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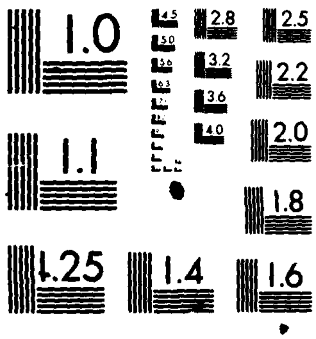

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UTILITY ASSESSMENT METHODS

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This paper integrates existing and new methods for assessing unidimensional expected utility functions in a comprehensive study of utility assessment. We describe briefly the utility assessment process in decision analysis and then review problem formulations, sources of bias in preference judgments, and the analysis of risk attitudes. We critically examine about two dozen utility assessment methods of which half appear for the first time. These methods are grouped under preference comparison methods, probability equivalence methods, value equivalence methods, certainty equivalence methods, hybrid methods, paired-gamble methods, and other approaches.

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Abstract

This paper integrates existing and new methods for assessing unidimensional expected utility functions in a comprehensive study of utility assessment. We describe briefly the utility assessment process in decision analysis and then review problem formulations, sources of bias in preference judgments, and the analysis of risk attitudes. We critically examine about two dozen utility assessment methods of which half appear for the first time. These methods are grouped under preference comparison methods, probability equivalence methods, value equivalence methods, certainty equivalence methods, hybrid methods, paired-gamble methods, and other approaches. We emphasize the nature of judgmental biases in comparing different assessment procedures. Since most multiattribute utility functions are decomposed into single-attribute functions, this study should facilitate such applications. We conclude with several directions for further developmental, empirical, and applied research.

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UTILITY ASSESSMENT METHODS

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1. Introduction

Many decision problems involve considerable uncertainty about the outcomes of alternative decisions. Often it is important not only to evaluate the possible outcomes in a decision problem but also to judge the riskiness of various alternatives. One approach that has received broad application in analyzing preferences for risky decisions is expected utility theory.

There is a sizable literature on utility theory and applications. Fishburn [47] provides a comprehensive survey of contributions to expected utility theory by von Neumann and Morgenstern [161], Savage [134], Pratt, Raiffa, and Schlaifer [125, 126], Fishburn [41, 50], and many others [40, 53, 54, 65, 73, 86, 96, 97, 103, 124]. On the other hand, critical reviews of expected utility models for decision making are in [1, 27, 29, 34, 35, 37, 67, 76, 98, 128, 138, 139, 142]. Recent utility research has focused on generalizations and alternatives to expected utility theory [11, 17, 19, 30, 45, 46, 48, 75, 76, 77, 102, 158], various behavioral issues [10, 27, 29, 35, 36, 64, 67, 75, 98, 128, 138, 142, 145, 155, 156, 157], multiattribute utility analysis [12, 31, 32, 71, 83], and related topics in decision making. Practical applications of utility analysis are summarized in [2, 12, 31, 40, 57, 63, 71, 81, 82, 83, 115, 117, 146, 150].

The main purpose of this paper is to provide an integration of existing methods for assessing single-attribute utility functions and to present some new assessment methods that may be appropriate for further applications and research. Several aspects of utility assessment are covered in previous studies by Fishburn [39], Hull et al. [72], Kneppreth et al. [85], Keeney and Raiffa [83], Johnson and Huber [74], and others, but these studies do not cover many recent research contributions and technological developments. Our emphasis is on investigating different steps in the utility assessment process and, in particular, on studying various methods of comparing gambles.

There are usually five steps in assessing unidimensional expected utility functions [83]:

1. *Preparation for a utility assessment* typically includes structuring the decision problem, developing an appropriate scale for measuring the attribute to be evaluated, and then acquainting the decision maker with various aspects of assessment procedures.

2. *Identification of relevant qualitative characteristics*, such as monotonicity, boundedness, continuity, or risk properties, provides useful information on the form of admissible utility functions.

3. *Specification of quantitative restrictions* on the decision maker's utility function is accomplished by comparing various gambles over the attribute in question.

4. *Selection of a utility function* satisfying the quantitative restrictions is usually made from a parametric family of functions derived from the qualitative characteristics identified earlier.

5. *Checks for consistency* are essential in reconciling possible inconsistencies and in ensuring that the overall results adequately reflect the decision maker's preferences for risky decisions.

The order of these steps varies in particular assessment applications. Recycling through some of these steps is common as a decision maker refines his judgments. Further details and alternate descriptions of the process of assessing utility functions are found in various references on decision analysis [2, 24, 68, 70, 80, 82, 83, 89, 125, 126, 127, 135, 136, 137].

Although a few multiattribute utility assessment procedures use trade-off methods involving two or more attributes directly [39, 99, 100, 101], most multiattribute utility functions are decomposed into several simpler components that require the assessment of only single-attribute utility functions [31, 32]. Thus the methodologies presented in this paper have wider application than decision problems involving only one attribute. Indeed, the most frequent use of single-attribute procedures is likely to be in facilitating the assessment of multiattribute utility functions.

A drawback of many recent applications is the heavy reliance on chaining methods of utility assessment. These assessment methods are very simple analytically, though behavioral research has shown that they often produce distortions in judgments [29, 64, 75, 108, 109, 111, 138, 139, 145, 155, 156]. One conclusion from empirical studies is the importance of counteracting possible biases by using more than one methodology in the assessment process [35, 36]. Some analysts recommend that different methods also should be used to check the consistency and adequacy of a utility function. Therefore, a second purpose of this paper is to promote broader usage of utility assessment methods now available.

The paper is organized as follows. Section 2 describes assumptions, terminology, and basic methods for making a single comparison of two gambles. Section 3 examines several background issues in utility assessment, such as assessment problem formulation, sources of bias in preference judgments, and regulating the context of a comparison of gambles. Section 4 considers the qualitative analysis of risk and other properties as a preliminary step in utility assessment. Section 5 deals with methods for assessing utility functions; this section covers preference comparison methods, probability equivalence methods, value equivalence methods, and certainty equivalence methods. Section 6 considers hybrid methods of utility assessment, paired-gamble methods, and several other topics. Section 7 concludes the paper with a brief summary and some directions for further research.

2. Preliminaries

Let X be a subset of real numbers representing the possible levels of a single *attribute*, such as market share, net asset value, units defective, response time, body weight, etc. We assume that each possible outcome, or consequence, in a given decision problem can be identified with a particular level x in the attribute set X. If multiple attributes are needed to describe the decision outcomes, then all attributes except X are presumably held fixed at some implicit level.

In examining risky decision problems, we treat alternative decisions as gambles (or lotteries) over finite sets of outcomes from X, though more general formulations are possible. Thus a decision *alternative* is defined here as a gamble which assigns probabilities $\alpha_1, \alpha_2, \ldots, \alpha_m$, adding to one, to some outcomes $x_1, x_2, \ldots, x_m$ in X, respectively. Let P denote the collection of all such gambles over X.

We further assume that an individual decision maker has a *preference relation* $\succ$ over the gambles in P satisfying an appropriate set of expected utility axioms [41, 47]. Thus, there exists a real function u on X, called a *utility function* for $\succ$ on P, such that for all gambles $p,q \in P$, p is preferred to q (i.e., $p \succ q$) if and only if the expected utility of p is greater than the expected utility of q (i.e., $\sum_{x \in X} p(x)u(x) > \sum_{x \in X} q(x)u(x)$).[1] In later applications it may be necessary to impose some additional restrictions on the utility function.

A utility function is unique up to positive, linear transformations: if u and v are both utility functions for the preference relation $\succ$ on P, then $v = a + bu$ for some constants a and b with $b > 0$. This uniqueness property allows one to choose a convenient origin and scale unit in assessing a utility function. Further discussion on expected utility theories and properties of utility functions is in [40, 41, 47, 50, 73, 96, 125, 161].

Most utility assessment methods covered in this paper consider gambles over just two outcomes in X, since extensions to more than two outcomes appear to be straightforward. For $x,y \in X$ and $0 \leq \alpha \leq 1$, the *gamble* which yields outcome x with probability α and outcome y with probability $1 - \alpha$ is denoted by $[x, \alpha, y]$. If either $x = y$, $\alpha = 0$, or $\alpha = 1$, the gamble $[x, \alpha, y]$ is *degenerate* because a particular outcome occurs for certain. For convenience, $[x, 1/2, y]$ is abbreviated as $[x, y]$.

A single comparison of gambles in most utility assessment procedures involves the expression

$$[x, \alpha, y] \; R \; [w, \beta, z], \tag{1}$$

[1]To each outcome x in X, the gamble p assigns probability p(x), the gamble q assigns probability q(x), and the function u assigns utility u(x).

where all items but one are fixed beforehand and the individual decision maker specifies the remaining item such that (1) holds, if possible. Note that in (1), $x,y,w,z \in X$; $0 < \alpha,\beta < 1$; and R is one of the following relations, $\succ$ (is more preferred than), ~ (is indifferent to), or $\prec$ (is less preferred than), where these relations are derived from $\succ$ in the usual way [40, 41].

The basic methods for comparing two gambles are listed in Table 1 under the categories of *standard-gamble* methods and *paired-gamble* methods. Under the first category one of the two gambles is degenerate in each comparison, while under the second category neither gamble is degenerate in each comparison. Note that the item underlined in each expression in Table 1 is to be specified by the individual decision maker, since all other items are presumably fixed beforehand.

Table 1 goes here

There are many methods for obtaining a sequence of gamble comparisons from each single comparison method listed in Table 1. Further descriptions of these varied assessment methods are presented in Sections 5 and 6.

3. Background for Utility Assessment

A reasonable background for subsequent discussions of assessment methods includes some consideration of different formulations of the assessment problem, possible sources of bias in preference judgments, and regulating the comparisons of gambles.

TABLE 1

Methods for Making a Single Comparison of Two Gambles

I. Standard-Gamble Methods

1. *Preference comparison*: $[x, \alpha, y]\ \underline{R}\ w$
2. *Probability equivalence*: $[x, \underline{\alpha}, y] \sim w$, where w is between x and y
3. *Value equivalence*: $[\underline{x}, \alpha, y] \sim w$
4. *Certainty equivalence*: $[x, \alpha, y] \sim \underline{w}$

II. Paired-Gamble Methods

5. *Preference comparison*: $[x, \alpha, y]\ \underline{R}\ [w, \beta, z]$
6. *Probability equivalence*: $[x, \underline{\alpha}, y] \sim [w, \beta, z]$, where both w and z are between x and y
7. *Value equivalence*: $[\underline{x}, \alpha, y] \sim [w, \beta, z]$

3.1 Assessment Problem Formulation

The assessment data available to a decision analyst typically consists of a set of preliminary assumptions and properties and a set of constraints derived from a sequence of gamble comparisons of the form in (1). A utility function that is consistent with particular assessment data is said to be *admissible*.

The assessment problem may be formulated then in several ways. If the assessment data is known to be internally consistent, the analytical problem might be to (1) specify the set of all admissible utility functions, (2) determine whether or not a given parametric family of functions is admissible, (3) find any admissible utility function, or (4) select a "best" admissible utility function, according to some additional criteria. Other assessment problems could be formulated, too. The literature on consistent assessment of utility functions is quite limited; Meyer and Pratt [104] and Bradley and Frey [16] use linear programming techniques to address some of these problems, but many issues remain unresolved. Somewhat related problems occur in probability assessment [66, 91, 94, 106, 141, 147, 163, 166, 167, 168].

Decision analysts have observed that many individuals do not immediately exhibit consistent preference responses in complicated decision situations [19, 82, 83, 164]. As a decision maker further understands the implications of his expressed preferences, modifications of previous responses are frequently encountered in a utility assessment. Experienced analysts rely on their intuition and a variety of means to spot possible inconsistent responses during the assessment process and thereby allow a decision maker the opportunity to change his mind [35, 37, 81]. Thus, efforts are directed at promoting consistent assessment data. As noted in a later section, some assessment methods readily provide consistency checks on the responses, while other methods do not.

Despite practical efforts to promote consistency, it is not uncommon to end an assessment with data that might be inconsistent. The analytical problem may then be formulated to (1) find a maximal consistent subset of data and perhaps attempt to reconcile any suspected inconsistencies, (2) approximately satisfy the data with a "quasi-admissible" utility function, (3) adopt a probabilistic utility model of the data that accounts for random errors in the responses, or (4) pursue some other approach. Research on these problems is scattered and often not directed specifically at assessing utility functions. Notable exceptions include research on probabilistic utility models [8, 30, 46, 97] and a few other approaches [11, 17, 19, 43, 45, 61, 76, 77, 120, 121, 138, 139, 157]. One promising approach is that of Novick et al. [112, 114] who use overspecification and least-squares fitting procedures to identify inconsistencies in utility judgments (also see Spetzler [146]). Some of their assessment methods are aimed at avoiding the biases in preference responses resulting from certainty effects and anchoring heuristics.

3.2 Potential Sources of Bias in Preference Judgments

Behavioral research on decision making demonstrates the labile nature of preference judgments. Seemingly subtle changes in problem structure, question format, response mode, individual perspective, or other aspects of the assessment process can sometimes dramatically change the preference responses of an individual decision maker. For example, Fischhoff, Slovic, and Lichtenstein [37] describe how decision analysts might unavoidably shape the assessment process in (1) formulating the decision problem, the alternatives, the measurement scales, and other structural features; (2) controlling the response mode and perspectives of the decision maker; and perhaps even (3) altering the value structure at various steps in the assessment process. Fischhoff [35, 36] comments further on the influence of the decision analyst in the assessment process.

A major recommendation from behavioral studies on individual decision making is to utilize more than one assessment procedure. In complicated or unfamiliar decision problems, consistent responses with a single assessment procedure may result primarily from a convenient heuristic rule or from a salient contextual effect. By systematically varying assessment procedures and contextual elements in behavioral research, it has been possible to examine some of their effects on preference judgments. This information can be helpful to decision analysts in choosing appropriate utility assessment procedures for particular decision situations.

There are many potential biases in making preference judgments. Hogarth [67, pp. 166-170] provides a compact summary of judgmental biases, so only two are emphasized here. Tversky [155, p. 212] argues, "human preferences are subject to a major bias, the *certainty effect,* according to which the utility (or the disutility) of an outcome looms larger when it is certain than when it is uncertain." Bias from the certainty effect may account for sharp differences (even inconsistencies) between responses obtained from standard-gamble methods and paired-gamble methods. For example, in one experiment a large number of individuals exhibited the following preferences,

$$\$45 \succ [\$100,\ 1/2,\ \$0] \text{ and } [\$100,\ 1/20,\ \$0] \succ [\$45,\ 1/10,\ \$0]; \qquad (2)$$

yet these preference relations are incompatible with any expected utility function. "Furthermore, the respondents whose preferences were inconsistent with utility theory were given the opportunity to revise their preferences. While most of the respondents expressed mild embarrassment for violating utility theory, very few were inclined to modify their choices" [155, p. 211]. Further studies by Tversky and Kahneman [156, 157] and others [1, 29, 64, 67, 75, 98, 138, 142, 145] describe other persistent judgmental biases.

The *anchoring and adjustment* heuristic described in [67, 142, 144, 147, 156] is another potential source of bias in utility assessment. The first response, a readily accessible piece of information, or a salient contextual factor can provide an individual with an initial basis for making judgments. Subsequent responses are determined by adjustments in this basis, or "anchor;" biases occur whenever these adjustments are insufficient. For example, if the median is elicited first in making probability judgments about some uncertain event, insufficient adjustments often lead to a central bias in the assessed probabilities [66, 75, 147, 156, 163]. Similar results can be expected in using probability comparisons or midpoint chaining methods for assessing utility functions (see Section 5). Insufficient adjustments can lead to skewed risk attitudes in preference judgments. Some utility assessment protocols for fitting positive, decreasing risk averse functions encounter inconsistent responses from individuals who seem too risk averse for gambles with small spreads between the outcomes and much less risk averse for gambles with large spreads [137]. Other biases due to *range effects* have been noted also [88, 108, 109]. Since anchoring and adjustment is such a persistent heuristic, individuals may have to work quite hard in an assessment to resolve any biases or inconsistencies in their responses [36; 83, p. 210].

Hershey, Kunreuther, and Schoemaker [65] summarize five sources of potential bias in utility assessment procedures. Across a variety of experiments, they observe

> ...First, the certainty equivalence method generally yields more risk-seeking preferences than the probability equivalence method. Second, the probability and outcome levels used in reference lotteries induce systematic bias. Third, combining gain and loss domains yields different utility measures than separate examinations of the two domains.

> Fourth, whether a risk is assumed or transferred away exerts a strong influence on people's preferences in ways counter to expected utility theory. Finally, context or framing differences strongly affect choice in a non-normative manner.

Observations from such studies highlight the contextual dependency of many utility assessment procedures. Other research [4, 15, 37, 58, 64, 75, 76, 92, 93, 95, 98, 118, 119, 129, 138, 139, 143, 153, 154, 157, 164, 169] on response mode effects and related topics in preference judgment further emphasizes the need for a better understanding of how to elicit and calibrate utility functions. More research is clearly warranted in this important area.

3.3 Regulating the Comparison of Gambles

The abstract descriptions of gamble comparison methods in Table 1 and in subsequent sections do not specify the context in which judgments are made. Since the context of an assessment can have significant effects on the derivation and interpretation of a utility function, some of these issues are collected here for discussion. The following discussion distinguishes between the *sale*, *purchase*, *gift*, or *transfer* of gambles. Although these contexts are illustrated with the certainty equivalence method of assessment on a monetary attribute, they apply generally to equivalence methods and arbitrary attributes (see Table 1).

For convenience, let X be measured in dollars and let G be a gamble over the monetary outcomes in X. A common procedure for assessing the utility of G requires the decision maker to specify a *certainty equivalent* \$w for which he is indifferent between the sure outcome \$w and the gamble G in a given situation. One situation assumes that the decision maker owns the rights to G; the certainty equivalent \$w is determined then from the decision maker's *minimum* *selling price*[2] for G. By the definition of \$w in this sale, the decision maker is indifferent between the amount \$0 and the gamble G (the status quo) and the amount \$w and no gamble G (the alternative).

Table 2 characterizes four contexts for assessing gambles with equivalence methods. Each context is distinguished by a different status quo position, although the decision maker always chooses a \$w so that the following two alternatives are equivalent.

(the amount \$0 and the gamble G) ~ (the amount \$w and no gamble G) (3)

Table 2 goes here

Starting with neither money nor gamble in hand, the decision maker in the *gift* situation of Table 2 is offered his choice of an amount of money or the gamble G. He is asked to specify an amount \$w for which he is indifferent between receiving the money or the gamble as a gift in (3). On the other hand, the decision maker in the *transfer* situation begins with both money and gamble in hand and then is forced to give up one or the other. The decision maker is

[2]The minimum selling price for a gamble is occasionally difficult to elicit from decision makers who wish to profit from the exchange. The following scenario helps to overcome this *inertial effect* [64]. The decision maker is asked to sell the gamble at an auction where he must provide the sales agent with a "reservation price" \$w in advance of the bidding. Once the reservation price is set, no negotiations are allowed at the auction. If the highest bid is less than \$w, then no sale takes place; if the highest bid is at least \$w, then the gamble is sold and the decision maker receives the amount of the highest bid. With very weak restrictions on the probability distribution of bids and the decision maker's utility function, Toda and MacCrimmon [151] prove that the reservation price which maximizes expected utility is indeed the decision maker's minimum selling price. This technique is analogous to a "proper scoring rule" for motivating an individual to report his true values in probability assessment [66, 91, 147, 163, 168]. Although similar *auction techniques* are available for other situations in Table 2, few applied studies report using them [9].

TABLE 2

Status Quo Positions for Four Situations in Assessing Equivalent Gambles

	No money in hand	Money in hand
No gamble in hand	Gift	Purchase
Gamble in hand	Sale	Transfer

asked to specify an amount $w such that he is indifferent between transferring the money or the gamble in (3). In both of these situations, the status quo position is bound to change, because it is not among the two alternatives from which the decision maker chooses.

The *purchase* situation is analogous to the *sale* situation described earlier. Starting with money in hand and no gamble, the decision maker is asked to specify his *maximum buying price* $w for the purchase of the gamble G. In this case, his status quo position (the amount $w and no gamble G) is judged equivalent to the alternative position (the amount $0 and the gamble G), as in (3). Recall that in the *sale* situation $w is interpreted as the minimum selling price of the gamble G, and the status quo position is reversed in (3). In both situations, the status quo position is one of the two alternatives in (3), so inertial effects are unavoidable in the assessments [64].

For convenience, we used the same symbol $w to denote the certainty equivalent in each context above. Raiffa [127, pp. 89-91] and others note that in general $w is not constant across contexts. On the contrary, if for all gambles G the minimum selling price equals the maximum buying price, then the individual has a constant risk attitude and hence the underlying utility function must be either linear or exponential [122, 124]. One can derive many qualitative properties of the utility function by further exploring relationships between certainty equivalents in other contexts.

Before ending this section, we note that the regulation of gamble comparisons also involves the ways in which outcomes are represented to the decision maker. For many years the accepted practice in utility analysis has been to consider gambles over final assets in dealing with monetary outcomes [40, 127, 135, 136, 150]. Recent research has considered various models for evaluating gambles over gains and losses when initial assets are given. Further discussion of these and other approaches is in [11, 45, 49, 51, 64, 76, 120, 121, 157].

4. Preliminary Analysis of Risk Attitudes and Other Properties

Before comparing gambles in a utility assessment, it is useful to examine risk attitudes and other factors affecting an individual's preferences. This preliminary analysis provides a basis for making reasonable assumptions about the form of a utility function.

4.1 Basic Properties

A preliminary analysis begins with basic questions about monotonicity, boundedness, continuity, and other properties. For example, monotonicity is a common assumption because it guarantees the uniqueness of indifference points in gamble comparisons and thereby simplifies the utility assessment. Moreover, monotonicity is easily checked by asking an individual to rank outcomes according to increasing preference. For some attributes, like body temperature or caloric intake, preferences are "single-peaked" instead of monotonic, that is, preferences increase up to some "ideal point" and decrease thereafter [4, 18]. Although more complicated preference relationships can be described, it is usually possible to restructure the attribute or to partition the assessment to obtain monotonic preferences. With little practical loss in generality, we assume throughout that the utility function u is a strictly increasing function on X.

In most situations, bounds on the utility function are simple to determine. Bounds are obvious for many attributes, like market share, units defective, etc., which naturally have a best and worst outcome. Other attributes, like response time, have preferences asymptotically approaching an upper or lower bound. Even in situations where preferences might be unbounded, often the underlying attribute is restricted in range for practical purposes. Further discussion and references are in [42, 44, 79].

Preliminary analyses rarely check continuity, differentiability, or other technical properties; instead, these properties are usually regarded as axioms [22, 52, 56]. Further discussion about the preliminary analysis of basic properties is in Keeney and Raiffa [83], LaValle [89], and other books on decision analysis.

4.2 Risk Attitudes

Much recent research has focused on the investigation of various risk properties as a key aspect of analyzing preferences. While some properties provide only weak characterizations, other risk properties restrict the admissible utility functions to particular parametric families, such as linear, exponential, logarithmic, or power functions. In these latter cases, the utility assessment process is considerably shortened, because one need only determine an appropriate set of parameters from a few gamble comparisons to complete the assessment. In any event, an individual's risk attitudes provide convenient consistency checks on the assessed utility function, so a preliminary analysis of risk attitudes is an important step in the assessment process.

Basic risk attitudes can be described using the following comparison.

$$x - \pi(x, t) \sim [x + t, x - t] \text{ where } t > 0. \tag{4}$$

The difference between the expected value of the gamble $[x + t, x - t]$ and its certainty equivalent is called the *risk premium*, $\pi(x, t)$, for this gamble. An individual's preferences are (1) *risk averse* if $\pi(x, t) > 0$, (2) *risk neutral* if $\pi(x, t) = 0$, or (3) *risk prone* if $\pi(x, t) < 0$, for all x. Note that the corresponding utility function must be (1) concave, (2) linear, or (3) convex, respectively. Alternatively, one can define these basic risk attitudes in terms of the *risk function* $r(x) = -u''(x)/u'(x)$, but we shall not pursue this approach here [3, 26, 62, 83, 124, 130, 170].

Furthermore, for a given basic risk attitude one can define a *comparative risk attitude* that is (1) *decreasing* if $\pi(x, t) > \pi(y, t)$, (2) *constant* if $\pi(x, t) = \pi(y, t)$, or (3) *increasing* if $\pi(x, t) < \pi(y, t)$, for all $x < y$. Pfanzagl [122] and Pratt [124] show that a constant risk attitude, for example, implies that the corresponding utility function is either linear or exponential.

$$u(x) = \begin{cases} a - be^{-rx} & \text{for } r > 0 \text{ (u is risk averse),} \\ a + bx & \text{for } r = 0 \text{ (u is risk neutral),} \\ a + be^{-rx} & \text{for } r < 0 \text{ (u is risk prone),} \end{cases} \tag{5}$$

where $r(x) \equiv r$ is a risk parameter and a and b are just scaling constants with $b > 0$. Keeney and Raiffa [83, pp. 191-193] and Novick and Lindley [114, p. 307] describe the use of "adjacent gambles" for checking basic and comparative risk attitudes before a utility assessment (see Section 5.3(b)).

Other risk attitudes typically yield many possible parametric families of admissible utility functions. Tables of some common functions that satisfy various risk attitudes are available in [83, 89, 124, 137, 159]. For example, in developing an interactive computer program to assess a utility function that is decreasingly risk averse, Schlaifer [137] considers a *sumex* function $u(x) = -e^{-ax} - be^{-cx}$ for $a > 0$ and $bc > 0$. These parameters can be determined easily from three gambles of the form in (4) with t fixed. On the other hand, Spetzler [146] describes the process of selecting an appropriate utility function for decreasingly risk averse individuals facing investment decisions. He begins with a logarithmic function $u(x) = a + b \cdot \log(x + c)$ for $b > 0$ and adds two more parameters before obtaining a satisfactory fit. Other examples are in [51, 57, 63, 80, 81, 83, 115, 150, 164].

For some attributes, one can define *proportional risk attitudes* by replacing (4) with the comparison $x - \pi^*(x, s) \cdot x \sim [x + sx, x - sx]$ for $s > 0$. For instance, a *constant proportional risk attitude,* which has $\pi^*(x, s) = \pi^*(y, s)$ for all x and y, yields a corresponding utility measure that is either a power function or a logarithmic function [83, 124]. Recently, Harvey [62] has defined a class of *linear risk attitudes* that includes all the risk attitudes mentioned above. His general approach produces appropriate functional forms for utility measures corresponding to a variety of risk attitudes that are easily checked before an assessment.

A new direction of research attempts to separate attitudes toward risk from an individual's strength-of-preference (or marginal value) for sure outcomes. Dyer and Sarin [26] introduce the concept of *relative risk attitude* in comparing an individual's von Neumann-Morgenstern utility function with his measurable value function [25, 41, 133, 149]. Krzysztofowicz [87] reviews some of the implications of this research for modifying the Arrow-Pratt risk measure mentioned above. Other research in this area includes [13, 33, 78, 139, 162].

4.3 Other Approaches Using Risk

Meyer and Pratt [104] describe an empirical procedure for assessing utility functions that uses risk attitudes to specify constraints in a linear programming model. They allow changes in risk attitude across different regions of an attribute and do not make a priori assumptions about the functional form of the utility. Further research by Bradley and Frey [16] shows how to use information from risk attitudes and previous responses to establish bounds on the probability responses and certainty equivalents for additional standard-gamble comparisons. Their methodology is helpful in avoiding inconsistent responses in interactive computer assessments of utility functions. Keeney and Raiffa [83] describe simple uses of such techniques in practical applications.

Hammond [60] presents an approach that eliminates the need for assessing a utility function when the decision problem involves relatively few uncertain alternatives. This approach examines how the graphs of the cumulative probability functions (or *risk profiles* [116]) of two alternatives cross one another. By using basic risk attitudes and restrictions on the pattern of crossings, one can determine the preference order of uncertain alternatives. Related research in the area of stochastic dominance provides somewhat similar approaches [5, 6, 165]. Further work in this area may be useful in reducing the need for full assessments of utility functions.

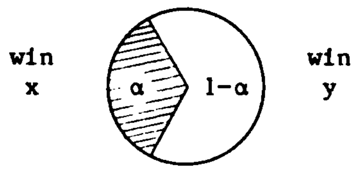
5. Utility Assessment Methods

This section examines different sequences of standard-gamble comparisons that can be used to assess a utility function. The various methods are described under four principal categories: (1) preference comparison methods, (2) probability equivalence methods, (3) value equivalence methods, and (4) certainty equivalence methods. Section 6 considers additional methods.

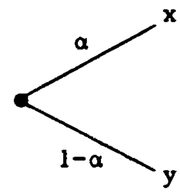
We need to say a few words about the representation $[x, \alpha, y]$ which denotes the gamble yielding outcome x with probability α and outcome y with probability $1 - \alpha$. Although $1 - \alpha$ is implicit in this notation, a variety of empirical studies indicate that the actual gamble should be displayed to the decision maker in a format that makes all components explicit [15, 75, 98, 118, 119, 143, 156]. Thus, the gamble [x, α, y] is symbolic for displays such as

event	probability	outcome
A	α	x
not A	1 − α	y

(Table)



(Chart)



(Tree)

or others. Moreover, we assume for simplicity that both x and y are positive outcomes (i.e., pure gains) to avoid any ambiguity about gains and losses. This restriction has important behavioral implications, but extensions to other domains are straightforward [45, 49, 51, 64, 76, 157].

5.1 Preference Comparison Methods

In a *preference comparison* between the gamble [x, α, y] and the sure outcome w, an individual specifies the relation R (either $\succ$, $\prec$, or ~) such that the expression [x, α, y] $\underline{R}$ w holds. Preference comparison methods involve a sequence of such comparisons, $[x_i, \alpha_i, y_i]\ \underline{R_i}\ w_i$ for $i = 1, 2, \ldots, n$, where the probabilities, values, and standards are chosen in particular ways.

There are three common uses of preference comparison methods in utility assessment. The first use is in investigating risk attitudes in a preliminary analysis. For example, risk averse individuals prefer w to [x, y] whenever $(x + y)/2 < w$. A second use is in checking on the consistency of an assessed utility function. Each preference comparison provides a linear constraint that the utility function must satisfy. With a sufficiently large set of constraints, one might use this approach to develop fairly tight bounds on either admissible utility functions or consistent future responses [16, 20, 23, 104, 148]. A third use of preference comparison methods is to converge on an indifference point where $[x_n, \alpha_n, y_n] \sim w_n$. Such convergence techniques iteratively adjust either the probabilities, values, or standards until indifference occurs [54, 72, 80, 81, 85, 111].

5.2 Estimation of Indifference Points

The following equivalence methods for utility assessment require indifference in the standard-gamble comparisons. The estimation of an indifference point (either a probability, value, or standard) can be accomplished in several ways. The *direct estimation technique* asks for an individual's indifference point in a single response [18, 74, 86, 123, 152]. The *convergence technique* successively adjusts points in a sequence of preference comparisons until indifference is established [72, 80, 81, 85, 111]. The *bounding technique* develops only upper and lower bounds on an indifference point through a sequence of responses. Such bounds are refined until a clear set of preferences emerges for the alternatives under consideration [38, 131, 132, 160].

The sequel presumes that an appropriate estimation technique is used to generate the indifference points in gamble comparisons and that proper attention is given to the background issues raised in Section 3.

5.3 Probability Equivalence Methods

Probability equivalence methods require that an individual specify an indifference probability α for which $[x, \underline{\alpha}, y] \sim w$, where w is between x and y. These methods apply to either discrete or continuous attributes X. We begin by selecting two distant reference points x_0 and x_{n+1} in X, where $x_0 < x_{n+1}$. These points may be either a worst and best outcome in X or some other convenient benchmarks. The task is to assess the utilities of the points $x_0 < x_1 < \dots < x_{n+1}$ using one of the following methods.

5.3(a) *Extreme gambles* (or *farthest neighbors*):[3] $[x_{n+1}, \underline{\alpha_i}, x_0] \sim x_i$

This method uses the reference points of the attribute X as the extremes in every gamble. If $u(x_0) \equiv 0$ and $u(x_{n+1}) \equiv 1$, then obviously $u(x_i) = \alpha_i$. Thus the elicited indifference probabilities themselves are the utilities of the x_i values. If x_0 and x_{n+1} are not the endpoints of X and it is necessary to find the utilities of points above x_{n+1} or below x_0, one can ask additional questions of the form $[y, \underline{\alpha}, x_0] \sim x_{n+1}$ for $y > x_{n+1}$ or $[x_{n+1}, \underline{\alpha}, y] \sim x_0$ for $y < x_0$.

Although the extreme gambles method is easy to use [39, 72, 74, 83, 85, 136], there are susceptibilities to serial dependence in the responses [111, 169] and to biases from range effects if x_0 and x_{n+1} are too extreme [108, 109]. An analyst might try to alleviate these potential problems by permuting the sequence 1, 2, ..., n of comparisons and taking other precautions to debias the responses [36, 37, 38, 156].

5.3(b) *Adjacent gambles* (or *nearest neighbors*): $[x_{i+1}, \underline{\alpha_i}, x_{i-1}] \sim x_i$

Instead of possibly using gambles over the best and worst values, this method uses gambles over the "locally best and worst" values for each x_i [114, p. 307]. Each of the individual's n responses generates an equation of the form $u(x_i) = \alpha_i u(x_{i+1}) + (1 - \alpha_i)u(x_{i-1})$. With $u(x_0) \equiv 0$, $u(x_{n+1}) \equiv 1$, $f_0 \equiv 1$, and $f_i \equiv (1 - \alpha_i)/\alpha_i$, Novick and Lindley [114] solve the resulting system of n equations in n unknowns to get

$$u(x_i) = \sum_{j=0}^{i-1} \prod_{k=0}^{j} f_k \Bigg/ \sum_{j=0}^{n} \prod_{k=0}^{j} f_k, \qquad \text{for } i = 1, \ldots, n. \qquad (6)$$

[3]Unless stated otherwise, the index i goes from 1 to n.

A key advantage of this method over the previous one is "provided we do not ask the subject to assess probabilities near zero or one (numerically large log-odds), the utilities will be relatively insensitive to a lack of precision in probability assessments [114, p. 308]." Points outside the range are easily determined by additional comparisons of the form $[y, \underline{\alpha}, x_n] \sim x_{n+1}$ if $y > x_{n+1}$ or $[x_1, \underline{\alpha}, y] \sim x_0$ if $y < x_0$.

Novick and Lindley recommend asking for additional comparisons, such as $[x_{i+2}, \underline{\beta_i}, x_{i-2}] \sim x_i$, to provide consistency checks on the assessed utilities. Rather than having an individual revise earlier responses to eliminate inconsistencies, Novick and Lindley admit the inconsistencies and use a least-squares procedure to estimate a best-fitting utility function [30, 63, 146]. Novick et al. [112] describe an interactive computer program for eliciting an individual's responses and estimating a utility function using this approach. Other research on the adjacent gambles method is found in [83, 85, 89].

5.3(c) *Assorted gambles:* $[x_{k_i}, \underline{\alpha_i}, x_{j_i}] \sim x_i$, where $j_i < i < k_i$

Although this method generalizes others in this category, the method has received no systematic treatment in the literature. In contrast to the previous methods, this method may require the numerical solution of n or more equations to determine $u(x_1), \ldots, u(x_n)$. For this reason, the method requires further structure on the gamble comparisons before it would be appealing for field applications. For example, one method related to the multiplication method in 5.4(c) is the *anchored gambles* method $[x_{i+1}, \underline{\alpha_i}, x_0] \sim x_i$, whose solution is straightforward. If $u(x_0) \equiv 0$ and $u(x_{n+1}) \equiv 1$, then $u(x_i) = \Pi_{j=i}^{n} \alpha_j$. In any case, the format of "assorted gambles" comparisons is quite appropriate for making consistency checks.

Probability equivalence methods offer several advantages over other methods of utility assessment. Since chaining of responses (i.e., the use of earlier responses in subsequent gamble comparisons) does not occur, serial dependence between comparisons can be sharply reduced with permuted sequences. Probability responses are less susceptible to risk distortions [64, 108, 109, 136], path dependence [169], and some other cognitive biases [67, 75]. Although many individuals find difficulty in making probability judgments [66, 77, 105, 115, 129], training procedures and aids are available [66, 91, 141, 147, 163, 166, 167, 168]. Moreover, the adjacent gambles method appears to be robust for probabilities not close to zero or one, so probability judgments need not be precise in that case.

5.4 Value Equivalence Methods

Value equivalence methods ask for an indifference value x such that $[\underline{x}, \alpha, y] \sim w$. These methods assume a continuum of values in X so that an x exists satisfying this indifference relation. Usually α equals 1/2 for convenience, though Karmarkar [77] and others [20, 64, 156] observe biases in the elicited utilities as α is systematically varied. Since x and y are assumed to be positive and unrelated, it is not necessary to examine $[x, \alpha, \underline{y}] \sim w$ separately. A more complete treatment, however, would address the situations where x and y are both gains, both losses, or a gain and a loss [45, 49, 64, 120, 121, 139, 157].

For the most part, the following value equivalence methods have not appeared before in the utility literature.

5.4(a) *Uniform sequence:* $[\underline{x_{i+1}}, x_{i-1}] \sim x_i$

In many measurement situations, it is helpful to construct a scale of equally-spaced values using a *uniform sequence* approach [59, 86, 123, 152]. There are two basic variations of this approach for gamble comparisons. The *bottom-up* method begins by fixing x_0 and x_1 not far apart in X with $x_0 < x_1$; the method then obtains additional values from $[\underline{x_{i+1}}, x_{i-1}] \sim x_i$ for $i = 1, \ldots, n$. If $u(x_0) \equiv 0$ and $u(x_1) \equiv 1$, obviously $u(x_i) = i$ so the values are equally-spaced in utility. Another variation is the *top-down* method which begins by fixing x_0 and x_1 with $x_1 < x_0$. If $u(x_0) \equiv 0$ and $u(x_1) = -1$, then $u(x_i) = -i$ for a top-down uniform sequence. These methods seem most appropriate for unidirectional attributes, like media exposure or waiting time, that are bounded from below or above in utility. Bipolar attributes, such as net assets, can employ both the bottom-up and the top-down methods to obtain a uniform sequence on X.

5.4(b) *Balanced values:* $[\underline{x_{i'}}, x_i] \sim x_0$, where $u(x_0)$ and $u(x_i)$ are known

This method is adapted from a scaling procedure reported by Guilford [59] and Horst [69]. Suppose that $\{x_0, x_1, \ldots, x_n\}$ is a uniform sequence anchored at a "neutral value" x_0; for definiteness, let $u(x_i) = i$ for $i = 0, 1, \ldots, n$. Although the top-down method might be used to construct a uniform sequence below x_0, the method of balanced values can also be used to derive $u(x_{i'}) = -i$. Similarly, if the uniform sequence $\{x_0, x_1, \ldots, x_n\}$ has $u(x_i) = -i$, then the method of balanced values yields $u(x_{i'}) = i$. This method can also provide consistency checks on equally-spaced utilities.

It is not necessary to assume that $\{x_0, x_1, \ldots, x_n\}$ is a uniform sequence to apply the method of balanced values. For example, if another procedure is used to find the utilities of these points, then this method gives $u(x_{i'}) = 2u(x_0) - u(x_i)$. Hence, the method can be used to find the utilities of several additional points in X.

5.4(c) *Multiplication:* $[\underline{x_{i+1}}, \alpha, x_0] \sim x_i$

The multiplication method for comparing gambles is related to a ratio scaling procedure suggested by Galanter [55] and to the fractionation methods described in Torgerson [152]. Let x_0 and x_1 be some reference points not far apart in X with $u(x_0) \equiv 0$. The gamble comparisons above produce successive multiplications of utilities resulting in $u(x_i) = (1/\alpha)^{i-1}\ u(x_1)$. One can obviously generalize this method using probabilities α_i instead of α, but in most situations the probability is fixed at 1/2. For example, with $\alpha = 1/2$ each x_i has twice the utility of the previous point. Applications of such power functions are given in [39, 55, 86, 152]. This method is related to the anchored gambles method in 5.3(c) and to the fractionation methods described in Sections 5.5(b) and 6.2(c).

5.4(d) *Equisection:* find $\underline{x_1}, \ldots, \underline{x_n}$ such that $[x_{i+1}, x_{i-1}] \sim x_i$

The equisection method for gamble comparisons is similar to a scaling procedure described by Torgerson [152] and others [39, 55]. Let x_0 and x_{n+1} be distant reference points in X, where $x_0 < x_{n+1}$. The method asks an individual to divide the interval from x_0 to x_{n+1} into n equal sections. For $n = 2$, the *bisection* method calls for $\underline{x_1}$ such that $[x_2, x_0] \sim x_1$; successive bisections comprise the midpoint chaining method described later. For $n = 3$, the *trisection* method seeks $\underline{x_1}$ and $\underline{x_2}$ such that the following relationships both hold.

$$[x_2, x_0] \sim x_1 \quad \text{and} \quad [x_3, x_1] \sim x_2 \tag{7}$$

In specifying x_1 and x_2 for the trisection method, an individual could use a convergence procedure of adjusting the unit (i.e., a section length) until $\{x_0, x_1, x_2, x_3\}$ forms a uniform sequence. Similarly, one can describe a *quadrisection* method for $n = 4$ or an *n-section* method in general.

Value equivalence methods appear to be useful tools for assessing utility functions, but further empirical research is needed to determine their strengths and weaknesses. The methods are simple analytically, but the use of chained responses in the sequences of gamble comparisons may have undesirable effects in some applications. On the other hand, value equivalence methods are not as susceptible to possible biases from range effects as the extreme value method of probability equivalence or the fractile method of certainty equivalence. An important question for further study, however, is the interpretation and effects of gains and losses in using value equivalence methods for utility assessment. This question also bears on other assessment methods.

5.5 Certainty Equivalence Methods

These methods ask an individual to specify a sure outcome w, called a *certainty equivalent,* for which $[x, \alpha, y] \sim \underline{w}$. Although weaker assumptions are possible, we assume a continuum of values in X so that w exists and strictly increasing preferences on X so that w is unique [39, 89].

Before discussing the fractile method, we fix a set of probabilities $0 < \alpha_1 < \ldots < \alpha_n < 1$ and select two reference points x_0 and x_* in X where $x_0 < x_*$ with $u(x_0) \equiv 0$ and $u(x_*) \equiv 1$.

5.5(a) *Fractile:* $[x_*, \alpha_1, x_0] \sim \underline{x_1}$

The fractile method has been described under a variety of names [2, 38-41, 68, 72, 83, 127, 135-137]. The method is easy to implement, and utility calculations are immediate since $u(x_i) = \alpha_i$. Since the fractile method and the extreme gambles method of probability equivalence are so similar, they share many of the same advantages and disadvantages.

The fractile method often takes the endpoints of X as x_0 and x_*, because any points below x_0 or above x_* need to be determined by another assessment procedure. Biases from range effects can distort the certainty equivalents in many situations where the endpoints are far apart [83, 88, 108, 109, 136]. The fractile method may incur other biases from certainty effects [64, 155], distortions in risk behavior [64, 156, 169], and probabilities too close to zero or one [1, 76, 78, 98]. Despite these potential drawbacks, the fractile method is a popular assessment procedure.

5.5(b) *Chaining methods: fractionation and midpoints*

If previously elicited values are used in subsequent gamble comparisons, the responses are *chained*. In formal terms, let S_i denote the set of values elicited from an individual before the i-th response. Then S_i is defined recursively by $S_i = S_{i-1} \cup \{x_{i-1}\}$ for $i = 1, \ldots, n$, where $S_0 \equiv \{x_*\}$. Thus for a given probability $0 < \alpha_i < 1$ and values $x_{i'}, x_{i''} \in S_i$, the i-th comparison is $[x_{i''}, \alpha_i, x_{i'}] \sim \underline{x_i}$. After each comparison, the new value x_i is easily assessed from $u(x_i) = \alpha_i u(x_{i'}) + (1 - \alpha_i)u(x_{i''})$. Like a few other methods, chaining methods allow one to assess additional values one at a time until either enough points are available to estimate the utility function satisfactorily or the assessment process must be terminated for some reason. The number of responses does not have to be specified beforehand.

One example of chaining is the *fractionation* method, which is analogous to procedures described in Torgerson [152]. Let x_0 and x_1 be two reference points not far apart in X with $u(x_0) \equiv 0$. The fractionation method uses the following gamble comparisons.

$$[x_i,\ \alpha,\ x_0] \sim \underline{x_{i+1}} \tag{8}$$

Each value x_i has a fraction α of the utility of the previous point, since (8) yields $u(x_i) = \alpha^{i-1}\, u(x_1)$. The fractionation method may be compared to the multiplication method in Section 5.4(c). If the probability $\alpha = 1/2$, then this method is a special case of the midpoint chaining method described next.

The *midpoint chaining* (or midvalue splitting) method has $\alpha_i \equiv 1/2$, so each comparison involves a bisection [24, 83, 104, 127]. A characteristic sequence for the midpoint chaining method is

$$\begin{aligned} &\text{Comparison 1:} \quad [x_*,\ x_0] \sim \underline{x_1}, \\ &\text{Comparison 2:} \quad [x_1,\ x_0] \sim \underline{x_2}, \\ &\text{Comparison 3:} \quad [x_*,\ x_2] \sim \underline{x_3}, \end{aligned} \tag{9}$$

where x_0 and x_* are as above. Note that $u(x_1) = 1/2$, $u(x_2) = 1/4$, and $u(x_3) = 3/4$, and that further midpoints can be assessed similarly if needed. (One might compare this procedure with the quadrisection method or the fractile method using $\alpha_1 = 1/2$, $\alpha_2 = 1/4$, and $\alpha_3 = 3/4$, since all three methods yield the same assessed utilities.) Furthermore, one can make an additional comparison $[x_2,\ x_3] \sim \underline{x_4}$ as a consistency check to determine whether or not $x_4 = x_1$.

The midpoint chaining method has its share of drawbacks: it can suffer biases from certainty effects, serial dependence, range effects, distortions in risk behavior, and others. Krzysztofowicz and Duckstein [88] and Novick et al. [111] review several sources of potential biases in using the midpoint chaining method above.

To alleviate bias from the first comparison, which uses a gamble over the extreme values x_0 and x_*, Krzysztofowicz and Duckstein [88] propose the *variable range gamble* method. This method first partitions the range from x_0 to x_* into two arbitrary subintervals and then applies the midpoint method separately to each subinterval. The utility function over the entire range is derived by "linking" the subintervals with a gamble comparison involving one elicited value from each subinterval [82, p. 198; 88]. An advantage of this approach is the avoidance of posing unrealistically extreme questions that can lead to strongly biased responses.

The certainty equivalence methods have been widely used because of their computational simplicity. Recent behavioral studies, however, have uncovered a variety of dysfunctional biases associated with these methods. Further research may provide improvements in these methods and reduce their susceptibility to some sources of biases. Hybrid assessment methods that combine different fundamental procedures may be one useful approach.

6. Further Methods in Utility Assessment

This section examines (1) hybrid methods that combine two or more basic methods for utility assessment, (2) paired-gamble methods of preference comparison, probability equivalence, and value equivalence, and (3) other approaches to utility assessment.

6.1 Hybrid Assessment Methods

Many of the basic assessment methods in Section 5 have sequences of gamble comparisons that generate triangular systems of equations from which one can immediately assess the utility of each successive point. The computational convenience of these methods is sometimes outweighed by assessment biases arising from several sources. In contrast, methods like adjacent gambles, equisection, etc., require a *block* of n comparisons before any utilities can be assessed; such methods appear less prone to certain forms of assessment bias. Computational simplicity is no longer paramount, however, with the development of interactive computer programs for utility assessment [84, 98, 110, 112, 114, 137, 140], so block methods are practical possibilities now.

One approach in examining block methods considers the coefficient matrix of the set of homogeneous equations generated by a particular sequence of gamble comparisons. This approach facilitates the design of hybrid utility assessment protocols from more basic methods, tests of the determinacy of a given sequence of comparisons, and the identification of serial dependence and other potential sources of bias. Work is in progress to develop this approach further.

As an illustration, we consider the variable range gamble (VRG) method in [88]. The VRG method yields an indeterminate coefficient matrix because the subintervals are not really linked together by the final midpoint comparison as claimed in [88]. This flaw is remedied, for example, by using a probability equivalence comparison between a gamble with one value from each subinterval and a standard given by the point which divides the two subintervals. Our revision links the subintervals and yields a determinate set of utilities. This modified VRG procedure is a hybrid of midpoint chaining and assorted gambles comparisons that is designed to alleviate the extreme range biases characterizing midpoint chaining methods.

There are many possibilities for merging basic assessment methods to produce hybrid methods that are robust against different forms of bias. Promising directions include combining a midpoint chaining method with either an equisection method or the adjacent gambles method. One significant gap is the need for a way of identifying and measuring biases associated with various utility assessment methods, so hybrid designs can be better evaluated for their effectiveness.

6.2 Paired-Gamble Methods

All standard-gamble methods suffer from a fundamental asymmetry in comparing a sure outcome with a risky alternative. Since individuals tend to disproportionately overvalue outcomes that are certain in comparison to outcomes that are only probable [154], standard-gamble comparisons are likely to skew a utility assessment. Therefore, one might consider comparisons involving pairs of gambles to eliminate possible biases from this certainty effect.

There is relatively little research on assessing utilities from paired-gamble comparisons. Most of the previous work involves preference comparisons; some research covers probability equivalence and value equivalence methods, but many of the following methods are presented for the first time.

6.2(a) *Preference comparison:* $[x_i, \alpha_i, y_i] \; \underline{R}_i \; [w_i, \beta_i, z_i]$

The preference comparison method often uses only even-chance gambles, $\alpha_i \equiv \beta_i \equiv 1/2$. One use of this method is in constructing an *ordered metric scale* for utilities [18, 21, 38-41, 86, 123, 149]. Other researchers apply linear programming procedures to the constraints generated by the preference comparisons above to estimate a utility function [16, 20, 23, 104, 148]. Although the method offers some advantages in elicitation simplicity and bias reduction, the only reported applications are consistency checks and multiattribute independence tests [83].

6.2(b) *Probability equivalence:* $[x_i, \underline{\alpha_i}, y_i] \sim [w_i, \beta_i, z_i]$,
where both w_i and z_i are between x_i and y_i

Further restrictions on the above gambles are imposed to simplify the assessments and obtain a determinate solution for the utilities. We begin as in Section 5.3 by specifying two distant reference points x_0 and x_{n+1} in X. The task is to assess the utilities of the points $x_0 < x_1 < \ldots < x_{n+1}$. We illustrate only a few of the possibilities for such probability equivalence methods using paired-gamble comparisons.

- *Extreme gamble:* $[x_{n+1}, \underline{\alpha_i}, x_0] \sim [x_{i+1}, x_{i-1}]$
- *Adjacent gambles:*[4] $[x_{i+2}, \underline{\alpha_i}, x_{i-2}] \sim [x_{i+1}, x_{i-1}]$
- *Anchored gambles:* $[x_{i+1}, \underline{\alpha_i}, x_0] \sim [x_i, x_0]$

These methods are generalizations of standard-gamble procedures with the same names in Section 5.3; many other generalizations are possible, too.

Some paired-gamble methods have no direct analogs among standard-gamble methods. For example, a simple restriction for probability equivalence comparisons is to *interlock* the gambles in each pair with $\alpha_i \equiv \beta_i$. Some illustrations of this approach are the following methods.

---

[4] Let $x_{-1} \equiv x_0$ and $x_{n+2} \equiv x_{n+1}$ for this method.

- *Interlocking extreme gambles:* $[x_{n+1}, \underline{\alpha_i}, x_0] \sim [x_{i+1}, \underline{\alpha_i}, x_{i-1}]$
- *Interlocking adjacent gambles:* $[x_{i+2}, \underline{\alpha_i}, x_{i-2}] \sim [x_{i+1}, \underline{\alpha_i}, x_{i-1}]$

Novick, Dekeyrel, and Chuang [111, pp. 563-564] call the latter method the *paired binary gamble* (PBG) procedure and report on preliminary results of its usefulness.

> The obvious hope is that the PBG procedure will avoid the certainty effect because the comparison is between two sets of gambles, and thus does not involve the for-sure option. We have used PBG in some informal assessments but have not yet been convinced of its usefulness. First, it is difficult even for experienced subjects. Fatigue and boredom are definite problems. We are not sure that there is no bias in that one situation always compares two adjacent states while the other always describes two states twice removed. We have not discarded this procedure, but we feel that refinements may be necessary if it is to be useful.

Further research on variations of probability equivalence methods using paired-gambles seems likely (see Section 6.3).

6.2(c) *Value equivalence:* $[\underline{x_i}, \alpha_i, y_i] \sim [w_i, \beta_i, z_i]$

There are several ways of further structuring the above gambles to facilitate utility assessment. One way is to have common probabilities, $\alpha_i \equiv \beta_i \equiv 1/2$. Davidson, Suppes, and Siegal [20] and Kneppreth, Gustafson, Leifer, and Johnson [85] report using gamble comparisons of the form $[\underline{x_i}, y_i] \sim [w_i, z_i]$. This *equal-differences* method usually requires some additional comparisons to determine the utilities. A variation with the gamble $[w_i, z_i] \equiv [w, z]$ in all comparisons is analogous to the balanced values method in 5.4(b). Other variations are based on the methods in Section 5.4.

Another approach in developing paired-gamble methods using value equivalence is to have common values, $y_i \equiv z_i \equiv x_0$. Let x_0 and x_1 be two reference points in X where $x_0 < x_1$; then let $u(x_0) \equiv 0$ and $u(x_1) \equiv 1$. Also, define ρ as the ratio of probabilities β/α. The following *anchored value* method reduces to the multiplication standard-gamble method in 5.4(c) when $\beta = 1$ or the fractionation standard-gamble method in 5.5(b) when $\alpha = 1$.

- *Multiplication:* $[\underline{x_{i+1}}, \alpha, x_0] \sim [x_i, \beta, x_0]$ where $\rho > 1$
- *Fractionation:* $[\underline{x_{i+1}}, \alpha, x_0] \sim [x_i, \beta, x_0]$ where $\rho < 1$

The utilities are easily computed from $u(x_i) = \rho^{i-1}$. Thus, the multiplication sequence x_i has increasing utility, while the fractionation sequence x_i has decreasing utility.

Not all paired-gamble methods are included in the above discussion, but the main procedures and a representative sample of possible variations are covered. A key question for further empirical research is whether the additional assessment effort required by these paired-gamble methods is worth the anticipated reduction in assessment bias.

6.3 Other Approaches

Novick, Dekeyrel, and Chuang [111] are developing new assessment procedures for interactive computer implementation. They have experimented with hybrid methods that provide immediate consistency checks and alleviate some biases in the assessment process. For example, their *local coherence* (LC) procedure combines a standard-gamble and a paired-gamble in one format.

$$[x, \underline{\alpha}, y] \sim w \iff [x, \beta_1(\alpha), y] \sim [w, \beta_2(\alpha), y], \qquad (10)$$

where w is between x and y in X. The individual first uses a convergence procedure to determine the indifference probability for the standard-gamble comparison in (10). After a computer program [112] calculates the probabilities $\beta_1(\alpha)$ and $\beta_2(\alpha)$, the individual is told that his response to the standard-gamble implies equivalence between the gambles displayed in (10). If the individual does not concur, then he can adjust α until his preference judgments agree with the results in (10). Novick, Dekeyrel, and Chuang [111, p. 566] report that the LC procedure is a "powerful tool for locating the most desirable point in the probability range $\alpha \pm .025$" and suggest that the procedure is is useful in "largely eliminating anchoring and adjustment biases."

Novick, Dekeyrel, and Chuang describe a *regional coherence* (RC) procedure that combines two adjacent gambles comparisons to infer equivalences in two other standard-gamble comparisons. After responding with indifference probabilities for the first two comparisons, the computer program displays all four comparisons. Individuals may then adjust any two indifference probabilities at a time until the four displayed standard-gambles are consistent with their preferences.

7. Summary and Directions for Future Research

Table 3 summarizes all of the utility assessment methods described in Sections 5 and 6. Note that the index i goes from 1 to n in each case and that the assumptions behind each method are stated earlier in the paper.

Table 3 goes here

In conclusion, we mention some topics of on-going or prospective research in utility assessment. We expect to see further research in computer-aided utility assessment [84, 99, 110, 112, 137, 140]; consistency, coherence, and errors in judgment [16, 30, 37, 91, 94, 104, 106, 141, 147, 163, 167]; heuristics, biases, and debiasing [1, 35-37, 64, 75, 88, 91, 98, 108, 109, 127, 142, 155, 156]; display format and response mode effects [4, 15, 19, 58, 64, 67, 75-77, 92, 93, 95, 98, 105, 118-121, 129, 138-139, 153-157, 164, 169]; and related topics.

On the other hand, we anticipate research in developing alternate assessment approaches. The research on risk attitudes [62], strength-of-preference and relative risk [13, 33, 25, 26, 87, 162], and risk profiles [6, 60, 116, 165] represents an important direction for further investigation. Similarly, the current interest in generalizations and alternatives to expected utility theory is likely to have a significant effect on assessment procedures [11, 17, 19, 45, 46, 48, 76, 139, 157].

Prospective research topics might include the study of assessment procedures using gambles with three or more outcomes [27, 90, 119, 120, 141], the study of comparisons with three or more gambles [18, 101, 152], the estimation of utility functions with probability densities or other curves [14, 113, 137], extensions of utility assessment procedures to determine scaling constants and trade-offs in multiattribute decision problems [31, 34, 39, 63, 74, 83, 85, 88, 99-101], and many others.

Table 3

Utility Assessment Methods

I. Standard-Gamble Methods

1. **Preference Comparison Methods:** $(x_i, \alpha_i, y_i)\ \underline{R_i}\ w_i$

2. **Probability Equivalence Methods:**
   - a) Extreme gambles: $(x_{n+1}, \underline{\alpha_i}, x_0) \sim x_i$
   - b) Adjacent gambles: $(x_{i+1}, \underline{\alpha_i}, x_{i-1}) \sim x_i$
   - (Assorted gambles) $(x_{k_i}, \underline{\alpha_i}, x_{j_i}) \sim x_i$, where $j_i < i < k_i$
   - c) Anchored gambles: $(x_{i+1}, \underline{\alpha_i}, x_0) \sim x_i$

3. **Value Equivalence Methods:**
   - a) Uniform sequence: $(\underline{x_{i+1}}, x_{i-1}) \sim x_i$
   - b) Balanced values: $(\underline{x_i}, x_i) \sim x_0$, for known $u(x_0)$ and $u(x_i)$
   - c) Multiplication: $(\underline{x_{i+1}}, \alpha, x_0) \sim x_i$
   - d) Equisection: find $\underline{x_1}, \ldots, \underline{x_n}$ such that $(x_{i+1}, x_{i-1}) \sim x_i$ (bisection, trisection, quadrisection, and n-section)

4. **Certainty Equivalence Methods:**
   - a) Fractile: $(x_*, \alpha_i, x_0) \sim \underline{x_i}$
   - (Chaining methods)
   - b) Fractionation: $(x_i, \alpha, x_0) \sim \underline{x_{i+1}}$
   - c) Midpoint chaining: $(x_*, x_0) \sim \underline{x_1}$, $(x_1, x_0) \sim \underline{x_2}$, $(x_*, x_2) \sim \underline{x_3}$.

II. Paired-Gamble Methods

5. **Preference Comparison Methods:** $(x_i, \alpha_i, y_i)\ \underline{R_i}\ (w_i, \beta_i, z_i)$

6. **Probability Equivalence Methods:** $(x_i, \underline{\alpha_i}, y_i) \sim (w_i, \beta_i, z_i)$ where both w_i and z_i are between x_i and y_i
   - a) Extreme gambles: $(x_{n+1}, \underline{\alpha_i}, x_0) \sim (x_{i+1}, x_{i-1})$
   - b) Adjacent gambles:[4] $(x_{i+2}, \underline{\alpha_i}, x_{i-2}) \sim (x_{i+1}, x_{i-1})$
   - c) Anchored gambles: $(x_{i+1}, \underline{\alpha_i}, x_0) \sim (x_i, x_0)$
   - d) Interlocking extreme gambles: $(x_{n+1}, \underline{\alpha_i}, x_0) \sim (x_{i+1}, \underline{\alpha_i}, x_{i-1})$
   - e) Interlocking adjacent gambles: $(x_{i+2}, \underline{\alpha_i}, x_{i-2}) \sim (x_{i+1}, \underline{\alpha_i}, x_{i-1})$

7. **Value Equivalence Methods:** $(\underline{x_i}, \alpha_i, y_i) \sim (w_i, \beta_i, z_i)$
   - a) Equal differences: $(\underline{x_i}, y_i) \sim (w_i, z_i)$
   - b) Anchored values: $(\underline{x_{i+1}}, \alpha, x_0) \sim (x_i, \beta, x_0)$ (multiplication for $\alpha < \beta$; fractionation for $\alpha > \beta$)

III. Hybrid Methods

8. **Block methods:** Modified variable range gamble method

IV. Other Methods

9. **Coherence methods:** Local and regional

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