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In this report, an algorithm for adaptive control of continuous time single-input single-output systems is presented. With the hybrid approach, the control structure involves a continuous as well as a discrete time part, instead of being totally discrete or totally continuous as in previous approaches. The system is sampled and the adaptive gains updated at a variable rate varying with the magnitude of the error itself.

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MODEL REFERENCE ADAPTIVE CONTROL SYSTEMS: THE HYBRID APPROACH

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In this report, an algorithm for adaptive control of continuous time single-input single-output systems is presented. With the hybrid approach, the control structure involves a continuous as well as a discrete time part, instead of being totally discrete or totally continuous as in previous approaches.

The system is sampled and the adaptive gains updated at a variable rate varying with the magnitude of the error itself.

Introduction

The theory and application of Adaptive Control Systems have been a center of discussion in the last few years. Continuous-time [1], [6], [7], [8], as well as discrete-time [2], [5], [9], [10] schemes have been devised, and stability has been proved.

In spite of the continuous-time nature of real systems, from a point of view of applications, discrete-time algorithms are preferred to continuous-time, due to recent advances in digital technology.

However, the discrete approach is not closely coupled to the continuous-time behavior of real plants, making a "hybrid" approach (partly discrete, partly continuous) desirable. It is a well known result [1], [6], that, for a given plant, poles and zeroes can be arbitrarily placed with appropriate compensators as in Fig. 1. If the plant parameters are known exactly, then the control input which gives the desired behavior is of the form

$$u(t) = \underline{K}^{*T}\,\underline{w}(t),$$

$\underline{w}(t)$ being filtered versions of the plant input and output, and $\underline{K}^*$ an array of constants. In case of plant unknown, or partially known, the input assumes the form

$$u(t) = \underline{K}(t)^T\,\underline{w}(t),$$

where $\underline{K}(t)$ are adapted in order to have $\underline{K}(t) \to \underline{K}^*$.

In the hybrid scheme which will be the subject of this paper, the set of parameters $\underline{K}(t)$ are updated by a digital computer at discrete intervals of time $\{t_k\}$, and the continuous-time nature of $u(t)$ is preserved.

The overall scheme of the control system is shown in Fig. 2.

Recently, hybrid algorithms for adaptive control [4] as well as self-tuning regulators [11], have been devised. In [4] the adaptive gains $\underline{K}(t)$ are discretely updated at a fixed rate, in base of samples taken from the plant in a random fashion.

It turns out that the sampling scheme is crucial in order to establish stability of the closed loop system. In this paper, the system is sampled, and the adaptive gains updated, at a variable rate according to the magnitude of the continuous time error itself. It is shown that the continuous time error becomes smaller than any bound, arbitrarily determined, after a finite number of adaptation steps.

The problem is stated in Section 1, with the error model in Section 2. The adaptive law is discussed in Section 3, and Section 4 describes the sampling scheme, with proof of stability.

Notation

The following notation will be used:

- vectors: $\underline{a} = [a_1, a_2, \ldots, a_n]^T$;
- time delay operator: z;
- differential operator: $p = \frac{d}{dt}$
- $x(t) = O[y(t)]$ iff there exists a positive constant M such that $|x(t)| \leq M|y(t)|$, for any t;
- $x(t) = o[y(t)]$ iff $|x(t)| \leq \beta(t)|y(t)|$ for some function $\beta(t)$ such that $\beta(t) \to 0$;
- $x(t) \sim y(t)$ iff $x(t) = O[y(t)]$ and $y(t) = O[x(t)]$;
- $\mathcal{L}$ denotes Laplace Transform operation.

1. Statement of the Problem

A continuous time dynamic system (plant) can be described by the linear time invariant, non-autonomous differential equation

$$(1.1)\quad D_p(p)\,x(t) = D_u(p)\,u(t)$$

with $D_p(p) = p^n + a_1 p^{n-1} + \ldots + a_n$

$D_u(p) = b_0 p^m + b_1 p^{m-1} + \ldots + b_m$

The following assumptions are made on the plant parameters:

(i) the values of a_i, $i=1, \ldots, n$ and b_i, $i=0, \ldots, m$, are unknown;
(ii) $m \leq n-1$ is known;
(iii) the plant is minimum phase; i.e., the polynomial $D_u(p)$ is Hurwitz;
(iv) the sign of b_0 is known, as are bounds b_{0m}

and b_{oM}, where $b_{oM} \geq b_o \geq b_{om}$.
Without loss of generality, $b_{om} > 0$ will be assumed.

Given a model

(1.2) $D_m(p)\, x_m(t) = K_o r(t)$,

with $D_m(n) = p^n + a_{m1}p^{n-1} + \ldots + a_{mn}$, Hurwitz, the design objective is to determine an input to the plant $u(t)$ such that, for some $E_0 > 0$, $t_F > 0$

(1.3) $|e(t)| \leq E_0$, for every $t \geq t_F$,

where

(1.3') $e(t) \triangleq x_m(t) - x(t)$

In particular, we restrict the input $u(t)$ to be of the form

(1.4) $u(t) = \sum_{1}^{n} K_i(k)\psi_i(t)$, for $t\epsilon[t_k, t_{k+1})$

where $K(k)$, $i=1, 2, \ldots n$ is a set of gains updated only at discrete instants $\{t_k\}$, and $\psi_i(t)$ are continuous time, observable state variables of the system.

2. The Error Model

It has been shown in [1] that constant vectors $\underline{\beta}_u$ and $\underline{\beta}_x$ exist such that

(2.1) $D_m(p)e(t) = D_w(p)[-b_o u_f(t) + \underline{\beta}_u{}^T\underline{\phi}_u(t) + \underline{\beta}_x{}^T\underline{\phi}_x(t) + K_o\phi_o(t)]$

where the following definitions pertain:

- $D_w(p) \triangleq p^{n-1} + c_1p^{n-2} + \ldots + c_{n-1}$ is a Hurwitz polynomial such that $\frac{D_w(p)}{D_m(p)}$ is Strictly Positive Real (S.P.R.);
- $u_f(t)$ is such that $D_f(p)u_f(t) = u(t)$ where $D_f(p) = p^{n-m-1} + F_1p^{n-m-2} + \ldots + F_{n-m-1}$ is any Hurwitz polynomial of degree $n-m-1$;
- $\phi_u{}^i(t)$, $i=0, \ldots, n-2$ are solutions of $D_w(p)D_f(p)\phi_u{}^i(t) = p^i u(t)$;
- $\phi_x{}^i(t)$, $i=0, \ldots, n-1$ are solutions of $D_w(p)D_f(p)\phi_x{}^i(t) = p^i x(t)$;
- $\phi_o(t)$ is solution of $D_w(p)\phi_o(t) = r(t)$.

If we choose $D_m(p) = (p+\alpha)D_w(p)$, with $\alpha > 0$, a sequence $\{t_k\}$, and

(2.2) $u_f(t) = \underline{K}_u{}^T(k)\underline{\phi}_u(t) + \underline{K}_x{}^T(k)\underline{\phi}_x(t) + K_o(k)\phi_o(t) + w_1(t)$, for $t\epsilon[t_k, t_{k+1})$,

we can write (2.1) as

(2.3) $(p+\alpha)e(t) = \underline{\delta}_u{}^T(k)\underline{\phi}_u(t) + \underline{\delta}_x{}^T(k)\underline{\phi}_x(t) + \delta_o(k)\phi_o(t) - b_o w_1(t)$, for $t\epsilon[t_k, t_{k+1})$

where $\underline{\delta}_j(k) \triangleq \underline{K}_j(k) - b_o\underline{\beta}_j$, $j = u,x,o$.

In what follows, the sequences $\underline{K}_j(k)$ will be called the Adaptive Gains, and will be updated at the sampling instants $\{t_k\}$ only. Furthermore, the input $u(t)$ has to be determined such that (1.3) is satisfied.

If (2.3) is sampled at instants $\{t_k\}$, the samples of the error are related by the linear, time variant difference equation

(2.4) $e(t_k) = A_k e(t_{k-1}) + \underline{\delta}_u{}^T(k-1)\bar{\underline{\phi}}_u(k) + \underline{\delta}_x{}^T(k-1)\bar{\underline{\phi}}_x(k) + \delta_o(k-1)\bar{\phi}_o(k) - b_o\bar{w}_1(k)$

where we define

$T_k = t_k - t_{k-1}$;

$A_k = \exp(-\alpha T_k)$;

(2.5) $\bar{\underline{\xi}}_j(k) = \underline{\xi}_j(t_k) - A_{k-1}\,\underline{\xi}_j(t_{k-1})$, $j = o,u,x$;

$(p+\alpha)\,\underline{\xi}_j(t) = \underline{\phi}_j(t)$, $j = o,u,x$;

$$\bar{w}_1(k) = \int_{t_{k-1}}^{t_k} \exp -\alpha(t_k-\tau)w_1(\tau)d(\tau)$$

Introducing the auxiliary network

(2.6) $y(k) = A_k y(k-1) + q(k) + w(k)$

with $\eta(k) \triangleq e(t_k) + y(k)$, equations (2.4) and (2.6) yield

(2.7) $\eta(k) = A_k\eta(k-1) + \underline{\delta}^T(k-1)\,\bar{\underline{\phi}}(k) + w(k) - b_o\bar{w}_1(k) + q(k)$

where

$$(2.8)\quad \begin{aligned} \underline{\delta}^T(k) &\triangleq [\underline{\delta}_u{}^T(k) \quad \underline{\delta}_x{}^T(k) \quad \delta_o(k)]; \\ \bar{\underline{\phi}}(k) &\triangleq [\bar{\underline{\phi}}_u{}^T(k) \quad \bar{\underline{\phi}}_x(k) \quad \bar{\phi}_o(k)] \end{aligned}$$

Let us choose

(2.9) $w(k) = K_w(k-1)\bar{w}_1(k)$,

then (2.7) becomes

(2.10) $\eta(k) = A_k(k-1) + \underline{\delta}^T(k-1)\,\bar{\underline{\phi}}(k) + \delta_w(k-1)\bar{w}_1(k) + q(k)$,

which is the augmented error equation.

3. Adaptive Law

The equations in the previous section hold for any sampling sequence $\{t_k\}$, on which no hypothesis has been made so far.

If we suppose $\{t_k\}$ be a sequence with an infinite number of elements, then it is a well known result--[2], [3]--that equation (2.10) and the following adaptive law

$$(3.1)\quad \begin{aligned} \underline{\delta}(k) &= \underline{\delta}(k-1) - F\,\bar{\underline{\phi}}(k)\,\eta(k) \\ q(k) &= -\gamma_1\,||\bar{\underline{\phi}}(k)||^2\,\eta(k) \end{aligned}$$

$$\delta_w(k) = \delta_w(k-1) + \frac{1}{\lambda_w}\bar{w}_1(k)\,\eta(k)$$

with $F = \text{diag}\,\{\frac{1}{\lambda_i}, i = 1,N\}$, $\gamma > 1/2 \min(\lambda_i, \lambda_w)$, $\gamma_i, \gamma_w > 0$, yield $\{\underline{\delta}(k)\}$ be a uniformly bounded sequence, and moreover

(3.2) $\lim_{k\to\infty} \eta(k) = 0$

(3.3) $\lim_{k\to\infty} \bar{\underline{\phi}}(k)\,\eta(k) = 0$

Let us define the control input as

(3.4) $u(t) = \underline{K}^T(k)\;\underline{\psi}(t)$, $t \in [t_k, t_{k+1})$

where

(3.5) $\underline{\psi}(t) \triangleq D_f(n-m-1)\;\underline{\phi}(t)$;

equations (3.4) and (2.2) then yield

(3.6) $w_1(t) = u_f(t) - \underline{K}^T(k)\;\underline{\phi}(t)$, $t \in [t_k, t_{k+1})$,

which, together with (2.5), gives the remaining input to the auxiliary network

(3.7) $\bar{w}_1(k) = \bar{u}_f(k) - \underline{K}^T(k-1)\;\bar{\underline{\xi}}(k)$

$$(3.8)\quad \bar{u}_f(k) \triangleq \int_{t_{k-1}}^{t_k} \exp -\alpha(t_k-\tau)\; u_f(\tau)\; d\tau.$$

4. Stability and Sampling Scheme

A suitable choice of the sampling sequence $\{t_k\}$ is crucial to prove stability of the closed loop system. It is evident, in fact, from (3.1) that if the output of the plant grows without bound in an oscillating fashion, we might choose $\{t_k\}$ such that $\eta(k) = 0$ for every k, and the gains never be updated.

A sufficient requirement on the sampling sequence can be stated as follows:

Theorem 4.1. Let the sampling sequence $\{t_k\}$ have an infinite number of terms, and be such that

(4.1) $\sup_{s<t_k} |e(s)| = O[\sup_{n<k} |e(t_n)|]$.

Then the hybrid system described in the previous sections is uniformly stable and

(4.2) $\lim_{k\to\infty} e(t_k) = 0$

Proof: see [12].

The central idea contained in Theorem 4.1 is that stability of the overall system is guaranteed if the sampled error $\{e(t_k)\}$ grows at the same rate as the continuous time error itself--as stated in (4.1).

Notice that the random sampling scheme discussed in [4] satisfies (4.1)--as in [4, lemma 2]--and then can be implemented to obtain stability in an almost sure sense.

In what follows a variable rate sampling scheme will be discussed, in which the sampling instants are determined by the comparison of a filtered version of the error with the error itself.

Let $\varepsilon(\cdot)$ be such that

(4.3) $\varepsilon(t) \triangleq c_0 \int_0^t e^{-\lambda(t-\tau)} \, |e(\tau)| d\tau$

with

(4.4) $0 < c_0 < \lambda < \mathrm{Re}[\alpha_i]$, $i=1, \ldots, 2n-1$,

and $\{\alpha_i, i=1, 2n-1\}$ being the zeroes of the Hurwitz polynomial $D_m(s)D_f(s)D_u(s)$. Then the following can be proved:

Theorem 4.2. If the sampling sequence $\{t_k\}$ is chosen such that

(4.5) $t_{k+1} = \min_{\tau\ge t_k+T} \{\tau \,\big|\, |e(\tau)| \ge \max(E_0, \varepsilon(\tau))\}$

with E_0, T arbitrary positive constants, and $\varepsilon(\cdot)$ as in (4.3), then the Hybrid Adaptive Control System discussed in the previous sections has the following properties:

i) the error $|e(\cdot)|$ is uniformly bounded;
ii) the sampling sequence $\{t_k\}$ is a finite sequence;
iii) an instant t_F exists such that
(4.6) $|e(t)| \le E_0$, for every $t \ge t_F$.

Before going into the details of this Theorem some technical lemmas need to be proved. An example of sequence as in (4.5) is shown in Fig. 3.

Lemma 4.1. If the error $|e(\cdot)|$ grows without bound then $\{t_k\}$ as in (4.5) is an infinite sequence.

Proof. Suppose $\{t_k\}$ is a finite sequence. Then an integer N exists such that

(4.7) $|e(t)| < \max(E_0, \varepsilon(t))$, for $t>t_N+T$.

Combining this with (4.3) we obtain, for $t>t_N+T$

(4.8) $\dot\varepsilon(t) \le -\lambda\, \varepsilon(t) + c_0 \max(E_0, \varepsilon(t))$.

Inequalities (4.8), (4.7), (4.4) contradict the error growing without bounds.

QED.

In the following lemmas, further results are obtained when the error grows unbounded. If this is the case, an infinite sequence of instants $\{\xi_j\}$ exists such that

(4.9) $|e(\xi_j)| = \sup_{\tau\le\xi_j} |e(\tau)|$

$0 < |e(\xi_j)| < |e(\xi_{j+1})|$

Moreover, let us define a sequence of integers $\{k_j\}$ such that

(4.10) $t_{k_j-1} \le \xi_j < t_{k_j}$

with $\{t_k\}$ as in (4.5).

Then we can prove the following

Lemma 4.2. For $\{t_{k_j}\}$ as in (4.10) we can write

(4.11) $|e(t_{k_j})| \ge M_0\, e^{-\lambda T_{k_j}} |e(\xi_j)|$

for some constant $M_0 > 0$, λ as in (4.4) and $T_k \triangleq t_k - t_{k-1}$.

Proof. The definition of $\{t_k\}$ in (4.5) implies that, for every sampling instant

(4.12) $|e(t_k)| \ge \varepsilon(t_k)$.

Combining this with (4.3) we obtain

(4.13) $|e(t_{k_j})| \ge c_0\, e^{-\lambda T_{k_j}} \int_{t_{k_j-1}}^{t_{k_j}} |e(\tau)| d\tau$.

The adaptive gains being bounded and the error growing without bound yields the inequalities

(4.14) $|e(t)| \le M_1 \sup_{\tau\le t} |e(\tau)|$

for some positive constant M_1, and

(4.15) $\int_{t_{k_j-1}}^{t_{k_j}} |e(\tau)| d\tau \ge \frac{1}{2} \min\left(\frac{1}{M_1}, T_{k_j}\right) |e(\xi_j)|$.

Finally, inequalities (4.13), (4.15) and $T_{k_j} \ge T$ prove the lemma.

QED

Lemma 4.3. If the error grows without bound then the sequence $\{T_{k_j}\}$ is uniformly bounded.

Proof. By equations (1.3), (2.4), (2.7) the sampled error at instant t_{k_j} satisfies the equation

(4.16) $e(t_{k_j}) = e^{-\alpha T_{k_j}} e(t_{k_j-1}) - \eta(k_j) + e^{-\alpha T_{k_j}} \eta(k_j-1) + w(k_j) + \gamma ||\tilde{\phi}(k_j)||^2 \eta(k_j)$

It is shown in [12] that

(4.17) $|w(k_j) + \gamma ||\tilde{\phi}(k_j)||^2 \eta(k_j)| \le \delta(j)|e(\xi_j)|$

for some sequence $\{\delta(j)\}$ such that $\lim_{j\to\infty} \delta(j)=0$.

Furthermore, by (4.10), (4.9) and (4.11), the following inequality holds

(4.18) $|e(t_{k_j-1})| \le |e(\xi_j)| \le \frac{1}{M_0} e^{\lambda T_{k_j}} |e(t_{k_j})|$

Combining equations (4.16), (4.17), (4.18) we obtain

(4.19) $|e(t_{k_j})| \le \left[\frac{1}{M_0} e^{-(\alpha-\lambda)T_{k_j}} + \delta(j)\right] |e(t_{k_j})| + |\eta(k_j)| + |\eta(k_j-1)|$

where $\alpha - \lambda > 0$ by (4.4). By the result in (3.2), and being $|e(t_k)| \ge E_0 > 0$, boundedness of T_{k_j} follows from inequality (4.19).

QED

Proof of Theorem 4.2. Let us suppose the error $|e(\cdot)|$ grows without bound. Then the sequence $\{k_j\}$ as in (4.10) is an infinite sequence, and by lemmas 4.2 and 4.3 the sampled error satisfies (4.1). But Theorem 4.1 contradicts the error growing without bound. Then i) is proved.

To prove ii) suppose the sampling sequence $\{t_k\}$ to be an infinite sequence. If this is the case, by Theorem 4.1 equation (4.2) holds, which contradicts with the fact that $|e(t_k)| \geq E_0 > 0$ for every k. Then $\{t_k\}$ cannot be an infinite sequence.

Finally iii) comes from the fact that, by ii), an instant t^* exists such that

(4.20) $|e(t)| < \max(E_0, \varepsilon(t))$, for every $t > t^*$.

Combining (4.20) with (4.3) we obtain

(4.21) $\dot{\varepsilon}(t) \leq -\lambda\, \varepsilon(t) + c_0 \max(E_0, \varepsilon(t))$, $t > t^*$

and then an instant $t_F \geq t^*$ exists such that

(4.22) $\varepsilon(t) < E_0$, for every $t > t_F$

Inequalities (4.21) and (4.22) prove the last part of Theorem 4.1.

QED

Conclusions

An algorithm for hybrid adaptive control for single-input, single-output systems has been described.

The gains are updated at a variable rate, and the minimum time between samples can be set arbitrarily large. Uniform stability of the closed loop system is guaranteed, and the magnitude of the error can be driven smaller than an arbitrary threshold, in a finite number of adaptation steps.

Nothing has been said on the performance of the algorithm in presence of disturbances and unmodeled dynamics, which is the subject of current research.

References

[1] R.V. Monopoli: "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Trans. Automat. Contr. vol. AC-19, pp. 474-484, Oct. 1974.

[2] T. Ionescu, R.V. Monopoli: "Discrete Model Reference Adaptive Control with an Augmented Error Signal," Automatica, pp. 507-517, vol. 13, 1977.

[3] R. Cristi, R.V. Monopoli: "Computational Aspects of Discrete Time Adaptive Control," 19th Annual Allerton Conference, Monticello, IL, Sept. 1981.

[4] H. Elliott: "Hybrid Adaptive Control of Continuous Time Systems," Technical Report, Colorado State Univ., Sept. 1980.

[5] K.S. Narendra, Y.H. Lin: "Stable Discrete Adaptive Control," IEEE Trans. Autom. Contr., vol. AC-25, No. 3, pp. 456-461, June 1980.

[6] K.S. Narendra, L.S. Valavani: "Stable Adaptive Controller Design-Direct Control," IEEE Trans. Autom. Contr., vol. AC-23, pp. 570-582, Aug. 1978.

[7] K.S. Narendra, Y.H. Lin, L.S. Valavani: "Stable Adaptive Controller Design, Part II: Proof of Stability," IEEE Trans. Autom. Contr., vol. AC-25, June 1980.

[8] A.S. Morse: "Global Stability of Parameter-Adaptive Control Systems," IEEE Trans. Autom. Contr., vol. AC-25, June 1980.

[9] G.C. Goodwin, P.J. Ramadge, P.E. Caines: "Discrete Time Multi-Variable Adaptive Control," IEEE Trans. Autom. Contr., vol. AC-25, June 1980.

[10] I.D. Landau, H.M. Silveira: "A Stability Theorem with Applications to Adaptive Control," IEEE Trans. Autom. Contr., vol. AC-24, pp. 305-312, April 1979.

[11] P.J. Gawthrop: "Hybrid Self-Tuning Control," IEEE Proceedings, vol. 127, Pt. D, No. 5, Sept. 1980.

[12] R. Cristi, R.V. Monopoli: "Stable Algorithms for Hybrid Adaptive Control Systems," Technical Report UMASS AFOSR-82-1, March 1982.

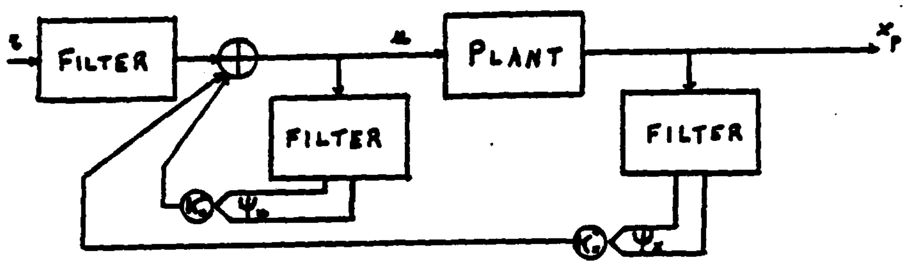

Fig 1 : Pole Placement Scheme.

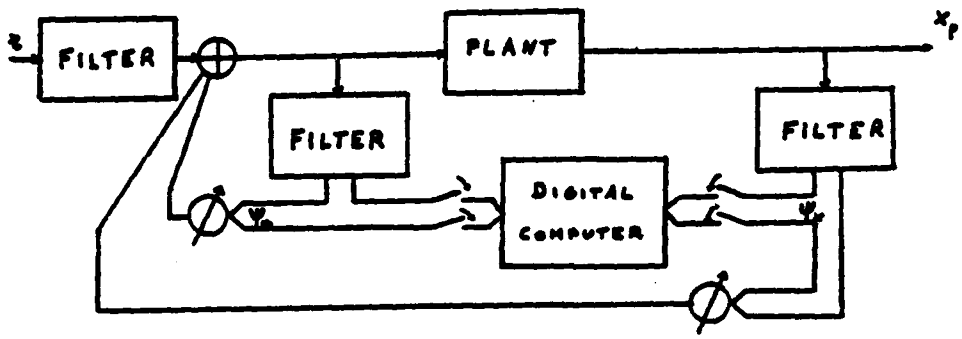

Fig 2 : Adaptive Controller Structure.

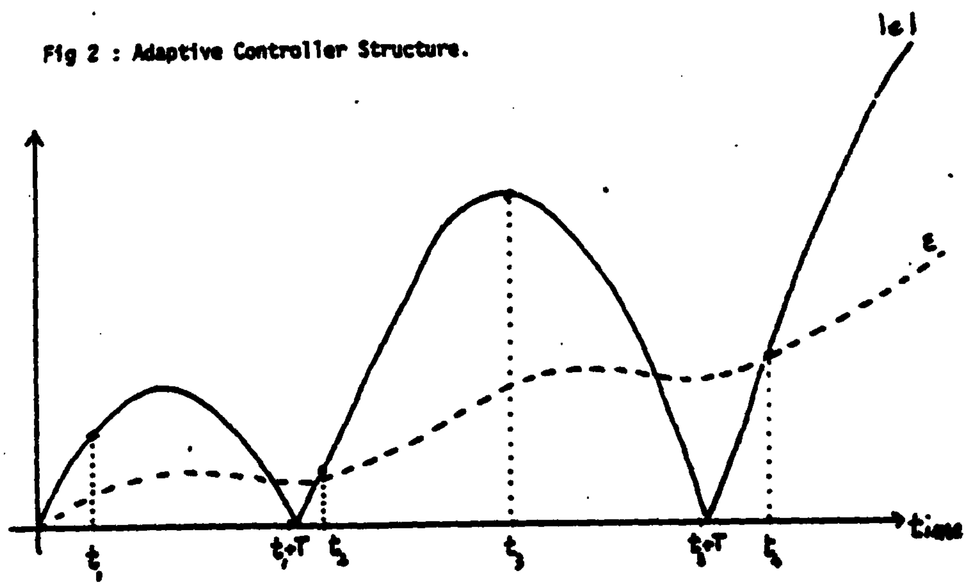

Fig 3 : Sampling Sequence.

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