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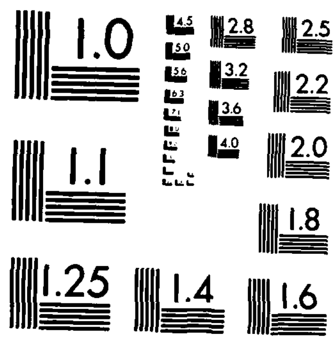

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ON DIGITAL CIRCLES

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ON DIGITAL CIRCLES

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ABSTRACT

This paper characterizes sets of lattice points which are digitizations of circles.

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1. Introduction

Image processing and pattern recognition are mainly concerned with classifying shapes or patterns which appear in pictures, and the classification is based on geometric properties of the patterns. To do such tasks by digital computers, the pictures must be digitized, i.e., converted into arrays of lattice points. Due to this conversion, it is not always obvious how to recognize that a set of lattice points must have arisen from a real pattern that has a given geometric property. For example, how do we characterize sets of lattice points which are the digitizations of real straight line segments? This and some related questions have been discussed in [1].

In this paper, we will consider the same problem for circles (i.e., disks). First, we discuss the case where the center of the circle is at a lattice point. For this case, we give conditions for a set of lattice points to be a digital circle. By making use of this result, it is also proved that the set of digitized circles is accepted by a deterministic tape-bounded array acceptor. Next, we consider the case where the center of the circle is not at any lattice point. The discussion for this case is not so easy, but is interesting. For this case, we give an algorithm to decide whether or not a given set of lattice points is a digital circle.

It is assumed that the readers are familiar with the basic definitions and notations of digital pictures.

2. Digitization of circles

Let S be a bounded subset of the plane. For purposes of computer analysis, it is usual to represent S by a finite set of lattice points, i.e., points with integer coordinates. This set, $\hat{S}$, is called the digital image of S, and the mapping that takes S into $\hat{S}$ is called digitization.

$\hat{S}$ can be defined in various ways; we list here several of them:

(a) $\hat{S}$ is the set of lattice points contained in S; this is called the subset digitization of S.

(b) $\hat{S}$ is the set of lattice points such that S comes closer than the city block distance ½ to them, i.e., $\{(i,j)\,|\,\exists(x,y)\in S\colon\ \max(|x-i|,|y-j|) < \tfrac{1}{2}\}$
This is called the open cell digitization of S. (If we imagine an open unit square P^C centered at each lattice point P, we have $P\in\hat{S}$ iff $S\cap P^C\neq\emptyset$.)

(b') Analogous to (b), using half-open cells P*, i.e.,

$$i-\tfrac{1}{2}\le x<i+\tfrac{1}{2},\ j-\tfrac{1}{2}\le y<j+\tfrac{1}{2}.$$

(b") Analogous to (b), using closed cells P.

In this paper, we will use the subset digitizaion of S. The finite set of lattice points B is called a digital circle iff there exists a circle A such that $B=\hat{A}$ under the subset digitization.

2.1 Circle with its center at a lattice point

In this subsection we assume that the center of the circle is at a lattice point. In this case, we can give a necessary and sufficient condition for a given set S of lattice points to be a digital circle. Let $\overline{PQ}$ represent the distance between points P and Q. Further, referring to Figure 1, the octants of the plane defined by a center are eight 45° wedges. They are labelled in counterclockwise order with Roman numerals, starting immediately above the positive x-axis. We will consider the following conditions on a finite set $\hat{S}$ of lattice points:

(1) $\hat{S}$ is connected and has no holes.

(2) There exists a center C satisfying the following conditions:

(a) Let C's coordinates be (0,0). Then the border of $\hat{S}$ is symmetric with respect to the four lines x=0, y=0, y=x, y= -x. Also, in octant I the border of $\hat{S}$ does not decrease along the x-axis by more than one step at a time; and for the other octants, symmetric conditions hold.

(b) Referring to Figure 2, for every two border corner points P_i, P_j, we have $\max_i \overline{CP_i}^2 < \min_j \overline{CP'_j}^2$, where P'_j is defined as follows: If P_j is in octant I, then P'_j is the lattice point immediately above P_j. If P_j is in octant II, then P'_j is the lattice point immediately to the right of P_j. For the other octants, P'_j is similarly defined.

We will denote the above conditions on $\hat{S}$ by <u>CDC</u>. Then we have the following theorem:

<u>Theorem 1</u> A given finite set $\hat{S}$ of lattice points is a digital circle iff it satisfies the conditions CDC.

<u>Proof</u>: Condition (2)(a) is explained in Figure 3. It is obvious that $\overline{CP}_i > \overline{CP}_{i+1}$.

Therefore, it is sufficient for the proof that we consider a circle with radius r such that $\max_i \overline{CP_i} \leq r < \min_j \overline{CP'_j}$. //

For the case where the center is at a lattice point, we can show by making use of Theorem 1 that the set of digital circles is accepted by a deterministic tape-bounded array acceptor (DTBA). The definition of a DTBA is given in [2]. Roughly speaking, a DTBA is a deterministic acceptor T over the cell space whose behavior is restricted as follows:

(i) The movement of T is bounded to the area of the input array.

(ii) T can rewrite any symbol in the cell that it reads.

<u>Theorem 2</u> There exists a deterministic tape-bounded array acceptor T which accepts the set of digital circles.

<u>Proof</u>: First, T marks a cell with C. This cell corresponds to the center. If this cell fails to be the center, T marks another cell. T continues this movement until it finds a center satisfying the conditions CDC. This is possible, since T is a DTBA. Next, T checks the conditions CDC. In particular,

CDC (2)(b) is checked by making use of the square area B as shown in Figure 4. That is, CDC (2)(b) is examined by marking consecutively all cells of B, since $\overline{CP_i}^2$ and $\overline{CP'_j}^2$ are not greater than kr^2, where k is a constant. T goes into the accepting state iff the conditions CDC are satisfied. //

From Theorem 2 and Theorem 8.3.3 of [2], we also have the following corollary:

Corollary 3. The set of digital circles is generated by a monotonic array grammar.

2.2 Circle with its center at a non-lattice point

Now, let us consider the case where the center of the circle is not at any lattice point. The discussion for this case is complicated. First, we treat the case where the center of circle is not at any lattice point but is on the x-axis. (The discussion for the y-axis is similar.) The case is shown in Figure 5. In this case, we have two points denoted by C_0 and C_1 in Figure 5. We call them centers. Without loss of generality, it is possible to assume that the real center is between C_0 and C_1. The case where the real center is on the left of C_0 is similarly treated. Then, it is obvious that by moving some border points by distance 1 to the left we get the case mentioned in Subsection 2.1. Also, it is obvious in this case that the border is symmetric with respect to the x-axis.

Thus, we have the following theorem:

Theorem 4. Assume that a given set $\hat{S}$ of lattice points is symmetric with respect to the x-axis. Then there is an algorithm to decide whether or not the set $\hat{S}$ is a digital circle.

Proof: First, we mark with C_0 and C_1 two cells that are horizontally adjacent. These cells correspond to the centers. If these cells fail to be centers, we mark other cells (after erasing the previous marks) until we find centers satisfying the following condition (M):

(M): We move some border points systematically by distance 1 to the left. Border points on the y-axis may move up (and down) by distance 1. The resulting set $\hat{S}'$ of lattice points satisfies the CDC conditions.

We repeat this procedure by moving other border points until the following condition is satisfied. Namely, if $\hat{S}'$ satisfies the CDC condition, there are two real circles which are the largest one and the smallest one corresponding to $\hat{S}'$. Let the radii of these circles be r_1 and r_2, respectively. We draw two circles of radii r_1 and r_2 with the center at the midpoint C_{21} between C_0 and C_1. If for every $\overline{C_{21}P_i}$ we have $r_2 \leq \overline{C_{21}P_i} < r_1$ and $\max_i \overline{C_{21}P_i} < \min_j \overline{C_{21}P_j'}$, then there exists a circle corresponding to the original set $\hat{S}$. At this stage, our procedure finishes. If not, we have the following two cases:

(i) There is no circle with center C_2 which corresponds to the original set $\hat{S}$.

(ii) We can repeat this procedure, making use of another center (i.e., the midpoint C_{31} between C_0 and C_{21} and the midpoint C_{32} between C_{21} and C_1).

Conditions (i) and (ii) are tested by comparing the radii r_1, r_2 and every $\overline{C_{21}P_i}$ and $\overline{C_{21}P_j'}$. When there is no possibility of (ii), we consider another movement (M), if possible, and repeat the procedure.

This procedure eventually finishes. The reason is as follows: The set of border points is finite. Thus, the number of moving actions (M) is finite. Also, if the given set S is a digital circle, the condition is eventually satisfied. This is because, since the difference of the largest radius r_1 and the smallest one r_2 is positive, for a sufficiently large number N C_N can be a real center. Further, if the given set $\hat{S}$ is not a digital circle, it is shown as described below that condition (ii) does not hold after any movement (M).

Let us consider the situation shown in Figure 6. Here, BD is a perpendicular line passing through the midpoint D of $\overline{P_i P'_j}$. Also, $\overline{AP_i}$ is the smallest radius of a circle with center A which contains every border point, and $\overline{AB}$ is the largest distance such that P'_j is contained in this circle. Then, it is a ncecessary and sufficient condition for the existence of a circle corresponding to $\hat{S}$ that the center be in BC_M. Similarly, in the case shown in Figure 7, the center must be in AB. Next, we move the center to A, and then repeat the process. This procedure finishes in a *finite number* of steps, because the number of $\overline{P_i P'_j}$ for all $i \neq j$ is finite. Therefore, if the given set $\hat{S}$ is not a digital circle, we eventually reach a contradiction. Thus the procedure eventually finishes, and we have the theorem. //

Now, let us consider the general case where the center is not a lattice point. Without loss of generality, the case is as shown in Figure 8.

Then we have the following theorem:

Theorem 5. There is an algorithm to decide whether or not a given set $\hat{S}$ of lattice points is a digital circle.

Proof: This theorem is provable by a similar technique. First, we take four cells C_0, C_1, C_2, and C_3 as in Figure 8, and call them centers. Then, we consider a moving process (M'), by which some border points move downward by distance 1. This (M') corresponds to (M) of the previous theorem. Let $\hat{S}''$ be the new set obtained from S by the moving (M'). By a similar technique to that used in the proof of Theorem 4, but using the square $C_0C_1C_2C_3$ instead of $\overline{C_0C_1}$, we get the desired algorithm. //

As mentioned above, there is an array acceptor (DTBA) for digital circles with their centers at lattice points. But for the general case of this subsection it is difficult t((e-fine such an acceptor. The reason is that any acceptor cá ɔt compare two distances whose values are real numbers. How- this is possible by considering a nondeterministic automaton with counters. For example, it is possible to define a Turing array acceptor (TA) which accepts digital circles. Thus, it seems to be an interesting question to find out a weaker acceptor than TA which accepts the set of digital circles.

References

1. Rosenfeld, A. and C. E. Kim, How a digital computer can tell that a straight line is straight, TR-1072, Computer Science Center, University of Maryland, College Park, MD, July 1981.

2. Rosenfeld, A., Picture Languages, Academic Press, New York, 1979.

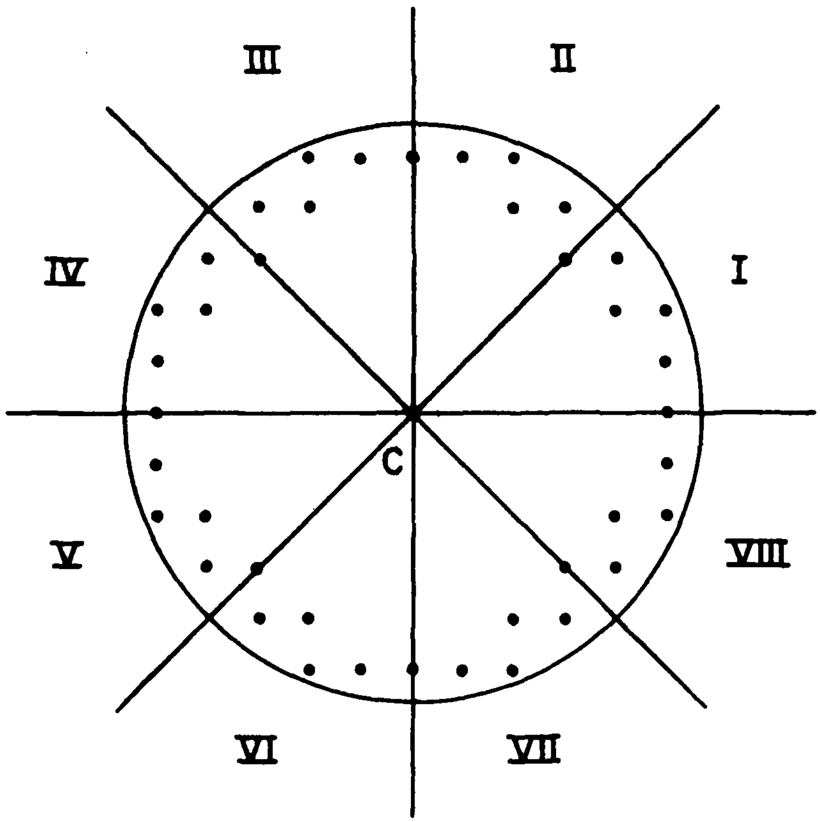
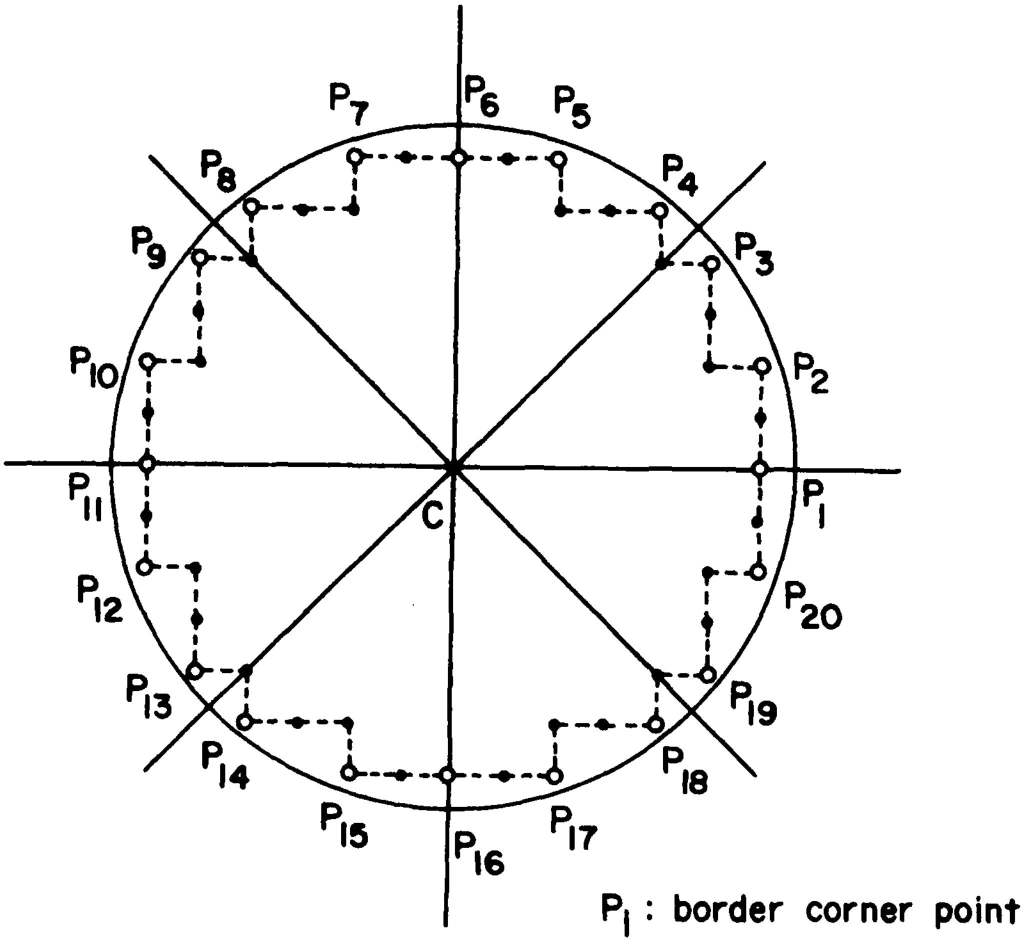


Figure 1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P1 : border corner point

Figure 2

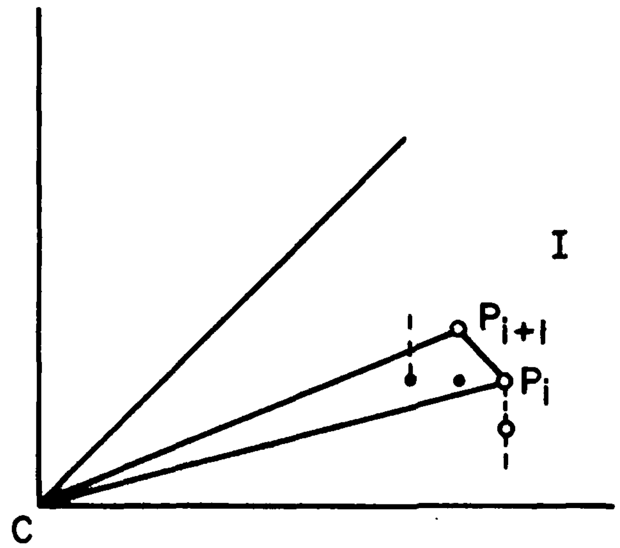


Figure 3

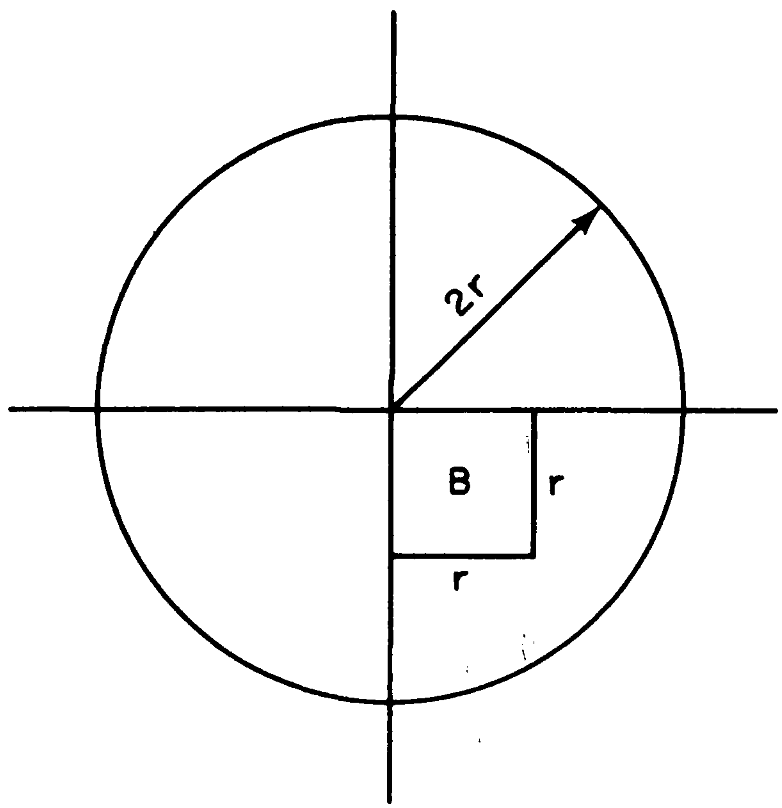


Figure 4

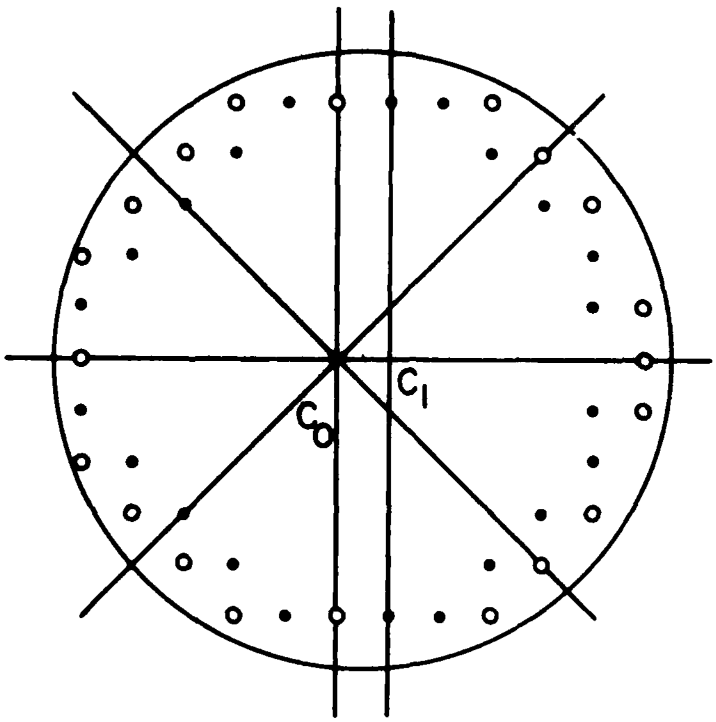


Figure 5

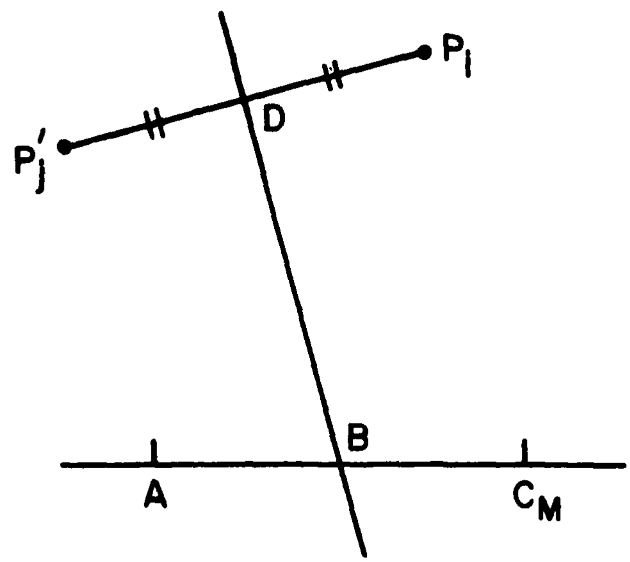


Figure 6

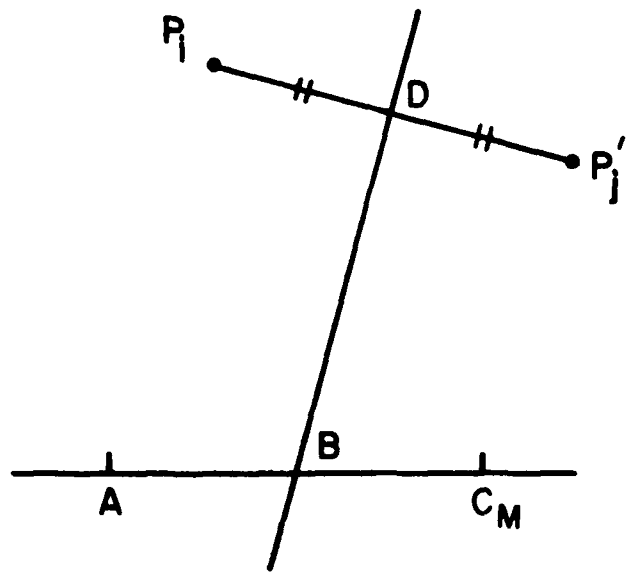
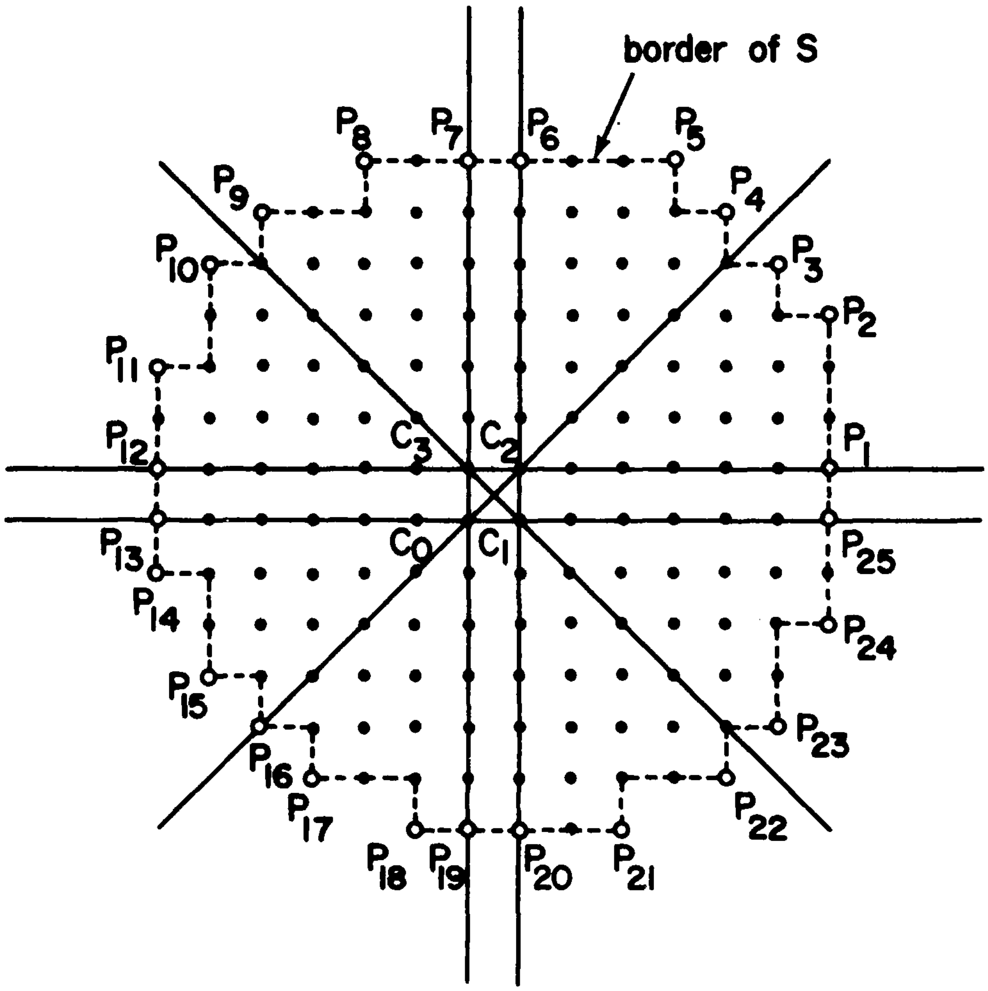


Figure 7



P_i : border corner point

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