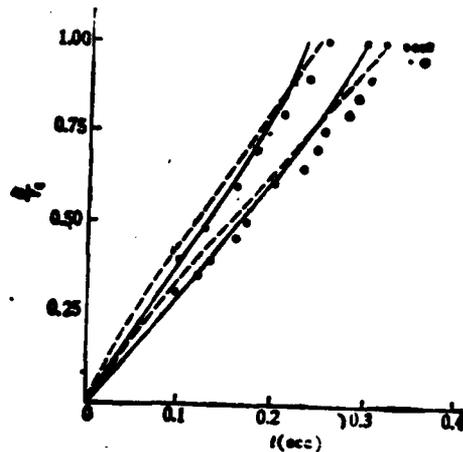


Figure 2. Comparison of calculated results with experimental results of Goettingen  
 □ experimental values for  $M_3=0.2$   $Re_d=5.1 \times 10^4$   
 o experimental values for  $M_3=0.3$   $Re_d=7.3 \times 10^4$   
 ----H. Ludwig's calculation  
 ——our calculation

2. Analysis of the starting process

The tube tunnel is an equipment for short-duration operations. The total process takes place within a time period on the order of  $10^2$  msec, while the starting time for the flow in the transonic test section to reach equilibrium can sometimes be on the same order of magnitude, and takes up a sizeable portion of the total time. To increase the effective operation time, one must shorten the starting time. Reducing the starting time, however, results in an increase

in the aerodynamic load on the wall of the test section and the regulating plate. Therefore, it is important to study the starting process, analyze the major factors and obtain simple and convenient computation methods.

We employ the method of concentrating parameters [6]. We found out from analysis that an analytical solution can be obtained by using an ordinary differential equation to describe the starting process. Under the conditions of supersonic flow, the equation of conservation of mass within the storage chamber is

$$\frac{d}{dt} \left( \frac{p_p}{p_{e0}} \right) + \frac{a_{e0}}{1.73} \frac{A_{pe}}{V_p} \left( \frac{p_p}{p_{e0}} \right) = \frac{a_{e0}}{1.73} \frac{A_e - A_t}{V_p} \quad (11)$$

The solution satisfying the initial conditions is

$$\frac{p_p}{p_{e0}} = \left( \mu_{e0} + \frac{A_e - A_t}{A_{pe}} \right) e^{-\frac{a_{e0}}{1.73} \frac{A_{pe}}{V_p} (t - t_0)} - \frac{A_e - A_t}{A_{pe}} \quad (12)$$

At the end of the starting process,  $p_p = A_1 p_t$ , and we obtain

$$\frac{a_{e0}}{1.73} \frac{A_{pe}}{V_p} (t_1 - t_0) = \ln \frac{\mu_{e0} + (A_e - A_t)/A_{pe}}{A_1 \mu_1 + (A_e - A_t)/A_{pe}} \quad (13)$$

In the above equation,  $\mu_{e0} = (p_p/p_{e0})|_{t=t_0}$ ,  $\mu_1 = (p_1/p_{e0})|_{t=t_1}$ ,  $A_1$  reflect the resistive effect of the wall.

When  $A_{pe} = 0$ , we have  $\frac{p_p}{p_{e0}} = \mu_{e0} - \frac{A_e - A_t}{A_{pe}} \frac{a_{e0}}{1.73} (t - t_0)$ . Except for  $A_e = A_t$ , there is no balance between the two sides of the above equation.

For transonic flows, there is no closed ordinary differential equation that can be used. On the basis of experimental and calculated results, we assume that [7]

$$\frac{p_1}{p_e} = \mu + (1 - \mu) e^{-kt}, \quad (14)$$

$$\left. \begin{aligned} \text{Let } k &= 1.24 \frac{a_{e0}}{A_{pe}} \left( \frac{A_e}{k_w} + \frac{A_t}{k_j} \right) \\ k' &= 1.24 \frac{a_{e0}}{A_{pe}} \left( \frac{A_e}{k_w} A_1 + \frac{A_t}{k_j} A_2 \right) \\ N &= (1 + K) \frac{a_{e0}}{1.73} \frac{A_{pe}}{V_p} \end{aligned} \right\} \quad (15)$$

The mass conservation equation in the storage chamber is

$$\text{Its solution is } \frac{d}{dt} \left( \frac{p_p}{p_s} \right) + N \frac{p_p}{p_s} = K' \frac{a_{\infty}}{1.73} \frac{A_{pe}}{V_p} [\mu + (1 - \mu)e^{-\theta t/\tau_s}]$$

$$\frac{p_p}{p_s} = \left( 1 - \frac{K'}{1+K} \mu - \frac{K'}{1+K} \frac{1-\mu}{1 - \frac{\theta}{N\tau_s}} \right) e^{-Nt}$$
(16)

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$$+ \frac{K'}{1+K} \mu + \frac{1-\mu}{1 - \frac{\theta}{N\tau_s}} \frac{K'}{1+K} e^{-\theta t/\tau_s}$$
(17)

When  $t = t_s, p_p = A_1 p_t$ , and equation (17) is a transcendental equation which can only be solved by numerical methods. However, for the case where  $\tau_s \rightarrow 0$  and the thin plate is used for starting, or for the case where  $\tau_s \rightarrow \infty$  and the valve is used for starting, we can obtain, respectively

$$(1+K) \frac{a_{\infty}}{1.73} \frac{A_{pe}}{V_p} t_s = \ln \frac{1 - [K'/(1+K)]\mu}{\mu[A_1 - K'/(1+K)]}$$
(18)

and

$$(1+K) \frac{a_{\infty}}{1.73} \frac{A_{pe}}{V_p} t_s = \ln \frac{1 - [K'/(1+K)]}{A_1 \mu - K'/(1+K)}$$
(19)

To obtain meaningful results, we must place restrictions on  $K'/(1+K)$ . This places restrictions on the relations among  $A_f, A_w$  and  $A_{pe}$ . In other words, the geometrical parameters of the storage chamber are interrelated.

Similar equations and relations hold for the case where  $A_{pe} = 0$ . Analysis shows that only when  $A_f$  is large and there is sufficiently strong induced emission in the pressure expansion section will the equipment start smoothly.

From the above analysis, we know that the major factors affecting the starting process are  $A_{pe}/A_1, V_p/V_1, A_1/A_2, A_2/A_3$  and  $L_1/a_1$ . With respect to the tube wind tunnel, the auxiliary exhaust is very important, and the duration and frequency of opening the exhaust has a very large effect on the starting load.  $A_2 - A_1 \ll A_{pe}$  is a condition favorable for the flow.

We performed calculations on several published sets of data for AEDC model tube wind tunnels using our analytical method of solution. The method is easy and convenient to apply and except for certain special cases, the results obtained agree very well with those obtained via numerical methods. The discrepancies are generally 10% or less. For example, for the case of starting with the value where  $M_\infty = 1.10$ , the AEDC value  $t_s = 76$  ms, while the analytical solution gives  $t_s = 80$  ms.

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SIMPLIFIED NAVIER-STOKES EQUATIONS AND COMBINED SOLUTION  
OF NON-VISCOUS AND BOUNDARY LAYER EQUATIONS\*

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ABSTRACT

This paper presents a part of the technical report [2] in which the author studied the simplified Navier-Stokes equations and the combined solution of nonviscous and boundary layer equations. From the full Navier-Stokes equations and an analysis of the combined solution of nonviscous and boundary layer equations, simplified Navier-Stokes equations were worked out. A perturbation analysis which differs slightly from the match-perturbation expansions of inner-outer layers developed by Van Dyke [1] shows that the solution of the simplified Navier-Stokes equations is uniformly valid with accuracy of  $O(\text{Re}_\infty^{-1/2})$  in the whole flow field, where  $\text{Re}_\infty = \frac{\rho_\infty U_\infty L}{\mu_\infty}$ ,  $\rho_\infty$  is the density of free stream,  $U_\infty$  the x-component of velocity,  $L$  the characteristic length,  $\mu_\infty$  the dynamic viscosity of free stream.

The simplified N-S equations possess the properties of parabolic-hyperbolic equations. Under equilibrium conditions, it is much easier to use the method of the forward-progressing calculation to solve the simplified N-S equations than to solve the full elliptical N-S equations by means of numerical methods. While solving the simplified N-S equations, one simultaneously obtains the non-viscous external flow as well as the viscous boundary layer flow. Theoretically, this is superior to the conventional procedure of first computing for the non-viscous flow and then computing for the viscous boundary layer flow. With respect to many types of flow fields, the simplified N-S equations can realistically reflect their mechanical

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\* received in May 1981

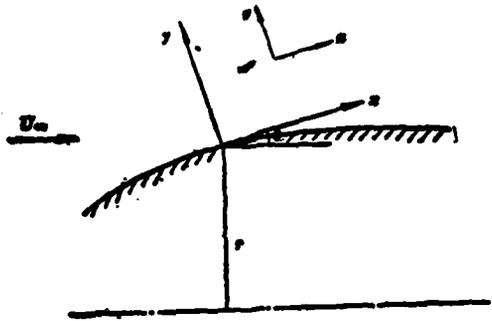


Figure 1

aspects. For instance, one can use these equations to accurately calculate for the complex flow diagram of the interaction among viscous boundary layer, the high-entropy layer and the non-viscous external flow of hypersonic diffracted flow fields.

Methods for solving the simplified N-S equations are currently under research. In fact, satisfactory answers have not yet been found for problems regarding the mathematical properties and stability of these equations, the correct way of presenting the Cauchy problem, and the degree of accuracy that can be reached by the flow calculations. Besides, different authors hold different points of view. This paper presents part of the technical report [2] the author gave in 1967 in which he obtained the simplified N-S equations from an analysis of the combined solution of non-viscous flow and viscous boundary layer equations, and used the perturbation method to show that one can obtain from that set of equations a solution that is accurate to the order  $O(\text{Re}_\infty^{-1/2})$ . It should be pointed out here that the author later became aware of a similar derivation of the simplified equations, given in [3].

### 1. Simplified N-S equations

Let  $x$  and  $y$  be the orthogonal coordinates along the wall and perpendicular to the wall, respectively (see Figure 1). Let  $u$  and  $v$  be the corresponding velocity components and  $\rho$ ,  $p$ ,  $T$ ,  $\mu$  and  $\lambda$  be the density, the pressure, the temperature, the viscosity coefficient and the coefficient of thermal conduction, respectively. Take the parameters  $U_\infty$  and  $\rho_\infty U_\infty^2$  of the oncoming stream to be the characteristic values of velocity and pressure, respectively, and the fixed wall length  $L$  to be the characteristic value of length. Assume that the order of magnitude of  $\rho$  and  $\mu$  can be estimated from  $\rho_\infty$  and  $\mu_\infty$ . From the standpoint of non-viscous external flow and that of viscous boundary layer flow, we make an estimate of the various terms in the full N-S equations as follows.

$$\frac{\rho}{H} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\lambda \mu}{H} = \frac{-1}{\rho H} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{\pi \mu}{\rho} \frac{\partial u}{\partial y} - \frac{\lambda \mu}{\rho} \frac{\partial}{\partial y} \left( \frac{\mu}{H} \right) + \theta,$$

	$\frac{U_\infty}{L}$	$\frac{U_\infty}{L}$	$\frac{U_\infty}{L}$	$\frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$
non-viscous flow	$\frac{U_\infty}{L}$	$\frac{U_\infty}{L}$	$\frac{U_\infty}{L}$	$\frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$
boundary layer flow	$\frac{U_\infty}{L}$	$\frac{U_\infty}{L}$	$\frac{\delta U_\infty}{L^2}$	$\frac{U_\infty}{L}$	$\text{Re}_\infty^{-1} \frac{L U_\infty^2}{\delta^2}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{\delta}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{\delta}$	$\text{Re}_\infty^{-1} \frac{U_\infty}{L}$

(1.1)

$$\begin{array}{l}
\frac{u}{H} \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - \frac{k u^2}{H} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{4}{3\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right) + \frac{1}{\rho H} \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial y} \right) + j \frac{\sin \theta}{\rho H} \mu \frac{\partial u}{\partial y} + \Phi_n \\
\text{non-viscous flow } \frac{U_0^2}{L} \quad \frac{U_0^2}{L} \quad \frac{U_0^2}{L} \quad \frac{U_0^2}{L} \quad \text{Re}_0^{-1} \frac{U_0^2}{L} \quad \text{Re}_0^{-1} \frac{U_0^2}{L} \quad \text{Re}_0^{-1} \frac{U_0^2}{L} \quad \text{Re}_0^{-1} \frac{U_0^2}{L} \\
\text{boundary layer flow } \frac{\delta U_0^2}{L^2} \quad \frac{\delta U_0^2}{L^2} \quad \frac{U_0^2}{L} \quad \text{Re}_0^{-1} \frac{U_0^2}{\delta} \quad \text{Re}_0^{-1} \frac{U_0^2}{\delta} \quad \text{Re}_0^{-1} \frac{U_0^2}{\delta} \ll \text{Re}_0^{-1} \frac{U_0^2}{L}
\end{array}
\tag{1.2}$$

Here  $x = j \frac{\cos \theta}{r} + \frac{k}{H}$ ,  $j = 0$  (two-dimensional flow), 1 (axially symmetric flow);  $H = 1 + ky$ , where  $k$  is the curvature of the wall surface;  $\text{Re}_0 = \frac{\rho U_0 L}{\mu}$ ,  $\frac{\delta}{L} = O(\text{Re}_0^{-1/2})$  where  $\delta$  is the thickness of the boundary layer;  $\Phi_t$  and  $\Phi_n$  represent all the terms in the tangential and normal momentum equations, respectively, that are of order of magnitude equal to or less than  $O\left(\text{Re}_0^{-1} \frac{U_0^2}{L}\right)$ . We discuss the case for  $\text{Re}_0 \gg 1$  as follows:

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1) For  $y > \delta$ , omitting the terms in the full N-S equations that are of order  $O\left(\text{Re}_0^{-1} \frac{U_0^2}{L}\right)$  we obtain the Euler equations. For  $y < \delta$ , we omit the terms in the full N-S equations that are of order equal to or less than  $O\left(\text{Re}_0^{-1/2} \frac{U_0^2}{L}\right)$  and obtain Prandtl's boundary layer equations.

2) When solving the Euler's equations and the boundary layer equations simultaneously, a mathematical singularity exists on the boundary line  $y = \delta$ . The asymptotic value of the solution of the Euler's equations as  $y(>\delta) \rightarrow \delta$  will not agree completely with that of the solution of the boundary layer equations as  $y(<\delta) \rightarrow \delta$ . The actual order of magnitude of this singularity is  $O\left(\text{Re}_0^{-1/2} \frac{U_0^2}{L}\right)$ . One should note that as  $y_A \xrightarrow{(>\delta)} \delta$ , the various inertial terms in the normal Euler equation are also of the order of magnitude  $O\left(\text{Re}_0^{-1/2} \frac{U_0^2}{L}\right)$ .

3) Therefore, in order to eliminate the singularity that arises in the simultaneous solution of the non-viscous external flow and the viscous boundary layer flow, we should omit all terms in the  $y < \delta$ , full N-S equations that are of order equal to or less than  $O\left(\text{Re}_0^{-1} \frac{U_0^2}{L}\right)$ . The resulting equations are the simplified N-S equations. Solving these equations simultaneously with the Euler equations, we can remove the singularity and obtain solutions with accuracies on the order of  $O\left(\text{Re}_0^{-1/2} \frac{U_0^2}{L}\right)$  that are uniformly valid throughout the entire

flow field.

As described above, the simplified N-S equations are obtained by neglecting all terms in the full N-S equations that are of order equal to or less than  $O\left(\text{Re}^{-1} \frac{U_0^2}{L}\right)$  as estimated from the standpoint of the boundary layer. These equations are given below (along with the continuity and energy equations).

$$\frac{\partial}{\partial x} (\rho u r) + \frac{\partial}{\partial y} (\rho v H r) = 0 \quad (1.3)$$

$$\rho \left( \frac{u}{H} \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{k u v}{H} \right) = - \frac{1}{H} \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + \mu \frac{\partial u}{\partial y} - k u \frac{\partial}{\partial y} \left( \frac{\mu}{H} \right) \quad (1.4)$$

$$\rho \left( \frac{u}{H} \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - \frac{k v^2}{H} \right) = - \frac{\partial p}{\partial y} + \frac{4}{3} \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right)$$

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$$+ \frac{1}{H} \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial y} \right) + \frac{\mu \sin \theta}{r H} \frac{\partial u}{\partial y} \quad (1.5)$$

$$\rho c_p \left( \frac{u}{H} \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) - \left( \frac{u}{H} \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} \right) - \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \mu \left( \lambda \frac{\partial T}{\partial y} + \mu \frac{\partial T}{\partial y} \right) + \mu \Phi$$

where

$$\Phi = \left( \frac{\partial u}{\partial y} \right)^2 - \frac{\partial u}{\partial y} - \frac{2 k u}{H} \frac{\partial u}{\partial y} + k u^2 \frac{\partial}{\partial y} \left( \frac{\mu}{H} \right) \quad (1.6)$$

$c_p$  is the specific heat under constant pressure. The simplified N-S equations (1.3)-(1.6) do not contain  $\frac{\partial^2}{\partial x^2}$  terms. When  $\mu \neq 0$ , they have four redundant characteristic values and two non-zero real characteristic values. Therefore, these equations possess the characteristics of parabolic-hyperbolic equations [4] and the method of forward-advancing calculation can be applied under equilibrium conditions. It should also be noted that in the simplified N-S equations, when  $y(<\delta) \rightarrow \delta$ ,  $\frac{\partial}{\partial y}$  from  $O(\text{Re}^{-1}) \rightarrow O(1)$  and the order of magnitude of all viscous terms become  $O\left(\text{Re}^{-1} \frac{U_0^2}{L}\right)$ . Therefore, the simplified N-S equations, accurate to the order  $O\left(\text{Re}^{-1/2} \frac{U_0^2}{L}\right)$ , will smoothly change into a set of Euler equations for  $y \geq \delta$ . In other words, to obtain the solutions for the entire field that are accurate

to the order  $O\left(\text{Re}^{-1/2} \frac{U_0^2}{L}\right)$ , it is not necessary to solve the simplified N-S equations simultaneously with the Euler's equations. In principle, it will be sufficient to solve only the simplified N-S equations, given the initial and boundary values, i.e., the properties of the oncoming stream (or shockwave) and the wall surface.

## 2. Discussion on accuracy

A uniformly valid and consistent solution on the order of  $O\left(\text{Re}^{-1/2} \frac{U_0^2}{L}\right)$  can be obtained when one applies the simplified N-S equations to the computation of the entire flow field, including the non-viscous external flow and the viscous boundary layer. In order to prove the validity of this statement, we need to make use of a perturbation expansion that is slightly different from the usual match-perturbation-expansions of the inner-outer layers [1]. Within the viscous boundary layer  $0 \leq \gamma \leq \delta$ , the solution of the full N-S equations (1.1) and (1.2) can be expanded in terms of  $\epsilon = \text{Re}^{-1/2}$  as follows:

$$\left. \begin{aligned} u/U_0 &= u_{10} + \epsilon u_{11} + \epsilon^2 u_{12} + \dots \\ v/sU_0 &= v_{10} + \epsilon v_{11} + \epsilon^2 v_{12} + \dots \\ \rho/\rho_0 &= \rho_{10} + \epsilon \rho_{11} + \epsilon^2 \rho_{12} + \dots \\ \mu/\mu_0 &= \mu_{10} + \epsilon \mu_{11} + \epsilon^2 \mu_{12} + \dots \\ [\rho(X, Y) - \rho_0(X)]/\epsilon \rho_0 U_0^2 &= \rho_{10} + \epsilon \rho_{11} + \epsilon^2 \rho_{12} + \dots \end{aligned} \right\} \quad (2.1)$$

in the above  $X = x/L$ ,  $Y = y/sL$ ,  $\epsilon = \text{Re}^{-1/2}$ ,  $\frac{\partial}{\partial Y} = O(1)$ ;  $u_{10}, u_{11}, \dots$

and the quantities  $u_{10}, u_{11}, \dots$  and their partial derivatives with respect to  $X$  and  $Y$  are all of order  $O(1)$ . Substituting the expansions in equation (2.1) into the N-S equations (1.1) and (1.2) and comparing terms in the same power of  $\epsilon$ , we obtain

$$\begin{aligned} O(\epsilon^0) \quad \rho_{10} \left( u_{10} \frac{\partial u_{10}}{\partial X} + v_{10} \frac{\partial u_{10}}{\partial Y} \right) &= - \frac{d\bar{p}_0(X)}{dX} + \frac{\partial}{\partial Y} \left( \mu_{10} \frac{\partial u_{10}}{\partial Y} \right) \\ & \quad k_1 \rho_{10} u_{10}^2 = \frac{\partial \rho_{10}}{\partial Y} \end{aligned} \quad (2.2)$$

$$O(\epsilon^1) \quad \rho_{10} \left[ \frac{\partial(u_{10} u_{11})}{\partial X} + v_{10} \frac{\partial u_{11}}{\partial Y} + u_{10} \frac{\partial v_{10}}{\partial Y} + k_1 u_{10} v_{10} \right] + \rho_{11} \left( u_{10} \frac{\partial u_{10}}{\partial X} + v_{10} \frac{\partial u_{10}}{\partial Y} \right)$$

$$\begin{aligned}
& -k_1 Y \left( \rho_{10} \mu_{10} \frac{\partial u_{10}}{\partial X} + \frac{d\bar{p}_w(X)}{dX} \right) - \frac{\partial p_{10}}{\partial X} + \frac{\partial}{\partial Y} \left( \mu_{10} \frac{\partial u_{11}}{\partial Y} + \mu_{11} \frac{\partial u_{10}}{\partial Y} \right) \\
& + \mu_{10} \frac{\partial u_{10}}{\partial Y} - k_1 \mu_{10} \frac{\partial \mu_{10}}{\partial Y} \\
\rho_{10} \left( u_{10} \frac{\partial v_{10}}{\partial X} + v_{10} \frac{\partial v_{10}}{\partial Y} - 2k_1 \mu_{10} u_{11} + k_1 Y u_{10}^2 \right) - \rho_{11} k_1 u_{10}^2 - \frac{\partial p_{11}}{\partial Y} & \quad (2.3) \\
& + \frac{4}{3} \frac{\partial}{\partial Y} \left( \mu_{10} \frac{\partial v_{10}}{\partial Y} \right) + \frac{\partial}{\partial X} \left( \mu_{10} \frac{\partial u_{10}}{\partial Y} \right) + j \frac{\sin \theta}{r_1} \mu_{10} \frac{\partial u_{10}}{\partial Y}
\end{aligned}$$

In the above,  $k_1 = kL$ ,  $r_1 = r/L$ ,  $\bar{p}_w(X) = p_w(X)/\rho_\infty U_\infty^2$ ; . Similar expansions can be obtained for the continuity equation and the energy equation. It should be pointed out that in the present expansion, the first-order equations (2.2) of order  $O(1)$  and the second-order equations (2.3) of order  $O(\epsilon)$  of the N-S equations for  $y \leq \delta$  are different from those obtained from the usual match-perturbation-expansions of inner outer layers [1]. In the usual expansions, the first and second order equations for the normal momentum for  $y \leq \delta$  are, respectively:

$$\begin{aligned}
\frac{\partial p_{10}}{\partial Y} &= 0 \\
\frac{\partial p_{11}}{\partial Y} &= k_1 \rho_{10} u_{10}^2
\end{aligned}$$

Hence, in the usual expansions, a mathematical singularity of order  $O\left(\text{Re}^{-1/2} \frac{U_\infty^2}{L}\right)$  exists on the boundary  $y = \delta$  when either the first order equations or the second order equations of the inner and outer layers are solved simultaneously.

In the non-viscous external flow region where  $y > \delta$ , the solution of the N-S equations (1.1) and (1.2) can be expanded in terms of  $\epsilon$  as follows:

$$\left. \begin{aligned}
u/U_\infty &= u_0 + \epsilon u_1 + \epsilon^2 u_2 + \dots \\
v/U_\infty &= v_0 + \epsilon v_1 + \epsilon^2 v_2 + \dots \\
\rho/\rho_\infty &= \rho_0 + \epsilon \rho_1 + \epsilon^2 \rho_2 + \dots \\
p(X, Y, \epsilon)/\rho_\infty U_\infty^2 &= p_0 + \epsilon p_1 + \epsilon^2 p_2 + \dots
\end{aligned} \right\} \quad (2.4)$$

In the above equations  $Y_e = y/L$ , and the quantities  $u_{e0}$ ,  $u_{e1}$ , ... and their corresponding partial derivatives with respect to  $X$  and  $Y_e$  are all of order  $O(1)$ . Substituting (2.4) into equations (1.1) and (1.2), and comparing terms in the same power of  $\epsilon$ , we obtain

$$O(\varepsilon^0) \left. \begin{aligned} \rho_\infty \left( \frac{u_\infty}{H} \frac{\partial u_\infty}{\partial X} + v_\infty \frac{\partial u_\infty}{\partial Y} + \frac{k u_\infty v_\infty}{H} \right) - - \frac{1}{H} \frac{\partial p_\infty}{\partial X} \\ \rho_\infty \left( \frac{u_\infty}{H} \frac{\partial v_\infty}{\partial X} + v_\infty \frac{\partial v_\infty}{\partial Y} - \frac{k u_\infty^2}{H} \right) - - \frac{\partial p_\infty}{\partial Y} \end{aligned} \right\} \quad (2.5)$$

$$O(\varepsilon^1) \left. \begin{aligned} \rho_\infty \left[ \frac{1}{H} \frac{\partial (u_\infty u_\infty)}{\partial X} + v_\infty \frac{\partial u_\infty}{\partial Y} + v_\infty \frac{\partial u_\infty}{\partial Y} + \frac{k}{H} (u_\infty v_\infty + u_\infty v_\infty) \right] \\ + \rho_\infty \left( \frac{u_\infty}{H} \frac{\partial u_\infty}{\partial X} + v_\infty \frac{\partial u_\infty}{\partial Y} + \frac{k u_\infty v_\infty}{H} \right) - - \frac{1}{H} \frac{\partial p_\infty}{\partial X} \\ \rho_\infty \left[ \frac{u_\infty}{H} \frac{\partial v_\infty}{\partial X} + \frac{u_\infty}{H} \frac{\partial v_\infty}{\partial X} + \frac{\partial}{\partial Y} (v_\infty v_\infty) - \frac{2k u_\infty u_\infty}{H} \right] \\ + \rho_\infty \left( \frac{u_\infty}{H} \frac{\partial v_\infty}{\partial X} + v_\infty \frac{\partial v_\infty}{\partial Y} - \frac{k u_\infty^2}{H} \right) - - \frac{\partial p_\infty}{\partial Y} \end{aligned} \right\} \quad (2.6)$$

It can be readily seen that in the region  $y > \delta$  of non-viscous flow, the  $O(1)$  approximation of the N-S equations, i.e., the set of first-order equations (2.5), is the Euler equations, while the approximation accurate to  $O(\varepsilon)$  i.e., the second-order equations (2.6), is a modified solution that takes into account the effect of the displacement thickness of the boundary layer. Solving equations (2.2) and (2.5) simultaneously; one can obtain a solution accurate to  $O(1)$  that is uniformly valid in the entire flow field. Solving equations (2.2), (2.3) and (2.5), (2.6) simultaneously via match-perturbation-expansions, one can obtain a solution accurate to  $O(Re^{-1/2})$  that is uniformly valid in the entire flow field. 610

Comparing the simplified N-S equations (1.4) and (1.5) as well as equations (2.2) and (2.3) with equations (2.5) and (2.6), one can prove that the solution to the simplified N-S equations satisfies the following relation, taking  $u$  as an example

$$\left. \begin{aligned} y < \delta & \quad \left| \frac{u}{U_\infty} - (u_\infty + \varepsilon u_\infty) \right| < O(\varepsilon^2 - Re_\infty^{-1}) \\ y > \delta & \quad \left| \frac{u}{U_\infty} - (u_\infty + \varepsilon u_\infty) \right| < O(Re_\infty^{-1}) \end{aligned} \right\} \quad (2.7)$$

Equation (2.7) holds for the other variables as well. Hence, the solution to the simplified N-S equations is accurate to  $O(Re_\infty^{-1/2})$  and is uniformly valid in the entire flow field.

### 3. Conclusion

The actual form taken by the simplified N-S equations will vary slightly, depending on whether the viscous terms of order  $O(Re^{-1/2})$  are neglected or not. In general, such variations are considered small [3]-[7]. Nevertheless, to obtain a solution that is accurate to the order  $O(Re^{-1/2})$  and uniformly valid in the entire flow field, it would probably be advisable to use the simplified N-S equations (1.3)-(1.6) presented by the author [2].

The simplified N-S equations can be used to obtain an accurate description of the complex interference phenomenon among the non-viscous flow, the high-entropy layer and the boundary layer flow in the shockwave layer [3]-[7]. They can also be applied to other flow field calculations, such as non-equilibrium flow, flow in a tube, distant wake flow, shock-boundary layer interference and the viscous diffracted flow near pointed head and blunt head three-dimensional bodies. The simplified N-S equations have also been applied to the computation of such complex flow fields as segregated flow, near wake flow and compressed flow near corners. One should note, however, that the simplified N-S equations possess the properties of parabolic-hyperbolic equations only when the viscous effect is dominant, i.e., when  $\mu \neq 0$ . For  $y > \delta$  where one can neglect the viscous effect, i.e.,  $\mu = 0$ , these equations become the Euler's equations with corresponding characteristic roots

$$\lambda_1 = \lambda_2 = H \frac{\rho}{n}, \quad \lambda_{3,4} = H \frac{u \pm c \sqrt{M^2 - 1}}{n^2 - C^2}$$

Here  $C$  is the speed of sound and  $M$  is Mach number. For  $M > 1$ ,  $\lambda_{3,4}$  are real characteristic roots belonging to the hyperbolic family, and the simplified N-S equations retain the same mathematical classification. For  $M < 1$ ,  $\lambda_{3,4}$  are complex characteristic roots, belonging to the elliptic family, and there is a change in the mathematical classification of the simplified N-S equations. In the region where the simplified N-S equations are elliptic equations, the Cauchy problem is not applicable. It is, therefore, necessary to obtain the solution by an iterative method or some other special treatment.

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