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Adding Asymmetrically Dominated Alternatives: Violations of Regularity and the Similarity Hypothesis Joel Huber, John W. Payne Christopher Puto ONR Technical Report 81-2 (Revised) Journal of Consumer Reseach (Forthcoming)

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ADDING ASYMMETRICALLY DOMINATED ALTERNATIVES: VIOLATIONS OF REGULARITY AND THE SIMILARITY HYPOTHESIS

One of the most important issues in marketing is understanding how the introduction of a new brand into a market will be reflected in choice probabilities or market shares. A standard model that is used in such situations is to assume that a new offering will take from others in proportion to their original shares. This assumption of proportionality is incorporated in the Luce (1959) model of choice and is central to a number of models of consumer behavior. For example, Pessemier et. al. (1971) and Reibstein (1978) use this assumption as a basis for transforming affect scores into choice probabilities for soft drinks, while Silk and Urban (1978) use a similar method to predict share for packaged goods. The assumption has also been central to models of college choice (Punj and Staelin 1978), and transportation mode choice (McFadden 1974).

It is not hard, however, to identify situations in which the assumption of proportionality fails (Debreu 1960; McFadden 1974). Generally there is agreement that a new product takes disproportionately more share from those similar to it than from <u>dissimilar</u> items. This idea, which has come to be called the similarity hypothesis (Tversky 1972), is reflected in the managerial belief that one can minimize cannibalization by designing a new product to be as dissimilar from the firm's current offerings as possible. The similarity hypothesis has served as a basis for a number of alternative theories of choice (Tversky 1972; Hauseman and Wise 1978; Batsell 1980; McFadden 1980). These models are increasingly being used to aid marketing managers making marketing entry decisions (e.g., Urban, Johnson and Brudnick 1981).

While substantially different in their underlying assumptions, the Luce choice model and the proposed revisions do share a common assumption that the addition of a new alternative cannot increase the probability of choosing a member of the original set. This condition, called regularity, is necessary for the validity of most probabilistic choice models and has been found to hold empirically.

It will be shown that both the similarity hypothesis and the regularity condition can be consistently violated by the addition of an asymmetrically dominated alternative. An alternative is "asymmetric" if it is dominated by at least one alternative in the set but is not dominated by at least one other. We show that the addition of such alternatives increases the share of the item that dominates it, thus violating regularity. Furthermore, since the new alternative is typically closest to the item that dominates it, this result implies that the new alternative set "helps" the items closest--a reversal of the similarity hypothesis which would predict the opposite.

If accepted, the results have managerial and theoretical importance. Managerially, the results lead to the counter-intuitive conclusion that there are times when profitability of a product line can be increased by adding a (dominated) alternative that virtually no one ever chooses. This unexpected conclusion is due to the fact that the function of the dominated alternative is ' draw attention to a more profitable item rather than to generate direct sales. Theoretically, the results indicate that there is a limit to the range of applicability of most discrete choice models. These models will either have to be modified to accept the distortion of dominated items or limited in their range to collections without dominated alternatives. Furthermore, the

results have implications for those who estimate the similarity effect (e.g., Batsell 1982). These researchers may wish to include a term that accounts for the dominance structure of the subsets. If such a term is not included the similarity effect may be artificially attentuated, since its effect is reversed when dominance is present.

The paper is organized as follows. First, the concepts of regularity, similarity and dominance are briefly reviewed, and a method is presented which tests the hypothesized violations. Then some explanations are provided that may account for these expected violations. Finally, the results are examined with respect to the explanations they support and the future research they suggest.

Regularity and Choice Models

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Regularity is a minimum condition of most existing choice models. Formally, for any item which is a part of set A where A is in turn a subset of B, then the probability of choosing X from A must not be less than from B, or

for all
$$x \in A \subseteq B$$
, (1)
 $Pr(x;A) \ge Pr(x;B)$.

If this inequality is satisfied, one cannot increase the probability of choosing an item by adding other items. Regularity is a rather weak condition that is required by both Luce's (1959) choice model and by Tversky's (1972) elimination by aspects model. Empirically, it has been found to be satisfied. For example, Becker, DeGroot and Marshak (1963) found that, in choices among gambles, regularity was satisfied even though proportionality was not. Luce summarized by remarking that the "only property of general choice probabilities that has not been empirically disconfirmed is regularity" (Luce 1977, p. 229).

On the other hand, it is easy to think of examples that violate regularity, particularly if higher-order rules are imposed on the decision. Corbin and Marley (1974) give two such examples. The first involves a woman in a small town having to decide between two hats. In this case, the probability of choosing a hat would decrease if its duplicate were also available. Presumably, the woman would not want a hat someone else could buy. Thus the probability of purchasing a hat could increase if one of its competitors were duplicated, violating regularity. The second example concerns the probability of choosing an entree where the decision rule is to choose from a set excluding the most expensive. The probability of choosing the most expensive entree could then be increased by simply adding one to the list that is more expensive still. Note that both of these exceptions involve higherorder rules where the value of alternatives depends on the choice set. That is, one has to have a rule about the desirability of having a unique hat or the undesirability of the most expensive entree for these exceptions to be plausible.

The exceptions to regularity we shall illustrate below do not depend on the existence of such higher-order rules. Moreover, the effect will be shown to occur in a number of different product categories.

Relative Similarity and Choice Models

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As noted earlier, the similarity hypothesis asserts that a new alternative takes disproportionate share from those with which it

is most similar. Researchers have shown that the similarity effect is operant for individual (Rumelhart and Greeno 1971) or aggregate (Huber and Sewall 1978) choice probabilities. Unfortunately, as Luce and Suppes (1965) show, the similarity hypothesis is logically incompatible with either constant utility, or independent random utility, models of choice.

Accordingly, several authors have attempted to modify these choice models to allow for the similarity effect. For example, working with a random utility framework, Hauseman and Wise (1978) modified the Thurstone model to -ccept covariances between alternatives. The similarity effect can then be represented by a positive covariance in the preferences among similar alternatives. Tversky's (1972) elimination by aspects model, arising as a multinominal generalization of Restle's (1961) model, accounts admirably for the similarity effect. Finally, work by Batsell (1980) provides a procedure for directly accounting for the similarity effect on choice probabilities from different choice sets.

In all of these modifications the addition of an alternative lowers the choice probability of similar items proportionately more than dissimilar ones. As will be shown, however, the addition of a dominated alternative appears to have the <u>opposite</u> effect, increasing choice of the similar item that dominates it. Further, this effect is stronger as relative similarity increases, thus limiting the applicability of the similarity hypothesis to choice sets where such dominance does not occur.

Dominance and Choice Models

Dominance is not easily modeled by most choice models. For example, it is easy to show that the existence of an asymmetrically dominated alternative in a choice set implies that pairwise probabilities cannot

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be modeled by either a constant or a random utility choice model. In both models, the probability of one item being chosen over another is a function of their distance on a one-dimensional utility scale. Dominated items, having zero probability of being chosen, are represented in the limit as being an infinite distance below those items that dominate them. The contradiction arises in that distances among non-dominating pairs (whose pairwise probabilities do not equal zero or one) must be finite. In the asymmetric case, then, there can be no one-dimensional scale that simultaneously accounts for the finite and infinite distances implied by the paired probabilities.

Previous models of choice have handled the issue of dominated alternatives in a number of ways. Both Restle's model (1961) and elimination by aspects (Tversky, 1972) account quite well for extreme probabilities. Since the probability of choosing an item is a function of its unique aspects, a dominated alternative lacking unique aspects has no probability of being chosen. Luce (1959) simply restricted the choice set to non-dominated alternatives. Consequently, many of the subsequent tests of choice models have not included dominated alternatives. It can also be reasonably argued that respondents initially delete dominated alternatives, leaving the choice along the efficient frontier unaffected (Coombs and Avrunin 1977). However, as will be shown, the very presence of the dominated alternative results in quite different choice probabilities among the remaining alternatives than in the pristine state where such items are never considered.

To summarize, the purpose of this paper is to demonstrate violations of regularity and the similarity hypothesis due to the addition of an asymmetrically dominated alternative. These violations will be illustrated in the context of a particularly simple product choice task.

Method

One hundred and fifty-three students in graduate and undergraduate business classes were asked to make choices from among six product categories: cars, restaurants, beers, lotteries, film, and T.V. sets. Decisions involved either two or three alternatives, with each alternative being defined on two attributes, thus providing a simple decision environment and a straightforward test of the hypotheses. The alternatives in each product class were designed to represent a target, a competitor and a decoy as shown in Figure 1. The target and the competitor are positioned so that neither dominates the other--each has a dimension on which it is superior. The decoy is then a stimulus anywhere in the shaded region of Figure 1 where it is dominated by the target but not the competitor.

The test of the effect of the decoy was made by comparing the percentage of times the target was chosen against the competitor with and without a decoy present. This test was performed within subjects by having a subset of the students repeat the task two weeks later with the decoys removed. Across respondents, the test was made by positioning the decoy in different corners of the space for matched groups. Thus the decoy effect is the difference in shares for an item when it is the target as opposed to the times when the competitor takes that role.

Why Decoys Can be Expected to Distort Choice

Before examining the results of the experiment it is useful to consider reasons why the addition of an asymmetrically dominated alternative (decoy) increases the proportion of choices in favor of the target. Notice, first that

if the added dominated alternative is <u>never</u> chosen, then any change in the proportion of choices between the target and the competitor is a technical violation of regularity (since the probability of choosing one of the original options increases). Such violations of regularity would be rather uninteresting even if they could be shown to be statistically significant. In the present case, however, the prediction is directional—adding the decoy is hypothesized to <u>increase</u> the percent of choices to the target, violating regularity in a predicted direction and, since decoys are typically closer to targets than competitors, reversing the similarity effect.

There are several, possibly interacting, reasons why a decoy placed in the shaded region of Figure 1 might be expected to increase the share of the target at the expense of the competitor. These reasons include the perceptual framing of the decision problem and the evaluation processes used. Consider the effect on weighting of attributes and scaling of alternatives of the four different decoy placement strategies shown in Figure 1. Figure 1 provides a graphical description of the strategies, and Table 1 provides examples of each strategy with six-packs of beer as the choice options.

The four strategies have the effect of (1) increasing the range of the dimension on which the target is weakest, <u>R</u>, (2) strongly increasing that range, <u>R</u>*, (3) increasing the frequency of the dimension on which the target is superior <u>F</u>, and (4) combining both a range and a frequency strategy, <u>RF</u>. Increasing the range of the dimension on which the competitor is superior is hypothesized to decrease the importance of a fixed difference on that dimension. Thus in the example given, the increase in the range of quality from 20 to 30 points may make the 20 point advantage of the competitor over the target seem less extreme. Such an effect would be similar to the result that

increasing the range of stimuli tends to narrow the category ratings on that dimension (Parducci, 1974). Notice further that a range effect would predict that an increase in the range (\underline{R} versus $\underline{R^*}$) would increase the biasing effect, thus permitting an evaluation of the efficacy of this explanation.

Increasing the frequency of items on the dimension on which the target is superior might increase the weight of that dimension. Such an effect could occur in two ways. First by adding another price level, more attention may be drawn to the dimension (Currin, Weinberg and Wittink, 1981). Second, the addition of a beer with a price of \$2.20 might tend to spread the psychological distance of the 80c price advantage the target has over its competitor. Adding such a decoy would lower the variance along the price dimension, thus making the standardized differences greater. This result is once again analogous to the finding by Parducci (1974) that adding alternatives within the range of others tends to spread out their distances on subjective category ratings.

The combination range-frequency strategy, <u>RF</u>, adds a decoy that simultaneously increases the range of the dimension on which the target is inferior while increasing the frequency on which it is superior. Although such a strategy should combine the biasing powers of both, as the example in Table 1 makes clear, it may be harder to detect dominance if one has to consider both dimensions. Thus, the biasing effect may be attenuated with such a strategy.

Finally, a reweighting to favor the target could occur simply because of a misplaced popularity inference on the part of the respondent. Before being aware of the dominance relations in the set, a subject may believe that all of the choices are popular, viable options. However, if the subject wishes to make a choice that others would make, the belief that the decoy is popular may shift votes toward the target.

In addition to perceptual types of effects discussed above, there are certain evaluation strategies which, if followed, would bias choice towards the target in the presence of the decoy. Suppose the choice process involves a series of paired comparisons and that each pair is evaluated on an attribute-by-attribute basis (Russo and Rosen 1975). Under such an evaluation process, an initial pairing of the decoy with the competitor could eliminate the competitor so that it could no longer compete, thereby increasing the target's probability of being chosen. A more subtle form of this process would involve a round-robin tournament where each stimulus is compared with all other stimuli in the set, and the item with the most wins is chosen. If subjects either count the number of wins or the number of attribute wins (c.f. Russo and Dosher 1980) then it is easy to show that addition of the decoy helps the target.

A consideration of the cost of thinking (Shugan, 1980) would also lead to an advantage to the target. Under the cost of thinking model, the hypothesized cost of making decisions between dominated pairs is much less than between non-dominated pairs. The easy choice between the target and the decoy might be more likely to be made by the simplifying decision maker than either decision involving the competitor, thus leaving the target as the choice.

In sum, consideration of either perceptual biases or certain evaluation strategies leads one to predict the diversion of choices to the target due to the presence of the dominated decoy. This hypothesized effect, leading to a violation of regularity and a reversal of the similarity effect, is tested on both a between- and a within-subject basis (c.f. Einhorn and Hogarth, 1981).

Within-subjects a count is made of the preference reversals between the target and the competitor due to the decoy. These results are aggregated across six product classes and four placement strategies according to a balanced design detailed in the Appendix. The between-subject analysis estimates the effect of each of the four (\underline{R} , $\underline{R^*}$, \underline{F} , \underline{RF}) decoy placement strategies across the six product categories. This analysis provides more detail as to the mechanism driving the effects.

Results: Within-Subject Preference Reversals

Two weeks after the initial test ninety-three subjects were asked to choose again between the competitor and the target with the decoy removed. Out of the 558 choices (6 product classes x 93 subjects), Table 2 tallies the number of reversed preferences. Two tests were made on the distorting effect of the decoy. The first was based on the 98% of the choices where the decoy was not chosen. In that sample, 63% of the 109 reversals (CELLS <u>b</u> and <u>d</u>) were to the target and 37% to the competitor. That difference is statistically significant (McNemar Test Siegel, 1956) at a $p \le 0.05$ level. The second test codes switching to the dominated decoy as switching from the target, thus merging the decoy and the competitor groups (CELLS b, d and c). In that test, 59% switched to the target, while 41% switched away. The difference was marginally significant at a $p \le 0.10$ level.

Within subjects the decoy effect was significant but not strongly so. Regularity was violated--the target's share jumped from 53% to 56% with the addition of the decoy. The relative weakness of the distortion can be attributed to a carryover effect where subjects simply repeated choices made two weeks earlier. Indeed the cross-subjects analysis, which does not share pretest sensitization, resulted in a much stronger decoy effect.

Results: Between-Subject Share Changes

Between subjects a test of the decoy effect was made by comparing the change in the proportion of subjects choosing the target over the competitor with different experimental placements of the decoy. Table 3 gives these proportions broken down by the six product classes and the four strategies. Looking across the top row, for example, Car <u>A</u> was chosen 44% of the time when there was no decoy but 66% of the time when a range increasing (<u>R</u>) decoy was added. The second set of columns gives the results when the other car, <u>B</u>, was the target. Regularity is violated when the percent choosing the item was greater with the decoy than in the no decoy condition. This occurred in 18 out of the 24 different/cases (P<0.05). The final column gives the average change due to adding the decoy. This is in the hypothesized direction for all of the product classes and overall has an average value of 9.2%.

A simple way to summarize the effectiveness of the various strategies is to compute from Table 3 the average share or gain to the target due to adding the decoy. The two range increasing strategies $(\underline{R} \text{ and } R^*)$ increased the average target's penetration by 13 points; next was the range-frequency (\underline{RF}) strategy with a gain of 8 percentage points, followed by the frequency strategy (\underline{F}) with a net gain of 4 points. A test of the statistical significance of these gains was made by comparing the within-product gain due to a strategy. For example, the \underline{R} strategy was tested using a Fisher Exact Test, testing if the two \underline{R} strategies for beer (each with a different

target) could have been drawn from the same population. The tests on both the <u>R</u> strategies and the <u>RF</u> strategy were significant at $P \leq .05$. The frequency increasing strategy was not significant at that level. The same test was used to compare the significance of differences between strategies. Both moderate and extreme range strategies were significantly more effective than the frequency strategy, $(P \leq .05)$ but all other differences were not.

Summary of Results

To summarize, overall asymmetric dominance appeared to have a strong effect in violating regularity. This effect was stronger (9 points) across subjects than it was within subjects (3 points). The fact that the range increasing strategies produced a 13 point change that did not differ with the degree of the range extention, suggests that a simple range extention explanation is not sufficient, and that other factors must be found to account for this effect. The weakness of the range-frequency strategy may be due to dominance not being as readily apparent in such situations. Finally, the weakness of the frequency strategy suggests that this strategy is not as successful in revising weights as had been expected; it also indicates that dominance <u>per se</u> may not be as critical as the particular placement of the decoy.

The concept of relative similarity could account in part for the results found, since the frequency strategy decoy is closest to the competition followed by the decoy for range-frequency and the two range strategies. It may be that the effectiveness of the decoy is related to its degree of relative closeness to the target. Such an explanation could account for the increasing effectiveness of the strategies as one moves away from the competition.

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DISCUSSION

The fact that regularity was found to be violated here but not in other studies can be attributed to their choice sets not containing asymmetrically dominated alternatives. In such tests the added alternative typically took substantial share from the items in the original set so that a <u>substitution</u> effect may have outweighed any <u>distortion</u> effect due to the presence of the new alternative. Thus regularity may have been satisfied because the substitution effect, tending to take share away from the original objects, was stronger than any consistent distortion effect in rearranging share. With the asymmetrically dominated alternatives studied here, by contrast, the substitution effect was virtually negligible (22), and so the distortion effect became clearly evident. It should be emphasized, however, that even though a distortion effect may be masked by a substitution effect, it still occurs, and should be part of our models of choice.

The violations of the similarity hypothesis found here took two forms. First, to the extent that asymmetrically dominating alternatives tend to be similar to the items they dominate, any help from such items results in a reversal of the standard similarity effect. The second violation of the similarity hypothesis occured in that those decoys whose relative similarity to the target was greatest had the greatest positive effect on the target. While this last violation must be considered to be speculative until a more precise measure of relative similarity can be tested, both results together have rather strong implications for the interpretation of any test of the similarity effect. Specifically, if stimulus sets are mixtures of dominated and non-dominated alternatives then the similarity effect is likely to be attenuated because

of the reversals due to the dominated alternatives. Thus, such tests should account for this interaction with dominance or restrict their applicability to sets of non-dominating objects.

The results here, while powerful, are limited in their scope. In particular, choice in this experiment was limited to three alternatives per product class defined on two dimensions. It is expected that increasing the complexity of the decision task would increase the error in the choices and thereby limit the effect of adding any alternative. In particular, the effect of dominance per se may be lessened with more alternatives or more dimensions per alternative, simply because it would be harder to recognize. Other effects found, however, such as the distorting effect of range or frequency of items on each dimension, may paradoxically be stronger since these aspects of a choice set may be relatively easy to acquire and use given a quick scan of relatively complex data.

If the results do extend to more complex and realistic task environments, however, the managerial implications of such distortions of choice probabilities could be very important. Consider, for example, the following hypothetical consumer choice situations.

A store owner has two camel hair jackets priced at \$100 and \$150 and finds that the more expensive jacket is not selling. A new camel hair jacket is added and displayed for \$250; the new jacket does not sell, but sales of the \$150 jacket increase.

'A seller of tours to Disney World for \$500 might also offer a tour to a theme park in Europe costing \$2,500. Few tickets for the European tour would be sold but penetration would increase for the domestic tour.

"A manufacturer of cars with relatively poor gas mileage (e.g. 20 MPG) might decrease the effect of this dimension by first showing prospects a highpowered car in the showroom with much worse (8 MPG) m.leage.

The preceding choice situations are interesting in that they are not clear cases of dominance, but rather near-dominance, where the decision from the decoy to the target is easy to make, and the range effect favors the target. In terms of the experimental paradigm such decoys would be positioned just the right of the R or R* strategies in Figure 2.

The examples given above reinforce the need to validate the dominance effect in the context of actual choice. As an example of such a study, a decoy camel hair coat could be experimentally added to the offerings of a retail firm, and its effect on jacket and total

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sales measured. Similarly, catalogues provide a particularly fruitful mechanism for field research into the effect of near or totally dominated alternatives. A sample of catalogues can be experimentally modified by adding decoys. The large mailings would then result in a very powerful test of the phenomenon.

In terms of the development of a comprehensive theory of choice, the empirical results given here cannot be accounted for by current theories of choice represented by the Luce model or its extensions. What is missing is a unique explanation for the effects found. Research is needed to determine the relative efficacy of various explanations as well as their applicability under different conditions (such as adding more stimuli). Such research could either emphasis tests of weight shifting or more directly examine the evaluation processes. Tests of weight shifting would be more appropriate as a way to examine the range or frequency explanations. Weights could be shown to depend systematically on the placement of the decoys, where the weights could be elicited either by direct or by statistical methods (e.g. see Currim, Weinberg and Wittink 1981). A process oriented research stream (c.f., Payne, Braunstein and Carroll, 1978) could provide the appropriate tests of the validity of the pair comparison explanation. For example, verbal protocals or eye tracking

methods might be used to assess the effect of a decoy placement on the order in which pairs are considered. Such results might indicate that decoys alter the implicit choice agenda.

It is likely that a thorough understanding of the phenomena reported here will come as a result of both statistical estimation methods and process tracing methods. The result of such will are a beeded in order to build a comprehensive theory of choice that explains the empirical results found here rather than leaving them as exceptions to current theories.



FIGURE 1

PLACEMENT OF ASYMETRICALLY DOMINATED DECOY

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Where:

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R =	Noderate range increasing
R*=	Extreme range increasing
F =	Frequency increasing

RF = Range and frequency increasing

TABLE 1

EXAMPLES OF CHOICE SETS For different strategies

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Price/Six Pack	Quality Rating
\$1.80	50
\$2.60	70
\$1.80	40
\$1.80	50
\$2.60	70
\$1.80	30
\$1.80	50
\$2.60	70
\$2.20	50
\$1.80	- <u>-</u> 50
\$2.60	70
\$2.20	40
	Price/Six Pack \$1.80 \$2.60 \$1.80 \$2.60 \$1.80 \$1.80 \$2.60 \$2.20 \$1.80 \$2.60 \$2.20

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TABLE 2

INDIVIDUAL CHOICE REVERSALS

DUE TO ADDITION OF DECOY

		Target	Competitor	Decoy	Total
	Target	242 (a)	40 (b)	8 ^(c)	290
2-It en Choice		(44%)	(7%)	(17)	(53%)
Set	Competitor	69 ^(d)	190 ^(e)	3 (f)	262
		(122)	(34%)	(17)	(47%)
	Total	311 ·	230	11	552
		(56%)	(422)	(2%)	(100%)

3 Item Choice Set

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TABLE 3

SUMMARY OF CHOICE PROBABILITIES FOR

ALTERNATIVE DECOY PLACEMENT STRATEGIES

			Probab	ility	of Choo	sing Tar	get Giv	761		
Product Class	A Decoy	is Ta Places	rget a ent St	ind rategy	15	Decoy	B is 1 Places	larget sent Si	and trategy i	.5
	No Decoy	R ^(a)	F	RF	<u></u>	No Decoy	R	F	RF B	Point Change * Due to Decoy
Cars (n)	.44 (102)	.66 (38)	.52 (33)			.56 (102)		.67 (40)	.67 (36)	13.0
Beer (n)	.43 (102)	.63 (39)		.35 (37)		.57 (102)	.75 (38)	.67 (36)		10.0
Restaurants (n)	.30 (102)		.21 (39)	.43 (37)		.70 (102)	.91 (34)		.87 (39)	10.5
Lotteries (a)	.75 (101)	.81 (36)			.68 (37)	.25 (101)	.41 (37)	.18 (38)		2.0
Film (n)	.24 (102)	.20 (40)	.19 (37)			.76 (102)		.84 (37)	.92 (37)	3.8
TV Sets (n)	.75 (102)		.37 (38)		.83 (35)	.25 (102)	.32 (38)		.62 (37)	16.0
Average	.485			.532		.515		. 5.53		9.2

(a) R = Moderate Range Expanding

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F = Frequency Expanding

RF = Range and Frequency Expanding

R* = Extreme Range Expanding

APPENDIX

DETAIL OF EXPERIMENTAL PROCEDURE

I. Sample Choice Problem

Below you will find three brands of beer. You know only the price per six-pack and the average quality ratings made by subjects in a blind taste test. Given that you had to choose one brand to buy on this information along, which one would it be?

Brand	Price, Six-Fack	Average Quality Rating (100 = Best; 0 = Worst)
I	\$1.80	50
II	\$2.60	70
III	\$3.00	70

I would prefer Brand - (Check one only)

I_____ II_____ III_____

			Prod	uct Cat	egories						
Product			Dimens	ion 1			Dir	nensio	<u>n 2</u>		
Beer		P	rice/Si	Lx-Pack			Q	ualit	у		
Value:	\$3.40	3.00	2.60	2.20	1.80	30	40	50	60	70	
Level:	1	2	3	4	5	1	2	3	4	5	
Cars:		 (1	ide Qua 00 = Li 60 = Li	ility ike a Ro ike a Jo	olls; mep)		 G	as Mi	leage		•
Value	60	70	80	90	100	21	24	27	30	33 Mpg	
Level:	1	2	3	4	5	1	2	3	4	5	
Restaurants:		 C		Time			 F	 ood Qu	·	- /- -	•
Value:	45	35	25	15	5 1	fin 1	2	3	4	5 Star	:s
Level:	1	2	3	4	5	1	2	3	4	5	
• • • • • • • • •			• • • •		 .						•
Volues:	284	4.7 9			0/4	616	37	26	OI W	54	
Level:	1	2	5	4	3	318 1	2	5	4	3	
		 C		ing Time	·		 c	olor 1 (100 -	Fidel: Bes	 ity t)	
Value:	6	44	3	14	41	tin 89	91	9 3	95	97	
Level:	1	2	3	4	5	1	2	3	4	5	
TV Sets:		7 Dis	tortion	0=Best			(Avg.	Rel: Time	iabil: to Bi	Lty reakdown)	•
Value:	4.5	3.5	2.5	1.5	. 5%	2	3	4	5	6 Year	s
Level:	1	2	3	4	5	-1	2	3	4	5	

II. Attribute Values for Product Categories

			Lev	els o	f Eac	h Di	iensi	on by	Grou	1 9:	5 = Bl	EST	
Product		Gre	gue	1*	Ģ	TOUD	2	G	roup	3	G	roup	4
<u>Class</u>	Dimension	T	ċ	<u>D</u>	T	c	D	<u> </u>	Ċ	D	T	c	D
	Strategy		R			R			F			RF	
P	Dl	3	5	2 "	5	3	5	3	5	3	5	3	4
	D2	5	3	5	3	5	2	5	3	4	3	5	2
	Strategy		F			F			RF			R	
C	D1 ·	3	5	3	5	3	4	3	5	2	5	3	5
Lars	D2	D1 3 5 2 5 3 5 3 5 3 5 3 5 3 5 3 5 3 5 3 5 3 5 3 5 3 5 3 5 3 5 3 4 3 5 3 5 3 4 3 5 3 4 3 5 3 4 3 5 3 4 3 5 3 4 3 5 3 4 3 5 3 4 3 5 2 5 D1 3 5 2 5 3 4 3 5 2 5 3 4 3 5 2 5 D1 3 5 2 5 3 4 3 5 2 5 3	3	5	2								
	Strategy		RF			RF	•	• • •	R			F	
	Dl	3	5	2	5	3	4	3	5	2	5	3	4
Kestaurants	D2	5	3	4	3	5	2	5	3	5	<u>5 = BEST</u> Group <u>T C</u> RF 5 3 3 5 R 5 3 - 3 5 - 5 3 R 3 5 - 7 - 7 - 7 - 7 - 7 - 7 - 7 - 7	5	3
	Strategy		R*	• • •		F	·		R		•	R	
	١u	5	3	5	3	5	3	5	3	5	3	5	2
Lotteries	D2	3	5	1	5	· 3	4.	3	5	2	5	3	5
	Strategy		R			R*			F	• • •		F	
	D1	5	ذ	5	3	5	1	5	3	4	3	5	3
Film	D2	3	5	2	5	3	5	3	5	3	5	3	4
	Strategy		F			R			R*			R*	
	Dl	5	3	4	3	5	2	5	3	5	3	5	1
TV Sets	-D2	3	5	3	5	3	-5	3	5	1	5	3	5

III. Assignment of Stimulus (Decoy) to Groups

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*Read as follows: For the product class Beer, the attribute values for the <u>Target</u> were determined by selecting level 3 of Dimension 1 (Price) and level 5 of Dimension 2 (Quality the attribute values for the <u>Competitor</u> were determined by selecting level 5 of Dimension 1 and level 3 of Dimension 2; and the attribute values for the <u>Decov</u> were determined by selecting level 2 of Dimension 1 and level 5 of Dimension 2. Each level is as defined on the preceeding page. The Strategies are:

- R . Moderate Range Increasing
- R* = Extreme Range Increasing
- F . Frequency Increasing
- RF = Range and Frequency Increasing

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