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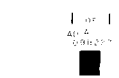

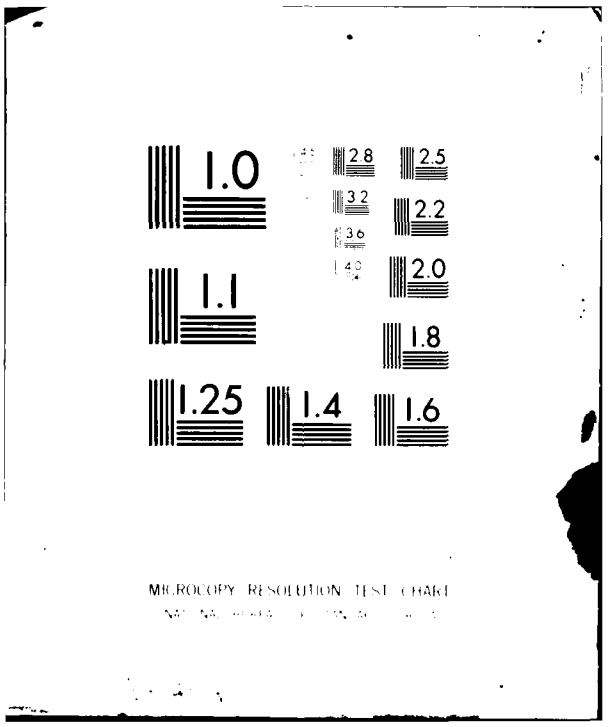
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FIXED STATE UTILITY SCALING: COMPARISON OF DIFFERENT CRITERIA

SHIN-ICHI MAYEKAWA
THE UNIVERSITY OF IOWA

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Several methods of fitting utilities using fixed state gambles and least-squares criteria are compared. It was found that similar results are found when the miximization is done in the probability , log-odds, or root arc-sine matric.

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Fixed State Utility Scaling

Comparison of Different Criteria

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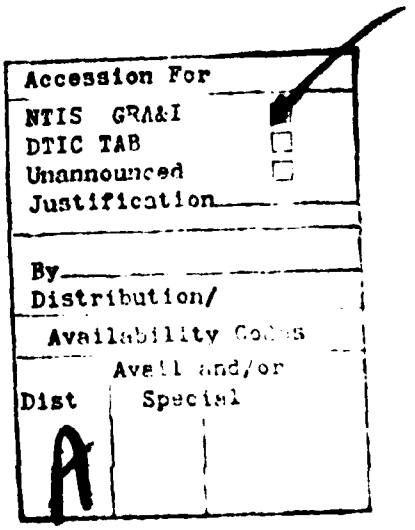

UTILITY SCALING

I. Introduction

In the Bayesian view of decision making, the posterior probability distribution of the variable of interest and its utility function are the essential ingredients of a decision process. The method used to assess the utility function installed in the CADA monitor (Novick, Hamer, Libby, Chen, and Woodworth, 1980) is called the fixed-state utility assessment procedure (Novick and Lindley, 1979) in which the investigator is asked to specify subjective indifference probabilities for gamble pairs. The assessment of the utility function or the scaling of the subjective utility is then done through a least squares procedure based on the log-odds transformed observations (subjective indifference probabilities).

In Novick and Lindley (1979), the following notation is used:

Observations

p_{ijk}, $i = 0,1,\ldots, N-1$

$j = 1,2,\ldots, N$

$k = 2,3,\ldots, N+1$

$i < j < k$

where $N + 2$ is the number of ordered outcomes. p_{ijk} denotes the numerical values of the subjective probability with which the subject receives the outcome k in a gamble between outcome i and outcome k, which makes the gamble indifferent to receiving the outcome j for sure.

Parameters

If u_i, $i=0,1,2,\ldots, N+1$, $u_0 = 0$, $u_{N+1} = 1$, is the utility of the

i-th outcome, then the true indifference probabilities are

(1) $\hat{p}_{ijk} := (u_j - u_i)/(u_k - u_i)$.

Given N observed indifference probabilities the N unspecified utilities can be calculated under certain conditions. However, in most situations the observations contain errors and it is therefore useful to obtain additional observations and to attempt to fit a utility function.

In this paper several different methods of fitting (scaling) will be discussed. Throughout this paper, we assume that the utilities are monotonic to the outcomes: that is, under the appropriate ordering of the outcomes, we assume that if $i < j$ then $u_i < u_j$. We further assume that p_{ijk}'s are measured in the absolute scale, that is, p_{ijk}'s are assumed to have a unique origin and a unique unit.

II. Least Squares Method

The simplest way to estimate the utilities is to solve the least squares problem:

(2) $Q_2 := \sum_{ijk} [(p_{ijk} - \hat{p}_{ijk})^2] \rightarrow \min$ w.r.t. u's, where $\sum_{ijk}$ denotes summation over all the observed p_{ijk}'s, and $\hat{p}_{ijk}$ is the function of the utilities as defined by (1). This solution, although straight forward, lacks the consideration of the comparative magnitude of the deviations or the relative weight of each observation. As indicated by Novick and Lindley (1979), the value of p_{ijk} close to 1/2 seems to have a relatively large magnitude of deviations, that is, the closer p_{ijk}'s are to 1/2, the less reliable they are. Thus, observed values p_{ijk} distant from 1/2 would have little influence on the solution.

One way to handle this problem is to define the least squares criterion using transformed observations. The transformation which removes the heterogeneity of the deviations of the observations must be chosen. Among the transformations which serve this purpose, we only consider the following two transformations:

(3) $f_L(x) = \ln(x/(1-x))$,

and

(4) $F_A(x) = \sin^{-1}(x^{1/2})$.

f_L and f_A are called log-odds transformation and arcsine transformation, respectively.

The least squares criterion defined on these transformed values are:

(5) $Q_3 := \sum_{ijk} [(f_L(p_{ijk}) - f_L(\hat{p}_{ijk}))^2] \rightarrow \min$,

and

(6) $Q_4 := \sum_{ijk} [(f_A(p_{ijk}) - f_A(\hat{p}_{ijk}))^2] \rightarrow \min$.

Another way to handle this problem is to define the weighted least squares criterion such that the heterogeneity of the deviations may be absorbed in the weights. Because the observations are assumed to be less reliable around 1/2, we may define the weighted least squares criterion as follows:

(7) $Q_1 := \sum_{ijk} [w_{ijk}(p_{ijk} - \hat{p}_{ijk})^2] \rightarrow \min$,

where

(8) $w_{ijk} = 1/[\hat{p}_{ijk}(1-\hat{p}_{ijk})]$.

In general the least squares criteria can be written as

(9) $Q_a := \sum_{ijk} [w_{ijk}(f_a(p_{ijk}) - f_a(\hat{p}_{ijk}))^2] \rightarrow \min$,

where

$$w_{aijk} = 1 \qquad \text{for } a = 2,3, \text{ and } 4$$
$$= 1/\hat{p}_{ijk}\,(1-\hat{p}_{ijk}) \qquad \text{for } a = 1,$$

and

$$f_a(x) = x \qquad \text{for } a = 1 \text{ and } 2,$$
$$= f_L(x) \qquad \text{for } a = 3,$$
$$= f_A(x) \qquad \text{for } a = 4.$$

III. Solution

Rewriting (9) in a matrix form, we have

(10) $Q_a := [\underline{f}_a(\underline{p}) - \underline{f}_a(\hat{\underline{p}})]' W_a [\underline{f}_a(\underline{p}) - \underline{f}_a(\hat{\underline{p}})] \to \min,$

where denoting the number of the observations by K, $\underline{f}_a(\underline{x})$ is the K x 1 vector whose e-th element is $f_a(x_e)$, W_a is the K x K diagonal matrix whose e-th diagonal element is w_{ae}, and $\underline{p}$ and $\hat{\underline{p}}$ are the K x 1 vectors consisting of p_{ijk}'s and $\hat{p}_{ijk}$'s, respectively. The subscript "e" stands for any ijk triad.

Because u_0 and u_{N+1} are fixed, there are only N free values of u which must be estimated. The solution of the problem is given by solving

(11) $$\frac{\partial Q_a}{\partial \underline{u}} = \left[\frac{\partial \underline{f}_a(\hat{\underline{p}})}{\partial \underline{u}}\right] W_a [\underline{f}_a(\hat{\underline{p}}) - \underline{f}_a(\underline{p})]$$

$$= \underline{0}$$

where $\underline{u}$ is the N x 1 vector of the utility scale, i.e.

$$\underline{u} = [u_1, u_2, \ldots, u_N]',$$

and

$$\left[\frac{\partial \underline{f}_a(\hat{\underline{p}})}{\partial \underline{u}}\right]$$

is the K x n matrix whose (e,c) element is

$$\frac{\partial f_a(\hat{p}_e)}{\partial u_c} = \frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c} \quad .$$

[The weight matrix is treated as a constant even when a = 1.]

Specifically, in a scalar form $\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c}$ can be written as follows:

(12) For a = 1 and 2,

$$\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c} = \frac{\partial \hat{p}_{ijk}}{\partial u_c} = \begin{cases} -(u_k-u_j)/(u_k-u_i)^2 & \text{if } c = i \\ 1/(u_k-u_i) & \text{if } c = j \\ -(j_j-u_i)/(u_k-u_i)^2 & \text{if } c = k, \end{cases}$$

for a = 3,

$$\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c} = \left(\frac{1}{\hat{p}_{ijk}} + \frac{1}{1-\hat{p}_{ijk}}\right) \frac{\partial \hat{p}_{ijk}}{\partial u_c}$$

and for a = 4,

$$\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c} = \frac{1}{2\sqrt{\hat{p}_{ijk}(1-\hat{p}_{ijk})}} \frac{\partial \hat{p}_{ijk}}{\partial u_c} \quad .$$

Among the various numerical methods for solving (11), the Gauss-Newton method seems to be appropriate here because it does not require the evaluation of the second derivatives and yet is as efficient as the Newton-Raphson method. The direction for search in the Gauss-Newton method is defined by

$$(13) \quad \underline{S}(\underline{u}) = -\left[\left(\frac{\partial \underline{f}_a(\hat{\underline{p}})}{\partial \underline{u}}\right)' W_a \left(\frac{\partial \underline{f}_a(\hat{\underline{p}})}{\partial \underline{u}}\right)\right]^{-1} \frac{\partial Q_a}{\partial \underline{u}}$$

The Gauss-Newton method with the step-size halving is always convergent. However, we must consider the monotonicity restrictions.

Usually, the restrictions can be enforced by setting the parameters which fall out of the proper region equal to the boundary surface. However, we cannot apply this simple method because our restrictions do not include the equalities: that is, u_i must be strictly less than u_{i+1}. To enforce the monotonicity restrictions, therefore, we modified the step-size halving procedure of the Gauss-Newton method so that all the updated parameters may satisfy the restrictions in each iteration. As long as the initial estimates satisfy the restrictions, this method is guaranteed to converge to the restricted minimum.

For the weighted least squares case (a = 1), the W matrix must be successively updated in each iteration by

$$(14)\quad w_{1ijk} = 1/\hat{p}_{ijk}(1 - \hat{p}_{ijk})$$

using the latest value of $\hat{p}_{ijk}$.

IV. Maximum Likelihood Method Based on the Normal Distribution

In the previous sections, we used the word "relative magnitudes of the deviations" without introducing the statistical concept. In this section, however, we assume that the observations are the random variables. Under the assumption that $f_a(p_{ijk})$ has the normal distribution with the mean $f_a(\hat{p}_{ijk})$ and the constant variance σ^2,

$$(15)\quad f_a(p_{ijk}) \sim N(f_a(\hat{p}_{ijk}), \sigma^2),$$

for a=2,3, and 4,

or under the assumption that p_{ijk} has the normal distribution with the mean $\hat{p}_{ijk}$ and the variance $c\hat{p}_{ijk}(1-\hat{p}_{ijk})$, where c is the unknown parameter to be estimated,

(16) $p_{ijk} \sim N(\hat{p}_{ijk}, c\hat{p}_{ijk}(1-\hat{p}_{ijk}))$,

it can be shown that the least squares solutions described in the previous sections coincide with the maximum likelihood solutions. That is, the derivatives of the log likelihood function for the assumption (15),

(17)

$$\ln L = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{ijk}(f_a(p_{ijk}) - f_a(\hat{p}_{ijk}))^2$$

for a = 2,3, and 4,

with respect to σ^2 and u_c's result in the equations

(18) $$\sigma^2 = \sum_{ijk}[(f_a(p_{ijk})-f_a(\hat{p}_{ijk}))^2]/N$$

and

(19) $$\frac{\partial \ln L}{\partial u_c} = \frac{1}{\sigma^2}\sum_{ijk}(f_a(p_{ijk}) - f_a(\hat{p}_{ijk}))\frac{\partial f_a(\hat{p}_{ijk})}{\partial u_c}$$

and for the assumption (16), the derivatives of the log likelihood function

(20) $$\ln L = -\frac{N}{2}\ln(2\pi) - \frac{1}{2}\sum_{ijk}\ln c^2 d^2_{ijk} - \frac{1}{2}\sum_{ijk}\frac{(p_{ijk}-\hat{p}_{ijk})^2}{c^2 d^2_{ijk}}$$

where

$$d^2_{ijk} = \hat{p}_{ijk}(1-\hat{p}_{ijk}),$$

with respect to c^2 and u_c's result in the equations

(21) $$c^2 = \frac{1}{N}\sum_{ijk}\frac{(p_{ijk} - \hat{p}_{ijk})^2}{\hat{p}_{ijk}(1 - \hat{p}_{ijk})}$$

and

$$(22)\quad \frac{\partial \ln L}{\partial u_c} = \frac{1}{c^2} \sum_{ijk} \frac{(p_{ijk} - \hat{p}_{ijk})}{d^2_{ijk}} \frac{\hat{p}_{ijk}}{\partial u_c}$$

These equations can be written as (11) using the same notation, and the Gauss-Newton method is also appropriate as the numerical method.

One of the merits of the maximum likelihood solution is the inverse matrix of

$$(23)\quad I_a(\underline{u}) := \left(\frac{\partial \underline{f}_a(\hat{\underline{p}})}{\partial \underline{u}}\right)' W_a \left(\frac{\partial \underline{f}_a(\hat{\underline{p}})}{\partial \underline{u}}\right) ,$$

which appeared in (12), can provide the asymptotic variance/covariance matrix of the estimated parameters (see Jennrich and Moor, 1975). From the Bayesian point of view this is equivalent to saying that with the uniform prior distribution, the posterior distribution of the parameter $\underline{u}$ is approximately multivariate normal with the mean vector $\hat{\underline{u}}$, the maximum likelihood estimates, and the variance/covariance matrix $I_a(\hat{\underline{u}})$ (See Lindley 1965). That is, using the approximate posterior distributions, we can draw rough inferences concerning the imprecision of the estimated utility scale. For example, we can draw the approximate 95 percent HDR of the utility function using the posterior marginal distributions.

V. Maximum Likelihood Method Based on the Beta Distribution

Under the assumption that each observation has the beta distribution with parameters $a\hat{p}_{ijk}+b$ and $a(1-\hat{p}_{ijk})+b$,

$$(24)\quad \hat{p}_{ijk} \sim \beta(a\hat{p}_{ijk}+b,\ a(1-\hat{p}_{ijk})+b),$$

where a is the unknown parameter related to the variance and b is the constant related to the location, the solution which maximizes

$$(25)\quad \ln L := \sum_{ijk} \ln \left[\frac{p_{ijk}^{a\hat{p}_{ijk}+b} \, (1 - p_{ijk})^{a(1-\hat{p}_{ijk}) + b}}{B[a\hat{p}_{ijk} + b,\ a(1-\hat{p}_{ijk}) + b]} \right],$$

where $B[\alpha,\beta]$ denotes the beta function with the arguments α and β, was proposed by Mayekawa (1980). In this formulation, the variance of p_{ijk} can be written as

$$(26)\quad \sigma^2_{ijk} = \frac{(a\hat{p}_{ijk} + b)\ (a(1 - \hat{p}_{ijk}) + b)}{(a + 2b)^2\ (a + 2b + 1)}$$

which shows that this assumption is compatible with the previous consideration of the errors.

In this section we will consider the two specific cases in which b is set equal to zero and one, respectively. When b is set equal to zero, the mean of the distribution is equal to $\hat{p}_{ijk}$ and when b is set equal to one, the mode of the distribution is equal to $\hat{p}_{ijk}$.

Fischer's information matrix under this assumption is given by

(27)

$$I_{\ell\ell} = a^2 \sum_{ijk} \left(\frac{\partial \hat{p}_{ijk}}{\partial u_\ell} \right)^2 [\psi^{V}(a\hat{p}_{ijk}+b) + \psi^{V}(a(1-\hat{p}_{ijk}) +b)] ,$$

$$I_{\ell m} = a^2 \sum_{ijk} \left(\frac{\partial \hat{p}_{ijk}}{\partial u_l} \right) \left(\frac{\partial \hat{p}_{ijk}}{\partial u_m} \right) [\psi^{V}(a\hat{p}_{ijk} + b) + \psi^{V}(a(1-\hat{p}_{ijk}) +b)],$$

$$I_{N+1,\ N+1} = \sum_{ijk} [\hat{p}^2_{ijk}\, \psi^{V}(a\hat{p}_{ijk} + b) + (1-\hat{p}_{ijk})\, \psi^{V}(a(1-\hat{p}_{ijk}) + b) - \psi^{V}(a + 2b)],$$

and

$$I_{\ell,N+1} = I_{N+1,\ell}$$

$$= a \sum_{ijk} \left[\hat{p}_{ijk} \left(\frac{\partial \hat{p}_{ijk}}{\partial u_\ell} \right) \psi^{\nabla}(a\hat{p}_{ijk} + b) - (1-\hat{p}_{ijk}) \left(\frac{\partial \hat{p}_{ijk}}{\partial u_\ell} \right) \right.$$

$$\left. \psi^{\nabla}(a(1 - \hat{p}_{ijk}) + b) \right] ,$$

where

$$\psi^{\nabla}(x) = \frac{d^2 \ln \Gamma(x)}{d x^2} .$$

To estimate the parameters Fisher's scoring method, with the modified step-size halving as before, is used.

VI. Results

Using the nine-point fixed-state method in Component 31 of the CADA monitor, in which N = 7, fourteen indifference probabilities were collected for seventeen subjects (K = 14). The outcome used in the assessment procedure was the GPA and the subjects were the students of The University of Iowa. For each subject, six utility scales based on the different criteria were estimated. In order to see the differences of the utility scales the interscale correlation coefficients (CC) and the sums of the interscale absolute differences (SAD) were calculated for each subject.

Generally speaking, the differences were very little for all the subjects. Among seventeen subjects, the minimum of the mean correlation coefficients (MCC) over six scales was .96732 and the maximum of the mean of the sums of the absolute differences (MSAD) was .09056, both of which were obtained for the subject #4. However, close

examination shows that while the differences between the scale based on the beta distribution with b = 1 and the other scales are fairly large, the differences among the other scales are very small for this subject, which may indicate that the beta scales with b = 1 converged at the local minimum for the subject #4.

The minimum of MCC and the maximum of MSAD among the sixteen subjects excluding the subject #4 was .99325 for the subject #9 and .04912 for the subject #3 respectively. The estimated utility scales and the 95 percent approximate HDR for the subject #3 are shown in Fig. 1 and Fig. 2. As indicated above, the differences of the six utility scales are the largest for this subject except subject #4. For all the other fifteen subjects the utility scales and their HDR's were very similar to each other. One of the typical results (subject #8) are shown in Fig. 3 and Fig. 4.

While there seems to be no consistent qualitative differences among the different scales, it is clear that the fit of the model and the interscale differences are negatively correlated. That is, if the fit is good (lower sum of the residuals for the normal models, or higher likelihood for the beta models), the differences among the scales become smaller. In Table 1 the correlation coefficients among the least squares criterions (Q_1 through Q_4) minimized, the log likelihoods (with b = 0 and b = 1), CC's, and SAD's over seventeen subjects are shown. It is also clear that the fit and the size of the HDR's for the beta scales with b = 1 seem to be very sensitive to the fit and always resulted in the largest intervals when the fit was poor. (The HDR for the subject #3 in Fig. 2 lies out of the range of the graph.)

VII. Discussion

As shown in the previous sections, for almost all the subjects, the differences among the utility scales based on the different criteria or distributional assumptions are very small. The fact that the least square solution under the assumption that p_{ijk}'s have the constant variances all over the range showing no qualitative differences tells us that the least squares solutions are very robust. While the other scales seem to be theoretically more sound, the least squares solution based on the probability itself might be recommended as the simplest method. However, as for the computations, there are no great differences among the least squares solutions, which indicates the choice is very arbitrary.

On the contrary, the maximum likelihood solutions based on the beta distributions require a great deal of effort to evaluate the beta function, psy function, and the derivative of the psy function, which may affect the computational time on small computers seriously.

Finally, we may mention the reliability of the utility scales. Unlike the area of the mental test theory, little attention has been paid to the reliability concept in psychological scaling.
The reasons are:

1) In the mental test theory, the observations and the estimated scales are essentially the same quantity, which leads to the variance-ratio definition of the reliability coefficient.

2) In the psychological scaling, usually, the observations and the scale have a functional relation. For example, we have to estimate the utility scale through the subjective probabilities. This fact makes it difficult to calculate the observed variance.

If we have the repeated observations, it is possible to calculate the interscale correlation coefficient among the two scales based on the different sets of observations like split-half methods. On the contrary, it is possible to define the reliability of the scale as

$$(28) \quad p = \frac{\mathrm{Var}(p_{ijk})}{\mathrm{Var}(\hat{p}_{ijk})}$$

which can be calculated without repeated observations. However, because this quantity is defined on the probability, rather than the utility, it may not be appropriate to call this the reliability of the utility scale. Further study of the reliability seems to be necessary.

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Utility

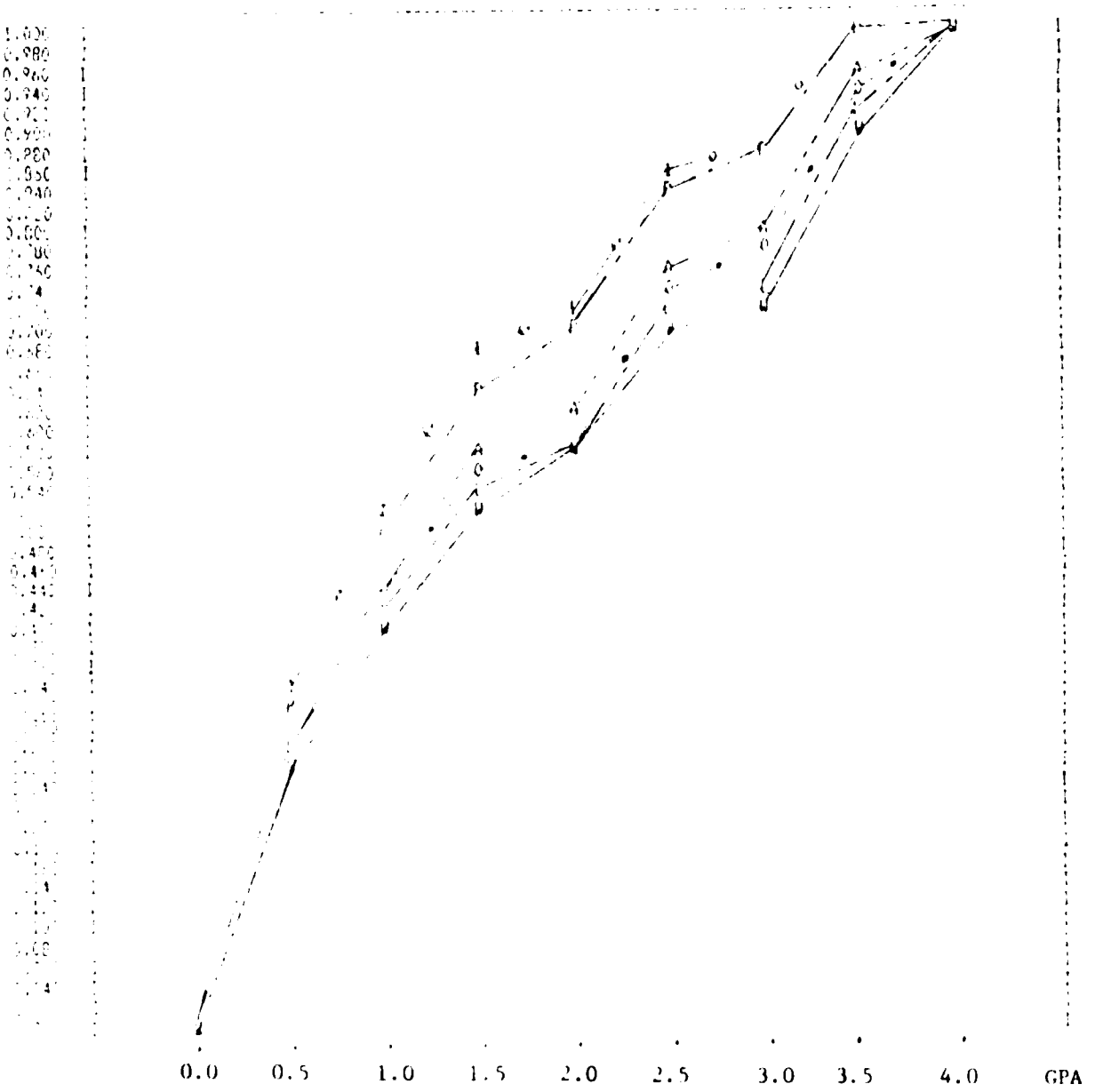

Fig. 1 Utilities (subject #3)

W Q_1 (weighted L.S.)
P Q_2 (probability)
L Q_3 (log-odds) K (# of gambles) = 14
A Q_4 (arc sin)
0 Beta (b=0)
1 Beta (b=1)

The goodness of fit for each model (the values of Q_a and the likelihoods) is: .6737, .2772, 1.4458, .3144, -2.6456, and 1.2851, respectively.

Utility

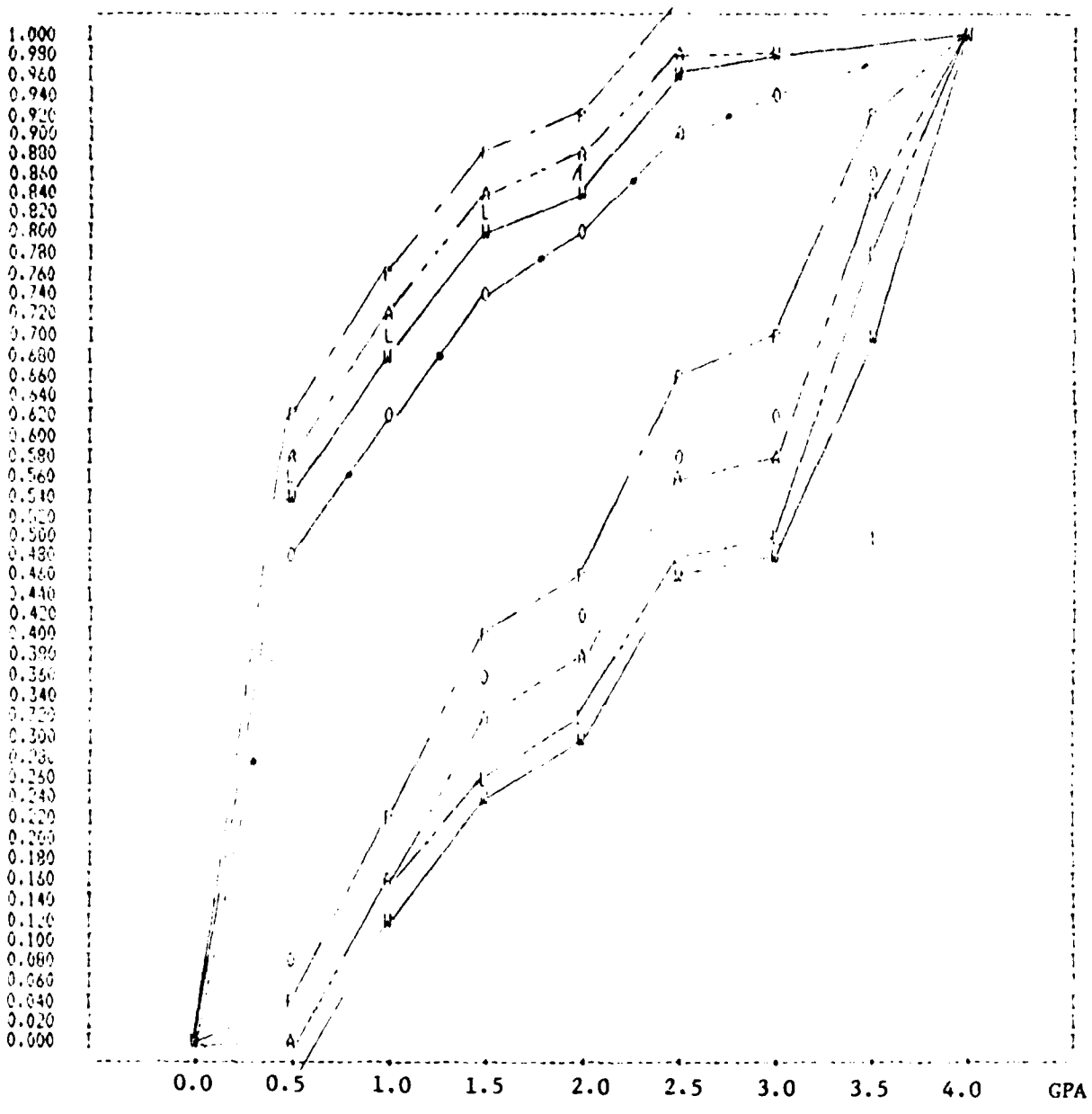

Fig. 2 Approximate 95% HDR's

HDR for Beta (b=1) model lies out of the range of this graph.

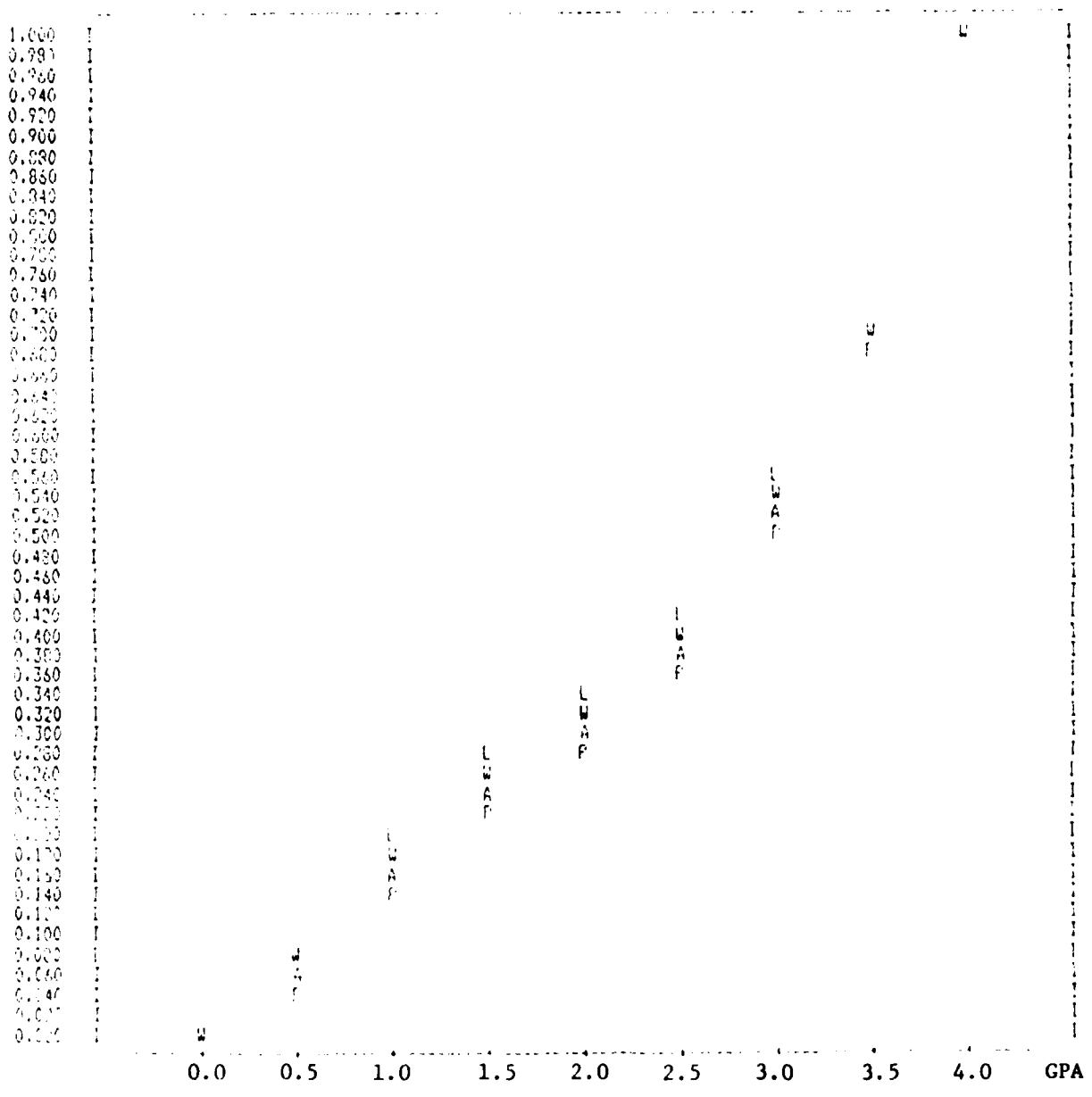

Fig. 3 Utilities (subject #8)

The goodness of fit for each model is: .2894, .0941, .5744, .1175, 14.0572, 16.8665, respectively. Beta models are close to the weighted L.S. Model.

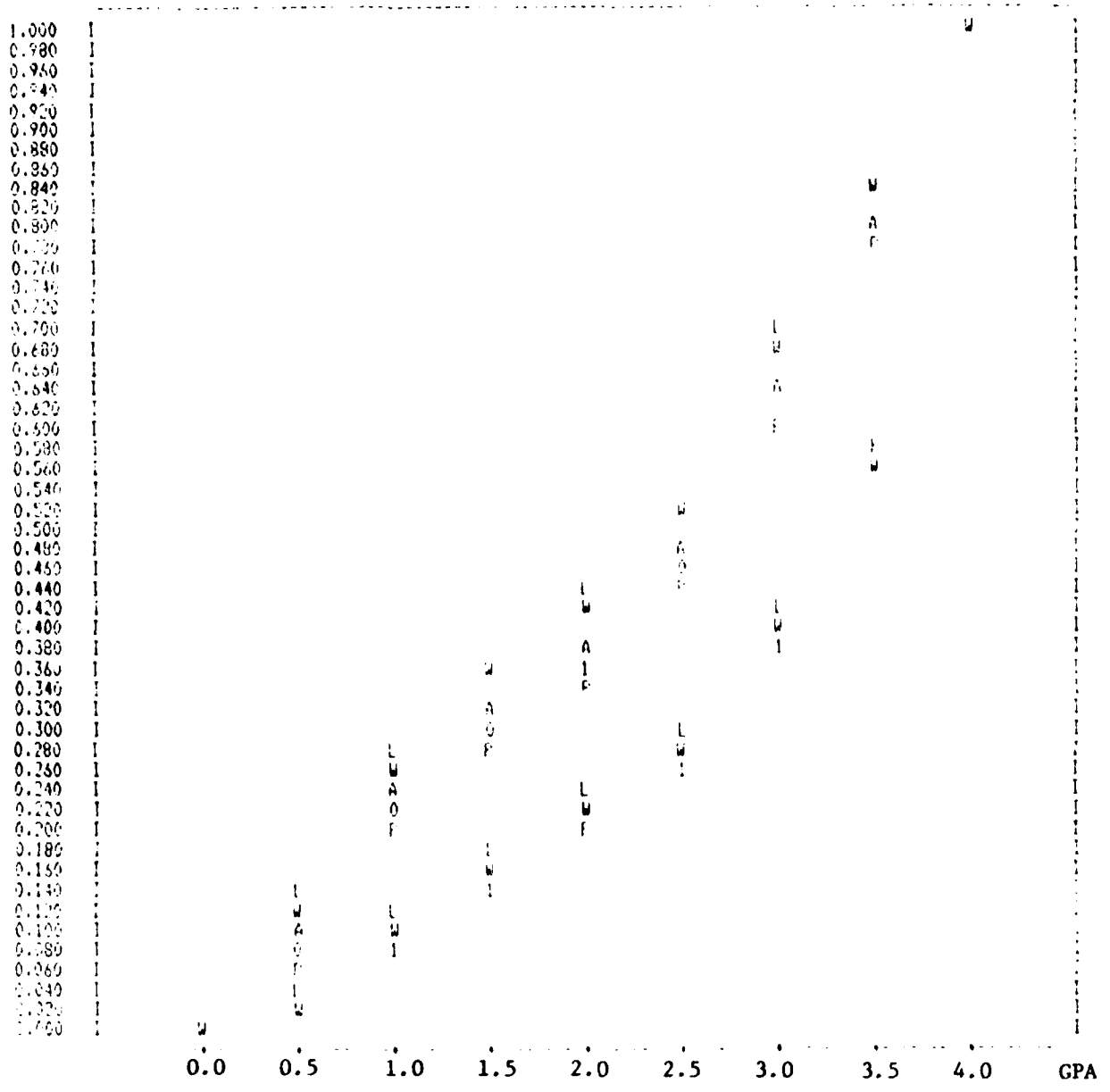

Fig. 4 Approximate 95% of HDR's

	L.S. criterions				log likelihoods			
	Q_1	Q_2	Q_3	Q_4	b = 0	b = 1	MSAD	MCC
Q_1	[illegible]	0.9366	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Q_2	[illegible]	1.0000	[illegible]	0.9760	0.0047	[illegible]	[illegible]	[illegible]
Q_3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Q_4	[illegible]	[illegible]	[illegible]	1.0000	[illegible]	[illegible]	[illegible]	[illegible]
b=0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b=1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
MSAD	[illegible]	[illegible]	0.7331	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
MCC	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 1. Correlation coefficients among the fits and the differences

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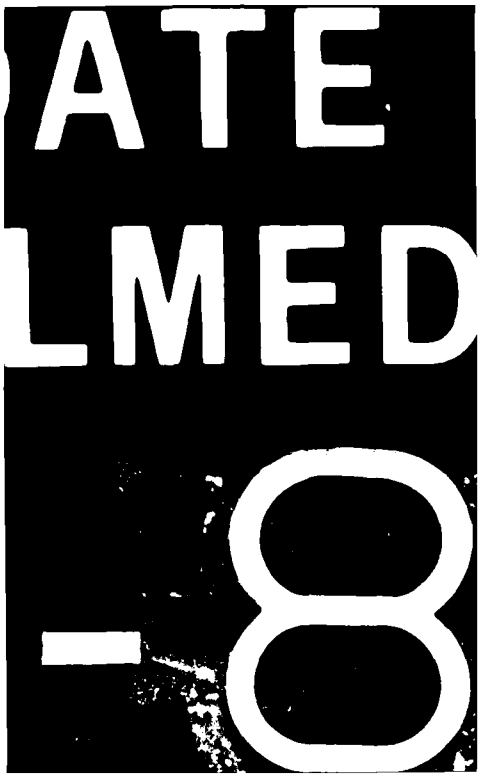
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