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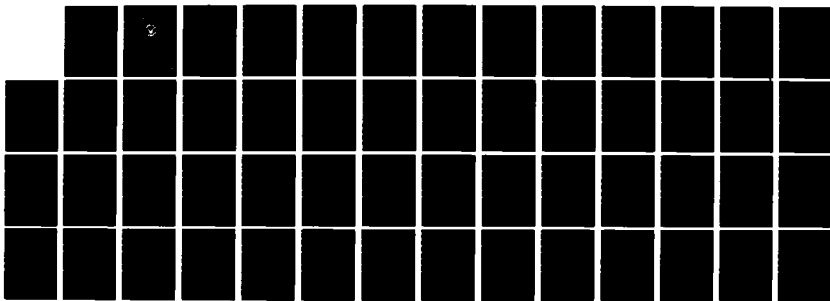
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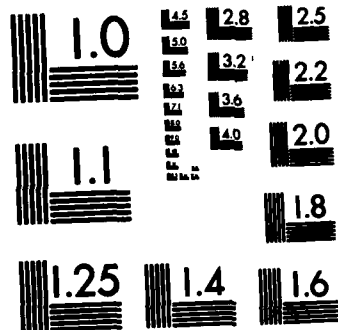
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## THESIS

AN EXAMINATION OF THE UNITED STATES AIR FORCE OPTIMAL  
NONNUCLEAR MUNITIONS PROCUREMENT MODEL

by

Paul H. Lord

October 1982

Advisor:

G. G. Brown

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An Examination of the United States Air Force Optimal  
Nonnuclear Munitions Procurement Model

by

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Captain, United States Marine Corps  
B.S., State University College at Oneonta, New York, 1974

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

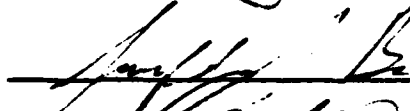
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## ABSTRACT

This evaluates the United States Air Force (USAF) Nonnuclear Armament Program (NAP) models and specifically the Heavy Attack (HA) model. Particular attention is paid to the optimization techniques incorporated in Heavy Attack, to the validity of the inputs being optimized, and to the implications of underlying model assumptions. An examination is made of the validity of using target values as model inputs for not only the beginning of a conflict, but also for times extending into the conduct of a conflict. New technology has been applied to the problem and the success achieved is reviewed. Reformulations aimed at improving model capabilities and/or solution speeds are described.

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## I. THE PROBLEM

Military Planning for the United States primarily involves preparation for an aggressive response to an attack made against the United States or allied forces. This basically defensive posture all but denies the United States the advantages inherent in surprise, and requires, as a counter to the initiative granted an attacker, that the United States maintain operational effectiveness and vigilance second to none.

To achieve maximum military effectiveness given fiscal constraints requires careful consideration in planning and budgeting. Men without weapons will neither intimidate nor stop an enemy, neither will men with weapons but without munitions or a means of maneuver (transport). As an example, the United States Air Force must weigh each dollar spent to see if it should be best utilized for personnel (acquisition, retention, and training), aircraft (acquisition and maintenance), fuel, or munitions in order to best fulfill its many missions.

The research described herein addresses the Air Force nonnuclear munitions procurement model. This model requires as input:

1. A target list to include target military values, defensive capabilities, and factors describing the potential for confirming target kill;
2. Numbers of friendly aircraft and munitions available;
3. Probabilities of encountering differing weather conditions; and
4. The effectiveness of specific munitions against various targets when delivered in a weather condition by a specified aircraft.

Given these inputs, the model seeks to maximize the damage done to the enemy by planning the use of the most effective munitions.

The model is played separately for each major Air Force theatre. Individual theatre results comprise the single largest input for non-nuclear weapons procurement decision-making for the Air Force. The amount of money spent with the aid of this model is currently in excess of one billion dollars a year. Not only is the model central to Air Force budgeting and planning, but it is also relied upon by the major theatre commanders for insight into scenarios involving their current missions.

With so much at stake in its use, an examination is in order to better understand the assumptions, formulation, processing, mathematical solution, and solution report generation for this model [Ref. 1].

## II. DESCRIPTION OF THE HEAVY ATTACK (HA) MODEL

Heavy Attack (HA) is a program which uses an imbedded nonlinear optimizer to identify a set of sortie allocations which maximizes the military worth of targets killed. It has been used since 1974 as part of a set of computer programs that have been known as Saber Mix or, more recently, as the Nonnuclear Armament Plan (NAP) models. These models together attempt to provide an optimal munitions mix for a given specification of available sorties, targets, and other factors.

The HA model can be viewed as consisting of an internal (optimization) model and an external model consisting of input and the output sequences. This view will be adopted in describing the model and its variations primarily because the optimization is still modeled, if not processed, exactly as it first was eight years ago, while input and output models have changed (grown) continuously. Careful attention will be paid to the optimization model while the input and output models will be examined only to achieve an understanding of the consequences of model input aggregation and output unraveling for solution interpretation.

### A. DESCRIPTION OF THE HA OPTIMIZATION FORMULATION (INTERNAL MODEL)

HA solves a sequence of internal models. Each of these models has the same mathematical structure (formulation), and each is a nonlinear optimization problem by virtue of its objective function. The internal optimization model is formulated as follows:

$$\begin{aligned}
& \text{maximize:} && \sum_{j=1}^J V_j (K_j \{S_{ij}\} - D_j) \\
& S_{ij} \\
& \text{subject to:} && \sum_{j=1}^J S_{ij} = S_i, \quad i = 1, 2, \dots, I; \\
& && a_j \leq K_j \leq T_j, \quad j = 1, 2, \dots, J; \\
& && \delta_m \left( \sum_{j \in J_m} (1 - \theta_m) S_{ij} + \sum_{j \notin J_m} (-\theta_m) S_{ij} \right) \leq 0, \\
& && m = 1, 2, \dots, M; \\
& && S_{ij} \geq 0.
\end{aligned}$$

The objective function, which is convex with respect to  $S_{ij}$  [Ref. 2: pp. 8, 9], quantifies the value of all targets killed in a time period as follows:

$$\sum_{j=1}^J V_j (K_j \{S_{ij}\} - D_j),$$

where,

- $i$  = aircraft type index ( $i = 1, \dots, I$ );
  - $j$  = target type index ( $j = 1, \dots, J$ );
  - $S_{ij}$  = number of sorties, *the independent variables*;
  - $V_j$  = value (military worth) of target type  $j$ ;
  - $D_j$  = cumulative number of targets killed in prior time periods; and
  - $K_j \{S_{ij}\}$  = number of kills of target type  $j$ , a nonlinear function of  $S_{ij}$ .
- $K_j$  is defined to be a function of the independent variables  $S_{ij}$ :

$$K_j \{S_{ij}\} = \frac{T_j}{C_j} (1 - \exp(-\frac{C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}))),$$

where,

- $T_j$  = number of type  $j$  targets;

$C_j$  = target kill confirmability parameter ( $0 \leq C_j < 1$ ) controlling the extent to which the law of diminishing productivity (as described later) applies;

$\alpha_j = \frac{T_j}{C_j} \log (1 - \frac{C_j}{T_j} D_j)$ ; a term (to be explained) added for mathematical convenience; and

$P_{ij}$  = expected number of type  $j$  targets killed per type  $i$  aircraft sortie when no other targets of type  $j$  have been previously killed and when conditions of kill confirmability are perfect.

HA problem constraints are of three forms:

1. Sorties available:

$$\sum_{j=1}^J S_{ij} = S_i, \quad i = 1, 2, \dots, I;$$

where  $S_i$  = the number of sorties for aircraft type  $i$ .

2. Target:

$$L_j \leq K_j \leq T_j, \quad j = 1, 2, \dots, J;$$

where  $L_j$  = the lower bound on targets of type  $j$  which must be killed.

3. Flight composition with the general form:

$$\delta_m \left( \sum_{j \in J_m} (1 - \theta_m) S_{ij} + \sum_{j \notin J_m} (-\theta_m) S_{ij} \right) \leq 0, \quad m = 1, 2, \dots, M;$$

where,

$m$  = flight composition constraint index ( $m = 1, 2, \dots, M$ );

$i_m$  = aircraft type;

$\delta_m$  = +1, maximum (-1, minimum);

$J_m$  = set of targets for which a maximum (minimum) flight composition is required; and

$\theta_m$  = maximum (minimum) proportion of sorties flown by aircraft type  $i_m$  against targets included in set  $J_m$   
[Ref. 2: pp. 5, 8, 10].

$K_j$  is computed assuming "a law of diminishing marginal productivity . . . i.e., the number of targets per sortie decreases for each successive sortie"

[Ref. 2: p. 1]. This underlying assumption is based on a belief that targets will be harder to find, harder to surprise, and harder to kill as the battle continues.

By restating target constraint inequalities to solve for  $\sum P_{ij}S_{ij}$ , the problem can be formulated (see Appendix A) with linear constraints [Ref. 2: pp. 6-7, 10-12] of the form:

$$R_j \leq \sum_{i=1}^I P_{ij}S_{ij} \leq \bar{R}_j$$

where,

$$R_j = \frac{-T_j}{C_j} (\log (1 - C_j)) - \alpha_j; \text{ and,}$$

$$\bar{R}_j = \frac{-T_j}{C_j} (\log (1 - \epsilon_j \frac{C_j}{T_j})) - \alpha_j.$$

Further examination of  $K_j$  provides significant insight into the nature of the function which is being maximized. If

$$K_j = \frac{T_j}{C_j} (1 - \exp (-\frac{C_j}{T_j} (\alpha_j + \sum_{i=1}^I P_{ij}S_{ij}))),$$

then noting,

$$\exp (-\frac{C_j}{T_j} \alpha_j) = 1 - \frac{C_j}{T_j} D_j,$$

and (for convenience of exposition) letting,

$$X_j = \sum_{i=1}^I P_{ij}S_{ij},$$

the term  $K_j - D_j$  (from the objective function) can be restated (see Appendix B) as follows:

$$K_j - D_j = (\frac{T_j}{C_j} - D_j)(1 - \exp (X_j (-\frac{C_j}{T_j}))).$$



The function  $K_j - D_j$  is a composition of two embedded functions. The constant parameter  $C_j$  can be used to select either component. When  $C_j$  is equal to one,

$$K_j - D_j = (T_j - D_j) \left( 1 - \exp \frac{-X_j}{T_j} \right).$$

When  $C_j$  is zero,

$$K_j - D_j = X_j.$$

Regardless of  $C_j$ , if  $X_j/T_j$  is small (a target-rich environment exists), then  $K_j - D_j = X_j$ .

$C_j$  can be interpreted to be a coefficient of confirmability. That is, if conditions allow a pilot to confirm the effect of his first ordnance drop before delivering any others then  $C_j$  should be zero. This case can be called shoot-look-shoot.

If conditions are such that a pilot must deliver his ordnance without reference to the success or failure of any weapons that might have been delivered earlier, then  $C_j$  should be one. This case could be called dump-all-ordnance.

While the composition of  $K_j - D_j$  can be understood for the boundary conditions where  $C_j$  is equal to one or zero, it is not at all certain how  $K_j - D_j$  should be interpreted in cases where  $C_j$  lies between the two boundaries, except that it will behave as some mixture of the two embedded functions.

Summarizing, HA solves a problem with a nonlinear, but convex, objective function subject to three sets of linear constraints. Functionally it selects sorties, *not weapons*, to inflict the greatest damage upon an enemy. Constraints demand each aircraft be utilized when available, that a specified range of each target type must be killed, and that

specified sets of targets, associated with an aircraft, must be attacked by a minimum (or maximum) percentage of the sorties available for that aircraft.

#### B. DESCRIPTION OF THE JUNE 1982 HA EXTERNAL MODEL

Because the optimization model is formulated in terms of *sorties* (aircraft/target combinations) while the purpose of HA is to provide preferred *weapon* information, the HA external model largely concerns itself with building sortie information ( $P_{ij}$ 's) from inputs and extracting from sortie solutions ( $S_{ij}$ 's) information regarding preferred weapons.

The HA model, external to the optimization model, has been modified repeatedly in the years since 1974 for a variety of reasons. Determining the history of these changes might prove interesting, but what is more important is to understand HA in its present form. (Inputs, processes, and outputs for the original HA model, as described in the seminal paper for the model [Ref. 2: p. 2], are described in Appendix C.)

Preceding HA in the execution of the Nonnuclear Armament Plan (NAP) model programs are *Weaponer*, *Survivor*, and *Selector* [Refs. 3: p. 5 and Ref. 4: p. 2].

"The first model, *Weaponer*, computes the expected target kills per pass for various aircraft/munition/target combinations. . . . The second model, *Survivor*, computes the attrition of delivery aircraft on an iterative basis and determines the expected kills per sortie . . . . The third model, *Selector*, selects the preferred long-list weapon for each aircraft/target/delivery band [weather type] combination. . . ." [Ref. 4: p. 2]

The models preceding HA in the NAP sequence are of current interest only insofar as they provide inputs to the HA optimization. What should be noted, however, is the concern for the passage of time implicit in the short descriptions of *Weaponer* and *Survivor* given above (and in the prior discussion of diminishing marginal productivity), as well as the use

of weather types in Selector. While Clasen, Graves, and Lu [Ref. 2] never discuss use of the model over multiple time periods or delivery (weather) types, these two elements are currently incorporated in the HA model. (Multiple time period use is implied in [Ref. 2] by inclusion of the term  $D_j$  in the objective function and by the adjective "interval" describing model inputs and outputs, however, no "iteration-over-time" scheme is described. The effects of weather on sortie effectiveness are never mentioned in [Ref. 2].)

Weather conditions have a profound impact on aircraft/weapon effectiveness. The HA external model expresses weather condition as six discrete "bands," associated with extremely poor to essentially unlimited flying visibility. The June 1982 HA input model builds sorties and associated expected kills using a weighted average based on expectation of weather types in the theatre being modeled. The  $P_{ij}$  associated with each  $S_{ij}$  is formed as follows:

$$P_{ij} = \sum_{w=1}^{numw} P_w \max_k P_{ijkw}$$

where,

$k$  = set of all weapon types;

$P_w$  = probability of a particular weather type;

$P_{ijkw}$  = expected number of kills by aircraft  $i$  loaded with weapon  $k$  against target  $j$  in weather condition  $w$ ; and

$numw$  = number of weather condition types modeled.

The effect of this averaging prior to the optimization model is two-fold. Sorties are not limited to a single weapon type; that is, sorties are evaluated as aircraft, not as aircraft/weapon combinations. More importantly, an average quantity representing the aircraft's effectiveness

is being sent to the optimizer. The result is a selection of extremal averages rather than the averaging of selected extremal values.

This can lead to paradoxical recommendations. Consider a paradigm with two equally probable weather conditions, two aircraft types, and a single target type. Given one aircraft is moderately effective against the target in both weather types while the second aircraft is near certain to kill the target in one weather type and as certain to miss it in the other, the first aircraft is likely to be selected by the model (to be flown in both weather types) despite the dominance of the other aircraft in the second weather type.

In HA the effects of weighted averaging may be subtle. A situation is likely in which for four of six weather types, the effectiveness of an aircraft/best weapon combination against a target type is quite high while in the other two weather types, the effectiveness of this aircraft is negligible. If another aircraft has mediocre success in all weather types against the same target, it might well be selected by the optimizer because the optimizer is not allowed to see the "extreme" effectiveness of the first aircraft against the target type in particular weather conditions. A weighted average model will not necessarily provide valid answers for any HA scenario.

A second major function of the external model is to accommodate processing over multiple time periods. The number of time periods is limited to seven in the June 1982 model, and each time period is typically defined to last fourteen to thirty days. Model parameters do change over time. The number of dead targets ( $D_j$ ) is accumulated following optimization

for each time period. Other parameters (in particular, target values) are reinitialized at specified time intervals.

The use of multiple time periods in HA is myopic. That is, optimization proceeds forward in time, in one pass, with no backtracking. (Each optimization seeks to inflict maximum damage upon enemy targets *in that time period* without regard for the effect "decisions" made in the current time period may have on outcomes for future time periods.)

It is apparent that a target type with greater than average defenses will generally have a higher than average value. Having a higher target value will increase the likelihood the target will be attacked--at least in a short duration "conflict"--since we are simply trying to inflict the greatest damage upon the enemy. However, by avoiding this target type in a longer duration conflict and killing other less valuable and less heavily defended targets, the attacking aircraft might be used to inflict more damage on the enemy, over time, by surviving longer and killing a large number of less valuable targets.

Lack of *logistical constraints* poses the most significant difficulty of a practical nature for the June 1982 model. HA uses a preferred weapon without regard to its actual availability. The internal model never "sees" weapons--it only "sees" sorties. During a time interval, the model will continue to "fly" a sortie type as long as the objective function value is improved (subject to the model constraints). Even when it is evident the supply of a preferred weapon has been exhausted in previous time periods, there is no way to prevent the June 1982 model from continuing to use that preferred weapon in subsequent solutions.

### C. DESCRIPTION OF THE JUNE 1982 HA PROGRAM CODE

Appendix D is a hierarchy chart of the June 1982 HA programs. Sub-program NONLIN and all those programs strictly subordinate to it on that chart comprise the nonlinear optimizer, while the other programs manage input and/or output.

Calling disciplines, variable naming conventions, and documentation of the HA code are neither standardized nor consistent. Variable names in the system frequently do not agree with current system documentation or with [Ref. 2], nor are they consistent among subroutines. (Variable names within the optimization code are, for the most part, consistent with [Ref. 2].) Appendix E details the name changes of some of the variables pertinent to the interface between the main program and the optimizer. Appendix F [Ref. 4: pp. 4-5] provides a data flow for the entire NAP process as it was executed in June 1982.

### III. DISCUSSION OF TARGET VALUES IN HEAVY ATTACK

#### A. CRITICISM OF THE USE OF TARGET VALUES

In a recent analysis of HA by G. Jenkins [Ref. 5] (an Air Force civilian employee and user of HA) the manner in which inputs are prepared for HA, as well as the optimization technique used in the model, are discussed. He concludes the optimization is straightforward and infers all inputs except one are derived objectively and correctly. The one input he expresses concern over is target values.

" . . . the entire process is based on optimization of firepower scores. This model calls it military worth which is probably more correct, since command centers and runways really don't fit in the context of firepower scores as do tanks. Nevertheless the purpose of this entire process, which is pursued with meticulous objectivity throughout each set . . . boils down to optimizing the relative subjective worth of target values. Granted, this methodology is probably more credible than the proverbial smoke-filled-room approach; however, it indicates that there is still room for improvement" [Ref. 5: p. 12]

He continues,

" . . . there is room for concern over whether the model's solution is credible at all. Maybe the model merely serves as a guide to quantify and substantiate some decision-maker's intuitive feeling, so that he may proceed with what he always wanted or 'knew' to be true. Or, maybe there is a sincere interest in gaining insights into the combat process. If so, there is plenty of room for improvement in this model. . . . "[Ref. 5: p. 13]

Jenkins is not alone in expressing concern over the use of target values in computing an optimal munitions mix for air forces.

NATO first implemented the entire Sabre Mix (now called NAP) methodology with few alterations, but later removed target values from the formulation. The new formulation simply seeks to maximize the total number of targets killed subject to a (new) constraint for each target

type mandating predetermined target type kill proportions relative to other target types (a proportionality constraint). The new objective function, new constraints, and other problem constraints (much as originally found in HA) are optimized with a linear programming code [Ref. 6: pp. 4-11]. The reasons for embarking on the reformulation were given as follows:

". . . While it is recognized that the concept of military worth plays a central role in tactical air mission planning models with short planning periods, this approach was abandoned for the following reasons:

1. Considerable difficulty arose from trying to assign credible military worth functions to the different target types.
2. It was considered advantageous to let proportions in which targets of different target types are killed constitute input to, and not output from, a model designed to contribute to the solution of a logistical problem." [Ref. 6: p. 3]

The concerns expressed regarding the use of target values in HA can be summarized as follows. Target values are subjectively derived and, perhaps, invite manipulation of the model by those executing it. They are hard to assign in a credible fashion. They remain the only subjective input in an otherwise objective process, and their use might confuse the distinction between tactical and logistical decision making.

Discussions with Major F. Cooper, the officer currently charged with executing the NAP models [Ref. 7], reveals another perspective on the use of target values in HA. The HA model is run separately for each major theatre (Europe, Pacific, S.E. Asia, etc.) by a theatre project team consisting of modeling personnel (from Major Cooper's office, AF/XOX) and military contingency planners and intelligence experts currently assigned to the command responsible for the theatre being examined. Projected targets are identified and target values are



assigned for the first time in a given scenario. Targets are evaluated using a scale of zero to twenty (relative to a tank platoon which is given a base score of one). The model is then executed for one time period. Based on the results for that first time period and an examination of the projected replenishment capabilities of the allied and opposing forces, target values are assigned for the second time period.

This process is repeated until the model has been run for all time periods. This cycle--process, evaluate, modify, and reprocess--has historically developed credibility for the model among its theatre users. Manual intervention also allows target values to be manipulated so as to discourage inadmissible solutions produced by the June 1982 HA model (because of missing constraints). Such manual, judgmental manipulation is done with the full knowledge and concurrence of the theatre project team, and its impact on model realism is carefully evaluated before continuing with the next time period. This careful evaluation of results for each time period often requires twenty or more model runs for a given scenario. However, the model is thus not allowed to use weapons no longer procurable, or to use weapons in quantities greater than can be procured, or to fly an aircraft to targets outside the aircraft's range. When a theatre study is completed, the results reflect the project management team's consensus and its total combined military judgment.

#### 8. A CASE IN BEHALF OF THE USE OF TARGET VALUES IN HA

The use of target values in HA is viewed in vastly different ways by knowledgeable people familiar with the model. In reviewing the methodologies used by USAF nonnuclear munitions mix models, Jenkins found the use of target values in HA to be a weak link in an otherwise strong

chain. Loritzen finds their use difficult and inappropriate, while Cooper finds them absolutely necessary if the model is to reflect both the commander's priorities (as understood by his contingency planners and intelligence experts) and real-world constraints not accommodated by the model's formulation [Refs. 5, 6, 7].

Jenkins infers HA should replace the use of target values with a two-sided game reflecting optimal strategies,

" . . . we can see that this model is not totally unlike the other models . . . . However, there is no direct gaming structure. This model is purely a one-sided affair and the only service provided by the opponent is the supply of targets and an unaffected attrition rate. The staging of the scenario and reconstitution features of the model could involve certain optimal strategy games, but there is no evidence of this in the process . . ." [Ref. 5: p. 12]

While the above is intriguing, developing the two-sided game preferred by Jenkins would require a scale, a set of values with which both sides will be originally endowed and by which each side will either gain or lose, depending on their strategies and initial endowments. It is apparent that this scale of values will have to relate to target values. If it is granted that values for potential U.S. targets are not derived easily, then how much harder must it be to evaluate targets on a scale applying to both the U.S. and its opponent(s)?

NATO removed target values from the formulation for its strategic aircraft munitions mix model, but then apparently had a difficult time validating the proportionality constraint used in the new formulation. Studying the linear programming dual of their formulation reveals the dual variables are "marginal implied military worths of the targets" [Ref. 6: p. 15]. In fact, Loritzen concludes that in order to get valid results from the new formulation "the adjustment of proportionality

factors will necessarily have to be done manually by military experts until a situation is reached where the implied military worths do not differ significantly from their estimates . . . " [Ref. 6: p. 15].

If reformulating HA as a game or reformulating it using target proportionality constraints provides no relief from the use of target values, then perhaps running the engagement as a simulation model, from the first day of the engagement, can provide a more objective model input. It is this author's opinion that while the use of simulation model outputs for optimization inputs may be more objective, it is doubtful anyone would vouch for the output produced. Further, the cost of running such a simulation in all its required iterations might be prohibitive.

Finally, an analytical model might be used to provide input in lieu of "subjective" target values; however, some significant, though more subtle, aspects of battle would prove particularly difficult to model. In evaluating alternatives for incorporating target activated munitions in the NAP models, Cudney and Bloomquist state,

"The development of value curves *must* be based upon the judgment of military commanders and analysts who are experienced in estimating the effects that casualties *and delay* might have on the success or failure of specific military missions."<sup>1</sup> [Ref. 8: p. 65]

Combatant morale is also directly related to choices regarding target destruction priorities. Military commanders and analysts are the sole credible source for evaluating the effects of such factors.

Without consideration of such factors as delay and combatant morale, this author would find it hard to accept any substitute for target values produced by an analytic or simulation model.

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<sup>1</sup>Emphasis added.

If no apparent, preferable alternative to the use of target values exists, why is their use viewed so negatively? It is likely that the practice of manipulating target values to compensate for model shortcomings contributes to their poor reputation. If target values represent just that, the value of a target, then military analysts would find their use easier to accept as valid. It is difficult to stand behind model output when inputs must be manipulated so violently.

Still, the issue of subjective model input remains. It is this author's belief the use of such input is allowable, even necessitated, because the use of military judgment is still the preferred alternative when trying to account for all the imponderables existing on a battlefield. Unfortunately, while admitting military science is a "soft science," analysts still seek to "exorcise" their models of subjective military inputs (judgments) rather than incorporating and exploiting their use.

#### IV. HEAVY ATTACK OPTIMIZATION ALGORITHMS AND THEIR PERFORMANCE

##### A. THE ORIGINAL (JUNE 1982) NONLINEAR PROGRAMMING ALGORITHM

The optimization code present in the June 1982 HA program (consisting of those programs strictly subordinate to subprogram OPTMUM in Appendix D) appear to be a faithful translation from the original FORTRAN (to PL1 and back to FORTRAN) of the optimizing code described in [Ref. 2]. That code solves a general nonlinear program by a sequence of local linear programs. The algorithm employed,

"... is a 'local,' 'gradient,' 'stepwise' correction descent algorithm . . . . By a 'stepwise' procedure we mean that given a point  $y^0$  in the domain of the functions, a 'correction' vector  $\Delta y$  is determined and a new point  $y = y^0 + k \Delta y$  is used for the successor 'step.' It is a 'local' method because the correction direction  $\Delta y$  and its length (determined by the skalar  $k$ ) are obtained from the behavior of the system in a 'sufficiently' small neighborhood of the current point  $y^0$ . It is a 'gradient' technique inasmuch as the gradients of the function  $g_i(y)$ <sup>2</sup> are principally used to obtain the correction direction. . . ." [Ref. 2: pp. 13-14]

Use of this algorithm to perform an HA optimization process, while precise and correct, involves significant expenditure of computing resources (typically five to six CPU minutes on an IBM 3022). Anticipating a state-of-the-art optimizer would perform the optimization at less expense, the Air Force requested [Ref. 1] the June 1982 HA optimizer be replaced with the X-System.

##### B. THE X-SYSTEM

Like the original optimizer, the X-System solves a nonlinear problem with a sequence of local linear programs. (The HA internal model consists

---

<sup>2</sup>The objective function.

of a nonlinear objective function subject to entirely linear constraints so that for each linear program, all that is required is that a linear approximation be made of the objective function and then a "standard" linear program is run.)

The X-System has been operational, but under continuing development since 1974 [Ref. 9]. It is a general-purpose, state-of-the-art optimization system which is used as both a vehicle for research and as the basis for a number of commercially installed, customized applications optimizers. It consists of open FORTRAN subroutines and is implemented in FORTRAN IV. The subset of FORTRAN with which it is coded is accepted by a majority of FORTRAN compilers.

The X-System is designed to solve large-scale optimization problems, and is especially effective on mixed integer problems. Decomposition issues have been an area of major interest to the designers; however, the core linear programming module has received the most design effort. It exhibits many unique features including:

1. Hyper-sparse data representation [Ref. 10];
2. Complete, constructive degeneracy resolution [Ref. 11];
3. Basis factorization [Ref. 12]; and
4. Elastic range constraints [Ref. 9].

In order to best support the wide variety of applications using the X-System, the system is designed to support all other optimization features simultaneously with the nonlinear feature (e.g., sortie constraints form an intrinsic Generalized Upper Bound, or GUB, set which has been exploited in the nonlinear solution).

### C. NUMERICAL EXPERIENCE WITH THE X-SYSTEM IN HA

The X-System was imbedded in HA as a subprogram, without changing the internal or external models, and delivered to the Air Force in July 1982. Headquarters, United States Air Force (XOX/FM) has been testing this version of the HA code (called "Fast" Attack by its users) since then. The X-System returns solutions whose objective function values agree with those provided by the June 1982 code to the second or third significant digit *in less than one fiftieth of the time* while using a fraction of the compute region. However, the numbers of particular sorties chosen differ significantly in some instances. These differences result from the level of precision specified for the optimal objective function value which permits early termination of the algorithm with an acceptable solution.

Since both the optimizers described are supposed to deliver correct solutions, one can hardly help but be surprised when their optimal solutions for the same problem differ, no matter how small the difference. One fundamental difference in the two algorithms is that while the original optimizer included a coded gradient function for the objective function, the X-System uses an automatic numerical difference approximation to estimate gradients. Use of this approximation in the X-System to enhance robustness, that is, to eliminate the errors and frustration associated with coding derivative functions, appears to be responsible for the small differences in objective function value.

A solution precision factor is used by the X-System. This algorithm parameter directs the X-System to stop optimization at the first point where the tolerance of the solution is estimated to be comparable to user

requirements, or confidence in the precision of input data. (This avoids wasting computing resources extracting the "last few pennies" in optimality from a problem for which input coefficients have been rounded to the next higher dollar.) By tightening this factor somewhat, perceived instability in total numbers of individual sorties can be eliminated. However, if this tolerance factor is out of proportion with user confidence in input data, any "stability" achieved is illusory.

The X-System has enabled HA to be used on a *time-sharing* system. Internal models with 81 constraints (13 GUB) and 793 variables typically yield solutions in 10-15 CPU seconds on an IBM 3033 processor, using approximately 250K bytes. The much faster response of the enhanced system has not only enabled speedier evaluations for the theatres, but has encouraged a critical review of the HA model. It is now technically possible to add model enhancements because neither space nor time constrain the problem as they did with the prior optimizer.



## V. A NEW HEAVY ATTACK EXTERNAL MODEL (WEATHER WARS)

In an attempt to ameliorate some of the major problems in the HA model, a new external process was designed in July and partially implemented in August 1982. It is still undergoing testing and refinement.

The new model, known as "Weather Wars" (WW), does away with the use of weighted average sorties and attempts to logistically constrain the problem.

WW avoids the use of weighted averages by building sorties for a particular weather type. Optimization is performed and inputs are updated and reinitialized as in the former HA; however, at the end of processing for the last time period, another "weather war" is "fought" in which sorties are built for a different weather type.

Running separate "wars" for each anticipated weather type (typically six) is costly. However, the expense is offset by the speed with which the X-System provides internal model solutions. When weighted averaging of weapons expended is performed *following* completion of all "weather war" processing, the resulting *weighted average* solution is provided and, at no additional cost, so is the *maximax* solution, the number of weapons required to meet all constraints and inflict great damage upon the enemy even if the weather becomes the enemy's consistent ally.

WW logistically constrains the problem by noting maximum procurable quantities for each weapon type and by reading in a user-defined number of "best" weapon types for an aircraft operating against a particular target type in a given weather condition. Prior to each optimization,

sorties are built and probabilities of effectiveness ( $P_{ij}$ 's) are assigned using the "best" weapon type still available. If all the "best" weapon types for a particular aircraft/target combination are no longer available, the model builds sorties using a fictitious weapon type with a zero coefficient of effectiveness.

The internal model still has no explicit logistical constraint, and given time periods of fourteen to thirty days, will include sortie totals in some time period solutions which consume weapons in quantities greater than will be available. However, this "over-use" of a weapon type can be made arbitrarily small by reducing time period duration (and subsequently increasing total numbers of optimizations). Another possible method for strictly limiting the numbers of a weapon type used is to run the entire WW model iteratively, reducing the maximum procurable quantity of a weapon type to the number used in the time period prior to the one in which the "over-use" occurred.

WW permits analysis of realistic scenarios in which sorties are to be selected *after* weather conditions are known. The June 1982 HA infers an assumption that sorties must be planned with only synoptic weather forecasts, or that various weather conditions will exist throughout the theatre in specified proportions. WW presents opportunities for decision analysis incorporating *weather-dependent* target values and damage confirmability.

## VI. POTENTIAL FUTURE DIRECTIONS FOR HEAVY ATTACK

### A. HA INTERNAL MODEL AS A GENERALIZED NETWORK

It is often advantageous to reformulate an optimization problem into an equivalent model which may be easier to solve. Recent computational advances in the efficient solution of generalized networks [Refs. 13, 14] have allowed these relatively specialized linear programs to be solved in a fraction of the time required to solve them with "standard" linear programming techniques.

The internal HA model can be viewed as a generalized network if flight composition constraints are ignored and the objective function is simplified (by letting  $C_j = 0$  for all  $j$ , or assuming a target rich environment) as follows:

$$\text{maximize: } \sum_{j=1}^J \sum_{i=1}^I V_j P_{ij} S_{ij}.$$

The model can then be viewed as displayed in Figure 1.

More general views of HA, including weapons and time periods, can be formulated as multicommodity compositions of generalized networks.

The preceding perspective of the HA problem as a generalized network is of more than purely academic interest. Work has been published [Ref. 15] and research continues [Ref. 16] solving generalized networks with side constraints (such as flight composition constraints). A commercial quality optimization system exploiting this new technology could provide a new vehicle for enhancing the efficiency with which HA is processed.

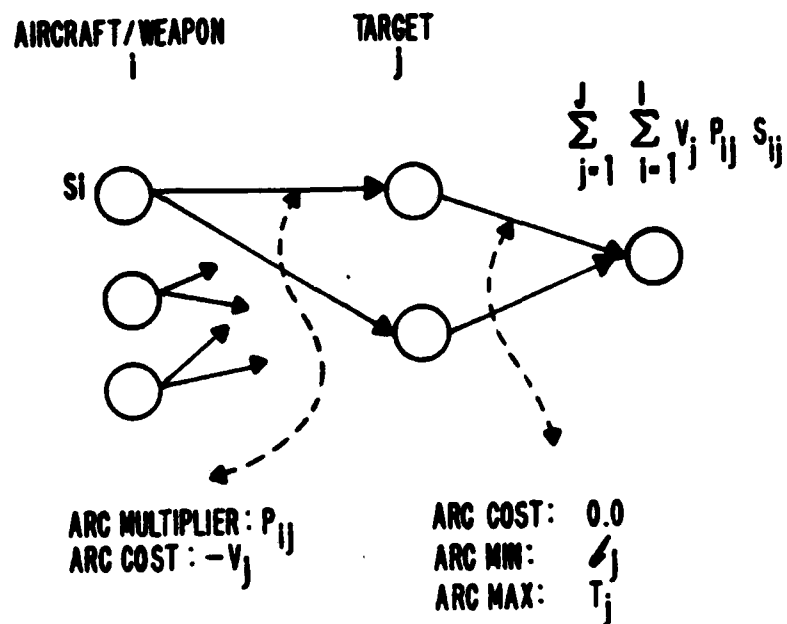


Figure 1. Generalized Network View of the Simplified HA Internal Model

B. AN EXPANSION OF THE HA INTERNAL MODEL TO INCLUDE WEAPONS AND TIME

Headquarters Armament Division (AFSC/XR) proposed to Headquarters United States Air Force in January 1982 that HA be reformulated as a nonlinear mixed integer problem. Appendix G presents the basic concept for the reformulation. As reported by Dean [Ref. 17: pp. 56, 59], the X-System has solved nonlinear integer and mixed integer programs. (However, the size of the integer problems Dean reported were considerably smaller than the typical HA internal model.) The proposed formulation is basically an expansion of the dimension of the internal model to include weapons.

Unfortunately, the model provided in Appendix G is intractable as stated and does not address all the issues pertinent to HA. It

entirely overlooks two model components described previously: weather and optimization over time. Also, it cannot ensure weapon selection of a "best" weapon for a particular aircraft/target combination prior to a "second best" weapon and so on to the "n<sup>th</sup> best." A prioritization constraint is presented composed, in part, of binary indicator variables. However, the mechanism for enforcing the use of these binary variables is not presented.

While a device for enforcing the use of the binary variables can be stated, this problem may be solved at less expense by allowing the selection of sorties to occur over the entire range of weapons considered by the problem. If the selection priority is ordered by  $P_{ijk}$  (expected type j target kills by a type i aircraft loaded with type k weapon), prioritization is then enforced intrinsically by the presence of  $P_{ijk}$  in the objective function. This alleviates the need for both prioritization constraints and binary variables, so that by expanding the size of the problem, the use of binary variables can be eliminated and the problem is made much easier to solve.

Using the AFXR/SC proposal, without integer variables, as basis for a new approach, optimization over time could be addressed by further expanding the dimension of the problem to include time. The resulting formulation would be:

$$\text{Maximize: } \sum_{j=1}^J \sum_{n=1}^I V_{jn}(K_{jn})$$

$S_{ikn}$

subject to:

1. Sorties available.

$$(1 - r) \sum_{n=1}^n RS_{in} \leq \sum_{k=1}^{NW} \left( \sum_{j=1}^J S_{ijkn} + \sum_{\rho=1}^{n-1} F_{ijk\rho} S_{ijk\rho} \right)$$

$$\leq (1 + r) \sum_{n=1}^n RS_{in} \quad i = 1, \dots, I; n = 1, \dots, N;$$

2. Target constraints.

$$\bar{R}_{jn} \leq \sum_{i=1}^I \sum_{k=1}^{NW} P_{ijkn} S_{ijkn} \leq \bar{R}_{jn}, \quad j = 1, \dots, J; n = 1, \dots, N;$$

3. Weapon logistics.

$$\sum_{i=1}^I \sum_{j=1}^J \sum_{n=1}^N L_{ijk} S_{ijkn} \leq W_k, \quad k = 1, \dots, NW;$$

4. Flight composition criteria.

$$\delta_m \left( \sum_{j \in J_m} (1 - \theta_m) S_{i_m j k n} + \sum_{j \notin J_m} (-\theta_m) S_{i_m j k n} \right) \leq 0,$$

$k = 1, 2, \dots, k; m = 1, 2, \dots, M;$

5. Target losses.

$$K_{jn} = D_j(n + 1).$$

All parameters retain their previous definitions unless (re)defined in the following:

$$K_{jn} = \frac{T_{jn}}{C_j} (1 - \exp(\frac{-C_j}{T_{jn}} (\alpha_{jn} + \sum_{i=1}^I \sum_{k=1}^{NW} P_{ijkn} S_{ijkn})));$$

$k$  = weapon type index ( $k = 1, \dots, NW$ );

$n$  = time period index ( $n = 1, \dots, N$ );

$r$  = sortie equality range restriction factor;

$W_k$  = number of available weapons of type  $k$ ;

$RS_{in}$  = resupply quantity (initial quantity, if  $n = 1$ ) of sorties available for aircraft  $i$  in period  $n$ ;

$$a_{jn} = \frac{-T_{jn}}{C_j} \log \left( 1 - \frac{C_j}{T_{jn}} D_{jn} \right);$$

$L_{ik}$  = number of weapons of type  $k$  loaded on aircraft of type  $i$ ;

$S_{ijk0} = 0.0$ ; and

$F_{ijkn}$  = friendly aircraft attrition factor.

Time periods would be connected by a friendly aircraft attrition factor,  $F_{ijkn}$ , associated with each aircraft/target/weapon combination. The target losses constraints equate dead targets in period  $n + 1$  ( $D_{j(n+1)}$ ) with those killed by the end of period  $n$  ( $K_{jn}$ ).

Unfortunately, if any  $C_j \neq 0$ , the target losses constraint is nonlinear and the model becomes somewhat more difficult to solve. Also, the attrition factor,  $F_{ijkn}$ , might realistically be defined as a nonlinear function of time period or prior attrition. However, it is believed these nonlinearities can be accommodated by a state-of-the-art optimizer.

The Weapons Logistic constraint is appropriate when HA is used as a munitions procurement model. In HA's secondary role, providing theatre commanders with insights into scenarios, a weapon logistics resupply constraint might be preferred. Let  $W_{kn}$  represent the *resupply* of weapon type  $k$  arriving in the theatre at the beginning of time period  $n$ . Such a constraint can be stated as follows:

### 3.1 Weapon Logistics resupply.

$$\sum_{n \leq CN} W_{kn} - \sum_{i=1}^I \sum_{j=1}^J \sum_{n \leq CN} L_{ik} S_{ijkn} \geq 0,$$

$CN = 1, \dots, N; k = 1, \dots, NW.$

However, this constraint, if used as stated, produces a dense problem matrix. Model clarity and optimization performance would improve with

the addition of variables  $w_{kn}$  to represent the unused weapons of type  $k$  remaining at the end of time period  $n$ . This new constraint can be stated as follows:

3.1.2 Weapon logistics resupply.

$$w_{k(n-1)} + W_{kn} - \sum_{i=1}^I \sum_{j=1}^J L_{ik} S_{ijkn} = w_{kn},$$

$n = 1, \dots, N; k = 1, \dots, NW;$

where,

$$w_{k0} = 0.0.$$



## VII. CONCLUSIONS AND RECOMMENDATIONS

This research was undertaken with the belief that HA was a single optimization model. Two models were identified and characterized. An internal (optimization) model was found, and an accompanying external model was discovered to principally constitute a circumvention of internal model shortcomings. Both models have been analyzed in detail, with special emphasis on the context of their use: scenario evaluation for theatres. The potential strengths and weaknesses of HA have been investigated.

HA has been provided with a new, fast optimizer. A new prototype external model, Weather Wars, which corrects some of the more obvious shortcomings of HA, has been designed and coded. To a degree, the prototype suffers from the same lack of standard calling disciplines, variable naming conventions, and documentation as its predecessor, precisely because it is a modification of its predecessor. Any further attempts to modify the existing external model are unlikely to provide continuing user satisfaction. However, an entirely new HA can be written to exploit the insights gained in this research.

What HA has always needed is a single model, a single formulation, which encompasses enough of the real problem to be adjudged realistic. While this may have been impossible in 1974, given the state-of-the-art in optimization, it is feasible now. A single model such as that proposed in the preceding section would not only be easier to understand, but when implemented should actually run faster than the current HA, providing better solutions.

## APPENDIX A. LINEAR RESTATEMENT OF TARGET CONSTRAINTS

Target constraints are of the form:

$$k_j \leq K_j \leq T_j, \quad j = 1, 2, \dots, J;$$

which is restated:

$$k_j \leq \frac{T_j}{C_j} (1 - \exp ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}))) \leq T_j, \\ j = 1, 2, \dots, J.$$

First the upper inequality is solved for the linear term  $\sum P_{ij} S_{ij}$ :

$$\frac{T_j}{C_j} (1 - \exp ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}))) \leq T_j,$$

$$(1 - \exp ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}))) \leq C_j,$$

$$1 - C_j \leq \exp ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij})),$$

$$\log (1 - C_j) \leq ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij})),$$

$$\frac{-T_j}{C_j} (\log (1 - C_j)) \geq (a_j + \sum_{i=1}^I P_{ij} S_{ij}),$$

$$\frac{-T_j}{C_j} (\log (1 - C_j)) - a_j \geq \sum_{i=1}^I P_{ij} S_{ij}.$$

Similarly, the lower inequality is solved:

$$k_j \leq \frac{T_j}{C_j} (1 - \exp ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}))),$$

$$k_j \frac{C_j}{T_j} \leq (1 - \exp ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}))),$$

$$1 - e_j \frac{C_j}{T_j} \geq (\exp ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}) )),$$

$$\log (1 - e_j \frac{C_j}{T_j}) \geq ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}) ),$$

$$\frac{-T_j}{C_j} (\log (1 - e_j \frac{C_j}{T_j})) - a_j \leq ( \sum_{i=1}^I P_{ij} S_{ij} ).$$

APPENDIX B. ALGEBRAIC MANIPULATION OF  $K_j - D_j$

If

$$K_j = \frac{T_j}{C_j} (1 - \exp ( \frac{-C_j}{T_j} (a_j + \sum_{i=1}^I P_{ij} S_{ij}) )),$$

then noting

$$\exp ( \frac{-C_j}{T_j} a_j ) = 1 - \frac{C_j}{T_j} D_j,$$

and letting

$$X_j = \sum_{i=1}^I P_{ij} S_{ij},$$

the term  $K_j - D_j$  (from the current HA internal model objective function) can be restated as follows:

$$\begin{aligned} K_j - D_j &= \frac{T_j}{C_j} (1 - \exp ( \frac{-C_j}{T_j} (a_j + X_j) )) - D_j \\ &= \frac{T_j}{C_j} (1 - (1 - \frac{C_j}{T_j} D_j) (\exp ( \frac{-C_j}{T_j} X_j ))) - D_j ( \frac{C_j}{T_j} ) ( \frac{T_j}{C_j} ) \\ &= \frac{T_j}{C_j} (1 - (1 - \frac{C_j}{T_j} D_j) (\exp ( \frac{-C_j}{T_j} X_j ))) - D_j ( \frac{C_j}{T_j} ) \\ &= \frac{T_j}{C_j} ( (1 - D_j \frac{C_j}{T_j} ) - (1 - \frac{C_j}{T_j} D_j) (\exp ( X_j \frac{-C_j}{T_j} )) ) \\ &= \frac{T_j}{C_j} (1 - D_j \frac{C_j}{T_j} ) - ( \frac{T_j}{C_j} - D_j ) (\exp ( X_j \frac{-C_j}{T_j} )) \\ &= ( \frac{T_j}{C_j} - D_j ) - ( \frac{T_j}{C_j} - D_j ) (\exp ( X_j \frac{-C_j}{T_j} )) \end{aligned}$$

$$= \left( \frac{T_j}{C_j} - D_j \right) \left( 1 - \exp \left( X_j \frac{-C_j}{T_j} \right) \right).$$

If  $C_j$  has a value of one, then  $K_j - D_j$  simplifies further to:

$$K_j - D_j = (T_j - D_j) \left( 1 - \exp \frac{-X_j}{T_j} \right).$$

If  $C_j$  has a value approaching zero, then  $K_j - D_j$  approaches  $X_j$ . Recalling  $(1 - \exp(-x))$  is approximately  $x$  for small  $x$ , and noting

$$\left( \frac{-C_j}{T_j} X_j \right) \rightarrow 0 \text{ as } C_j \rightarrow 0,$$

and further noting that as  $C_j$  approaches zero, the quotient  $T_j/C_j$  will become so large as to leave the effect of  $D_j$  negligible,  $K_j - D_j$  then becomes

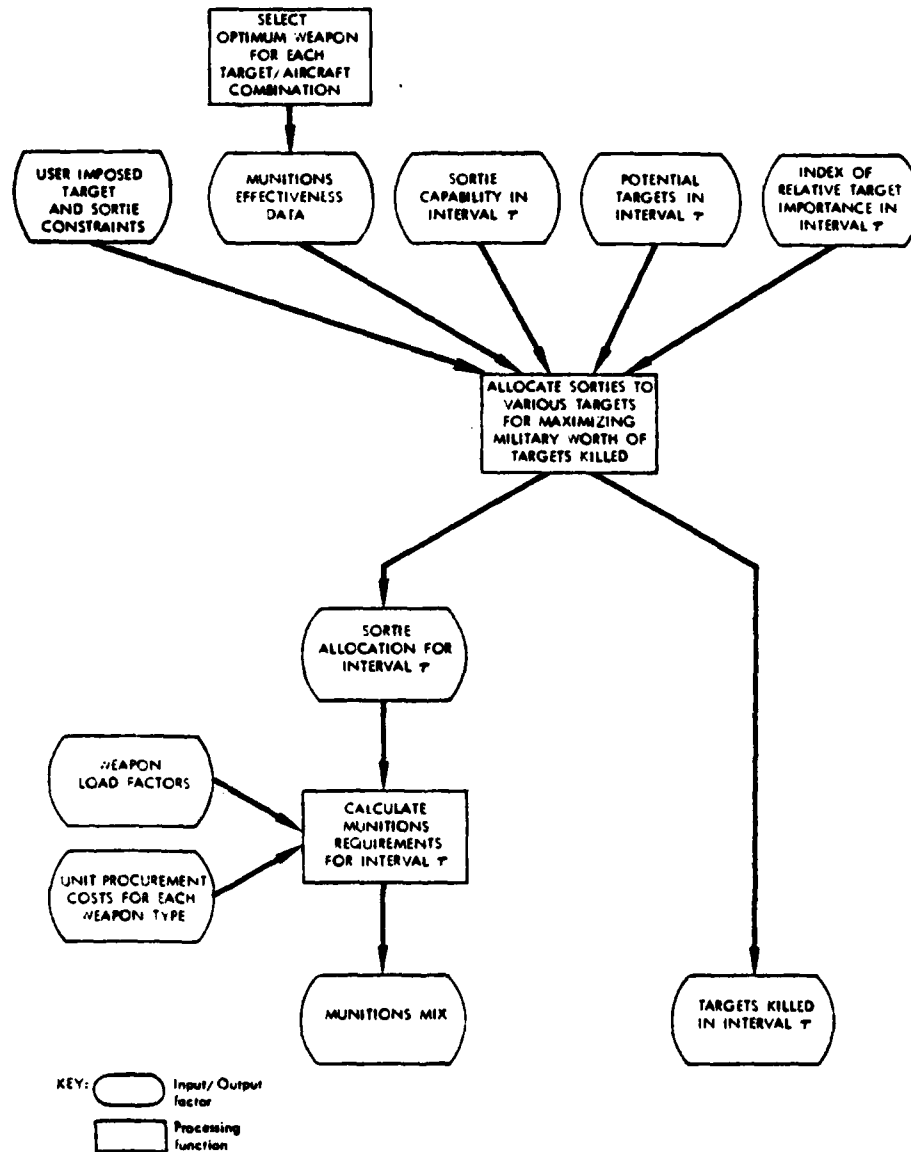
$$K_j - D_j = \left( \frac{T_j}{C_j} \right) \left( \frac{C_j X_j}{T_j} \right) = X_j, \text{ when } C_j = 0.$$

In a target rich environment, no matter what the value of  $C_j$ ,  $X_j/T_j$  will be quite small. Noting the fact that  $(1 - \exp(-x))$  is approximately  $x$  for small  $x$ ,  $K_j - D_j$  is finally approximated by:

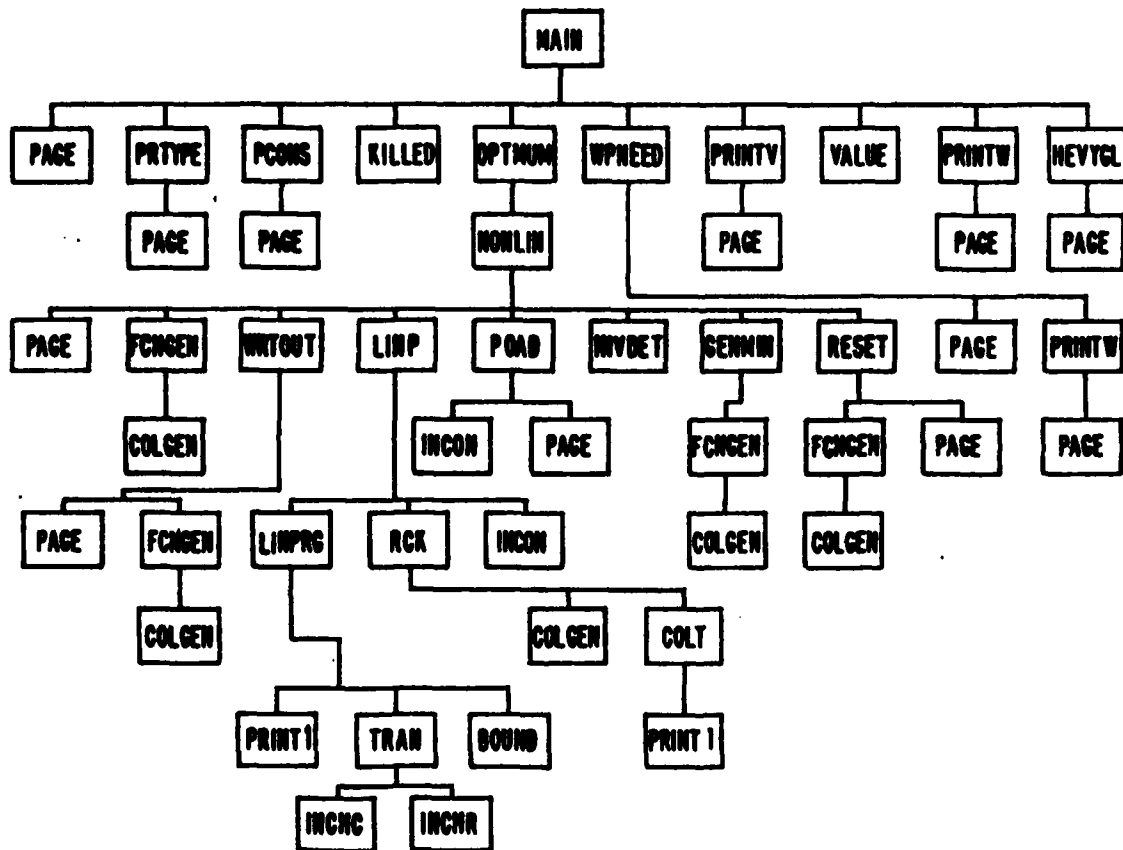
$$\begin{aligned} K_j - D_j &= (T_j - D_j) \frac{X_j}{T_j} = X_j - \frac{X_j D_j}{T_j} \\ &= X_j, \text{ for small } \frac{X_j}{T_j}. \end{aligned}$$

Thus,  $K_j$  is nearly *linear* in a target rich environment.

# APPENDIX C. ORIGINAL HEAVY ATTACK INPUT-PROCESS-OUTPUT



APPENDIX D. HIERARCHY CHART OF HA PROGRAMS IN USE AS OF 1 JUNE 1982

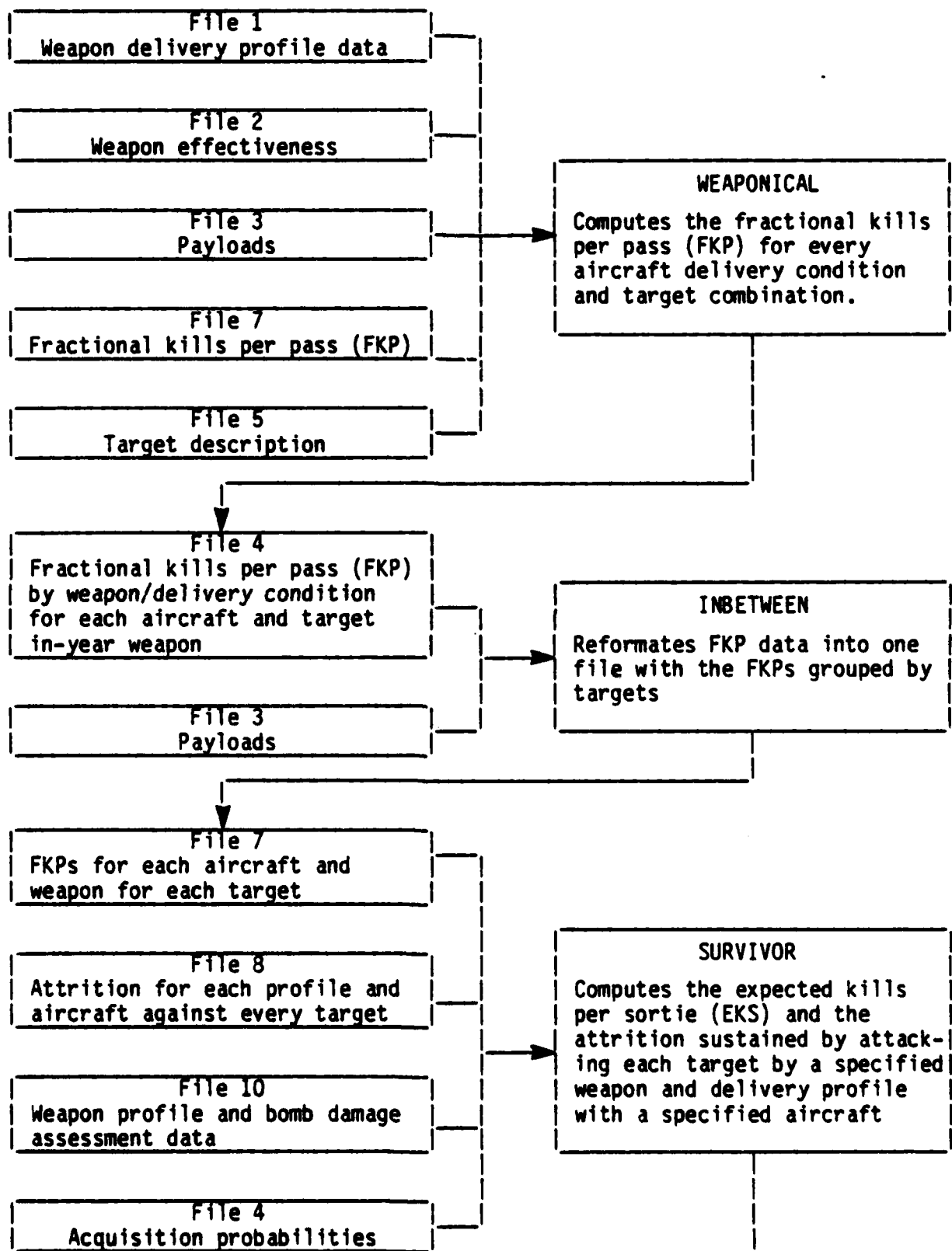


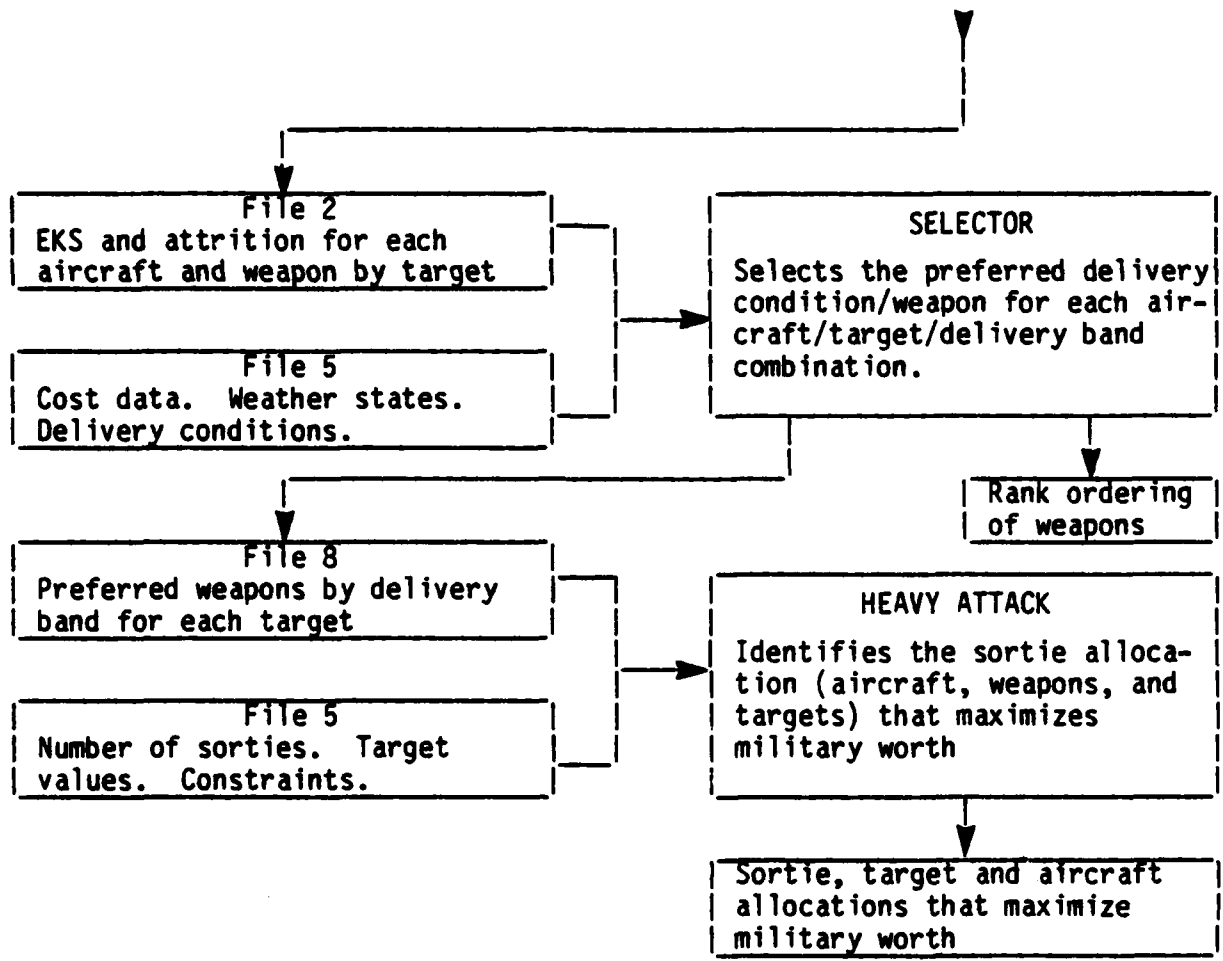
APPENDIX E. VARIABLE NAME CHANGES IN HA PRIOR TO OPTIMIZATION

Formulation Variable	Subroutine NONLIN	Subroutine OPTMUM	Main Program
$S_i$	SMAX	S	SORT
I	XNAT	NAC	NAC
J	XNTT	NTGTS	NTGTS
$T_j$	T	TTGT	----
$D_j$	D	DTGT	TD
$a_j$	SL	TL	----
$C_j$	CC	CC	CC
$V_j$	VT	VT	VT
$P_{ij}$	PROB	EKS	EKS
no. side constraints	XNADC	NADC	----
m	ROW	IDAC	----
$\theta_m$	PCTX	PCT	----
# in set $J_m$	NTSX	NUMTGT	----
set $J_m$	IARRY	IARRY	----
$S_{ij}$	YS	SOL	NSORT/SOL
Obj Fcn Value	GM	OBJ	OBJ



APPENDIX F. NAP MODELS INPUT-PROCESS-OUTPUT RELATIONSHIPS





APPENDIX G. AFSC/XR PROPOSED NEW HA METHODOLOGY

Definition of Variables:

- $i$  = Aircraft type index ( $i = 1, \dots, I$ );
- $j$  = Target type index ( $j = 1, \dots, J$ );
- $k$  = Weapon type index ( $k = 1, \dots, K$ );
- $T_j$  = Number of  $j$  type targets;
- $D_j$  = Cumulative number of targets killed in prior time periods;
- $P_{ijk}$  = Expected number of type  $j$  targets killed per type  $i$  aircraft loaded with type  $k$  weapon when no other targets of type  $j$  have been previously killed and when conditions of confirmability are perfect;
- $S_{ijk}$  = Number of sorties of type  $i$  aircraft flown with weapon type  $k$  against target type  $j$ ;
- $F_i$  = Quantity of sorties (fragable) available for aircraft type  $i$ ;
- $V_j$  = Value (Military Worth) of target type  $j$ ;
- $\epsilon_j$  = Lower bound on targets of type  $j$  to be killed;
- $Q_j$  = Number of type  $j$  targets killed;
- $C_j$  = Target kill confirmability parameter for  $j$  type targets (as defined in old formulation);
- $W_k$  = Quantity of type  $k$  weapons available;
- $L_{ijk}$  = Standard loadout of weapon type  $k$  on aircraft type  $i$  used against target type  $j$ ;
- $n$  = Number of members in the weapon prioritization sets;
- $n_{(i,j)p}$  = The  $p^{\text{th}}$  priority member (a  $k$  value) of the ordered weapon priority set of  $k$  values valid for the  $i^{\text{th}}$  aircraft type,  $j^{\text{th}}$  target type combination ( $p$  ranges from 1 to  $m^{\text{th}}$  best);
- $m$  = flight composition constraint index ( $m = 1, 2, \dots, M$ );
- $i_m$  = aircraft type;

- $\delta_m$  = +1, maximum (-1, minimum);  
 $J_m$  = set of targets for which a maximum (minimum) flight composition is required; and  
 $\theta_m$  = maximum (minimum) proportion of sorties flown by aircraft type  $i_m$  against targets included in set  $J_m$

The formulation:

$$\text{Maximize } Z = \sum_{j=1}^J V_j(Q_j - D_j)$$

where

$$Q_j = \frac{T_j}{C_j} \left( 1 - \exp \frac{-C_j}{T_j} \left( a_j + \sum_{i=1}^I \sum_{k=1}^K P_{ijk} S_{ijk} \right) \right)$$

$$a_j = \frac{-T_j}{C_j} \log \left( 1 - \frac{C_j D_j}{T_j} \right)$$

subject to:

Sortie constraint;

$$\sum_{j=1}^J \sum_{k=1}^K S_{ijk} \leq F_i; \quad i = 1, \dots, I;$$

Target constraints;

$$L_j \leq Q_j \leq T_j; \quad j = 1, \dots, J;$$

Side (flight composition) constraints;

$$\delta_m \sum_{j \in J_m} (1 - \theta_m) S_{ijmjk} + \sum_{j \notin J_m} (-\theta_m) S_{ijmjk} \leq 0,$$

$m = 1, 2, \dots, M;$   
 $k = 1, 2, \dots, K;$

Weapon constraints;

$$\sum_{i=1}^I \sum_{j=1}^J L_{ijk} S_{ijk} \leq W_k; \quad k = 1, \dots, K;$$

Prioritization constraints;

$$B_{ij^n(i,j)_{p+1}} - B_{ij^n(i,j)_p} \leq 0,$$

$$\begin{aligned} i &= 1, \dots, I; \\ j &= 1, \dots, J; \\ p &= 1, \dots, n; \end{aligned}$$

where this indicator variable is defined as:

$$B_{ij^n(i,j)_{p+1}} = \begin{cases} 0; & \text{if } S_{ij^n(i,j)_p} = 0 \\ 1; & \text{otherwise} \end{cases}$$

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