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**INVESTIGATIONS TO DEFINE ACCEPTABILITY
TOLERANCE RANGES IN VARIOUS
REGIONS OF COLOR SPACE**

by

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Current acceptance of goods for color by the Army depends on visual comparison against a standard and eight limit samples. The Army wished to have a numerical method of setting color tolerances to be used with instrumental measurement. The limit samples, on examination, were seen to be somewhat erratic as guides for instrumental tolerances. Instead, we selected pairs of samples from a large number of previous submissions by industry. These pairs showed four graduated lightness steps, four graduated chroma steps, and four graduated hue steps. Six observers looked at each pair 10 times, randomly interspersed with other pairs,		

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and issued a pass or fail judgement each time. From these data we established lightness, chroma, and hue tolerance limits. For an olive green and a tan shade, these tolerances were roughly in the ratio 3:2:1; for a dark blue, the ratios were roughly 2:2:1. We wrote simple equations that can be used with instrumental measurement in order to determine quickly whether a sample passes or fails. We recommend similar treatment of all samples for which the Army desires acceptability limits.

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PREFACE

This work was performed under Contract Number DAAK 60-78-C-0084 for which Mr. Alvin O. Ramsley and Miss Therese R. Commerford were project officers. We wish to thank them and Mr. Charles R. Williams for the support and guidance they provided.

We greatly appreciate the constructive suggestions made by the Committee on Color Measurement of the National Research Council during the course of this investigation. The committee members are David L. MacAdam, chairman; Michael E. Breton, Ellen G. Carter, Franc Grum, John J. Hanlon, and Robert F. Hoban. We are particularly indebted to Dr. Breton for his recommendation of the method of constant stimuli and the logistic function, and to Dr. MacAdam for his suggestion that ellipsoids be used as a basis for the acceptability equations.

We also wish to thank the observers, who gave of their time to cooperate in making this work a success.

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TABLE OF CONTENTS

	Page
Preface	1
List of Figures	5
List of Tables	5
I. Introduction	7
II. Tests on Standard and Limit Samples.	7
A. Relationship between standard and limit samples in color space.	7
B. Determination of minimum ellipsoid containing standard and all limit samples.	8
1. Non-tilting ellipsoid, unconstrained axis angle.	
2. Non-tilting ellipsoid, axis angle equal to hue angle.	
C. Results of ellipsoid tests; hue-chroma-value axis ratios.	17
III. Tests on Multiple Sample Collections	17
A. Selection of sample pairs.	17
B. Presentation of sample pairs.	21
C. Scoring by logistic function.	23
D. Results of tests.	23
1. Differences between plus and minus directions.	
2. Differences between hue, chroma and lightness; axis ratios.	
3. Differences between inspectors from industry and from government service.	
IV. Acceptability Equations	32
V. Conclusions and Recommendations	37
References	38

	Page
Appendices	
A. Computer program for study of relative positions of standard and limit samples (Program RAMSLEY).	39
B. Computer program for determination of minimum non-tilting ellipsoid containing standard and limit samples (Programs ELPSOID and ELPSOD2).	45
C. Computer program for rearranging file in numerical order (Program REARRAN).	57
D. Computer program for selecting sample pairs for inspectors' evaluation (Program SIXWAYS)	58
E. Computer program for randomizing order of presentation of sample pairs (Program FUNFER).	64
F. Computer program for calculation of 50% acceptability limits by logistic function (Program LOGIT).	65
G. Derivation and meaning of the acceptability equation.	70

LIST OF FIGURES

Figure 1. b^* vs. a^* plot of standard and limit samples for Olive Drab 7 untreated cotton duck, 8.25 oz.	12
Figure 2. L^* vs. a^* plot of standard and limit samples for Olive Drab 7 untreated cotton duck, 8.25 oz.	13
Figure 3. Illustration in two dimensions of the two minim ellipsoids.	16
Figure 4. Sample pairs for Olive Green selected for observers' evaluation plotted on a^*-b^* diagram.	22
Figure 5. Vectorial representation of tetrahedron created by four samples in CIELAB color space.	40

LIST OF TABLES

	Page
Table 1. Designations, CIELAB Coordinates, and Geometric Orientations of Standards and Limit Samples (D75, CIE 1931).	9
Table 2. Minimum Ellipsoid Characteristics.	15
Table 3. Angle of Principal Horizontal Axis of Ellipsoid with a^* Coordinate Axis, in Degrees.	18
Table 4. Axis Ratios and Ellipsoid volumes for Ellipsoids Printing in Isohue Direction.	18
Table 5. Raw Results for Observers A, B, and C, Together with 50% Acceptance Limits.	24
Table 6. Raw Results for Observers D, E, and F, Together with 50% Acceptance Limits.	27
Table 7. Raw Results for Observer G on Dark Blue, Together with 50% Acceptance Limits.	30
Table 8. Summary of Tolerance Results.	31
Table 9. Application of Tolerance Equations to Limit Samples.	35

INVESTIGATIONS TO DEFINE ACCEPTABILITY TOLERANCE RANGES IN VARIOUS REGIONS OF COLOR SPACE

I. INTRODUCTION

The establishment of acceptability tolerance limits for purchased goods is a problem as old as the practice of colorimetry. Basing the tolerance on a perceptibility criterion has become discredited, as it is now generally realized that perceptibility and acceptability are not the same. A method of setting up tolerance limits that really represent the informed judgment of experienced inspectors is greatly to be desired. This need becomes all the more cogent as instrumental color measurement gradually displaces visual methods.

The United States Army has been accepting shipments of textiles to be made into army uniforms on a visual basis. The inspector is provided with a standard sample and eight limit samples representing maximum allowable deviations from the standard in each of eight directions in color space. Armed with these nine samples, the inspector examines the submitted sample and issues a pass or fail verdict. This visual method has proved troublesome, as evidenced by frequent disagreements between the Army and the textile mills submitting the samples. The Army therefore wishes to change its acceptance procedures to one using instrumental measurement and valid tolerance limits for acceptance.

The object of this investigation was to develop a procedure for setting up these valid acceptance limits for use with instrumental measurement. The Army submitted the standards and visually selected limit samples for twelve colors of long-term interest. In addition, some 200-300 samples were submitted for each of three other shades. These samples were routine submissions for acceptance or rejection received from various textile mills. We were not requested to set up tolerances for any of these standards; we were rather asked to develop a modus operandi that the Army could use to set up such tolerances for any sample in the future.

II. TESTS ON STANDARD AND LIMIT SAMPLES

A. Relationships between standard and limit samples in color space.

Ideally, the standard and the eight limit samples for any color should form a three-dimensional figure in color space, with the standard somewhere in the interior of the figure and all the limit samples on the boundary. We wished to see if this was indeed the case for limit standards that had been chosen visually and been used successfully for several years in Government procurement.

We obtained measurements of all the standard and limit samples on a Diano-Hardy spectrophotometer, together with calculated CIELAB coordinates L^* , a^* and b^* . We then wrote a computer program that created a three-dimensional figure in CIELAB color space for each of the twelve colors. This figure was constructed

from the standard and the eight limit samples all taken as a group. The program in effect created all possible tetrahedra that could be constructed from all the nine samples taken four at a time, and then put these tetrahedra together to form the three-dimensional figure. The program then determined which of the nine samples (the standard and the eight limit samples) lay inside the figure and which lay on the boundary. Appendix A describes the program and presents the mathematical details, as well as lists of the computer program itself (program RAMSLEY).

Table 1 presents the computer printout from this program. For each of the twelve shades the printout shows the designations of the limit samples, the CIELAB coordinates of the limit samples as well as the standard, and an indication of whether or not each of the standards and limit samples was on the boundary of the three-dimensional figure. We see that in only three of the twelve cases did the standard lie inside the figure; in the other nine cases it was on the boundary. Furthermore, in ten out of the twelve cases one or more of the limit samples fell inside the figure; in only two sets were all the limit samples on the boundary.

Figures 1 and 2 show CIELAB color space plots of the standard and the limit samples for Olive Drab 7, untreated cotton duck, 8.25 oz. Figure 1 shows a plot of b^* against a^* . The isohue line indicated on the figure is the line joining the standard with the origin; the isochroma line is a segment of a circle drawn through the standard around the origin. The letters shown are abbreviations for the designations of the limit samples; their meaning can be inferred from the designations listed for this sample in Table 1. Although they are useful to visual graders, we see little correlation between the color names applied to some of the limit samples and their position on this diagram. In Figure 2, which is a plot of L^* against a^* , we see that there is good correlation between the thin or full designation and the lightness of the sample; the thin samples are uniformly lighter than standard and the full samples darker. A glance at the L^* values in Table 1, however, shows that this correlation does not strictly hold in all cases; there are cases where the thin samples are darker than standard or the full samples lighter.

Summarizing the results of this section, we see that the standard and the limit samples do not, in many cases, have the expected orientation of standard in the middle and limit samples on the outside. Also, the descriptive designations of the limit samples do not always show a correlation with their plotted points in CIELAB color space. These factors probably reflect the experience of dyers, their practices in matching shades, and their terminology.

B. Determination of minimum ellipsoid containing standard and limit samples.

1. Non-tilting ellipsoid, unconstrained axis angle. In order to learn more about the orientation of the standard and limit samples in CIELAB

Table 1. Designations, CIELAB Coordinates, and Geometric Orientations of Standards and Limit Samples (D75, CIE 1931)

OLIVE DRAB 7, UNTREATED COTTON DUCK, 8.25 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	34.12	-2.71	10.85	ON BOUNDARY
THIN STANDARD	34.45	-2.64	11.07	NOT ON BOUNDARY
THIN YELLOW	36.05	-3.15	11.75	ON BOUNDARY
THIN GREEN	36.25	-2.70	11.04	ON BOUNDARY
THIN RED	36.56	-2.22	10.75	ON BOUNDARY
FULL STANDARD	32.65	-2.34	10.65	ON BOUNDARY
FULL YELLOW	33.19	-2.49	10.84	ON BOUNDARY
FULL GREEN	33.53	-2.73	10.82	ON BOUNDARY
FULL RED	32.63	-1.70	10.13	ON BOUNDARY

TAN 46, COTTON POPLIN, 4 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	68.90	1.03	13.69	ON BOUNDARY
THIN STANDARD	71.13	.69	13.47	ON BOUNDARY
THIN RED	67.88	1.15	13.21	NOT ON BOUNDARY
THIN BLUE	69.57	1.20	12.13	ON BOUNDARY
FULL STANDARD	67.72	.76	13.20	ON BOUNDARY
FULL YELLOW	68.40	.31	13.83	ON BOUNDARY
FULL RED	67.10	1.30	12.86	ON BOUNDARY
FULL BLUE	69.23	.81	13.09	NOT ON BOUNDARY

TAN M-1, CL. POLY/WOOL TROP. 9 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	54.86	2.96	15.73	NOT ON BOUNDARY
THIN STANDARD	55.28	2.77	15.75	NOT ON BOUNDARY
THIN YELLOW	55.99	2.62	15.82	ON BOUNDARY
THIN GREEN	54.67	2.75	15.30	ON BOUNDARY
THIN RED	54.74	3.15	15.62	ON BOUNDARY
FULL STANDARD	54.76	2.85	15.99	NOT ON BOUNDARY
FULL YELLOW	54.42	2.92	16.06	ON BOUNDARY
FULL GREEN	54.65	2.41	15.82	ON BOUNDARY
FULL RED	54.19	3.17	15.85	ON BOUNDARY

ARMY GREEN 44, WL GAB., 130Z.

SAMPLE	L	A	B	POSITION
STANDARD	26.49	-3.36	1.34	ON BOUNDARY
THIN STANDARD	26.68	-3.39	1.12	ON BOUNDARY
THIN BLUE	26.51	-3.53	1.05	ON BOUNDARY
THIN GREEN	26.20	-3.61	1.07	ON BOUNDARY
THIN RED	26.52	-3.11	1.52	ON BOUNDARY
FULL STANDARD	25.87	-3.15	1.30	ON BOUNDARY
FULL BLUE	25.83	-3.38	.88	ON BOUNDARY
FULL GREEN	26.13	-3.51	1.42	ON BOUNDARY
FULL RED	26.34	-3.09	1.63	ON BOUNDARY

Table 1. (continued)

OLIVE GREEN 107, NYLON/CTN (50/50) POPLIN, 5 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	32.26	-3.46	11.37	NOT ON BOUNDARY
THIN STANDARD	33.59	-3.65	11.93	ON BOUNDARY
THIN YELLOW	33.84	-3.66	11.95	ON BOUNDARY
THIN GREEN	32.95	-3.68	11.32	ON BOUNDARY
THIN RED	33.52	-3.56	11.74	ON BOUNDARY
FULL STANDARD	31.18	-3.21	11.21	ON BOUNDARY
FULL YELLOW	31.89	-3.28	11.73	ON BOUNDARY
FULL GREEN	32.04	-3.65	11.55	ON BOUNDARY
FULL RED	31.95	-3.31	11.32	ON BOUNDARY

BLUE 150TROP. WL. 10 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	15.46	1.12	-4.52	ON BOUNDARY
THIN STANDARD	15.70	1.31	-4.61	ON BOUNDARY
THIN BLUE	15.61	1.23	-4.91	ON BOUNDARY
THIN GREEN	15.67	1.11	-4.50	NOT ON BOUNDARY
THIN RED	15.60	1.24	-4.60	ON BOUNDARY
FULL STANDARD	15.70	1.03	-4.18	ON BOUNDARY
FULL BLUE	15.70	1.04	-4.56	ON BOUNDARY
FULL GREEN	15.41	1.08	-4.24	ON BOUNDARY
FULL RED	15.58	1.20	-4.28	ON BOUNDARY

ARMY GREEN 344, POLYESTER/WOOL GABARDINE 9 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	27.12	-3.52	1.47	ON BOUNDARY
THIN STANDARD	27.13	-3.39	1.40	NOT ON BOUNDARY
THIN BLUE	27.30	-3.42	1.20	ON BOUNDARY
THIN GREEN	27.26	-3.60	1.42	ON BOUNDARY
THIN RED	26.47	-3.47	1.31	NOT ON BOUNDARY
FULL STANDARD	27.27	-3.13	1.34	ON BOUNDARY
FULL BLUE	26.65	-3.50	1.14	ON BOUNDARY
FULL GREEN	26.39	-3.68	1.31	ON BOUNDARY
FULL RED	26.44	-3.19	1.34	ON BOUNDARY

ARMY GREEN 344, POLYESTER/WOOL TROPICAL, 10 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	27.33	-3.29	1.43	ON BOUNDARY
THIN STANDARD	26.63	-3.66	1.12	ON BOUNDARY
THIN BLUE	26.37	-3.36	1.08	NOT ON BOUNDARY
THIN GREEN	27.15	-3.79	1.10	ON BOUNDARY
THIN RED	27.10	-3.13	1.34	ON BOUNDARY
FULL STANDARD	25.35	-3.01	.78	ON BOUNDARY
FULL BLUE	24.98	-3.00	.72	ON BOUNDARY
FULL GREEN	26.07	-3.05	1.00	ON BOUNDARY
FULL RED	25.76	-2.75	1.13	ON BOUNDARY

Table 1. (continued)

ARMY GREEN 44, WL. SERGE 15 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	26.25	-3.18	1.11	ON BOUNDARY
THIN BLUE	26.10	-3.53	1.18	ON BOUNDARY
THIN GREEN	26.21	-3.52	1.17	ON BOUNDARY
THIN RED	26.21	-3.46	1.24	ON BOUNDARY
FULL STANDARD	26.15	-3.43	1.00	ON BOUNDARY
THIN STANDARD	26.24	-3.49	1.22	ON BOUNDARY
FULL BLUE	26.47	-3.46	.97	ON BOUNDARY
FULL GREEN	25.79	-3.35	1.09	ON BOUNDARY
FULL RED	26.22	-3.36	1.13	NOT ON BOUNDARY

OLIVE GREEN 108, WOOL/NYLON, 16 OZ. SHIRTING

SAMPLE	L	A	B	POSITION
STANDARD	29.60	-2.66	11.77	ON BOUNDARY
THIN STANDARD	30.51	-2.88	12.35	ON BOUNDARY
THIN YELLOW	30.35	-2.92	12.58	ON BOUNDARY
THIN GREEN	30.52	-3.41	12.21	ON BOUNDARY
THIN RED	30.19	-2.74	12.06	ON BOUNDARY
FULL STANDARD	28.52	-2.75	11.44	NOT ON BOUNDARY
FULL YELLOW	28.48	-2.72	11.64	ON BOUNDARY
FULL GREEN	29.28	-3.16	11.52	ON BOUNDARY
FULL RED	27.81	-2.41	10.70	ON BOUNDARY

OLIVE GREEN 107, CTN. SATEEN, 8.2 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	31.30	-3.46	10.31	ON BOUNDARY
THIN STANDARD	31.78	-4.11	11.10	ON BOUNDARY
THIN YELLOW	31.78	-3.86	11.58	ON BOUNDARY
THIN RED	31.60	-3.58	11.58	ON BOUNDARY
THIN GREEN	30.64	-4.13	10.84	ON BOUNDARY
FULL STANDARD	30.84	-3.56	10.74	NOT ON BOUNDARY
FULL YELLOW	30.62	-3.14	10.84	ON BOUNDARY
FULL GREEN	30.35	-3.73	10.70	ON BOUNDARY
FULL RED	31.53	-3.04	10.88	ON BOUNDARY

BLUE 151, TROP. WL., 12 OZ.

SAMPLE	L	A	B	POSITION
STANDARD	23.48	4.48	-17.33	NOT ON BOUNDARY
THIN STANDARD	23.85	4.68	-17.47	ON BOUNDARY
THIN BLUE	23.54	4.39	-17.10	ON BOUNDARY
THIN GREEN	24.44	4.65	-18.13	ON BOUNDARY
THIN RED	24.18	4.61	-17.72	ON BOUNDARY
FULL STANDARD	23.07	4.40	-17.07	NOT ON BOUNDARY
FULL BLUE	22.84	4.34	-16.72	ON BOUNDARY
FULL GREEN	23.26	4.43	-17.52	ON BOUNDARY
FULL RED	23.12	4.62	-17.19	ON BOUNDARY

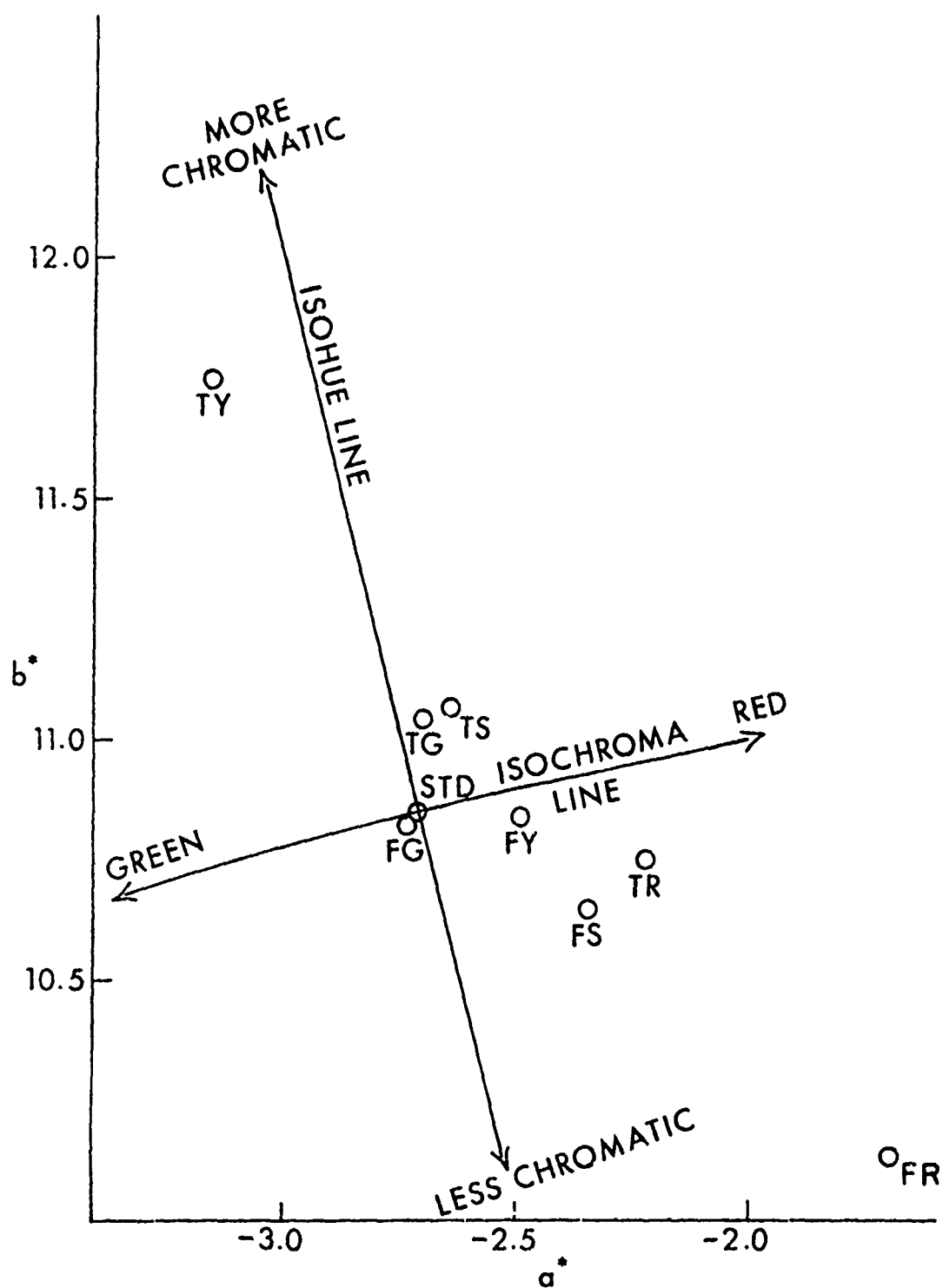


Figure 1. b^* vs. a^* plot of Standard and limit samples for Olive Drab 7
Untreated cotton duck, 8.25 oz.

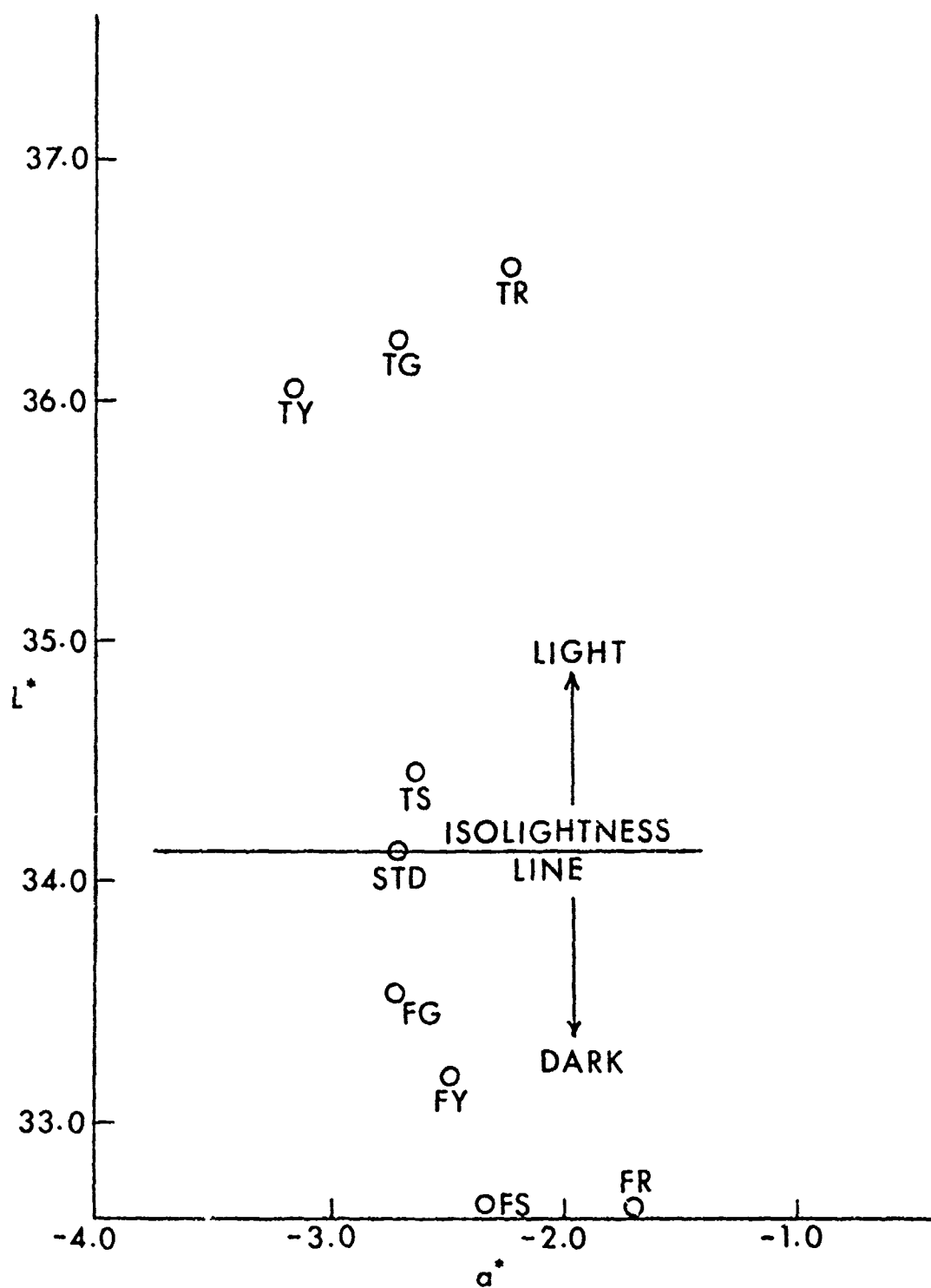


Figure 2. L* vs. a* plot of Standard and limit samples for Olive Drab 7 untreated cotton duck, 8.25 oz.

color space, we wrote a computer program that determined the minimum non-tilting ellipsoid that could be constructed around these samples. The non-tilting feature meant that the ellipsoid had a horizontal major axis, a horizontal minor axis, both in the chromaticity plane, and a vertical axis; the two horizontal axes lay entirely in the a^*b^* plane and the vertical axis was perpendicular to the other two. We propose this non-tilting restriction in the spirit of the observations of Schultze and Gall (1) and of MacAdam (2), as applied to perceptibility ellipsoids.

Each ellipsoid was therefore defined by seven parameters; the three position coordinates of the center, the length of the three semiaxes, and the angle that the major axis in the chromaticity plane makes with the a^* axis. The computer program varied these seven parameters one at a time in a systematic fashion, each time determining whether a smaller ellipsoid could be constructed that contained all the points for the standard and limit samples. The program stopped when no parameter variation was possible that would shrink the ellipsoid without excluding one of the points. Appendix B describes the mathematics used and presents the computer program, called ELPsOID. The results from this program are given in Table 2. In this table the 12 colors are presented in the same order as they are in Table 1.

2. Non-tilting ellipsoid, axis angle equal to hue angle. We wrote another minimum ellipsoid program, similar to the first except that this time we had only six adjustable parameters instead of seven. The axis angle was set equal to the hue angle; the latter was the angle between the line connecting the center of the ellipsoid with the L^* axis and the a^* axis. Thus, one of the horizontal axes of the ellipsoid always coincides with the isohue direction, the other horizontal axis with the isochroma direction. Figure 3 serves as an illustration, in two-dimensional space, of the difference between the two programs. The color points shown were plotted arbitrarily. The two ellipses drawn on the figure, however, were calculated by a variation of the two programs that constructs ellipses instead of ellipsoids. The dashed ellipse is the smallest ellipse of any orientation that can be drawn surrounding all the points whose major axis coincides with an isohue line.

The reason for this second minimum ellipsoid program was that we wished to take advantage of the commercial experience which has been built up in the

1. W. Schultze and L. Gall, "Experimentelle Überprüfung Mehrerer Farbabstandsformeln bezüglich der Helligkeits- und Sättigungsdifferenzen bei gesättigten Farben," Farbe 18, 131-148 (1969).
2. D. L. MacAdam, in Color Metrics, edited by J. J. Vos, L.F.C. Friele and P.L. Walraven (AIC/Holland, Soesterburg, 1972) pp 160-170.

Table 2. Minimum Ellipsoid Characteristics

	COLOR NO.											
	1	2	3	4	5	6	7	8	9	10	11	12
<u>Unconstrained Azimuthal Angle</u>												
Ellipsoid												
Center: L *	34.44	68.91	54.89	26.25	32.51	15.60	26.84	26.27	26.24	29.11	31.23	23.64
a *	-2.42	.77	2.82	-3.33	-3.43	1.13	-3.41	-3.27	-3.36	-2.86	-3.66	4.49
b *	10.84	12.98	15.73	1.18	11.58	-.44	1.31	1.01	1.10	11.74	10.97	-17.42
Semiaxis												
Lengths:												
Maj. Horiz.	1.58	.99	.44	.53	.55	.48	.39	.61	.19	1.35	.78	1.17
Min. Horiz.	.26	.65	.43	.27	.36	.17	.19	.50	.19	.47	.60	.19
Vertical	2.56	2.62	1.29	.55	2.15	.22	.63	1.78	.45	2.40	.99	1.05
< between												
Horiz. axis &												
a* axis, deg.	-48.2	-62.0	73.6	68.8	-58.7	-76.4	1.8	-23.3	a	-65.8	-55.7	-80.4
Vol.	4.40	7.07	1.03	.33	1.78	.08	.19	2.28	.07	6.36	1.95	.98
<u>Constrained Azimuthal Angle</u>												
Ellipsoid												
Center: L *	34.48	68.79	54.81	26.22	32.44	15.60	26.89	26.40	26.21	29.47	31.16	23.65
a *	-2.42	.91	2.79	-3.35	-3.50	1.15	-3.43	-3.23	-3.37	-2.85	-3.63	4.51
b *	10.86	13.18	15.77	1.26	11.47	-4.49	1.33	1.03	1.07	11.41	10.96	-17.36
Semiaxis												
Lengths:												
Maj. Horiz.	1.55	1.27	.48	.21 _b	.66	.44	.45	.63	.22	1.54	.91	1.06
Min. Horiz.	.93	.74	.43	.60 _b	.31	.15	.22	.43	.17	.52	.62	.16
Vertical	3.15	2.54	1.30	.63	2.21	.24	.69	2.04	.43	2.37	.90	1.37
< between												
Horiz. axis &												
a* axis, deg.	-77.4	86.1	80.0	-20.6	-73.0	-75.6	-21.1	-17.7	-17.7	-76.0	-71.7	-75.4
Vol.	19.04	9.96	1.12	.34	1.86	.07	.28	2.32	.07	7.94	2.14	.99

a Two horizontal axes equal; therefore angle cannot be specified.

b Iso(hue-lightness) axis (considered by computer program to be major horizontal axis) is shorter than iso(chroma-lightness) axis.

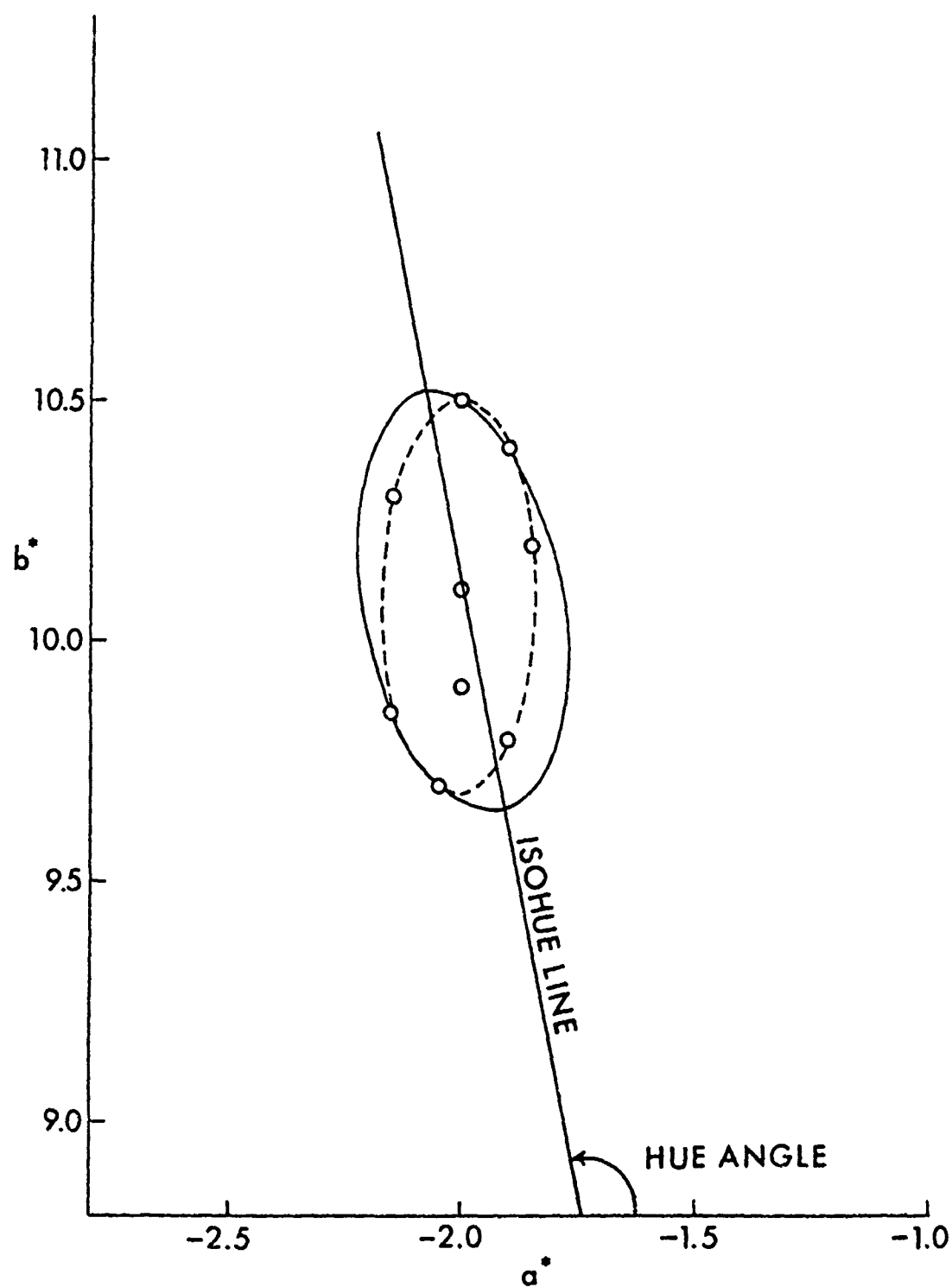


Figure 3. Illustration in two dimensions of the two minimum ellipsoid.

textile industry relating to acceptance of production lots. This experience has indicated that acceptability tolerances in a given uniform color space should be so arranged as to allow the greatest leeway in the lightness direction, intermediate latitude in the chroma direction, and the most stringent tolerances in the hue direction (3). We can thus picture an acceptability ellipsoid with the largest axis pointing in the L^* direction, the intermediate axis in the chromaticity plane along an isohue line, and the shortest axis in the chromaticity plane perpendicular to the isohue line. The ratios between these three tolerances have typically been found to range between 3:2:1 and 4:?:1, in CIELAB space.

This computer program was called ELPSOD2, and is given in Appendix B.

Results of this program are presented in Table 2.

C. Results of ellipsoid tests; hue-chroma-value axis ratios.

Comparison of the axis angles given by the two minimum ellipsoid programs is interesting and is shown in Table 3, which was abstracted from the results in Table 2. We see a remarkable agreement between the two programs with respect to the axis angle of the ellipsoid. In six out of eleven cases the angles agreed to 10° or less; in eight out of eleven cases the agreement is to 20° or less. Since in one of the programs, one horizontal axis was constrained to the isohue direction, whereas in the other program the axes were unconstrained, we conclude that the acceptability ellipsoids, at least insofar as they are defined by the standard and limit samples, naturally tend to orient themselves in the isohue and isochroma directions.

Table 4, calculated from the figures in Table 2, shows the ratios of the three principal axes for the constrained ellipsoids, as well as the ellipsoid volumes. We see that in nine out of the twelve cases we have the expected size relationship of the largest axis being in the lightness direction, the intermediate axis in the chroma direction, and the smallest axis in the hue direction. In one of the other three cases the lightness and chroma axes were equal; in the second of the three exceptions the chroma and hue axes were reversed from the expected order, and in the third case the lightness and chroma axes were reversed. These results seem to bear out the commercial experience for textile industry color acceptances mentioned above.

III. TESTS ON MULTIPLE SAMPLE COLLECTIONS

A. Selection of sample pairs

As a general policy, we thought that acceptability standards should be based on the visual judgment of trained and experienced observers. In view of the ellipsoid orientation indicated by the work just described, we thought that it would be appropriate to establish six tolerances: two in the plus and minus

3. "The Hue Angle Sort Program," pamphlet issued by Hunter Associates Laboratory, Inc.

TABLE 3. ANGLE OF PRINCIPAL HORIZONTAL AXIS OF ELLIPSOID WITH a* COORDINATE AXIS, IN DEGREES.

	Angle unconstrained	Angle constrained along isohue line	Corrected Difference
Olive Drab 7, Cotton Duck	-48	-77	29
Olive Green 107, Nylon/cotton	-59	-73	14
Olive Green 108, Wool/nylon	-66	-76	10
Olive Green 107, Cotton sateen	-56	-72	16
Army Green 44, Wool Gabardine	69	-21a	0
Army Green 344, Polyester/Wool Gabardine	2	-21	23
Army Green 344, Tropical Wool	-23	-18	5
Army Green 44, Wool Serge	b	-18	
Tan 46, Cotton Poplin	-62	86	32
Tan M-1, Polyester/Wool Tropical	74	80	6
Blue 150, Tropical	-76	-76	0
Blue 151, Tropical	-80	-75	5

a Axis in isohue direction smaller than axis in isochroma direction; angle must therefore be increased by 90° before comparison with value in left column.

b. Two horizontal axes equal; therefore angle cannot be specified.

TABLE 4. AXIS RATIOS AND ELLIPSOID VOLUMES FOR ELLIPSOIDS POINTING IN ISOHUE DIRECTION

	Lightness:Chroma:Hue	Ellipsoid Volume
Olive Drab 7, Cotton Duck	3:37 : 1.66 : 1.00	19.04
Olive Green 107, Nylon/Cotton	7.23 : 2.16 : 1.00	1.86
Olive Green 108, Wool/Nylon	4.59 : 2.98 : 1.00	7.94
Olive Green 107, Cotton Sateen	1.45 : 1.45 : 1.00	2.14
Army Green 44, Wool Gabardine	1.05 : 0.36 : 1.00	0.34
Army Green 344, Polyester/Wool	3.23 : 2.10 : 1.00	0.28
Army Green 344, Tropical	4.72 : 1.45 : 1.00	2.32
Army Green 44, Wool Serge	2.56 : 1.29 : 1.00	0.07
Tan 46, Cotton Poplin	3.44 : 1.72 : 1.00	9.96
Tan M-1, Polyester/Wool Tropical	3.00 : 1.10 : 1.00	1.12
Blue 150, Tropical	1.62 : 3.01 : 1.00	0.07
Blue 151, Tropical	8.41 : 6.53 : 1.00	0.99

chroma directions, two in the plus and minus lightness directions, and two in the plus and minus hue directions. The tolerances would be set up on the basis of the responses of the inspectors to samples shown to them. We would then establish an acceptability ellipsoid that incorporates these tolerances and that has its three axes oriented in the hue, chroma, and lightness directions.

The problem then arises as to how to obtain samples that differ from the standard only in hue, chroma, or lightness and in varying amounts for each. We rejected a preliminary plan involving the dyeing of samples of various degrees of color difference in each of the six directions when we realized how difficult it would be to prepare such dyeings. In order to obtain samples having the precise location in color space that we want, we would have to have at least two and probably more trial dyeings for each sample. Such a procedure would take an inordinate amount of time.

A much better plan seemed to be to try to find the desired samples among previous submissions of samples from textile mills. For each shade that is actively used by the Army, textile mills are continually submitting samples that are either accepted or rejected by the inspectors. Fortunately, a set of some 200 to 300 samples was available for each of three shades of interest to the Army: olive green, tan, and dark blue. These samples are located in a cloud of points close to the standard, since they are intended to match the standard. It should be possible to find, among all these samples, test cases that could be shown to the observers and that would serve to define the required tolerances.

We wished to find samples representing four color differences of increasing amount in each of the six directions mentioned above. These color differences would ideally extend from completely acceptable (for the smallest color difference) to completely unacceptable (for the largest color difference). The two intermediate color differences would presumably be close to the tolerance limits. The procedure would be to show each of the color differences to an observer ten times (randomly mixed with other samples so that the inspector does not realize that he is looking at the same sample repeatedly). Each time that the observer is shown the color difference, he is asked whether he would accept or reject such a difference if it were an incoming shipment against standard. The experimental regime is known as the method of constant stimuli.

To explain the procedure, let us assume that the four color differences chosen are 0.5, 1.0, 1.5, and 2.0 CIELAB units. Let us observe that the observer accepts the 0.5 color difference ten times out of ten; the 1.0 color difference seven times out of ten; the 1.5 color difference three times out of ten. It can then easily be seen that the 50% acceptance limit should be close to 1.25 color difference units. In fact, in the more general case, the method of logistic functions (see below) is used to determine the 50% acceptability limit.

The question then arises as to how to select these color differences out of the cloud of 200 to 300 samples. We realize that it is not necessary to have only a single standard against which all the judgments would be made. All that is necessary is a selection of sample pairs, of which one would play the role of standard and the other would play the role of sample submitted for acceptance. Since we have six directions in color space with four degrees of color difference for each direction, we need 24 sample pairs. These pairs were chosen in the following way:

We first measured all the samples for each of the three colors on the Diano-Hardy spectrophotometer, and obtained the CIELAB coordinates (D75, CIE 1931). We then created a computer file with an entry for each of the 200 to 300 samples for each color; the file contained the numerical sample designations and the three CIELAB coordinates $L^*a^*b^*$. We rearranged this file so that it was in increasing numerical order for the sample designations. This rearrangement was carried out by Program REARRAN, given in Appendix C.

We then wrote a computer program which we called Program SIXWAYS. For each of the three collections of samples, the program took each individual sample, one at a time, and constructed three lines in CIELAB color space passing through the sample. The first line was an iso (hue-lightness) line, which means that it was a line in the $a^* - b^*$ plane that joined the origin with the point in question. Since the line passed through the point for the sample, it extended from the sample in two directions: the direction of increasing chroma and the direction of decreasing chroma. The second line was an iso (chroma-lightness) line, or a line in the $a^* - b^*$ plane that was perpendicular to the first. This line also passed through the sample and extended from the sample in two opposite hue directions. The third line was an iso (hue-chroma) line, or a vertical line through the sample perpendicular to the $a^* - b^*$ plane and extending from the sample in a plus lightness and a minus lightness direction.

The computer program thus contained an outer loop that was traversed n times, where n is the number of samples in the collection. For each of these samples, the program constructed the six lines in CIELAB color space just described. The program then went through an inner loop, with $n - 1$ passages, during which it again considered every sample in the collection and determined whether it lay on one of the six lines that was constructed (the program actually determined whether the sample deviated from these lines by no more than 0.1 CIELAB unit). If the sample did so lie, the program calculated the color difference between the two samples. We thus had, for each of the samples in the collection considered temporarily as a standard, a subset of samples extending from the standard in each of the six directions, together with the associated color differences between each sample in the subset and the temporary standard. It was therefore relatively easy to select pairs of samples for which the "sample" deviated from the "standard" in each of six directions and with four different color differences in each of the directions. Notice that there was no single fixed standard chosen; the standard varies with each pair. Notice also that the sizes of the color differences available were determined and limited by the particular samples in the collection.

Appendix D shows the mathematics that was used to determine how far a given sample deviated from the six iso lines radiating from another sample. The same appendix presents the listing of programs SIXWAYS. Also included in the appendix is a typical page from the output of program SIXWAYS.

Figure 4 shows an $a^* - b^*$ plot of the samples making up the four color differences of increasing chroma and the four color differences of increasing hue for the olive green collection. The points represent actual samples in the collection; the solid lines join pairs of points for the various color differences. The dashed lines are the true constant chroma lines and constant hue lines, and it can be seen that in no case does a sample deviate from these lines by more than 0.1 CIELAB unit.

We were able to physically reduce the number of pairs of samples chosen by almost half, because in all but a few cases, the same pair of samples was used for a minus difference as for a plus difference. For example, let us suppose that Samples A and B have the same hue and lightness but differ by 1.0 CIELAB unit in chroma. In order to determine whether there is a difference in acceptability in the plus chroma or the minus chroma direction, the pair will be shown to the inspector in two different ways; in the first way, Sample A is the standard and Sample B is the sample to be judged against the standard; in the second way, Sample B is the standard and Sample A is the sample. In fact, if Sample B would appear under the Sample A heading in the printout from program SIXWAYS as being more chromatic than Sample A, then Sample A would appear under the Sample B heading as being less chromatic.

B. Presentation of Sample Pairs

As a result of work described in the preceding section, we had 72 sample pairs to show to the observers (three shades, six directions in color space for each shade, that is + chroma, + lightness, and four degrees of color difference for each direction). We wished to show each sample pair to each observer ten times, making a total of 720 individual observations by each inspector. In order not to confuse the observers more than was absolutely necessary, we did one shade at a time, involving 240 judgments per shade. We wrote a computer program, entitled Program FUNFER, that used a random number generator to randomize the order of presentation of these 240 trials (see Appendix E). The observers were not told that there were replicate sample presentations, although they probably guessed that from the fact that there were many fewer sample pairs than there were presentation. In order to prevent the inspector being influenced by the location of the sample pair as he saw it being removed, we had a "lazy susan" sample holder constructed, on which the samples were arranged circularly and accessed by rotating a tray. In this way, distinctive sample positions were abolished.

We administered the test to six observers, three from the government service and three from industry. In addition, a seventh observer, previously from industry but now in the government service, looked at only one of the three shades--the dark blue. The observer was presented with a sample pair

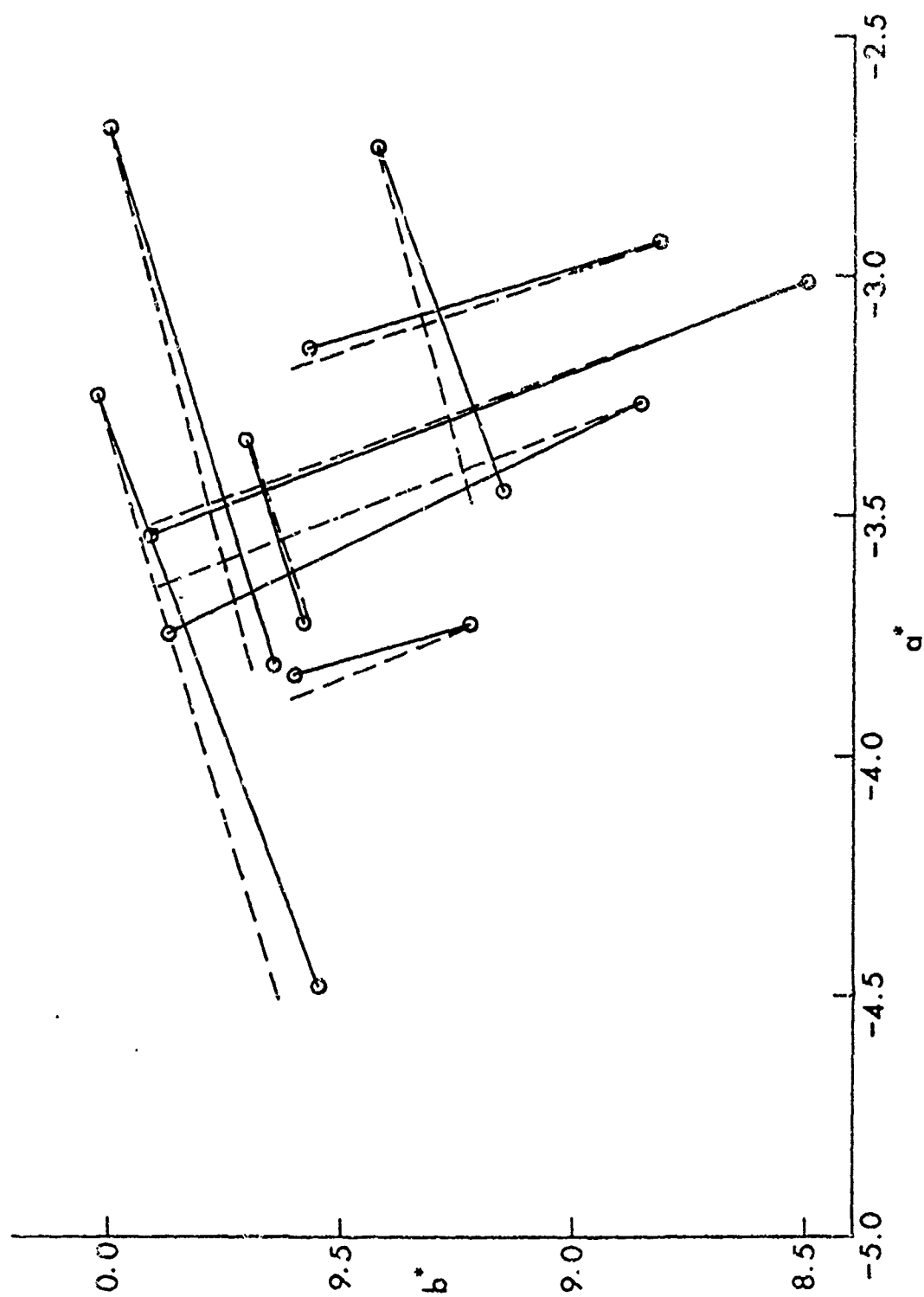


Figure 4. Sample pairs for Olive Green selected for observers' evaluation plotted on a^* - b^* diagram.

and was told which was the standard and which was the sample to be compared with standard. He or she was told to either accept or reject the sample against standard; no intermediate judgments were allowed. The observers' judgments were tabulated, and the percentage of passes for each pair was noted.

Tables 5, 6 and 7 show the raw results of the observers for all the samples. In the leftmost column, the CIELAB total color difference for each pair is given. In the columns to the right the percent pass judgments for each pair is shown. A, B and C were the three observers from the government service; D, E, and F were the three observers from industry. G was the observer who was previously in industry and is now in the government service, who looked at only the dark blue series of samples.

We see that in most cases the percentage of samples passed decreases with increasing color difference. Occasionally there are reversals, as indicated by a larger color difference giving a greater percentage of acceptances. In some cases these reversals are severe enough to invalidate that particular set of judgments, as will be seen later. For the most part, however, the results of these tests are surprisingly good.

C. Scoring by Logistic Function

The raw data in Tables 5, 6 and 7 take the form of percent acceptance as a function of color difference. We now have to calculate from these data, on a valid statistical basis, the color difference that would correspond to a 50% acceptance rate. We used the method of logistic functions, explained in Appendix F. This method not only gives the 50% point but also gives some idea of the precision of this determination in the form of a standard deviation.

In Tables 5, 6 and 7 we see next to each group of four acceptance percentages the calculated color difference corresponding to a 50% acceptance rate, together with the standard deviation of this color difference. This standard deviation is really a measure of the self-consistency of the acceptance judgments. If there are serious reversals in the judgments, the standard deviation may be quite large in comparison with the value itself (in some cases larger).

We decided, quite pragmatically, to accept only those values that were at least about four times as large as their standard deviations. In addition, in view of the fact that observer F accepted 18 out of the 24 tan pairs and 20 out of the 24 dark blue pairs on a 100% basis, we considered him unfamiliar with colors and used his results only for the olive green pairs.

D. Results of Tests

Table 8 summarizes the tolerance results that were accepted. The standard deviations shown refer to the values given in this table and do not involve the standard deviations shown in Tables 5, 6 and 7. The following observations can be made:

Table 5. Raw Results for Observers A, B, and C, Together with 50% Acceptance Limits

COLOR: OLIVE GREEN

DELTA E	A		B		C	
	PCT. PASS	BOUNDARY ± STD. DEV.	PCT. PASS	BOUNDARY ± STD. DEV.	PCT. PASS	BOUNDARY ± STD. DEV.

PLUS CHROMA

.40	100		100		100	
.80	40		50		70	
1.13	20	.78 ± .11	80	1.31 ± .46	90	4.02 ± 7.01
1.52	20		40		70	

MINUS CHROMA

.40	100		100		100	
.80	100		50		80	
1.13	100	0.00 ± 0.00	70	1.05 ± .19	80	1.47 ± .38
1.52	100		20		40	

PLUS HUE

.40	100		70		100	
.77	70		0		40	
1.17	10	.83 ± .08	0	.46 ± .07	0	.71 ± .08
1.32	0		0		0	

MINUS HUE

.40	40		90		90	
.77	20		0		0	
1.17	0	.34 ± .16	0	.53 ± .07	0	.53 ± .07
1.45	0		0		0	

PLUS LIGHT

.60	100		90		100	
1.17	70		100		100	
1.82	30	1.76 ± .38	10	1.64 ± .34	70	6.70 ± 13.73
2.29	50		40		90	

MINUS LIGHT

.60	100		100		100	
1.17	100		100		100	
1.82	100	4.46 ± 3.11	10	1.78 ± .29	100	2.29 ± .15
2.29	90		40		50	

Table 5. (continued)

COLOR: TAN

DELTA E	A		B		C	
	PCT. PASS	BOUNDARY & STD. DEV.	PCT. PASS	BOUNDARY & STD. DEV.	PCT. PASS	BOUNDARY & STD. DEV.
PLUS CHROMA						
.50	80		100		80	
.92	80		40		80	
1.33	10	.95 ± .15	0	.85 ± .09	20	.98 ± .15
1.92	10		0		10	
MINUS CHROMA						
.50	90		100		100	
.92	20		20		30	
1.33	0	.73 ± .10	10	.77 ± .13	0	.80 ± .10
1.92	0		0		0	
PLUS HUE						
.25	40		70		60	
.47	40		60		30	
.69	0	.21 ± .17	0	.43 ± .10	0	.30 ± .08
1.01	0		0		0	
MINUS HUE						
.25	50		70		60	
.48	10		0		70	
.69	0	.25 ± .07	0	.29 ± .04	10	.42 ± .15
1.01	0		0		0	
PLUS LIGHT						
.66	60		100		100	
1.32	30		50		20	
1.95	20	.84 ± .25	0	1.27 ± .14	10	1.09 ± .18
2.65	0		0		0	
MINUS LIGHT						
.66	100		100		100	
1.32	50		20		30	
1.95	30	1.41 ± .17	70	1.50 ± .38	10	1.15 ± .16
2.65	10		0		0	

Table 5. (continued)

COLOR: DARK BLUE

DELTA E	A		B		C	
	PCT. PASS	BOUNDARY & STD. DEV.	PCT. PASS	BOUNDARY & STD. DEV.	PCT. PASS	BOUNDARY & STD. DEV.
PLUS CHROMA						
.36	70		90		80	
.69	20		80		80	
1.07	0	.47 ± .14	0	1.77 ± 1.17	10	.93 ± .23
1.42	20		70		40	
MINUS CHROMA						
.36	80		100		40	
.69	30		90		80	
1.07	10	.59 ± .15	100	6.64 ± 14.74	10	.42 ± .22
1.42	30		80		10	
PLUS HUE						
.18	70		100		100	
.38	80		100		90	
.57	60	1.12 ± 1.29	100	1.67 ± 1.48	80	.69 ± .20
.75	50		90		0	
MINUS HUE						
.18	70		100		80	
.38	50		90		90	
.57	100	85.62 ± 1.01	100	.61 ± .14	100	.90 ± .68
.75	60		10		0	
PLUS LIGHT						
.50	50		100		60	
1.00	30		100		10	
1.50	0	.53 ± .20	10	1.29 ± .12	0	.56 ± .13
2.01	0		0		0	
MINUS LIGHT						
.50	90		100		70	
1.00	70		80		10	
1.50	10	1.04 ± .13	20	1.17 ± .13	0	.62 ± .12
2.01	0		0		0	

Table 6. Raw Results for Observers D, E, and F, Together with 50% Acceptance Limits

COLOR: OLIVE GREEN

DELTA E	D		E		F	
	PCT. PASS	BOUNDARY & STD. DEV.	PCT. PASS	BOUNDARY & STD. DEV.	PCT. PASS	BOUNDARY & STD. DEV.
PLUS CHROMA						
.40	100		100		100	
.80	90		90		100	
1.13	50	1.38 ± .32	40	1.13 ± .17	100	1.42 ± .10
1.52	50		30		30	
MINUS CHROMA						
.40	100		100		100	
.80	40		60		100	
1.13	100	.92 ± .18	20	.85 ± .09	70	1.21 ± .11
1.52	20		10		10	
PLUS HUE						
.40	80		100		100	
.77	0		0		70	
1.17	0	.49 ± .07	0	.55 ± .08	10	.83 ± .08
1.32	0		0		0	
MINUS HUE						
.40	100		100		100	
.77	0		0		10	
1.17	0	.55 ± .08	0	.55 ± .08	0	.60 ± .10
1.45	0		0		0	
PLUS LIGHT						
.60	100		100		100	
1.17	100		90		100	
1.82	0	1.46 ± .12	30	1.50 ± .18	40	3.15 ± 3.52
2.29	0		0		90	
MINUS LIGHT						
.60	100		100		100	
1.17	100		90		100	
1.82	0	1.58 ± .17	20	1.42 ± .16	60	4.97 ± 8.13
2.29	10		0		90	

Table 6. (continued)

COLOR: TAN

DELTA E	D		E		F	
	PCT. PASS	BOUNDARY ± STD. DEV.	PCT. PASS	BOUNDARY ± STD. DEV.	PCT. PASS	BOUNDARY ± STD. DEV.
PLUS CHROMA						
.50	100		100		100	
.92	0		90		100	
1.33	100	1.04 ± .26	100	667.20 ± 5.07	80	378.07 ± 3.62
1.92	0		100		100	
MINUS CHROMA						
.50	100		100		100	
.92	0		100		100	
1.33	70	1.22 ± .28	100	1.92 ± .20	100	5.60 ± 7.94
1.92	0		50		90	
PLUS HUE						
.25	100		100		100	
.47	90		90		100	
.69	0	.54 ± .08	20	.57 ± .07	100	076.48 ± 8.40
1.01	0		0		100	
MINUS HUE						
.25	100		100		10	
.48	100		90		100	
.69	10	.59 ± .04	10	.53 ± .07	20	.65 ± .18
1.01	0		0		100	
PLUS LIGHT						
.66	100		100		100	
1.32	20		100		100	
1.95	0	1.07 ± .16	100	2.55 ± .20	100	2.47 ± .17
2.65	0		40		30	
MINUS LIGHT						
.66	100		100		100	
1.32	0		90		100	
1.95	0	.93 ± .14	100	2.64 ± .71	100	6.47 ± 6.89
2.65	0		40		90	

Table 6. (continued)

COLOR: DARK BLUE

DELTA E	D		E		F	
	PCT. PASS	BOUNDARY & STD. DEV.	PCT. PASS	BOUNDARY & STD. DEV.	PCT. PASS	BOUNDARY & STD. DEV.
PLUS CHROMA						
.36	10		100		100	
.69	100		100		100	
1.07	0	2.71 ± 7.00	0	1.41 ± .53	100	088.70 ± 0.32
1.42	0		60		100	
MINUS CHROMA						
.36	50		100		100	
.69	100		100		100	
1.07	10	.43 ± .16	0	1.24 ± .32	100	088.70 ± 0.32
1.42	0		50		100	
PLUS HUE						
.18	90		100		100	
.38	100		100		100	
.57	100	.72 ± .31	100	.65 ± .04	100	472.55 ± 2.88
.75	0		0		100	
MINUS HUE						
.18	100		100		90	
.38	100		100		100	
.57	100	.67 ± .04	100	.67 ± .04	100	21.81 ± 0.04
.75	10		10		80	
PLUS LIGHT						
.50	100		100		100	
1.00	10		100		100	
1.50	10	.80 ± .15	0	1.22 ± .09	100	1.77 ± .12
2.01	0		0		10	
MINUS LIGHT						
.50	100		90		100	
1.00	10		90		100	
1.50	0	.77 ± .13	0	1.15 ± .23	100	1.74 ± .12
2.01	0		0		0	

Table 7. Raw Results for Observer G on Dark Blue, Together with
Calculated 50% Acceptance Limits

PCT. BOUNDARY
PASS ± STD. DEV.

80
100
10 .91 ± .24
40

100
90
0 1.82 ± 1.45
100

100
100
90*443.87 ±5.12
100

100
90
100 .68 ± .16
30

90
90
20 1.15 ± .16
10

100
100
60 1.55 ± .14
10

Table 8. Summary of Tolerance Results

Olive Green						
	<u>+ Chroma</u>	<u>- Chroma</u>	<u>+ Hue</u>	<u>- Hue</u>	<u>+ Lightness</u>	<u>- Lightness</u>
A	0.78		0.83		1.76	
B		1.05	0.46	0.53	1.64	1.78
C		1.47	0.71	0.53		2.29
D	1.38	0.92	0.49	0.55	1.46	1.58
E	1.13	0.85	0.55	0.55	1.50	1.42
F	1.42	1.21	0.83	0.60		
Av. <u>±</u> St. D.	1.13 ± 0.26		0.60 ± 0.13		1.68 ± 0.28	
	<u>+ Chroma</u>	<u>- Chroma</u>	<u>+ Hue</u>	<u>- Hue</u>	<u>+ Lightness</u>	<u>- Lightness</u>
A	0.95	0.73		0.25		1.41
B	0.85	0.77	0.43	0.29	1.27	1.50
C	0.98	0.80	0.30		1.09	1.15
D	1.04	1.22	0.54	0.59	1.07	0.93
E		1.92	0.57	0.55	2.55	2.64
Av. <u>±</u> St. D.	1.03 ± 0.37		0.44 ± 0.14		1.51 ± 0.64	
	<u>+ Chroma</u>	<u>- Chroma</u>	<u>+ Hue</u>	<u>- Hue</u>	<u>+ Lightness</u>	<u>- Lightness</u>
A		.59				1.04
B				.61	1.29	1.17
C	.93				.56	.62
D				.67	.80	.77
E		1.24	.65	.67	1.22	1.15
G	.91			.68	1.15	1.55
Av. <u>+</u> St. D.	0.92 ± 0.27		0.66 ± 0.03		1.03 ± 0.31	

1. Differences between plus and minus directions. There is no significant difference between the plus hue and minus hue tolerances or between the plus lightness and minus lightness tolerances. The difference between the plus chroma and minus chroma tolerances, however, is debatable. If we apply the Student's t test to the unpaired data, there is no significant difference between plus and minus chroma. If, however, we pair the plus chroma and minus chroma data by observer, we see that three observers had a larger plus chroma tolerance than a minus chroma tolerance for the olive green, whereas no observer had the reverse. Application of the Student's t test to the paired data showed significance for this color. But for the tan, even the paired data did not show a significant difference; three observers favored plus chroma and one favored minus chroma. We decided to consider that no difference existed between any plus and minus directions. We therefore averaged all the values, plus and minus alike in calculating averages and standard deviations, and these latter are included in Table 8.

2. Differences Between Hue, Chroma, and Lightness; Axis Ratios. There is strong confirmation of the 3:2:1 lightness: chroma: hue tolerance ratios in the cases of the olive green and the tan. For the olive green, the ratios are 2.8:1.9:1.0; for the tan, they are 3.4:2.3:1.0. But for the dark blue, the ratios are 1.6:1.4:1.0; although the differences between lightness and hue or between chroma and hue are significant, the difference between lightness and chroma is not. We see that it is dangerous to generalize, and that we must work out individual tolerances for each color.

The absolute values for the hue tolerances lie somewhere in the neighborhood of half a CIELAB unit; those for chroma tolerances are near one CIELAB unit. The lightness tolerances are near one and one-half CIELAB units for the olive green and the tan, and near one CIELAB unit for the dark blue.

3. Differences between Observers from Industry and from Government Service. We might possibly expect that the observers from industry would permit larger tolerances than those from government service. Table 8 shows that this is not the case for the values shown therein; there is no significant difference between the results of the observers from industry and those of the government observers, both taken as a group. An exception is Observer G on the tan and the dark blue, but his results for these colors were not included in Table 8.

IV. ACCEPTABILITY EQUATIONS

We use the tolerances for chroma, hue and lightness given in Table 8 to construct an acceptability ellipsoid for each color. The equation of this ellipsoid determines whether a given sample does or does not pass.

This equation takes the following form:

$$\Delta A = \left[g_{11} (\Delta a^*)^2 + 2g_{12} \Delta a^* \Delta b^* + g_{22} (\Delta b^*)^2 + g_{33} (\Delta L^*)^2 \right]^{1/2} \quad (1)$$

where ΔA is an acceptability figure, so scaled that normally if ΔA is less than 1, the sample is accepted; if ΔA is greater than 1, it is rejected (see further discussion on this point below). The quantities Δa^* , Δb^* , ΔL^* are sample minus standard in CIELAB coordinates.

Appendix G explains the derivation and meaning of the acceptability equation. The coefficients g_{11} , $2g_{12}$, g_{22} and g_{33} are given by the following equations where a_0^* and b_0^* represent the CIELAB a^* and b^* values of the standard, c is the chroma tolerance, h is the hue tolerance, and v (for value) is the lightness tolerance:

$$\theta = \tan^{-1} \left(b_0^* / a_0^* \right), \quad (2)$$

$$g_{11} = \left(\cos^2 \theta / c^2 \right) + \left(\sin^2 \theta / h^2 \right), \quad (3)$$

$$2g_{12} = 2 \sin \theta \cos \theta \left[\left(1/c^2 \right) - \left(1/h^2 \right) \right], \quad (4)$$

$$g_{22} = \left(\sin^2 \theta / c^2 \right) + \left(\cos^2 \theta / h^2 \right), \quad (5)$$

$$g_{33} = 1/v^2. \quad (6)$$

To illustrate how these equations are used, let us apply them to the olive green. The olive green standard has the CIELAB coordinates $L_0^* = 31.71$, $a_0^* = 3.76$, $b_0^* = 9.31$ (see Table 9). We have $\theta = \tan^{-1} (9.31 / 3.76) = 68.01^\circ$.

This gives $g_{11} = [\cos^2 (-68.01) / 1.13^2] + [\sin^2 (-68.01) / 0.60^2] = 2.50$.

Similarly, $2g_{12} = 1.39$; $g_{22} = 1.06$, $g_{33} = 0.354$. We therefore have

$$\Delta A = \left[2.50 (a^* + 3.76)^2 + 1.39 (a^* + 3.76) (b^* - 9.31) + 1.06 (b^* - 9.31)^2 + 0.354 (L^* - 31.71)^2 \right]^{1/2}, \quad (7)$$

where a^* , b^* and L^* are the CIELAB coordinates of the sample being judged. This equation will have to be modified each time the standard is changed. The values for c , h and v , of course, were taken from Table 8 and would remain constant irrespective of the standard.

The equations for the other colors are derived in the same way. The CIELAB coordinates for the standards are given in Table 9. We have data for two different sets of limit samples that were used with the tan; each set had its own standard. Following are the equations to be used with each of the tan standards and the equation for the dark blue.

For the standard of tan set no. 1:

$$\Delta A = \left[5.09 (a^* - 2.05)^2 - 1.09 (a^* - 2.05) (b^* - 15.64) + 1.01 (b^* - 15.64)^2 + 0.439 (L^* - 57.48)^2 \right]^{\frac{1}{2}}, \quad (8)$$

For the standard of tan set no. 4:

$$\Delta A = \left[5.06 (a^* - 2.47)^2 - 1.35 (a^* - 2.47) (b^* - 15.09) + 1.05 (b^* - 15.09)^2 + 0.439 (L^* - 56.43)^2 \right]^{\frac{1}{2}}, \quad (9)$$

For the dark blue:

$$\Delta A = \left[2.30 (a^* - 0.205)^2 + 0.041 (a^* - 0.205) (b^* + 11.12) + 1.18 (b^* + 11.12)^2 + 0.943 (L^* - 22.98)^2 \right]^{\frac{1}{2}}, \quad (10)$$

We tried these equations on the limit samples for each color. Results are shown in Table 9. We see that all of the limit samples for the olive green and most of those for tan set no. 1 would fail the acceptability test as given. The average ΔA values are 1.65 for the olive green and 1.63 for tan set no. 1. For tan set no. 2 the average ΔA drops to 1.21, whereas for the dark blue the average ΔA is 0.76, with all but one of the limit samples passing the test.

It is widely believed in industry that for commercial customers the range of limit samples is influenced by the state of the marketplace at the time of choice. If goods are needed urgently, the limit sample might be chosen so as to reflect wider tolerances; if there is enough material and the concern is more with high quality, the reverse might be true. This is rarely true of military procurement; other factors are far more important. Industrial experience in matching a standard may result in tightening the range, as may be seen by comparing tan set no. 1 with tan set no. 2; the tolerance is considerably tighter in the latter set. It is tighter still in the dark blue set because the end use of the fabric is more critical.

Table 9 Application of Tolerance Equations to Limit Samples

DESIG.	L*	a*	b*	ΔA
<u>Olive Green</u>				
STD	31.71	-3.76	9.31	2.29
FY	32.29	-5.29	9.76	1.78
FS	31.72	-4.92	9.45	1.39
FG	31.87	-3.85	8.03	1.44
FB	32.35	-4.28	8.51	1.34
FR	32.10	-4.65	9.57	1.30
TG	32.23	-3.74	8.07	1.21
TY	32.43	-4.26	8.78	2.38
TB	33.79	-4.87	8.80	1.74
TS	32.65	-4.89	9.75	1.65 $\pm$ 0.43
Av. $\pm$ St. D.				
<u>Tan, Set No. 1</u>				
STD	57.48	2.05	15.64	2.84
FS	56.47	3.31	16.38	2.07
FY	57.26	2.92	16.91	1.08
FG	57.54	2.49	16.36	3.09
FR	58.52	3.41	15.91	0.91
FB	58.00	2.35	15.27	1.11
TS	58.96	2.21	15.34	1.51
TY	58.50	2.53	16.74	1.53
TR	58.42	2.67	16.30	0.56
TB	57.94	2.14	15.27	1.63 $\pm$ 0.87
Av. $\pm$ St. D.				
<u>Tan, Set. No. 2</u>				
STD	56.43	2.47	15.09	0.79
FS	56.06	2.80	15.07	1.66
FR	56.89	3.21	15.23	1.64
TY	56.92	1.86	13.60	0.98
FY	55.66	2.24	15.60	1.50
FG	55.97	1.82	15.12	1.26
TG	56.34	1.93	15.21	0.73
TB	57.23	2.57	14.71	1.16
FB	56.84	2.82	14.50	1.09
TS	57.83	2.56	14.63	1.26
TR	57.67	2.88	15.72	1.21 $\pm$ 0.33
Av. $\pm$ St. D.				

Table 9 (cont'd.)

DESIG.	L*	a*	b*	ΔA
<u>Dark Blue</u>				
STD 1 *	22.90	0.40	-11.49	
STD 2 *	23.06	0.01	-10.75	
FG 1	20.93	0.16	-10.81	2.02
FG 2	22.18	-0.06	-10.79	0.94
TG	23.09	0.05	-11.48	0.47
TS	22.53	0.23	-10.89	0.51
TB	23.56	-0.03	-11.16	0.67
FB	22.70	-0.25	-11.25	0.76
FR	22.34	-0.16	-10.92	0.86
FS	22.59	0.04	-10.96	0.48
TR	22.85	0.13	-11.12	<u>0.17</u>
		Av. $\pm$ St. D.		0.76 $\pm$ 0.53

* Two readings on same standard; values were averaged.

The tolerances for the calculation method presented here can be varied by merely changing the limit for ΔA at which we are willing to accept samples upward or downward from 1.0. For example, if the ΔA limit is set at 1.6 instead of 1.0, we would have approximately the tolerances that obtain for the olive green limit samples. If, on the other hand, we set the ΔA limit at 0.8, we would have the more stringent tolerances that hold for the dark blue limit samples. A tolerance limit of 1.0 represents average practice, as determined by the judgments of unbiased inspectors obtained as described in the sections above. We should remember that any change in tolerance limits as just described will still maintain the relative importance of the hue, chroma and lightness directions; we are merely changing the overall tolerance.

V. CONCLUSIONS AND RECOMMENDATIONS

The methods described in this report for selecting samples for inspection, for presenting the samples to the observers, and for scoring the results and setting up the acceptability equations appear to be reliable. For the present, we recommend that every color for which tolerances are desired by the Army be handled in the same way as we have illustrated for the olive green, the tan and the dark blue. The procedure allows for flexibility in setting up the tolerances by simply changing the ΔA acceptability limit upward or downward from 1.0. This choice can be made anew each time any change in procurement requirements occurs.

It may eventually happen that enough history and experience will accumulate to make it unnecessary to obtain observers' judgments in setting up equations for each new color. We may see enough regularity in the ellipsoids to make it unnecessary to determine many more. Considerably more data like those in Table 8 would be needed, however, before we could make any such generalizations.

References

1. W. Schultze and L. Gall, "Experimentelle Überprüfung mehrerer Farbabstandsformeln bezüglich der Helligkeits- und Sättigungsdifferenzen bei gesättigten Farben," *Farbe* 18, 131-148 (1969).
2. D. I. MacAdam, in *Color Metrics*, edited by J. J. Vos, L.F.C. Friele and P. L. Walraven (AIC/Holland, Soesterberg, 1972), pp. 160-170.
3. "The Hue Angle Sort Program," pamphlet issued by Hunter Associates Laboratory, Inc.
4. J. Berkson, "Application of the Logistic Function to Bio-Assay," *Am. Stat. Assn. J.* 39, 257-365 (1944).
5. J. Berkson, "A Statistically Precise and Relatively Simple Method of Estimating the Bio-Assay with Quantal Response, Based on the Logistic Function," *Am. Stat. Assn. J.* 48, 565-599 (1953).

Appendix A. Computer program for study of relative positions of standard and limit samples (Program RAMSLEY)

This program constructs all possible tetrahedra from the nine points in color space corresponding to the standard and the eight limit samples (seven limit samples in one case). There are $\binom{9}{4} = 126$ such tetrahedra that can be constructed from the nine points.

The program takes each of the nine points in turn, and determines whether it falls inside any one of the $\binom{8}{4} = 70$ tetrahedra that are not constructed from the point in question. If the point falls inside at least one of the tetrahedra, it is obviously on the inside of the geometric figure determined by the nine points; if not, it is on the boundary. (We define the "geometric figure determined by the nine points" as the union of all 126 tetrahedra.)

The determination of whether a point falls inside a tetrahedron is made as follows: Call the four corners of the tetrahedron A, B, C and D; the point in question is called P. Choose arbitrarily the D corner of the tetrahedron and construct vectors to the other three corners (see Figure 5). Also construct a vector from point D to the point in question (dotted line). Call the vectors to the three corners $\vec{DA}$, $\vec{DB}$ and $\vec{DC}$; the vector to the point in question is $\vec{DP}$. Now the point is inside the tetrahedron if

$$p\vec{DA} + q\vec{DB} + r\vec{DC} = \vec{DP}; \quad p > 0;$$

$$q > 0;$$

$$r > 0;$$

$$1-p-q-r > 0.$$

Expressing the vector equation in algebraic notation, if L_A^* indicates the L^* value of point A, etc., we have the following matrix equation:

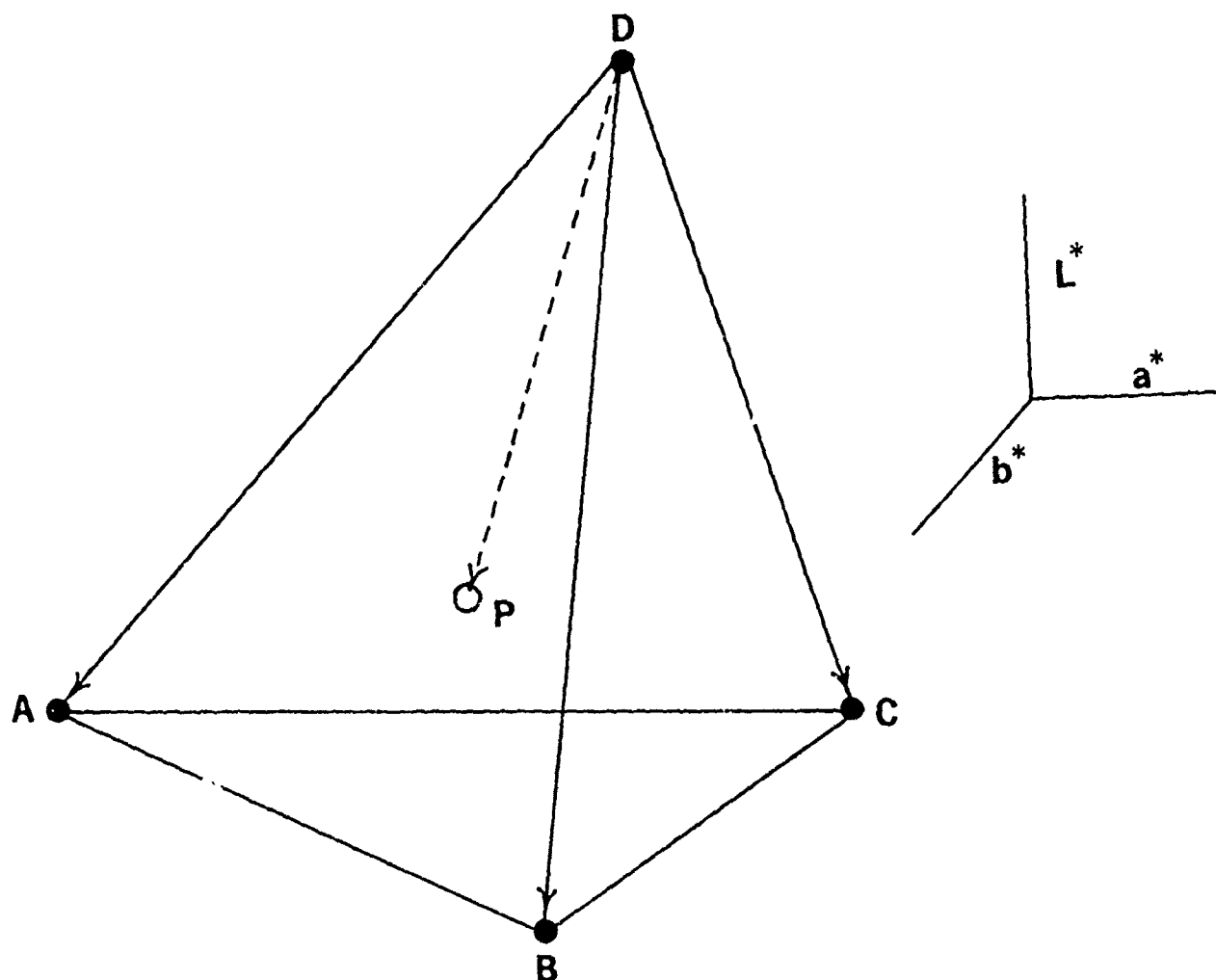


Figure 5. Vectorial representation of tetrahedron created by four samples in CIELAB Color space.

$$\begin{bmatrix} L_A^* - L_D^* & L_B^* - L_D^* & L_C^* - L_D^* \\ a_A^* - a_D^* & a_B^* - a_D^* & a_C^* - a_D^* \\ b_A^* - b_D^* & b_B^* - b_D^* & b_C^* - b_D^* \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} L_P^* - L_D^* \\ a_P^* - a_D^* \\ b_P^* - b_D^* \end{bmatrix}$$

The computer program forms all 126 matrices shown on the left above, and inverts them. It then chooses the 70 inverted matrices to be used for each point, multiplies them in turn into the appropriate right-hand side vector, and calculates p , q and r . If the inequalities shown above for p , q and r are satisfied in at least one of the 70 cases, the point in question is inside the geometric figure formed by the nine points.

The computer program follows.

PROGRAM RAMSLEY (INPUT, OUTPUT, TAPE1, TAPE2)

FOR A SERIES OF 12 COLORS, READS THE
SPECTROPHOTOMETRIC CURVES OF THE STANDARDS
AND THE LIMIT STANDARDS, AND FOR EACH OF
THESE DETERMINES WHETHER ITS POINT IN
CIELAB COLOR SPACE LIES ON THE BOUNDARY OF
THE THREE-DIMENSIONAL FIGURE DETERMINED
BY ALL THE POINTS.

DIMENSION NAME(6, 12), LIMNUM(9, 12), A(9), B(9),
+ID(4, 126), RHS(3), C(4), LABEL(9), WORK(3), LIM(2, 9)

REAL L(9), MAT(3, 3), MINJ(3, 3), MINJS(3, 3, 126)

DATA NAME/8HOLIVE DR,8HAB 7, UN,8HTREATED ,8HCOTTON D,
+8HUCK, 8.2,8H5 OZ. ,8HTAN 46, ,8HCOTTON P,8HOPLIN, 4,
+8H OZ. ,8H ,8H ,8HTAN M-1,,8H CL. POL,
+8HY/WOOL T,8HROP. 9 O,8HZ. ,8H ,8HARMY GRE,
+8HEN 44, W,8HL GAB., ,8H13OZ. ,8H ,8H ,
+8HOLIVE GR,8HEEN 107,,8H NYLON/C,8HTN (50/5,8H0) POPLI,
+8HN, 5 OZ.,8HBLUE 150,8HTROP. WL,8H. 10 OZ.,8H ,
+8H ,8H ,8HARMY GRE,8HEN 344, ,8HPOLYESTE,
+8HR/WOOL G,8HABARDINE,8H 9 OZ. ,8HARMY GRE,8HEN 344, ,
+8HPOLYESTE,8HR/WOOL T,8HROPICAL,,8H 10 OZ. ,8HARMY GRE,
+8HEN 44, W,8HL. SERGE,8H 15 OZ. ,8H ,8H ,
+8HOLIVE GR,8HEEN 108,,8H WOOL/NY,8HLON, 16 ,8HOZ. SHIR,
+8HTING ,8HOLIVE GR,8HEEN 107,,8H CTN. SA,8HTEEN, 8.,
+8H2 OZ. ,8H ,8HBLUE 151,8H, TROP. ,8HML., 10 ,
+8HOZ. ,8H ,8H /

DATA LIMNUM/1,2,3,4,5,7,8,9,10,1,2,5,6,7,8,10,11,3,
+1,2,3,4,5,7,8,9,10,1,2,6,4,5,7,11,9,10,1,2,3,4,5,7,8,9,10,
+1,2,6,4,5,7,11,9,10,1,2,6,4,5,7,11,9,10,1,2,6,4,5,7,11,9,10,
+1,6,4,5,7,2,11,9,10,1,2,3,4,5,7,8,9,10,1,2,3,5,4,7,8,9,10,
+1,2,6,4,5,7,11,9,10/

START LARGE DO LOOP BASED ON 12 SAMPLES

REWIND 1
DO 500 NSTD = 1, 12

START DO LOOP BASED ON NINE STANDARDS AND LIMIT
SAMPLES (EIGHT IN THE CASE OF SAMPLE NO. 2)

NP = 9
IF (NSTD .EQ. 2) NP = 8
DO 190 M = 1, NP
READ (1, 1001) L(M), A(M), B(M)

ASSIGN NAMES TO STANDARDS AND LIMIT STANDARDS.

INDEX = LIMNUM(M, NSTD)
GO TO (171, 172, 173, 174, 175, 176, 177, 178, 179, 180,
+181), INDEX
171 LIM(1, M) = 8HSTANDARD
LIM(2, M) = 8H
GO TO 190
172 LIM(1, M) = 8HTHIN STA
LIM(2, M) = 8HNDARD
GO TO 190
173 LIM(1, M) = 8HTHIN YEL
LIM(2, M) = 8HLOW

001000
001010
001020
001030
001040
001050
001060
001070
001080
001090
001100
001110
001140
001150
001160
001170
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001190
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001930
001940
001950
001960
001970
001980
001990
002000
002010
002020
002030
002040

	GO TO 190	002050
174	LIM(1, M) = 8HTHIN GRE	002060
	LIM(2, M) = 8HEN	002070
	GO TO 190	002080
175	LIM(1, M) = 8HTHIN RED	002090
	LIM(2, M) = 8H	002100
	GO TO 190	002110
176	LIM(1, M) = 8HTHIN BLU	002120
	LIM(2, M) = 8HE	002130
	GO TO 190	002135
177	LIM(1, M) = 8HFULL STA	002140
	LIM(2, M) = 8HNDARD	002150
	GO TO 190	002160
178	LIM(1, M) = 8HFULL YEL	002170
	LIM(2, M) = 8HLOW	002180
	GO TO 190	002190
179	LIM(1, M) = 8HFULL GRE	002200
	LIM(2, M) = 8HEN	002210
	GO TO 190	002220
180	LIM(1, M) = 8HFULL RED	002230
	LIM(2, M) = 8H	002240
	GO TO 190	002250
181	LIM(1, M) = 8HFULL BLU	002260
	LIM(2, M) = 8HE	002270
190	CONTINUE	002310
C		002320
C	CREATE MATRICES REPRESENTING TETRAHEDRA FOR ALL	002330
C	THE POSSIBLE COMBINATIONS OF THE POINTS TAKEN	002340
C	FOUR AT A TIME	002350
C		002360
	NMAT = 0	002370
	NPM1 = NP - 1	002380
	NPM2 = NP - 2	002390
	NPM3 = NP - 3	002400
	DO 200 J1 = 1, NPM3	002410
	J1P = J1 + 1	002420
	DO 200 J2 = J1P, NPM2	002430
	J2P = J2 + 1	002440
	DO 200 J3 = J2P, NPM1	002450
	J3P = J3 + 1	002460
	DO 200 J4 = J3P, NP	002470
	NMAT = NMAT + 1	002480
	ID(1, NMAT) = J1	002490
	ID(2, NMAT) = J2	002500
	ID(3, NMAT) = J3	002510
	ID(4, NMAT) = J4	002520
	MAT(1, 1) = L(J1) - L(J4)	002530
	MAT(1, 2) = L(J2) - L(J4)	002540
	MAT(1, 3) = L(J3) - L(J4)	002550
	MAT(2, 1) = A(J1) - A(J4)	002560
	MAT(2, 2) = A(J2) - A(J4)	002570
	MAT(2, 3) = A(J3) - A(J4)	002580
	MAT(3, 1) = B(J1) - B(J4)	002590
	MAT(3, 2) = B(J2) - B(J4)	002600
	MAT(3, 3) = B(J3) - B(J4)	002610
C		002620
C	INVERT MATRICES AND STORE THEM IN MEMORY.	002630
C		002640
	CALL LINJIF(MAT, 3, 3, MINV, 0, WORK, IER)	002650
	DO 200 J = 1, 3	002660
	DO 200 K = 1, 3	002670
200	MINVS(J, K, NMAT) = MINV(J, K)	002680
C		002690
C	TRY EACH POINT SEPARATELY WITH EACH OF THE	002700
C	INVERTED MATRICES, TO SEE IF IT FALLS WITHIN	002710
C	THE TETRAHEDRON DEFINED BY THE FOUR POINTS THAT	002720

C	WENT INTO THE MATRIX: IF SO, LABEL THE POINT	002730
C	AS BEING INSIDE THE BOUNDARY.	002740
C		002750
	DO 300 J = 1, 9	002760
	DO 270 NMAT = 1, 126	002770
	IF (J .EQ. ID(1, NMAT) .OR. J .EQ. ID(2, NMAT) .OR.	002780
	+ J .EQ. ID(3, NMAT) .OR. J .EQ. ID(4, NMAT)) GO TO 270	002790
	JJ = ID(4, NMAT)	002800
	RHS(1) = L(J) - L(JJ)	002810
	RHS(2) = A(J) - A(JJ)	002820
	RHS(3) = B(J) - B(JJ)	002830
	DO 250 K = 1, 3	002840
	C(K) = 0.	002850
	DO 250 KK = 1, 3	002860
250	C(K) = C(K) + MINVS(K, KK, NMAT) * RHS(KK)	002870
	C(4) = 1. - C(1) - C(2) - C(3)	002880
	IF (C(1) .GT. 0. .AND. C(2) .GT. 0. .AND.	002890
	+ C(3) .GT. 0. .AND. C(4) .GT. 0.) GO TO 280	002900
270	CONTINUE	002910
	LABEL(J) = 2	002920
	GO TO 300	002930
280	LABEL(J) = 1	002940
300	CONTINUE	002950
C		002960
C	PRINT RESULTS	002970
C		002980
	PRINT 1002, (NAME(J, NSTD), J = 1, 6)	002990
	PRINT 1008	003000
	DO 400 J = 1, NP	003010
	INDEX = LABEL(J)	003020
	GO TO (310, 320), INDEX	003030
310	PRINT 1003, LIM(1, J), LIM(2, J), L(J), A(J), B(J)	003040
	GO TO 400	003050
320	PRINT 1004, LIM(1, J), LIM(2, J), L(J), A(J), B(J)	003060
400	CONTINUE	003105
	IF (NSTD .EQ. 4 .OR. NSTD .EQ. 8 .OR.	003110
	+ NSTD .EQ. 12) PRINT 1010	003115
	IF (NSTD .EQ. 2) PRINT 1006	003116
500	CONTINUE	003120
	STOP	003130
C		003140
C	FORMAT STATEMENTS	003150
C		003160
	1001 FORMAT(3F8.2)	003170
	1002 FORMAT(///6X, 6A8)	003180
	1003 FORMAT(1X, 2A8, F7.2, 2F8.2, 3X, *NOT ON BOUNDARY*)	003190
	1004 FORMAT(1X, 2A8, F7.2, 2F8.2, 3X, *ON BOUNDARY*)	003200
	1006 FORMAT(1H)	003220
	1008 FORMAT(/3X, *SAMPLE*, 12X, *L*, 7X, *A*, 7X, *B*,	003240
	+7X, *POSITION*/)	003250
	1010 FORMAT(/)	003270
	END	003280

Appendix B. Computer program for determination of minimum non-tilting ellipsoid containing standard and limit samples (Programs ELPSOID and ELPSOD2)

A non-tilting ellipsoid (in the sense described in the main section of this report) is completely determined by seven parameters: the three position coordinates of the center, the three semiaxis lengths, and the angle between the major horizontal axis and the a^* axis. For Program ELPSOID, all seven parameters are adjustable. For Program ELPSOD2, only six parameters are adjustable; the axis angle is constrained by the fact that the major horizontal axis lies along a line connecting the center of the ellipsoid with the L^* axis.

Consider a non-tilting ellipsoid located somewhere in the $a^*-b^*-L^*$ coordinate system. Set up another coordinate system to coincide with the three axes of the ellipsoid. The equation of the ellipsoid in this second coordinate system is

$$\frac{x^2}{p^2} + \frac{y^2}{q^2} + \frac{z^2}{r^2} = 1, \quad (B-1)$$

where x , y and z are the coordinates of any point on the ellipsoid relative to the second coordinate system, and p , q and r are the semiaxis lengths. We consider that x and y are the axes parallel to the a^*-b^* plane and z is the axis that is parallel to the L^* axis. The major horizontal semiaxis, of length $2p$, lies along the x axis.

If θ is the angle going from the positive branch of the a^* axis counter-clockwise to the positive branch of the x axis, and if a_c^* , b_c^* and L_c^* are the coordinates of the center of the ellipsoid in the $a^*-b^*-L^*$ system, then the following transformation equations apply:

$$\begin{aligned} x &= (a^* - a_c^*) \cos \theta + (b^* - b_c^*) \sin \theta, \\ y &= -(a^* - a_c^*) \sin \theta + (b^* - b_c^*) \cos \theta, \\ z &= L - L_c \end{aligned} \quad (B-2)$$

Substituting these equations into the equation of the ellipsoid given above and collecting terms, we have

$$\begin{aligned}
 & \left(\frac{\cos^2 \theta}{p^2} + \frac{\sin^2 \theta}{q^2} \right) (a^* - a_c^*)^2 \\
 & + 2 \sin \theta \cos \theta \left(\frac{1}{p^2} - \frac{1}{q^2} \right) (a^* - a_c^*) (b^* - b_c^*) \\
 & + \left(\frac{\sin^2 \theta}{p^2} + \frac{\cos^2 \theta}{q^2} \right) (b^* - b_c^*)^2 \\
 & + \frac{1}{r^2} (L^* - L_c^*)^2 = 1.
 \end{aligned} \tag{B-3}$$

This is the equation for the ellipsoid in the $a^*-b^*-L^*$ coordinate system incorporating the seven parameters mentioned above. The position parameters for the center are a_c^* , b_c^* and L_c^* ; the semiaxis lengths are p , q and r ; the angle between the major horizontal axis and the a^* axis is θ .

Program ELPSOID works in the following way: The seven parameters are systematically varied, one at a time. After each change in the parameter, they are inserted in Equation B-3 to give what may be called a reference ellipsoid. All the points (a^*, b^*, L^*) that satisfy Equation B-3 fall on the surface of this reference ellipsoid.

For each parameter change and associated reference ellipsoid, the program then enters a subroutine called SIZER. This subroutine goes through the nine points for the standard and the limit samples, and substitutes the coordinates of each point at a time into the equation for the reference ellipsoid. Consider one of these points. Since this point is probably not on the reference ellipsoid, the right hand side of the equation will not be unity but some value that we will call S^2 . Dividing Equation B-3 through by S^2 , and setting $p' = pS$, $q' = qS$ and $r' = rS$, we obtain a new equation just like Equation B-3 but with p' replacing p , q' replacing q , and r' replacing r , and with unity back on the right hand side.

This new equation defines a new ellipsoid that is concentric with the reference ellipsoid, that has the point in question on its surface and that has the same ratios of axis lengths as those of the reference ellipsoid, since the new axes are merely the old axes multiplied by S. The volume of an ellipsoid is equal to $4/3 \pi$ multiplied by the product of the three semiaxes; the new ellipsoid therefore has a volume equal to $4/3 \pi$ times the product of the p, q and r parameters times S^3 .

The subroutine goes through all nine points in this way, and computes nine corresponding ellipsoids, all concentric with and similar (in the geometric sense) to the reference ellipsoid. It then returns to the main program the volume of the largest of these concentric ellipsoids, since that is the one that contains all the points. If this volume is smaller than the smallest volume previously determined for other parameter variations, the current variation was favorable; otherwise not. The systematic parameter variation continues until no parameter change can produce a smaller ellipsoid returned by the subroutine. The program then normalizes the current parameters by multiplying the p, q and r values by the S value for the ellipsoid that was chosen, and finally prints out the ellipsoid characteristics.

In Program ELPSOD2, the angle θ is always fixed by

$$\theta = \tan^{-1} (b_c^*/a_c^*) . \quad (B-4)$$

It is therefore obviously a hue angle. Each time the a_c^* or b_c^* coordinate is varied, the angle θ must be recalculated. Otherwise, the program runs like Program ELPSOID.

It is convenient to define the coefficients in Equation B-3 as follows:

$$g_{11} = (\cos^2 \theta / p^2) + (\sin^2 \theta / q^2) , \quad (B-5)$$

$$2g_{12} = 2 \sin \theta \cos \theta \left[(1/p^2) - (1/q^2) \right] , \quad (B-6)$$

$$g_{22} = (\sin^2 \theta / p^2) + (\cos^2 \theta / q^2) , \quad (B-7)$$

$$g_{33} = 1/r^2 . \quad (B-8)$$

With these substitutions, Equation B-3 comes

$$g_{11} (a^* - a_c^*)^2 + 2 g_{12} (a^* - a_c^*) (b^* - b_c^*) + g_{22} (b^* - b_c^*)^2 + g_{33} (L^* - L_c^*)^2 = 1. \quad (B-9)$$

This is the terminology that is used in the computer programs.

	PROGRAM ELPSOID (INPUT, OUTPUT, TAPE3)	001000
C		001010
C	FOR A SERIES OF POINTS IN CIELAB COLOR SPACE,	001020
C	DETERMINES THE SMALLEST NON-TILTING ELLIPSOID	001030
C	THAT CONTAINS THE POINTS. THE ELLIPSOID MAY	001040
C	BE INCLINED WITH RESPECT TO THE HORIZONTAL AXIS,	001050
C	BUT MAY NOT TILT EITHER FORWARD OR SIDEWAYS.	001060
C		001070
	DIMENSION PAR(7), HOWFAR(9), DELTA(7), DI/(4), NAME(6, 12),	001080
	+SIGN(2)	001085
C		001090
	COMMON CIE(3, 9), NP	001100
C		001110
	LOGICAL CHANGE, ACTION	001120
C		001130
	DATA DELTA/0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.017453293/,	001140
	+DI//1., 2., 5., 10./, SIGN/1., -1./	001150
C		001160
	DATA NAME/8H01 TVE DR, 8HAB 7, UN, 8HTREATED, 8HCOTTON D,	001170
	+8HUCK, 8.2, 8H 0Z., 8HTAN 46, 8HCOTTON P, 8HOPLIN, 4,	001180
	+8H 0Z., 8H 8H 8HTAN M-1, 8H CL. POL,	001190
	+8HY/WOOL T, 8HROP. 9 0, 8HZ., 8H 8HARMY GRE,	001200
	+8HEN 44, W, 8HL GAB., 8H130Z., 8H 8H	001210
	+8HOLIVE GR, 8HEEN 107, 8H NYLON/C, 8HTN (50/5, 8H0) POPLI,	001220
	+8HN, 5 OZ., 8HBLUE 150, 8HTROP. WL, 8H. 10 OZ., 8H	001230
	+8H 8H 8HARMY GRE, 8HEN 344, 8HPOLYESTE,	001240
	+8HR/WOOL G, 8HABARDINE, 8H 9 OZ., 8HARMY GRE, 8HEN 344, ,	001250
	+8HPOLYESTE, 8HR/WOOL T, 8HROPICAL., 8H 10 OZ., 8HARMY GRE,	001260
	+8HEN 44, W, 8HL. SERGE, 8H 15 OZ., 8H 8H	001270
	+8HOLIVE GR, 8HEEN 108, 8H WOOL/NY, 8HLON, 16 8HOZ. SHIR,	001280
	+8HTING 8HCLIVE GR, 8HEEN 107, 8H CTN. SA, 8HTEEN, 8.,	001290
	+8H2 OZ., 8H 8HBLUE 151, 8H, TROP. 8HHL., 10 ,	001300
	+8HOZ., 8H 8H /	001310
C		001320
C	READ LAB VALUES OF STANDARDS (BUT IN THE ORDER ABL)	001330
C		001340
	REWIND 3	001345
	DO 600 NSTD = 1, 12	001350
	IF (NSTD . NE . 2) NP = 9	001360
	IF (NSTD . EQ . 2) NP = 8	001370
	READ (3, 1001)(CIE(3, J), CIE(1, J), CIE(2, J), J = 1, NP)	001380
C		001390
C	ASSIGN STARTING VALUES TO THE SEVEN PARAMETERS	001400
C		001410
	HOMAX = 0.	001420
	VOMAX = 0.	001430
	NPM1 = NP - 1	001435
	DO 150 J = 1, NPM1	001440
	JP1 = J + 1	001445
	DO 150 K = JP1, NP	001450
	HD = SQRT((CIE(1, J) - CIE(1, K)) ** 2 + (CIE(2, J)	001460
	+ - CIE(2, K)) ** 2)	001470
	IF (HD . LE . HOMAX) GO TO 130	001500
	HOMAX = HD	001510
	JHMAX = J	001520
	KHMAX = K	001530
130	VD = ABS(CIE(3, J) - CIE(3, K))	001540
	IF (VD . LE . VOMAX) GO TO 150	001550
	JOMAX = J	001560
	JVMAX = J	001570
	KVMAX = K	001580
150	CONTINUE	001590
	PAR(1) = HOMAX / 2.	001600

C	HORIZONTAL MAJOR SEMIAXIS	001610
	PAR(2) = PAR(1) / 2.	001620
C	HORIZONTAL MINOR SEMIAXIS	001630
	PAR(3) = JDMAX / 2.	001640
C	VERTICAL SEMIAXIS	001650
	PAR(4) = 0.5 * (CIE(1, JHMAX) + CIE(1, KHMAX))	001660
C	A STAR COORDINATE OF CENTER OF ELLIPSOID	001670
	PAR(5) = 0.5 * (CIE(2, JHMAX) + CIE(2, KHMAX))	001680
C	B STAR COORDINATE OF CENTER OF ELLIPSOID	001690
	PAR(6) = 0.5 * (CIE(3, JHMAX) + CIE(3, KHMAX))	001700
C	L STAR COORDINATE OF CENTER OF ELLIPSOID	001710
	PAR(7) = ATAN((CIE(2, JHMAX) - CIE(2, KHMAX)) /	001720
	+ (CIE(1, JHMAX) - CIE(1, KHMAX)))	001730
C	ANGLE OF HORIZONTAL MAJOR AXIS TO A STAR AXIS	001740
C		001750
C	DO LOOP FOR IDIOT SEARCH PROCEDURE. CHANGE ONE	001760
C	PARAMETER AT A TIME AND SEE IF THE ELLIPSOID	001770
C	GETS SMALLER. IF SO, START OVER. EVENTUALLY	001780
C	MAKE THE PARAMETER CHANGE INCREMENTS SMALLER	001790
C	AND SMALLER.	001800
C		001830
	CALL SIZER (PAR, ELLMIN)	001840
	DO 450 NDIV = 1, 4	001860
160	CHANGE = .FALSE.	001865
	DO 400 NPAR = 1, 7	001870
	DEL = DELTA(NPAR) / DIV(NDIV)	001880
	DO 400 NSIGN = 1, 2	001890
	IF (NSIGN .EQ. 1) GO TO 170	001900
	IF (ACTION) GO TO 400	001910
170	ACTION = .FALSE.	001920
180	PAR(NPAR) = PAR(NPAR) + DEL * SIGN(NSIGN)	001930
	IF (PAR(1) .LT. 0. .OR. PAR(2) .LT. 0.	001935
	+ .OR. PAR(3) .LT. 0.) GO TO 390	001936
	CALL SIZER (PAR, ELLIE)	001940
	IF (ELLIE .GT. ELLMIN) GO TO 390	001950
	ELLMIN = ELLIE	001960
	CHANGE = .TRUE.	001990
	ACTION = .TRUE.	002000
	GO TO 180	002010
390	PAR(NPAR) = PAR(NPAR) - DEL * SIGN(NSIGN)	002020
400	CONTINUE	002030
	IF (CHANGE) GO TO 160	002040
450	CONTINUE	002050
C		002060
C	NORMALIZE ELLIPSOID SEMIAXES BEFORE PRINTING OUT	002070
C	INFORMATION. SEMIAXES ARE ADJUSTED SO AS TO MAKE	002080
C	RIGHT HAND SIDE OF ELLIPSOID EQUATION UNITY.	002090
C		002100
	CALL SIZER (PAR, ELLIE)	002110
	ETEMP = PAR(1) * PAR(2) * PAR(3)	002120
	SCFACTR = (ELLIE / ETEMP) ** (1. / 3.)	002130
	DO 500 J = 1, 3	002140
500	PAR(J) = PAR(J) * SCFACTR	002150
	IF (PAR(1) .GT. PAR(2)) GO TO 510	002153
	TEMP = PAR(2)	002156
	PAR(2) = PAR(1)	002159
	PAR(1) = TEMP	002162
	PAR(7) = PAR(7) - 0.5 * 3.141592654	002165
C		002168
C	CALCULATE ELLIPSOID PARAMETERS FOR OUTPUT	002170
C		002180
510	ANG = PAR(7) * 180. / 3.141592654	002190
	IF (ANG .GT. 90) ANG = ANG - 180.	002195
	SINE = SIN(PAR(7))	002200
	COSINE = COS(PAR(7))	002210
	G11 = (COSINE / PAR(1)) ** 2 + (SINE / PAR(2)) ** 2	002220

TWOG12 = 2. * SINE * COSINE * (1. / PAR(1) ** 2 -	002230
+ 1. / PAR(2) ** 2)	002240
G22 = (SINE / PAR(1)) ** 2 + (COSINE / PAR(2)) ** 2	002250
G33 = 1. / PAR(3) ** 2	002260
VOL = (4. / 3.) * 3.141592654 * PAR(1) * PAR(2) * PAR(3)	002270
C	002310
C	002320
C	002330
PRINT 1002, (NAME(J, NSTD), J = 1, 6)	002340
PRINT 1003, PAR(6), PAR(4), PAR(5)	002350
PRINT 1004, G11, TWOG12, G22, G33	002360
PRINT 1005, PAR(1), ANG, PAR(2), PAR(3)	002370
PRINT 1006, VOL	002380
DO 530 J = 1, NP	002385
DEL1 = CIE(1, J) - PAR(4)	002386
JEL2 = CIE(2, J) - PAR(5)	002387
DEL3 = CIE(3, J) - PAR(6)	002388
530 HOWFAR(J) = G11 * DEL1 ** 2 + TWOG12 * DEL1 * DEL2 +	002390
+ G22 * DEL2 ** 2 + G33 * DEL3 ** 2	002395
PRINT 1007, (CIE(1, J), CIE(2, J), CIE(3, J), HOWFAR(J),	002400
+ J = 1, NP)	002403
600 CONTINUE	002405
PRINT 1008	002406
C	002408
C	002409
C	002410
FORMAT STATEMENTS	002420
1001 FORMAT(3F8.2)	002430
1002 FORMAT(////////* ACCEPTABILITY ELLIPSOID*/1X,6A8)	002440
1003 FORMAT(//24H CENTER OF ELLIPSOID: L*, F18.2/22X,	002450
+ 2HA*, F18.2/22X, 2HB*, F18.2)	002460
1004 FORMAT(//* COEFFICIENTS:*,8X, *G11*, E23.4/21X,	002470
+ *G22*, E23.4/22X, *G33*, E23.4/22X, *G33*, E23.4)	002480
1005 FORMAT(//* SEMIMAJOR HORIZONTAL AXIS, LENGTH*, F8.2/	002490
+ * SEMIMAJOR HORIZONTAL AXIS, ANGLE*, F8.1/* SEMIMINOR*,	002500
+ * HORIZONTAL AXIS, LENGTH*, F8.2/* SEMIVERTICAL AXIS,*	002510
+ * LENGTH*, F16.2)	002520
1006 FORMAT(//* VOLUME OF ELLIPSOID*,F22.2)	002523
1007 FORMAT(//* DISTANCE OF POINT FROM CENTER OF ELLIPSOID*	002525
+ * IN ELL UNITS*//10X, *COORDINATES OF POINT DISTANCE*,	002527
+ //11X,2HA*,6X,2HB*,6X,2HL*/(6X, 3F8.2, F11.5))	002528
1008 FORMAT(//////////)	002530
END	002540
C	002550
C	002560
SUBROUTINE SIZER (PAR, ELLIE)	002570
C	002580
C	002590
C	002600
C	002610
C	002620
DETERMINES THE SIZE OF THE ELLIPSOID PASSING THROUGH	002630
EACH OF THE STANDARDS WITH THE CURRENT PARAMETERS, AND	002640
SELECTS THE ELLIPSOID OF LARGEST SIZE AMONG THEM.	002650
C	002660
C	002670
C	002680
C	002690
C	002700
C	002710
C	002720
C	002730
C	002740
C	002750
C	002760
C	002770
DIMENSION PAR(7)	
COMMON CIE(3, 9), NP	
SINE = SIN(PAR(7))	
COSINE = COS(PAR(7))	
G11 = (COSINE / PAR(1)) ** 2 + (SINE / PAR(2)) ** 2	
TWOG12 = 2. * SINE * COSINE * (1. / PAR(1) ** 2 -	
+ 1. / PAR(2) ** 2)	
G22 = (SINE / PAR(1)) ** 2 + (COSINE / PAR(2)) ** 2	
G33 = 1. / PAR(3) ** 2	
RHSMAX = 0.	
DO 150 J = 1, NP	
DELTA = CIE(1, J) - PAR(4)	
DELTB = CIE(2, J) - PAR(5)	
DELTL = CIE(3, J) - PAR(6)	

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RHS = G11 * DELTA ** 2 + TWOG12 * DELTA * DELTB +
+ G22 * DELTB ** 2 + G33 * DELTL ** 2
IF (RHS . LT . RHSMAX) GO TO 150
RHSMAX = RHS
150 CONTINUE
ELLIE = RHSMAX ** 1.5 * PAR(1) * PAR(2) * PAR(3)
RETURN
END

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002780
002790
002800
002810
002820
002830
002840
002850

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C	PROGRAM ELPSOD2 (INPUT, OUTPUT, TAPE3)	001000
C		001010
C	FOR A SERIES OF POINTS IN CIELAB COLOR SPACE,	001020
C	DETERMINES THE SMALLEST NON-TILTING ELLIPSOID	001030
C	THAT CONTAINS THE POINTS. THE ELLIPSOID MAY	001040
C	NOT TILT EITHER FORWARD OR SIDEWAYS, AND MUST	001050
C	HAVE ITS MAJOR AXIS INCLINED IN THE DIRECTION	001060
C	OF THE HUE ANGLE OF THE STANDARD.	001070
C		001080
C	DIMENSION PAR(7), HOWFAR(9), DIV(4), NAME(6, 12), SIGN(2)	001090
C		001100
C	COMMON CIE(3, 9), NP	001110
C		001120
C	LOGICAL CHANGE, ACTION	001130
C		001140
C	DATA DELTA/0.1/, DIV/1., 2., 5., 10./, SIGN/1., -1./	001150
		001160
	DATA NAME/8HOLIVE DR,8HAB 7, UN,8HTREATED ,8HCOTTON D,	001170
	+8HUCK, 8.2,8H5 OZ. ,8HTAN 46, ,8HCOTTON P,8HOPLIN, 4,	001180
	+8H OZ. ,8H ,8H ,8HTAN M-1,8H CL. POL,	001190
	+2HY/WOOL T,8HROP. 9 O,8HZ. ,8H ,8HARMY GRE,	001200
	+8HEN 44, W,8HL GAB., ,8H13OZ. ,8H ,8H	001210
	+8HOLIVE GR,8HEEN 107,,8H NYLON/C,8HTN (50/5,8H0) POPLI,	001220
	+8HN, 5 OZ.,8HBLUE 150,8HTROP. HL,8H. 10 OZ.,8H	001230
	+8H ,8H ,8HARMY GRE,8HEN 344, ,8HPOLYESTE,	001240
	+6HR/WOOL G,8HABARDINE,8H 9 OZ. ,8HARMY GRE,8HEN 344, ,	001250
	+8HPOLYESTE,8HR/WOOL T,8HROPICAL,,8H 10 OZ. ,8HARMY GRE,	001260
	+8HEN 44, W,8HL. SERGE,8H 15 OZ. ,8H ,8H	001270
	+8HOLIVE GR,8HEEN 103,,8H WOOL/NY,8HLON. 16 ,8HOZ. SHIR,	001280
	+8HTING ,8HOLIVE GR,8HEEN 107,,8H CTN. SA,8HTEEN, 8.,	001290
	+8H2 OZ. ,8H ,8HBLUE 151,8H, TROP. ,8HHL., 10 ,	001300
	+8HOZ. ,8H ,8H /	001310
		001320
C	READ LAB VALUES OF STANDARDS (BUT IN THE ORDER AGL)	001330
C		001340
C	REWIND 3	001350
	DO 600 NSTD = 1, 12	001360
	IF (NSTD . NE . 2) NP = 9	001370
	IF (NSTD . EQ . 2) NP = 8	001380
	READ (3, 1001)(CIE(3, J), CIE(1, J), CIE(2, J), J = 1, NP)	001390
C		001400
C	ASSIGN STARTING VALUES TO THE SEVEN PARAMETERS	001410
C		001420
	PAR(4) = 0.	001430
	PAR(5) = 0.	001440
	PAR(6) = 0.	001450
	DO 120 J = 1, NP	001460
	PAR(4) = PAR(4) + CIE(1, J) / NP	001470
C	A STAR COORDINATE OF CENTER OF ELLIPSOID	001480
	PAR(5) = PAR(5) + CIE(2, J) / NP	001490
C	B STAR COORDINATE OF CENTER OF ELLIPSOID	001500
	PAR(6) = PAR(6) + CIE(3, J) / NP	001510
C	L STAR COORDINATE OF CENTER OF ELLIPSOID	001520
	120 CONTINUE	001530
	D1MAX = 0.	001540
	D2MAX = 0.	001550
	D3MAX = 0.	001560
	DO 150 J = 1, NP	001570
	D = SQRT((PAR(4) - CIE(1, J)) ** 2 + (PAR(5)	001580
	+ - CIE(2, J)) ** 2)	001590
	ALPHA = ATAN((PAR(5) - CIE(2, J)) / (PAR(4)	001600
	+ - CIE(1, J)))	001610
	THETA = ATAN(PAR(5) / PAR(4))	001620

PHI = ALPHA - THETA	001630
D1 = ABS(D * COS(PHI))	001640
IF (D1 . LE . D1MAX) GO TO 130	001650
D1MAX = D1	001660
130 D2 = ABS(D * SIN(PHI))	001670
IF (D2 . LE . D2MAX) GO TO 140	001680
D2MAX = D2	001690
140 D3 = ABS(PAR(6) - CIE(3, J))	001700
IF (D3 . LE . D3MAX) GO TO 150	001710
D3MAX = D3	001720
150 CONTINUE	001730
PAR(1) = D1MAX	001740
C HORIZONTAL MAJOR SEMIAXIS	001750
PAR(2) = D2MAX	001760
C HORIZONTAL MINOR SEMIAXIS	001770
PAR(3) = D3MAX	001780
C VERTICAL SEMIAXIS	001790
PAR(7) = THETA	001800
C ANGLE OF HORIZONTAL MAJOR AXIS TO A STAR AXIS	001810
C	001820
C DO LOOP FOR IDIOT SEARCH PROCEDURE. CHANGE ONE	001830
C PARAMETER AT A TIME AND SEE IF THE ELLIPSOID	001840
C GETS SMALLER. IF SO, START OVER. EVENTUALLY	001850
C MAKE THE PARAMETER CHANGE INCREMENTS SMALLER	001860
C AND SMALLER.	001870
C	001880
CALL SIZER (PAR, ELLMIN)	001890
DO 450 NDIV = 1, 4	001900
160 CHANGE = .FALSE.	001910
DO 400 NPAR = 1, 6	001920
DEL = DELTA / DIV(NDIV)	001930
DO 400 NSIGN = 1, 2	001940
IF (NSIGN . EQ . 1) GO TO 170	001950
IF (ACTION) GO TO 400	001960
170 ACTION = .FALSE.	001970
180 PAR(NPAR) = PAR(NPAR) + DEL * SIGN(NSIGN)	001980
IF (PAR(1) . LT . 0. . OR . PAR(2) . LT . 0.	001990
+ . OR . PAR(3) . LT . 0.) GO TO 390	002000
IF (NPAR . EQ . 4 . OR . NPAR . EQ . 5)	002010
+ PAR(7) = ATAN(PAR(5) / PAR(4))	002020
CALL SIZER (PAR, ELLIE)	002030
IF (ELLIE . GT . ELLMIN) GO TO 390	002040
ELLMIN = ELLIE	002050
CHANGE = .TRUE.	002060
ACTION = .TRUE.	002070
GO TO 180	002080
390 PAR(NPAR) = PAR(NPAR) - DEL * SIGN(NSIGN)	002090
IF (PAR . EQ . 4 . OR . NPAR . EQ . 5)	002100
+ PAR(7) = ATAN(PAR(5) / PAR(4))	002110
400 CONTINUE	002120
IF (CHANGE) GO TO 160	002130
450 CONTINUE	002140
C	002150
C NORMALIZE ELLIPSOID SEMIAXES BEFORE PRINTING OUT	002160
C INFORMATION. SEMIAXES ARE ADJUSTED SO AS TO MAKE	002170
C RIGHT HAND SIDE OF ELLIPSOID EQUATION UNITY.	002180
C	002190
CALL SIZER (PAR, ELLIE)	002200
ETEMP = PAR(1) * PAR(2) * PAR(3)	002210
SCFACTR = (ELLIE / ETEMP) ** (1. / 3.)	002220
DO 500 J = 1, 3	002230
500 PAR(J) = PAR(J) * SCFACTR	002240
C	002250
C CALCULATE ELLIPSOID PARAMETERS FOR OUTPUT	002260
C	002270
C	002280
ANG = PAR(7) * 180. / 3.141592654	

	IF (ANG . GT . 90) ANG = ANG - 180.	002290
	SINE = SIN(PAR(7))	002300
	COSINE = COS(PAR(7))	002310
	G11 = (COSINE / PAR(1)) ** 2 + (SINE / PAR(2)) ** 2	002320
	TWOG12 = 2. * SINE * COSINE * (1. / PAR(1)) ** 2 -	002330
	+ 1. / PAR(2)) ** 2)	002340
	G22 = (SINE / PAR(1)) ** 2 + (COSINE / PAR(2)) ** 2	002350
	G33 = 1. / PAR(3)) ** 2	002360
	RATIO1 = PAR(3) / PAR(2)	002370
	RATIO2 = PAR(1) / PAR(2)	002380
	VOL = (4. / 3.) * 3.141592654 * PAR(1) * PAR(2) * PAR(3)	002390
C		002400
C	PRINT OUT RESULTS	002410
C		002420
	PRINT 1002, (NAME(J, NSTD), J = 1, 6)	002430
	PRINT 1003, PAR(6), PAR(4), PAR(5)	002440
	PRINT 1004, G11, TWOG12, G22, G33	002450
	PRINT 1005, PAR(1), ANG, PAR(2), PAR(3)	002460
	PRINT 1006, RATIO1, RATIO2	002470
	PRINT 1007, VOL	002480
	DO 530 J = 1, NP	002490
	DEL1 = CIE(1, J) - PAR(4)	002500
	DEL2 = CIE(2, J) - PAR(5)	002510
	DEL3 = CIE(3, J) - PAR(6)	002520
	530 HOWFAR(J) = G11 * DEL1 ** 2 + TWOG12 * DEL1 * DEL2 +	002530
	+ G22 * DEL2 ** 2 + G33 * DEL3 ** 2	002540
	PRINT 1008, (CIE(1, J), CIE(2, J), CIE(3, J), HOWFAR(J),	002550
	+ J = 1, NP)	002560
	600 CONTINUE	002570
	PRINT 1009	002580
C		002590
C	FORMAT STATEMENTS	002600
C		002610
	1001 FORMAT(3F8.2)	002620
	1002 FORMAT(////////* ACCEPTABILITY ELLIPSOID*/1X,6A8)	002630
	1003 FORMAT(//24H CENTER OF ELLIPSOID: L*, F18.2/22X,	002640
	+ 2HA*, F18.2/22X, 2HB*, F18.2)	002650
	1004 FORMAT(/* COEFFICIENTS:*,8X, *G11*, E23.4/21X,	002660
	+ *G12*, E23.4/22X, *G22*, E23.4/22X, *G33*, E23.4)	002670
	1005 FORMAT(/* SEMIMAJOR HORIZONTAL AXIS, LENGTH*, F8.2/	002680
	+ * SEMIMAJOR HORIZONTAL AXIS, ANGLE*, F8.1/* SEMIMINOR*,	002690
	+ * HORIZONTAL AXIS, LENGTH*, F8.2/* SEMIVERTICAL AXIS,*	002700
	+ * LENGTH*, F16.2)	002710
	1006 FORMAT(/* LIGHTNESS : CHROMA : HUE RATIOS*,	002720
	+ F10.2, * :*, F5.2, * : 1.00*)	002730
	1007 FORMAT(/* VOLUME OF ELLIPSOID*, F22.2)	002740
	1008 FORMAT(/* DISTANCE OF POINT FROM CENTER OF ELLIPSOID*	002750
	+ * IN ELL UNITS*//10X, *COORDINATES OF POINT DISTANCE*,	002760
	+ //11X, 2HA*, 6X, 2HB*, 6X, 2HL*/(6X, 3F8.2, F11.5))	002770
	1009 FORMAT(//////////)	002780
	END	002790
C		002800
C	SUBROUTINE SIZER (PAR, ELLIE)	002810
C		002820
C	DETERMINES THE SIZE OF THE ELLIPSOID PASSING THROUGH	002830
C	EACH OF THE STANDARDS WITH THE CURRENT PARAMETERS, AND	002840
C	SELECTS THE ELLIPSOID OF LARGEST SIZE AMONG THEM.	002850
C		002860
C	DIMENSION PAR(7)	002870
C		002880
C	COMMON CIE(3, 9), NP	002890
C		002900
	SINE = SIN(PAR(7))	002910
	COSINE = COS(PAR(7))	002920
	G11 = (COSINE / PAR(1)) ** 2 + (SINE / PAR(2)) ** 2	002930
		002940

TWOG12 = 2. * SINE * COSINE * (1. / PAR(1) ** 2 -	002950
+ 1. / PAR(2) ** 2)	002960
G22 = (SINE / PAR(1)) ** 2 + (COSINE / PAR(2)) ** 2	002970
G33 = 1. / PAR(3) ** 2	002980
RHSMAX = 0.	002990
DO 150 J = 1, NP	003000
DELTA = CIE(1, J) - PAR(4)	003010
DELTB = CIE(2, J) - PAR(5)	003020
DELTL = CIE(3, J) - PAR(6)	003030
RHS = G11 * DELTA ** 2 + TWOG12 * DELTA * DELTB +	003040
+ G22 * DELTB ** 2 + G33 * DELTL ** 2	003050
IF (RHS . LT . RHSMAX) GO TO 150	003060
RHSMAX = RHS	003070
150 CONTINUE	003080
ELLIE = RHSMAX ** 1.5 * PAR(1) * PAR(2) * PAR(3)	003090
RETURN	003100
END	003110

Appendix C. Computer program for rearranging file in numerical order (Program (REARRAN))

PROGRAM REARRAN (INPUT, OUTPUT, TAPE1, TAPE3)	001000
DIMENSION I(200), A(200), B(200)	001010
REAL L(200)	001020
LOGICAL OK	001030
REWIND 3	001040
J = 1	001050
100 READ (3, 1001) I1, I2, L(J), A(J), B(J)	001060
1001 FORMAT(I5, 1X, I3, F11.0, F15.0, F10.0)	001070
IF (I1 .EQ. 999) GO TO 120	001080
I(J) = 1000 * I1 + I2	001090
J = J + 1	001100
GO TO 100	001110
120 NCOLORS = J - 1	001120
NMIN1 = NCOLORS - 1	001130
ITER = 0	001140
130 OK = .TRUE.	001150
ITER = ITER + 1	001160
PRINT 1004, ITER	001170
1004 FORMAT(1H5, I4)	001180
DO 160 J = 1, NMIN1	001190
IF (I(J) .LE. I(J + 1)) GO TO 160	001200
OK = .FALSE.	001210
ITEMP = I(J)	001220
I(J) = I(J + 1)	001230
I(J + 1) = ITEMP	001240
TEMP = L(J)	001250
L(J) = L(J + 1)	001260
L(J + 1) = TEMP	001270
TEMP = A(J)	001280
A(J) = A(J + 1)	001290
A(J + 1) = TEMP	001300
TEMP = B(J)	001310
B(J) = B(J + 1)	001320
B(J + 1) = TEMP	001330
160 CONTINUE	001340
IF (.NOT. OK) GO TO 130	001350
DO 180 J = 1, NCOLORS	001360
I1 = I(J) / 1000	001370
I2 = I(J) - 1000 * I1	001380
WRITE (1, 1002) I1, I2, L(J), A(J), B(J)	001390
1002 FORMAT(I5, 1H-, I3, F11.2, 2F10.2)	001400
180 CONTINUE	001410
STOP	001420
END	001430

Appendix D. Computer program for selecting sample pairs for observers' evaluation (Program SIXWAYS)

Following is the derivation of the equations used in Program SIXWAYS to determine if a given sample lay no more than 0.1 CIELAB unit from one of the six iso lines radiating from a given standard.

1. Distance between a point and an iso (hue-lightness) line

Let L_s, a_s, b_s be the L^*, a^*, b^* coordinates of the standard. The iso (hue-lightness) line is the straight line between the standard point and the achromatic axis at the L_s lightness level. Let L_p, a_p, b_p be the L^*, a^*, b^* coordinates of the point. The asterisks will be omitted for simplicity.

First, raise or lower the point vertically until it has the same lightness value as the standard. The distance moved in color space is $L_s - L_p$.

Now drop a perpendicular from the new position of the point to the iso (hue-lightness) line. The equation of the iso (hue-lightness) line in the constant L plane is

$$b = (b_s / a_s) a. \quad (D-1)$$

The equation of the perpendicular from the new position of the point is

$$(b - b_p) / (a - a_p) = -(a_s / b_s). \quad (D-2)$$

We solve Equations D-1 and D-2 simultaneously to get the point of intersection, which we will call (a_i, b_i) . We find that

$$a_i = a_s (a_p a_s + b_p b_s) / (a_s^2 + b_s^2), \quad (D-3)$$

$$b_i = b_s (a_p a_s + b_p b_s) / (a_s^2 + b_s^2). \quad (D-4)$$

Now the distance between the point and the iso (hue-lightness) line, which we will call D_1 , is

$$D_1 = \left[(L_s - L_p)^2 + (a_p - a_i)^2 + (b_p - b_i)^2 \right]^{1/2}. \quad (D-5)$$

Substituting Equations D-3 and D-4 into Equation D-5, we obtain

$$D_1 = \left[(L_s - L_p)^2 + (a_s b_p - a_p b_s)^2 / (a_s^2 + b_s^2) \right]^{1/2} \quad (D-6)$$

2. Distance between a point and an iso (chroma-lightness) line

We first raise or lower the point vertically until it has the same lightness value as the standard, as we did under 1.

We then drop a perpendicular from the point to the iso (hue-chroma) line. The equation of the iso (hue-chroma) line is

$$(b - b_s) / (a - a_s) = -(a_s / b_s). \quad (D-7)$$

The equation of the perpendicular from the new position of the point is

$$(b - b_p) / (a - a_p) = b_s / a_s. \quad (D-8)$$

We solve Equations D-7 and D-8 simultaneously to get the point of intersection, which we will call (a_i, b_i) . We find that

$$a_i = \left[a_s (a_s^2 + b_s^2) + b_s (a_p b_s - a_s b_p) \right] / (a_s^2 + b_s^2) \quad (D-9)$$

$$b_i = \left[b_s (a_s^2 + b_s^2) - a_s (a_p b_s - a_s b_p) \right] / (a_s^2 + b_s^2) \quad (D-10)$$

Substituting Equations D-9 and D-10 into Equation D-5 (with D_2 substituted for D_1) gives

$$D_2 = \left[(L_s - L_p)^2 + (a_s^2 + b_s^2 - a_p a_s - b_p b_s)^2 / (a_s^2 + b_s^2) \right]^{1/2} \quad D-11$$

3. Distance between a point and an iso (hue-chroma) line

The required distance is

$$D_3 = \left[(a_s - a_p)^2 + (b_s - b_p)^2 \right]^{1/2} \quad (D-12)$$

PROGRAM SIXWAYS (INPUT, OUTPUT, TAPE1, TAPE7)	001000
C	001010
C	001020
C	001030
C	001040
C	001050
C	001060
DIMENSION A(400), B(400), NUMBER(400), NUMB(40, 3, 2),	001070
+DE(40, 3, 2), NP(3, 2)	001080
C	001090
REAL L(400)	001100
C	001110
C	001120
C	001130
J = 1	001140
REWIND 1	001150
REWIND 7	001157
100 READ (1, 1001) NUMBER(J), L(J), A(J), B(J)	001160
IF (NUMBER(J) .EQ. 7H9999999) GO TO 120	001170
J = J + 1	001180
GO TO 100	001190
120 NCOLORS = J - 1	001200
C	001220
C	001230
C	001240
DO 300 J = 1, NCOLORS	001250
PRINT 1014, J	001260
WRITE (7, 1002) NUMBER(J)	001270
C	001280
C	001290
C	001300
C	001310
DO 220 IVIA = 1, 3	001320
M1 = 0	001330
M2 = 0	001340
C	001350
C	001360
C	001370
C	001380
DO 200 K = 1, NCOLORS	001390
IF (J .EQ. K) GO TO 200	001400
GO TO (130, 135, 140), IVIA	001410
C	001420
C	001430
C	001440
130 DIST = SQRT((L(J) - L(K)) ** 2 + (A(J) * B(K) - A(K) * B(J))	001450
+ ** 2 / (A(J) ** 2 + B(J) ** 2))	001460
GO TO 150	001470
135 DIST = SQRT((L(J) - L(K)) ** 2 + (A(J) ** 2 + B(J) ** 2	001480
+ - A(K) * A(J) - B(K) * B(J)) ** 2 / (A(J) ** 2 + B(J) ** 2))	001490
GO TO 150	001500
140 DIST = SQRT((A(J) - A(K)) ** 2 + (B(J) - B(K)) ** 2)	001510
C	001520
C	001530
C	001540
150 IF (DIST .GT. 0.1) GO TO 200	001550
C	001560
C	001570
C	001580
C	001590
C	001600
GO TO (160, 165, 170), IVIA	001610
160 BI = B(J) * (A(K) * A(J) + B(K) * B(J)) / (A(J) ** 2 + B(J)	001620

165	AI = A(J) + B(J) * (A(K) + B(J) - A(J) * B(K)) / (A(J) ** 2	001630
	+ + B(J) ** 2)	001640
	IF (AI . GT . A(J)) GO TO 190	001650
	GO TO 180	001660
170	IF (L(K) . GT . L(J)) GO TO 180	001670
	GO TO 190	001680
180	M1 = M1 + 1	001690
	NUMB(M1, IVIA, 1) = NUMBER(K)	001700
	DE(M1, IVIA, 1) = SQRT((L(J) - L(K)) ** 2 + (A(J) - A(K))	001710
	+ ** 2 + (B(J) - B(K)) ** 2)	001720
	GO TO 200	001730
190	M2 = M2 + 1	001740
	NUMB(M2, IVIA, 2) = NUMBER(K)	001750
	DE(M2, IVIA, 2) = SQRT((L(J) - L(K)) ** 2 + (A(J) - A(K))	001760
	+ ** 2 + (B(J) - B(K)) ** 2)	001770
200	CONTINUE	001780
	NP(IVIA, 1) = M1	001790
	NP(IVIA, 2) = M2	001800
220	CONTINUE	001810
C		001820
C	PRINT OUT RESULTS FOR EACH STANDARD POINT.	001830
C		001840
	NMAX = MAX0(NP(1, 1), NP(1, 2), NP(2, 1), NP(2, 2), NP(3, 1),	001850
	+ NP(3, 2))	001860
	IF (NMAX . EQ . 0) GO TO 270	001870
	WRITE (7, 1011)	001880
	DO 250 K = 1, NMAX	001890
	WRITE (7, 1004)	001900
	DO 250 M1 = 1, 3	001905
	DO 250 M2 = 1, 2	001910
	IF (K . GT . NP(M1, M2)) GO TO 250	001920
	IF (M1 . EQ . 1 . AND . M2 . EQ . 1) WRITE (7, 1005)	001930
	+ NUMB(K, M1, M2), DE(K, M1, M2)	001940
	IF (M1 . EQ . 1 . AND . M2 . EQ . 2) WRITE (7, 1006)	001950
	+ NUMB(K, M1, M2), DE(K, M1, M2)	001960
	IF (M1 . EQ . 2 . AND . M2 . EQ . 1) WRITE (7, 1007)	001970
	+ NUMB(K, M1, M2), DE(K, M1, M2)	001980
	IF (M1 . EQ . 2 . AND . M2 . EQ . 2) WRITE (7, 1008)	001990
	+ NUMB(K, M1, M2), DE(K, M1, M2)	002000
	IF (M1 . EQ . 3 . AND . M2 . EQ . 1) WRITE (7, 1009)	002010
	+ NUMB(K, M1, M2), DE(K, M1, M2)	002020
	IF (M1 . EQ . 3 . AND . M2 . EQ . 2) WRITE (7, 1010)	002030
	+ NUMB(K, M1, M2), DE(K, M1, M2)	002040
250	CONTINUE	002050
	GO TO 300	002060
270	WRITE (7, 1012)	002070
300	CONTINUE	002080
	STOP	002090
C		002100
C	FORMAT STATEMENTS	002110
C		002120
	1001 FORMAT(2X, A7, F12.2, 2F10.2)	002130
	1002 FORMAT(/////* SAMPLE NUMBER *, A7)	002140
	1003 FORMAT(I5)	002150
	1004 FORMAT(* *)	002160
	1005 FORMAT(1H+, A7, F6.2)	002170
	1006 FORMAT(1H+, 17X, A7, F6.2)	002200
	1007 FORMAT(1H+, 34X, A7, F6.2)	002210
	1008 FORMAT(1H+, 51X, A7, F6.2)	002220
	1009 FORMAT(1H+, 68X, A7, F6.2)	002240
	1010 FORMAT(1H+, 85X, A7, F6.2)	002250
	1011 FORMAT(* PLUS CHROMA*, 5X, *MINUS CHROMA*, 7X,	002260
	+ *PLUS HUE*, 9X, *MINUS HUE*, 7X, *PLUS LIGHT*, 7X,	002270
		002280
		002290
		002300

**MINUS LIGHT*)
1012 FORMAT(* NO POINTS FOUND IN ANY DIRECTION*)
1014 FORMAT(1H\$,I5)

C

END

002305
002310
002315
002320
002330

Typical portion of output from Program Sideways

SAMPLE NUMBER 519-152

PLUS CHROMA	MINUS CHROMA	PLUS HUE	MINUS HUE	PLUS LIGHT	MINUS LIGHT
	513- 10 .21	528-610 .09	967-140 .06	519-336 .46	528-610 .09
	528-610 .09	528-642 .16		526-753 2.08	967-140 .06
	967-140 .06	536-835 .12		528-574 .15	
				529-757 .29	
				529-765 .31	
				529-779 .61	

SAMPLE NUMBER 519-176

PLUS CHROMA	MINUS CHROMA	PLUS HUE	MINUS HUE	PLUS LIGHT	MINUS LIGHT
513- 30 .13	526- 40 .46	513- 30 .13	526-108 .11	51-989*00.20	526-123 .11
526-135 .11	526-123 .11	526-135 .11		511-167 1.00	526-135 .11
528-610 .10	528-584 .10	528-584 .10		512-496 .74	
	531-446 .29			513- 9 .19	
				513- 10 .17	
				526- 71 .56	
				526-748 1.66	
				526-748 1.68	
				526-753 1.49	
				528-607 .75	
				528-636 .82	
				529-776 .74	
				530-345 .87	

SAMPLE NUMBER 519-223

PLUS CHROMA	MINUS CHROMA	PLUS HUE	MINUS HUE	PLUS LIGHT	MINUS LIGHT
	513- 9 .23	529-748 .05	528-574 .10	526-753 1.96	536-835 .10
	513- 10 .29		536-835 .10	528-574 .10	
	526-165 .44			529-748 .05	
	528-114 .20			529-757 .18	
	528-574 .10			529-765 .19	
	529-748 .05			529-779 .49	
	530-351 .61				
	534-980 .68				
	536-834 .24				
	536-835 .10				
	536-929 .47				

Appendix E. Computer program for randomizing order of presentation
of sample pairs (Program FUNFER)

PROGRAM FUNFER (INPUT, OUTPUT, TAPE1)

```

C
C      PREPARES RANDOMIZED LIST OF COMPARISONS FOR THE INSPECTOR
C
C      DIMENSION NAME (2, 24), ITAL(24)
C
C      LOGICAL DONE
C
C      DATA NAME/
+1001,8444,8444,1001,7030,7430,7430,7030,
+7291,8443,8443,7291,7052,7000,7000,7052,
+0709,7023,7023,0709,0847,0979,0979,0847,
+0866,0650,0650,0866,0706,0837,0837,0706,
+0681,0984,0984,0681,0714,0644,0644,0714,
+5000,5327,5327,5000,1000,5310,5310,1000/
C
      DO 100 I = 1, 24
100  ITAL(I) = 0
150  K = IFIX(RANF(0.0) * 24. + 1.)
      IF (ITAL(K) .GE. 10) GO TO 150
      WRITE (1, 1001) (NAME(J, K), J = 1, 2)
      ITAL(K) = ITAL(K) + 1
      DONE = .TRUE.
      DO 200 J = 1, 24
      IF (ITAL(J) .LT. 10) DONE = .FALSE.
200  CONTINUE
      IF (DONE) STOP
      GO TO 150
1001 FORMAT(//1X, I4, I9)
      END

```

001000
001010
001020
001030
001040
001050
001060
001070
001075
001080
001090
001100
001110
001120
001130
001220
001230
001240
001250
001260
001270
001280
001290
001300
001310
001320
001330
001340
001350
001370

```

***** 17.25.57.      MANNYLW 000081 LINES PRINTED  /// END OF LIST ///      LQ  22
***** 17.25.57.      MANNYLW 000081 LINES PRINTED  /// END OF LIST ///      LQ  22

```

Appendix F. Computer program for calculation of 50% acceptability limits
by logistic function, (Program LOGIT).

The logistic function was used by Berkson (4, 5) to establish the dose of a drug that is lethal to 50% of the population exposed, the so-called L.D.50 value. The use in our work is analogous: we wish to determine the color difference at which a given inspector will pass examples 50% of the time. The papers of Berkson should be consulted for details of the function and its use; we will just present the equations here.

The logistic function is

$$P = 1 - Q = \frac{1}{1 + e^{-(\alpha + \beta x)}} \quad (F-1)$$

In this equation, x is an independent variable. In Berkson's work with drugs, it is the logarithm of the dosage; in our work, following Berkson, we used the logarithm of the color difference. The symbol P represents, in Berkson's work, the calculated fraction of test animals killed by the dose corresponding to x ; in our work it is the calculated fraction of times the inspector passes the pair of samples having a color difference corresponding to x . The quantities α and β are parameters.

Equation F-1 can be linearized by using the function $\ln (P/Q)$, which is equal to $\alpha + \beta x$. This function is known as the logit of P . Thus, plotting the logit of P against x should yield a straight line, if the assumptions on which the logistic function is based are valid.

4 J. Berkson, "Application of the Logistic Function to Bio-Assay," Am. Stat. Assn. J., 39, 357-365 (1944).

5 J. Berkson, "A Statistically Precise and Relatively Simple Method of Estimating the Bio-Assay with Quantal Response, Based on the Logistic Function," Am. Stat. Assn. J., 48, 565-599 (1953).

The following analysis applies to one inspector looking at a series of four color differences, where each color difference is shown to him 10 times. Let p_i be the observed fraction of passes at the color difference corresponding to x_i (remember that P was the theoretical fraction of passes), $q_i = 1 - p_i$, and $l_i = \ln (p_i/q_i)$. It can then be shown that good estimates of α and β (denoted by a and b) can be obtained by the following normal equations, where the summations are taken over the four color differences:

$$a = \frac{\sum p_i q_i l_i^2 - \sum p_i q_i l_i \sum p_i q_i x_i}{\sum p_i q_i \sum p_i q_i x_i^2 - (\sum p_i q_i x_i)^2}, \quad (F-2)$$

$$b = \frac{\sum p_i q_i \sum p_i q_i l_i x_i - \sum p_i q_i x_i \sum p_i q_i l_i}{\sum p_i q_i \sum p_i q_i x_i^2 - (\sum p_i q_i x_i)^2}. \quad (F-3)$$

Once a and b have been calculated, the x value corresponding to a 50% pass is given very simply. Since $\ln (P/Q) = \alpha + \beta x$, if $P = Q = 0.5$ then $\ln (P/Q) = 0$ and $x_{50} = -(\alpha/\beta)$. We thus have

$$\Delta E_{50} = e^{-(a/b)} \quad (F-4)$$

since x represented the logarithm of the color difference. The symbol ΔE_{50} refers to the color difference for which the inspector will pass the sample 50% of the time.

The standard deviation of the ΔE_{50} value is calculated by the following equations:

$$s_a^2 = 1/(10 \sum pq), \quad (F-5)$$

$$s_b^2 = 1/ \left[10 \sum pq (x - \bar{x})^2 \right], \quad (F-6)$$

where $\bar{x}$ is the x value averaged over the four color differences.

$$s_{x_{50}}^2 = (1/b^2) \left[s_a^2 + s_b^2 (x_{50} - \bar{x})^2 \right], \quad (F-7)$$

$$s_{\Delta E_{50}} = \Delta E_{50} s_{x_{50}}, \quad (F-8)$$

where $s_{\Delta E_{50}}$ is the desired standard deviation.

In entering the observers' data for fraction of passes, we avoided entering 1 for 100% passes or 0 for 0% passes, since the corresponding logits would have been infinite. Following Berkson, we used 0.95 and 0.05, respectively.

C	PROGRAM LOGIT (INFUT, OUTPUT, TAPE1)	001000
C		001010
C	COMPUTES 50 PER CENT ELLIPSOID BOUNDARIES	001020
C	BY THE METHOD OF LOGISTIC FUNCTIONS.	001030
C		001040
	DIMENSION DE(4, 6, 2), Q(4), IQ(4, 3), W(4), DS0(3),	001050
	*STDEV(3)	001060
C		001070
	REAL LOGIT(4), LOGDE(4)	001080
C		001090
	INTEGER CCHMLL, GUY, SIZE, COLOR	001100
C		001110
	DATA DE/.4, .8, 1.13, 1.52, .4, .8, 1.13, 1.52, .4, .77,	001120
	+1.17, 1.32, .4, .77, 1.17, 1.45, .6, 1.17, 1.82, 2.29,	001130
	+6, 1.17, 1.82, 2.29, .5, .92, 1.33, 1.92, .5, .92, 1.33,	001140
	+1.92, .25, .47, .69, 1.01, .25, .48, .69, 1.01, .66, 1.32,	001150
	+1.95, 2.65, .66, 1.32, 1.95, 2.65/	001160
C		001170
C	DO LOOPS FOR COLOR STANDARD, DIRECTION IN	001180
C	COLOR SPACE, AND INSPECTOR.	001190
C		001200
	REWIND 1	001205
	DO 300 COLOR = 1, 2	001210
	IF (COLOR . EQ . 1) PRINT 1003	001220
	IF (COLOR . EQ . 2) PRINT 1004	001230
	PRINT 1012	001240
	DO 300 CCHMLL = 1, 6	001250
	GO TO (101, 102, 103, 104, 105, 106), CCHMLL	001260
101	PRINT 1005	001270
	GO TO 110	001280
102	PRINT 1006	001290
	GO TO 110	001300
103	PRINT 1007	001310
	GO TO 110	001320
104	PRINT 1008	001330
	GO TO 110	001340
105	PRINT 1009	001350
	GO TO 110	001360
106	PRINT 1010	001370
110	AVX = 0.	001380
	DO 120 SIZE = 1, 4	001390
	LOGDE(SIZE) = ALOG(DE(SIZE, CCHMLL, COLOR))	001400
120	AVX = AVX + LOGDE(SIZE)	001410
	AVX = 0.25 * AVX	001420
	DO 250 GUY = 1, 3	001430
C		001440
C	READ PERCENT PASSES.	001450
C		001460
	READ (1, 1002) (Q(SIZE), SIZE = 1, 4)	001470
	DO 130 SIZE = 1, 4	001480
	IQ(SIZE, GUY) = 100. * Q(SIZE) + .0001	001490
	IF (IQ(SIZE, GUY) . GT . 90) IQ(SIZE, GUY) = 100	001500
	IF (IQ(SIZE, GUY) . LT . 10) IQ(SIZE, GUY) = 0	001505
130	CONTINUE	001510
C		001520
C	CALCULATION OF LOGISTIC FUNCTIONS AND WEIGHTS.	001530
C		001540
	DO 150 SIZE = 1, 4	001550
	P = 1. - Q(SIZE)	001560
	W(SIZE) = P * Q(SIZE)	001570
150	LOGIT(SIZE) = ALOG(P / Q(SIZE))	001580
C		001590
C	CALCULATION OF SUMMATION VALUES.	001600

C	SUMW = 0.	001610
	SUMWL = 0.	001620
	SUMWLX = 0.	001630
	SUMWX = 0.	001640
	SUMWX2 = 0.	001650
	SUMWXX2 = 0.	001660
	DO 200 SIZE = 1, 4	001670
	SUMW = SUMW + W(SIZE)	001680
	SUMWL = SUMWL + W(SIZE) * LOGIT(SIZE)	001690
	SUMWLX = SUMWLX + W(SIZE) * LOGIT(SIZE) * LOGDE(SIZE)	001700
	SUMWX = SUMWX + W(SIZE) * LOGDE(SIZE)	001710
	SUMWX2 = SUMWX2 + W(SIZE) * LOGDE(SIZE) ** 2	001720
200	SUMWXX2 = SUMWXX2 + W(SIZE) * (LOGDE(SIZE) - AVX) ** 2	001730
C		001740
C	CALCULATION OF SLOPE AND INTERCEPT OF LEAST SQUARES LINE.	001750
C		001760
	DENOM = SUMW * SUMWX2 - SUMWX ** 2	001770
	A = (SUMWL * SUMWX2 - SUMWLX * SUMWX) / DENOM	001780
	B = (SUMW * SUMWLX - SUMWX * SUMWL) / DENOM	001790
C		001800
C	50 PER CENT POINT.	001810
C		001820
	X50 = -A / B	001830
	D50(GUY) = EXP(X50)	001840
C		001850
C	STANDARD DEVIATION OF 50 PER CENT POINT.	001860
C		001870
	VARAPR = 1. / (10. * SUMW)	001880
	VARB = 1. / (10. * SUMWXX2)	001890
	JARX50 = (1. / B ** 2) * (JARAPR + JARB * (X50 - A/X) ** 2)	001900
	VARD50 = VARX50 * D50(GUY) ** 2	001910
	STDEV(GUY) = SQRT(VARD50)	001920
250	CONTINUE	001930
C		001940
C	PRINT RESULTS.	001950
C		001960
	PRINT 1011, (DE(SIZE, CCHHLL, COLOR), (IQ(SIZE, GUY),	001970
	+GUY = 1, 3), SIZE = 1, 2)	001980
	PRINT 1001, DE(3, CCHHLL, COLOR), (IQ(3, GUY),	001990
	+D50(GUY), STDEV(GUY), GUY = 1, 3)	002000
	PRINT 1011, DE(4, CCHHLL, COLOR), (IQ(4, GUY),	002005
	+GUY = 1, 3)	002010
300	CONTINUE	002020
	STOP	002030
C		002040
C	FORMAT STATEMENTS.	002050
C		002060
	1001 FORMAT(F5.2, I6, F7.2, 2H +/-1H+, 17X, 2H n, F5.2,	002070
	+I5, F7.2, 2H +/-1H+, 36X, 2H n, F5.2, I5, F7.2, 2H +/-	002080
	+1H+, 55X, 2H n, F5.2)	002090
	1002 FORMAT(F15.0)	002095
	1003 FORMAT(////22X, *COLOR: OLIVE GREEN*)	002100
	1004 FORMAT(////26X, *COLOR: TAN*)	002110
	1005 FORMAT(//6X, *PLUS CHROMA*/)	002120
	1006 FORMAT(//6X, *MINUS CHROMA*/)	002130
	1007 FORMAT(//6X, *PLUS HUE*/)	002140
	1008 FORMAT(//6X, *MINUS HUE*/)	002150
	1009 FORMAT(//6X, *PLUS LIGHT*/)	002160
	1010 FORMAT(//6X, *MINUS LIGHT*/)	002170
	1011 FORMAT(F5.2, I6, 2I19/F5.2, I6, 2I19)	002180
	1012 FORMAT(/// * DELTA*, 7X, *BILL*, 16X, *MOE*, 15X, *CAROL*	002190
	+ /3X, *E*/4X, 3(4X, *PCT.*, 3X, *BOUNDARY*/6X, 3(2X, *PASS*,	002200
	+2X, *^ STD. DEV.*))	002210
C		002220
	END	002230
		002240

Appendix G. Derivation and Meaning of the Acceptability Equation

If the acceptability equation, Equation 1, were to have the square root bracket removed, and if ΔA were set equal to unity, it would become an ellipsoid equation in which the center of the ellipsoid is placed at the standard point in CIELAB space, the three semiaxes are made equal in size to the tolerance values determined by the inspectors, and the angle θ is made equal to the hue angle of the standard. Thus, in Equation B-3, c , the chroma tolerance, replaces p ; h , the hue tolerance replaces q ; and v , the lightness tolerance, replaces r . The coordinates of the standard, a_0^* , b_0^* and L_0^* , replace those of the center of the ellipsoid, a_c^* , b_c^* and L_c^* . With these changes, Equations B-4 to B-8 become Equations 2 to 6, and Equation 1 modified as just indicated is equivalent to Equation B-9. This equation defines what we call the "acceptability ellipsoid."

Now if we substitute the coordinates of any point into this equation, we generally will no longer calculate unity but some other value that we have called S^2 in Appendix B and we will call $(\Delta A)^2$ here. We showed in Appendix B that we have defined a new ellipsoid that passes through the sample point in which the linear distances are expanded or contracted by a factor of S (or ΔA). We thus see the rationale of taking the square root of the expression in Equation 1; it is that we wish to calculate the factor (ΔA) by which the linear distance from the standard to the sample in question is greater or less than the linear distance along the same line but from the standard to the acceptability ellipsoid. If ΔA is greater than 1, the point lies outside of the acceptability ellipsoid; if it is less than 1, the point is inside the acceptability ellipsoid.