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EVALUATION OF TERMINALLY GUIDED REENTRY VEHICLE EFFECTIVENESS AGAINST UNDEFENDED HARD TARGETS

THESIS

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AFIT/GST/OS/82M-2

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THESIS

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Abstract

This thesis examines the effectiveness of various reentry vehicle configurations when they are targeted against buried, hard targets. The configurations are based on the reentry vehicle parameters of yield and CEP as well as the number of reentry vehicles per missile. An examination of the ground shock and overpressure kill radii resulted in the use of overpressure as the hard target kill mechanism. The methodology developed to examine reentry vehicle effectiveness was programmed on a Hewlett-Packard HP-41CV. The methodology allows variations in CEP, weapon system reliability, weapon yield, and number of reentry vehicles per missile, and the desired kill level. The measure of effectiveness of each reentry vehicle configuration is the number of missiles required to achieve a desired kill level on a user defined target matrix. The results of the methodology were generalized with a set of exponential equations. Each equation is based on a desired kill level and a fixed number of reentry vehicles per missile. A sensitivity analysis on the various configurations revealed the relative impact of equal percentage changes in the factors used in this study.

EVALUATION OF TERMINALLY GUIDED
REENTRY VEHICLE EFFECTIVENESS
AGAINST UNDEFENDED HARD TARGETS

I. Introduction

The objective of a wartime ballistic missile launch is to deliver a nuclear warhead to a specified location in order to destroy a given target. Consequently, the accuracy of a missile system is important. Missile accuracy is measured in terms of circular error probable (CEP), which is defined as the radius of the circle around a target within which 50 percent of the missiles aimed at it will impact (10:1101). Advances in a number of fields such as electronics, materials, computers, and geodesy have contributed to the continued increases in the accuracy of Soviet and United States intercontinental ballistic missile (ICBM) systems (11:1192).

Accuracy, however, is only a concept that involves the distance between the target and the impact point of the warhead. The effectiveness of an ICBM is its critical attribute. A warhead with a relatively large CEP may be ineffective, and thus considered inadequate, when deployed against a hard target, which is a target resistant to nuclear effects. However, it may be effective, and thus sufficiently accurate, for use against soft targets which are susceptible to nuclear effects. Therefore, the effectiveness of an ICBM depends on target hardness and weapon yield as well as CEP (15:14).

The CEP required of an ICBM depends upon the strategy under which it may be launched. As a countervalue weapon, an ICBM would be targeted against a city or an industrial complex. When deployed as a countervalue weapon, a missile system may be effective without an extremely small CEP. On the other hand, a counterforce missile system would attempt to destroy hard targets such as missile silos. Thus, an ICBM launched under a counterforce strategy would require a relatively small CEP to be considered effective (14:393).

Current Situation

According to a 1977 Time magazine article, the most advanced operational U.S. ICBM is the Minuteman III (MMIII) with a mark 12A warhead and an NS-20 guidance system. It carries three 350 kiloton (kt) warheads to three independently targeted locations with an estimated 600 foot (.1 NM) CEP (17). Assuming a counterforce strategy, the MMIII could be targeted against a fifth generation Soviet ICBM. This new generation of missiles, beyond the SS-17, SS-18, and SS-19, is capable of carrying ten 500 kt warheads over 10,000 kilometers to their targets with an estimated CEP of less than 840 feet (.14 NM). Its new silo design is capable of withstanding overpressures of 6,000 pounds per square inch (psi) (7:28). To be effective against such a target, the MMIII must have a combination of warhead yield and CEP that provides a hard target kill capability. Hard target kill capability is generally defined as $Y^{2/3}/CEP^2$ where yield (Y) is expressed in megatons and CEP is expressed in kilofeet (5:169). Since this measure of effectiveness of a RV deployed against a hard target increases much more rapidly with improvements in CEP than with equal percentage increases in yield, the very hard Soviet silos provide a motivation to improve the CEP to the U.S. ICBM force (15:16-17).

Current Developments

The Advanced Ballistic Reentry System (ABRES) office at the Air Force's Ballistic Missile Office (BMO) has spent 18 years providing a technological basis for improvements and advances in ballistic missile technology for the Air Force, Navy, and Army. The development of the advanced maneuverable reentry vehicle (AMARV) was a significant achievement in maneuvering strategic missile technology. However, throughout its development, the emphasis of the ABRES maneuvering reentry vehicle effort has shifted between a terminal avoidance capability to evade Soviet anti-ballistic missiles and a terminal guidance system to approach a near-zero CEP (6:22). As a result, the ABRES office is concurrently developing two classes of an AMARV: Evader AMARV and Accuracy AMARV (12).

Evader AMARV. The Evader AMARV will be able to perform preprogrammed maneuvers to avoid interception during the reentry phase. It would be about the size of the MMIII Mark 12 reentry vehicle (RV), possessing an accuracy at least as good as the Mark 12. The Evader AMARV would contain a full three-axis, self-contained inertial guidance system, enabling it to continuously update its position with respect to prestored target coordinates. Following an evasion-type maneuver, the Evader AMARV would make the necessary change in its course to compensate for the maneuver. Such a system promises to eliminate some of the error factors such as boost separation errors, wind shears, and atmospheric behavior that contribute to the miss distance in the more conventional RVs that fly purely ballistic trajectories after separation from an inertially guided vehicle (6:22).

Accuracy AMARV. The Accuracy AMARV emphasizes a near-zero CEP. It would be similar in size and shape to the Evader AMARV, but it would not be capable of flying preprogrammed, avoidance-type maneuvers. It would use a

Terminal Fix System (TFS) to update its position, theoretically resulting in a very small CEP (12).

A typical flight profile would consist of a dive to 30,000 feet where the AMARV would begin near horizontal flight. There the TFS would determine the AMARV's position and update its navigation system. The navigation system would then steer the AMARV to its correct location and initiate a dive to the target (13).

Goodyear, McDonnell Douglas, and Raytheon hold contracts to develop TFS sensors for the Accuracy AMARV (12). Goodyear is developing a range-only correlation system that locates the RV by correlating radar ranging information with stored data. McDonnell Douglas is developing a Tercom mapping system that measures height above the terrain, compares these data with prestored profiles, and makes necessary position corrections. Raytheon is developing a pulse Doppler radar and map matching scheme in which a Doppler sensor would map the scene along the RV's path, compare it with prestored maps, and make required location corrections (6:23). By flying low over miles of terrain, these sensors update and correct the RV's position (12). This system, like the Evader AMARV, promises to eliminate some of the previously mentioned error factors prevalent in the more conventional RVs.

Significance

The primary objective of U.S. strategic forces is to deter a nuclear attack against the United States or its allies. This strategic deterrence depends heavily on a credible retaliatory counterforce capability which must be both survivable and effective (3:123-124).

It has been estimated that with a .14 NM CEP, the Soviets would have achieved by 1981 the capability to destroy 95 percent of the U.S. ICBM force (7:28). The vulnerability of the Minuteman system implies that a significant portion of the U.S. strategic TRIAD has eroded with respect to the perceived Soviet threat. This apparent weakness in the Minuteman system could possibly encourage the Soviets to attempt to undermine the rest of the TRIAD (3:6). In addition to the improved Soviet ICBM accuracy, the new generation of Soviet ICBMs are survivable, being either mobile or hardened to withstand overpressures of 6,000 psi (7:28). Thus, Soviet ICBM advances seriously question the United States' ability to conduct an effective retaliatory counterforce attack. To reinstate a credible retaliatory counterforce capability necessitates improvements in both the survivability and the accuracy of the U.S. ICBM force.

The AMARV program is contributing to the technology base required to improve ICBM accuracy, which is necessary to offset increases in Soviet target hardness levels. Use of terminally guided RVs may provide the capability to implement a highly accurate, mobile ICBM which would increase ICBM survivability as well as effectiveness (8:16). The U.S. submarine launched ballistic missile (SLBM) fleet is considered survivable but ineffective against very hard targets (3:124). AMARV technology would provide the SLBM fleet with a credible missile silo kill capability, thereby enhancing the flexibility and effectiveness of the U.S. strategic TRIAD.

An operational AMARV would improve the posture of the U.S. strategic TRIAD; however, the combination of warhead yield and CEP must be carefully considered. Even a very small CEP may offer no advantage if the warhead yield is too small just as a large warhead yield may be ineffective when coupled with a poor guidance system. The challenge of implementing an AMARV is determining the proper combination of warhead yield and CEP required to form an effective weapon system.

II. Problem Statement

The problem posed by the ICBM Requirements Division at Strategic Air Command Headquarters (HQ SAC/XPQ) was to assess the effectiveness of a terminally guided RV used as a hard target weapon. Major Schankel, of HQ SAC/XPQ, stated the division was interested in determining the effectiveness of a terminally guided RV with a presumably smaller yield and CEP than a ballistic RV. His office showed no interest in an anti-ballistic missile avoidance capability (9).

Objectives

The objective of this study was to develop a methodology to compare the effectiveness of RVs with different yield and CEP combinations when they are deployed against hard targets. Through this comparison, the determination of whether or not to deploy a terminally guided RV could be made. Should the decision to deploy a terminally guided RV be made, this methodology could assist in analyzing which specific yield and CEP configuration would be most effective.

Specific Goals

To accomplish the stated objective, the following specific goals were set:

1. Develop an analytical technique to compare the effectiveness of variously configured RVs.

2. Automate the analytical technique to the extent practicable.

Criterion

In developing the analytical technique, or methodology, the criterion of equal effectiveness was adopted. The methodology allocated RVs to targets such that variously configured RVs would inflict approximately equal levels of kill on a defined set of targets. The cost associated with each RV configuration was the number of missiles required to achieve the desired level of kill. In this sense, cost is defined as the expenditures of a scarce resource, an ICBM, and not necessarily a monetary amount. Thus, the most cost-effective system would be the RV configuration requiring the fewest number of missiles to achieve the desired targeting objective.

Scope and Limitations

1. This study examined only the tradeoff between yield and CEP of a single hypothetical ICBM, developing a methodology for specific use.

2. This study did not examine the astronautics or navigation involved in terminal guidance systems, but rather used assumed yields and CEPs.

3. This study only considered buried, hard targets, and, consequently, did not consider multiple target kills with a single RV.

4. The targets were considered to be of equal value.

5. Anti-ballistic missile systems were not considered.

6. This study did not take targeting complications such as footprint patterns, target location, or RV timing into account.

7. To avoid unrealistically high allocations of RVs, each single target was allocated five or fewer RVs.

III. Methodology

Introduction

In order to compare the effectiveness of variously configured RVs, a methodology for specific use was developed. Careful consideration of the appropriate kill mechanisism led to the expression for the single shot probability of kill (SSPK) from which the probability of kill for multiple shots could be determined. Using these probabilities, a target matrix kill level was calculated by targeting a sample ICBM force against a proportionately-sized target matrix, which is defined as a representative set of the targets of interest. Targeting the alternative ICBM forces to ensure equal target matrix kill levels provided the basis for comparing the variously configured RVs.

Kill Mechanism

In considering the effectiveness of any weapon against a particular target type, the kill mechanisms considered are critically important. The kill mechanism employed in this study is overpressure. However, the damage to buried structures inflicted by nuclear bursts detonated near the surface of the earth is in general difficult to predict. Glasstone and Dolan recommend a simple and practical approach, consisting of examining three distinct regions about the impact point of the RV (4:241-242).

The first region is the true crater, which is larger than the apparent, or observed, crater. While the apparent crater is the depression in the ground caused by the explosion, the true crater extends to the point at which a definite shear in the earth has occurred (4:253). There is essentially complete destruction of objects in this region.

The second region consists of the rupture zone and the plastic deformation zone. Its size is approximately the area affected by major ground displacement, which can have a radius as large as two and one-half times the radius of the apparent crater (R_a). Damage to underground structures in this region is most frequently calculated with computer codes for each target and burst encounter condition. However, even though structural damage depends on the structure's size, shape, flexibility, and orientation to the burst, as well as soil characteristics; evidence indicates that the degree of damage to the structure is related to the radius of the apparent crater. Hard targets within $1.25R_a$ of the explosion should fail due to collapse (4:266). This ground shock kill radius also corresponds to the crest of the crater lip (4:253).

Beyond the plastic deformation zone is the third region. Ground shock is not a significant damage mechanism for hard targets in this zone.

In addition to ground shock, hard targets such as ICBM launch facilities are also vulnerable to overpressure effects as they are partially exposed to the atmosphere. Assuming an overpressure kill mechanism, the hard target kill radius was developed in Appendix A. The kill radius is

$$R_k = \frac{Y^{1/3}}{(.0680(H) - .2272(H)^{1/2} + .1899)^{1/3}} \qquad (3\text{-}1)$$

where R_k is the kill radius in NM, Y is the yield in megatons, and H is the target hardness in psi. The overpressure kill radii for values of interest in this study are listed in Table I.

TABLE I

OVERPRESSURE KILL RADIUS (FT)

Yield (kt)	Target Hardness (psi)		
	6000	8000	10000
25	240	218	202
50	302	274	254
150	436	395	367
250	517	469	435
350	578	524	486

To compare overpressure effects with ground shock effects, the ground shock kill radii, defined as $1.25R_a$, were calculated. The apparent crater radius, R_a, may be approximated by

$$R_a \simeq R_o Y^{0.3} \qquad (3\text{-}2)$$

where R_o is the apparent radius in feet for a one-kiloton explosion, and Y is the weapon yield expressed in kilotons (3:253). Assuming a ground burst, Table II displays the ground shock kill radius for the values of interest.

TABLE II

GROUND SHOCK KILL RADIUS (FT)

Yield (kt)	Soil Type			
	1	2	3	4
25	269	200	190	161
50	331	246	235	198
150	461	343	325	275
250	537	480	456	320
350	594	537	511	355

Soil Type 1: Wet soil or wet soft rock
Soil Type 2: Dry soil or dry soft rock
Soil Type 3: Wet hard rock
Soil Type 4: Dry hard rock

Examination of the data in Tables I and II reveals that the overpressure kill radii either contain or roughly approximate the ground shock kill radii for soil types two, three, and four. In these cases, consideration of overpressure effects alone should provide a close estimation of the number of targets destroyed in an attack scenario. For target hardness values up to 8000 psi, the data suggests this is an excellent assumption. However, the larger yields on a 10,000 psi target in soil type two result in overpressure kill radii that are only about 90 percent as large as the ground shock kill radii. Careful examination of the kill radii would be required if one were to consider target hardness values above 10,000 psi.

For soil type one, the overpressure kill radii are all smaller than the ground shock kill radii. The failure to consider ground shock in this case would result in underestimating the number of targets killed in an attack scenario. However, Soviet planners would probably attempt to avoid the deployment of an ICBM force in soil type one for two reasons: greater vulnerability and the deleterious effects of moisture on launch facilities, propellants, and electronic equipment. Thus, use of overpressure effects alone as the hard target kill mechanism will provide a sound basis for comparing the effectiveness of various RV configurations.

An additional consideration is appropriate if strategic reconnaissance is available in such a scenario. It would classify targets as either killed or not killed.

Targets that were in the crater would undoubtedly be recognized as having been killed. Outside the crater radius, the only discernible damage would have been inflicted by overpressure. Thus, from an operational perspective, the overpressure kill radii define the areas of discernible damage. Therefore, the overpressure kill radii should provide a close approximation of the number of targets killed or, at least, considered killed in an attack scenario.

Probability of Kill

Assuming normally distributed targeting errors in two dimensions and the kill radius described by equation 3-1, the single shot probability of kill is

$$SSPK=1-e^{-.6391\left(\frac{Y^{2/3}}{(.0680(H)-.2272(H)^{1/2}+.1899)^{2/3}(CEP)^{2}}\right)} \qquad (3\text{-}3)$$

where Y is the yield in megatons, H is the target hardness in psi, and CEP is expressed in nautical miles. The derivation of the SSPK equation is contained in Appendix B. If the weapon system was 100 percent reliable, the SSPK value would represent the probability of a target kill with a single shot. However, to account for weapon system failures, SSPK must be multiplied by an appropriate reliability factor, R. Thus, the probability of a target kill with a single shot is SSPK*R. The probability of kill for n shots is (17:71)

$$P_k = 1 - (1\text{-SSPK*R})^n \qquad (3\text{-}4)$$

<u>Target Matrix</u>

One may be tempted to use the SSPK*R value mentioned above and a single target to evaluate the effectiveness of various RV configurations. However, examination of a single target is not sufficient. Table III demonstrates that there exists a band of SSPK*R values that requires the same number of RVs on target to achieve a desired P_k on a particular target.

TABLE III

MINIMUM SSPK*R VALUES REQUIRED TO EFFECT AN EXPECTED KILL LEVEL WITH n RVs

	P_k		
n	.70	.80	.90
1	.7000	.8000	.9000
2	.4523	.5528	.6838
3	.3306	.4152	.5358
4	.2599	.3313	.4377
5	.2140	.2752	.3690
6	.1818	.2353	.3187
7	.1580	.2054	.2803
8	.1397	.1822	.2501
9	.1252	.1637	.2257
10	.1134	.1487	.2057

For example, if the desired P_k is .80, three RVs are required if the SSPK*R value lies within the range from .4152 to $.5528^{-}$. Thus, while one RV configuration may exhibit a higher SSPK*R than another, it does not necessarily require fewer RVs on a single target to at least achieve the desired P_k.

Although the number of RVs required is unchanged, differing SSPK*R values within that range will result in different P_k values. Using equation 3-4, three RVs with a SSPK*R of .55 would result in a P_k of .9089, while the P_k figure for three RVs with a SSPK*R of .42 is .8049. The RV with a SSPK*R of .55 appears to be the most cost-effective system when cost is defined as the expenditure of a RV; it achieves a greater P_k for the same cost. However, the desired P_k was .8. If the excess P_k of the more cost-effective system represents a near-zero marginal utility, then the utility-to-cost ratio is essentially equal for the two alternatives. Increases in the P_k above a desired level may be of little value to a planner, targeter, or commander. Thus, the analysis of RV alternatives should focus on the desired level of effectiveness in order to avoid the use of potentially misleading cost-effectiveness ratios. The simple examination of a single target clearly prevents this type of analysis.

As an alternative, a representative target matrix could be used to determine how many missiles would be required to effect the desired P_k over the entire target matrix. The examination of many targets allows the development of a linear combination of single target P_k values that equals the desired target matrix kill level. The variables in the equation are the proportions of targets attacked by the number of RVs that corresponds to each P_k value. Solving the equation provides the proportions of targets that should be attacked with specific quantities of RVs. From these proportions, the number of RVs and missiles required to achieve the desired target matrix kill level can be determined. Thus, RV configurations can be compared with respect to a desired level of effectiveness.

To properly employ the target matrix approach, the target matrix should be a representative sample of expected target types. In addition, the ratio of targets in the matrix to missiles available to attack them should be realistic.

Targeting Technique

The targeting technique assumes all targets are equally valued. Thus, the RVs are allocated to ensure the kill level for each target category is approximately equal to the desired kill level for the entire target matrix. The resulting target matrix kill levels for different RV configurations then represent approximately equal levels of effectiveness on which to base RV configuration comparisons.

To initiate this procedure, one must determine a pair of P_k values for a given target and RV combination that bounds the desired P_k. $P_k(n)$, the P_k based on n RVs, must be greater than or equal to the desired P_k while $P_k(n-1)$ must be less than the desired P_k. The desired P_k is then equal to a linear combination of $P_k(n)$ and $P_k(n-1)$. The variable X is defined as the proportion of targets attacked with n RVs and 1-X is the proportion attacked by n-1 RVs. The desired P_k is represented by

$$(P_k(n-1))(1-X) + (P_k(n))(X) = P_k \qquad (3\text{-}5)$$

Solving equation 3-5 for X yields the proportion of targets that should be attacked with n RVs. Rounding the product of X times the number of targets in the category to the nearest integer provides $T_{i,n}$, the number of targets in category i attacked by n RVs. The difference between the total number of targets in the category and $T_{i,n}$ is the number of targets attacked by n-1 RVs, identified by $T_{i,n-1}$. Since $_{i,n}$ was rounded to the nearest integer, the P_k for the particular category may not exactly equal the desired P_k. Letting T_i be the total number of targets in category i, equation 3-6 provides the calculated P_k for the i^{th} target category.

$$P_k(i) = ((P_k(n-1))(T_{i,n-1}) + (P_k(n))(T_{i,n}))/T_i \quad (3\text{-}6)$$

Repeating this process for each target category yields a kill level $P_k(i)$ for each target category. The $P_k(i)$ values can be used to determine the expected target matrix kill level for a particular RV combination by

$$P_k = \sum_i T_i P_k(i) / \sum_i T_i \quad (3\text{-}7)$$

This P_k value will probably not exactly equal the desired P_k due to rounding errors. However, in order to compare different RV comparisons, the assumption that the calculated and desired P_k values are equal is made. The maximum possible difference between these values is derived in Appendix D. For a 100 target matrix with five or fewer target categories, the calculated P_k will be within .00625 of the desired P_k.

Evaluation of Effectiveness

For the RV configurations capable of attaining the desired P_k, the number of missiles expended can be calculated by dividing the number of RVs allocated by the number of RVs per missile. Since fractional missiles cannot be launched, the number of missiles required should be rounded to the next higher integer to ensure the computed P_k is achieved. The extra unassigned RVs are slack RVs. They are not required by the analysis, but the multiple RV configuration of the missiles required their expenditure. They may or may not be deployed against the 100 target matrix under consideration. Thus, the P_k will be greater than or equal to the computed P_k.

Considering the expenditure of a missile as a single unit of cost, the RV combination requiring the fewest missiles is the most efficient and, thus, the preferred alternative. By requiring fewer missiles to accomplish the targeting objective, it provides for greater targeting and retargeting flexibility as well as the capability of destroying more hard targets than any of the alternatives. Hence, it is the most effective as well as the most efficient alternative. It is also the most survivable offensive threat as fewer missiles would be required to survive a first strike in order to maintain the capability to inflict an unacceptable level of damage on a potential adversary.

Sensitivity Analysis

The above methodology depended on fixed values for the weapon CEP, yield, and reliability for each RV configuration, the target hardness levels, and the desired target matrix kill level. Changes in these values could alter the effectiveness evaluation of the various RV configurations. Thus, when there is uncertainty in any of the fixed values, an analysis is required to determine if the ranking of alternatives is sensitive to the uncertainties.

Recall that a simple evaluation of the SSPK*R was not sufficient to compare RV configurations. Neither is such a simple evaluation adequate in conducting a sensitivity analysis. In conducting a sensitivity analysis, one should vary the appropriate parameters, in turn, and examine the effects of the changes.

A sensitivity analysis may also provide key insights into which system improvements may be most effective. Tables representing the number of missiles required under various hypothesized configurations will provide a direct measure of the impact on a RV's effectiveness due to changes in one of its parameters. These insights can help focus engineering and development efforts on those parameters or components that have a direct, significant impact on system effectiveness. Similarly, such insights can also assist in identifying efforts to improve parameters or components that have little impact on the number of missiles required to achieve a desired P_k.

HP-41CV Implementation

The methodology presented above was programmed on a Hewlett-Packard HP-41CV handheld, programmable calculator. The program is described in Appendix C-1 which also contains a detailed set of user instructions.

IV. Application of Methodology

Introduction

For illustrative purposes, an example application of the methodology developed in the previous section is given. In the example, a target matrix and an ICBM force were assumed. A discussion of alternative RV configurations reveals how dominated alternatives may be eliminated from further consideration prior to application of the methodology. The effectiveness of the remaining RV configurations was compared and a sensitivity analysis on the results was accomplished.

Target Matrix

The assumed target matrix consists of 100 targets of three hardness levels. Fifty targets have a hardness rating of 6,000 psi, thirty of 8,000 psi, and twenty targets of 10,000 psi.

RV Configurations

The assumed ICBM force, consisting of 100 missiles, is solely responsible for attaining the desired kill level on the target matrix as strategic bomber and SLBM forces will be ignored. One could interpret the desired target matrix kill level as the ICBM contribution to a strategic attack on the target matrix, realizing that strategic bombers and SLBMs may participate in a concerted attack. The assumed alternatives for RV configurations are listed in Table IV.

TABLE IV

ALTERNATIVE RV CONFIGURATIONS

Configuration	Yield (Mt)	CEP (NM)	RVs per Missile
1	.025	.0167	2
2	.050	.0250	2
3	.150	.0333	3
4	.250	.0500	3
5	.350	.0833	3

Table V displays the single shot kill probabilities for the RV configurations shown in Table IV. The values in Table V were calculated using equation 3-3.

In examining the data in Table V, an obvious grouping of alternatives is configurations one and two, possessing two RVs per missile, and configurations three, four, and five, with three RVs per missile. Within the first grouping, configuration one appears superior to configuration two. Its SSPK values for each target category exceed those of configuration two. In a similar manner, configuration three appears to be superior to configuration four

TABLE V

SSPK VALUES

Configuration	6000 psi	8000 psi	1000 psi	RVs per Missile
1	.9813	.9620	.9398	2
2	.9401	.9013	.8634	2
3	.9631	.9339	.9030	3
4	.8724	.8160	.7666	3
5	.6049	.5339	.4811	3

which, in turn, seems superior to configuration five. However, the effectiveness of these alternatives must be evaluated in order to determine how large a difference in the number of missiles required to achieve the desired target matrix kill level results from the different SSPK values. An exception is configuration two. It is clearly dominated by configuration three which has more RVs per missile and greater SSPK values for each target category. Although configuration five does not appear competitive, it will be evaluated. Thus, configurations one, three, four, and five will be compared.

Evaluation of Effectiveness

In applying the methodology described in Chapter III, a target matrix kill level of .80 and a reliability factor of .70 were used. For configuration one, the following SSPK values were calculated for the three target categories:

$$SSPK(6000) = .9813 \tag{4-1}$$

$$SSPK(8000) = .9620 \tag{4-2}$$

$$SSPK(10000) = .9398 \tag{4-3}$$

Multiplying the SSPK values by the reliability factor yielded the reliability adjusted SSPK values shown below:

$$SSPK(6000)*R = .6869 \tag{4-4}$$

$$SSPK(8000)*R = .6734 \tag{4-5}$$

$$SSPK(10000)*R = .6579 \tag{4-6}$$

<u>6000 psi Target Category</u>. In order to bracket the desired value of .80, $P_k(1)$ and $P_k(2)$ were calculated.

$$P_k(1) = .6869 \tag{4-7}$$

$$P_k(2) = .9028 \tag{4-8}$$

Note that $P_k(2)$ is greater than or equal to .80 while $P_k(1)$ is less than .80.

Thus,

$$(.6869)(1-X) + (.9028)(X) = .80 \tag{4-9}$$

Solving for X yields

$$X = .5260 \tag{4-10}$$

With 50 targets

$$T_{1,2} = (50)(.5260) = 26.30 \qquad (4\text{-}11)$$

which rounds to 26. Thus, 26 targets are attacked with 2 RVs, and 24 (50-26=24), are attacked with 1 RV. The P_k value for the 6000 psi target category is

$$(.6869)(24/50) + (.9028)(26/50) = \underline{.7987} \qquad (4\text{-}12)$$

The total number of RVs used for this target category is

$$(24)(1) + (26)(2) = 76 \qquad (4\text{-}13)$$

<u>8000 psi Target Category</u>. Repeating the procedure yielded the following results for this target category:

$$P_k(1) = .6734 \qquad (4\text{-}14)$$

$$P_k(2) = .8933 \qquad (4\text{-}15)$$

$$(.6734)(1-X) + (.8933)(X) = .80 \qquad (4\text{-}16)$$

$$\therefore X = .5757 \qquad (4\text{-}17)$$

$$T_{2,2} = (30)(.5757) - 17.17 \simeq 17 \qquad (4\text{-}18)$$

$$T_{2,1} = 30 - 17 = 13 \qquad (4\text{-}19)$$

$$P_k = (.6734)(13/30) + (.8933)(17/30) = \underline{.7980} \qquad (4\text{-}20)$$

$$\# \text{ RVs} = (13)(1) + (17)(2) = 47 \qquad (4\text{-}21)$$

<u>10000 psi Target Category</u>. The methodology produced the following results for the final target category:

$$P_k(1) = .6579 \tag{4-22}$$

$$P_k(2) = .8829 \tag{4-23}$$

$$(.6579)(1-X) + (.8829)(X) = .80 \tag{4-24}$$

$$\therefore X = .6315 \tag{4-25}$$

$$T_{3,2} = (20)(.6315) = 12.63 \simeq 13 \tag{4-26}$$

$$T_{3,1} = 20 - 13 = 7 \tag{4-27}$$

$$P_k = (.6579)(7/20) + (.8829)(13/20) = \underline{.8042} \tag{4-28}$$

$$\# \text{ RVs} = (7)(1) + (13)(2) = 33 \tag{4-29}$$

<u>Effectiveness of Configuration One</u>. Using the P_k values for each target category, the target matrix kill level is

$$(50/100)(.7987) + (30/100)(.7980) + (20/100)(.8042) = \underline{.7996} \tag{4-30}$$

The total number of RVs used for all three target categories is

$$76 + 47 + 33 = 156 \tag{4-31}$$

Containing two RVs per missile, this configuration requires 156/2 or 78 missiles to attain the desired target matrix kill level. Thus, this configuration leaves 22 of the 100 missile ICBM force in attaining the desired kill level.

Comparison of RV Alternatives. Configurations three, four, and five were analyzed in a similar manner. The HP-41CV output for the analysis of each of these configurations is displayed in Appendix C-5. Table VI summarizes these results.

TABLE VI

SUMMARY OF RV EVALUATION

Configuration	Effectiveness	Slack RVs	# Missiles Left
1	$P_k = .7996$	0	22
3	$P_k = .7989$	0	46
4	$P_k \geq .8009$	1	36
5	$P_k \geq .8033$	2	-11

Configuration five required 111 missiles to achieve the desired target matrix kill level as evidenced by the -11 entry in the # Missiles Left column of Table VI. Exceeding the number of missiles available renders configuration five infeasible. By not using more than the available number of missiles, configurations one, three, and four present feasible alternatives. Since configuration three provided the greatest number of remaining missiles, it appears to be the preferred alternative.

Generalization of the Effectiveness Evaluation

An attempt was made to develop a relationship between yield, CEP, reliability, P_k, and hardness that would predict the number of missiles required by the methodology developed in Chapter III. A multiple linear regression was accomplished using the Statistical Package for the Social Sciences (SPSS). Although the resulting coefficient of determination (R^2) was greater than .90 for two of eight different regression models, the coefficient of variability translated into errors as large as 34 missiles per target matrix. Thus, the multiple linear regression approach did not produce acceptable results.

A second attempt involved fixing the number of RVs per missile and the desired P_k. An examination of a plot of the number of missiles required to achieve a desired target matrix kill level appeared to be exponentially related to SSPK*R. Thus, an exponential curve fit was accomplished over the SSPK*R ranges of the data. The lower limit of the SSPK*R range was .2752. A lesser value would require more than five RVs per target, and allocations of six or more RVs per target were not considered in this study. The upper value, limited by the reliability factor, was .7000. The resulting equation for the two RV per missile case with a desired P_k of .80 was

$$N = 497.9247\, e^{-2.7744(SSPK*R)} \qquad (4-32)$$

where N is the number of missiles required to effect the desired P_k. The corresponding R^2 value was .9920. The three RV per missile curve is given by

$$N = 333.8287\, e^{-2.7940(SSPK*R)} \qquad (4-33)$$

with R^2 equal to .9898.

To further generalize the equations, the upper limit of SSPK*R was increased from .7000 to 1.0. Additional data points were generated, and the exponential curve fit procedure was reaccomplished to include these new data points. The resulting expression for the two RV per missile case, fixing P_k equal to .80, is

$$N = 456.5356\, e^{-2.5915(SSPK*R)} \qquad (4-34)$$

with R^2 equal to .9883. The corresponding expression for the three RV per mission configuration was

$$N = 300.1703\, e^{-2.5925(SSPK*R)} \qquad (4-35)$$

with R^2 equal to .9748.

Note that although the independent variables of the equations are SSPK and R, SSPK is a function of yield, CEP, and hardness. Referencing equation 3-3, one could develop the general exponential relationshp below which relates the required number of missiles to yield, CEP, hardness, and reliability for a given P_k and number of RVs per missile.

$$N-ae^{bR\left(1-e^{-.6391\left(\frac{Y^{2/3}}{(.0680(H)\ -\ .2272(H)^{1/2}+.1899)^{2/3}(CEP)^{2}}\right)}\right)} \tag{4-36}$$

These equations approximate the number of missiles the methodology in Chapter III would require for a homogeneous target matrix. Thus, the equations could be used to find N_i for each category i. N_i is defined as the number of missiles required to achieve the desired P_k on a homogeneous target matrix *consisting of 100 category i* targets. By summing the products of the N_i values and the proportion of the targets in the i^{th} category, p_i, the number of missiles required for a mixed-target-category matrix could be approximated. The equation below provides the approximate values of N for the RV alternatives considered in this chapter.

$$N = p_1N_1 + p_2N_2 + p_3N_3 \tag{4-37}$$

These approximate values were compared to the values generated by the methodology developed in Chapter III. Table VII summarizes these results. This limited sample, which only spans SSPK*R values from .3368 to .6579,

TABLE VII

COMPARISON OF N VALUES FROM METHODOLOGY AND APPROXIMATING EQUATIONS

Configuration	Methodology	Approximation
1	78	79
2	85	87
3	54	54
4	64	66
5	111	109

demonstrates that the equations closely approximate the values predicted by the methodology. Thus, the approximating equations were used to perform the sensitivity analysis on varying levels of yield, CEP, hardness, and reliability. Since the approximating equations did not incorporate differing levels of P_k values, the methodology was used to accomplish the sensitivity analysis on this factor.

Sensitivity Analysis

A sensitivity analysis on configurations one, three, and four was performed. The values of CEP, hardness, yield, reliability, and P_k were examined at their predicted value and at levels 5 percent above and below that value. In addition, a higher and lower value of interest was examined. Tables VIII through XII present this data.

TABLE VIII

CONFIGURATION SENSITIVITY TO CEP

Config.	CEP	SSPK*R 6000 psi	SSPK*R 8000 psi	SSPK*R 10000 psi	N
1	.0125	.6994	.6980	.6954	75
	.0159	.6913	.6810	.6685	78
	.0167	.6869	.6734	.6579	79
	.0175	.6813	.6644	.6458	81
	.0209	.6447	.6132	.5836	91
3	.0250	.6980	.6943	.6889	50
	.0316	.6821	.6657	.6476	53
	.0333	.6742	.6537	.6321	54
	.0350	.6648	.6401	.6153	56
	.0416	.6156	.5771	.5431	65
4	.0450	.6994	.6980	.6954	49
	.0475	.6285	.5928	.5604	63
	.0500	.6107	.5712	.5366	66
	.0525	.5919	.5493	.5130	70
	.0550	.5723	.5272	.4897	74

TABLE IX

CONFIGURATION SENSITIVITY TO HARDNESS

	Hardness			SSPK*R			
Config	H1	H2	H3	H1	H2	H3	N
1	5400	7200	9000	.6902	.6791	.6658	78
	5700	7600	9500	.6886	.6763	.6619	78
	6000	8000	10000	.6869	.6734	.6579	79
	6300	8400	10500	.6851	.6704	.6538	80
	6600	8800	11000	.6831	.6673	.6498	80
2	5400	7200	9000	.6799	.6621	.6429	53
	5700	7600	9500	.6771	.6579	.6375	54
	6000	8000	10000	.6742	.6537	.6321	54
	6300	8400	10500	.6713	.6494	.6268	55
	6600	8800	11000	.6683	.6451	.6215	55
4	5400	7200	9000	.6234	.5864	.5533	64
	5700	7600	9500	.6170	.5787	.5448	65
	6000	8000	10000	.6107	.5712	.5366	66
	6300	8400	10500	.6045	.5639	.5287	67
	6600	8800	11000	.5984	.5568	.5211	69

TABLE X

CONFIGURATION SENSITIVITY TO YIELD

		SSPK*R			
Config	Yield	6000 psi	8000 psi	10000 psi	N
1	.0200	.6773	.6582	.6379	82
	.0238	.6851	.6704	.6539	80
	.0250	.6869	.6734	.6579	79
	.0263	.6886	.6762	.6617	79
	.0300	.6921	.6826	.6707	77
3	.1300	.6652	.6407	.6161	56
	.1425	.6712	.6492	.6266	55
	.1500	.6742	.6537	.6321	54
	.1575	.6769	.6576	.6372	54
	.1700	.6807	.6634	.6446	53
4	.2000	.5813	.5373	.5002	72
	.2375	.6043	.5637	.5284	67
	.2500	.6107	.5712	.5366	66
	.2625	.6166	.5782	.5443	65
	.3000	.6316	.5965	.5646	62

TABLE XI

CONFIGURATION SENSITIVITY TO RELIABILITY

		SSPK*R			
Config	Rel	6000 psi	8000 psi	10000 psi	N
1	.6500	.6378	.6253	.6109	89
	.6650	.6525	.6397	.6250	86
	.7000	.6869	.6734	.6579	79
	.7350	.7212	.7071	.6908	72
	.7500	.7359	.7215	.7049	70
3	.6500	.6261	.6070	.5870	61
	.6650	.6405	.6210	.6005	59
	.7000	.6742	.6537	.6321	54
	.7350	.7080	.6864	.6638	50
	.7500	.7224	.7004	.6773	48
4	.6500	.5671	.5304	.4983	74
	.6650	.5802	.5427	.5098	71
	.7000	.6107	.5712	.5366	66
	.7350	.6412	.5998	.5635	61
	.7500	.6543	.6120	.5750	59

TABLE XII

CONFIGURATION SENSITIVITY TO DESIRED KILL LEVEL

P_k	Config 1	Config 3	Config 4
.70	56	40	50
.76	69	49	58
.80	78	54	64
.84	88	61	72
.90	104	74	91

An examination of the sensitivity tables revealed that the ranking of alternatives remained the same over all levels of each factor. Thus, configuration three remained the preferred alternative.

In addition to examining the ranking of alternatives, the sensitivity analysis provided an insight into the relative impact that the 5 percent changes in the predicted parameters had on RV effectiveness. Table XIII summarizes the effects of these changes. The entries in the table represent the differences in the number of missiles required for the values of the parameter at 5 percent above and 5 percent below its predicted value. Thus, the table entries are the change in the number of missiles required over a specified 10 percent parameter interval. In Table XII, for example, the desired kill level was .80. With a 5 percent change in kill level equal to .04, the

TABLE XIII

SENSITIVITY TO 5 PERCENT FACTOR CHANGES

	Configuration		
Parameters	1	3	4
CEP	3	3	7
Hardness	2	1	2
Yield	1	1	2
Reliability	14	9	10
P_k	19	12	14

10 percent interval of interest is bounded by .76 and .84. The resulting difference in missiles required over this interval for configuration one is 88 - 69 or 19. Thus, in Table XIII, the entry for P_k under configuration one is 19.

The most significant impacts occur due to changes in the desired target matrix kill level. Careful consideration must be given to the appropriate P_k value when considering RV alternatives. Variation in this parameter will significantly affect the expected force size required to meet desired target matrix kill levels. The reliability factor is almost as important. Changes in system reliability also translate into significant differences in the expected number of missiles required to accomplish the targeting objectives. The yield, CEP, and hardness parameters are not quite as sensitive as P_k and reliability. However, the 5 percent changes in CEP become fairly significant as CEP increases. Thus, as the CEP decreases, sensitivity to variations in CEP will also decrease. The number of missiles required to inflict the desired P_k on a target matrix is least sensitive to variations in yield and hardness. They represent approximately equal levels of sensitivity in the three RV alternatives examined. However, as yields become very small, variations in target hardness may become more significant as is the case with configuration one.

Further examination of the sensitivity data revealed that the relative sensitivity of a given parameter was also related to the SSPK*R term. The parameters that have the greatest affect on SSPK*R will be the most sensitive parameters. For example, a given percentage change in reliability induces the same percentage change in the SSPK*R term. Thus, reliability is a highly sensitive parameter. On the other hand, the same percentage change in CEP, hardness, or yield does not translate into an equal percentage change in SSPK. Thus, the SSPK*R value changes at a rate less than the parameter changes, rendering those parameters less sensitive than the reliability parameter. The P_k term cannot be analyzed in this manner as the SSPK*R values do not change for varying P_k levels. The change in a desired P_k changes the targeting objective, and would require a different exponential equation to analytically relate SSPK*R to the required number of missiles. However, as previously mentioned, P_k is a highly sensitive parameter.

V. Conclusions and Recommendations

Conclusions

The analytical technique developed in this study provides a means for comparing the number of missiles that RVs with different yields and CEPs require to achieve desired targeting objectives. This methodology was programmed on a Hewlett-Package HP-41CV for ease of use. However, the assumptions of the methodology and the kill mechanism used should be carefully examined by potential users. The methodology was developed for a specific scenario, and its validity does not extend beyond that scenario. Blind application of the methodology could result in misleading data and incorrect conclusions.

The HP-41CV program is limited to target matrixes of five or fewer target categories. In addition, to avoid unrealistically high RV allocation, the program will notify the user when a RV configuration requires more than five RVs per target. After notifying the user, the program ceases processing. When one employs the HP-41CV program while considering its assumptions and limitations, it can be a valuable aide in assessing the relative effectiveness of various RV configurations that are used against hard targets.

In addition, approximating equations were developed to relate RV parameters to the number of missiles required to attain a desired target matrix kill level. These equations, or tables and graphs based on these equations, could possibly be of assistance to planners or decision makers.

The sensitivity analysis revealed that RV effectiveness was highly sensitive to changes in the desired kill level and the reliability parameter. If one has fixed the desired kill level, RV effectiveness is most sensitive to changes in reliability. Thus, reliability engineering should receive a great deal of attention. Unless CEP is greater than .04 or .05 NM, CEP is not a very sensitive parameter, and the yield and hardness factors are less sensitive than CEP.

Recommendations

A recommended area of additional study is further investigation into the approximating exponential equations. It appears that the coefficient in the exponent, b, may be a constant for a given kill level. The coefficient of the constant e term, a, seems to vary with the number of RVs per missile. Examination of these relationships may lead to a simpler development of the approximating equations and a greater understanding of the impact of RV parameters on RV effectiveness.

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Appendix A

Development of the Overpressure Kill Radius

Tsipis stated that an overpressure of Δp psi at a distance r kilofeet from a nuclear detonation of Y megatons is governed by equation A-1 (14:395):

$$\Delta p = (3.3)(10^3)\left(\frac{Y}{r^3}\right) + (192)\left(\frac{Y}{r^3}\right)^{1/2} \qquad \text{(A-1)}$$

The values from equation A-1 correspond very closely to the "knee curves" in Glasstone and Dolan for overpressures on the ground caused by a nuclear burst at ground level (4:111-2). Thus, equation A-1 provides an appropriate value for the overpressure inflicted on a hard target. In converting the measure of distance from kilofeet to nautical miles, note that

$$1 \text{ kilofoot} = .1646 \text{ NM} \qquad \text{(A-2)}$$

so a conversion factor of 6.0761 is required in the distance term to offset the change in units. The resulting equation is

$$\Delta p = (14.71)\left(\frac{Y}{R^3}\right) + (12.8192)\left(\frac{Y}{R^3}\right)^{1/2} \qquad \text{(A-3)}$$

or

$$(\Delta p)/(12.8192) = (1.1475)(\frac{Y}{R^3}) + (\frac{Y}{R^3})^{1/2} \qquad \text{(A-4)}$$

Solving for $(Y/R^3)^{1/2}$ using the qudratic formula yields

$$(\frac{Y}{R^3})^{1/2} = \frac{-1 \pm (1 + .3581\ \Delta p)^{1/2}}{2.2950} \qquad \text{(A-5)}$$

where the negative solution has no physical meaning. Note that for $\Delta p \geq 6{,}000$, omitting the one under the square root induces an error of less than .03 percent. Thus the simplified expression is

$$(\frac{Y}{R^3})^{1/2} = -.4357 + .2607(\Delta p^{1/2}) \qquad \text{(A-6)}$$

Squaring both sides yields

$$\frac{Y}{R^3} = .0680(\Delta p) - .2272(\Delta p^{1/2}) + .1899 \qquad \text{(A-7)}$$

Solving in terms of R yields

$$R^3 = \frac{Y}{.0680(\Delta p) - .2272(\Delta p^{1/2}) + .1899} \qquad \text{(A-8)}$$

or

$$R = \frac{Y^{1/3}}{(.0680(\Delta p) - .2272(\Delta p^{1/2}) + .1899)^{1/3}} \qquad \text{(A-9)}$$

For a target hardness, H, any warhead of yield Y inside a circle of radius R_k will destoy the target. Thus, the kill radius of the warhead for a specified hardness level is (14:395):

$$R_k = \frac{Y^{1/3}}{(.0680(H) - .2272(H^{1/2}) + .1899)^{1/3}} \qquad \text{(A-10)}$$

Appendix B

Development of the Single Shot Probability of Kill

Assuming targeting errors in two dimensions (range and azimuth or deflection), a bivariate normal distribution describes the probability of hitting a target within a circle of prescribed radius. The bivariate distribution is

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\, e^{\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\bar{x}}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\bar{x}}{\sigma_x}\right)\left(\frac{y-\bar{y}}{\sigma_y}\right) + \left(\frac{y-\bar{y}}{\sigma_y}\right)^2\right]\right\}} \qquad \text{(B-1)}$$

When the variables x and y are independent, the correlation factor, ρ, equals zero. Assuming the range and azimuth errors are independent, the equation simplifies to the product of two one-dimensional functions as shown below.

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y}\, e^{\left\{-\frac{1}{2}\left[\left(\frac{x-\bar{x}}{\sigma_x}\right)^2 + \left(\frac{y-\bar{y}}{\sigma_y}\right)^2\right]\right\}} \qquad \text{(B-2)}$$

To employ the circular error concept, the assumption that the target is circular and range error equals azimuth error will be made (16:86). A cartesian coordinate system placed at the desired impact point requires that $\bar{x}=0$ and $\bar{y}=0$. Thus, the equation reduces to

$$f(x,y) = \frac{1}{2\pi\sigma_x^2}\left\{e^{-\frac{1}{2}\left[\frac{x^2+y^2}{\sigma_x^2}\right]}\right\} \tag{B-3}$$

Converting the cartesian coordinate system to a polar coordinate system,

$$r^2 = x^2 + y^2 \tag{B-4}$$

and

$$f(r) = \frac{1}{2\pi\sigma_r^2}\left\{e^{-\frac{1}{2}\left(\frac{r}{\sigma_r}\right)^2}\right\} \tag{B-5}$$

The probability of a RV striking within a distance R of the target is:

$$P(r\leq R) = \frac{1}{2\pi\sigma_r^2}\int_0^{2\pi}\int_0^R e^{-\frac{1}{2}\left(\frac{r}{\sigma_r}\right)^2} r\,dr\,d\theta \tag{B-6}$$

$$= \frac{1}{\sigma_r^2}\int_0^R e^{-\frac{1}{2}\left(\frac{r}{\sigma_r}\right)^2} r\,dr \tag{B-7}$$

By letting $t = r/\sigma_r$, then $dr = \sigma_r dt$

$$P(r \leq R) = \int_0^t e^{-\frac{1}{2}t^2} \, tdt \qquad \text{(B-8)}$$

$$= -e^{-\frac{1}{2}t^2} \Big]_0^t \qquad \text{(B-9)}$$

$$= -e^{-\frac{t}{2}} + e^0 \qquad \text{(B-10)}$$

Since $t = r/\sigma_r$,

$$P(r \leq R) = 1 - e^{-\frac{1}{2}\left(\frac{R}{\sigma_r}\right)^2} \qquad \text{(B-11)}$$

By the definition of CEP,

$$.5 = \frac{1}{\sigma_r^2} \int_0^{CEP} e^{-\frac{1}{2}\left(\frac{R}{\sigma_r}\right)^2} \, rdr \qquad \text{(B-12)}$$

$$= 1 - e^{-.5\left(\frac{CEP}{\sigma_r}\right)^2} \qquad \text{(B-13)}$$

$$= e^{-.5\left(\frac{CEP}{\sigma_r}\right)^2} \qquad \text{(B-14)}$$

Solving for σ_r results in the following:

$$\ln(.5) = -.5\left(\frac{CEP}{\sigma_r}\right)^2 \qquad \text{(B-15)}$$

$$-2 \ln(.5) = \left(\frac{CEP}{\sigma_r}\right)^2 \qquad \text{(B-16)}$$

$$2 \ln 2 = \left(\frac{CEP}{\sigma_r}\right)^2 \qquad \text{(B-17)}$$

$$(2 \ln 2)^{1/2} = \frac{CEP}{\sigma_r} \qquad \text{(B-18)}$$

$$\sigma_r = \frac{CEP}{(2 \ln 2)^{1/2}} \qquad \text{(B-19)}$$

$$\sigma_r = \frac{CEP}{1.774} \qquad \text{(B-20)}$$

Substituting equation B-20 into B-11 yields the single shot probability of kill (1:356-360)

$$SSPK = P(r \leq R) \qquad \text{(B-21)}$$

$$= 1 - e^{-\frac{1}{2}\left(\frac{1.774R}{CEP}\right)^2} \qquad \text{(B-22)}$$

$$= 1 - e^{-.6391\left(\frac{R}{CEP}\right)^2} \qquad \text{(B-23)}$$

Substituting equation A-10, the kill radius, for R in equation B-24 yields the following expression for the single shot probability of kill:

$$SSPK = 1 - e^{-.6391\left[\frac{Y^{2/3}}{(.0680H - .2272H^{1/2} + .1899)^{2/3}(CEP)^2}\right]} \qquad \text{(B-24)}$$

where Y is the yield in megatons, CEP is the circular error probable in nautical miles, and H is the target hardness in pounds per square inch.

Appendix C-1

HP-41CV Program Description

The analytical technique described in Chapter III of this report was programmed on a Hewlett-Packard HP-41CV. Program RV uses program SSPK as a subprogram, allowing one to determine SSPK values without accessing program RV. Program RV is listed in Appendix C-2, while program SSPK is listed in Appendix C-3. Note that program RV is limited to RV configurations requiring five or less RVs per target. Additionally, it will only accommodate target matrixes consisting of five or fewer target types. The use of data storage registers are shown in Appendix C-4 if one wished to alter the program or its limitations in any way. The user instructions follow.

HP-41CV PROGRAM RV USER INSTRUCTIONS

SIZE:038

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Initialize the program		XEQ RV	KILL LVL?
2	Enter kill level	0<x<1	R/S	REL FACTOR?
3	Enter total weapon system reliability factor	0<x<1	R/S	NO. MISSILES?

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
4	Enter the number of missiles available	X	R/S	HARD-PSI?
5	Enter the target category hardness in psi	X	R/S	CEP-NM?
6	Enter the weapon CEP in nautical miles (1NM ≃ 6000ft)	X	R/S	YIELD-MT?
7	Enter the weapon yield in megatons	X	R/S	SSPK=<u>SSPK</u>
8	Determine the reliability adjusted SSPK		R/S	SSPK*R=<u>(SSPK) (R)</u>
9			R/S	NO. TGTS?
10	Enter the number of targets in the target category	X	R/S	RVS/TGT:<u>n</u>
11	Determine the number of targets in the category attacked by n RVs		R/S	NO. TGTS:$\underline{T_{i,n}}$
12			R/S	RVS/TGT:<u>n-1</u>
13	Determine the number of targets in the category attacked by (n-1) RVs		R/S	NO. TGTS:$\underline{T_{i,n-1}}$
14	Determine the P_k for the target category		R/S	PK(I)=$\underline{P_k}$
15			R/S	MORE TGTS?Y:N
15a	Y implies another target category. Repeat steps 5-14.	Y	R/S	HARD-PSI?

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
15b	N implies no additional target categories. If the total number of RVs used is a multiple of the number of RVs per missile, PK=P_k is displayed. Otherwise, PK$\geq P_k$ is displayed.	N	R/S	PK=$\underline{P_k}$ or PK$\geq\underline{P_k}$
15c	Determine the number of slack RVs if step 15b displayed PK>P_k		R/S	SLACK RVS:$\underline{Y}$
16	Determine the number of *remaining missiles*		R/S	M↑ LEFT:$\underline{Z}$

HP-41CV PROGRAM SSPK USER INSTRUCTIONS

SIZE:003

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
1	Initialize the program		XEQ SSPK	HARD-PSI?
2	Enter the target category hardness in psi	X	R/S	CEP-NM?
3	Enter the weapon CEP in nautical miles (1NM=6000ft)	X	R/S	YIELD-MT?
4	Enter the weapon yield in megatons	X	R/S	SSPK=SSPK

Appendix C-2

Listing of Program RV

```
PROGRAM RV

01♦LBL "RV"
02 FIX 4
03 CLRG
04 "KILL LVL=?"
05 PROMPT
06 STO 21
07 "REL FACTOR=?"
08 PROMPT
09 STO 22
10 "RVS/MISSILE?"
11 PROMPT
12 STO 30
13 "NO. M↑=?"
14 PROMPT
15 STO 35
16 10
17 STO 20
18 15
19 STO 26
20♦LBL 01
21 3
22 STO 08
23 XEQ "SSPK"
24 PROMPT
25 RCL 22
26 *
27 STO 06
28 "SSPK*R="
29 ARCL Y
30 PROMPT
31♦LBL 02
32 1
33 RCL 00
34 -
35 RCL 08
36 2
37 -
38 Y↑X
39 CHS
40 1
41 +
42 STO IND 08
43 RCL 21
44 X<=Y?
45 GTO 03
46 1
47 ST+ 08
48 RCL 08
49 8
50 Y>Y?
51 GTO 02
52 "NO. RVS > 8
53 PROMPT
54 GTO 02
55♦LBL 03
56 "NO. TARGETS?"
57 PROMPT
58 STO IND 20
59 RCL 08
60 1
61 -
62 STO 09
63 3
64 X<=Y?
65 GTO 07
66 37
67 STO 09
68♦LBL 07
69 RCL 21
70 RCL IND 09
71 -
72 RCL IND 08
73 RCL IND 09
74 -
75 /
76 STO 23
77 RCL IND 20
78 *
79 FIX 0
80 RND
81 STO 24
82 RCL 06
83 2
84 -
85 STO 09
86 *
87 STO 32
88 "RVS/TGT: "
89 ARCL 09
90 PROMPT
91 "NO. TGTS: "
92 ARCL 24
93 PROMPT
94 RCL IND 20
95 RCL 24
96 -
97 STO 25
98 RCL 08
99 3
100 -
101 STO 09
102 *
103 STO 36
104 "RVS/TGT "
105 ARCL 09
```

```
106 PROMPT
107 "NO. TGTS: "
108 ARCL 25
109 PROMPT
110 FIX 4
111 RCL 06
112 1
113 -
114 STO 09
115 3
116 X<=Y?
117 GTO 03
118 37
119 STO 09
120♦LBL 03
121 RCL 24
122 RCL IND 08
123 *
124 RCL 25
125 RCL IND 09
126 *
127 +
128 RCL IND 28
129 /
130 STO IND 26
131 "PK(I)= "
132 ARCL IND 26
133 PROMPT
134 RCL 32
135 RCL 36
136 +
137 ST+ 31
138 "MORE TGTS?Y,N"
139 AON
140 PROMPT
141 ASTO Y
142 "N"
143 ASTO X
144 AOFF
145 X=Y?
146 GTO 04
147 1
148 ST+ 28
149 1
150 ST+ 26
151 GTO 01
152♦LBL 04
153 RCL IND 26
154 RCL IND 28
155 ST+ 27
156 *
157 ST+ 28
158 1
159 ST- 26
160 1
161 ST- 28
162 RCL 28
163 10
164 X<=Y?
165 GTO 04
166 RCL 28
167 RCL 27
168 /
169 STO 29
170 RCL 31
171 RCL 30
172 /
173 STO 33
174 FRC
175 STO 34
176 X=0?
177 GTO 05
178 "PK>= "
179 ARCL 29
180 PROMPT
181 1
182 ENTER↑
183 RCL 34
184 -
185 RCL 30
186 *
187 FIX 0
188 "SLACK RVS: "
189 ARCL X
190 PROMPT
191 FIX 4
192 GTO 06
193♦LBL 05
194 "PK= "
195 ARCL 29
196 PROMPT
197♦LBL 06
198 RCL 33
199 .9999
200 +
201 INT
202 CHS
203 RCL 35
204 +
205 FIX 0
206 "M" LEFT "
207 ARCL X
208 PROMPT
209 END
```

Appendix C-3

Listing of Program SSPK

```
PROGRAM SSPK        22 SQRT
                    23 .2272
                    24 *
 01♦LBL "SSPK"      25 -
 02 "HARD-PSI?"     26 .1899
 03 PROMPT          27 +
 04 STO 01          28 2
 05 "CEP-NM?"       29 ENTER↑
 06 PROMPT          30 3
 07 STO 02          31 /
 08 "YIELD-KT?"     32 Y↑X
 09 PROMPT          33 /
 10 2               34 .6931
 11 ENTER↑          35 *
 12 3               36 CHS
 13 /               37 E↑X
 14 Y↑X             38 CHS
 15 RCL 02          39 1
 16 X↑2             40 +
 17 /               41 CLA
 18 RCL 01          42 "SSPK=
 19 .0686           43 ARCL X
 20 *               44 AVIEW
 21 RCL 01          45 END
```

Appendix C-4

Data Registers Used by Programs RV and SSPK

Register	Register Comments
00	Reliability adjusted SSPK
01	Target hardness
02	Weapon CEP
03	P_k with 1 RV
04	P_k with 2 RVs
05	P_k with 3 RVs
06	P_k with 4 RVs
07	P_k with 5 RVs
08	Counter pointing to registers 03 - 07
09	Utility register used for calculations
10	Number of targets in category 1
11	Number of targets in category 2
12	Number of targets in category 3
13	Number of targets in category 4
14	Number of targets in category 5
15	P_k for category 1
16	P_k for category 2
17	P_k for category 3
18	P_k for category 4
19	P_k for category 5

Register	Register Comments
20	Counter pointing no ro registers 10 - 14
21	Desired target matrix kill level
22	Weapon system reliability
23	Desired proportion of targets attacked by n RVs for category i
24	Number of targets attacked by n RVs for category i
25	Number of targets attacked by n-1 RVs for category i
26	Counter pointing to registers 15 - 19
27	$\Sigma_i T_i$, the total number of targets through category i
28	$\Sigma_i T_i P_k(i)$, the sum of $P_k(i)$ weighted by T_i through category i
29	P_k
30	Number of RVs per missile
31	Total number of RVs used
32	Number of RVs deployed at the rate of n RVs per target in category i
33	Number of RVs deployed divided by the number of RVs per missile
34	Fractional part of register 33
35	Number of missiles
36	Number of RVs deployed at the rate of n-1 RVs per target in category i
37	Zero

Appendix C-5

Program RV Output

```
CONFIG THREE

                 XEQ "RV"
KILL LVL=?
           .8000    RUN
REL FACTOR=?
           .7000    RUN
RVS/MISSILE?
          3.0000    RUN
NO. MY=?
        100.0000    RUN
HARD-PSI?
       6000.0000    RUN
CEP-NM?
           .0333    RUN
YIELD-MT?
           .1500    RUN
SSPK= 0.9632
SSPK= 0.9632
                    RUN
SSPK*R=0.6742
                    RUN
NO. TARGETS?
         50.0000    RUN
RVS/TGT: 2
                    RUN
NO. TGTS: 29
                    RUN
RVS/TGT: 1
                    RUN
```

```
NO. TGTS: 21
                    RUN
PK(I)=0.8016
                    RUN
MORE TGTS?Y:N
Y                   RUN
HARD-PSI?
       8000.0000    RUN
CEP-NM?
           .0333    RUN
YIELD-MT?
           .1500    RUN
SSPK= 0.9339
SSPK= 0.9339
                    RUN
SSPK*R=0.6537
                    RUN
NO. TARGETS?
         30.0000    RUN
RVS/TGT: 2
                    RUN
NO. TGTS: 19
                    RUN
RVS/TGT: 1
                    RUN
NO. TGTS: 11
                    RUN
PK(I)=0.7970
                    RUN
MORE TGTS?Y:N
```

```
Y                   RUN
HARD-PSI?
      10000.0000    RUN
CEP-NM?
           .0333    RUN
YIELD-MT?
           .1500    RUN
SSPK= 0.9031
SSPK= 0.9031
                    RUN
SSPK*R=0.6321
                    RUN
NO. TARGETS?
         20.0000    RUN
RVS/TGT: 2
                    RUN
NO. TGTS: 14
                    RUN
RVS/TGT: 1
                    RUN
NO. TGTS: 6
                    RUN
PK(I)=0.7943
                    RUN
MORE TGTS?Y:N
N                   RUN
PK= 0.7989
                    RUN
MY LEFT: 46
```

CONFIG FOUR

```
                XEQ "RV
KILL LVL=?
           .8000    RUN
REL FACTOR=?
           .7000    RUN
RVS/MISSILE?
          3.0000    RUN
NO. Mt=?
        100.0000    RUN
HARD-PSI?
       6000.0000    RUN
CEP-NM?
           .0500    RUN
YIELD-MT?
           .2500    RUN
SSPK= 0.8724
SSPK= 0.8724
                    RUN
SSPK*R=0.6107
                    RUN
NO. TARGETS?
         50.0000    RUN
RVS/TGT: 2
                    RUN
NO. TGTS: 40
                    RUN
RVS/TGT: 1
                    RUN
NO. TGTS: 10
```

```
                    RUN
PK(I)=0.8003
                    RUN
MORE TGTS?Y.N
Y                   RUN
HARD-PSI?
       8000.0000    RUN
CEP-NM?
           .0500    RUN
YIELD-MT?
           .2500    RUN
SSPK= 0.8160
SSPK= 0.8160
                    RUN
SSPK*R=0.5712
                    RUN
NO. TARGETS?
         30.0000    RUN
RVS/TGT: 2
                    RUN
NO. TGTS: 28
                    RUN
RVS/TGT: 1
                    RUN
NO. TGTS: 2
                    RUN
PK(I)=0.7955
                    RUN
MORE TGTS?Y.N
Y                   RUN
HARD-PSI?
```

```
      10000.0000    RUN
CEP-NM?
           .0500    RUN
YIELD-MT?
           .2500    RUN
SSPK= 0.7666
SSPK= 0.7666
                    RUN
SSPK*R=0.5366
                    RUN
NO. TARGETS?
         20.0000    RUN
RVS/TGT: 3
                    RUN
NO. TGTS: 3
                    RUN
RVS/TGT: 2
                    RUN
NO. TGTS: 17
                    RUN
PK(I)=0.8226
                    RUN
MORE TGTS?Y:N
N                   RUN
PK> 0.8065
                    RUN
SLACK RVS: 1
                    RUN
Mt LEFT: 36
```

CONFIG FIVE

```
          XEQ "PK"
KILL LVL-?
        .8000   RUN
REL FACTOR=?
        .7000   RUN
RVS/MISSILE?
       3.0000   RUN
NO. MX=?
     100.0000   RUN
HARD-PSI?
    6000.0000   RUN
CEP-NM?
        .0833   RUN
YIELD-MT?
        .3500   RUN
SSPK= 0.6843
SSPK= 0.6843
                RUN
SSPK*R=0.4[illegible]
                RUN
NO. TARGETS?
      52.0000   RUN
RVS/TGT: 3
                RUN
NO. TGTS: 47
                RUN
RVS/TGT: 2
                RUN
```

```
NO. TGTS: 3
                RUN
PK(I)=0.7998
                RUN
MORE TGTS?Y:N
Y               RUN
HARD-PSI?
    8000.0000   RUN
CEP-NM?
        .0633   RUN
YIELD-MT?
        .3500   RUN
SSPK= 0.5339
SSPK= 0.5339
                RUN
SSPK*R=0.3737
                RUN
NO. TARGETS?
      30.0000   RUN
RVS/TGT: 4
                RUN
NO. TGTS: 15
                RUN
RVS/TGT: 3
                RUN
NO. TGTS: 15
                RUN
PK(I)=0.8263
                RUN
MORE TGTS?Y:N
Y               RUN
```

```
HARD-PSI?
   10000.0000   RUN
CEP-NM?
        .0233   RUN
YIELD-MT?
        .3500   RUN
SSPK= 0.4811
SSPK= 0.4811
                RUN
SSPK*R=0.3368
                RUN
NO. TARGETS?
      20.0000   RUN
RVS/TGT: 4
                RUN
NO. TGTS: 17
                RUN
RVS/TGT: 3
                RUN
NO. TGTS: 1
                RUN
PK(I)=0.8210
                RUN
MORE TGTS?Y:N
N               RUN
PK)= 0.8045
                RUN
SLACK RVS: 2
                RUN
MX LEFT -11
```

Appendix D

Equal Effectiveness Assumption

In assuming the calculated and desired P_k values are equal, one assumes the difference between them is negligible. This assumption can be evaluated by determining the maximum difference between these values. The following discussion calculates this maximum difference and presents the error tolerances of the methodology developed in Chapter III.

Suppose $T_{i,n}$ was rounded to the next lower integer for category i in accordance with the targeting technique discussed in Chapter III. The value of $P_k(i)$ will be less than the desired P_k. However, if $T_{i,n}$ targets were attacked with n RVs, $T_{i,n-1} - 1$ targets were attacked with n-1 RVs, and one target was attacked with some non-integer number of RVs between n and n-1, then $P_k(i)$ would exactly equal the desired P_k. The difference in the desired P_k and $P_k(i)$ is caused by a single target, and the maximum difference can be calculated.

The targeting technique allocates quantities of n and n-1 RVs. Examination of the marginal increases in the single target P_k due to increasing the number of RVs from n-1 to n reveals the sizes of the probability of kill intervals that could possibly contain the desired P_k. Larger intervals represent a source of greater possible error. It is assumed that SSPK*R will be less than the desired P_k for realistic applications of the methodology. Thus, at least one RV will be allocated to each target. It can be easily verified that if a single RV is already allocated to a target, the greatest marginal increase in P_k occurs when the second RV is assigned. This greatest marginal increase in the single target P_k corresponds to the maximum possible deviation between the desired P_k and $P_k(i)$. Thus, the maximum difference between the desired P_k and $P_k(i)$ occurs when n equals two and n-1 equals one.

Given that one RV is assigned to a target, the maximum marginal increase in the single target P_k is given by

$$m = 1-(1-SSPK*R)^2 - (1-(1-SSPK*R)) \tag{D-1}$$

which simplifies to

$$m = SSPK*R - (SSPK*R)^2 \tag{D-2}$$

The first derivative of m with respect to SSPK*R is

$$d(m)/d(SSPK*R) = 1-2SSPK*R \tag{D-3}$$

While the second derivative is equal to negative two. Setting the first derivative equal to zero reveals that the maximum difference in P_k for a single target occurs when SSPK*R equals .50. With one RV, equation 3-4 provides a single target P_k of .50, while two RVs results in a single target P_k of .75. Thus, the maximum marginal increase in the single target P_k for the allocation of a second or subsequent RV is .25.

The difference in the desired P_k and $P_k(i)$ is a result of the noninteger number of RVs required by the single target of interest. Error is induced by allocating either n or n-1 RVs to the target because the number of targets attacked by n RVs is rounded to the nearest integer. If the fractional portion of the required number of RVs is close to one or zero, the rounded number of RVs is close to the required number. However, as the fractional portion approaches .50, the maximum rounding error of .50 RV occurs. If the required number of RVs, $T_{i,n}$, is rounded to the next higher integer, the maximum contribution to the difference between P_k and $P_k(i)$ by the n RV targets is $+(.5)P_k(i,n)$, while the maximum contribution of the n-1 RV targets is $-(.5)P_k(i,n-1)$. Similarly, when the required number of RVs, $T_{i,n}$, is rounded down, the maximum contributions to error are the additive inverses of those discussed above. Therefore, the absolute value of the maximum contribution to the difference between the desired P_k and the computed $P_k(i)$ is expressed by

$$D = (.5)P_k(i,n) - (.5)P_k(i,n-1) \qquad (D-4)$$

The contribution to error is positive when the number of targets attacked by n RVs is rounded to the next higher integer and negative when it is rounded down. Using the SSPK*R value that maximizes m results in the maximum contributing error of .125.

The impact of the contributing error on $P_k(i)$ will depend on the number of targets in the category. For 50 targets the maximum difference in the desired P_k and $P_k(i)$ is .125/50 or .0025, while the target category error is .0125 for a 10 target category. Note, however, that a linear combination of the error terms can be formulated by substituting the target category error terms for the $P_k(i)$ terms in equation 3-7. This linear combination equals the difference between the computed and desired P_k for the entire target matrix. In this linear combination, the target category error term, which is the contributing error divided by T_i, is multiplied by T_i. Thus, the linear combination of target category error terms can be simplified. The resulting expression is the sum of the contributing error terms divided by the total number of targets in the matrix. Therefore, the maximum deviation of the computed P_k from the desired P_k is the product of the maximum contributing error, which is .125, and the number of target categories divided by the number of targets in the matrix. For a 100 target matrix, any given category will not contribute more than .125/100 or .00125 to the difference between the desired and computed target matrix kill levels. Thus, the maximum deviation between the calculated and desired P_k for a 100 target matrix is the product of .00125 and the number of target categories in the matrix. For example, the computed P_k of a homogeneous target matrix will be within .00125 of the desired P_k. For a three category target matrix, the computed P_k will not deviate more than .00375 from the desired P_k. The range of the deviation of computed P_k values for a four category target matrix is .005 while the range for the five category matrix is .00625. Thus, to two significant digits, the methodology generates equal levels of effectiveness for target matrixes with three or fewer target categories. For target matrixes of four or five categories, the methodology generates levels of effectiveness within .01 of the desired P_k. The calculated P_k values within these error tolerances represent the approximately equal levels of effectiveness the methodology uses to compare RV alternatives.

Vita

Paul Fernand Auclair was born on 2 August 1953 at Westover AFB, Massachusetts. He was graduated from Merritt Island High School in Merritt Island, Florida in 1971. Following high school, he attended Brevard Community College in Cocoa, Florida for two semesters. He entered the USAF Academy in 1972 where he was awarded a Bachelor of Science degree and a commission in the USAF. Upon graduation in June 1976, he was assigned to the 341st Strategic Missile Wing at Malmstrom AFB, Montana. There he served as a Select Deputy Missile Combat Crew Commander Evaluator, a Missile Combat Crew Commander, and a Missile Procedures Trainer Operator. While stationed at Malmstrom AFB, he earned a Master of Science degree in Systems Management from the University of Southern California. Subsequent to his tour in missile operations, he entered the School of Engineering, Air Force Institute of Technology, in August 1980.

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CEP	Target Matrix
Hardness	Handheld Calculator Program
Kill Level	

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This thesis examines the effectiveness of various reentry vehicle configurations when they are targeted against buried, hard targets. The configurations are based on the reentry vehicle parameters of yield and CEP as well as the number of reentry vehicles per missile. An examination of the ground shock and overpressurekill radii resulted in the use of overpressure of the hard target kill mechanism. The methodology developed to examine reentry vehicle effectiveness was programmed on a Hewlett-Packard HP-41CV. The methodology allows variations in CEP, weapon system reliability, weapon yield, and number

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of reentry vehicles per missile as well as the desired kill level. The measure of effectiveness of each reentry vehicle configuration is the number of missiles required to achieve a desired kill level on a user defined target matrix. The results of the methodology were generalized with a set of exponential equations. Each equation is based on a desired kill level and a fixed number of reentry vehicles per missile. A sensitivity analysis on the various configurations revealed the relative impact of equal percentage changes in the factors used in this study.