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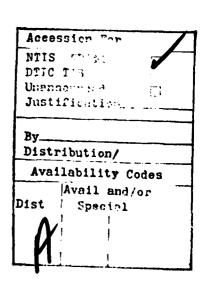


# Le Catastrophe by any other name. . . .

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## Le Catastrophe by any other name....

### R. V. BUCK

The application of the techniques of mathematical physics to the analysis of <u>discontinuities</u> is well known, but it rests on a detailed understanding of the continuous behavior of other parts of the process or system. In many cases, the "soft" sciences especially, such detailed knowledge is unlikely.

As a part of mathematics, Catastrophe Theory (a subset of singularity and bifurcation theories) deals with the properties of discontinuities without reference to any specific underlying mechanism. It is thus most appropriate for the study of systems whose inner workings are not known, and for situations in which the only reliable observations are of the discontinuities (or sudden changes in the state of the system).

René Thom, creator of CT, has admitted that it does not provide a precise quantitative description of what is taking place when a discontinuity/catastrophe occurs, but it does offer a rigorous analytical technique - by classification and analogy - for providing a qualitative interpretation. The instrument sometimes may not fit, or may give misleading interpretations, but it is one of only a very few ways of handling complex systems which tend to "settle" into something like minimum or maximum potential energy states. A major difficulty is in not always knowing whether there is an underlying potential function.

In applying CT, we are limited in the amount of model tinkering we can do, since so few assumptions are made about the system being studied. If we reach unsatisfactory conclusions, we can 1) reject the theory, 2) try to more completely observe and describe the system mechanisms, or 3) change our choice of control variables. Sometimes, as we fit the method to the problem, great insight follows a fortuitous choice of variables.

Borrowing from morphologists, who concern themselves with shapes adopted by natural objects, Thom considered what sorts of surfaces one would observe when there were only one or two behavior variables and no more than four control variables for potential systems posessed of discontinuities. His theory of elementary catastrophes tells us several remarkable things:

- The number of qualitatively different configurations of discontinuities depends not on a potentially large number of state variables but on a generally small number of control variables.
- If the number of control variables is no more than four then there are only seven distinct types of catastrophes.

The seven elementary catastrophes have been described as follows:

Control Factors	Behavior one axis	Variables two axis
1	Fold	-
2	Cusp	-
3	Swallowtail	Hyperbolic Umbilic Elliptic Umbilic
4	Butterfly	Parabolic Umbilic

Since the <u>topological</u> approach provides no scale, an act of faith is required to identify a mathematical jump between stable potential states on a catastrophe behavior surface with an observed discontinuity in nature. In some cases where more is known of the underlying process, some success has been made in developing statistical techniques for fitting CT surfaces to experimental data. Generally, only two assumptions are necessary to suggest a CT application:

- System behavior depends on a limited number of important control factors,
- The system is governed by a potential such that the system is structurally stable except at isolated points.

The elementary catastrophes have been found to model - in the sense of organizing existing observation and suggesting where to look for interesting behavior - a wide variety of processes. One might compare this result to that discovered by the ancient Greeks: only three regular polygons (square, triangle, and hexagon) can be packed edge to edge to fill the plane. This is so, not because geometry dictates to nature, but because there is no other way for the natural process to turn out.

Four quite good introductory readings for those interested in pursuing the geometry, mathematics and applications of CT are:

Woodcock and Davis, Catastrophe Theory, E. P. Dutton, 1978

- P. T. Saunders, An Introduction to Catastrophe Theory, Cambridge Press, 1980.
- E. C. Zeeman, "Catastrophe Theory", Scientific American, April 1976, p.65-82

Behavioral Science, September 1978 issue (applications of Catastrophe Theory in the Behavioral and Life Sciences)

## A Combat Example

The combat "laws" of F.W. Lanchester use differential equations and postulate continuous attrition. A difficulty arises, however, in defining when a battle has ended. One way out is to assume a "fight to the finish" rule, though actual combat results suggest there are more generally points at which the "weaker" force breaks off fighting (and perhaps runs away). Many reasons are advanced for such behavior: leadership, training, morale, casualty level, mission, terrain, and C<sup>3</sup>.

Let us consider some combination of factors which, together, produce unit <u>cohesiveness</u> (C). Let C, along with <u>strength</u> (S) of the unit (inversely proportional to casualties), be considered important control variables which influence combat behavior along a spectrum from "fight" to "run". The behavior variable is called <u>break point</u> (B).

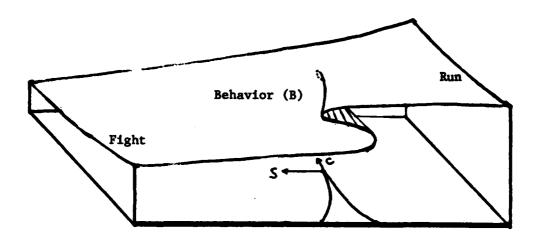
We might then postulate a potential liklihood function which seems to fit a wide variety of combat situations:

Strength	Cohesiveness	Probability
high	normal	
1ow	normal	
high	high	
high/low	low	
		Run Fight (high B) Behavior (low B)

We are perhaps led to consider CT by observing that there are localized discontinuities of behavior in combat, that past history influences whether panic or stiff resistance result from a given force, that sudden flight does not occur at the same values of S and C as sudden resistance, that a slight difference in strength can stimulate a major change in behavior, and that low cohesiveness tends to produce extremes in behavior. All the elements of a cusp catastrophe are now present. We can write the behavior surface equation (origin directly above the cusp vertex) as:

$$B^3 - S - (C \cdot B) = 0$$

with C acting as a splitting factor (bimodal behavior resulting) and S acting as a normal factor (continuous change in behavior results).



One might next proceed to consider tradeoffs between the quality and the quantity of a force, the transient effect of casualties and/or reinforcements during a battle, whether observed data supports collapsing several factors into a "cohesiveness" variable, and what the effect would be of assuming 3 control variables (i.e. casualties, weather, morale) and 2 behavior variables (i.e. perceived ferocity and coherent plan of battle).

One might also speculate whether, for a given estimate of enemy behavior, firepower or propaganda would be the more effective to defeat him.

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