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A SYSTEMATIC PROCEDURE FOR CALCULATING THE AVERAGE ILLUMINANCE --ETC(U)
MAY 82 J B MURDOCH, J E NETTLETON N62583-81-MR-307
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NAVAL CIVIL ENGINEERING LABORATORY
Port Hueneme, California

Sponsored by
CHIEF OF NAVAL MATERIAL
NAVAL FACILITIES ENGINEERING COMMAND

A SYSTEMATIC PROCEDURE FOR CALCULATING THE AVERAGE ILLUMINANCE ON A WORK PLANE FROM SKYLIGHTS LOCATED IN A PITCHED ROOF

May 1982

An Investigation Conducted by
DEPARTMENT OF ELECTRICAL ENGINEERING
University of New Hampshire
Durham, New Hampshire

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METRIC CONVERSION FACTORS

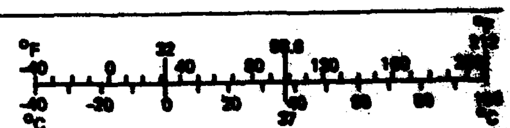
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A systematic design procedure has been developed to calculate the average horizontal illuminance upon a work plane due to skylights in a pitched roof. The calculation procedure allows for pitched roofs with slopes up to 30 degrees from horizontal, and for both clear and overcast sky conditions. The procedure is divided into two basic steps: (1) the calculation

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cont → of total external daylight illuminance upon the sloped skylight surface, and (2) the calculation of average horizontal illuminances upon an interior workplane due to the sloped skylights. Worksheets are used to systematically step through the procedure. All necessary tables and equations are included.

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FIGURES

TABLES

A. INTRODUCTION

One of the most effective ways to save electrical energy is to turn off or dim electric lights whenever possible. Properly designed daylighting can supplement or, in some cases, completely replace electric lighting during the daylight hours while maintaining the predetermined necessary level of illumination.

Design techniques have been developed to calculate the horizontal illuminance on the work plane in a room, based on luminous flux being furnished by vertical openings (windows) or horizontal openings (skylights). This report discusses the procedures taken to calculate the average illuminance upon a workplane due to sloped openings (i.e. skylights in a pitched roof).

The sloped skylight problem is divided into two sections:

1) determination of total external daylight illuminance upon a sloped surface.
2) determination of average illuminance, upon an interior workplane, due to the sloped skylight.

The division of the problem in this way mades it relatively easy to develop a simple procedure for calculating the average illuminance on a workplane due to skylights located in a pitched roof.

B. DETERMINATION OF TOTAL EXTERNAL ILLUMINANCE ON A SLOPED SURFACE

To calculate the external illuminance upon a sloped surface many factors must be known, such as where the sun is relative to the surface's position, cloud distribution of the sky (i.e. cloudy-clear), time of the year, and the angle the sloped surface makes with the earth's surface (a horizontal plane).

Several solar quantities are calculated to determine the sun's position relative to the surface's position on earth. The cloud distribution of the sky is picked to be the two extreme cases of completely clear sky and completely overcast sky.

Once the above mentioned variables are known the external illuminance on a sloped surface can be determined for the completely overcast sky and the completely clear sky cases.

1) Determination of Solar Quantities:

a) Solar Time (Reference 1)

Solar time (ST) is defined as the time based on the apparent angular motion of the sun across the sky, with solar noon the time the sun crosses the meridian (longitude) of the observer. Solar time does not coincide with the local standard clock time. It is necessary to convert standard time to solar time by using Equation 1a.

$$ST = \text{Standard Time} + \Delta M \qquad (1a)$$

where ST = solar time

ΔM = correction to solar time in minutes.

ΔM is given in Equation 1b.

$$\Delta M = 4(L_{ST} - L_{LOC}) + ET \qquad (1b)$$

where L_{ST} = Standard Meridian for the local time zone. See Chart 2 of Figure 1 for a listing of Standard Meridians of Time Zones.

L_{LOC} = longitude (degrees west) of site location.

ET = Equation of Time (Reference 2). The function is depicted in Chart 1 of Figure 1 and can be calculated by Equation 1c.

$$ET = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B \quad (1c)$$

where $B = (n-81)/58$ rad: n is the number of the day in the year and a partial listing of n is given by Table 1.

The equation of time may be read directly from Chart 1 of Figure 1 to get a crude approximation. For example, on March 21 ET is about -8 minutes. Table 2 gives the approximate ET's for the two equinoxes and solstices.

TABLE 1

MONTH	DAYS IN MONTH*	n
JAN	31	i
FEB	28	31 + i
MARCH	31	59 + i
APR	30	90 + i
MAY	31	120 + i
JUNE	30	151 + i
JULY	31	181 + i
AUG	31	212 + i
SEPT	30	243 + i
OCT	31	273 + i
NOV	30	304 + i
DEC	31	334 + i

where i is the day of tne month in question.

*This table is for the average year of 365 days. For leap year n should be increased by 1 for the months after FEB.

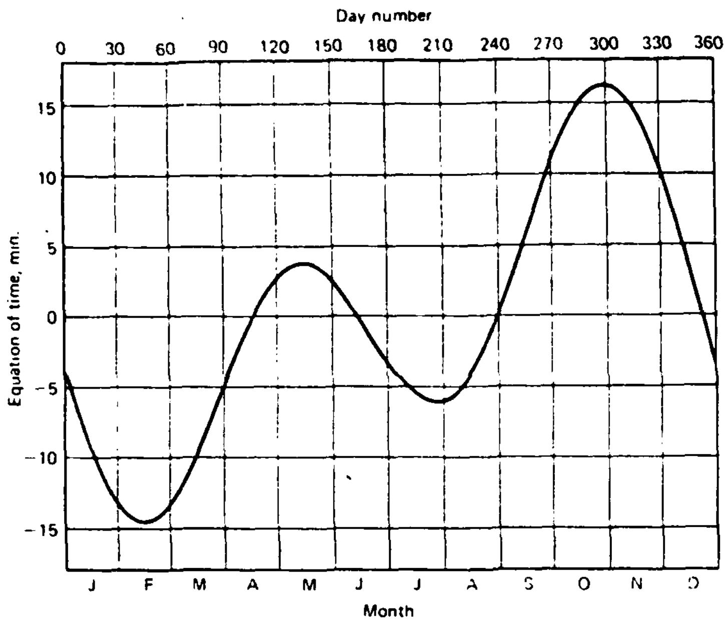

Chart 1

Zone	Position	Standard Meridian (Longitude)
+4	Atlantic	60° W
+5	Eastern	75° W
+6	Central	90° W
+7	Mountain	105° W
+8	Pacific	120° W

Chart 2

Standard Meridians of Time Zones

FIG. 1. CHART 1 THE PERIODIC SHIFT FOR THE SOLAR DAY

CHART 2 A LIST OF THE STANDARD MERIDIANS OF TIME ZONES

TABLE 2

APPROXIMATE VALUES OF ET

SEASON	DATE	ET(MIN)
SPRING	MARCH 21	-8
SUMMER	JUNE 21	-1
FALL	SEPTEMBER 21	+8
WINTER	DECEMBER 21	+1

b) Solar Altitude Angle

The solar altitude angle is the vertical angle of the sun above the horizon, as seen in Figure 2. The solar altitude (SA) may be found by using Equation 2.

$$SA = ARCSIN(\sin\delta \sin(L) - \cos(L)\cos\delta\cos(ST \times 15°)) \quad (2)$$

where L = latitude of the site

δ = the declination angle, which is defined as the angular position of the sun at solar noon with respect to the plane of the equator. δ can be found by using Equation 3.

$$\delta = 23.45° \sin(360 \times \frac{284+n}{365}) \quad (3)$$

where n = the number of the day in the year (listed in Table 1)

c) Solar Azimuth Angle

The solar azimuth angle (SAZ) is defined as the angular displacement from south of the projection of the line from sun to site on the horizontal plane, as shown in Figure 2. The SAZ may be calculated using Equation 4.

$$SAZ = ARCSIN[(\cos\delta \sin(-15° \times ST)/\cos SA) \quad (4)$$

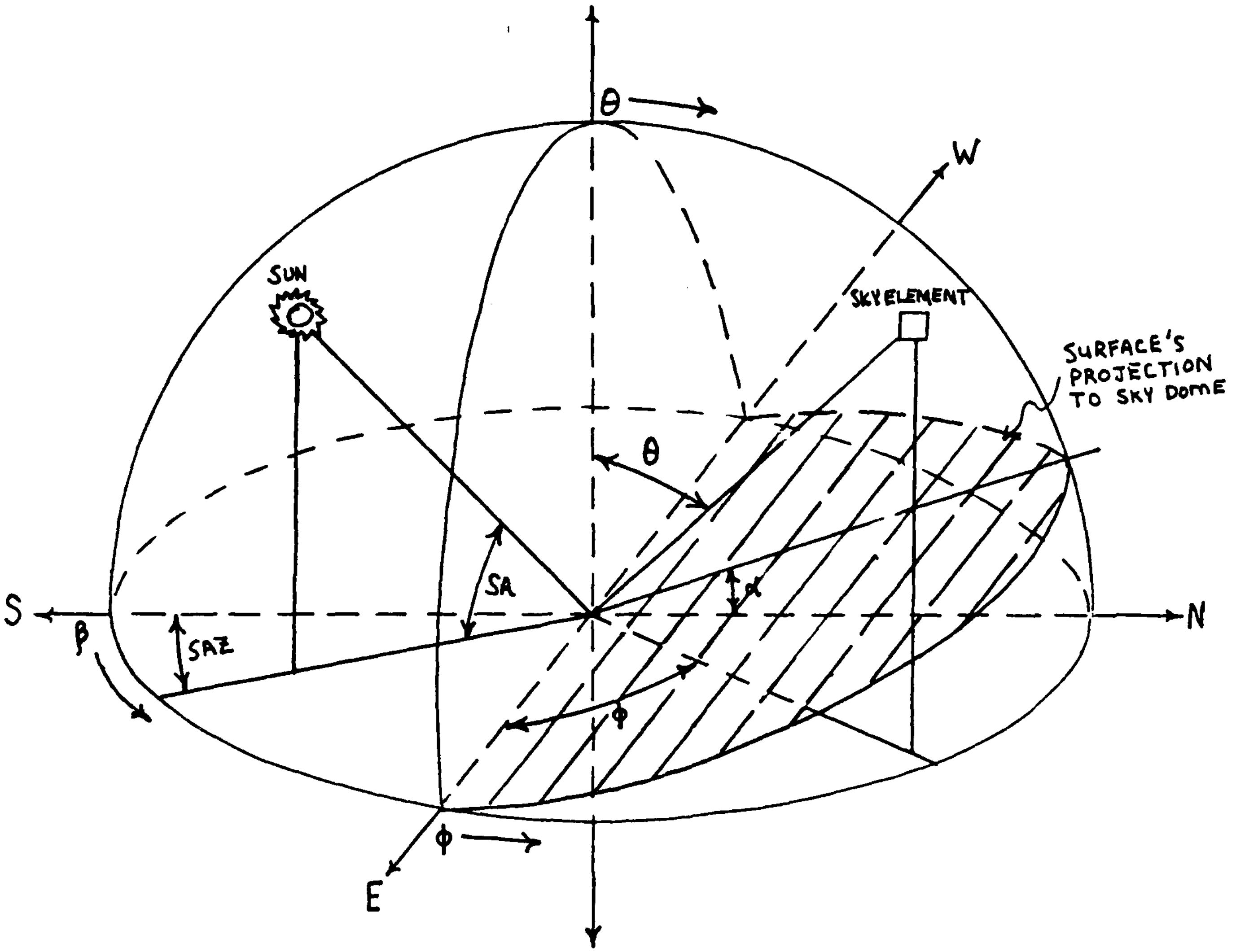

Figure 2. Angular representations.

2) Illuminance from an Overcast Sky:

a) Overcast Sky Luminance

The International Commission on Illumination (CIE) adopted the Moon and Spencer formula, given in Equation 5, to represent the luminance distribution of the completely overcast sky (Reference 3).

$$L = (L_z/3)\ [1+2\cos\theta] \tag{5}$$

where L_z = sky zenith luminance

θ = Angular distance from zenith to sky element (see Figure 2).

The sky zenith luminance is also given by the CIE as Equation 6.

$$L_z = 123 + 8600 \sin(SA)\ \frac{\text{candela}}{m^2} \tag{6}$$

b) Overcast Sky Illuminance

The illuminance on a sloped surface under completely overcast sky conditions is given in Equation 7 (see Appendix A for derivations of overcast sky equations)

$$E_{OC} = E_{HOC} \cos\alpha + E_{VOC} \sin\alpha \tag{7}$$

where E_{HOC} = Horizontal illuminance for overcast skies

E_{VOC} = Vertical illuminance for overcast skies

α = angle of tilt of sloped surface with respect to earth surface

The completely overcast sky's horizontal illuminance is calculated by Equation 8.

$$E_{HOC} = \frac{L_z}{3}\left[\frac{7\pi}{3} - \sum_{\theta = \frac{\pi}{2}-\alpha,\ \theta_{inc}}^{\pi/2\,*} (\pi-2\phi')\left[\frac{\sin^2\theta_2-\sin^2\theta_1}{2} - \frac{2}{3}(\cos^3\theta_2 - \cos^3\theta_1)\right]\right] \tag{8}$$

[Angles in Radians]

*The summation is read as: θ is summed from $\frac{\pi}{2} - \alpha$ to $\frac{\pi}{2}$ by θ_{inc} increments

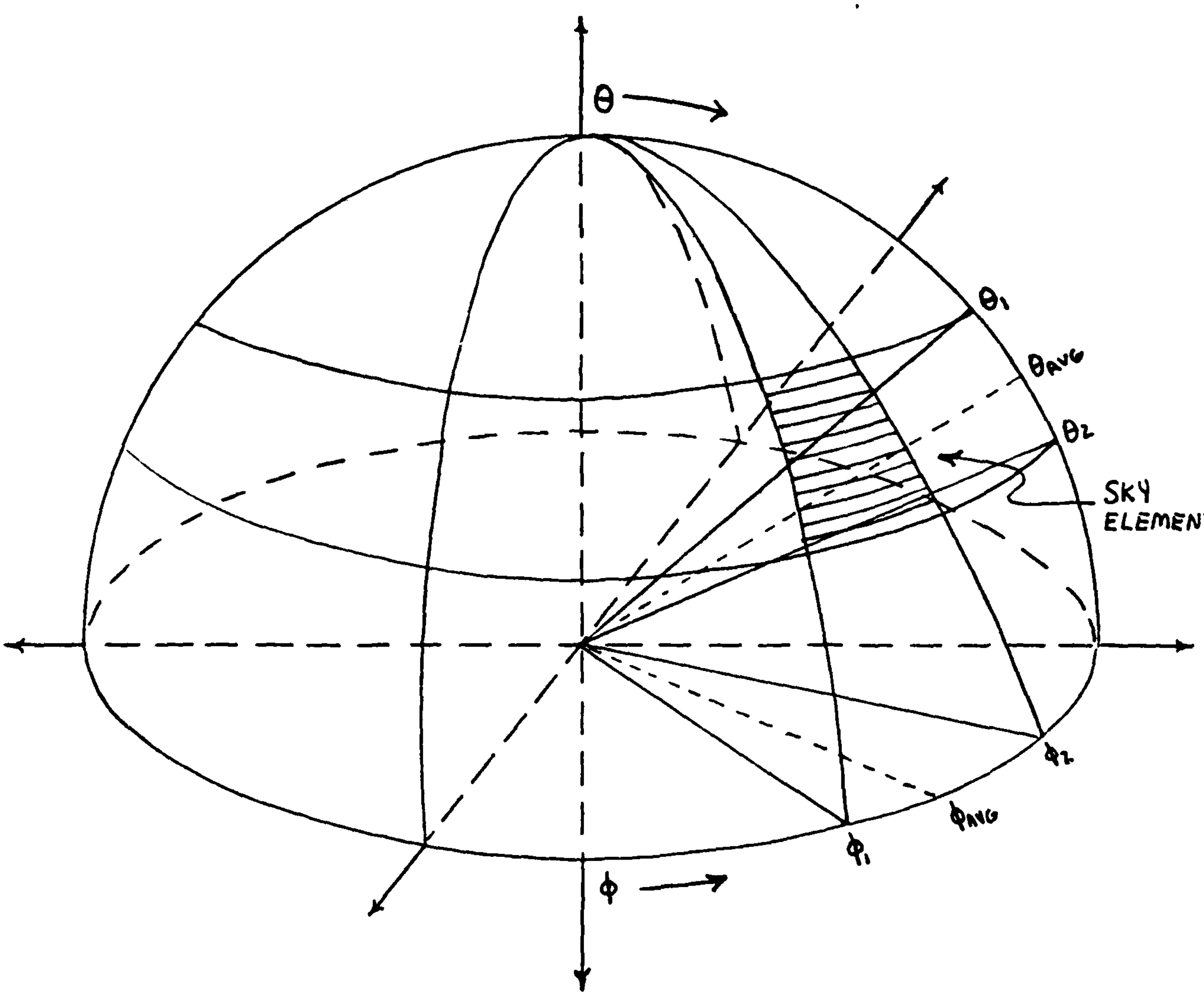

Figure 3. Sky element representation

where L_z = zenith luminance for overcast skies

θ_1, θ_2 = upper and lower bounds from the zenith (see Figure 3) to sky element.

ϕ' = Average angular distance from East to the surface's projected intersection with the sky sphere (see Figure 4).

ϕ' is calculated by Equation 9 (see Appendix B)

$$\phi' = \text{ARCSIN}(\text{CTN}\theta_{AVG}\text{CTN}\alpha) \text{ Radians} \tag{9}$$

where θ_{AVG} = Average angular distance from zenith to sky element

$= \left(\frac{\theta_1 + \theta_2}{2}\right)$

The vertical illuminance component of the completely overcast sky is calculated using Equation 10.

$$E_{VOC} = \frac{L_z}{3} \left[\sum_{\theta = \frac{\pi}{2} - \alpha, \theta_{inc}}^{\pi/2} \cos\phi' \left[(\theta_1 - \theta_2) + \frac{4}{3}(\sin^3\theta_1 - \sin^3\theta_2) - \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2) \right] \right] \tag{10}$$

[Angles in Radians]

A listing for the overcast sky illuminance (E_{OC}) is given in Table 3. It is listed with respect to time of day, day of the year, latitude of site, and slope of the surface in question.

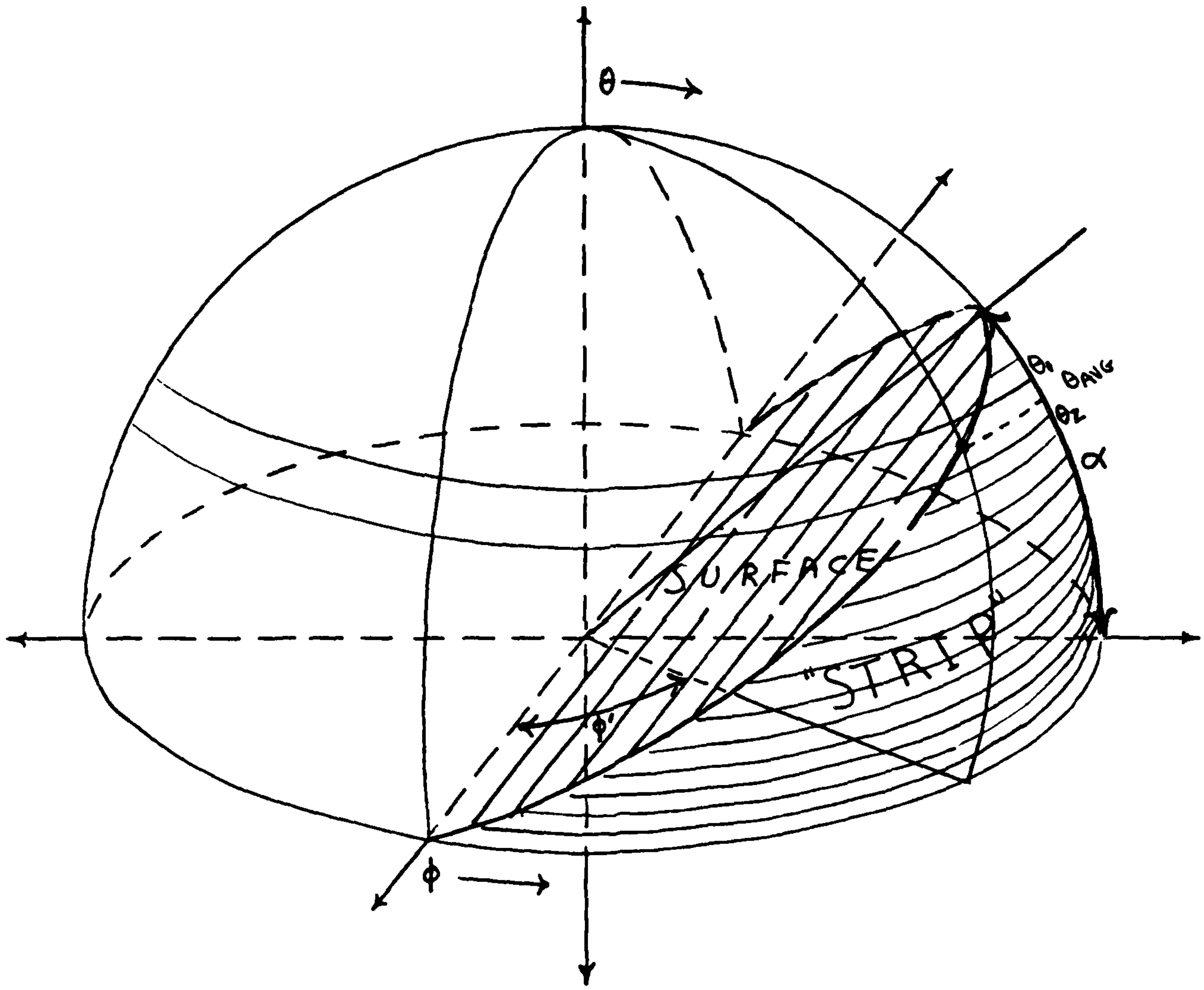

Figure 4. "STRIP"Representation

TABLE 3 OVERCAST SKY ILLUMINANCE (Ft-c)

LATITUDE (degrees)	SLOPE (degrees)	8 am 4 pm				9 am 3 pm				10 am 2 pm				11 am 1 pm				Noon			
		DEC 21	MAR 21	JUN 21	SEPT 21	DEC 21	MAR 21	JUN 21	SEPT 21	DEC 21	MAR 21	JUN 21	SEPT 21	DEC 21	MAR 21	JUN 21	SEPT 21	DEC 21	MAR 21	JUN 21	SEPT 21
30	0	415	871	1192	871	737	1221	1514	1221	983	1490	1760	1490	1138	1658	1915	1658	1191	1716	1968	1716
	15	405	848	1161	848	718	1189	1474	1189	958	1451	1715	1451	1109	1615	1866	1615	1160	1672	1917	1672
	30	375	786	1077	786	665	1102	1367	1102	888	1345	1590	1345	1028	1498	1730	1498	1076	1550	1777	1550
34	0	336	834	1205	834	644	1169	1513	1169	880	1427	1749	1427	1028	1588	1897	1588	1079	1644	1948	1644
	15	327	812	1174	812	627	1139	1474	1139	857	1390	1704	1390	1002	1547	1848	1547	1051	1601	1897	1601
	30	304	753	1088	753	581	1056	1366	1056	794	1288	1576	1288	928	1434	1713	1434	974	1484	1759	1484
38	0	256	794	1212	794	548	1112	1505	1112	772	1357	1729	1357	913	1511	1870	1511	961	1563	1918	1563
	15	249	773	1181	773	534	1084	1466	1084	752	1322	1684	1322	890	1471	1822	1471	937	1523	1869	1523
	30	231	717	1095	717	495	1004	1359	1004	697	1225	1561	1225	825	1364	1689	1364	868	1411	1732	1411
42	0	174	749	1214	749	450	1050	1489	1050	661	1281	1701	1281	794	1425	1834	1425	840	1475	1879	1475
	15	169	730	1182	730	438	1023	1451	1023	644	12[illegible]	1657	1247	774	1389	1787	1389	818	1437	1831	1437
	30	157	677	1096	677	406	948	1345	948	597	1156	1536	1156	717	1287	1656	1287	758	1337	1697	1332
46	0	91	702	1209	702	349	983	1467	983	547	1198	1665	1198	671	1334	1789	1334	714	1380	1832	1380
	15	89	684	1178	684	340	957	1429	957	533	1167	1622	1167	654	1299	1743	1299	695	1344	1784	1344
	30	82	634	1092	635	315	887	1325	887	494	1082	1503	1082	606	1205	1616	1205	644	1246	1654	1246

3) Illuminance from a Clear Sky:

a) Solar Illuminance (Direct Sunlight)

The direct perpendicular (normal to the earth's surface) solar illuminance is listed in Figure 7-8 of the IES Handbook, 6th ed. (Reference 4), or can be calculated by Equation 11, which is a polynomial "fit" of Figure 7-8.

$$E_{PSI} = -17.9 + 427.5(SA) - 8.7(SA)^2 + 0.086(SA)^3 - 0.3306E\text{-}3(SA)^4 \qquad (11)$$

where SA = Solar altitude in degrees.

The horizontal and vertical solar illuminances are given by Equations 12a and 12b, respectively.

$$E_{HSI} = E_{PSI}\sin(SA) \qquad (12a)$$

$$E_{VSI} = E_{PSI}\cos(SA) \qquad (12b)$$

The total solar illuminance on a sloped surface with any orientation is given by Equation 13 (see Appendix C).

$$E_{TSI} = E_{HSI}\cos\alpha + E_{VSI}\sin\alpha\cos(\beta - SAZ). \qquad (13)$$

where E_{HSI} = horizontal solar illuminance

E_{VSI} = vertical solar illuminance

α = angle of surface tilt

β = surface orientation with respect to south. A counter-clockwise direction is positive.

SAZ = solar azimuth angle (Equation 4)

b) Clear Sky Luminance

A standardized luminance distribution of clear skies accepted by the CIE is based on the Kittler equation (angles in radians) (Reference 5).

$$L = L_Z \frac{(1-e^{-.32/\cos\theta})(.91+10e^{-3\gamma}+.45\cos^2\gamma)}{.247(.91+10e^{-3(\pi/2-SA)}+.45\cos^2\gamma)} \quad (14)$$

where L = sky luminance

L_Z = zenith luminance

γ = angular distance from sun to sky element

θ = angular distance from zenith to sky element

SA = solar altitude

For the clear sky condition, the zenith luminance of Equation 15a is used. (Reference 6).

$$L_Z = 507e^{2.35SA}(cd/m^2) \quad (15a)$$

The angle γ is calculated by Equation 15b.

$$\gamma = \text{ARCCOS}[\sin SA\cos\theta_{AVG} - \cos SA\sin\theta_{AVG}\sin(\phi_{AVG} - SAZ)] \quad (15b)$$

where θ_{AVG} = average angular distance from zenith to sky element $(\frac{\theta_1+\theta_2}{2})$. (See Figure 4).

ϕ_{AVG} = average angular distance from East to sky elements ground projection $(\frac{\phi_1+\phi_2}{2})$(see Figure 4).

c) Clear Sky Illuminance

The total illuminance for a completely clear sky is composed of a sky luminance component and a solar illuminance component as given by Equation 16.

$$E_C = [E_{HC}\cos\alpha + E_{VC}\sin\alpha] + E_{TSI} \quad (16)$$

where E_{HC} = horizontal illuminance due to sky luminance

E_{VC} = vertical illuminance due to sky luminance

E_{TSI} = total solar illuminance (Equation 13)

α = tilt of the surface

E_{HC} is given in Equation 17 while E_{VC} is calculated in Equation 18.

$$E_{HC} = \sum_{\theta=0,\theta_{inc}}^{\pi/2} [(\frac{\sin^2\theta_2-\sin^2\theta_1}{2}) \sum_{\phi=0,\phi_{inc}}^{2n} L(\phi_2-\phi_1)] -$$

$$\sum_{\frac{\pi}{2}-\alpha,\theta_{inc}}^{\pi/2} [(\frac{\sin^2\theta_2-\sin^2\theta_1}{2}) \sum_{\phi=\phi'+\beta,\phi_{inc}}^{\pi-\phi'+\beta} L(\phi_2-\phi_1)] \qquad (17)$$

$$E_{VC} = \sum_{0,\theta_{inc}}^{\pi/2} (\frac{\theta_2-\theta_1}{2} + \frac{\sin 2\theta_1-\sin 2\theta_2}{4}) \sum_{\phi=\pi+\beta,\phi_{inc}}^{2\pi+\beta} L|(\cos\phi_1-\cos\phi_2)|$$

$$- \sum_{\theta=0,\theta_{inc}}^{\pi/2} (\frac{\theta_2-\theta_1}{2}) + \frac{\sin 2\theta_1-\sin 2\theta_2}{4}) \sum_{\phi=\beta,\phi_{inc}}^{\pi+\beta} L|(\cos\phi_1-\cos\phi_2)|$$

$$+ \sum_{\theta=\frac{\pi}{2}-\alpha,\theta_{inc}}^{\pi/2} (\frac{\theta_2-\theta_1}{2} + \frac{\sin 2\theta_1-\sin 2\theta_2}{4}) \sum_{\phi=\phi'+\beta,\phi_{inc}}^{\pi-\phi\ +\beta} L|(\cos\phi_1-\cos\phi_2)| \qquad (18)$$

where L = clear sky luminance

θ_1,θ_2 = upper and lower bounds from the zenith to the sky element (see Figure 4).

ϕ_1,ϕ_2 = upper and lower bounds, from East to the sky element's projection on the earth's surface

ϕ' = average angular distance from East to the ground projection of the surface's intersection with the sky sphere (see Figure 5)

β = surface orientation with respect to South. A counter-clockwise direction is positive.

The clear sky illuminance of Equation 16 is tabulated in Tables 4, 5, and 6. Table 4 contains E_c's for March 21 and September 21 (Spring & Fall), Table 5 contains E_c's for June 21 (Summer), and Table 6 contains E_c's for December 21 (Winter). The E_c's are tabulated with respect to the time of day, day of the year, latitude of the site, slope of the surface, and surface orientation. The surface's orientation, which is the direction the slope faces, is given in these tables as the main compass points N,S,E or W. Figure 6 gives a division of the compass into four quadrants of N,S,E or W to be used for rough approximations of surface orientation.

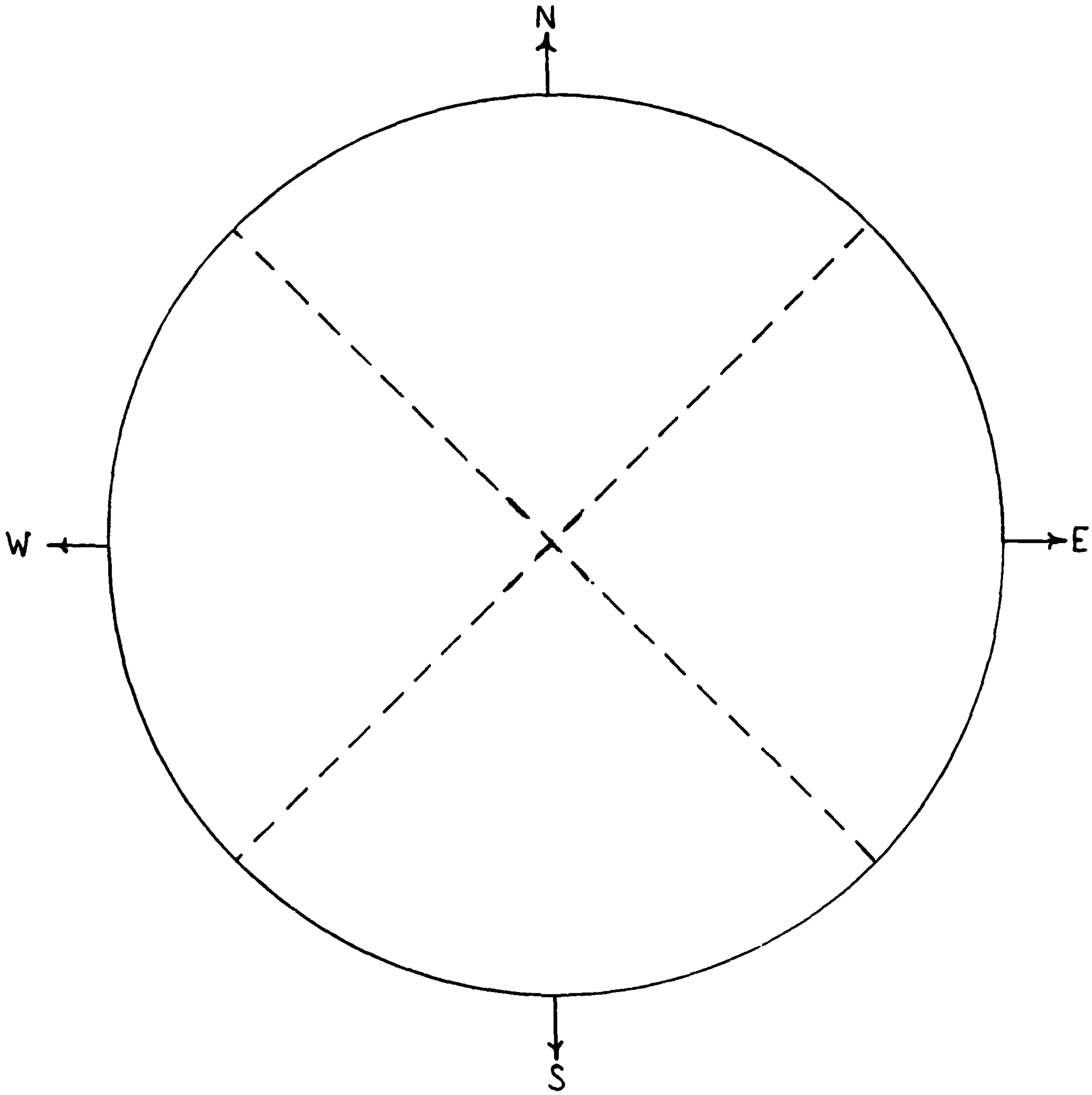

Figure 5. Division of the four compass points into four quadrants.

TABLE 4 CLEAR SKY ILLUMINANCE (Ft-C) - MAR & SEPT 21

Latitude (degrees)	Slope (degrees)	8 am				10 am				Noon				2 pm				4 pm			
		N	S	E	W	N	S	E	W	N	S	E	W	N	S	E	W	N	S	E	W
30	0	3718	3718	3718	3718	7692	7692	7692	7692	9190	9190	9190	9190	7692	7692	7692	7692	3718	3718	3718	3718
	15	3133	4098	5159	2211	6395	8531	8633	6265	7619	10207	8897	8897	6395	8531	6263	8633	3133	4098	2210	5159
	30	2389	4236	6266	975	4744	8839	9014	4438	5620	10580	8031	8030	4744	8839	9014	2389	4236	4236	975	6266
34	0	3498	3498	3498	3498	7277	7277	7277	7277	8708	8708	8708	8708	7277	7277	7277	7277	3498	3498	3498	3498
	15	2878	3929	4911	1874	5890	8238	8223	5874	7023	9873	8431	8431	5890	8238	5874	8222	2878	3929	1872	4911
	30	2116	4127	6006	610	4184	8686	8635	4105	4956	10416	7614	7615	4184	8686	4104	8635	2116	4127	609	6006
38	0	3260	3260	3260	3260	6821	6821	6821	6820	8176	8176	8176	8176	6821	6821	6821	6821	3260	3260	3260	3260
	15	2612	3732	4638	1682	5355	7890	7767	5448	6394	9475	7918	7918	5355	7890	5448	7766	2612	3732	1683	4638
	30	1841	3985	5716	584	3613	8486	8211	3739	4277	10179	7155	7155	3613	8468	3739	8210	1841	3985	582	5716
42	0	3003	3003	3003	3003	6322	6322	6322	6322	7597	7597	7597	7597	6322	6322	6322	6322	3003	3003	3003	3003
	15	2337	3511	4341	1485	4796	7485	7266	4987	5735	9016	7359	7361	4796	7485	4987	7267	2337	3511	1485	4341
	30	1567	3813	5400	554	3035	8183	7740	3348	3590	9807	6654	6654	3035	8185	3348	7740	1567	3813	554	5401
46	0	2733	2733	2733	2733	5788	5788	5788	5788	6973	6973	6973	6973	5788	5788	5788	5788	2733	2733	2733	2733
	15	2059	3266	4022	1280	4221	7028	6724	4495	5055	8491	6755	6755	4221	7028	4494	6724	2059	3266	1281	4022
	30	1300	3610	5055	525	2460	7832	7228	2936	2903	9478	6114	6115	2460	7832	2936	7228	1300	3610	525	5055

TABLE 5 CLEAR SKY ILLUMINANCE (Ft-C) - JUNE 21

Latitude (degrees)	Slope (degrees)	8 am				10 am				Noon				2 pm				4 pm			
		N	S	E	W	N	S	E	W	N	S	E	W	N	S	E	W	N	S	E	W
30	0	5748	5748	5748	5748	9485	9485	9485	9485	10833	10833	10833	10833	9485	9485	9485	9584	5748	5748	5748	5748
	15	5332	5838	7232	3900	9057	9330	10299	8059	10189	10801	10480	10480	9057	9330	8059	10299	5332	5838	3900	7232
	30	4587	5578	8245	1796	8954	8600	10440	6109	8915	10091	9442	9442	8954	8600	6109	10440	4587	5578	1796	8245
34	0	5831	5831	5831	5831	9408	9408	9408	9408	10724	10724	10724	10724	9408	9408	9408	9408	5831	5831	5831	5831
	15	5495	5825	7319	3975	8824	9416	10225	7980	9901	10881	10376	10376	8824	9416	7980	10225	5495	5825	3975	7319
	30	4838	5472	8329	1856	7704	8841	10373	6044	8470	10352	9350	9350	7704	8841	6044	10373	4838	5472	1856	8329
38	0	5878	5878	5878	5878	9277	9277	9277	9277	10538	10538	10538	10538	9277	9277	9277	9277	5878	5878	5878	5878
	15	5635	5774	7367	4016	8540	9445	10097	7858	9542	10882	10197	10200	8540	9445	7859	10097	5635	5774	4016	7367
	30	5060	5329	8375	1890	7290	9027	10258	5930	7968	10540	9191	9191	7290	9027	5932	10258	5060	5329	1889	8375
42	0	5886	5886	5886	5886	9090	9090	9090	9090	10282	10282	10282	10282	9090	9090	9090	9090	5886	5886	5886	5886
	15	5684	5738	7376	4024	8208	9417	9915	7680	9122	10809	9950	9950	8208	9417	7680	9915	5684	5738	4024	7376
	30	5154	5251	8384	1896	6838	9160	10094	5774	7417	10654	8972	8972	6338	9160	5774	10094	5154	5251	1896	8589
46	0	5859	5859	5859	5859	8849	8849	8849	8849	9963	9963	9963	9963	8849	8849	8849	8849	5859	5859	5859	5859
	15	5565	5806	7346	3998	7830	9332	9681	7451	8649	10666	9642	9642	7830	9332	7451	9681	5565	5806	3998	7346
	30	4946	5409	8056	1875	6350	9233	9882	5572	6829	10696	8696	8696	6350	9233	5572	9882	4946	5409	1875	8056

TABLE 6 CLEAR SKY ILLUMINANCE (Ft-C) - DEC 21

		8 am				10 am				Noon				2 pm				4 pm			
		N	S	E	W	N	S	E	W	N	S	E	W	N	S	E	W	N	S	E	W
30	0	1310	1310	1310	1310	4413	4413	4413	4413	5740	5740	5740	5740	4413	4413	4413	4413	1310	1310	1310	1310
	15	638	1938	2149	463	2786	5809	5230	3334	3785	7381	5565	5564	2786	5809	3334	5230	638	1938	463	2149
	30	427	2459	2857	385	1072	6847	5716	2078	1689	8561	5047	5049	1072	6847	2074	5716	428	2459	383	2857
34	0	1001	1001	1001	1001	3775	3775	3775	3775	5019	5019	5019	5019	3775	3775	3775	3775	1001	1001	1001	1001
	15	431	1544	1709	420	2198	5163	4561	2769	3085	6686	4867	4868	2198	5163	2769	4562	431	1544	420	1709
	30	391	2007	2315	351	635	6236	5061	1623	1063	7938	4422	4422	635	6236	1622	5060	391	2007	351	2315
38	0	734	734	734	734	3135	3135	3135	3135	4278	4278	4278	4278	3135	3135	3135	3135	734	734	734	734
	15	388	1175	1294	380	1640	4483	3880	2215	2402	5937	4152	4155	1640	4483	2214	3880	388	1175	380	1290
	30	357	1556	1777	321	570	5559	4383	1192	659	7230	3779	3780	570	5559	1192	4383	357	1556	321	1778
42	0	524	524	524	524	2508	2508	2508	2508	3532	3532	3532	3532	2508	2508	2508	2508	524	524	524	524
	15	351	839	914	344	1133	3779	3197	1687	1754	5143	3431	3431	1133	3779	1686	3197	351	839	344	915
	30	325	1116	1252	293	510	4822	3689	797	586	6439	3131	3137	510	4822	797	3690	325	1116	293	1250
46	0	382	382	382	382	1959	1959	1959	1959	2799	2799	2799	2799	1959	1959	1959	1959	382	382	382	382
	15	316	551	580	310	695	3066	2530	1205	1166	4316	2722	2722	695	3066	1203	2530	316	551	310	580
	30	295	701	750	267	454	4036	2993	460	520	5570	2493	2495	454	4036	459	2995	295	701	267	750

4) Examples:

a) Overcast Sky Example

What is the illuminance on a 15° tilted surface in Boston, Ma., for an overcast sky on Dec. 21 at 2 pm standard time?

First find solar time: Equation 1. Need site's latitude and longitude, the standard meridian, and ET, as well as standard time.

Site Lat. = 42° } from map
Site Long. = 71° }

Std Merd. = 75° (chart 2 of Figure 1)

ET = +1 min (Table 2)

Eq. 1:

Solar time = 14:00 + [4(75-71) + 1] = 14:00 + :17

Solar time = 14:17 or 2:17 pm

Now use 21 Dec column and 15° slope row of Table 3. Interpolating between 2 pm and 3 pm to obtain a value for 2:17 pm yields.

$$E_{oc} \text{ (2:17)} = 584.5 \text{ ft-c}$$

b) Clear Sky Example (1)

What is the exterior horizontal illuminance for Washington, DC, on June 21 at 10 am Daylight Savings Time under clear sky conditions.

Find solar time: Use Equation 1

Standard Time = 10-1 = 9 am

Site Lat. = 39° } from map
Site Long. = 77° }

Std Merd. = 75°

ET = -1 min

Solar time = (9:00) + [4(75-77) + (-1)]

: = 9:00 + [-9]

Solar time = 8:51 am

With double interpolation from Table 5, as shown below, the horizontal illuminance (0° slope) is found to be 7305 ft-c. Note that surface orientation is <u>not</u> a factor in the horizontal illuminance case.

Lat	8:00	8:51	10:00
	Solar Time		
38	5878↓		9277↓
39	5880 →	7305	← 9230
42	5886↑		9090↑

Clear Sky Example (2)

Repeat Example (1) but with a 15° sloped surface with a northern exposure. Use Table 5 but with a 15° slope and a north orientation. From before, the solar time is 8:51 am and latitude is 39°. Using double interpolation we find an exterior illuminance of 6841 ft-c.

North at 15°

Lat	8:00	8:51	10:00
	Solar Time		
38	5635↓		8540↓
39	5647 →	6841	← 8457
42	5684↑		8208 ↑

Note that for this case that the orientation is very important, it determines which column to use in Table 5.

C. INTERIOR ILLUMINANCE FROM SLOPED SKYLIGHTS

This section is composed of two parts. First, a calculation procedure, similar to that in Navy Tech Data Sheet 80-09, but for pitched, rather than horizontal, skylights, will be presented. Second, a simplifying straight line equation for calculating room coefficients of utilization will be developed.

The geometry to be considered is shown in Figure 6. It consists of a room of length ℓ, width w and wall height h, with a ceiling sloped at an angle ϕ. Skylights of length a and width b are imbedded in the roof, n per side. It is assumed that these skylights are perfectly diffusing and have a transmittance τ, a reflectance from their interior side ρ_s and a dirt depreciation factor SDD. A procedure to determine the average maintained illuminance on a work plane 2.5 feet above the floor is desired, assuming the illuminance on the exterior of the skylights is known.

The procedure will be presented through three work sheets. Worksheet 1 deals with the skylights and asks the user to enter the data described in the previous paragraph. Worksheet 2 concerns the room. The first six items are data entries. Items 7-10 involve finding the effective reflectance of the ceiling cavity at the upper dashed line in Figure 6. This converts the room interior into a rectangular parallepiped of height h, to which the customary zonal cavity method of interior lighting design may be applied. The room cavity ratio is calculated and entered in location 9 on Worksheet 2. Then the room coefficient of utilization is obtained from Table 8 on page 34, a table to be developed in the next portion of this report. The result is entered in location 10.

Finally, the last step on Worksheet 2 is the entry of the expected room surface dirt depreciation (RSDD).

The equations in steps 7 and 8 on Worksheet 2 may be justified as follows: The weighted average reflectance of the ceiling surface is

$$\rho_{av} = \frac{\rho_c A_c + \rho_s A_s}{A_c + A_s} \quad (19)$$

where A_s is the skylight area and A_c is the area of the nonskylight portion of the ceiling. A_s and A_c are given by

$$A_s = 2nab$$
$$A_c = \frac{w\ell}{\cos\phi} - 2nab \quad (20)$$

Substituting Equations 20 into Equation 19 yields

$$\rho_{av} = \rho_c - \frac{2nab\ \cos\phi}{w\ell} (\rho_c - \rho_s) \quad (21)$$

Equation 21 ignores the triangular ends of the ceiling cavity, which are usually relatively small.

With ρ_{av} calculated, we can proceed to obtain ρ_{cc}. The general form for the effective cavity reflectance of a non-horizontal ceiling is

$$\rho_{cc} = \frac{1}{\frac{A}{A_o} \left(\frac{1-\rho_{av}}{\rho_{av}}\right)} \quad (22)$$

where A is the area of the ceiling cavity and A_o is the area of the ceiling cavity opening. From Equation 20, Equation 22 is rewritten as

$$\rho_{cc} = \frac{1}{1 - \frac{1}{\cos\phi} \left(\frac{1-\rho_{av}}{\rho_{av}}\right)} \quad (23)$$

A comment on the two dirt depreciation factors is in order. SDD represents dirt accumulation on the exterior and interior of the skylights, which will reduce the transmittance τ over time. The authors are not aware of any data on this factor. It is suggested that, in the absence of such data, a value of .75 be assumed and that the skylights be cleaned at least once annually.

To determine RSDD, the method in IES Handbook VI, Reference Volume, Chapter 9 may be used, with the skylight interpreted as a direct luminaire. Otherwise an average value of .90 may be assumed.

We turn next to Worksheet 3, which requires the entry of sky data. The first four items are self-explanatory. For now, the sky condition is either overcast or clear, but, in the future, data for other sky conditions, such as partly cloudy may become available.

Item 5 on Worksheet 3 asks that the illuminance on the tilted skylight be entered for either of two building orientations. If the walls labeled "ℓ" in Figure 4 face north and south, the illuminance on the north-facing skylights is entered in the E_1 slot and that on the south-facing skylights in the E_2 slot. If the "ℓ" walls face east and west, E_1 is the illuminance on the east-facing skylights and E_2 is the illuminance on those skylights facing west. The values for E_1 and E_2 are taken from the appropriate tables in Section B of this report.

All information for the calculation of average maintained interior illuminance on the work plane has now been assembled and we turn to Worksheet 4. The work plane area and skylight area per side are entered. Next the overall utilization coefficient, which is the product of the skylight transmittance and the room coefficient of utilization, is calculated and entered. Following this the overall light loss factor, including both skylight and room surface dirt depreciation is calculated and entered. Finally, the desired illuminance is obtained. The quantity $(E_1+E_2)\ A_s$ is the luminous flux impinging on the skylights. $(E_1+E_2)A_sK_u$ is the luminous flux impinging on the work plane if all surfaces are clean. Multiplying this flux by K_m gives the average luminous flux over time reaching the work plane. Division by the work plane area gives the average maintained work plane illuminance.

A table of room coefficients of utilization for skylights is presented in Figure 9-76 of IES Handbook VI, Reference Volume. These are given for ceiling reflectances of 75% and 50% and wall reflectances of 50% and 30%, as a function of a quantity called Room Ratio (RR). These data may be made a function of room cavity ratio through the relation

$$RCR = \frac{5}{RR} \tag{24}$$

If the data are plotted against RCR for the various reflectances, four curves result which are very close to each other and may be approximated by a single straight line equation as

$$RCU = -.057RCR+.965 \tag{25}$$

Use of this equation eliminates the need for calculating ρ_{av} and ρ_{cc} in Steps 7 and 8 of Worksheet 2.

RCU's from Equation 25, when compared with those in Figure 9-76 in the IES Handbook, differ by a maximum of 6% in the range $1<RCR<8$ and by slightly higher percentages outside this RCR range.

SLOPED-ROOF SKYLIGHTS

DIAGRAM AND WORKSHEETS

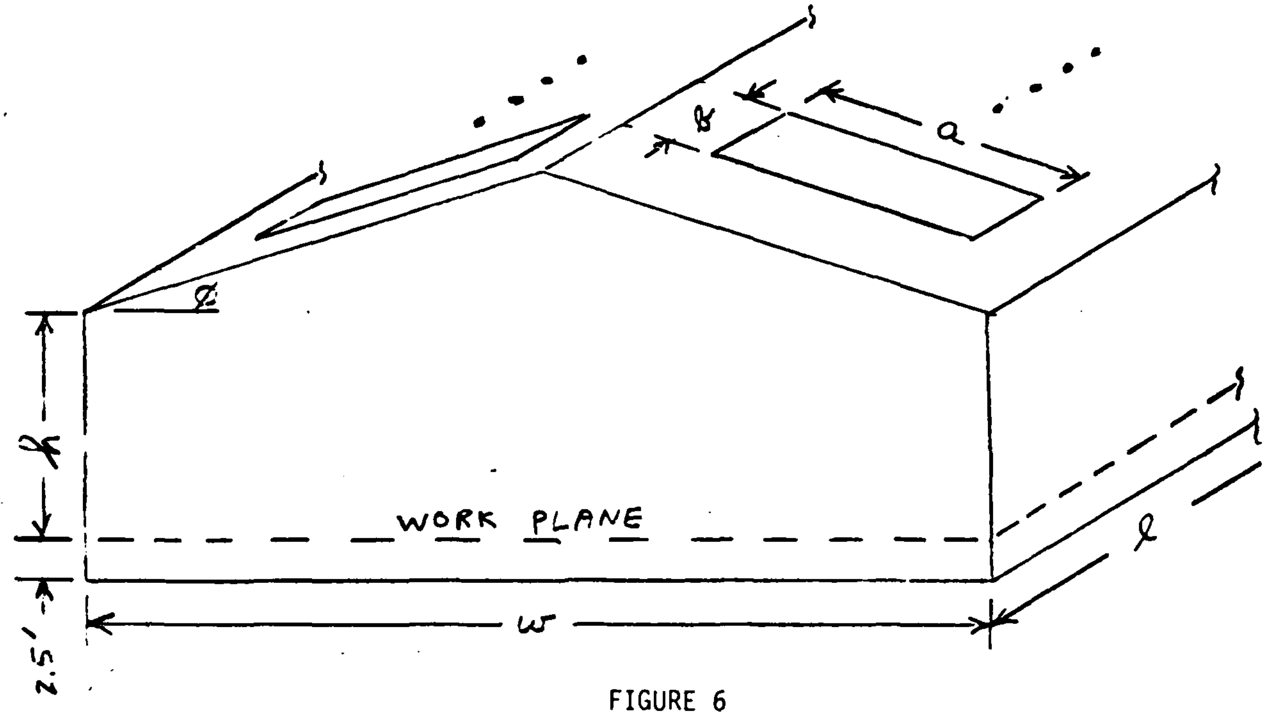

FIGURE 6

WORKSHEET 1 - SKYLIGHTS

1.	Length	a	= ____	ft.
2.	Width	b	= ____	ft.
3.	Number Per Side	n	= ____	
4.	Dirt Depreciation Factor	SDD	= ____	
5.	Transmittance	τ	= ____	
6.	Reflectance	ρ_s	= ____	

WORKSHEET 2 - ROOM

1. Directions Roof Faces N-S _____, E-W _____

2. Roof Pitch ϕ = _____ deg.

3. Work Plane Length ℓ = _____ ft.

4. Work Plane Width w = _____ ft.

5. Room Cavity Height h = _____ ft.

6. Room Surface Reflectances

   ρ_c = _____, ρ_w = _____, ρ_f = _____

7. Average Ceiling Reflectance

   $$\rho_{av} = \rho_c - \frac{2nab\cos\phi}{w\ell}(\rho_c - \rho_s) = \_\_\_\_\_$$

8. Effective Ceiling Cavity Reflectance

   $$\rho_{cc} = \frac{1}{1+\frac{1}{\cos\phi}\left(\frac{1-\rho_{av}}{\rho_{av}}\right)} = \_\_\_\_\_$$

9. Room Cavity Ratio $RCR = \frac{5h(w+\ell)}{w\ell}$ = _____

10. Room Coefficient of Utilization RCU = _____

11. Dirt Depreciation Factor RSDD = _____

BLANK

WORKSHEET 3 - SKY ILLUMINANCE

1. Date ______
2. Time ______
3. Sky Condition ______
4. Latitude ______
5. Skylight Illuminance $E_1 =$ ______ ft-c

   $E_1 = E_n$ or E_e $E_2 =$ ______ ft-c

   $E_2 = E_s$ or E_w

WORKSHEET 4 - INTERIOR ILLUMINANCE

1. Work Plane Area $A_{wp} = w\ell =$ ______ ft^2
2. Skylight Area Per Side $A_s = nab =$ ______ ft^2
3. Utilization Coefficient $K_u = RCU x \tau =$ ______
4. Light Loss Factor $K_m = SDD x RSDD =$ ______
5. Average Maintained Work Plane Illuminance

$$E_{wp} = \frac{(E_1 + E_2) x K_u x K_m x A_s}{A_{wp}} = \_\_\_\_\_\_ \text{ ft-c}$$

D. ROOM COEFFICIENTS OF UTILIZATION FOR SKYLIGHTS IN HORIZONTAL ROOFS

In this section a suggested revision of Table 2 in Navy Tech Data Sheet 80-09 will be presented. New values for the room coefficients of utilization appearing in this table will be calculated using the procedure in IES Handbook VI, Reference Volume, pages 9-37 to 9-39. This procedure may be adapted to skylights as follows:

1. For each skylight, define nine 10-degree flux zones. Let n be the zone index, where $1 \leq n \leq 9$, with zone 1 adjacent to the nadir and zone 9 adjacent to the roof.

2. Compute the lumens in each zone from

$$\phi_n = 2\pi I(\cos\theta_2 - \cos\theta_1) \tag{26}$$

where θ_2 and θ_1 are the angles bounding the 10-degree zone and I is the mid-zone intensity.

3. Determine the total (downward) lumens leaving the skylight.

$$\phi = \sum_{n=1}^{9} \phi_n \tag{27}$$

4. Separate the total lumens into two parts, those traveling directly to the walls and those going directly to the work plane. The fraction of the flux which reaches the work plane directly is

$$D_m = \frac{1}{\phi} \sum_{n=1}^{9} k_{mn}\phi_n \tag{28}$$

where k_{mn} is called a zonal multiplier and is a function of cavity ratio (m), zone (n), and spacing to mounting height ratio (S/MH). Zonal multipliers have been published only for S/MH values of .4, .7 and 1.0. In skylight installations, the S/MH ratio is often larger. With the aid of unpublished data assembled by R.E. Levin (GTE Sylvania), we have developed a set of zonal multipliers for an S/MH ratio of 1.5. These appear in Table 7 and are valid for room length to width ratios between 1 and 2 and for an edge ratio of 1/2 (distance from skylight to wall is one-half distance between skylights).

TABLE 7 ZONAL MULTIPLIERS, S/MH = 1.5

Zone (n)	1	2	3	4	5	6	7	8	9	10
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
4	.98	.96	.94	.92	.90	.88	.86	.84	.82	.80
5	.87	.74	.60	.47	.34	.21	.08	0	0	0
6	.82	.64	.48	.32	.18	.06	0	0	0	0
7	.75	.52	.33	.18	.09	.01	0	0	0	0
8	.56	.22	.04	0	0	0	0	0	0	0
9	.10	0	0	0	0	0	0	0	0	0

5. Obtain the form factor (f). Form factors for various room cavity ratios, based on L/W = 1.6, are given in Figure 9-17 in the IES Handbook.

6. Calculate the room coefficient of utilization from

$$CU = \frac{2.5\rho_1 C_1 C_3 (1-D_m)}{m(1-\rho_1)(1-\rho_3)C_o} + \left[1 - \frac{\rho_3 C_3 (C_1 + C_2)}{(1-\rho_3)C_o}\right] \frac{D_m}{1-\rho_3} \qquad (29)$$

where

$$C_1 = \frac{(1-\rho_1)(1-f^2)m}{2.5\rho_1(1-f^2)+mf(1-\rho_1)}$$

$$C_2 = \frac{(1-\rho_2)(1+f)}{1+\rho_2 f} \qquad (30)$$

$$C_3 = \frac{(1-\rho_3)(1+f)}{1+\rho_3 f}$$

$$C_o = C_1 + C_2 + C_3$$

and ρ_1, ρ_2 and ρ_3 are the reflectances of the walls, ceiling and floor, respectively.

In carrying out this procedure to determine room coefficients of utilization for skylights, two normalized intensity distributions were assumed namely,

$$I = \cos\theta, \; I = \cos^3\theta \qquad (31)$$

The first represents a perfectly diffusing skylight, whereas the second is a more directed distribution. The resulting coefficients of utilization appear in Tables 8 and 9.

An examination of Tables 8 and 9 shows that the coefficients are consistently greater for $I = \cos^3\theta$, for a given RCR, ρ_c and ρ_w. This is because of the greater directivity of the $\cos^3\theta$ distribution, i.e. more flux reaching the work plane directly.

It should not be assumed that either of the intensity distributions in Equations 31 is typical of a particular skylight. Until intensity distribution curves for a variety of skylights under a variety of sun angle and atmospheric conditions become available, it is fruitless to speculate on what "typical" distributions might be. In the interim, it is suggested that the $\cos\theta$ (perfectly diffusing) distribution be used as the norm and that coefficients for that distribution be used (Table 8). Parenthetically, it should be noted that the coefficients in Table 8 are generally larger than the corresponding coefficients obtained from Figure 9-76 in the IES Handbook, which have been used in skylight calculations for a considerable number of years.

ρ_c	RCR	ρ_w		
		50%	30%	10%
80%	0	1.19	1.19	1.19
	1	1.05	1.00	0.97
	2	0.93	0.86	0.81
	3	0.83	0.76	0.70
	4	0.75	0.67	0.60
	5	0.67	0.59	0.53
	6	0.62	0.53	0.47
	7	0.57	0.49	0.43
	8	0.54	0.47	0.41
	9	0.53	0.46	0.41
	10	0.52	0.45	0.40
50%	0	1.11	1.11	1.11
	1	0.98	0.95	0.92
	2	0.87	0.83	0.78
	3	0.79	0.73	0.68
	4	0.71	0.64	0.59
	5	0.64	0.57	0.52
	6	0.59	0.52	0.47
	7	0.55	0.48	0.43
	8	0.52	0.46	0.41
	9	0.51	0.45	0.40
	10	0.50	0.44	0.40
20%	0	1.04	1.04	1.04
	1	0.92	0.90	0.88
	2	0.83	0.79	0.76
	3	0.75	0.70	0.66
	4	0.68	0.62	0.58
	5	0.61	0.56	0.51
	6	0.57	0.51	0.46
	7	0.53	0.47	0.43
	8	0.51	0.45	0.41
	9	0.50	0.44	0.40
	10	0.49	0.44	0.40

TABLE 8 COEFFICIENTS OF UTILIZATION

ρ_f = 20%, S/MH = 1.5, I = $\cos\theta$

ρ_c	RCR	ρ_w		
		50%	30%	10%
80%	0	1.19	1.19	1.19
	1	1.11	1.08	1.06
	2	1.03	0.99	0.96
	3	0.97	0.92	0.88
	4	0.91	0.86	0.82
	5	0.86	0.80	0.76
	6	0.81	0.75	0.71
	7	0.76	0.71	0.67
	8	0.74	0.68	0.65
	9	0.73	0.68	0.64
	10	0.72	0.67	0.63
50%	0	1.11	1.11	1.11
	1	1.05	1.03	1.01
	2	0.98	0.95	0.93
	3	0.93	0.89	0.76
	4	0.88	0.84	0.80
	5	0.83	0.79	0.75
	6	0.79	0.74	0.70
	7	0.74	0.70	0.66
	8	0.72	0.68	0.64
	9	0.71	0.67	0.64
	10	0.71	0.66	0.63
20%	0	1.04	1.04	1.04
	1	0.99	0.98	0.97
	2	0.94	0.92	0.90
	3	0.89	0.87	0.84
	4	0.85	0.82	0.79
	5	0.81	0.77	0.74
	6	0.77	0.73	0.70
	7	0.73	0.69	0.66
	8	0.71	0.67	0.64
	9	0.70	0.66	0.63
	10	0.69	0.66	0.63

TABLE 9 COEFFICIENTS OF UTILIZATION

$\rho_f = 20\%$, $S/MH = 1.5$, $I = \cos^3\theta$

E. APPENDIX

A. Derivation of overcast sky equations for illuminance.

B. Derivation of ϕ'.

C. Derivation of (E_{TSI}), total solar illuminance.

D. Derivation of clear sky equations for illuminance.

A. Derivation of Overcast Sky Equations for Illuminance Due to Sky luminance

1. The horizontal illuminance for a sloped surface under overcast sky conditions is given by Equation A1.

$$E_{HOC} = E_{HFLAT} - E_{HSTRIP} \tag{A1}$$

where E_{HFLAT} = horizontal illuminance due to overcast sky luminance of entire sky dome.

E_{HSTRIP} = horizontal illuminance due to overcast sky luminance of a strip of the sky dome (See Figure A1)

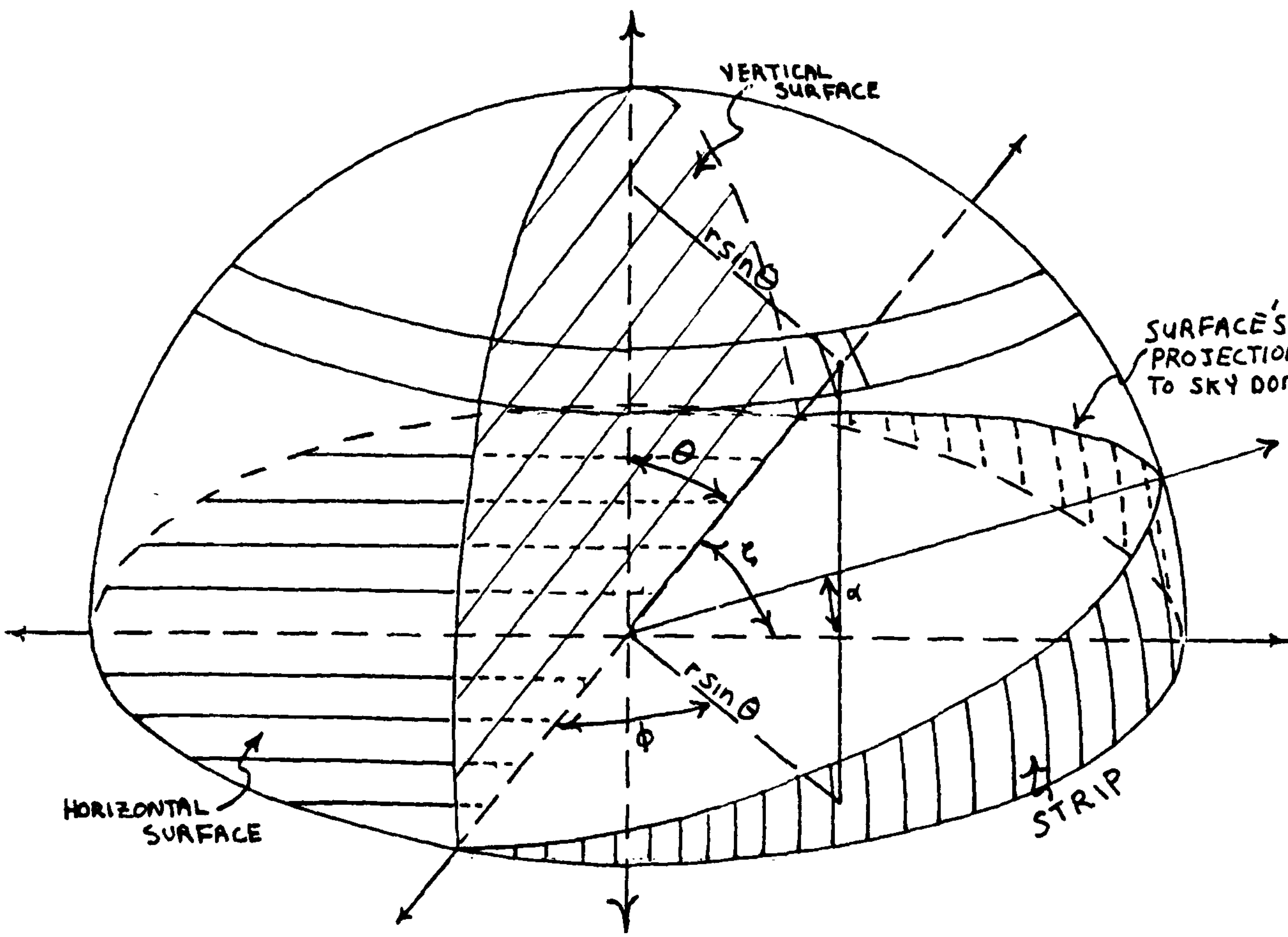

Figure A1

The portion of horizontal illuminance for each sky element is given by

$$dE_H = \frac{L\cos\theta dA}{r^2}$$

where $dA = (r\sin\theta)d\phi r d\theta$

$$L = \frac{L_z}{3}[1+L_z\cos\theta]$$

$$dE_H = \left(\frac{L_z}{3}\right)(1+L_z\cos\theta)(\cos\theta\sin\theta)d\theta d\phi$$

By selecting the correct limits of integration E_{HFLAT} and E_{HSTRIP} can be calculated as follows:

$$E_{HFLAT} = \int_0^{2\pi}\int_0^{\pi/2} (L_z/3)(1+2\cos\theta)(\cos\theta\sin\theta)d\theta d\phi$$

$$E_{HFLAT} = (7/9)\pi\ L_z$$

$$E_{HSTRIP} = \int_{\phi'}^{\pi-\phi'}\int_{\pi/2-\alpha}^{\pi/2} (L_z/3)(1+2\cos\theta)(\cos\theta\sin\theta)d\theta d\phi$$

which is not in a closed integral form due to ϕ' (see Equation 9), so by approximation

$$E_{HSTRIP} = \sum_{\theta=\pi/2-\alpha,\theta_{inc}}^{\pi/2} (\pi-2\phi')\frac{L_z}{3}\left[\frac{\sin^2\theta_2-\sin^2\theta_1}{2} - \frac{2}{3}(\cos^3\theta_2-\cos^3\theta_1)\right]$$

2. The vertical illuminance is due to the uniform luminance distribution with respect to ϕ. Each θ-ring has constant luminance all the way around. It is seen that the only vertical component is due to the strip's non-contribution (if the sky dome is divided into half with the left of the surface as positive and right of the surface negative) and is positive.

$$dE_V = \frac{L\cos\zeta dA}{r^2}$$

where $\cos\zeta = \sin\theta\sin\phi$

$$dE_V = \left(\frac{L_z}{3}\right)(1+2\cos\theta)\sin\phi\sin^2\theta d\theta d\phi$$

$$E_V = \int_{\phi'}^{\pi-\phi'} \sin\phi \int_{\pi/2-\alpha}^{\pi/2} \frac{L_z}{3}(1+2\cos\theta)\sin^2\theta d\theta d\phi$$

which is approximated by

$$E_{VOC} = \frac{L_z}{3} \sum_{\theta=\pi/2-\alpha,\theta_{inc}}^{\pi/2} \cos\phi'\left((\theta_1-\theta_2) + \frac{4}{3}(\sin^3\theta_1-\sin^3\theta_2) - \frac{\sin2\theta_1-\sin2\theta_2}{2}\right)$$

3. The total illuminance under overcast sky conditions is

$$E_{OC} = E_{HOC}\cos\alpha + E_{VOC}\sin\alpha.$$

B. <u>Derivation of ϕ'</u>

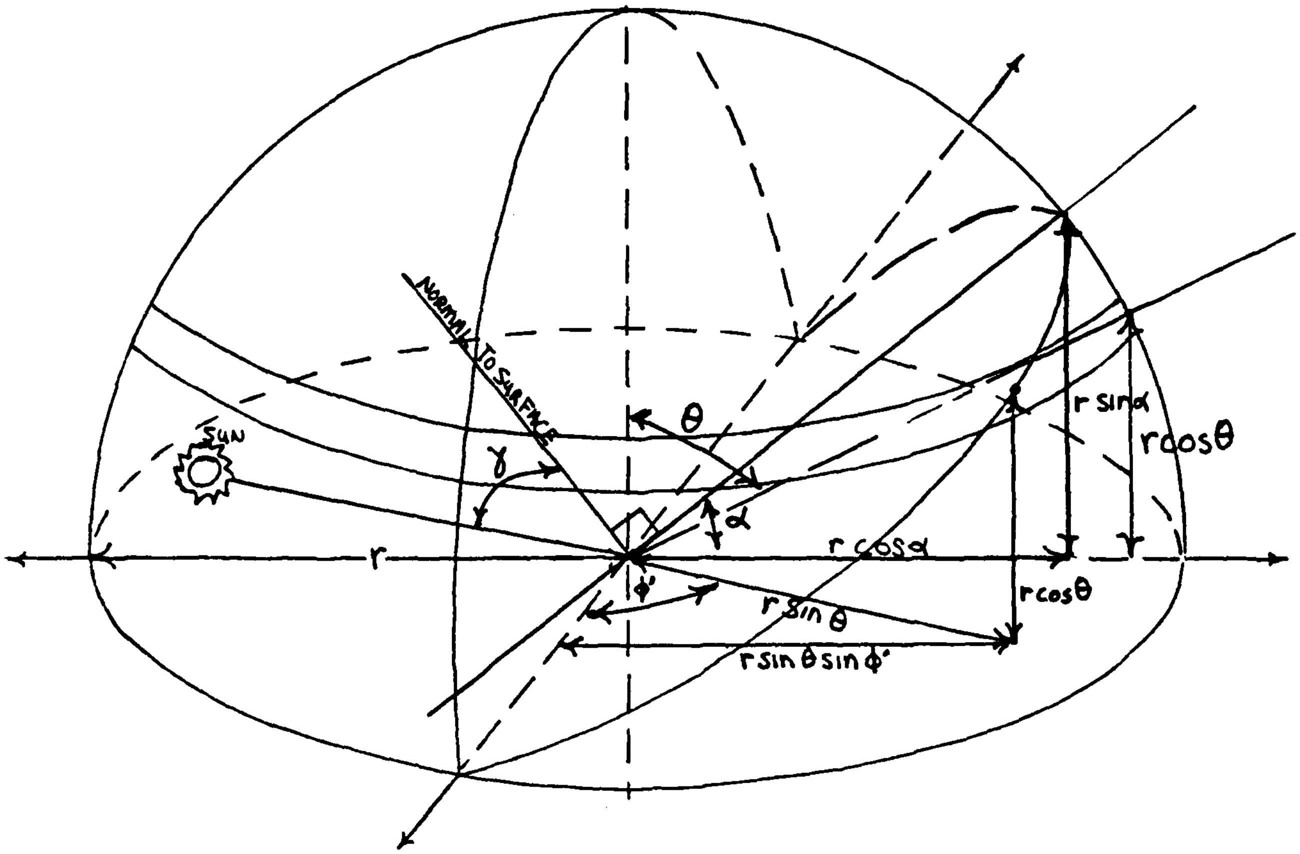

Figure A2

From the representations of Figure A2 we can find ϕ' *by:*

$$\tan\alpha = \frac{r\cos\theta}{r\sin\theta\sin\phi'} = \frac{r\sin\alpha}{r\cos\alpha}$$

$$\sin\phi' = ctn\theta ctn\alpha$$

or

$$\phi' = \arcsin(ctn\theta ctn\alpha)$$

C. Derivation of Total Solar Illuminance

By approximation of the sun as a point source and using the inverse square law

$$E = \frac{I\cos\gamma}{d^2}$$

where γ = angular distance between sun and surface normal and is given by

$$\gamma = \cos^{-1}(\sin SA\cos\alpha+\cos SA\sin\alpha\cos(\beta-SAZ))$$

(Reference 5)

$$E_{TSI} = \frac{I}{d^2}(\sin SA\cos\alpha+\cos SA\sin\alpha\cos(\beta-SAZ)$$

$$E_{PSI} = \frac{I}{d^2}$$

$$E_{TSI} = E_{PSI}\sin SA\cos\alpha+E_{PSI}\cos SA\sin\alpha\cos(\beta-SAZ)$$

But $E_{HSI} = E_{PSI}\sin SA$ and $E_{VSI} = E_{PSI}\cos SA$

$$\therefore\ E_{TSI} = E_{HSI}\cos\alpha+E_{VSI}\sin\alpha\cos(\beta-SAZ)$$

D. Derivation of Clear Sky Equations for Illuminance Due to Sky Luminance

1. As in the overcast sky case the horizontal illuminance can be calculated by

$$E_{HC} = E_{HCFLAT} - E_{HCSTRIP}$$

where E_{HCFLAT} = horizontal illuminance due to clear sky luminance of the entire sky dome

$E_{HCSTRIP}$ = horizontal illuminance due to clear sky luminance of a strip of the sky dome (see Figure A1)

The portion of horizontal illuminance for each sky element is

$$dE_{HC} = \frac{L_c\cos\theta dA}{r^2}$$

where $dA = (r\sin\theta)d\phi r d\theta$

$$L_c = \frac{L_z(1-e^{-.32/\cos\theta_{avg}})(.91+10e^{-3\gamma}+.45\cos^2\gamma)}{.279(.91+10e^{-3(\pi/2-SA)}+.45\sin^2 SA}$$

The clear sky luminance (L_c) is an approximation where

$$\gamma = ARC\cos[\sin SA\cos\theta_{Avg}-\cos SA\sin\theta_{avg}\sin(\phi_{avg}-SAZ)]$$

$$\theta_{avg} = \frac{\theta_1+\theta_2}{2}$$

$$\phi_{avg} = \frac{\phi_1+\phi_2}{2} \quad \text{(see Figure 4)}$$

The horizontal illuminance on a flat surface is

$$E_{HCFLAT} = \int_0^{2\pi}\int_0^{\pi/2} L_c \sin\theta\cos\theta d\theta d\phi$$

By using an approximation of L_c and summing by taking small increments, in first the θ then the ϕ direction, the above integral becomes

$$E_{HCFLAT} = \sum_{\theta=0,\theta_{inc}}^{\pi/2} [(\frac{\sin^2\theta_2-\sin^2\theta_1}{2}) \sum_{\phi=0,\phi_{inc}}^{2\pi} L(\phi_2-\phi_1)].$$

When the surface is not flat the surface's orientation becomes a factor because the contribution of that part of the sky that is "beneath" the surface's plane projection to the sky dome is lost. The clear sky luminance is not uniform with respect to the ϕ direction so the contribution to horizontal illuminance will vary with orientation of the surface.

The horizontal illuminance due to the strip beneath the surface's plane projection that intersects the sky dome (see Figure 5) is

$$E_{HCSTRIP} = \sum_{\phi+\beta}^{\pi-\phi'+\beta} \sum_{\frac{\pi}{2}-\alpha}^{\pi/2} L_c \sin\theta\cos\theta d\theta d\phi$$

where ϕ' = intersection of sky dome and surface plane projection for each "ring" of sky elements (see Figure 4)

β = angle of orientation with respect to south and counter-clockwise direction being positive (see Figure 2)

The approximate horizontal strip illuminance is

$$E_{HCSTRIP} = \sum_{\theta=\frac{\pi}{2}-\alpha,\theta_{inc}}^{\pi/2} \left[\left(\frac{\sin^2\theta_2-\sin^2\theta_1}{2}\right) \sum_{\phi=\phi'+\beta,\phi_{inc}}^{\pi-\phi'+\beta} L(\phi_2-\phi_1)\right]$$

2. The vertical illuminance of the clear sky is

$$E_{VC} = E_{VCPOS} - (E_{VCNEG} - E_{VCSTRIP}).$$

where E_{VCPOS} = positive vertical illuminance component (half of sky dome without strip).

E_{VCNEG} = negative vertical illuminance component (half of sky dome with strip).

$E_{VCSTRIP}$ = vertical illuminance due to strip.

The portion of vertical illuminance for each sky element is

$$dE_V = \frac{L_c\cos\zeta dA}{r^2}$$

where $\cos\zeta = \sin\theta\sin\phi$

$dA = (r\sin\theta)d\phi rd\theta$

L_c = sky luminance of clear sky (same as in Appendix D1).

The positive vertical component is then

$$E_{VCPOS} = \int_{\pi+\beta}^{2\pi+\beta} \int_{0}^{2\pi} L_c\sin\phi\sin^2\theta d\theta d\phi$$

and is approximated by

$$E_{VCPOS} = \sum_{\theta=0,\theta_{inc}}^{\pi/2} \left(\frac{\theta_2-\theta_1}{2} + \frac{\sin2\theta_1-\sin2\theta_2}{4}\right) \sum_{\phi=\pi+\beta,\phi_{inc}}^{2\pi+\beta} L|(\cos\phi_1-\cos\phi_2)|.$$

The negative vertical component is

$$E_{VCNEG} = \int_{\beta}^{\pi+\beta} \int_{0}^{\pi/2} L_c\sin\phi\sin^2\theta d\theta d\phi$$

and is approximated by

$$E_{VCNEG} = \sum_{\theta=0,\theta_{inc}}^{\pi/2} \left(\frac{\theta_2-\theta_1}{2} + \frac{\sin 2\theta_1 - \sin 2\theta_2}{4}\right) \sum_{\phi=\beta,\phi_{inc}}^{\pi+\beta} L|(\cos\phi_1 - \cos\phi_2)|.$$

The strip vertical component is

$$E_{VCSTRIP} = \int_{\phi'+\beta}^{\pi-\phi'+\beta} \int_{\pi/2-\alpha}^{\pi/2} L_c \sin\phi \sin^2\theta \, d\theta \, d\phi$$

and is approximated by

$$E_{VCSTRIP} = \sum_{\theta=\frac{\pi}{2}-\alpha,\theta_{inc}}^{\pi/2} \left(\frac{\theta_2-\theta_1}{1} + \frac{\sin 2\theta_1 - \sin 2\theta_2}{4}\right) \sum_{\phi=\phi'+\beta,\phi_{inc}}^{\pi-\phi'+\beta} L|(\cos\phi_1 - \cos\phi_2)|$$

3. The total illuminance due to sky luminance of the completely clear sky is

$$E_C = E_{HC} \cos\alpha + E_{VC} \sin\alpha$$

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