



MICROCOPY RESOLUTION TEST CHART NATIONAL BUREAU OF STANDARDS-1963-A

	ARO 13605,14-MA	
Unclassified	16465, 3-MA	
SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)		
REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM	6-
	N NO. 3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitio)	S. TYPE OF REPORT & PERIOD COVERED	\smile
Techniques for Modeling Stochastics	Final Report	
Dynamical Systems	4 Feb. 1976 - 1 Jan. 1983 5. PERFORMING ORG. REPORT NUMBER	
AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(+)	
R. W. Brockett	Grant DAAG29-76-G-0139 Grant DAAG29-79-C-0147	
PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
Division of Applied Sciences Harvard University		
CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE	
U. S. Army Research Office Post Office Box 12211	May 26, 1983	
Research Triangle Park, NC 27709	13. NUMBER OF PAGES 7	
MONITORING AGENCY NAME & ADDRESS(I dillerant from Controlling Of	lice) 15. SECURITY CLASS. (of this report)	
U.S. Army Research Office		
P.O. Box 12211	Unclassified 150. DECLASSIFICATION/DOWNGRADING	
Research Triangle Park, NC 27709	SCHEDULE	
Approved for public release; distribution unlin	CALECTE	
17. DISTRIBUTION STATEMENT (of the obstract entered in Block 20, 11 differ	JUN 2 3 1983	
18. SUPPLEMENTARY NOTES	A	
ITT, ANTELEMENTER INCLES		
The view, opinions, and/or findings contained i author(s) and should not be construed as an off position, policy, or decision, unless so design	ficial Department of the Army	
The view, opinions, and/or findings contained i author(s) and should not be construed as an off	ficial Department of the Army nated by other documentation	
The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design	ficial Department of the Army nated by other documentation umber)	
The view, opinions, and/or findings contained i author(s) and should not be construed as an off position, policy, or decision, unless so design 19. KEY WORDS (Continue on reverse elds if necessary and identify by block no Stochastic modelling, estimation, nonlinear sy noise, Poisson processes.	Ficial Department of the Army nated by other documentation mober) ystems, Wiener processes, white	
The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design 19. KEY WORDS (Continue on reverse elde if necessary and identify by block no Stochastic modelling, estimation, nonlinear sy	Ficial Department of the Army nated by other documentation mober) ystems, Wiener processes, white	
The view, opinions, and/or findings contained in author(s) and should not be construed as an off position, policy, or decision, unless so design 19. KEY WORDS (Continue on reverse elds if necessary and identify by block no Stochastic modelling, estimation, nonlinear sy noise, Poisson processes.	Ficial Department of the Army nated by other documentation (moder) (ystems, Wiener processes, white (mber) findings of 30 published	
 The view, opinions, and/or findings contained is author(s) and should not be construed as an off position, policy, or decision, unless so design 19. KEY WORDS (Continue on reverse elde if necessary and identify by block numbers of the state of the second state of the	Ficial Department of the Army nated by other documentation (moder) (ystems, Wiener processes, white (mber) findings of 30 published	
 The view, opinions, and/or findings contained is author(s) and should not be construed as an off position, policy, or decision, unless so design 19. KEY WORDS (Continue on reverse elds if necessary and identify by block numbers of the stimation, nonlinear sy noise, Poisson processes. 24. ABSTRACT (Continue on reverse elds H necessary and identify by block numbers) 25. ABSTRACT (Continue on reverse elds H necessary and identify by block numbers) 26. ABSTRACT (Continue on reverse elds H necessary and identify by block numbers) 27. This report summarizes the principle research papers and 3 Ph.D. theses completed with participation. 	Ficial Department of the Army nated by other documentation (moder) (ystems, Wiener processes, white (mber) findings of 30 published	
The view, opinions, and/or findings contained is author(s) and should not be construed as an off position, policy, or decision, unless so design 19. KEY WORDS (Continue on reverse olds if necessary and identify by block nu Stochastic modelling, estimation, nonlinear sy noise, Poisson processes. 36. ABSTRACT (Continue on reverse olds If necessary and identify by block nu > This report summarizes the principle research papers and 3 Ph.D. theses completed with parti- Research Office. B3. 06 22 DD - Forma 1073 Epthon of 1 moves is percenter.	ficial Department of the Army nated by other documentation mover) ystems, Wiener processes, white move findings of 30 published Lal support of the Army	

-

FINAL REPORT

U.S. ARMY RESEARCH GRANT/CONTRACT

DAAG29-76-G-0139 DAAG29-79-C-0147

1. Statement of the Problem Studied

Nonlinear stochastic systems arise in various areas of engineering including communications, vibrations and control. Under the grant and contract referenced above we undertook to produce rich class models which were, at the same time, analytically tractable. We studied these questions with respect to computing the statistical properties of solutions and with respect to finding sufficient statistics for nonlinear estimation problems. We also investigated the specific questions which arise in an attempt to adaptively control systems via system identification and subsequent control.

2. Summary of the Most Important Results

In the period covered by this final report we have studied stochastic modeling and the implications for estimation and control. This has resulted in a large number of publications. The most significant results obtained seem to be the following:

A. Our results which show how to compute all moments and all multipoint (in time) statistics associated with linear and bilinear stochastic differential equations whose coefficients are finite state jump processes or white noise. (See [5], [8], [11], and [18].) This work completely systemetizes earlier ad hoc treatments of special cases and has been used by others in the study of failure detection, reliability of electrical distribution grids, etc. It defines the role of the Poisson counter in modeling finite state processes and illustrates its utility in modeling.

B. Our results which describe the connection between sufficient statistics for nonlinear estimation problems and Lie algebras. (See [13], [16], [17], and [22].) This work succeeded in bringing together two major research activities -- nonlinear filtering and nonlinear realization theory -- in order to understand in a very natural way the intrinsic complexity of a given nonlinear filtering problem. A major part of the 1980 NATO Conference on Stochastic Systems was devoted to this work and it has been taken up by others subsequently.

C. Our research which has had significant conclusions concerning the role of geometry on system identification. (See [12], [14], and [31]. This work has given rise to a new point of view on the identification of linear systems and a reexamination of the performance of self timing regulations as applied to linear systems. D. Our results concerning the linearization of nonlinear systems by feedback. (See [7].) Here we take up the question of linearization by feedback and give a testable criterion for linearizability in terms of a certain Lie algebraic condition. This line of work has subsequently been pursued intensively by people at NASA-Ames and carried by them to the flight test stage. Our more recent paper [24] represents a first step toward making these local results global.

3. List of all Publications and Technical Reports Published

- [1] "The Lie Groups of Simple Feedback Systems," IEEE Decision and Control Conference, 1976.
- [2] "Convergence of Volterra Series on Infinite Intervals and Bilinear Approximations," <u>Nonlinear Systems and Applications</u> (V. Lakshmikanthan, ed.), Academic Press, pp. 39-46, 1977.
- [3] "Control Theory and Analytical Mechanics," in <u>Geometric Control Theory</u> (C. Martin and R. Hermann, eds.), Math. Sci. Press, Brookline, MA, pp. 1-48, 1977.
- [4] "Optimal Linear Systems with Polynomial Performance Measures," 1977 Joint Automatic Control Conference.
- [5] "Stationary Covariance Generation with Finite State Markov Processes," 1977 Joint Automatic Control Conference.
- [6] "Stochastic Bilinear Models," 1977 Joint Automatic Control Conference.
- [7] "Feedback Invariants for Nonlinear Systems," Proc. of the 1978 IFAC Congress, Helsinki, Finland.
- [8] "A Representation Theory for Linear Differential Equations with Markovian Coefficients," 1977 Allerton Conference (with G. Blankenship).
- [9] "Lie Algebras and Rational Functions: Some Control Theoretic Connections," in Lie Theories and Their Applications (W. Rossman, ed.), pp. 268-280, Dept. of Mathematics, Queen's University, Kingston, Ontario, 1978.
- [10] "On Improving the Circle Criterion," Proc. of the IEEE Decision and Control Conference, New Orleans, LA, 1977.
- [11] "Modeling and Estimation with Bilinear Stochastic Systems," <u>Trans. of the</u> <u>24th Conference of Army Mathematicians</u> (ARO Report 79-1), pp. 395-504, 1979.
- [12] "A Scaling Theory for Linear Systems," IEEE Trans. on Automatic Control, Vol. 25, pp. 197-207, 1980 (with P.S. Krishnaprasad).
- [13] "The Geometry of the Conditional Density Equations," in <u>Analysis and</u> <u>Optimization of Stochastic Systems</u> (O.L.R. Jacobs et al, eds.), Academic Press, N.Y., pp. 299-309, 1980 (with J.M.C. Clark).

- [14] "The Geometry of the Partial Realization Problem," Proc. of the 1978 Conference on Decision and Control, IEEE, N.Y.
- [15] "Stochastic Control and the Second Law of Thermodynamics," Proc. of the 1978 Conference on Decision and Control, IEEE, N.Y. (with J.C. Willems).
- [16] "Some Remarks on Finite Dimensional Nonlinear Estimation," <u>Asterique</u>, Societe de Matematique de France, Vol. 75-76, pp. 47-55, 1980.
- [17] "Classification and Equivalence in Estimation Theory," <u>1979 Conference on</u> Decision and Control, IEEE, N.Y., 1979.
- [18] "Stochastic Realization Theory and the Planck Law for Black Body Radiation," Ricerche di Automatica, Vol. 10, pp. 344-362, 1979.
- [19] "Polynomials, Bilinear Forms, and Representations of Lie Algebras," in <u>Algebraic and Geometric Methods in Linear System Theory</u> (AMS Lectures on <u>Applied Mathematics, Vol. 18 [C.I. Byrnes and C. Martin, eds.]</u>), pp. 1-6, 1980.
- [20] "Multivariable Nyquist Criterion, Root Locus, and Pole Placement by Output Feedback," <u>IEEE Trans. on Automatic Control</u>, Vol. AC-26, pp. 271-284, 1981 (with C.I. Byrnes).
- [21] "Control Theory and Singular Riemannian Geometry," in <u>New Directions in</u> <u>Applied Mathematics</u> (P. Hilton and G. Young, eds.), Springer-Verlag, pp. 11-27, 1981.
- [22] "Nonlinear Systems and Nonlinear Estimation Theory," in <u>Stochastic Systems</u> (M. Hazewinkel and J.C. Willems, eds.), Reidel Publishing Co., Dordrecht, The Netherlands, pp. 441-477, 1981.
- [23] "Asymptotically Optimal Estimation," Proc. of the 1981 Conference on Decision and Control, IEEE, pp. 76-79, 1981.
- [24] "The Global Description of Locally Linear Systems," <u>Springer-Verlag Lecture</u> <u>Notes on Control and Information Sciences</u>, Vol. 39 (D. Hinrichsen and A. Isidori, eds.), pp. 1-8, 1982.
- [25] "Asymptotic Stability and Feedback Stabilization," in <u>Differential</u> <u>Geometrics in Control Theory</u> (R.W. Brockett, et al, eds.), (Progress in <u>Mathematics</u>, Vol. 27), Birkhauser Publishers, Boston, MA, pp. 181-191, 1983.
- [26] "Linear Feedback Systems and the Groups of Lie and Galois," Linear Algebra and Its Applications, Vol. 24, (1983).

- [27] W.S. Wong, "Volterra Series, Universal Bilinear Systems and Fock Representations," Proc. of the Thirteenth Annual Conf. on Information Sciences and Systems, Johns Hopkins University, 1979.
- [28] P.S. Krishnaprasad, "Geometry of Parametric Models Some Probabilistic Questions," Proc. of the 15th Allerton Conf. on Control, Communicating and Computing, 1977, pp. 661-670.
- [29] M.A. Shayman, "Geometry of the Algebraic Riccati Equation, Part I," <u>SIAM J. Control and Optimization</u>, Vol. 21, No. 3, May 1983, pp. 375-394.
- [30] M.A. Shayman, "On the Variety of Invariant Subspaces of a Finite-Dimensional Linear Operator," <u>Transactions of the American Mathematical</u> <u>Society</u>, Vol. 274, No. 2, December 1982, pp. 721-747.

4a. List of All Participating Scientific Personnel

Roger W. Brockett Anthony Bloch Varda Haimo P.S. Krishnaprasad Josip Lonćarić Alan McIvor Mark Shayman Tom Taylor Wing Wong

A summary firstness processes to a sur-

a and a second state of the second second

- 4b. List of All Participating Scientific Personnel Showing Any Advanced Degrees Earned by Them While Employed on the Project
- [31] P.S. Krishnaprasad, "Geometry of Minimal Systems and the Identification Problem," Ph.D. Thesis, Harvard University, August 1977.
- [32] W.S. Song, "Operator Theoretic Methods in Nonlinear Systems," Ph.D. Thesis, Harvard University, August 1980.
- [33] M.A. Shayman, "Varieties of Invariant Subspaces and the Algebraic Riccati Equation," Ph.D. Thesis, Harvard University, December 1980.