AD-A127 948 TWO ORTHOGONAL (V/2)X2V F-RECTANGLE FOR ALL EVEN V(U) 1/1
ILLINOIS UNIV AT CHICAGO CIRCLE DEPT OF MATHEMATICS
A S HEDAYAT ET AL. JAN 83 AFOSR-TR-83-0339
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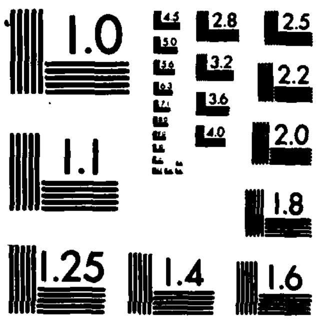

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 83-0339	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) TWO ORTHOGONAL (v/2) x 2v F-RECTANGLES FOR ALL EVEN v		5. TYPE OF REPORT & PERIOD COVERED TECHNICAL
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A.S. Hedayat and W.T. Federer		8. CONTRACT OR GRANT NUMBER(s) AFOSR-80-0170
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mathematics, Statistics, and Computer Science, University of Illinois, P.O. Box 4348, Chicago IL 60680		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS PE61102F; 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS Mathematical & Information Sciences Directorate Air Force Office of Scientific Research Bolling AFB DC 20332		12. REPORT DATE JAN 83
		13. NUMBER OF PAGES 5
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION ST. ENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY TES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Changeover; simultaneous experiments; surveys.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

It is shown how to construct a pair of orthogonal v/2 by 2v rectangles for all even v. Also, it is shown how to construct a set of t pairwise orthogonal v/2 by 2v F-rectangles for all even v for which a set of t pairwise orthogonal Latin squares of order v exists. Some situations where these designs are useful in practice are indicated.

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AFOSR-TR- 83-0339

TWO ORTHOGONAL (v/2)x2v F-Rectangle For
All Even v

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*Research is sponsored by Grant AFOSR 80-0170

Orthogonal F-Rectangles

Two Orthogonal $(v/2) \times 2v$ F-Rectangles for All Even v

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SUMMARY

It is shown how to construct a pair of orthogonal $v/2$ by $2v$ rectangles for all even v. Also, it is shown how to construct a set of t pairwise orthogonal $v/2$ by $2v$ F-rectangles for all even v for which a set of t pairwise orthogonal Latin squares of order v exists. Some situations where these designs are useful in practice are indicated.

Let $V = \{1, 2, \cdots, v\}$ be a set of v distinct symbols. A $k \times b$ array filled with the elements of V is said to be an F-rectangle if

(i) Every element of V appears the same number of times, $r = bk/v$, in the array,

(ii) The appearance of each element in each row and column is as uniform as possible.

Note that condition (ii) indicates that if $k \le v$, no element of V appears more than once in each column, and if $b = tv$, each element of V appears t times in each row.

Key words: Changeover; Simultaneous experiments; Surveys

<u>Definition</u>. Let F_1 and F_2 be two $k \times b$ F-rectangles. Then we say F_1 is orthogonal to F_2 (denoted by $F_1 \perp F_2$) if upon superposition of F_1 on F_2 every element of V in F_1 appears the same number of times with every element of V in F_2.

<u>Example</u>. Let $V = \{1,2,3,4\}$; then, the following F_1, F_2 and F_3 are pairwise orthogonal 2×8 F-rectangles.

F_1

1	2	3	4	3	4	1	2
2	1	4	3	4	3	2	1

,

F_2

1	2	3	4	4	3	2	1
3	4	1	2	2	1	4	3

F_3

1	2	3	4	2	1	4	3
4	3	2	1	3	4	1	2

We shall now prove the following theorem.

<u>Theorem 1</u>. There exists a pair of orthogonal $(v/2) \times 2v$ F-rectangles for all even v.

The proof is by construction. Construct a $(v/2) \times v$ F-rectangle, A, based on V with $1, 2, \cdots, v/2$ as its entries in the first column and fill the remaining cells cyclically. Construct another $(v/2) \times v$ F-rectangle, B, based on V with $(v/2)+1$, $(v/2)+2$, $\cdots$, v as its entries in the first column and fill the remaining cells cyclically. Construct another $(v/2) \times v$ F-rectangle, C, based on V with the odd numbers among $1, 2, \cdots, v$ as its entries in the first column and fill the remaining cells cyclically. Then,

$$F_1 = \boxed{A \mid B} \quad \text{and} \quad F_2 = \boxed{C \mid C}$$

form a pair of orthogonal $(v/2) \times 2v$ F-rectangles. It is obvious that F_1 and F_2 are F-rectangles. The fact that they are orthogonal follows from (a) the cyclic construction of A, B and C and (b) the property that each element of V in F_2 appears once with the element 1 in F_1.

<u>Example</u>. Let $V = \{1,2,3,4,5,6\}$. Then,

$$A = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \\ 3 & 4 & 5 & 6 & 1 & 2 \end{matrix}, \quad B = \begin{matrix} 4 & 5 & 6 & 1 & 2 & 3 \\ 5 & 6 & 1 & 2 & 3 & 4 \\ 6 & 1 & 2 & 3 & 4 & 5 \end{matrix}, \quad \text{and} \quad C = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 1 & 2 \\ 5 & 6 & 1 & 2 & 3 & 4 \end{matrix}.$$

Now form $F_1 = \boxed{A \mid B}$ and $F_2 = \boxed{C \mid C}$ as

$$F_1 = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 4 & 5 & 6 & 1 & 2 & 3 \\ 2 & 3 & 4 & 5 & 6 & 1 & 5 & 6 & 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 & 1 & 2 & 6 & 1 & 2 & 3 & 4 & 5 \end{matrix}, \quad F_2 = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 1 & 2 & 3 & 4 & 5 & 6 & 1 & 2 \\ 5 & 6 & 1 & 2 & 3 & 4 & 5 & 6 & 1 & 2 & 3 & 4 \end{matrix}.$$

The concept of orthogonal F-rectangles is closely related to the concept of orthogonal Latin squares. One such relation in the context of this note is indicated below.

<u>Theorem 2</u>. For v even the existence of t pairwise orthogonal Latin squares of order v implies the existence of t pairwise orthogonal $(v/2) \times 2v$ F-rectangles.

The proof is by construction. If $\{L_1, L_2, \cdots, L_t\}$ is a set of pairwise orthogonal Latin squares of order v (even), then split L_1 into halves as

$$L_1 = \begin{array}{|c|} \hline A_1 \\ \hline B_1 \\ \hline \end{array}$$

and let $F_i = \boxed{A_i \mid B_i}$. Then clearly $\{F_1, F_2, \cdots, F_t\}$ forms the required set of pairwise $(v/2) \times 2v$ F-rectangles.

Remark. Theorem 2 cannot be used when $v = 2$ and 6 since there is no pair of orthogonal Latin squares of order 2 and 6. Also, Theorem 2 requires the construction of orthogonal Latin squares, which is not easy for $v \equiv 2 \pmod 4$. If one is interested only in a pair of orthogonal $(v/2) \times 2v$ F-rectangles then Theorem 1 is useful and easy to implement for all even orders including $v = 6$. For $v = 2$ one may use

$$F_1 = \boxed{1 \quad 2 \quad 1 \quad 2} \qquad \text{and} \qquad F_2 = \boxed{1 \quad 2 \quad 2 \quad 1}$$

to obtain $F_1 \perp F_2$.

This class of experiment designs has usefulness in many areas of experimentation. In a variety of situations, it may be undesirable, or even impossible, to have as many treatment periods as there are treatments, but it is relatively easy to obtain more individuals or organizations for the experiment. For example, in a study of diet and aerobic dance exercise, it was undesirable to subject each individual to more than three exercise-diet treatments, but it was relatively easy, and desirable, to obtain 36 individuals for the study for the six diet-exercise treatments. It is necessary to have a class of individuals, 36 here, for aerobic dancing. It was considered essential to have every treatment follow every other treatment and vice versa. The above example for $v = 6$ and the 36 sequences from F_1 and F_2 forms such a changeover design. Some situations where these experiment designs may be useful are:

(i) In marketing and other experiments where the 2v sampling units (individuals, organizations, etc.) are used for v/2 time periods for two sets of v treatments,

(ii) In changeover experiments with 4v sequences and v/2 periods balanced for residual effects of v treatments,

(iii) In surveys where the order of the v questions (sensitive or otherwise) is to be in a balanced arrangement for residual effects for each set of 4v individuals and each individual answers v/2 questions.

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