FRANK J. SEILER RESEARCH LABORATORY

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MULTIPLE, INDEPENDENTLY TARGETED REENTRY VEHICLE (MIRV) TARGETING MODELS

FINAL REPORT

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A previously reported weapons allocation model was extended to include multiple independently targeted reentry vehicles (MIRV). Model feasibility is demonstrated through computational testing. Implementation considerations are discussed.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1 Introduction

Current ballistic missile weapon systems are capable of carrying multiple, independently targeted reentry vehicles (MIRV). The effective assignment of these independent weapons against individual targets is the subject of the research described below. This research is a continuation of the work described in SRL-TR-78-0005, Optimal Targeting of Ballistic Missiles in a Tiered Aimpoint System.

The earlier work described the modeling process which transformed a weapons data base and a target data base into a mathematical programming model whose solution was an 'optimal' allocation of weapons to targets. Optimality was measured by maximizing expected damage value initially defined by a scoring process. In this paper we will a) review the previously reported work, b) describe the model changes necessary to incorporate the concept of MIRVed vehicles, c) report the computational results of solving several MIRVed problems, d) discuss potential computational improvements, and e) recommend avenues for implementation.

Chapter 2. Model Formulation and Assumptions

2.1 UnMIRVed Model Description

There are several assumptions which are made in the transition from the data describing the basic target set and the weapon arsenal to the mathematical programming model which is used to construct a weapon laydown. To appreciate the utility of the model and understand its limitations, one must relate these assumptions to the basic model structure. The next sections review these modeling assumptions first for the target set, then for the weapon array, and finally for the structure of the model which ties these together.

2.2 Target Set

The basic set of targets are called installations and are characterized by the following:

a) location defined by coordinates

b) value or score (specified by targeting agency)

c) hardness

d) damage requirements

The Aimpoint Generation Algorithm (AGA) described in SRL-TR-78-0005 partitions these installations into aggregated targets called designated-ground-zeros (DGZs). For a particular weapon type (tier) these DGZs are nonoverlapping target clusters meeting the criteria specified by the AGA user. A DGZ is defined by the following:

a) location (coordinates)

b) eligible weapon type(s)

c) value extracted

d) height of burst

2.3 Weapon Data Set

A single weapon type is assumed to be launched from a uniquely specified launch site at a location defined by its coordinates. The assumption of a single weapon type per launch site is not a limitation since multiple launch sites can be defined at the same coordinates. The number of weapons at a launch site is determined by defining the number of launch vehicles per site and the number of weapons per launch vehicle. Any launch site which has more than one weapon per launch vehicle is called a MIRVed launch site. Thus, the defining data elements for a launch site are:

1) location (coordinates)
2) weapon type (W/CEP)
3) number of launch vehicles
4) number of weapons per launch vehicle
5) range limitation for different launch azimuths for the weapon type

2.4 UnMIRVed Model Definition

From the target/weapon data base defined in the previous two sections, a mathematical programming model can be formulated which, when optimized, will solve the problem of allocating weapons to targets. The fundamental decision variable (λ_{ij}) is defined for all DGZs which are eligible targets for launch site i under all restrictions such as range, launch azimuth, and weapon type. The set of eligible DGZs for launch site i is designated D(i). The decision variable, λ_{ij}, represents weapon assignment, thus:

$$\lambda_{ij} = \begin{cases} 1 & \text{If a weapon from launch site is allocated to DGZj} \\ 0 & \text{otherwise} \end{cases}$$

The value extracted is an output of the AGA and represents the expected collective damage to all the installations within a DGZ if one of the designated weapon type(s) is detonated according to the parameters specified when the AGA created the DGZ. The computation of the value extracted assumes no interaction with other DGZs, i.e., no collateral damage. It is an expected value figure taking into account weapon inaccuracies expressed as CEPs and a user designated probability of minimum damage (P_D) used in the expected value calculation.

Another set of data is required to completely describe the characteristics of the target set for model construction. Although the DGZs within a tier or weapon class are constructed as nonoverlapping, the DGZs defined for different tiers are either proper subsets of another tier or are overlapping covers of the target set. This DGZ/tier coverage interaction is described by identifying DGZs by tier and specifying for each installation which DGZs include it. Since tier definition is arbitrary, this is a flexible method adaptable for a wide range of operational situations.

To review the definitions used to construct the target set:

1) The DGZ represents the basic aimpoint or target for its associated weapon type.

2) The value-extracted represents the value of allocating the DGZ weapon type to that DGZ; no interaction with other DGZs or weapons occur.

3) The value-extracted is an expected value measure and incorporates probabilities quantifying inaccuracies in weapon delivery.

4) The collateral coverage between DGZs of different tiers is completely defined by including appropriate installations in the respective DGZs.

As described previously, each DGZ is assigned a value which is extracted by assignment of a specified weapon system. Thus, the objective of maximizing extracted value (in a sense, maximize expected damage) can be mathematically stated:

$$\text{MAXIMIZE} \sum_{i} \sum_{j \in D(i)} V_j \ \lambda_{ij}$$

Inherent in this additive definition of the objective function is the assumption that at most one weapon is assigned to each DGZ. Multiple weapon assignments would require a nonlinear objective function since the damage is not additive. Because our environment is "target rich", i.e., there are many more targets than available weapons, the restriction of, at most, one weapon per DGZ does not markedly degrade the quality of the resulting weapon allocation. In addition, the definition of a DGZ is essentially a grouping of installations. These installations are, in fact, the actual targets. The DGZ designations allow the inclusion of a particular installation in more than one DGZ. The nonadditive characteristic of damage precludes the assignment of weapons to two DGZs which overlap, i.e., which contain some of the same installations. All of these multiple coverage, overlap considerations can be included in a single type of restriction:

$$\sum_{(i,j) \in I(k)} \lambda \quad \leq \qquad \forall \ k$$

where I(k) is the set of launch site DGZ pairs which describe eligible weapons to a DGZ containing installation k.

Taken literally, this cover constraint would produce many redundant and trivial constraints. Thus, in implementing the model, only those explicit nonredundant, nontrivial constraints are included.

The weapon supply is limited by the number of launch vehicles (n_i) and the number of reentry vehicles per launch vehicle (μ_i) at each launch site. For an unMIRVed launch site, $\mu_i = 1$. Thus, the allocation for a particular launch site is limited:

$$\sum_{j \in D(i)} \lambda_{ij} \leq \mu_i n_i \qquad \forall i$$

The unMIRVed problem can be summarized as a zero/one integer programming problem:

$$\text{MAXIMIZE} \quad \sum_{i} \sum_{j \varepsilon D(i)} V_j \quad \lambda_{ij}$$

subject to

$$\sum_{(ij) \varepsilon I(k)} \lambda_{ij} \leq 1 \qquad \forall k$$

(UIP)

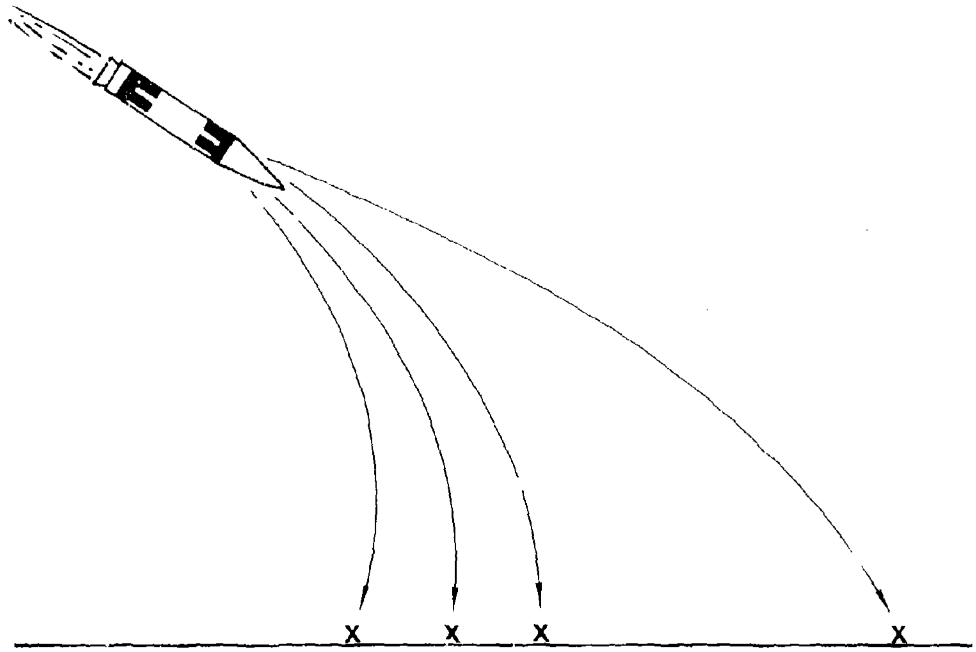
$$\sum_{j \varepsilon D(i)} \lambda_{ij} \leq \mu_i n_i \qquad \forall i$$

$$\lambda_{ij} = 0 \text{ or } 1$$

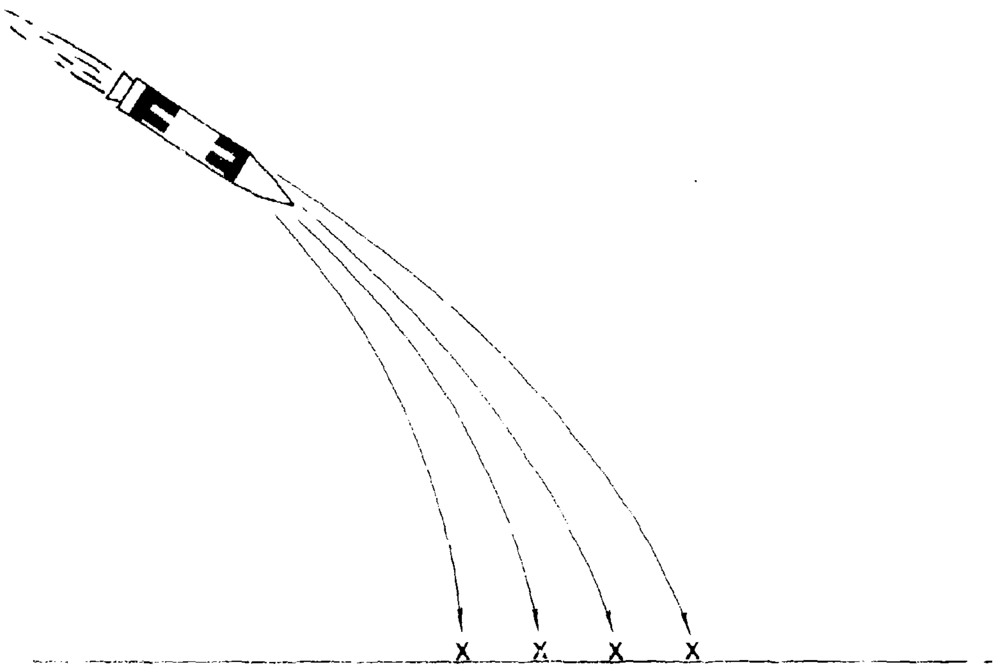
2.5 Footprinting

In the problem UIP, each reentry vehicle is treated as an independently targeted weapon, subject only to the range and weapon type restrictions imposed in the definition of the decision variables λ_{ij}. In reality, when several reentry vehicles (RV) are on a single launch vehicle, there are limitations imposed on the RVs as a group. Simply put, the total energy available for dispersing the group of RVs is bounded; implying that the collective range of the group is limited. This limitation could be satisfied in a variety of ways such as all but one RV landing in a tightly clustered group and the remaining RV using most of the energy to hit a distant target (Figure 1). In applying this proximity constraint to MIRVed weapons, one

FIGURE 1. POSSIBLE MIRV DISPERSAL PATTERNS



A. 4 MIRV'S ON A MLV - DISTANT DGZ REQUIRES MOST OF THE AVAILABLE DISPERSAL ENERGY



B. 4 MIRV'S ON A MLV DISPERSAL ENERGY USED EQUALLY BY UNIFORMLY LOCATED TARGETS could consider all possible sets of DGZs eligible for a specified MIRV launch vehicle (MLV), and identify those which meet the MIRV proximity condition. This would produce an unreasonably large number of target alternatives and would be computationally difficult to implement. A way to produce a rich but manageable number of MIRV target alternatives is to pick a geometric figure representative of a realistic MIRV laydown which can be varied by simple parameter specification, and assume that any pattern of RV laydown within the figure is feasible. This figure is designated the MIRV footprint.

An ellipse provides sufficient flexibility yet is easy to implement; thus, it was chosen as the MIRV footprint mechanism. A footprint ellipse is oriented with its major axis along the radius from a LS and has a dimension ratio which can be set to represent the desired pattern characteristics. For simplicity, the footprints are all taken as identical, however, the model requires only that the shape and dimension of footprints be identical for all LVs deployed from the same LS. A typical MIRV footprint might have an ellipse ratio of 5 with a largest dimension of approximately 200 miles.

Figure 2 shows the footprints for a 3 weapon MIRV ($u = 3$) targeted against a small target set.

2.6 Print Set and MIRV Target Set

By overlaying a footprint on the set of DGZs eligible to a particular LS, we define those DGZs within the footprint as a print set. Thus, a print set is a set of DGZs eligible for targeting from a single MLV and encompassed within a specific footprint. It is clear that many print sets with multiple redundancies could be defined by repositioning the footprint and defining the set of DGZs covered as a new print set. Even a scheme of defining an arbitrary grid and positioning the footprint on all points of the grid would generate an unmanageably large number of print sets. The technique used to define print sets is to position the center of a

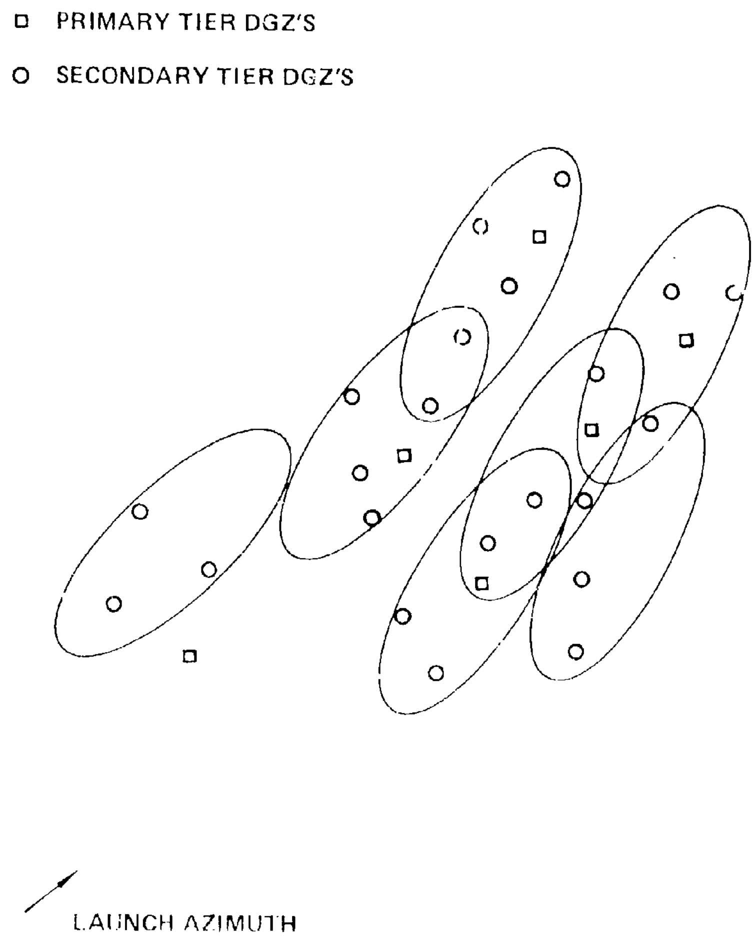


FIGURE 2. ILLUSTRATIVE FOOTPRINTS

footprint at each eligible DGZ and define the covered DGZs as a print set. Furthermore, we will only define print sets which conform to the reciprocity condition given as:

$$(X_i - X_j)^2 + S_k(Y_i - Y_j)^2 \leq R_k$$

where x and y are downrange and crossrange coordinates measured from LS k, and S_k and R_k are parameters of the footprint ellipse, and R_k and S_k define:

$\sqrt{R_k}$ Semimajor axis

$\sqrt{\frac{R_k}{S_k}}$ Semiminor axis

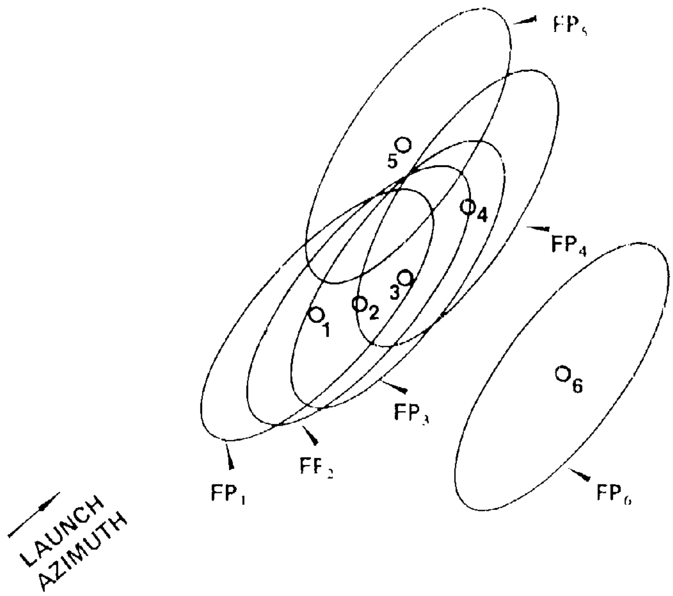
Thus, the set of decision variables from launch site k contained in the print set defined about DGZ i would be

$$\left\{ \lambda_{kj} \right\} \quad \forall_j \in P_k(i)$$

where $P_k(i)$ is the print set defined about DGZ i for a MLV from launch site k. This method of defining print sets provides a rich but computationally manageable number of potential MIRV targets. Figure 3 illustrates this concept of a print set.

The number of DGZs within a print set depends on the geometry and characteristics of a specific launch site/target set scenario. In general, the number of DGZs in a print set would exceed the number of weapons on a MLV. To distinguish those DGZs specifically targeted, the MIRV target set will be defined as that subset of N DGZs within a print set which the weapons on a MLV are allocated against. This is shown in Figure 4.

FIGURE 3. FOOTPRINT – PRINT SET – DGZ HIERARCHY



A. EACH FOOTPRINT IS ASSOCIATED WITH A DGZ ($FP_i \sim DGZ_i$)

$FP_1 \rightarrow PS_1 = DGZ_1, DGZ_2, DGZ_3$

$FP_2 \rightarrow PS_2 = DGZ_1, DGZ_2, DGZ_3, DGZ_4$

$FP_3 \rightarrow PS_3 = DGZ_1, DGZ_2, DGZ_3, DGZ_4$

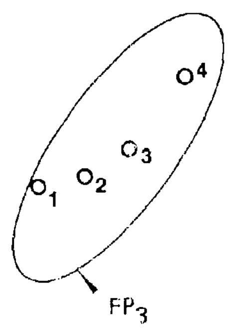
$FP_4 \rightarrow PS_4 = DGZ_2, DGZ_3, DGZ_4$

$FP_5 \rightarrow PS_5 = DGZ_5$

$FP_6 \rightarrow PS_6 = DGZ_6$

B. EXAMPLES OF PRINT SETS

FIGURE 4. ILLUSTRATION OF A MIRV TARGET SET



$PS_3 = DGZ_1, DGZ_2, DGZ_3, DGZ_4$

A. FOOTPRINT AND ASSOCIATED PRINT SET

FOR $\mu = 3$ POTENTIAL MIRV TARGET SETS ARE:

$MTS_{31} = DGZ_1, DGZ_2, DGZ_3$

$MTS_{32} = DGZ_1, DGZ_2, DGZ_4$

$MTS_{33} = DGZ_1, DGZ_3, DGZ_4$

$MTS_{34} = DGZ_2, DGZ_3, DGZ_4$

2.7 Modeling the MIRV Constraints

In characterizing the MIRV weapon, a hierarchy of terms has been required. The footprint is a geometric figure whose size, shape, and orientation are dependent on the specific MIRV weapon system and launch site and which defines an area of feasible coverage by a single MIRV. A print set is the specific set of eligible DGZs for a particular placement of a footprint. A MIRV target set is that subset of a print set designated to be hit by a reentry vehicle from the MLV.

In formulating the MIRV problem, the three following conditions must be satisfied by MIRV target sets:

a) Each MIRV target set must be a subset of at least one print set. Clearly, a MIRV target set can overlap two or more print sets, and any print set can contain more than one MIRV target set, but each MIRV target set must be a subset of at least one print set. (See Figure 4.)

b) Use of a single weapon from a MLV implies use of all the weapons from that MLV. That is if there are μ weapons on a MLV, targeting one implies targeting μ against a specific MIRV target set.

c) MIRV target sets must be disjoint; they cannot share a common DGZ. This is implied by the single coverage assumption necessary to assure linearity of the objective function.

These requirments will now be translated into modifications of the unMIRVed assignment problem. Whereas conditions (b) and (c) will be seen as variants of supply and single coverage constraints, condition (a) has no counterpart in the unMIRVed formulation. It introduces a new constraint. For the moment, we will drop the subscript l and consider a single MIRV LS with μ weapons per LV.

The number of assignments print set i is

$$\sum_{j \in P(i)} \lambda_j$$

Condition (a) states that an assignment is permitted to DGZ k only if there are at least μ assignments in some print set containing k; that is, only if

$$\sum_{j \in P(i)} \lambda_j \geq \mu \quad \text{for some } i \in P(k)$$

To distinguish print sets containing at least μ assignments from those containing less, we define a new variable, w_i, by the relation

$$\mu > \sum_{j \in P(i)} \lambda_i - \mu W_i \geq 0 \qquad (1a)$$

where

$$W_i \geq 0 \quad \text{and integer} \qquad (1b)$$

Then

$$W_i = \begin{cases} 0 & \text{if print set } i \text{ contains less than } \mu \text{ assignments} \\ > 0 & \text{if print set } i \text{ contains } \mu \text{ or more assignments} \end{cases}$$

Since $k \in P(j)$ implies $j \in P(k)$, the quantity

$$\sum_{j \in P(k)} W_j$$

is greater than zero only if at least one print set containing DGZ k contains μ or more assignments. The mathematical expression of (a) is therefore

$$\sum_{j \in P(k)} W_j - \lambda_k \geq 0$$

together with the defining relations (1). These relations (with the subscript 1 restored to all quantities appearing) comprise the print constraints. They are consistency relations ensuring that MIRV weapons are allocated in proximity clusters.

As indicated, condition (c) arises from the single coverage restriction, but adoption will require redefining the decision variables. Condition (b) will then follow with a slight corresponding modification of the supply constraints.

Condition (c) requires that MIRV target sets be disjoint. They must, therefore, be identifiable. To this point, weapons at a single LS were considered indistinguishable. The decision variable λ_{lj} was equal to 1 if any weapon from LS l was allocated to DGZ j and 0 otherwise. The single coverage constraint ensured that LS target sets were disjoint. If now every MLV is considered to be a separate LS (many with the same coordinates), then the corresponding LS target sets are MIRV target sets and will be disjoint by the single coverage constraint operating on the expanded set of decision variables, λ_{lj}, where l now denotes LS or MLV as the case may be. MIRV target sets are then identified by MLV and DGZs targeted. And finally, this modification enables the remaining requirement, condition (b), to be satisfied if the supply constraints are written

$$\sum_j \lambda_{lj} \le \begin{cases} n_l & \text{if } l \text{ is an unMirved LS} \\ \mu_l & \text{if } l \text{ is a MIRVed LS} \end{cases}$$

for the print constraints require at least μ_l assignments while not more than μ_l assignments are permitted by the supply constraints.

The MIRV problem can be now stated:

MAXIMIZE $$\sum_{i} \sum_{j \epsilon D(i)} V_j \lambda_{ij}$$

SUBJECT TO:

SINGLE COVER $$\sum_{(i,j) \epsilon J(k)} \lambda_{ij} \leq 1 \qquad k = 1, \ldots, \quad T_I$$

SUPPLY-UNMIRVED $$\sum_{j \epsilon D(i)} \lambda_{ij} \leq N_i \qquad i \epsilon L_u$$

SUPPLY-MIRVED $$\sum_{j \epsilon D(i)} \lambda_{ij} \leq \mu_i \qquad i \epsilon L_m$$

POINT SET ELIGIBILITY $$\sum_{j \epsilon R(k)} W_j - \lambda_{ij} \geq 0 \qquad i \epsilon L_m \quad k = 1, \ldots, N_D$$

ALLOCATION WITHIN A PRINT SET $$\sum_{k \epsilon Q(j)} \lambda_{ik} - \mu_i W_k \geq 0 \qquad i \epsilon L_m \quad k = 1, \ldots, P$$

$$W_k \geq 0, \text{ integer} \qquad \lambda_{ij} = 0 \text{ or } 1$$

Solution of MIP will yield a laydown of weapons which maximizes the value assigned to the targets designated to be hit within the supply constraints and the MIRV proximity constraints under the assumption that no installation will be hit with more than one weapon.

The price paid for incorporating these proximity constraints is the substantial increase in the number of variables required to represent the MIRV model. Table 1 shows the magnitude of the increase for several example problems. The actual increase depends on launch site and target geometry and MIRV characteristics since these determine the number of print sets, number of DGZs per print set, etc.

Table 1

COMPARISON OF UNMIRVED AND MIRV PROBLEM SIZE

PROBLEM	UNMIRVED SIZE	MIRVED SIZE
1	97 x 530	1057 x 1010
2	100 x 752	1504 x 1454
3	101 x 730	1561 x 1460
4	101 x 802	1515 x 1465
5	99 x 684	1367 x 1318
6	99 x 666	1331 x 1282
7	97 x 530	1057 x 1010

The size of the problem is difficult to estimate in general because it depends on the geometry of the launch sites and targets which dictate target eligibility. The number of constraints can be determined by:

1) one for each installation (single coverage)

2) one for each unMIRVed launch site

3) one for each MLV

4) one for each MIRV pseudo launch site and eligible DGZ pair

5) one for each print set and pseudo MIRV launch site pair

Four (4) and five (5) imply a potentially large number of constraints since there is a pseudo launch site for each MLV weapon. For example, 10 MIVR weapons and eligible DGZs would yield at least 600 constraints.

Likewise, the number of variables increases substantially when the MIRV aspect is modeled. Again the actual number of variables depends on the geometry and weapon characteristics, but is somewhat proportional to the number of launch sites times the number of eligible DGZs. For the unMIRVed problem with two launch sites and 20 DGZs, the number of variables was about 40 no matter how many launch vehicles were at each launch site. If each launch site has two launch vehicles, each with 4 MIRVs, the number of variables is now 160. Clearly the introduction of the MIRV restriction, while feasible, imposes practical limitations on the size of problems which can be solved.

Chapter 3. Computational Results

3.1 Introduction

Several problems formulated using the models developed in Chapter 2 were solved to demonstrate the utility of the models and investigate potential practical computational bounds. The problems were derived from a representative target/launch site data base provided by USAF/SA. The data were selected to exercise the range of real world attributes present in the model.

3.2 Results

Tables 2 and 3 provide detailed statistics for the solution of two test problems (TP1, TP2). The two problems show two important features: 1) the increase in computational complexity when the MIRV aspect is modeled, and 2) the amount of value degradation when the MIRV is imposed. In TP1 the size of the problem increases by a factor of at least ten and the solution time by a factor of over 30 while the value of the solution decreased by 15%. For TP2 the increase in size was again a factor of over ten while the solution time increased by a factor of over 75. The degradation in the solution value was only 1%.

Table 4 presents the results for solving several problems generated from the representative data base. As can be seen, the number of total weapons has been varied, the MIRV configurations have been varied and the launch azimuth has been varied. Experience with this set of problems indicate that the practical limitation on problem size which can be solved using the Burroughs integer/linear programming package TEMPO is between 1000 x 1000 and 2000 x 2000. As expected, the exponential increase in solution time vs problem size dictates the maximum solvable problem for any mathematical programming (MP) package. The exact limitation varies because of the MP system implementation and computer manufacturer.

Table 2

TEST PROBLEM 1

AREA - 5° LAT BY 8° LONG

TARGETS BY TIER

	NUMBER	VALUE	
PRIMARY	15	4930	
SECONDARY	21	4827	
	36	9757	TOTALS

UNMIRVED SOLUTION

WEAPONS 13
LINEAR PROGRAM 13 ROWS BY 36 COLUMNS
SOLUTION TIME .15 MIN SOLUTION VALUE 4909

MIRVED SOLUTION

WEAPONS 3 x 3 4 x 1 = 13
LINEAR PROGRAM 177 ROWS BY 161 COLUMNS

SOLUTION	TIME (min)	VALUE
LP	.75	4841.625
1st INT.	2.52	3734
BEST INT.	4.69	4164

TOTAL 6*

*OPTIMALITY NOT VERIFIED

Table 3

TEST PROBLEM 2

AREA 2° LAT BY 20° LONG

TARGETS BY TIER

	NUMBER	VALUE	
PRIMARY	50	19557	
S1	68	10461	
S2	81	13596	
	199	43614	TOTALS

(135 SEPARATE TARGETS)

UNMIRVED SOLUTION

WEAPONS 31
LINEAR PROGRAM 97 ROWS BY 530 COLUMNS
SOLUTION TIME .86 MIN SOLUTION VALUE 20341

MIRVED SOLUTION

WEAPONS 7x1 2x6 4x3 = 31
LINEAR PROGRAM 961 ROWS BY 915 COLUMNS

SOLUTION	TIME (MIN)	VALUE
LP	6.79	2010[illegible]
1st INT.	26.92	18702
BEST INT.	37.[illegible]2	19116

TOTAL 140*

*OPTIMALLY NOT VERIFIED

Table 4

COMPUTATIONAL RESULTS

PROBLEM	PR	WEAPONS S1	WEAPONS S2	TOTAL	UNMIRVED SIZE	UNMIRVED VALUE	MIRVED SIZE	MIRVED VALUE	COST OF MIRV
1	7x1	2x6	4x3	31	97x530	20341	1057x1010	19116	6.02%
2	10x1	4x6	5x4	54	100x752	21875	1504x1454	20903	4.44%
3	4x2	4x3	3x4	32	101x730	20343	1561x1460	19199	5.41%
4	7x1	6x2	4x3	31	101x802	20341	1515x1465	19568	3.8%
5	7x1	3x6	5x3	40	99x684	21453	1367x1318	20062	6.48%
6	7x1	4x2	4x4	31	99x666	20331	1331x1282	19634	3.45%
7 (45°)	7x1	2x6	4x3	31	97x530	20341	1057x1010	19133	5.9%
8 (15°)	7x1	2x6	4x3	31	97x530	20341	1057x1010	19075	6.22%

A significant observation is the "cost of MIRVing", that is the reduction in value extracted when the MIRV restrictions are imposed. Over the range of problems solved, this cost essentially remained at about 5% of the unMIRVed value extracted. This means that if one is able to "optimize" weapon allocation, then the cost of MIRVing is not too great.

Chapter 4. Conclusions and Recommendations

The research described in the previous chapters demonstrates the feasibility of incorporating the limitation of MIRV delivery systems into a weapons allocation model. Also, the flexibility and utility of such a model for use in a planning environment is indicated by the parameter variation and associated computational results. The principal tradeoff of this major model enhancement is the increased problem size and resultant increase in computer resources required to solve the linear/integer programming problem.

No attempt was made to research efficient solution algorithms for the MIRV problem. The standard linear/integer programming package, TEMPO, available on the USAF Academy B6700 was used. In fact, default options were used for setting algorithm parameters so the computer resources required represent a worst case.

If the MIRV model were to be used in a planning or operational environment, there are several things which could be done to improve solution times and to expand the size of problems which could be solved. First, there are several commercially available linear/integer programming computer packages which solve problems faster than TEMPO (5-10 times) and more efficiently store problems and thus allowing the solution of larger problems. Thus, implementing the MIRV model using one of these codes will increase the size of problems which can be solved and improve solution times. Also, in similar applications, significant improvement is gained by tailoring an algorithm to take advantage of the structure of a specific problem. This could certainly be the case of the MIRV model. It is not unreasonable to presume that a factor of 10 or more improvement in solution times and problem size could be gained with a special purpose algorithm. This would certainly be advisable for routine use of the model.