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DETERMINATION OF THEORETICAL SAMPLING EFFICIENCIES FOR ASPIRATED PARTICULATE MATTER THROUGH THE DRES LARGE-VOLUME SAMPLER (U)

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by

Irene Miskew and Stanley B. Mellsen

PCN 13E10

April 1981

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NOTATION

C	particle concentration in the sample, g cm^{-3}
C_o	particle concentration in the free stream, g cm^{-3}
d	particle diameter, cm
D	distance from the inlet to the outlet cross section of the collection tube, cm
h	thickness of the collection tube wall at the outlet cross section, cm
L	length of coaxial boundary tube, cm
r	radial co-ordinate of particle position, cm
r_A	radius of coaxial boundary tube, cm
r_B	radius of collection tube at exit, cm
r_C	inlet radius of cone, cm
$r_{p,\infty}$	radial co-ordinate of particle position far upstream, cm
$r_{s,\infty}$	far upstream radius of the stream tube that impinges on the collection tube circumference, cm
t	time, seconds
u_r	radial component of local fluid velocity, cm sec^{-1}
u_z	axial component of local fluid velocity, cm sec^{-1}
U	fluid velocity in collection tube, cm sec^{-1}
U_A	fluid velocity at boundary tube entrance, cm sec^{-1}
U_B	fluid velocity at collection tube exit, cm sec^{-1}
U_C	fluid velocity at boundary tube exit, cm sec^{-1}
U_i	fluid velocity at inlet of sampler, cm sec^{-1}
v_r	radial component of local particle velocity, cm sec^{-1}
v_z	axial component of local particle velocity, cm sec^{-1}
z	axial co-ordinate (origin at collection tube inlet) of particle position, cm

NOTATION (Cont'd)

z_o — axial co-ordinate of particle far upstream, cm

μ — absolute viscosity of fluid, poise

ρ — fluid density, g cm^{-3}

σ — particle density, g cm^{-3}

ψ — stream function, cm^3 sec^{-1}

The following are dimensionless

C_D — drag coefficient for spheres

G(1), G(2), G(3) and G(4) — dependent variables solved for by numerical integration they represent $\bar{v}_{\bar{z}}$, $\bar{v}_{Y}$, $\bar{z}$ and $\bar{r}$ respectively

E_m — collection efficiency of sampling tube

H — thickness of collection tube wall, h/r_A

i,j — grid point co-ordinates in the radial and axial directions respectively

i_B, j_B — grid point co-ordinates of the edge of the collection tube inlet

j_o — axial grid point co-ordinate of a particle at the far upstream position

K — inertia parameter of particle

$\bar{r}$ — radial co-ordinate of particle, r/r_c

$\bar{r}_{p,\infty}$ — radial co-ordinate of particle position far upstream, $r_{p,\infty}/r_c$

$\bar{r}_{s,\infty}$ — far upstream radius of the stream tube that impinges on the collection tube circumference, $r_{s,\infty}/r_c$

R — radial co-ordinate used in calculating the stream function field, r/r_A

Re — spherical particle Reynolds number in flow in the proximity of the collection tube

NOTATION (Cont'd)

Re_o	spherical particle Reynolds number in free stream
$\bar{u}_{\bar{r}}$	radial component of local fluid velocity, $d\bar{u}/d\bar{r}$
$\bar{u}_{\bar{z}}$	axial component of local fluid velocity, $d\bar{u}/d\bar{z}$
$\bar{v}_{\bar{r}}$	radial component of local particle velocity, $d\bar{r}/d\tau$
$\bar{v}_{\bar{z}}$	axial component of local particle velocity, $d\bar{z}/d\tau$
$\bar{z}$	axial co-ordinate (origin at collection tube inlet) of particle, z/r_c
$\bar{z}_o$	axial co-ordinate of particle far upstream, z_o/r_c
Z	axial co-ordinate used in calculation of the stream function field, z/r_A
α	ratio of collection tube radius to boundary tube radius, r_B/r_A
β	length of coaxial boundary tube, L/r_A
γ	distance from the inlet to the outlet cross section of the collection tube, D/r_A
δ	distance from inlet of boundary to inlet of collection tube, $\beta - \gamma$
τ	time, tU_A/r_c
ϕ	dimensionless group independent of particle position, $Re_o{}^2/K$
Ψ	stream function, $\psi/\frac{1}{2}U_A r_A{}^2$

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DETERMINATION OF THEORETICAL SAMPLING EFFICIENCIES FOR ASPIRATED PARTICULATE MATTER THROUGH THE DRES LARGE-VOLUME SAMPLER (U)

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ABSTRACT

Sampling and collection efficiencies are calculated for a large-volume air sampler under conditions of anisokinetic as well as isokinetic flow. A mathematical model developed to evaluate a tapered-tube sampling probe was modified to obtain results for the large-volume sampler, using various particle sizes and flow velocities. These results should facilitate the prediction or correction of sampling errors in field and laboratory experiments.

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1. INTRODUCTION

In order to assess the effectiveness of a specific large-volume air sampler (cyclone scrubber), the instrument's ability to collect samples of finely-divided particulate matter must be determined. These samples can come from still or moving airstreams, and can vary both in particle size distribution and in concentration. The bio-sampler under evaluation consists of an air inlet cone and collection unit, and is designed to operate at a capacity of 1000 litres (air) per minute. (It is described fully in Suffield Technical Note No. 311).

Sampling from streams of suspended particulates is representative only if the size distribution and content of particles in the sample are identical to those of particles in ambient air at the point of sampling. The sampling system may give rise to three different types of error (Vitols, 1964) due to:

(1) particles failing to enter the sampling cone in representative concentrations;

(2) particles being deposited between the air inlet cone and the collection location; and

(3) particles being shattered, agregated or incompletely retained by collection devices.

When the velocity of gas entering the inlet cone is exactly the same as the far-upstream velocity of the gas ('isokinetic' sampling), particles will enter the sampler in representative concentrations. Otherwise, errors of the first type will occur as the result of anisokinetic sampling.

The purpose of this report is to describe the modification of a mathematical model devised formerly for calculating the error due to anisokineticity (Mellsen, 1979) of a sampling probe developed and used at DRES. The model, previously applied to a straight, tapered tube is herein adapted to the funnel-shaped inlet cone of a specific large-volume air sampler, and as such, calculates the sampling and collection efficiencies produced by varying upstream gas velocity and particle size.

2. DEFINITION OF THE PROBLEM

As explained in Suffield Technical Paper No. 499 (Mellsen, 1979), the problem of finding the sampling and collection efficiencies is one of determining the values of the upstream particle and fluid radii. The upstream particle radius, $r_{p,\infty}$, is defined as the radius of the limiting particle trajectory envelope which encompasses all particles (of any given diameter) entering the sampler. The upstream fluid radius, $r_{s,\infty}$, is the radius of the stream tube impinging on the outer circumference of the inlet cone, and containing the total volume of air passing through the sampler. The sampling efficiency, proportional to the areas of upstream particle envelope and fluid stream tube, can then be calculated:

$$\frac{C}{Co} = \left(\frac{r_{p,\infty}}{r_{s,\infty}}\right)^2 \qquad \text{(Eq. 1)}$$

where Co is the upstream particle concentration and C is the particle concentration in the sample; the collection efficiency is given by:

$$Em = \left(\frac{r_{p,\infty}}{r_c}\right)^2 \qquad \text{(Eq. 2)}$$

where r_c is the radius at the inlet of the cone.

Inertial and drag forces may cause particles flowing far upstream of the collection inlet to deviate from stream lines on arriving at the cone, where the fluid velocity may be changing markedly. Thus, in obtaining the true free stream concentration of particles and the sampling efficiency, the two different values of upstream particle radius and upstream fluid radius must be known. When the free stream velocity, U_A, is less than the sampler inlet velocity, U_i $\left(\text{i.e. } \frac{U_A}{U_i} < 1\right)$, some particles originally inside the limiting stream tube will pass outside the sampler, whereas for $\frac{U_A}{U_i} > 1$, some particles originally outside the stream tube will be drawn into the sampler.

3. DESCRIPTION OF THE SAMPLER

The part of the large-volume air sampler which determines stream function values and hence, affects sampling and collection, is the air inlet cone (Figure 1). With an inlet radius of 2 1/2 inches, the cone converges to a straight tube of inside radius 3/8 inch, through a funnel shaped by the intersection of three circular arcs. The entire inlet cone is 6 inches long, the converging section being 4 inches and the straight tube, therefore, 2 inches. The wall of the cone is 1/16 inch thick, but although this was taken into account in the calculation of the velocity U_c, the wall thickness was neglected in the computations leading to the array of stream function values. Since a grid unit in the array represents 1/8 inch, the cone wall thickness of 1/16 inch would have little effect on stream function values, but would make computing procedures unnecessarily complicated.

The three circular arcs defining the shape of the inlet cone are (numbers in inches):

$$(X_1 - 2\ 5/16)^2 + (Y_1 - 3\ 5/8)^2 = (2\tfrac{1}{2})^2 \quad \text{at inlet} \quad \text{(Eq. 3)}$$
$$(X_2 - 4\ 3/8)^2 + (Y_2 - 6\ 15/32)^2 = (6)^2 \quad \text{in middle} \quad \text{(Eq. 4)}$$

$$(X_3 - 4)^2 + (Y_3 - 3)^2 = (2\ 19/32)^2 \quad \text{just before straight tube} \qquad \text{(Eq. 5)}$$

Newton's Method was used to determine the two intersection points (between Equations 3 and 4, and Equations 4 and 5), with initial values for the iterative technique found by inspection of a drawing of the curve.

4. EQUATIONS OF MOTION

The equations of motion were established in a previous report (Mellsen, 1979), but are included here for completeness.

The motion of an individual particle has been shown (Vitols, 1964 and Batchelor, 1956) to be determined by the following ordinary differential equations:

$$\frac{d\bar{v}_r}{d\tau} = \frac{C_D Re(\bar{u}_r - \bar{v}_r)}{24K} \qquad \text{(Eq. 6)}$$

$$\frac{d\bar{v}_z}{d\tau} = \frac{C_D Re(\bar{u}_z - \bar{v}_z)}{24K} \qquad \text{(Eq. 7)}$$

where $$Re = Re_o[(\bar{u}_r - \bar{v}_r)^2 + (\bar{u}_z - \bar{v}_z)^2]^{1/2} \qquad \text{(Eq. 8)}$$

$$K = \frac{\rho\, d^2 U_A}{18 \mu r_c} \quad \text{particle inertia parameter} \qquad \text{(Eq. 9)}$$

$$Re_o = \frac{U_A d \rho}{\mu} \quad \text{free stream Reynolds number} \qquad \text{(Eq. 10)}$$

The symbols are defined in the notation section near the front of this report and the basic geometry of the flow system is illustrated in Figure 2.

Several assumptions are inherent in the use of Eqs. 6 and 7 for calculating the collection and sampling efficiencies due to a stream of particles, including:

(a) uniform particle distribution;

(b) no gravitational or electrostatic forces of consequence;

(c) monodisperse spherical particles with diameter very small in relation to the inlet diameter of the sampler; and

(d) free stream flow that is steady, incompressible and irrotational.

The drag coefficient is a function of Reynolds number and is available in the form of definitive empirical equations (Davies, 1945). These equations are stated as follows:

$$Re = \frac{C_D Re^2}{24} - 2.3363 \times 10^{-4} (C_D Re^2)^2 + 2.0154 \times 10^{-6} (C_D Re^2)^3 - 6.9105 \times 10^{-9} (C_D Re^2)^4 \qquad \text{(Eq. 11)}$$

for $Re < 4$ or $C_D Re^2 < 140$

$$\log_{10} Re = -1.29536 + 9.86 \times 10^{-1} (\log_{10} C_D Re^2) - 4.6677 \times 10^{-2} (\log_{10} C_D Re^2)^2 + 1.1235 \times 10^{-3} (\log_{10} C_D Re^2)^3 \qquad \text{(Eq. 12)}$$

for $3 < Re < 10^4$ or $C_D Re^2 < 4.5 \times 10^7$

5. AIR FLOW FIELD EQUATIONS

These equations were stated and explained in an earlier report (Mellsen, 1979), but are again shown for the sake of thoroughness.

The equations of fluid velocity were derived from the stream function for ideal flow over and through the sampler. To solve the problem, an outer boundary was used around the collection cone in the form of a coaxial tube of radius r_A (Figure 3), which was chosen large enough so that the effect of the boundary tube on flow in the proximity of the sampler is negligible. The collection cone was inserted a distance D into the downstream end of the boundary tube. Since the flow is axisymmetric only a radial plane containing both tubes had to be considered.

The fluid enters the boundary tube with steady velocity U_A, and separates into a central stream with velocity U_B at the exit and U_I at the entrance of the sampler, and an annular stream, with velocity U_C, at the downstream end of the boundary tube. The axial velocities U_A, U_B, U_C and U_i are uniform. Also, there is no radial flow at the end cross sections.

The boundary conditions on the flow can now be completely specified so that the flow field can be obtained by solution of the equation of the stream function.

The axially symmetric stream function $\psi(r,a)$ (Batchelor, 1967) satisfies:

$$\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} = 0 \qquad \text{(Eq. 13)}$$

The two velocity components (Figure 2) are given by:

$$u_z = \frac{1}{r}\frac{\partial\psi}{\partial r} \qquad \text{(Eq. 14)}$$

$$u_r = -\frac{1}{r}\frac{\partial\psi}{\partial z} \qquad \text{(Eq. 15)}$$

When U_A and U_B are specified, continuity gives U_C as follows:

$$U_C = \frac{U_A - \alpha^2 U_B}{1 - \left[\left(\frac{r_B + h}{r_B}\right)\alpha\right]^2} \qquad \text{(Eq. 16)}$$

where $\alpha = \dfrac{r_B}{r_A}$ (Eq. 17)

and h is the thickness of the collection tube wall.

For uniform velocity profiles, the stream function is of the form:

$$\psi = \tfrac{1}{2}ur^2 \qquad \text{(Eq. 18)}$$

To allow for greater generality, the stream function and the geometric variables are restated in the following dimensionless form:

$$\psi = \frac{\Psi}{\frac{1}{2}U_A r_A{}^2} \qquad \text{(Eq. 19)}$$

$$R = \frac{r}{r_A} \qquad \text{(Eq. 20)}$$

$$Z = \frac{z}{r_A} \qquad \text{(Eq. 21)}$$

$$\beta = \frac{L}{r_A} \qquad \text{(Eq. 22)}$$

$$\gamma = \frac{D}{r_A} \qquad \text{(Eq. 23)}$$

$$H = \frac{h}{r_A} \qquad \text{(Eq. 24)}$$

The boundary values for the stream function and the geometric configuration in terms of the dimensionless variables are shown in Figure 4.

The axially symmetric stream function equation (Figure 13) becomes:

$$\frac{\partial^2\Psi}{\partial R^2} - \frac{1}{R}\frac{\partial\Psi}{\partial R} + \frac{\partial^2\Psi}{\partial Z^2} = 0 \qquad \text{(Eq. 25)}$$

6. DISCRETIZATION SCHEME FOR THE AIR FLOW FIELD

The equation for the axially symmetric stream function (Eq. 25) is discretized as follows:

$$\frac{\Psi_{i-1,j} - 2\Psi_{i,j} + \Psi_{i+1,j}}{\Delta R^2} - \frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{2i\Delta R^2} +$$

$$\frac{\Psi_{i,j-1} - 2\Psi_{i,j} + \Psi_{i,j+1}}{\Delta Z^2} = 0 \qquad \text{(Eq. 26)}$$

where i and j are the grid point numbers in the R and Z directions respectively (Figure 5). Eq. 26 can be rearranged to give a simple equation by choosing a square grid so that ΔR and ΔZ are equal. The resulting equation, which is suitable for Gauss-Seidel iteration

(Carnahan et al, 1969), is given as follows:

$$\Psi_{i,j} = \frac{\Psi_{i-1,j} + \Psi_{i+1,j} + \Psi_{i,j-1} + \Psi_{i,j+1}}{4} - \frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{8i} \qquad \text{(Eq. 27)}$$

Eq. 27 can be applied to all interior points, which are defined as points for which the nearest boundary is at least one grid unit away.

In dealing with points on or surrounding the boundary described by the sampling cone (for which the nearest boundary in either the horizontal or vertical direction is less than one grid square away), a Taylor series expansion was used (Carnahan et al, 1969) and the following finite difference equations derived. (The first applies to points below or to the left of the curved boundary, and the second, to points above or to the right of the boundary.)

$$\Psi_{i,j} = \frac{ab}{a+b}\left[\frac{\Psi_{i,j-1}}{b+1} + \frac{\Psi_{i-1,j}}{a+1} + \frac{\Psi_V}{a(a+1)} + \frac{\Psi_H}{b(b+1)} - \frac{\Psi_V - \Psi_{i-1,j}}{2i(a+1)}\right] \qquad \text{(Eq. 28)}$$

$$\Psi_{i,j} = \frac{ab}{a+b}\left[\frac{\Psi_{i,j+1}}{b+1} + \frac{\Psi_{i+1,j}}{a+1} + \frac{\Psi_V}{a(a+1)} + \frac{\Psi_H}{b(b+1)} - \frac{\Psi_{i+1,j} - \Psi_V}{2i(a+1)}\right] \qquad \text{(Eq. 29)}$$

where a is the vertical distance ($0 < a \leq 1$) to Ψ_V and Ψ_V represents (for points below the curve) either the boundary Ψ-value (if the boundary lies between $\Psi_{i,j}$ and $\Psi_{i+1,j}$) or the adjacent Ψ-value ($\Psi_{i+1,j}$). (For points above the curve, Ψ_V takes either the boundary value or the value of $\Psi_{i-1,j}$.) In the horizontal direction, b is similarly defined as the distance ($0 < b \leq 1$) to Ψ_H, and Ψ_H is the closer of the two Ψ-values, the boundary value and the adjacent value ($\Psi_{i,j+1}$ for points below the curve and $\Psi_{i,j-1}$ for points above it).

The grid size was chosen from past experience (Mellsen, 1979) so that each grid unit (both horizontally and vertically) represents 1/8 inch. Transferred to the grid (Figure 5), the straight tube radius, r_B, then corresponds to 3 units, the inlet radius, r_C, is 20 units, the boundary tube radius, r_A, is 120 units (to be located a distance of five times the inlet radius outward from the edge of the cone), the complete length of the inlet cone, γ, is 48 units, and the distance to the upstream end of the boundary tube, δ, is 152 units (so as to be more than seven inlet radii upstream of the collection inlet). Specifying the boundary tube radius and the distance to the upstream boundary in this way ensures that the behaviour of the flow be as if the inlet cone were situated in free space and the particles coming from such a distance upstream as not to be affected by the cone.

The stream function was obtained by Gauss-Seidel iteration using Equation 27, 28 and 29. The boundary conditions were set initially at the centerline, at the boundary tube wall and inlet, and at the outlet, and held constant throughout the iterative procedure. Any point not falling on either one of these boundaries or the wall of the inlet cone was initialized to zero. A Fortran program (listed in Appendix A) was written to perform the calculations on an IBM 370 computer.

A special routine (adapted from Carnahan et al, 1969) to handle points near the curved wall of the inlet cone had to be incorporated into the Fortran program. This routine first labels points as being one of four types (see Figure 6) by finding the highest point, JMAX (the maximum within the boundary), for each row, I, and classifying points according to the horizontal and vertical distances to the curve (B and A, respectively):

$$B = AK - \sqrt{RK^2 - ((I-1) - BK)^2} - (J-1) \qquad \text{(Eq. 30)}$$

$$A = BK - \sqrt{RK^2 - ((J-1) - AK)^2} - (I-1) \qquad \text{(Eq. 31)}$$

where I and J are the coordinates of the point, and AK, BK and RK assume the values of the α, β and r in the equation,

$$(X - \alpha)^2 + (Y - \beta)^2 = r^2 \qquad \text{(Eq. 32)}$$

from the particular circular arc (Eqs. 3, 4 and 5) defining the curve at that point. The distance, A, is then found for every point in each row, starting at JMAX and decreasing along the row until an interior point is reached, and the procedure is repeated on the right side of the curve, using JMIN(I) (the minimum above the boundary, neglecting the wall thickness of the cone):

$$JMIN(I) = JMAX(I) + 1 \qquad \text{(Eq. 33)}$$

and continuing until the upper interior point is reached. The horizontal and vertical distances are now defined by:

$$BO = 1 - B \qquad \text{(Eq. 34)}$$

$$AO = (I-1) - [BK - \sqrt{RK^2 - (J-AK)^2}] \qquad \text{(Eq. 35)}$$

The coefficients of the Ψ-values in Equations 28 and 29 are then calculated using A and B (for Eq. 28) or AO and BO (as a and b in Eq. 29).

Type IV points are assigned the boundary value and held fixed through the program. For the other types (I, II and III), the values of Ψ_V and Ψ_H can then be determined and the iteration performed according to Eq. 28 (for points below the curve) or Eq. 29 (for points above the curve). For example, for a Type II point, $\Psi_{i,j}$, below the curve, Ψ_V would assume the value of the boundary and Ψ_H, the value of Ψ_{i+j}, while if $\Psi_{i,j}$ were above the curve, Ψ_V would again assume the boundary value, but Ψ_H would become $\Psi_{i,j-1}$.

7. SOLUTION OF THE EQUATIONS OF MOTION

From Section 2 of this report, as in a prior paper (Mellsen, 1979), the problem is to find the upstream particle and fluid radii, $r_{p,\infty}$ and $r_{s,\infty}$, respectively, in order to calculate the sampling and collection efficiencies. In the same dimensionless form of Equations 6 and 7, the value of $\bar{r}_{p,\infty}$ (notation) was found by an iterative procedure called the half interval method (Carnahan et al, 1969). The value of $\bar{r}_{p,\infty}$ for a critical particle was estimated far upstream, the path followed to the plane of the cone inlet and the miss distance (from the edge of the inlet) calculated. Next, the aforementioned half interval method was applied to determine a better initial estimate, the path again followed to the plane of the inlet, and another miss distance calculated. This was repeated several times until sufficient accuracy was achieved. The initial upstream position in a plane perpendicular to the flow direction was located far enough from the inlet so that free stream conditions would prevail. A distance of seven inlet radii upstream of the inlet was considered adequate on the basis of the five inlet radii serving the case of straight tube sampling (Batchelor, 1956).

The path of an individual particle was determined step-by-step by applying a fourth order Runge-Kutta method (Carnahan et al, 1969) to the equations of motion (Eqs. 6 and 7). The values of Re and K in these equations were easily found for each new step by direct substitution of previously determined values into Eqs. 8, 9 and 10, but the value of $C_D Re$ in Eqs. 6 and 7 had to be calculated in each step by numerical solution of the definitive empirical equations (Eqs. 11 and 12). This was done using Newton's method (Carnahan et al, 1969) for finding the zero of a function. The values of $\bar{u}_r$ and $\bar{u}_z$ were calculated in each step from the stream function field as follows:

$$\bar{u}_r = \frac{\Psi_{i,j-1} - \Psi_{i,j+1}}{4(i-1)(\Delta R)^2} \qquad \text{(Eq. 36)}$$

$$\bar{u}_z = \frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{4(i-1)(\Delta R)^2} \qquad \text{(Eq. 37)}$$

where i and j define the grid point of the particle position. Since the inlet radius of the sampler was chosen to be 20 grid units, these are given by:

$$i = 1 + 20\bar{r} \qquad \text{(Eq. 38)}$$

$$j = j_o + 20(\bar{z} - \bar{z}_o) \qquad \text{(Eq. 39)}$$

where j_o and z_o are the starting point values of j and $\bar{z}$. The values of i and j obtained from Eqs. 38 and 39 were rounded off to the nearest lower integer value in each calculation. The value of $\bar{r}_{s,\infty}$ was obtained directly from the stream function by:

$$\bar{r}_{s,\infty} = \frac{\Delta R(i-2)r_A}{r_C}\sqrt{\frac{\Psi_{i_C,j_C}}{\Psi_{i-1,j_o}}} \qquad \text{(Eq. 40)}$$

calculated at the lowest value of i satisfying:

$$\Psi_{i,j_o} > \Psi_{i_C,j_C} \qquad \text{(Eq. 41)}$$

where i_C and j_C define the grid point at the edge of the collection cone inlet. The calculations to obtain the solutions were done with an

IBM 370 Computer by means of a Fortran program, the listing of which is shown in Appendix B. The sampling and collection efficiencies given by Eqs. 1 and 2 were also obtained by this program after the values of $\bar{r}_{p,\infty}$ and $\bar{r}_{s,\infty}$ had been calculated.

8. RESULTS

Method of Analysis

A stream function array was computed for each of the following ratios of $\frac{U_B}{U_A}$: $\frac{400}{1}$, $\frac{400}{3}$, $\frac{400}{9}$, $\frac{400}{27}$ and $\frac{400}{54}$. Because of the funnel shape of the sampler, tapering from an inlet radius of 2½ inches to a straight-tube radius of 3/8 inch, a velocity of U_B = 400 implies an inlet velocity of 9. This means that the sampling velocity ratios $\left(\frac{U_A}{U_i}\right)$ are 1/9, 1/3, 1, 3 and 6. When the sampler operates at its design capacity of 1000 ℓ/min, the values of U_B and U_i then become U_B = 5847.482 cm/s and U_i = 131.5683 cm/s, so that U_A varies from 14.62 cm/s $\left(\frac{U_A}{U_i} = \frac{1}{9}\right)$ to 789.4 cm/s $\left(\frac{U_A}{U_i} = 6\right)$.

A broad range of particle sizes, of diameters, 6, 10, 20, 50, 100, 200 and 500 microns, composing monodisperse fields, was analyzed for each stream function array. Results were tabulated and plotted in graphs of sampling efficiency versus inertia parameter (Figure 7; $\frac{C}{Co}$ vs log K), collection efficiency versus inertia parameter (Figure 8; Em vs log K), and sampling efficiency versus sampling velocity ratio (Figure 9; $\frac{C}{Co}$ vs $\frac{U_A}{U_i}$).

Discussion

The validity of the mathematical model has been discussed and reported (Mellsen, 1979). Results of the present work (Table of Results) show the sampling efficiency of a uniform field of 20 micron spherical particles, in a wind that is six times the sampling inlet velocity, to be in error by over 30% ($\frac{C}{Co}$ = 1.306). Since smaller particles are carried more readily with the air stream, the sampling of small particles gives rise to smaller error. The exact errors for very small particles cannot be determined by this model because computing errors increase with decreasing particle size (Mellsen, 1979), the reason being that as particle size decreases, a larger number of calculations is required.

The case of isokinetic sampling, where free stream velocity matches inlet velocity $\left(\frac{U_A}{U_1} = 1\right)$, should be characterized by both sampling and collection efficiencies equal to 1 $\left(\frac{C}{Co} = 1,\ Em = 1\right)$. This is displayed quite well by the predicted values of the model. For example (Table of Results), a 6 micron-particle field indicates an error of only 1.7% $\left(\frac{C}{Co} = 1.017\right)$, and a 100 micron-particle field, of 2.5% $\left(\frac{C}{Co} = .9748\right)$.

Although the model cannot be used for the prediction of efficiencies in completely still air, very low free stream velocities can be handled. The lowest free stream velocity currently tested and plotted is 14.62 cm/s, but if desired, lower velocities might be tried. The effect of varying free stream velocity while keeping the sampling velocity constant is clearly illustrated in Figure 9 for selected particle sizes.

9. CONCLUSIONS

The effect of anisokineticity on sampling with the DRES large-volume air sampler is sufficient to produce significant errors in sampling and collection efficiencies. A mathematical model, formerly applied to a straight, tapered-tube sampling probe, was modified to be applied to the specific large-volume sampler developed at DRES. The results from the model can serve to predict the magnitude of sampling errors. Several free stream velocities for a fixed sampling rate were evaluated with a number of monodisperse fields of suspended particles. Therefore, if the wind velocity and particle size and density are known, the results of this model can be used in correcting measured samples.

10. REFERENCES

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NOTES

1. CONTOUR DIMENSIONS ARE APPROXIMATE. TRANSITION AND POINTS OF TANGENCY SHOULD BE SMOOTH.

2. CONE TO HAVE 1/16" THICK WALL.

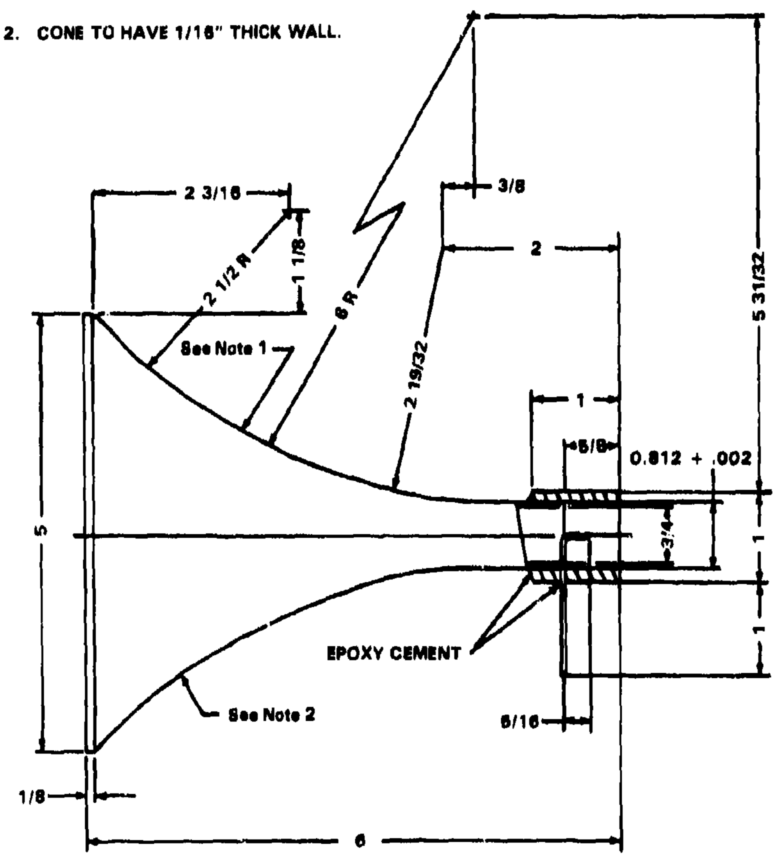

FIGURE 1: DESIGN DRAWING OF LARGE VOLUME AIR SAMPLER

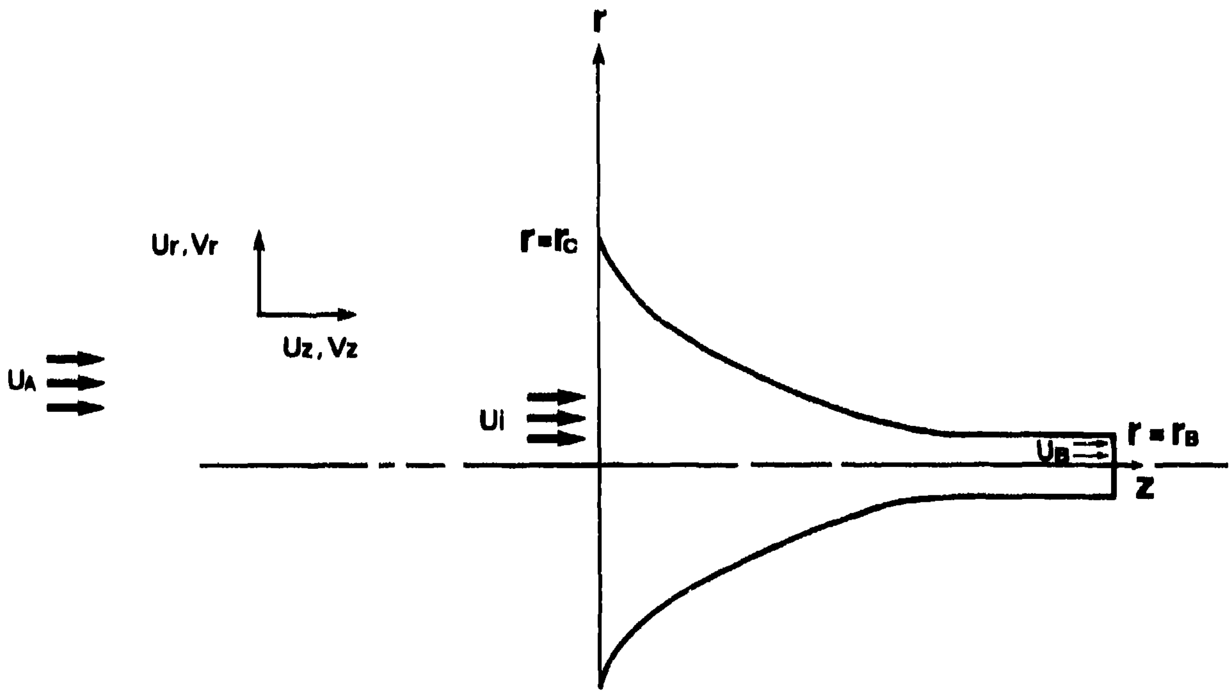

FIGURE 2: CO-ORDINATE SYSTEM FOR AXIAL FLOW IN THE PROXIMITY OF THE COLLECTION CONE

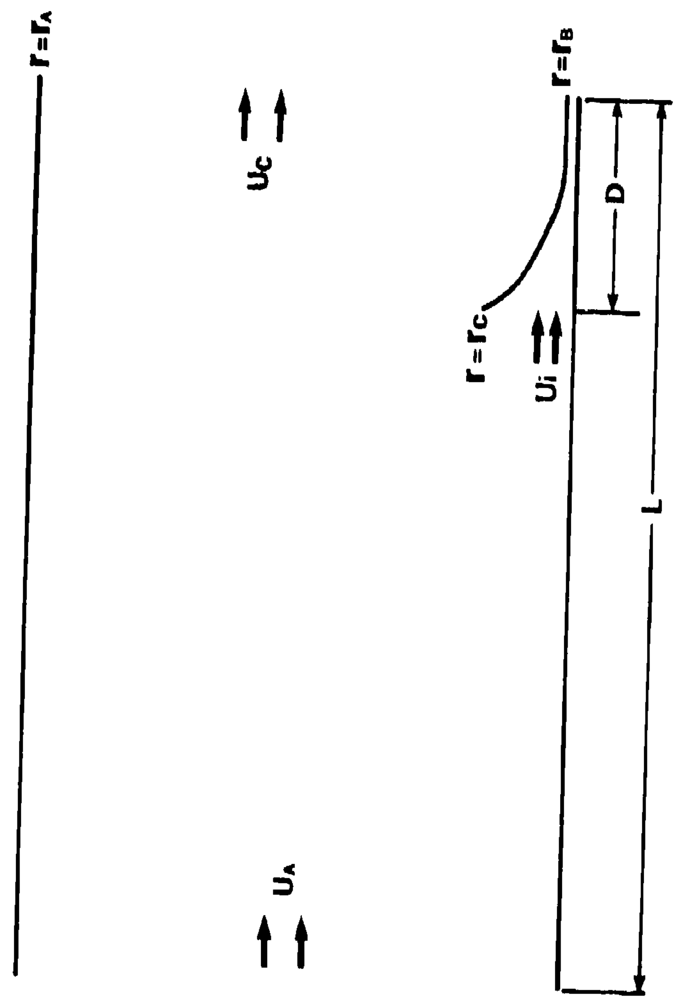

FIGURE 3: GEOMETRIC MODEL FOR FLOW FIELD IN THE PROXIMITY OF THE COLLECTION CONE

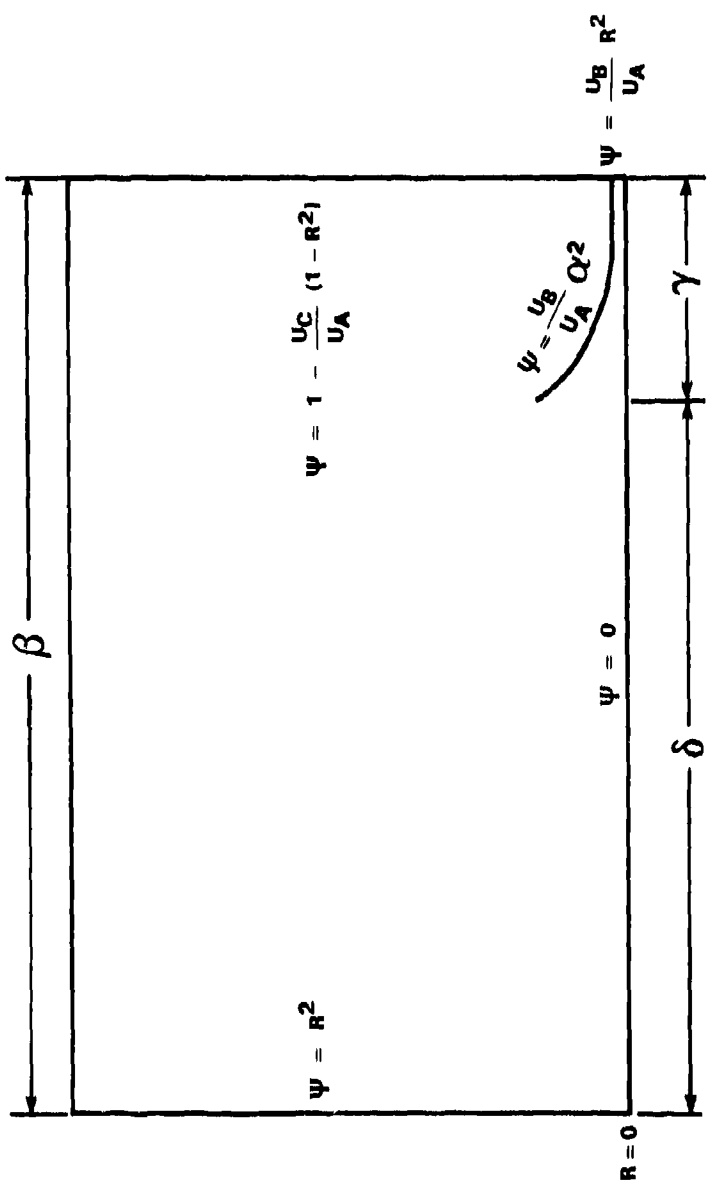

FIGURE 4: STREAM FUNCTION BOUNDARY CONDITIONS

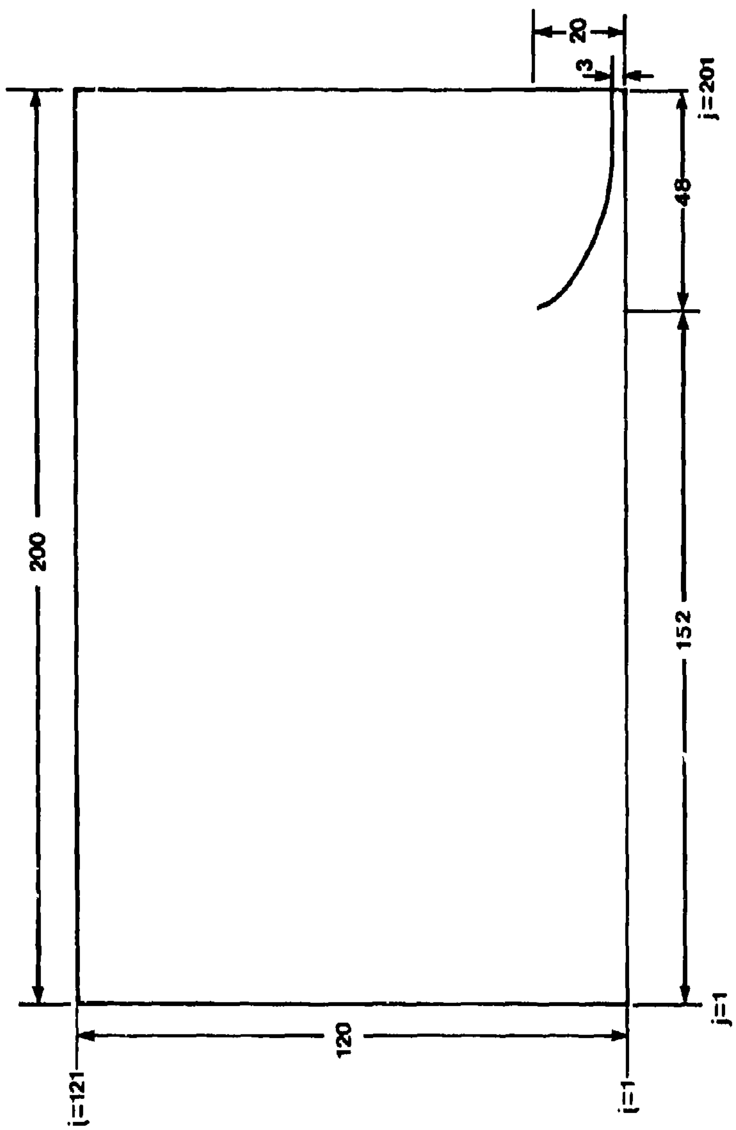

FIGURE 5: DIMENSIONS OF DISCRETIZATION GRID FOR AIR FLOW FIELD

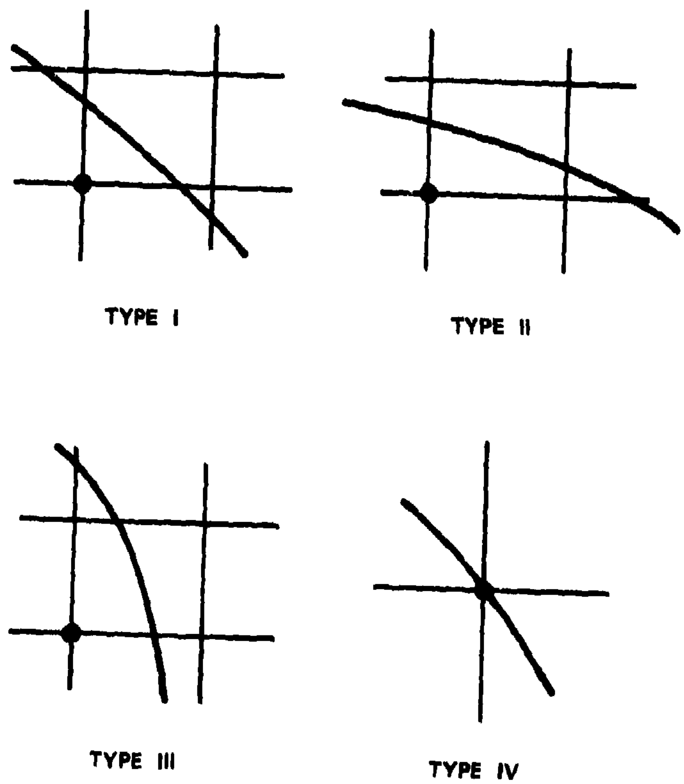

FIGURE 6: TYPES OF BOUNDARY POINTS

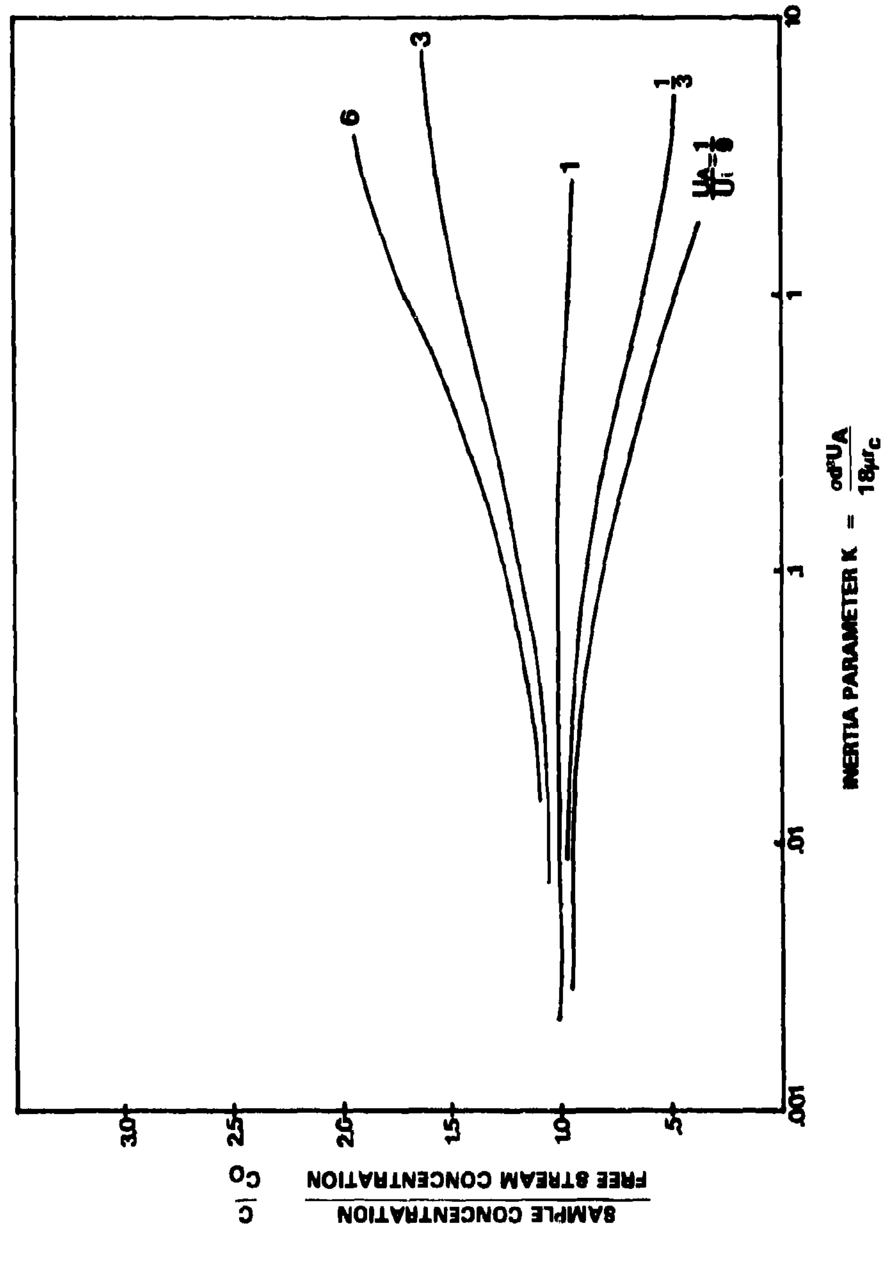

FIGURE 7: EFFECT OF VELOCITY RATIO ON SAMPLING EFFICIENCY

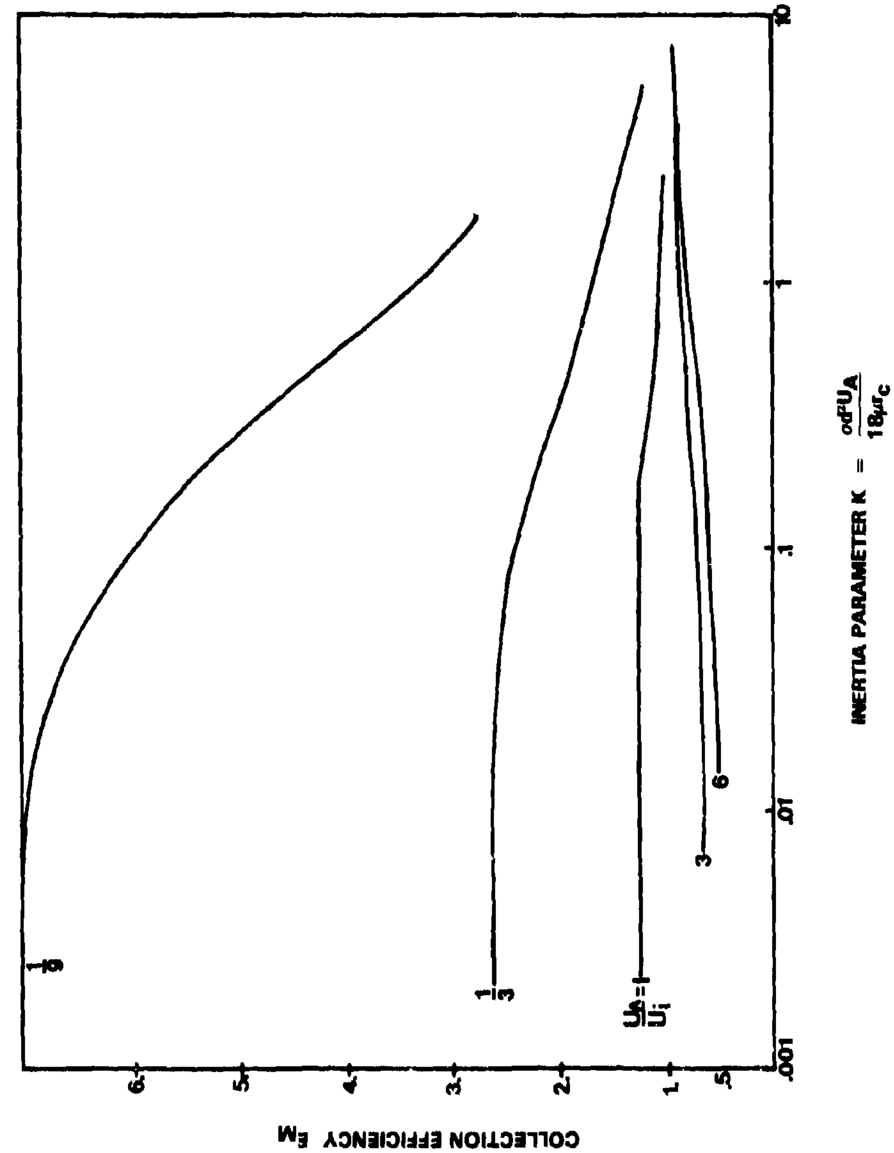

FIGURE 8: EFFECT OF VELOCITY RATIO ON COLLECTION EFFICIENCY

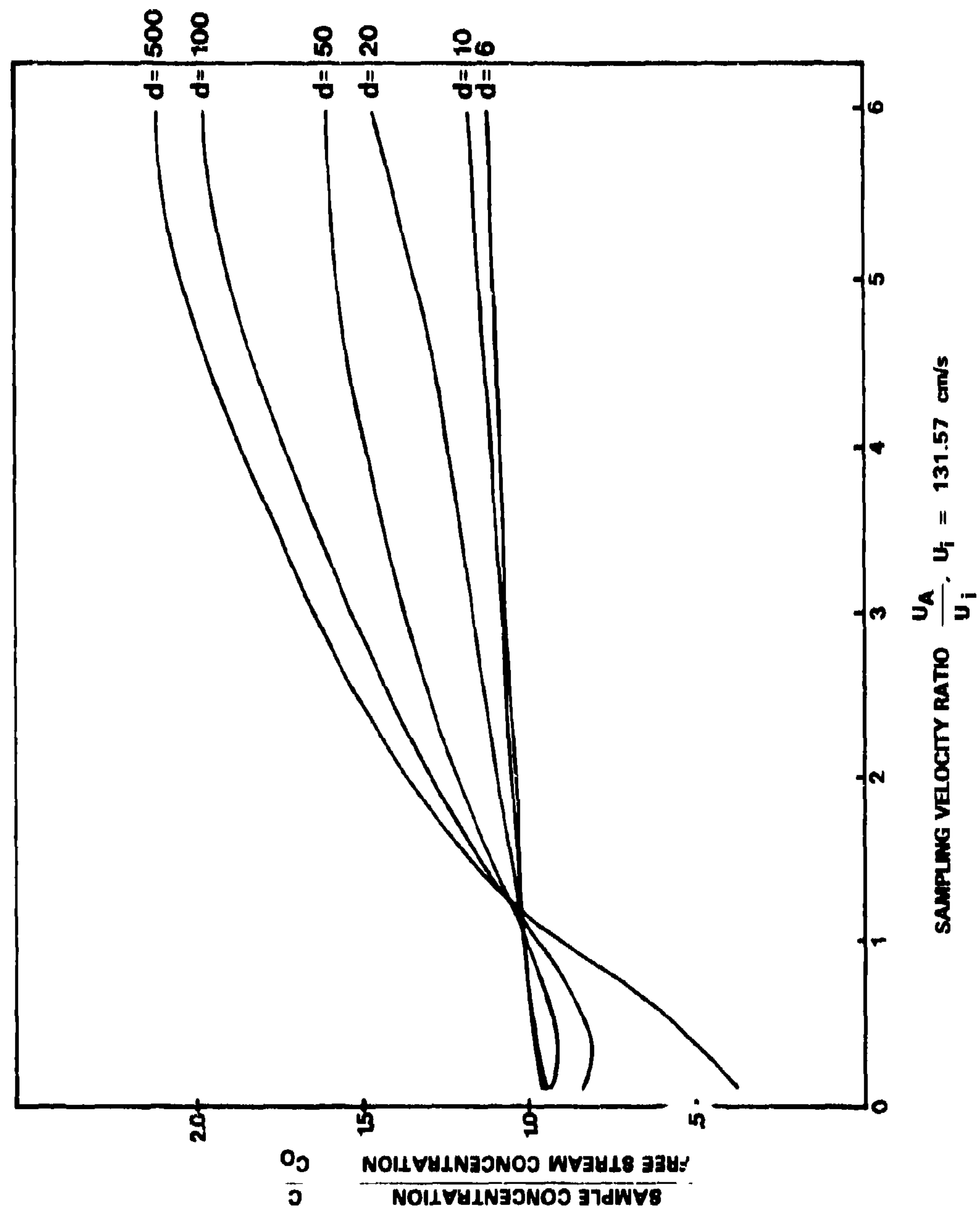

FIGURE 9: EFFECT OF FREE STREAM VELOCITY ON SAMPLING EFFICIENCY

TABLE I RESULTS

$\frac{U_B}{U_A}$	$\frac{U_A}{U_1}$	U_A (cm/s)	d (cm)	K	$\frac{C}{C_0}$	Em
$\frac{400}{1}$	$\frac{1}{9}$	14.62	.05	1.787	.3754	2.749
			.02	.2860	.6708	4.912
			.01	.07149	.8487	6.215
			.005	.01787	.9358	6.853
			.002	.002860	.9550	6.993
			.001	.0007149	.9592	7.024
			.0006	--	--	--
$\frac{400}{3}$	$\frac{1}{3}$	43.86	.05	5.362	.4645	1.243
			.02	.8579	.6418	1.717
			.01	.2145	.8014	2.144
			.005	.05362	.9223	2.486
			.002	.008579	.9681	2.590
			.001	.002145	.9773	2.615
			.0006	.0007721	.9804	2.623
$\frac{400}{9}$	1	131.6	.05	16.09	.8950	1.004
			.02	2.574	.9286	1.042
			.01	.6434	.9748	1.094
			.005	.1609	1.007	1.130
			.002	.02574	1.015	1.139
			.001	.006434	1.016	1.140
			.0006	.002316	1.017	1.141
$\frac{400}{27}$	3	394.7	.05	48.26	1.628	.9822
			.02	7.721	1.595	.9624
			.01	1.930	1.516	.9142
			.005	.4826	1.356	.8182
			.002	.07721	1.161	.7005
			.001	.01930	1.072	.6466
			.0006	.006949	1.063	.6413
$\frac{400}{54}$	6	789.4	.05	96.51	2.071	.9805
			.02	15.44	2.029	.9608
			.01	3.861	1.930	.9137
			.005	.9651	1.699	.8045
			.002	.1544	1.306	.6185
			.001	.03861	1.154	.5464
			.0006	.01390	1.102	.5217

U_B = 5847.482 cm/s, U_1 = 131.5683 cm/s

APPENDIX A

COMPUTER PROGRAM FOR CALCULATING THE STREAM FUNCTION

```
//IRENE2 JOB (0162,101911,CHOW),IRENE,NOTIFY=0357021,MSGCLASS=N,
// MSGLEVEL=1,CLASS=K,TIME=(0,29)
//*MAIN LINES=20,ORG=RM020
// EXEC FORTXCL,PARM.FORT='OPTIMIZE(2)'
//SYSPRINT DD SYSOUT=N
//SYSTERM DD SYSOUT=N
//FORT.SYSIN DD *
C
      COMMON PSI(121,201),ITERS,URAT,UA,RB,NPRB,NPZB,UB,RA,NPZ,NPR
      COMMON INTVL,INDEX,ITER,RC,NRC,NPRC,NPN
      READ(5,*) ITER,INTVL
      IF(ITER)1,1,2
    1 INDEX=-1
      CALL SBM24
      REWIND 8
      WRITE(8)PSI,ITERS,URAT,UA,RB,NPRB,NPZB,UB,RA,NPZ,NPR,RC,NPRC,NPN
      GO TO 3
    2 INDEX =1
      CALL REPSI
      CALL SBM24
      REWIND 8
      WRITE(8)PSI,ITERS,URAT,UA,RB,NPRB,NPZB,UB,RA,NPZ,NPR,RC,NPRC,NPN
    3 CONTINUE
      STOP
      END
C
      SUBROUTINE REPSI
      COMMON PSI(121,201),ITERS,URAT,UA,RB,NPRB,NPZB,UB,RA,NPZ,NPR
      COMMON INTVL,INDEX,ITER,RC,NRC,NPRC,NPN
      REWIND 8
      READ(8)PSI,ITERS,URAT,UA,RB,NPRB,NPZB,UB,RA,NPZ,NPR,RC,NPRC,NPN
      WRITE(6,200)
      WRITE(6,201)RA,RB,NPZ,NPR,NPZB,NPRB,URAT,UA,UB,ITERS,RC,NPRC,NPN
      WRITE(6,202)
      DO 24 J=1,NPZ,INTVL
   24 WRITE(6,203)(PSI(I,J),I=1,NPR,INTVL)
  200 FORMAT(20H1 DISK STORAGE CHECK)
  201 FORMAT(65H0STREAM FUNCTION FOR FLOW IN TWO CONCENTRIC PIPES WITH P
     1ARAMETERS/ 10H0RA      = ,F7.2/ 10H RB      = ,F7.2/ 10H NPZ     =,
     2I5 / 10H NPR     = ,I5/ 10H NPZB    = ,I5/ 10H NPRB    = ,I5 /
     310H URAT    = ,F9.4/10H UA      = ,F9.4/10H UB      = ,F9.4 /
     410H ITERS   = ,I5/10H RC      = ,F7.2/10H NPRC    = ,I5/10H NPN     =
     5,I5)
  202 FORMAT(45H1THE CURRENT VALUES OF PSI STORED ON DISK ARE)
  203 FORMAT('0',16F7.4)
      RETURN
      END
C
C     THIS SUBROUTINE CALCULATES THE STREAM FUNCTION FOR FLOW THROUGH
C     TWO CONCENTRIC PIPES WITH A FUNNEL-SHAPED INSIDE PIPE
C
      SUBROUTINE SBM24
      DIMENSION JR(20),JMAX(20),ITYPE(20,32),C(20,32),D(20,32),
     1E(20,32),F(20,32),G(20,32),H(20,32),JRO(20)
      COMMON PSI(121,201),ITERS,URAT,UA,RB,NPRB,NPZB,UB,RA,NPZ,NPR
      COMMON INTVL,INDEX,ITER,RC,NRC,NPRC,NPN
C
C     READ AND CHECK INPUT PARAMETERS
C
      READ(5,*)NZ,NR,ITMAX,EPSMX
      READ(5,*)RA,RB,XL,DL,UA,UB,RC,NRC,NPN
      WRITE(6,200)NZ,NR,ITMAX,EPSMX,RA,RB,XL,DL,UA,UB,INTVL,ITER,RC
C
C     CALCULATE AND WRITE DIMENSIONLESS PARAMETERS
C
      ALPHA=RB/RA
      BETA=XL/RA
      GAMMA=DL/RA
      DELTA =  BETA-GAMMA
      UC=(UA-ALPHA**2*UB)/(1.-(((RB+0.5)/RB)*ALPHA)**2)
      URAT = UB/UA
      UCRA=UC/UA
      PSIB=URAT*ALPHA**2
      WRITE(6,201)ALPHA,BETA,GAMMA,DELTA,URAT,UCRA
C
C     R=RD/RA,  Z=ZD/RA,  PSI=PSID/((1.0/2.0)*UA*RA**2)
C
C
C
```

```
      ESTABLISH BOUNDARY POINTS

      NPZ=NZ+1
      NPR=NR+1
      DELR=1.0/FLOAT(NR)
      XZB=((XL-DL)/XL)*FLOAT(NZ)
      NZB=IFIX(XZB + 0.1)
      NPZB=NZB+1
      XRB=ALPHA*FLOAT(NR)
      NRB=IFIX(XRB + 0.1)
      NPRB=NRB+1
      NPRB1=NPRB+1
      NPRC=NRC+1
C
C     IF PSI IS PARTIALLY CALCULATED AND IN FILE
C     GO DIRECTLY TO FURTHER ITERATIONS
C
      IF(INDEX)1,1,7
    1 CONTINUE
C
C     ESTABLISH INITIAL GUESSES FOR STREAM FUNCTION
C     AND SET BOUNDARY CONDITIONS ON CENTRE LINE
C     AND INLET OF OUTSIDE PIPE
C
      DO 2 I=1,NR
      RI=FLOAT(I-1)
      PSI(I,1)=(RI*DELR)**2
      DO 2 J=2,NPZ
    2 PSI(I,J)=0.0
C
C     SET BOUNDARY CONDITION AT OUTLET OF INSIDE PIPE
C
      DO 3 I = 2,NPRB
      RI=FLOAT(I-1)
    3 PSI(I,NPZ) = URAT*(RI*DELR)**2
C
C     SET BOUNDARY CONDITION AT OUTLET OF OUTSIDE PIPE
C
      DO 4 I = NPRB1,NR
      RI=FLOAT(I-1)
    4 PSI(I,NPZ)=1.0-UCRA*(1.0-(RI*DELR)**2)
C
C     SET BOUNDARY CONDITION AT NECK OF INSIDE PIPE
C
      DO 5 J=NPN,NPZ
    5 PSI(NPRB,J)=PSIB
C
C     SET BOUNDARY CONDITION AT WALL OF OUTSIDE PIPE
C
      DO 6 J=1,NPZ
    6 PSI(NPR,J)=1.0
C
C     SET BOUNDARY CONDITION AT FUNNEL WALL
C
    7 M=NPRC-NPRB
      N=NPN-NPZB-2
      READ(5,*) A1,B1,R1,A2,B2,R2,A3,B3,R3
      X23=23.
      CALL ARCEPT(X23,Y23,R2,R3,B2,B3,A2,A3)
      X12=9.
      CALL ARCEPT(X12,Y12,R1,R2,B1,B2,A1,A2)
      CALL BNDPTS(M,N,JR,JMAX,ITYPE,C,D,E,F,G,H,Y12,Y23,JRO,A1,B1,R1,A2,
     1B2,R2,A3,B3,R3)
      DO 63 I=NPRB,NPRC
      IM=I-NRB
      JN=JMAX(IM)
      J=JN+NPZB
      IF (ITYPE(IM,JN)-4) 63,62,62
   62 PSI(I,J)=PSIB
   63 CONTINUE
C
C     COMPUTE SUCCESSIVELY BETTER APPROXIMATIONS FOR
C     THE STREAM FUNCTION AT ALL GRID POINTS, ITERATING BY
C     THE GAUSS-SEIDEL METHOD UNTIL THE CONVERGENCE CRITERION
C     IS SATISFIED
C
    8 EPS=0.0
      ITER=ITER+1
      ITERS = ITER
      DO 70 I=2,NR
      IM=I-NRB
      DO 70 J=2,NZ
      JN=J-NPZB
```

```
C
      IF(I-NPRB)17,16,14
14    IF(I-NPRC) 15,15,17
15    IF(JN-JR(IM)) 17,17,19
16    IF(JN-JR(IM)) 17,17,18
17    HOLDT=PSI(I,J)
      PSI(I,J)=(PSI(I,J+1)+PSI(I,J-1)+PSI(I+1,J)+PSI(I-1,J))/4.0
     1-(PSI(I+1,J)-PSI(I-1,J))/(8.0*FLOAT(I-1))
      GO TO 69
C
18    IF(J-NPN) 19,70,70
19    IF(JN-JRD(IM)) 20,17,17
20    IF(JN-JMAX(IM)) 23,21,29
21    IF(ITYPE(IM,JN)-4) 23,70,70
23    IF(ITYPE(IM,JN)-3) 25,24,24
24    PSIV=PSI(I+1,J)
      PSIH=PSIB
      GOTO 28
25    IF(ITYPE(IM,JN)-2) 27,26,26
26    PSIV=PSIB
      PSIH=PSI(I,J+1)
      GOTO 28
27    PSIV=PSIB
      PSIH=PSIB
28    HOLDT=PSI(I,J)
      PSI(I,J)=C(IM,JN)*(D(IM,JN)*PSI(I,J-1)+E(IM,JN)*PSI(I-1,J)+
     1F(IM,JN)*PSIV+G(IM,JN)*PSIH-H(IM,JN)*(PSIV-PSI(I-1,J))/FLOAT(I-1))
      GO TO 69
C
29    IF(ITYPE(IM,JN)-3) 31,30,30
30    PSIV=PSI(I-1,J)
      PSIH=PSIB
      GOTO 34
31    IF(ITYPE(IM,JN)-2) 33,32,32
32    PSIV=PSIB
      PSIH=PSI(I,J-1)
      GOTO 34
33    PSIV=PSIB
      PSIH=PSIB
34    HOLDT=PSI(I,J)
      PSI(I,J)=C(IM,JN)*(D(IM,JN)*PSI(I,J+1)+E(IM,JN)*PSI(I+1,J)+
     1F(IM,JN)*PSIV+G(IM,JN)*PSIH-H(IM,JN)*(PSI(I+1,J)-PSIV)/FLOAT(I-1))
69    EPS=EPS+ABS(PSI(I,J)-HOLDT)
   70 CONTINUE
C
C     STOP ITERATIONS IF COMPUTED VALUES SHOW LITTLE FURTHER
C     CHANGE, OR IF NUMBER OF ITERATIONS IS TOO LARGE
C
      IF(EPS-EPSMX)83,83,82
   82 CONTINUE
      IF(ITER-ITMAX)8,8,85
C
C     PRINT VALUES OF ITERATION COUNTER
C     ITER AND THE FINAL STREAM FUNCTION FIELD
C
   83 WRITE(6,202)ITER
      DO 84 J=1,NPZ,INTVL
   84 WRITE(6,203)(PSI(I,J),I=1,NPR,INTVL)
      GO TO 300
C
C     COMMENT IN CASE ITER EXCEEDS ITMAX
C
   85 WRITE(6,204)
      DO 86 J=1,NPZ,INTVL
   86 WRITE(6,203)(PSI(I,J),I=1,NPR,INTVL)
      WRITE(6,206)
      WRITE(6,208) EPS
      WRITE(6,206)
      WRITE(6,207)((PSI(I,J),I=NRB,NPRC),J=NZB,NPN)
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  200 FORMAT(65H1STREAM FUNCTION FOR FLOW IN TWO CONCENTRIC PIPES WITH P
     1ARAMETERS/ 10H0NZ       = ,I5/ 10H NR       = ,I5/ 10H ITMAX    = ,I5,/
     110H EPSMX    = , E11.2/ 10H RA       = ,F7.2/ 10H RB       = ,F7.2/
     110H XL       = ,F7.2/ 10H DL       = ,F7.2/ 10H UA       = ,F7.2/
     110H UB       = ,F7.2/ 10H INTVL    = ,I5/ 10H ITER     = ,I5/10H RC
     1= ,F7.2)
  201 FORMAT(37H0CALCULATED DIMENSIONLESS PARAMETERS /
     110H0ALPHA =  ,F9.4/ 10H BETA   =  ,F9.4/ 10 H GAMMA =  ,F9.4/
     1 10H DELTA =  ,F9.4/ 10H URAT   =  , F9.4/ 10H UCRA   =  ,F9.4)
  202 FORMAT(46H1CONVERGENCE CONDITION HAS BEEN REACHED AFTER ,
     1I5, 11H ITERATIONS/ 38H THE STREAM FUNCTION FIELD IS GIVEN BY)
  203 FORMAT(1H0, 16F7.4)
  204 FORMAT(42H1NO CONVERGENCE. CURRENT VALUES OF PSI ARE)
206   FORMAT('0')
207   FORMAT(1X,19F6.3)
208   FORMAT(1X,E12.3)
  300 RETURN
C
      END
```

```
C
C      SUBROUTINE FOR DETERMINING BOUNDARY POINTS AND INTERCEPTS
C
       SUBROUTINE BNDPTS(M,N,JR,JMAX,ITYPE,C,D,E,F,G,H,Y12,Y23,JRO,A1,B1,
      1R1,A2,B2,R2,A3,B3,R3)
       DIMENSION JR(20),JMAX(20),ITYPE(20,32),C(20,32),D(20,32),
      1E(20,32),F(20,32),G(20,32),H(20,32),JRO(20),JMIN(20)
       EPS=1.0E-6
       MP1=M+1
C
C      LOCATE EXTREME RIGHT POINT
C      (I,JMAX(I)) AND DETERMINE ITS TYPE
C
       JMAX(1)=N+2
       JMIN(1)=N+2
       DO 7 I=2,MP1
       FIM1=I-1
       IF (FIM1-Y12) 12,12,13
12     IF (FIM1-Y23) 15,15,14
13     AK=A1
       BK=B1
       RK=R1
       GOTO 16
14     AK=A2
       BK=B2
       RK=R2
       GOTO 16
15     AK=A3
       BK=B3
       RK=R3
16     XORDX=AK-SQRT(RK**2-(FIM1-BK)**2)
       JM1=XORDX+EPS
       FJM1=JM1
       J=JM1+1
       JMAX(I)=J
       YORDX=BK-SQRT(RK**2-(FJM1-AK)**2)
       A=YORDX-FIM1
       B=XORDX-FJM1
       IF(ABS(A)-EPS)2,31,31
31     IF(ABS(B)-EPS)2,3,3
2      ITYPE(I,J)=4
       JMIN(I)=JMAX(I)
       GO TO 7
     3 A0=FIM1-(BK-SQRT(RK**2-(FLOAT(J)-AK)**2))
       B0=1.-B
       JMIN(I)=JMAX(I)+1
       IF(A-1.0)5,5,32
32     ITYPE(I,J)=3
       A=1.0
       GO TO 6
     5 CONTINUE
       ITYPE(I,J)=1
     6 CONTINUE
       C(I,J)=A*B/(A+B)
       D(I,J)=1.0/(B+1.0)
       E(I,J)=1.0/(A+1.0)
       F(I,J)=1./A/(A+1.)
       G(I,J)=1./B/(B+1.)
       H(I,J)=1./2./(A+1.)
       JP1=J+1
       IF (A0-1.) 36,36,35
35     ITYPE(I,JP1)=3
       A0=1.
       GOTO 37
36     ITYPE(I,JP1)=1
37     CONTINUE
       C(I,JP1)=A0*B0/(A0+B0)
       D(I,JP1)=1./(B0+1.)
       E(I,JP1)=1./(A0+1.)
       F(I,JP1)=1./A0/(A0+1.)
       G(I,JP1)=1./B0/(B0+1.)
       H(I,JP1)=1./2./(A0+1.)
     7 CONTINUE
```

```
C
C      LOCATE BOUNDARY POINTS OF TYPE 2
C
       DO 11 I=1,M
       FIM1=I-1
       J=JMAX(I)-1
    8  IF(JMAX(I+1)-J)34,10,10
 34    FJM1=J-1
       IF  (FIM1-Y12) 17,17,18
 17    IF  (FIM1-Y23) 20,20,19
 18    AK=A1
       BK=B1
       RK=R1
       GOTO 21
 19    AK=A2
       BK=B2
       RK=R2
       GOTO 21
 20    AK=A3
       BK=B3
       RK=R3
 21    A=BK-SQRT(RK**2-(FJM1-AK)**2)-FIM1
       ITYPE(I,J)=2
       C(I,J)=A/(A+1.0)
       D(I,J)=0.5
       E(I,J)=1.0/(A+1.0)
       F(I,J)=1./A/(A+1.)
       G(I,J)=0.5
       H(I,J)=1./2./(A+1.)
       J=J-1
       GO TO 8
   10  CONTINUE
   11  JR(I)=J
       JR(MP1)=JMAX(MP1)-1
       DO 41 II=2,M
       I=M+2-II
       FIM1=I-1
       J=JMIN(I)+1
 38    IF  (JMIN(I-1)-J) 40,40,39
 39    FJM1=J-1
       IF  (FIM1-Y12) 22,22,23
 22    IF  (FIM1-Y23) 25,25,24
 23    AK=A1
       BK=B1
       RK=R1
       GOTO 26
 24    AK=A2
       BK=B2
       RK=R2
       GOTO 26
 25    AK=A3
       BK=B3
       RK=R3
 26    A0=FIM1-(BK-SQRT(RK**2-(FJM1-AK)**2))
       ITYPE(I,J)=2
       C(I,J)=A0/(A0+1.)
       D(I,J)=.5
       E(I,J)=1./(A0+1.)
       F(I,J)=1./A0/(A0+1.)
       G(I,J)=.5
       H(I,J)=1./2./(A0+1.)
       J=J+1
       GOTO 38
 40    CONTINUE
 41    JR0(I)=J
       JR0(1)=N+2
       JR0(MP1)=JMIN(MP1)+1
       RETURN
C
       END
C
C      SUBROUTINE FOR FINDING INTERCEPTS OF CIRCULAR ARCS
C
       SUBROUTINE ARCEPT(X,Y,R1,R2,B1,B2,A1,A2)
       EPS=1.E-04
       DO 4 ITR=1,20
       F=B2-B1-SQRT(R2**2-(X-A2)**2)+SQRT(R1**2-(X-A1)**2)
       DF=(X-A2)/SQRT(R2**2-(X-A2)**2)-(X-A1)/SQRT(R1**2-(X-A1)**2)
       DX=F/DF
       X=X-DX
       IF(ABS(DX/X)-EPS) 3,3,4
 3     Y=B2-SQRT(R2**2-(X-A2)**2)
       RETURN
 4     CONTINUE
       WRITE(6,100)X,DX
 100   FORMAT(16H0 NO CONVERGENCE.',  X= ',E9.3,',  DX= ', E9.3)
       RETURN
       END
//LKED.SYSPRINT DD SYSOUT=N
//LKED.SYSLMOD DD DSN=G357421.STAN.LOAD(IRENE2),DISP=OLD
/*
// EXEC COMPROC,DSET='G357421.STAN.LOAD'
//SYSPRINT DD SYSOUT=N
/*
```

APPENDIX B

COMPUTER PROGRAM FOR SOLVING THE EQUATION OF MOTION

```
//MOTION JOB (0162,101911,CN04),IRENE,NOTIFY=G357421,MSGCLASS=N,
// MSGLEVEL=1,CLASS=K,TIME=(0,29)
//*MAIN LINES=20,ORG=RM028
// EXEC FORTXCL,PARM.FORT='OPTIMIZE(2)'
//SYSPRINT DD SYSOUT=N
//SYSTERM DD SYSOUT=N
//FORT.SYSIN DD *
      DIMENSION G(4),DG(4)
      COMMON PSI(121,201),URAT,RC,NPRC,NPZB,RA,NPR,INTVL
    1 READ(5,*)G4LFT,G4RIT,SIGNL,DTAU,NIBP,NBBP,NX
      WRITE(6,200)
      WRITE(6,201)G4LFT,G4RIT,SIGNL,DTAU,NIBP,NBBP,NX
C
C     ESTABLISH PHYSICAL PROPERTIES FOR CALCULATING COLLECTION
C     EFFICIENCY
C
C     DC     IS CYLINDER DIAMETER, CM
C     DP     IS PARTICLE DIAMETER, CM
C     RHO    IS FLUID DENSITY, GM/CC
C     SIGMA  IS PARTICLE DENSITY, GM/CC
C     XMU    IS ABSOLUTE VISCOSITY OF FLUID, POISE
C     UB IS FLUID VELOCITY IN INSIDE PIPE CM/SEC
C     ISR    IS STARTING RATIO Z/RC = -G(3)
C
      READ(5,*)DC,DP,RHO,SIGMA,XMU,UB,ISR
C
C     IF CORRECT STREAM FUNCTION VALUES ARE ALREADY IN TEMPORARY
C     STORAGE PASS BY RETRIEVAL FROM DISK STORAGE
C
      READ(5,*) ICONF
      IF(ICONF)2,3,3
    2 CONTINUE
C
C     CALL STREAM FUNCTION ARRAY FROM DISK STORAGE
C     AND SET CONSTANTS FOR FLUID VELOCITY CALCULATION
C
      READ(5,*) INTVL
      CALL STPSI
      WRITE(6,211) INTVL
    3 CONTINUE
C
C     UA     IS FREE STREAM VELOCITY, CM/SEC
C
      UA = UB/URAT
      REZ=RHO*DP*UA/XMU
      XK=SIGMA*DP**2*UA/(9.*XMU*DC)
      P = REZ**2/XK
      WRITE(6,204)
      WRITE (6,202)  REZ,XK,P,DC,DP,RHO,SIGMA,XMU,UA,UB,ISR
C
C     ESTABLISH GRID STEP SIZE
C
      NR = NPR - 1
      DELR = 1.0/FLOAT(NR)
      FORSQ = 4.0*DELR**2
      RR=RC+0.001
C
C     HALF INTERVAL ITERATION FOR INITIAL G4 VALUE
C
      DO 21 ITER=1,NX
C
C     SET AND PRINT INITIAL CONDITIONS
C
      M=0
      NSTEP=0
      TAU=0.0
      G3ZER = -FLOAT(ISR)
      G(3) = G3ZER
      G4ZER=(G4LFT+G4RIT)/2.0
      G(4)=G4ZER
      I = 1+IFIX(RR*G(4))
      JO=NPZB-(NPRC-1)*ISR
      J = JO
      RI=FLOAT(I-1)
      UZ=(PSI(I+1,J)-PSI(I-1,J))/(FORSQ*RI)
      UR=(PSI(I,J-1)-PSI(I,J+1))/(FORSQ*RI)
      G(1)=UZ
      G(2)=UR
      RE=REZ*((UR-G(2))**2+(UZ-G(1))**2)**0.5
      XCORE=CORE(RE)
      IP=ITER/NIBP*NIBP
      IF(IP-ITER)5,7,5
    5 CONTINUE
      IF(ITER-1)6,7,6
    6 CONTINUE
      IF(ITER-NX)8,7,8
    7 CONTINUE
      WRITE (6,205)
      WRITE(6,203)ITER,G4LFT,G4ZER,G4RIT,TAU,G(1),G(2),G(3),G(4),UZ,UR,
     1XCORE
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
```

```
    8 CONTINUE
      M=M+1
      CALL SBM22(4,G,DG,TAU,DTAU,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RE=REZ*((UR-G(2))**2+(UZ-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      DG(1)=((XCDRE)/(24.0*XK))*(UZ-G(1))
      DG(2)=((XCDRE)/(24.0*XK))*(UR-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      I = 1+IFIX(RC*G(4))
      J =J0+IFIX(RC*(G(3)-G3ZER))
      RI=FLOAT(I-1)
      UZ=(PSI(I+1,J)-PSI(I-1,J))/(FDRSQ*RI)
      UR=(PSI(I,J-1)-PSI(I,J+1))/(FDRSQ*RI)
C
C     PRINT SOLUTIONS
C
      IS = ITER/NIBP*NIBP
      IF(IS-ITER)11,13,11
   11 CONTINUE
      IF(ITER-1)12,13,12
   12 CONTINUE
      IF(ITER-NX)17,13,17
   13 CONTINUE
      NSTEP=NSTEP+1
      IF(NSTEP-NSBP)17,14,17
   14 CONTINUE
      NSTEP=0
      TAW = TAU + 0.0001
      WRITE(6,204)TAW,G(1),G(2),G(3),G(4),UZ,UR,XCDRE
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
   17 HITS=G(3)
      IF(HITS)8,18,18
   18 CONTINUE
C
C     FIND INTERVAL HALF WITH THE SIGN CHANGE
C
      IF((G(4)-1.0)*SIGNL-0.0)19,19,20
   19 G4RIT=G4ZER
      GO TO 21
   20 G4LFT=G4ZER
   21 CONTINUE
C
C     PRINT SOLUTIONS FOR FINAL VALUE OF TAU
C
      TAW = TAU + 0.0001
      WRITE(6,204)TAW,G(1),G(2),G(3),G(4),UZ,UR,XCDRE
C
C     CALCULATE THE COLLECTION EFFICIENCY
C
      WRITE(6,209) G4ZER
      EM = G4ZER**2
      WRITE(6,206)EM
C
C     CALCULATE THE SAMPLING EFFICIENCY
C
      RSINF=SQRT(PSI(NPRC,NPZB))*RA/RC
      PSIHT = PSI(NPRC,NPZB)
      DO 25 I=1,NPR
      IF(PSI(I,J0)-PSIHT)25,25,24
   24 RSINF = FLOAT(I-2)*SQRT(PSIHT/PSI(I-1,J0))*DELR*RA/RC
      GO TO 28
   25 CONTINUE
   28 CONTINUE
      WRITE(6,210) RSINF
      CR=(G4ZER/RSINF)**2
      WRITE(6,207) CR
      READ(5,*)NSTOP
      IF(NSTOP)1,30,30
   30 STOP
C
C
C
```

```
      FORMATS FOR OUTPUT STATEMENTS
  200 FORMAT( 1H1, 37X, 40HCOLLECTION EFFICIENCY OF A CIRCULAR TUBE/
     1    1H0 )
  201 FORMAT(  10H0G4LEF =  ,F10.6/ 10H G4RIT =  ,F10.6/10H SIGNL =  ,
     1    F4.0/ 10H DTAU   =  ,F10.6/ 10H NIBP   =  ,I4/ 10H NSBP   =  ,I4/
     2    10H NX     =  ,I4)
  202 FORMAT(  10H0REZ    =  ,F12.7/10H XK     =  ,E12.6/
     1    10H P      =  ,E10.4/
     2    10H0DC     =  , F10.5/10H DP     =  ,F10.7/10H RHO    =  ,F10.6/
     3    10H SIGMA  =  , F10.6/10H XMU    =  ,F10.7/10H UA     =  ,F10.4/
     4 10H UB     =  ,F10.4/10H ISR    =  ,I5)
  203 FORMAT( 10H0ITER   =  ,I3/ 10H G4LEF =  ,F10.6/ 10H G4ZER =  ,
     1    F10.6/ 10H G4RIT =  ,F10.6/ 7H0   TAU, 11X, 4HG(1), 12X,
     2    4HG(2), 12X, 4HG(3), 12X, 4HG(4), 13X, 2HUZ, 14X,  2HUR ,
     3    12X, 4HCORE /
     4    1H0, F7.4, 4F16.6, 3F16.4 )
  204 FORMAT ( 1H , F7.4, 4F16.6, 3F16.4 )
  205 FORMAT ( 46H1THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY )
  206 FORMAT( 30H0THE COLLECTION EFFICIENCY IS , E10.4)
  207 FORMAT( 29H0THE SAMPLING EFFICIENCY IS  ,E10.4)
  208 FORMAT(  28H1THE PHYSICAL PARAMETERS ARE )
  209 FORMAT(  34H0THE UPSTREAM PARTICLE RADIUS IS  ,E10.4)
  210 FORMAT( 31H0THE UPSTREAM FLUID RADIUS IS  ,E10.4)
  211 FORMAT(39H0THE INTERVAL OF THE WRITTEN VALUES IS ,I5)
C
      END
      SUBROUTINE GTPSI
C
C     THIS SUBROUTINE RETRIEVES THE STREAM FUNCTION ARRAY AND ASSOCIATED
C     CONSTANTS FROM DISK
C
      COMMON PSI(121,201),URAT,RC,NPRC,NPZB,RA,NPR,INTVL
      READ(8)PSI,ITERS,URAT,UA,RB,NPRB,NPZB,UB,RA,NPZ,NPR,RC,NPRC,NPN
      WRITE(6,200)
      WRITE(6,201)RA,RB,NPZ,NPR,NPZB,NPRB,URAT,UA,UB,ITERS,RC,NPRC,NPN
      WRITE(6,202)
      DO 24 J=1,NPZ,INTVL
   24 WRITE(6,203)(PSI(I,J),I=1,NPR,INTVL)
  200 FORMAT(20H0 DISK STORAGE CHECK)
  201 FORMAT(65H0STREAM FUNCTION FOR FLOW IN TWO CONCENTRIC PIPES WITH P
     1ARAMETERS/ 10H0RA     = ,F7.2/ 10H RB     = ,F7.2/ 10H NPZ    = ,
     2I5 / 10H NPR    = ,I5/ 10H NPZB   = ,I5/ 10H NPRB   = ,I5 /
     310H URAT   = ,F9.4/10H UA     = ,F9.4/10H UB     = ,F9.4 /
     410H ITERS  = ,I5/10H RC     = ,F7.2/10H NPRC   = ,I5/10H NPN    =
     5,I5)
  202 FORMAT(45H1THE CURRENT VALUES OF PSI STORED ON DISK ARE)
  203 FORMAT('0',16F7.4)
      RETURN
      END
C
      FUNCTION CDRE(RE)
C
C     THIS FUNCTION COMPUTES THE PRODUCT OF DRAG COEFFICIENT
C     AND REYNOLDS NUMBER FOR A SPHERE AS A FUNCTION OF
C     REYNOLDS NUMBER
C
C     CONSTANT COEFFICIENTS
C
      A1=1./24.
      A2=-2.3363*1.E-04
      A3=2.0154*1.E-06
      A4=-6.9105*1.E-09
      B0=-1.29536
      B1=9.86*1.E-01
      B2=-4.6677*1.E-02
      B3=1.1235*1.E-03
C
C     CHOOSE THE APPROPRIATE POLYNOMIAL
C
      IF(RE-4.0)2,7,7
C
C     INITIAL ESTIMATE
C
    2 IF(RE-0.00001)3,4,4
    3 CDRE = 24.0
      GO TO 30
    4 X=24.*RE
C
C     BEGIN NEWTON METHOD ITERATION
C
```

```
      DO 6 ITER=1,20
      FX=A1*X+A2*X**2+A3*X**3+A4*X**4-RE
      FPX=A1+2.*A2*X+3.*A3*X**2+4.*A4*X**3
      DELX=FX/FPX
      X=X-DELX
C
C     CHECK FOR CONVERGENCE
C
      EPS=1.E-06
      IF(ABS(DELX/X)-EPS)5,5,6
    5 CORE=X/RE
      GO TO 30
    6 CONTINUE
      GO TO 29
C
C     INITIAL ESTIMATE
C
    7 CD = 1.0
      ELOG = 0.434294481903252
      X=ALOG(CD*RE**2)*ELOG
C
C     BEGIN NEWTON METHOD ITERATION
C
      DO 24 ITER=1,20
      FX=B0+B1*X+B2*X**2+B3*X**3 - ALOG(RE)*ELOG
      FPX=B1+2.*B2*X+3.*B3*X**2
      DELX=FX/FPX
      X=X-DELX
C
C     CHECK FOR CONVERGENCE
C
      EPS=1.E-06
      IF(ABS(DELX/X)-EPS)22,22,24
   22 CORE=10.**X/RE
      GO TO 30
   24 CONTINUE
   29 WRITE(6,202)
   30 RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  202 FORMAT(16H0 NO CONVERGENCE)
C
      END
      SUBROUTINE SBM22(N,Y,F,X,H,IRUNG,M)
C
C     FOURTH ORDER RUNGE KUTTA METHOD
C     FOR N FIRST ORDER O.D.E.
      DIMENSION PHI(50),SAVY(50),Y(50),F(50)
C
      GO TO (2,3,4,5,6),M
C
C     PASS 1
C
    2 IRUNG=1
      RETURN
C
C     PASS 2
C
    3 DO 22 J=1,N
      SAVY(J)=Y(J)
      PHI(J)=F(J)
   22 Y(J)=SAVY(J)+0.5*H*F(J)
      X=X+0.5*H
      IRUNG=1
      RETURN
C     PASS 3
    4 DO 33 J=1,N
      PHI(J)=PHI(J)+2.0*F(J)
   33 Y(J)=SAVY(J)+0.5*H*F(J)
      IRUNG=1
      RETURN
C
C     PASS 4
    5 DO 44 J=1,N
      PHI(J)=PHI(J)+2.0*F(J)
   44 Y(J)=SAVY(J)+H*F(J)
      X=X+0.5*H
      IRUNG=1
      RETURN
C     PASS 5
    6 DO 55 J=1,N
   55 Y(J) = SAVY(J) + (PHI(J) + F(J))*H/6.0
      IRUNG=2
      RETURN
C
      END
//LKED.SYSPRINT DD SYSOUT=N
//LKED.SYSLMOD DD DSN=G357421.STAN.LOAD(MOTION),DISP=OLD
/*
// EXEC COMPROC,DSET='G357421.STAN.LOAD'
//SYSPRINT DD SYSOUT=N
/*
```

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