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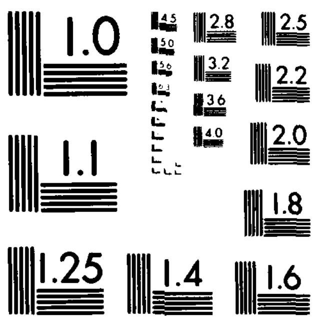

MICROCOPY RESOLUTION TEST CHART

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This report completes development of the mathematical error propagation model. A report called "Operational Positioning Model" will follow as part of the same project element

16. Abstract

A computer model was written to simulate errors associated with the Coast Guard aids to navigation positioning process with emphasis placed on finding average effects of errors in the system. A mathematical expectation approach was used to model random system elements. The model routines have been designed to study specific positioning situations or, by use of Monte Carlo Methods, a variety of situations as a group.

The model was designed for use in planning within the aid positioning macrostructure. For planning purposes, the probability that the resulting position lies within a specific circular region is a meaningful measure of positioning success. The model was exercised for numerous cases and offers a priority listing of errors.

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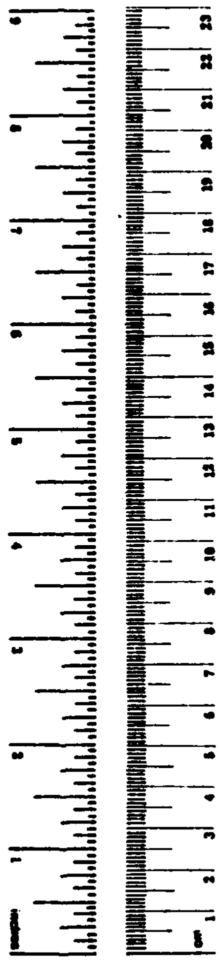
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

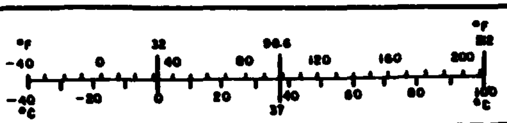
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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1.0 INTRODUCTION

1.1 Background

The Aids to Navigation Positioning Project (ANPP) was initiated in the Coast Guard to establish ways to improve the reliability of buoys in marking advertised positions. Support efforts in this task were assigned to the CG Research and Development Center at Groton, Connecticut, and are entitled "The Aids to Navigation Position Accuracy and Reliability (ANPAR) Project."

1.1.1 Philosophy

The Positioning/Error Model has been defined in three distinct phases:

I - Error Sensitivity Model
II - Operational Positioning Model
III - Uses of the Models

This report concerns itself with Phase I which adopted the following philosophy:

Error sources, both random and systematic in nature, are inherent in the positioning process. Each error source can cause an error in one or more of the observations from which position is determined. The resulting observation error will be propagated within the positioning process and result in a corresponding position error. This error propagation is a function of both the fix geometry (locations of landmarks or transmitting stations) and the observed parameter (sextant angle, radar range, gyroscope bearing, or LORAN time differences). It is reasonable to construct a mathematical model of error propagation routines which can include, where practical, the functional relationship between a measure of error source magnitude and a corresponding observation error. For the purpose of this report the term "error" will refer to either an error source magnitude or an observation error; the effect of an "error" is called "position error"; and the error propagation routine is called "modeled error." Errors with similar propagation functions can be grouped and referred to by a representative "modeled error" to avoid redundant model routines.

Studies of error in the positioning process can be conducted by constructing a model which can accommodate various modeled errors, error magnitudes, and fix geometries, as well as display various measures of position error. The model developed by this project allows the user to: (1) select the modeled error to be studied; (2) specify the range of error magnitude; (3) either define one particular geometry or specify the number of randomly generated geometries for an average effect; and (4) select the position error effect to be displayed.

Studies using this model of the positioning process can be conducted for ranking errors by their effect on a selected position error measure. Once errors are ranked, efforts can be directed toward minimizing the magnitudes of the priority errors. This model also can be used to study the effect blunders have on the positioning process. The most significant blunder effects can be identified and examined to ascertain the need for training efforts directed towards amelioration of these effects.

The Error Sensitivity Model (ESM) phase of this effort provides insight into the effect of the various error sources and provides a tool for use by ANPP to support further efforts. The result of such efforts should help Coast Guardsmen comply with the requirement that an aid mark accurately and precisely the geographic position advertised by the government. In this light, the Error Sensitivity Model is a management tool to help study the aid positioning macro-structure.

1.2 *Terminology and Symbology*

The terminology established by reference 1 has been adopted here and extended as necessary. In addition to the Glossary of reference 1, the following publications establish the terminology used herein:

a. American Practical Navigator, Bowditch (1977)
b. Aids to Navigation Manual, Positioning, Volume V, 1978
c. Observations and Least Squares, Mikhail, E.M.
d. Hydrographic Manual, 4th Edition, Department of Commerce
e. Definitions of Survey Terms, Mitchell, H.C., Special Pub. No. 242

Symbols are defined where they are first used in the text.

1.3 Summary of Previous Work

A review of the descriptive bibliography of reference 1 indicates that ANPP work related to the ESM has been in progress for about seven years. Items listed in this bibliography (along with some unpublished notes compiled within the Coast Guard) comprise the majority of previous work on the ESM Phase of the Positioning/Error Model.

2.0 MATHEMATICAL MODEL

The model simulates the inherent stochastic nature of observations through use of systematic and random error inputs making possible the observation of error propagation effects on important mathematical measures of position error.

The model employs the least squares principle to find a set of adjustments that will cause the observations to be compatible with the following functional model:

- Within the local region where the positioning process is being executed, the lines of position determined by any sufficient subset of functionally independent observations should cross at a single point.

2.1 Preliminary Considerations

Mathematical models, by necessity, must rely upon appropriate assumptions and/or approximations. When these are understood, the model is applicable to the domain defined by them. These considerations are defined and expanded in this and succeeding sections of the report. While every effort has been made to select error magnitudes that reflect those occurring in practice, the model is flexible enough to incorporate verified values as they become available.

While the basic transformation from observation space to real positioning space is well known, the statistical methods chosen for weighting each observation and simulating each error source in the system are dictated by the project element requirements. The requirements of phase I of this project element are to both model and study position error effects as a function of the systematic and the random measurement error components. Statistics which effectively deal with each of these error components are a necessity.

The basic tenet (reference 1), "that the conditions imposed upon any positioning task are unique to the station and not necessarily well defined," dictate procedures which statistically account for the uncertainty of a specific positioning evolution by calculating unbiased estimates of measurement precision from observations made on scene. These procedures deal with the total measurement error, not its components, and are not needed to satisfy the requirements of phase I of this project element. These procedures will be discussed in phase II of the Positioning/Error Model effort where emphasis is placed on mathematical tools for actual aid positioning. (See reference 1)

The results of ESM studies presented in this report are position error effects as a function of error magnitudes and provide an indication of which error to prioritize. The measurement and quantification of actual error magnitudes is the subject of other project elements of the ANPAR project. Definite error magnitudes can be input to the model for analysis when they are established.

2.2 Least Squares Adjustment Method

The error model is based on the least squares adjustment method of computing the Most Probable Position (MPP). Mathematical justification for the adjustment method is contained in references 4 and 8. Expressions for each matrix element are provided in appendix A.

The least squares method is used to provide probable corrections to a set of observations so that they will satisfy the functional model. The input is a set of inconsistent measurements with an inherent stochastic nature and the output is an adjusted set of measurements which are consistent but are not the true measurements. In a related fashion, the least squares method can be used to study the effect observation errors have on position error.

A "fix geometry" is defined so that the set of errorless measurements cause a consistent set of lines of position. The observations are then subjected to a known error, or set of errors, and the resulting inconsistent system is studied. If the system is subjected to systematic error, the MPP may change its location and that change will be dependent on the weight of the observations subjected to the error and the structure of the fix geometry. Mathematically, displacement of the MPP in positioning space is defined by the vector $X^T = [\Delta x, \Delta y]$. The vector X is determined through the transformation of n observations given by: (references 3 and 8)

$$X = -(A^TWA)^{-1}A^TWL \qquad (2\text{-}1)$$

Where L is an (n x 1) vector matrix of systematic observation errors.
W is an (n x n) diagonal matrix of observation weighting factors (inverse of covariance matrix).
A is an (n x 2) matrix of partial derivatives of measurements with respect to x and y dimensions in real positioning space.

It is upon this equation that the Error Sensitivity Model is based. The A matrix is completely determined by the "fix geometry" specified for modeling. The L matrix is the tool for systematic error input to the system and the W matrix is the weighting matrix which depends on the specified variances (and therefore weight) of each observation. It can be seen at this point that the effect a systematic error has on X is linear but is dependent heavily on the variance of the measurements to which it is subjected.

The MPP displacement vector X is not the only error effect of interest. The stochastic nature of the observations is transformed into a bivariate normal probability distribution (reference 11) centered around the MPP. Selected parameters of this distribution are calculated as a function of the stochastic nature of the observations.

The next step is to define the error sources within the positioning process and to functionally relate each in turn to the respective elements of either the L or W matrix. In this fashion, the errors of interest can be studied for any "fix geometry."

2.3 The Errors Modeled

Each element l_i of the L matrix in the adjustment equation (2-1) is a function of the systematic error sources present in making the i^{th} observation. In an actual positioning evolution each l_i is subject to random error sources present in making a measurement. Because of the random nature of l_i, each element is associated with the variance of the respective measurement type which is used to weight the observations. The diagonal elements w_{ii} of the W matrix in the adjustment equation are the weighting factors. They are determined by the equation:

$$w_{ii} = \frac{1}{\sigma_i^2} \qquad (2\text{-}2)$$

Where σ_i is the standard deviation of the i^{th} observation in units of the observation.

The matrix elements l_i and w_{ii} are the dependent variables of the functional relationships between errors and the adjustment equation. For any "fix geometry," the effect that varying the fundamental elements (l_i and w_{ii}) has on the resulting position error could have been studied. This, however, does not make the relative significance of the various error sources obvious. The output of the routine is a function of readily understandable errors which are described in words in the following sections and mathematically in appendix A.

Throughout this report, the measuring instruments are assumed to be the sextant (angles), the gyrocompass (bearings), the radar (ranges), and the LORAN receiver (time differences). These four measuring instruments were chosen because of field familiarity with the measurement terminology of each. The model is capable of simulating other range, bearing, angle, or time difference measuring instruments in the same manner it simulates the four chosen instruments.

2.3.1 Modeled Systematic Errors

The ERRORS column of table 2-1 lists readily apparent systematic errors subject to study through use of the ESM. Many errors have a similar effect on the position error. Therefore, errors with similar effect are grouped and listed under the heading - MODELED ERROR. The MODELED ERROR column of table 2-1 is a list of these groups. Each of the modeled errors is described briefly in the subsections to follow. For this report, an error maximum for each modeled error is given in table 2-1. The maximum error must be assigned by the user of the ESM. Because the model iterates through a range from zero to the maximum error, it is not necessary to specify an expected value of any systematic error. It is accepted that the list of systematic errors may not be entirely complete; but, any exclusions are of minor importance, since any measurement error omitted may be grouped with one of the errors listed for modeling.

2.3.1.1 Index Error

All angular measurement errors which add directly to the angular measure are grouped with Index Error for purposes of this model because Index Error is probably the best known error in making sextant measurements.

TABLE 2-1

SYSTEMATIC ERRORS

INSTRUMENT	ERRORS	MODELED ERROR	CODE#	ERROR MAXIMUM+
SEXTANT (Angles)	Index Personal Three-Arm Protractor Instrument Sextant	Index Error	n-IE	60 minutes
	Inclined Angle	Inclined Angle Error (C&NC)#	n-IA	100 meters
	Observer Coincidence	Observer Coincidence Error	n-OC	20 meters
	Survey Alignment Landmark Charting	Landmark Misplaced Error (C&NC)#	n-LMM	100 meters
RADAR (Ranges)	Radar *Variable Electronic Bias *Range Marker Alignment *Personal *Landmark Misplaced	Range Error	n-RE	100 meters
GYROCOMPASS (True Bearings)	*Gyro *Speed Personal Repeater Offset *Landmark Misplaced	Bearing Error	n-BE	3 degrees
LORAN (Time Differences)	*Prediction *Electronic Bias *Transmission	LORAN Error	n-LE	0.25 μsec

# C = Common landmark for several observations.
NC = No common landmark to any two observations.
n = The number of measurements in each simulated geometry subjected to the error.

* These errors were not modeled in detail, even though they may have a complicated functional relationship to the elements of the L matrix, because the weight of observations made by instruments other than the sextant is small compared to the weight of sextant observations.

+ This quantity was arbitrarily chosen for this study.

Personal Error - This error occurs when individual observers make measurements that, "on the average," are a little larger or a little smaller than they actually should be.

Three-Arm Protractor Error - This error is due to inaccuracies in a three-arm protractor which is used for plotting or unplotting angular values.

Instrument Error - The sum of all non-adjustable errors in a sextant including, prismatic, graduation, and centering errors, constitute Instrument Error.

Sextant Error - The total systematic error in a sextant measurement is considered sextant error. Index error, mirror perpendicularity, and instrument error are included in this group.

2.3.1.2 Inclined Angle Error

When a sextant is used to measure the horizontal angle subtended by two landmarks, an error may occur if the landmarks are of different altitudes. The error is called Inclined Angle Error. For purposes of modeling, the Inclined Angle is given the units of meters which represents the difference in altitudes of the two landmarks. The functional form of the model for Inclined Angle Error is provided in appendix A.

2.3.1.3 Observer Coincidence Error

When resection methods are used in positioning, the assumption is normally made that all angles are observed with all observers standing on the same point. This, however, is usually not the case, and results in position error. The error in angular measurement due to a lack of coincidence of the observers is called Observer Coincidence Error. The orientation of the observer separation with respect to the "fix geometry" is also important. For modeling purposes, the observer making the measurement in error is displaced in the direction that causes maximum effect. The functional relationship between observer coincidence error and the elements of the L matrix is provided in appendix A.

2.3.1.4 Landmark Misplaced Error

This modeled error can be used to simulate errors occurring when the geographic coordinates of a landmark differ from those used in computations or plotting. Survey Error falls in this group. Survey error represents any case where the published coordinates of the landmark are not in agreement with the coordinates of the landmark. This may be due to lower order survey work or, even worse, to rebuilding of landmarks in different locations without a new survey.

In using the sextant to measure horizontal angles, two landmarks must be aligned in the horizon mirror. If the two landmarks are aligned using points other than the published coordinates of the surveyed point of the landmark, Alignment Error will occur. For example, alignment error will occur when lining up the perceived centerlines of two tanks rather than the surveyed points on the platforms around the tanks. The effect of this error is identical to survey error but the source is the observer rather than the survey network.

Cartography is the science or art of making maps and charts. Though the techniques used by cartographers are quite accurate, there is always a small allowable error in charting any specific landmark. Charting error contributes to position error when graphical procedures are used in positioning. For purposes of modeling, this small error is called Landmark Charting Error and is placed in the Landmark Misplaced group.

The orientation of the Landmark Misplacement error with respect to the "fix geometry" can be important, as it is with Observer Coincidence. For the purpose of modeling, the misplacement is always perpendicular to the simulated line of sight to the landmark. The mathematical details of the Landmark Misplaced group are contained in appendix A.

2.3.1.5 Range Error

There are many possible sources of error in measuring ranges using either electronic or mechanical devices. Most errors do not have a direct relationship to the elements of the L matrix. Model requirements, however, do not specify a need for the exact formulation of each relationship. All errors in this group are simulated by direct addition to the measurement.

Radar Error - The difference between radar range output and a true geodetic range is called Radar Error. Unfortunately, calibration by using this comparison is often not performed; only an electronic "tune up" is performed. The lack of an electronic "tune up" may cause an Electronic Bias Error to be overlooked.

Variable Range Marker Alignment Error - This error occurs when the user does not correctly align the variable range marker with the blip that represents the landmark. This error is expected to be closely related to Personal Error because most observers have their own way of aligning the marker.

Landmark Misplaced Error - This error also applies to radar measurements, but the functional form for this error is used only in angular measurement simulation because of the relative weights of the measurement types.

2.3.1.6 Bearing Error

The determination of true bearings using the gyrocompass is subject to error from many sources. Most of the errors are related to the elements of the L matrix in a complex manner. Model requirements, however, did not specify a need for the exact formulation of the relationship. All errors in this group are simulated by direct addition to the true bearing measurement.

Gyro Error - The Gyro Error is the total combination of the gyrocompass errors at any time. It is expressed in degrees east or west to indicate the direction in which the axis of the compass is offset from true north. The errors generally associated with Gyro Error are: speed error, tangent latitude error, ballistic deflection error, ballistic damping error, quadrantal error, and gimballing error.

Reference 2 contains definitions and discussion of these error types. For purposes of modeling, they are grouped under Bearing Error.

Personal Error - This error represents the fact that the overall mean of an individual's observations may tend to be slightly larger or smaller than the true measurement.

Repeater Offset Error - This occurs when gyrocompass repeaters, which are located at a convenient location for measurement, do not present the same values as indicated on the master gyrocompass.

Landmark Misplaced Error - Because of the relative weights of bearing measurements to sextant measurements, the functional form for this error is used only in sextant measurement simulation.

2.3.1.7 LORAN Error

There are many possible sources for error in position location using LORAN (reference 3). Most do not have a simple direct relationship to the elements of the L matrix of the adjustment equation. Model requirements, however, did not specify a need for the exact formulation of each relationship. All errors in this group are simulated by direct addition to the measured LORAN time difference measurement. All receiver electronic misadjustments which could cause measurement error are represented by Electronic Bias for modeling purposes. The other group of error sources which could result in position error are those due to the differences in the predicted and actual transmission path of the LORAN signal. Any error source in this category is represented by Transmission Error. There is also a possibility for Charting Error if a chart is used in positioning with LORAN.

2.3.2 Modeled Random Errors

Table 2-2 provides a list of the random error sources subject to study through use of the ESM. The table also provides an arbitrary maximum value for each standard deviation (s.d.) of each modeled error as well as a default s.d. for each modeled error. Lacking an input value for s.d., the model assumes the default value for the s.d. The default s.d. values used are taken from references 3, 9, and 10. These arbitrary values are intended to represent the relative goodness of the measuring instruments and are presently being refined/verified by other ongoing work. Unless otherwise specified, the default s.d.'s associated with the measurement types are:

a. Sextant - 5 minutes
b. Radar - 30 meters
c. Bearing - 0.5 degree
d. LORAN - 0.1 μsec

TABLE 2-2

RANDOM ERRORS

INSTRUMENT	ERRORS	MODELED ERROR	CODE#	ERROR MAXIMUM+	ERROR DEFAULT VALUE
SEXTANT	Sextant/Observer	Sextant Observer Standard Deviation	n-S/O SD	10 minutes	5 minutes
	Landmark Definition	LM Definition (C&NC)#	n-LMD	100 meters	0.0 meters
RADAR (Ranges)	Electronic Noise Observer Variable Range Marker Atmospheric Noise	Range Standard Deviation	n-RSD	100 meters	30 meters
GYROCOMPASS (True Bearings)	Observer Alignment Electronic Noise Ship Motion	Bearing Standard Deviation	n-BSD	1.0 degree	0.5 degree
LORAN (Time Differences)	Electronic Noise Propagation Path Charting	LORAN Standard Deviation	n-LSD	0.25 μsec	0.1 μsec

\# C = Common landmark for several observations.
NC = No common landmark to any two observations.
n = The number of measurements in each simulated geometry subjected to the error.
\+ This quantity was arbitrarily chosen for this study.

In some cases, it is more appropriate to assign smaller standard deviations to chosen measurement types in order to see the effect of a projected improvement in the repeatability of an instrument.

The importance of random error assignment cannot be overemphasized as the effect of any particular systematic error is dependent upon the weight assigned to the observation. This is not a serious constraint because, as more refined measures of instrument precision are provided, the model can be modified and exercised accordingly.

All but Landmark Definition have a simple inverse square relationship to the diagonal elements of the weighting matrix, W. The relationship between landmark definition and the variance of the resulting measurement is provided in appendix A.

The list of random error sources may not be entirely complete but it is considered that any exclusions are of minor importance since omitted random errors may be grouped with one of the errors listed for modeling.

2.3.2.1 Sextant/Observer Standard Deviation

The inherent stochastic nature of sextant observations is modeled through use of this error. It combines random error due to Observer Standard Deviation and Sextant Standard Deviation. Observer s.d. represents the fact that even with a perfect sextant, an observer will not make the same observations with repeated measurements.

Sextant Standard Deviation is similar to Observer Standard Deviation except that the observer is considered perfect while the sextant is the random error source. Both errors have identical effect on position error.

2.3.2.2 Landmark Definition

A landmark observed while making resection measurements may not provide a clear and distinct image. This would cause even the perfect observer with the perfect sextant to have poor repeatability of a measurement. Some landmarks provide a very clear point for alignment while other landmarks, such as a wide water tower at close range, do not. These errors are represented by Landmark Definition. Any doubt in the quality of horizontal control can be modeled also through use of this error (for doubt associated only with the direction perpendicular to the line of sight to the landmark).

2.3.2.3 Radar, Bearing, and LORAN Standard Deviation

The precision of each radar, gyrocompass, and LORAN measurement is dependent on the observer and the equipment. Electronic noise, inherent mechanical fluctuations, and visibility conditions are sources of random error in each measurement. The random errors are modeled directly by user assignment of s.d.'s to measurements simulated. Since known values of s.d.'s are not yet available, the s.d.'s assigned for this report are estimates.

2.3.3 Modeled Outliers

A project requirement for the ESM is to introduce a high confidence outlier detection technique. A mathematical description of an outlier detection scheme, which can be investigated using the ESM, is provided in appendix F, and demonstrated in section 5.5 in this report. The technique is fashioned after the "χ^2 Goodness of Fit" statistical test used in the physical sciences. In this test the question is asked, "Does the observation deviate from the functional model by such a magnitude that it is unlikely due to random error alone?" The existence of a large blunder, or of a systematic error as a cause of the outlier, can be suspected using this method when a reasonable estimate of the measurement s.d. is known. However, small blunders or systematic errors will go unsuspected.

Examples of the most common mistakes are applying Index Correction improperly or of sighting a landmark which is not the one selected. Blunders are modeled similar to systematic errors.

Table 2-3 provides a list of often-discussed blunders (not all possible blunders) and the systematic error group in which they are assigned.

Because the least squares adjustment equation uses linear first-order approximations to a more complex system, large blunders cause higher order effects and cannot be modeled accurately.

TABLE 2-3

BLUNDERS

BLUNDER	SYSTEMATIC MODELED ERROR GROUP
Index Correction Applied Wrong	Index Error
Sextant Read Wrong	Index Error
Wrong Landmark Observed	Landmark Misplacement
Data Entry Mistake	Index Error
Incorrect Radar Reading	Radar Error
Incorrect Gyro Reading	Bearing Error
Incorrect LORAN Reading	LORAN Error
Angles Switched	Too drastic to model
Uncompensated-for Systematic Errors	All errors discussed in 2.3.1*

*Source of blunder need not be Coast Guardsmen (e.g., survey team).

3.0 GEOMETRY

The geometry of a fix consists of all the LOP's which represent the measurements that compose the fix, their respective measurement standard deviations, and their orientation with respect to each other. The "fix geometry" determines the strength of the fix. Effects of measurement errors and computational mistakes on position error depend on the strength of the fix, which implies that choice of the "fix geometry" is important part to ESM efforts. The pitfalls of assuming only a few geometry layouts become evident when considering the basic tenet "that all fixes are unique." Too much information (studying many selected "fix geometries") can be as harmful as too little if useful information is not apparent. The ESM model was created with two alternative approaches to the geometry problem. In the first alternative, called the "Fixed Geometry" alternative, representative positioning scenarios are selected for study. The three scenarios selected for this study are described in detail in the next section. Users of this model for other studies are cautioned that if too few geometry scenarios are considered, erroneous conclusions could be drawn and important results bypassed. The second alternative, called the "Random Geometry" alternative, generates information from many "fix geometries" without creating so much information that it is unmanageable. The "Random Geometry" alternative is described verbally in section 5.2 and expressed mathematically in appendices A and E.

3.1 Fixed Geometry

Although many scenarios can be analyzed using the ESM, only scenarios called Harbor, Near-Coast, and Offshore were selected for study in this report.

3.1.1 Harbor

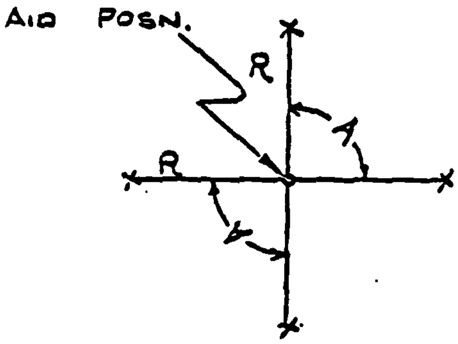
The Harbor scenario is composed of sextant measurements using landmarks that are relatively close (4000 meters) to the designated position, and which surround the position with equal angular spacing (90^{o}). Figure 3-1 depicts the geometry of the Harbor scenario. This scenario was chosen because many aids serviced by the Coast Guard are within rivers and harbors.

3.1.2 Near-Coast

This scenario is composed primarily of sextant measurements using landmarks equally spaced on a nearby (4000 meters) coastline. A fourth line of position is included using an additional gyrocompass measurement thereby further "checking" the fix. The effect of this additional "check" is studied. Figure 3-2 provides the fix geometry for the Near-Coast scenario.

3.1.3 Offshore

This scenario represents cases when sufficient landmarks are not available or visible for resection, and is composed of measurements made using combinations of instruments other than the sextant. Comparison between measurement types is made using a symmetrical geometry for each and then studying the respective position error. Figure 3-3 describes the Offshore geometries.

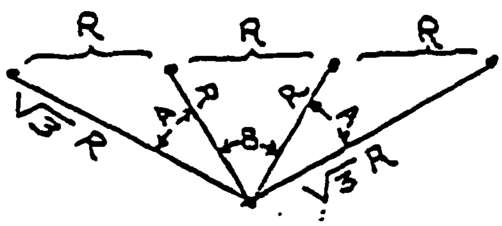


R = 4000 meters
A = 90 degrees

Three lines of position from sextant.
No other measurements.

FIGURE 3-1

HARBOR GEOMETRY

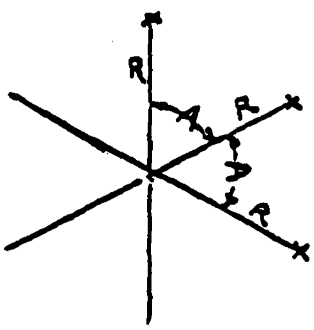


R = 4000 meters
A = 30 degrees
B = 60 degrees

Three lines of position from sextant.
One line of position from gyrocompass, sighting on second landmark from left.

FIGURE 3-2

NEAR-COAST GEOMETRY



R = 4000 meters
A = 60 degrees

Three lines of position from radar, gyrocompass, or LORAN (Gradient taken to be 300 meters/μsec).

FIGURE 3-3

OFFSHORE GEOMETRY

3.1.4 Other fix geometries

The operating instructions for using the ESM to study other scenarios of interest are contained in appendix B.

3.2 Random Geometry

It is seldom that all elements of a large population can be studied. A common method of studying large populations is to draw a sample from the population, perform tests on it, and infer the statistics apply to the entire population.

A sample of "fix geometries" using the sextant was drawn from the SANDS (reference 3) forms of USCGC REDWOOD (WLM 685), and sample statistics were calculated. The direct route of subjecting each "fix geometry" to analysis proved to be cumbersome, therefore, a Monte Carlo routine was created using the sample statistics. The mathematical equations used in the Monte Carlo simulation and the sample statistics are provided in appendix E.

The "random geometries" generated by the Monte Carlo routine were subjected to error analysis and selected frequency distributions of error effects were determined. The inference was made that the frequency distributions created represent the results obtainable by study of all "real" aid positioning geometries. The critical reader will question the validity of representing all possible fix geometries by a sample from the SANDS data of only one unit. The response to this valid criticism is that the requirements placed on the ESM efforts reported on herein calls for a tool to be used in judging relative error effects among some representative geometries. If a different set of representative geometries (such as one drawn randomly nationwide) is desired, the statistics of the sample can be entered into the model for further study. The results contained herein must be weighted by the fact that they are created through use of the USCGC REDWOOD sample.

The sample drawn from USCGC REDWOOD had too few "fix geometries" composed of radar, gyrocompass, and LORAN measurements. Because of this data deficiency, other modeling techniques were employed. A Monte Carlo simulation of all measurements is used adding reasonable constraints upon landmarks sighted and crossing angles. The simulated landmarks used for range and bearing measurements were generated using statistics of the REDWOOD sample. However, successive landmark bearings were distributed uniformally between 30 and 90 degrees. LORAN LOP's were simulated with similar crossing angles. Though the distributions used in this work are necessarily biased due to sampling from a single unit, the point that this error evaluation technique is a powerful one is effectively made. More precise distributions can be substituted if a substantial return is indicated.

4.0 COMPUTER ADAPTATION OF THE MODEL

A set of landmarks, measurements, and/or signals are chosen which define a "fix geometry" that satisfies the functional model introduced in section 2.0. The measurements are perturbed by subjecting them to an iteratively increasing modeled error from among those defined in section 2.3. The perturbed measurements are then transformed via the least squares adjustment equation into a two-dimensional location with an associated two-dimensional uncertainty. Various measures of position error are calculated following each iteration.

As inputs are required, the computer displays a message to the operator. The routine does not have a data entry error feedback system which means the operating instructions of appendix B should be followed strictly. The time required to study the effect of one error source on a specified "fix geometry" is a few minutes, including data entry time. The time required for a "random geometry" case increases linearly with the number of geometries desired in the sample.

4.1 Required Inputs

4.1.1 Fixed Geometry

Table 4-1 lists the inputs required by the routine in studying a "fix geometry" (refer to appendix B for proper data entry procedures).

4.1.2 Random Geometry

Table 4-2 lists the inputs required by the routine in studying "random geometries" (refer to appendix B for proper data entry procedures).

4.2 Outputs of the Error Sensitivity Model

The outputs differ significantly between the two geometry approaches. In all "fixed geometry" cases, the chosen error effect parameter is plotted as a function of the modeled error, whereas the "random geometry" outputs are frequency distributions or cumulative frequency distributions of the selected error effect parameter as a function of five values of the modeled error all equally spaced within the modeled error domain (see section 5.2). In either method the same modeled errors and error effect parameters are available. The modeled errors were discussed in section 2.3 and the error effect parameters are described in the following subsections. Refer to Results (section 5.0) for examples of the output of the model.

4.2.1 AP-to-MPP

The point at which all lines of position cross in an unperturbed "fix geometry" is called the assigned or assumed (designated) position, AP. The adjusted set of lines of position cross at a point called the most probable position, MPP. The vector on the two-dimensional positioning surface between these two points is an error effect parameter called the AP-to-MPP vector. The AP-to-MPP vector, following any disturbance of the initial "fix geometry," requires magnitude and direction to define it completely. The direction is not of value in meeting present project requirements. The routine

TABLE 4-1

FIXED GEOMETRY - INPUTS

(1) Number of sextant angular measurements

(2) Number of gyrocompass bearing measurements

(3) Number of radar range measurements

(4) Number of LORAN time difference measurements

(5) Number of landmarks that are common to sextant measurements (see section 5.6, Limitations of the Model)

(6) Standard Deviations of simulated sextant measurements (Default = 5 minutes) (Reference 9)

(7) Standard Deviations of simulated gyrocompass measurements (Default = 0.5 degrees) (Reference 9)

(8) Standard Deviations of simulated radar measurements (Default = 30 yards) (Reference 10)

(9) Standard Deviations of simulated LORAN measurements (Default = 0.1 μsec) (Reference 3)

(10) Error Effect Parameter Choice (see section 4.2)

(11) Error to be studied (see section 2.3)

(12) Radius of target circle (optional)

(13) Maximum of error to be studied

(14) Range and true bearing to all landmarks used in sextant angular measurements

(15) Range and true bearing to all landmarks used for gyro bearing measurements

(16) Range and true bearing to all landmarks used for radar range measurements

(17) Gradient magnitude and direction for all LORAN time difference measurements

TABLE 4-2

RANDOM GEOMETRY - INPUTS

(1) Number of sextant angular measurements

(2) Number of gyrocompass bearing measurements

(3) Number of radar range measurements

(4) Number of LORAN time difference measurements

(5) Number of landmarks that are common to sextant measurements (see section 5.6, Limitations of the Model)

(6) Standard Deviations of simulated sextant measurements (Default = 5 minutes)

(7) Standard Deviations of simulated gyrocompass measurements (Default = 0.5 degrees)

(8) Standard Deviations of simulated radar measurements (Default = 30 yards)

(9) Standard Deviations of simulated LORAN measurements (Default = 0.1 μsec)

(10) Error Effect Parameter Choice (see section 4.2)

(11) Error to be studied (see section 2.3)

(12) Radius of target circle (optional)

(13) Maximum of error to be studied

*(14) Number of "fix geometries" desired in sample

(15) Seed for random number generator (nine-digit fraction between 0 and 1)

* To change the parameters used to randomly generate sample geometries, the program needs slight modifications. Appendix A describes the program listing that requires change.

is not written to provide the direction, but a modification is not difficult if studies of this nature are desired. The mathematical expression for AP-to-MPP is provided in appendix A.

4.2.2 2-drms

The mathematical model results in an MPP and in the two-dimensional uncertainty in the MPP for each iteration through the modeled error. The two-dimensional uncertainty of the MPP is defined with respect to a rotated reference system centered on the MPP. The angle between the original reference axis and the new reference system is determined such that the coordinates are uncorrelated in the new system. The variance associated with the MPP in each of the two orthogonal directions defines completely the uncertainty in its location in the form of a bivariate normal probability distribution. Often, the two-dimensional uncertainty can be approximated by one number. One such number, 2-drms, is twice the square root of the sum of the variances in the orthogonal reference system. It is used to define the radius of a circle which contains at least 95 percent of the probability mass of the bivariate normal distribution.

4.2.3 Maj*Min (90%)

The contours of equal probability density of the bivariate normal distribution form ellipses. The area of an ellipse which contains 90 percent of the probability mass of the distribution is an error effect parameter, the Maj*Min (90%).

4.2.4 Maj (90%)

The major semi-axis of the ellipse described in section 4.2.3 is an error effect parameter, the Maj (90%).

4.2.5 P-in-R

The probability mass which is contained by a circle of radius R centered about the assigned position is called the P-in-R. Of course, P-in-R is always less than one. The probability is calculated by a two-dimensional integration of the bivariate probability distribution over the area of the designated circle. Details of this calculation are contained within appendix C.

The value of this error effect parameter can be realized by considering that it is a function of both random and systematic errors. A drawback to its usefulness is that the computations necessary to arrive at it are very time consuming.

4.2.6 Sum Sqd Res

The Sum Squared Residuals (Sum Sqd Res) is an error effect measure which indicates how far, in terms of multiples of assumed measurement standard deviation units, the lines of position of the perturbed system are from satisfying the functional model. A residual is the difference between a measurement of the perturbed "fix geometry" and the respective measurement of the adjusted "fix geometry." The residual of each measurement is squared then divided by the variance of that measurement. The weighted quantities are summed to obtain the final parameter. The value of this error effect parameter is demonstrated in section 5.5, Outlier Detection and Identification. The mathematics associated with its calculation and use are expressed in appendices A and F.

4.2.7 Min/Maj

The ratio of the minor and major semi-axis of the ellipse defined in section 4.2.3 is called the Min/Maj error effect parameter.

5.0 APPLICATION OF THE MODEL - RESULTS

The preliminary considerations of Section 2.1 should be reviewed before the reader becomes involved in studying the results presented in this section. A failure to appreciate the constraints of using specific geometries can easily lead to erroneous conclusions. On the other hand, to interpret that the expected values of error effect derived from the "random geometry" method are all inclusive, can just as easily lead to erroneous conclusions. It is a combination of both approaches that will prove useful in a general analysis of the positioning problem.

The primary result of this effort is that an Error Sensitivity Model has been created complete with operating instructions and example applications. The examples given do not represent the total of all information derivable by use of the model and should only be examined with regard to the "fix geometries" from which they were derived.

The key for figure numbers throughout this section is:

X-LL-X(L)

Report Section → X; Geometry Type → LL; Page of LL Data → X; Figure a, b, or c → (L)

X=number
L=Letter

Geometry Type
H=Harbor
NC=Near-Coast
OS=Offshore
R=Random
B=Blunder

Example: 5-OS-2(a) is figure (a) on page 2 of the offshore data in section 5 of the report.

5.1 Fixed Geometry

The results of error analysis on the fixed geometry scenarios described in section 3.1 are displayed in figures 5-H-1(a) through 5-OS-4(c). Refer to section 2.3 for a description of the modeled errors and section 4.2 for a description of the error effect parameters.

Equivalent modeled error is introduced to allow comparison of the effects of modeled errors pertinent to each measuring instrument. The equivalent modeled error is the error value which causes 100 percent degradation of the error effect parameter studied (i.e., either halves or doubles the error effect parameter - as appropriate - with respect to its value when the modeled error is zero.). Table 5-1 lists the equivalent modeled errors for the P-in-R error effect parameter. The equivalent modeled errors are grouped by their respective measuring instrument within the columns and by geometry type within the rows. The table is useful for comparing the different geometry scenarios and in finding the relative importance of each modeled error within each "fixed geometry" scenario. Examination of table 5-1 and figures 5-H-1(a) through 5-OS-4(c), pages 33-54, reveals information as follows:

Pages 33-50
The modeled errors have a more significant increased effect on the error effect parameters using the Near-Coast Geometry than they do in the Harbor Geometry. Equivalent modeled errors differ by as much as a factor of two in the case studied.

TABLE 5-1

FIXED GEOMETRY EQUIVALENT MODELED ERRORS

N/A = Not applicable   NWR = Not within range   NE = No effect

ERROR#	HARBOR	NEAR-COAST	OFFSHORE
Sextant			
1-IE	58 minutes	25 minutes	N/A
1-NCLMM	70 meters	50 meters	N/A
2-CLMM	32 meters	19 meters	N/A
1-NCIA	NE	NWR	N/A
2-CIA	NE	NWR	N/A
1-OC	NWR	NWR	N/A
+*3-S/OSD	8.2 minutes	7.6 minutes	N/A
+*1-NCLMD	NWR	NWR	N/A
+*2-CLMD	12 meters	8 meters	N/A
Gyrocompass			
3-BE	N/A	NWR	0.42 degrees
*3-BSD	N/A	NWR	0.74 degrees
Radar			
3-RE	N/A	N/A	24 meters
*3-RSD	N/A	N/A	46 meters
LORAN			
3-LE	N/A	N/A	0.08 usec
*3-LSD	N/A	N/A	0.15 usec

* The value listed is that which corresponds to a P-in-R which is one-half of that obtained using the default standard deviations for the simulated observations.

+ Target Circle 5-meter radius

The number represents the number of measurements to which the modeled error was applied.

Pages 33-35, 40-42

The P-in-R output parameter remains high (more than 0.9) until the AP-to-MPP distance is about 80 percent of the target circle radius.

Pages 33-96(a), 40-46(a), 51-53(a)

The AP-to-MPP error effect parameter is a linear function of systematic error.

Pages 34-35, 38-39, 41-44, 48-49

Landmark-related modeled errors have a more severe effect on position error when the geometry contains a landmark which is common to two measurements. In all cases presented, only one common landmark was simulated. The same effects would result if two landmarks, each used once, provided similar input errors.

Pages 36, 43-45

Inclined Angle Error and Observer Coincidence Errors have, within the range investigated, little effect on the error effect parameters studied.

Pages 35, 42, 39(c), 49(c)

Common Landmark Misplacement and Common Landmark Definition can significantly affect position error if the error magnitude is of the same order as the target circle radius.

Pages 37(b), 47(b)

The Maj (90%) error effect parameter is a linear function of S/O Standard Deviation.

Pages 37(a), 47(a)

The Maj*Min (90%) error effect parameter is a quadratic function of S/O Standard Deviation.

Pages 46(a), 50(c)

The simulated gyrocompass bearing measurement in the Near-Coast geometry adds very little to the strength of the fix. Unless the standard deviation of gyrocompass measurements is decreased to less than 0.25 degrees, it is often of little help in further "checking" a fix determined by three-angle resection (assuming a reasonably good geometry is used in the three-angle fix).

Pages 51-53

Given the default measurement standard deviations, three measurements using the gyrocompass, radar, and LORAN receiver have a small (less than 0.1) chance of positioning the observer within a target circle of 25 meters radius, if systematic error exceeds 0.5 degree, 40 meters, and 0.1 μsec, respectively.

Page 54

Gyrocompass, radar, and LORAN measurements must have standard deviations of less than 0.25 degrees, 20 meters, and 0.05 μsec, respectively, to achieve high P-in-R values (approximately 0.8) for a target circle of radius 25 meters with 60° crossing angles (provided all systematic errors are zero).

5.2 Random Geometry

Cumulative frequency distributions of error effect parameters are displayed in figures 5-R-1(a) through 5-R-6(c), pages 55-60. Refer to section 2.3 for modeled error definitions and section 4.2 for error effect parameter definitions. Each figure displays five cumulative frequency distributions (one for each of five different modeled error values) each of which was created using the following procedure:

The error effect parameter is assigned a maximum value as an optional user input (see appendix B) and the resulting range (0 to maximum) is divided into 26 data accumulation bins. Each geometry of the sample is studied iteratively, once for each fifth of the maximum modeled error, with the resulting error effect parameter tabulated in the appropriate bin. If the error effect parameter exceeds the specified maximum, it is tabulated in a twenty-sixth bin. After all geometries of the sample have been studied, the sample fraction within each bin is calculated. The modeled error effect is determined by comparing the cumulative frequency distributions and summary statistics which correspond to the respective modeled error values.

If desired, the frequency distribution can be plotted directly (not cumulative). The numbers which compose each frequency distribution on the graph represent the number of fifths of the maximum modeled error present in generating the respective frequency distribution.

The expected value of any error effect parameter is calculated from the frequency distribution and is used to rank order expected error effects.

At least 200 randomly generated geometries are required to sufficiently "smooth" the cumulative distributions, however, 100 geometries normally provide enough data to distinguish among distributions on a single graph. In each case, the number of sample geometries, as well as the number of each measurement type, are given in the figure. In each case, the number of angle (A), bearing (B), range (R), and LORAN (L) measurements is indicated on the right side of the graph. In addition, the target circle radius, R, is provided in the title block.

Examination of figures 5-R-1(a) through 5-R-6(c), pages 55 through 60 respectively, and the expected values of all error effect parameters reveals the following expected trends and quantified information:

<u>Page 55</u>

Subjecting one sextant measurement to Index Error is less severe than subjecting all three measurements to Index Error.

<u>Pages 56, 57, and 59</u>

Landmark-related errors have a more severe effect on P-in-R when the geometry contains a landmark which is common to two measurements (that is, when two measurements contain errors).

Pages 55-58

The following errors cause at least 90 percent of the geometries to have a P-in-R of less than 0.5. The target circle radius is 25 meters.

ERROR#	MAGNITUDE
1-IE	90 minutes (extrapolated value)
3-IE	48 minutes
1-NCLMM	100 meters
2-CLMM	60 meters
1-NCIA	NWR*
2-CIA	NWR*
1-OC	NWR*
3-BE	0.20 degrees
3-RE	Less than 10 meters
3-LE	Less than 0.05 μsec

\# See table 2-1 for list of modeled error abbreviations and codes.

* NWR - Not within range.

Pages 55-58

The following errors cause 50 percent of the geometries to have a P-in-R of less than 0.9. The target circle radius is 25 meters.

ERROR#	MAGNITUDE
1-IE	18 minutes
3-IE	12 minutes
1-NCLMM	20 meters
2-CLMM	18 meters
1-NCIA	450 meters
2-CIA	300 meters
1-OC	16 meters
3-BE	0 degrees
3-RE	0 meters
3-LE	0 μsec

\# See table 2-1 for modeled error abbreviations and codes.

Page 57

Inclined Angle Error and Observer Coincidence Errors have very little effect on the P-in-R under nearly all circumstances investigated.

Page 59(a)

There is a huge potential for increasing fix strength by reducing the Sextant/Observer Standard Deviation. Fifty percent of the P-in-R's are moved from below to above 0.8 by reducing Sextant/Observer Standard Deviation from 5 minutes to 2 minutes.

Page 59(c)

Landmark Definition can have a significant effect on P-in-R for many geometries if its magnitude is greater than the target circle radius. For a 5-meter target circle, a Common Landmark Definition Error of 10 meters causes P-in-R to be less than 0.2 for 80 percent of the geometries studied. A Common Landmark Definition error of 10 meters is unacceptably large by applicable survey standards. The average landmark is 4000 meters from the assigned position which, when Third Order Class II Geodetic Survey Standards (reference 19) are used, indicates a possible 1:5000 or 0.8 meter uncertainty in landmark location. Landmark alignment difficulties or poor visibility may cause landmark definition errors as large as 10 meters.

Page 60

s.d.'s of gyrocompass, radar, and LORAN measurements of greater than 0.5 degrees, 30 meters, and 0.1 μsec, respectively, make positioning within a 25-meter target circle very improbable (approximately 80 percent of geometries with P-in-R of less than 0.5 with no uncompensated systematic errors present).

Table 5-2

Table 5-2 summarizes the expected values of the error effect parameters as a function of the modeled errors.

5.3 Rank Order of Modeled Errors by Average Effect

There is no conclusive method for rank ordering modeled errors by average effect. Any rank ordering for one "fixed geometry" may not apply to any other "fixed geometry." In order to provide a rank order with far-reaching applicability, the modeled errors must be ranked with the following considerations in mind:

a. On what error source should resources be expended to most effectively decrease position error?

b. Are the modeled errors under consideration relatively negligible?

c. What is the frequency with which the modeled error occurs?

d. Is a suggested method of decreasing position error subject to other (possibly more severe) error sources?

Answers to these questions are very complicated. Modeled errors can be ranked to meet the project requirement by considering the expected values of the error effect parameters. The P-in-R error effect parameter was chosen to rank modeled errors by average effect.

Table 5-2 provides the information needed to rank modeled errors. P-in-R is given as a function of each modeled error over the domain of the modeled error selected. The function provided is a least squares curve fit of the "random geometry" derived P-in-R averages. The correlation coefficient was very good for nearly all error effect functions. In reviewing the functions, consideration must be given the relative ease in effecting a decrease

TABLE 5-2

EXPECTED ERROR EFFECT FUNCTIONS

EXPECTED ERROR EFFECT FUNCTION	$y = ax^b$ (y = error effect parameter, x = modeled error magnitude)															$y = ae^{bx}$			
ERROR EFFECT	AP-to-MPP (meter)			Sum Sqd Res			Maj*Min 90% ($meter^2$)			2-drms (meter)			Maj 90% (meter)			*P-in-R			
+MODELED ERROR	a	b	r^2	a	b	r^2	a	b	r^2	a	b	r^2	a	b	r^2	a	b	r^2	$-\frac{1}{b}$
1-IE (min)	0.9	1	1.0	0.013	2	1.0										1.18	-0.025	0.99	40
3-IE (min)	1.8	1	1.0	0.038	2	1.0										1.06	-0.039	0.98	26
1-NCLMM (met)	0.93	1	1.0	0.012	2	1.0										0.907	-0.022	0.95	45
2-CLMM (met)	1.0	1	1.0	0.025	2	1.0										1.13	-0.034	0.95	29
1-NCIA (met)	0.08	1	1.0	3.5×10^{-5}	2.5	1.0										1.01	-0.001	0.91	1000
2-CIA (met)	0.26	1	1.0	1.5×10^{-6}	3.4	1.0										1.06	-0.002	0.98	500
1-OC (met)	1.0	1	1.0	0.015	2	1.0										0.994	-0.015	0.93	67
3-BE (deg)	86	1	1.0	1.88	2	1.0										0.609	-2.50	1.0	0.4
3-RE (met)	1.3	1	1.0	4.8×10^{-4}	2	1.0										0.784	-0.062	0.97	16
3-LE (usec)	500	1	1.0	8.46	1.9	1.0										0.239	-12.87	0.99	0.08
3-S/OSD (min)#							6.74	2	1.0	3.28	1	1.0	3.55	1	1.0	1.15	-0.164	1.0	6.1
1-NCLMD (met)#							232	0.27	0.98	16.8	0.42	0.98	18.2	0.42	0.98	0.450	-0.012	0.85	83
2-CLMD (met)#							26.8	1.1	0.98	2.1	1	0.98	2.19	1	0.98	0.307	-0.023	0.85	43
3-BSD (deg)							13200	2	1.0	161	1	1.0	150	1	1.0	1.30	-2.29	0.99	0.44
3-RSD (met)							321	2	1.0	2.41	1	1.0	2.05	1	1.0	1.10	-0.034	0.98	29
3-LSD (μsec)							3.69×10^5	2	1.0	835	1	1.0	758	1	1.0	1.23	-11.9	0.99	0.08

* P-in-R curve fit does not include zero modeled error values. Target circle radius equals 25 meters unless otherwise stated. 1/b value described in section 5.3

\# Target circle with 5-meter radius

\+ See table 2-1, 2-2 for code

r = Correlation Coefficient

in the error throughout the aids to navigation macrostructure. For example, large improvements in P-in-R are possible by decreasing the S/O Standard Deviation (Slope = -0.08/min at S/O Standard Deviation of 5 minutes). However, decreasing this standard deviation by 1.0 minute throughout the Coast Guard may be very difficult compared to decreasing other errors. The error effect function for P-in-R is of the form $y=ae^{bx}$. The coefficient, b, in the exponent of each curve fit can be used to define another useful value, 1/b. A modeled error of magnitude 1/b decreases the expected P-in-R value to a/e, where a is determined by the curve fit and e is the natural base, 2.718. A review of table 5-2 allows selection of the modeled errors which often do not contribute significantly to position error. The following is a list of the modeled errors of least effect on position error:

a. Common and Non-Common Inclined Angles - Very seldom is an elevation difference of the range modeled large enough to cause significant increase in position error.

b. Observer Coincidence - Most often P-in-R is low only when observers are separated by distances greater than those physically possible on the ships used to position aids.

c. Non-Common Landmark Definition - The image of a non-common landmark has to be quite poor before any significant decrease in P-in-R is suffered.

The effectiveness of any decrease in an error from a given source is indicated by the slope of the error effect functions. Table 5-3 lists the first derivatives of the P-in-R error effect functions evaluated at selected modeled error magnitudes. This provides a measure of how effective a decrease of one model error unit is in increasing P-in-R.

The list in table 5-3 still does not allow exact ranking of errors by average effect because there is no data on the frequency of occurrence of the modeled errors. However, the following statements are made with reference to tables 5-2 and 5-3:

a. Index Error, whether it occurs on one or all of the sextant measurements, has a significant effect on P-in-R and should be compensated for in a regular fashion.

b. Landmark Misplacement Error contributes very little to position error for displacements acceptable to geodetic survey standards. However, unsurveyed landmarks can easily cause a large position error. Blunders caused in using landmarks differing from those selected are obvious violators of fix strength.

c. Inclined angles normally degrade position error very little.

d. Observer Coincidence is not significant enough to warrant any more than directing observers to stand as close as possible to one another.

TABLE 5-3

EFFECTIVENESS OF MODELED ERROR DECREASE

	MODELED ERROR MAGNITUDE		ERROR EFFECT FUNCTION DERIVATIVE*	EFFECTIVENESS P-in-R/unit
25-meter Target Circle Radius	1-IE	= 5 minutes	-0.030exp(-0.025(1-IE))	-0.026/min
	3-IE	= 5 minutes	-0.041exp(-0.039(3-IE))	-0.034/min
	1-NCLMM	= 10 meters	-0.020exp(-0.022(1-NCLMM))	-0.016/met
	2-CLMM	= 10 meters	-0.038exp(-0.034(2-CLMM))	-0.027/met
	1-NCIA	= 100 meters	-0.001exp(-0.001(1-NCIA))	-0.001/met
	2-CIA	= 100 meters	-0.002exp(-0.002(2-CIA))	-0.002/met
	1-OC	= 5 meters	-0.015exp(-0.015(1-OC))	-0.014/met
	3-BE	= 1.0 degree	-1.52exp(-2.50(3-BE))	-0.124/deg
	3-RE	= 50 meters	-0.049exp(-0.062(3-RE))	-0.002/met
	3-LE	= 0.1 μsec	-3.05exp(-12.7(3-LE))	-0.850/μsec
5-meter Target Circle Radius	3-S/O SD	= 5 minutes	-0.188exp(-0.164(3-S/OSD))	-0.082/min
	1-NCLMD	= 10 meters	-0.005exp(-0.012(1-NCLMD))	-0.004/met
	2-CLMD	= 10 meters	-0.007exp(-0.023(2-CLMD))	-0.006/met
25-meter Target Circle Radius	3-BSD	= 0.5 degree	-2.97exp(-2.29(3-BSD))	-0.940/deg
	3-RSD	= 30 meters	-0.038exp(-0.034(3-RSD))	-0.014/met
	3-LSD	= 0.1 μsec	-14.6exp(-11.9(3-LSD))	-4.40/μsec

* The coefficient (1-IE) in the exponent represents a modeled Index Error applied to one angle (not, 1 minus the Index Error); this is also true for other modeled errors.

e. Improvements in Bearing, Range, and LORAN Errors of 0.25 degree, 10 meters, and 0.05 μsec, respectively, increase P-in-R by less than 5 percent at points considered in table 5-3.

f. A decrease in S/O Standard Deviation can greatly increase P-in-R.

g. Common Landmark Definition, on the average, does not affect position error as drastically as Landmark Misplacement, but should not be neglected when either large or unclear landmarks are used.

h. Decreases in Bearing, Range, and LORAN Standard Deviations of 0.25 degrees, 10 meters, and 0.05 μsec, respectively, can in many cases increase P-in-R by more than 0.20. This can make them suitable for positioning aids when a 25-meter target circle is adequate.

5.4 Priority Error Designation

The discussion in sections 5.1 through 5.3 leads to designation of the following modeled errors as "Priority Average Errors" (based on assumptions made).

a. Index Error

b. Landmark Misplacement

c. S/O Standard Deviation

d. Bearing, Range, and LORAN Standard Deviation

5.5 Outlier Detection and Identification

An R&D project requirement called for formulation of a high confidence outlier detection technique for application when aid positioning. Two mathematical techniques are described in appendix F and one is discussed in this section. Section 2.3.3 provides a list of some of the blunders possible in the positioning process.

The mathematics in appendix F indicate that the presence of an outlier becomes apparent in many cases by performing a χ^2 test (reference 8) on the sum of the squared weighted residuals. The test is designed to indicate when the consistency of the lines of position is so poor, compared to what is expected, that it probably was not the result of only random error. An analogy to blunder detection is to consider the outlier as a signal, and random errors as the noise. If the signal-to-noise ratio is large enough, the χ^2 test should indicate an inconsistency in the measurements used to create the fix; the inconsistency may be large enough to be flagged as an outlier containing a blunder. If the fix is composed of three lines of position, further examination is required to identify the suspicious observations. In a fix containing four independent lines, all four can be tested and if found excessively inconsistent by failure of the χ^2 test, each of the four combinations of three can be tested in turn against the fourth line (see apppendix F). In cases where the geometry includes common landmarks (e.g., a four-line fix with only four landmarks, not independent lines of position), a blunder in using a landmark other than the selected common landmark could disturb two or more lines of position and the blunder identification procedure could break down because of this dependency. Four good landmarks are required to find an inconsistency due to one poorly located landmark. Fixes with more than four LOP's can be tested similarly, but the possible combinations grow rapidly.

Figure 5-B-1(a), Page 61, shows frequency distributions of the Sum Sqd Res of a three-angle fix using the random geometry method; there is one distribution for each of five Index Error (IE) values to which the fixes were subjected. One angle was subjected to error and the fixes were tested for excessive inconsistency at the 95 percent confidence level (i.e., $^{2}_{1}$ (0.95)=3.8). The reference s.d. used was 5 minutes (appendix F).

The following information is obtained by review of the figure:

a. Eighty percent of all fix geometries subjected to 12 minutes of Index Error go unsuspected.

b. Forty-five percent of all fix geometries subjected to 24 minutes of Index Error go unsuspected.

c. Thirty-two percent of all fix geometries subjected to 36 minutes of Index Error go unsuspected.

d. Twenty-two percent of all fix geometries subjected to 48 minutes of Index Error go unsuspected.

e. Twenty percent of all fix geometries subjected to 60 minutes of Index Error go unsuspected.

If the S/O Standard Deviation was improved, many more blunder candidates could be found. The ESM can be used in the fixed geometry mode to study the outlier rejection levels of any specified "fix geometry." Figures 5-B-1(b-c) are examples of such a study on the Harbor geometry, employing the chosen 5-minute standard deviation of sextant measurements. Figure 5-B-1(b) displays the effect Index Error has on the Sum Sqd Res when it is subjected to one measurement of the Harbor geometry. The outlier detection scheme would detect Index Errors greater than 15 minutes. This plot may be different when subjecting the error to other measurements in the "fix geometry." Figure 5-B-1(c) displays the effect Index Error has on the Sum Sqd Res when it is subjected to all measurements of the Harbor geometry. The inconsistency of the measurements becomes apparent at a much smaller value of Index Error (7 minutes).

The 5-minute measurement standard deviation used can be changed for any study to suit the requirements on outlier detection. Likewise, other confidence levels can be used in the ^{2} tests.

5.6 Limitations of the Model

The following list contains limitations of the Error Sensitivity Model. The limitations remain because solutions were either too time-consuming, unjustly complicated, or beyond the scope of this report.

a. Study of the effect of combinations of errors.

b. Study of the effect of errors using various combinations of common and non-common landmarks.

c. Study of geometry scenarios using various sets of default standard deviations.

d. The instability of numerical integration of very peaked probability distributions (appendix C) exists and has not been dealt with adequately.

e. The model has no built-in landmark selection algorithm and therefore should not be considered a Landmark Selection Routine.

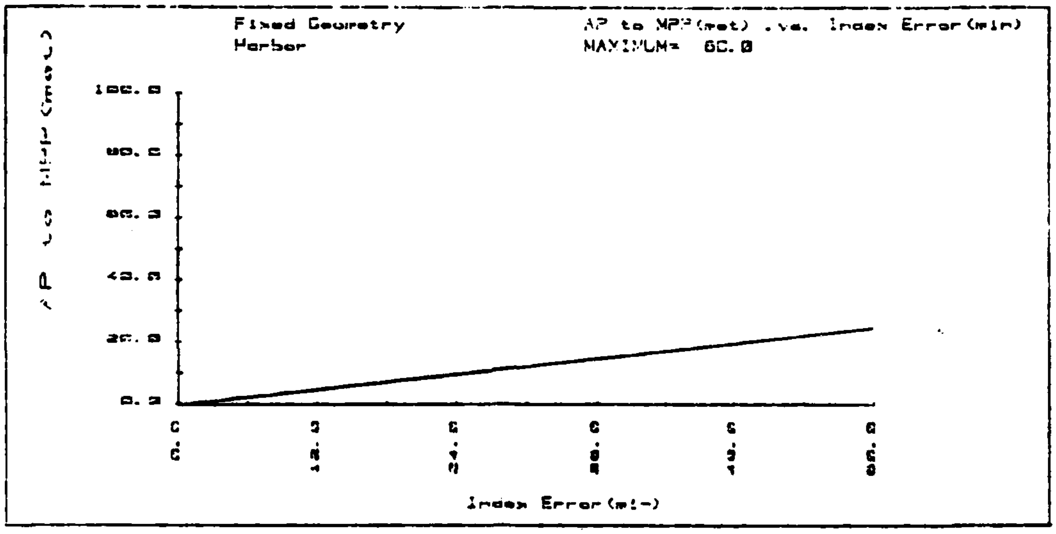


Figure 5-H-1(a)

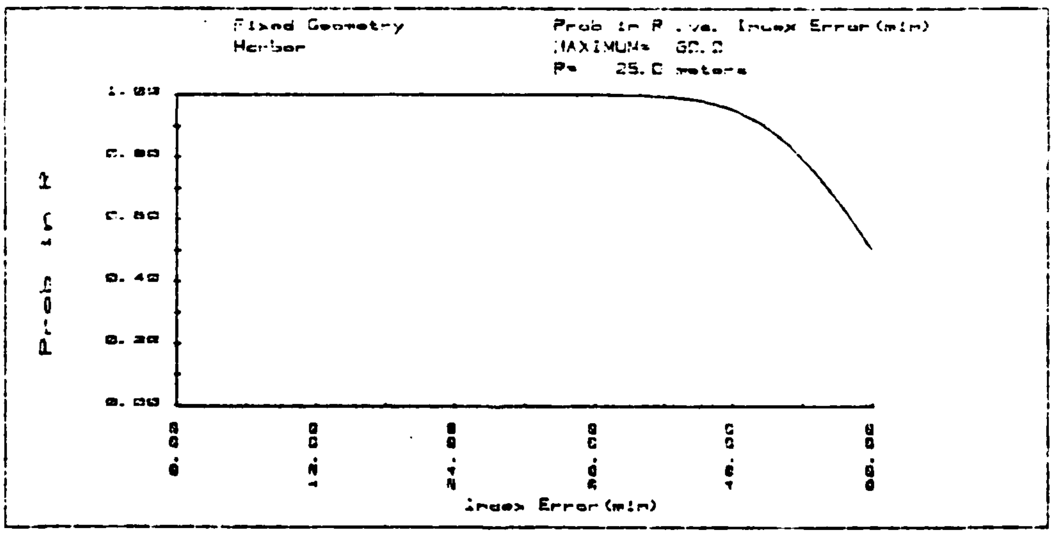


Figure 5-H-1(b)

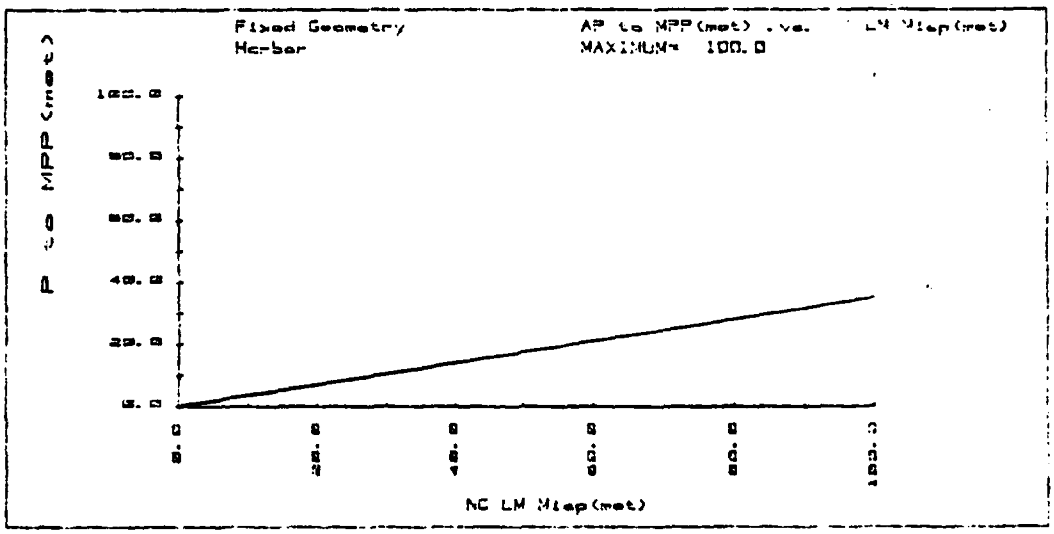
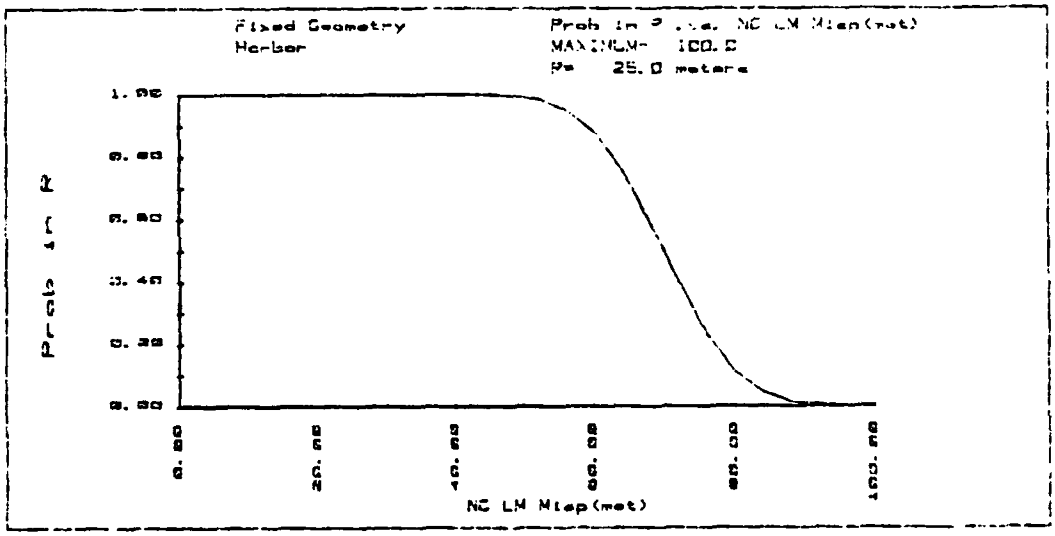


Figure 5-H-2(a)



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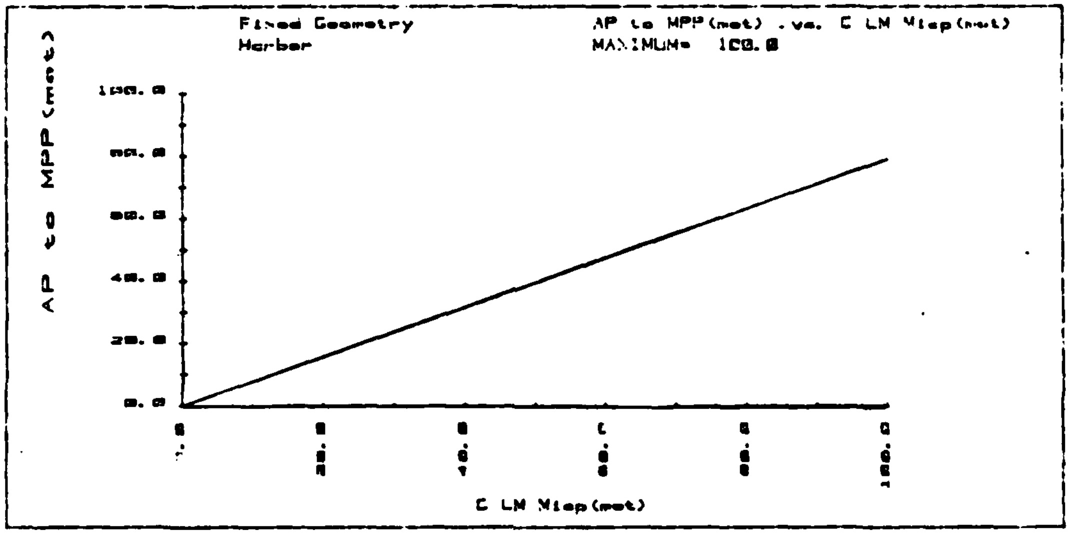
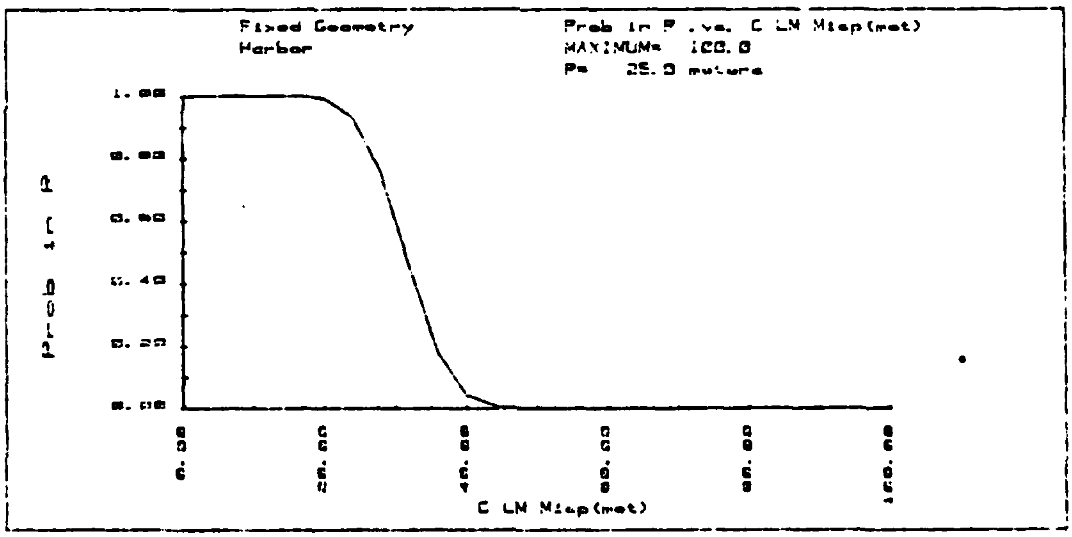


Figure 5-H-3(a)



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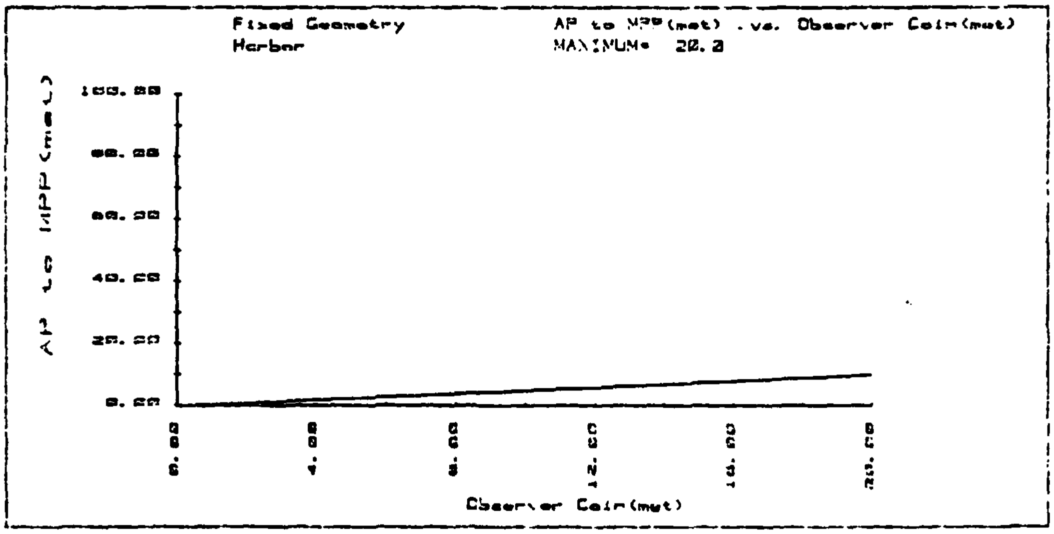
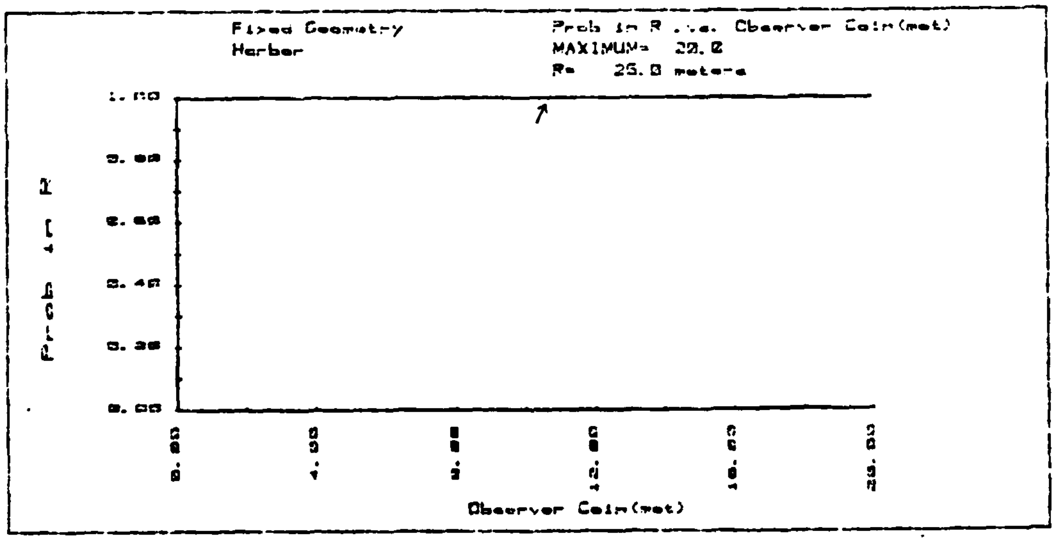


Figure 5-H-4(a)



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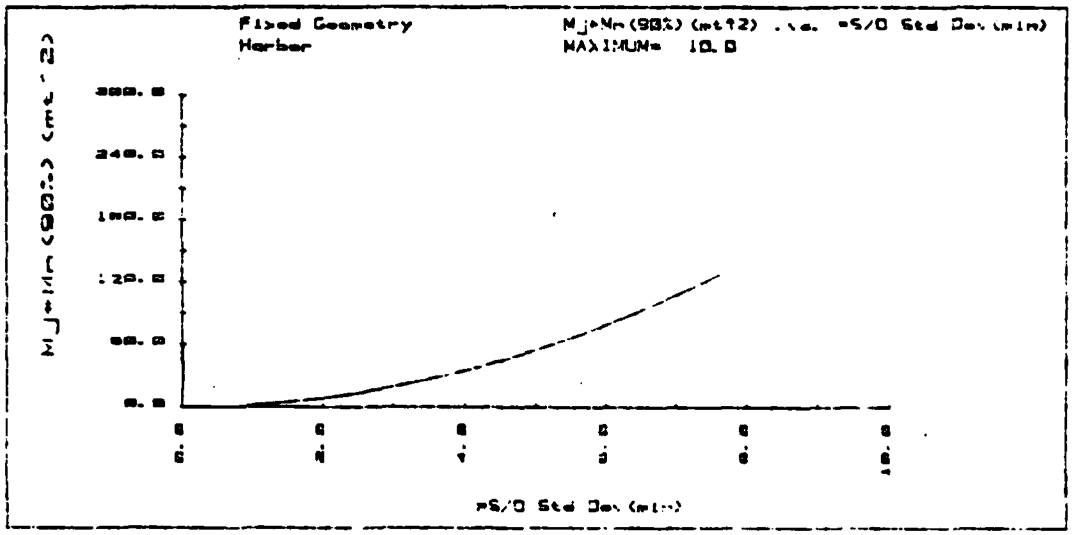

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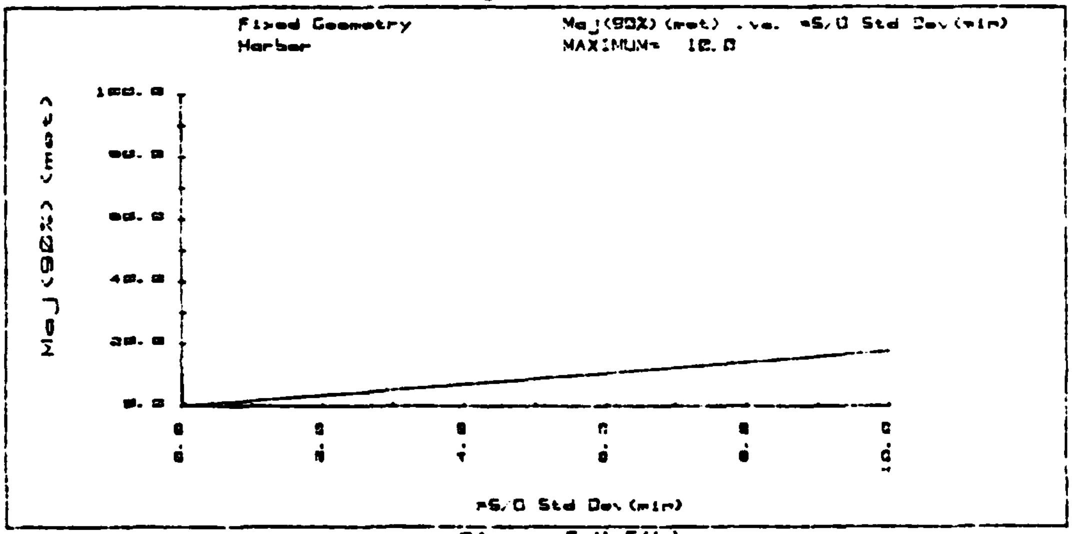

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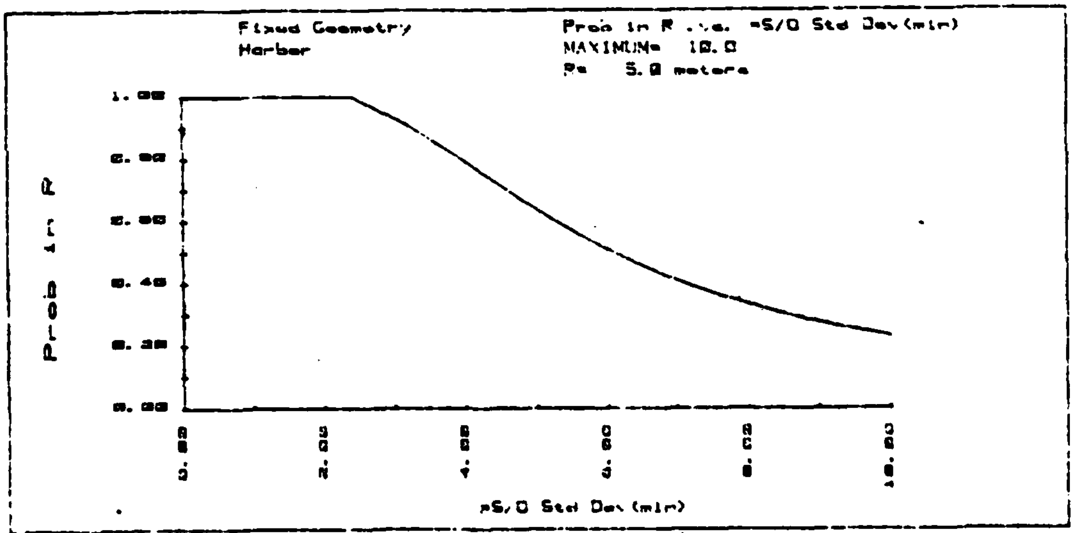

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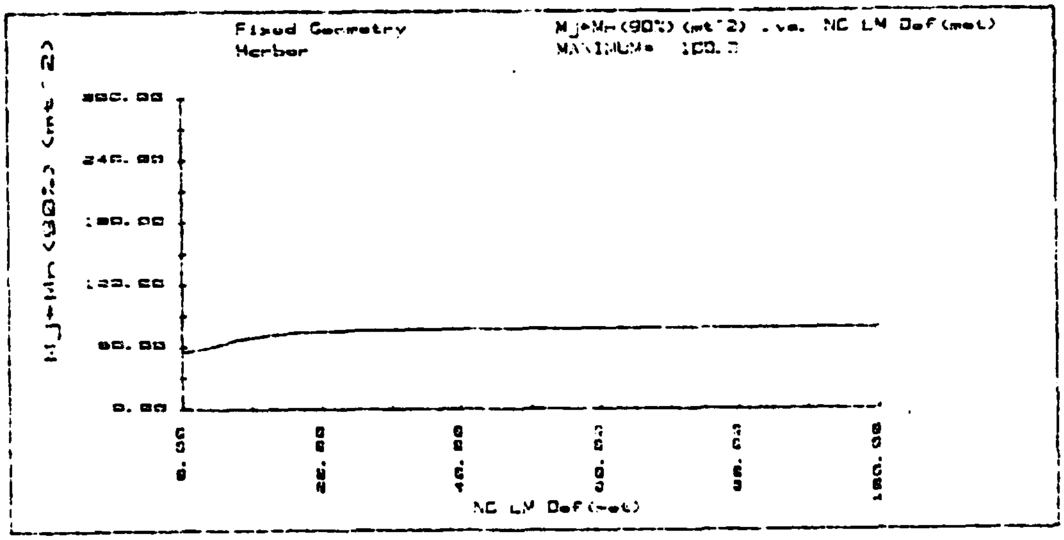

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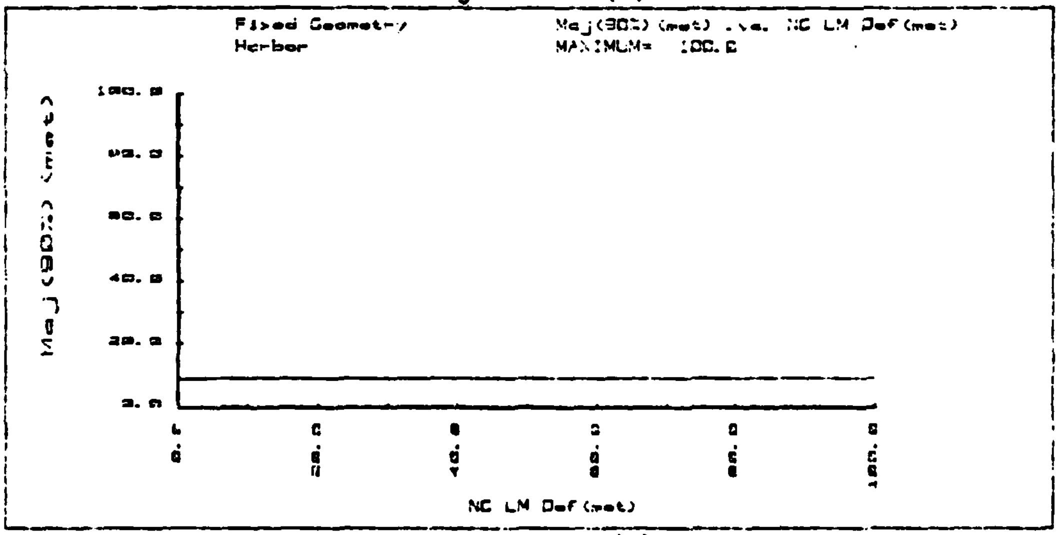

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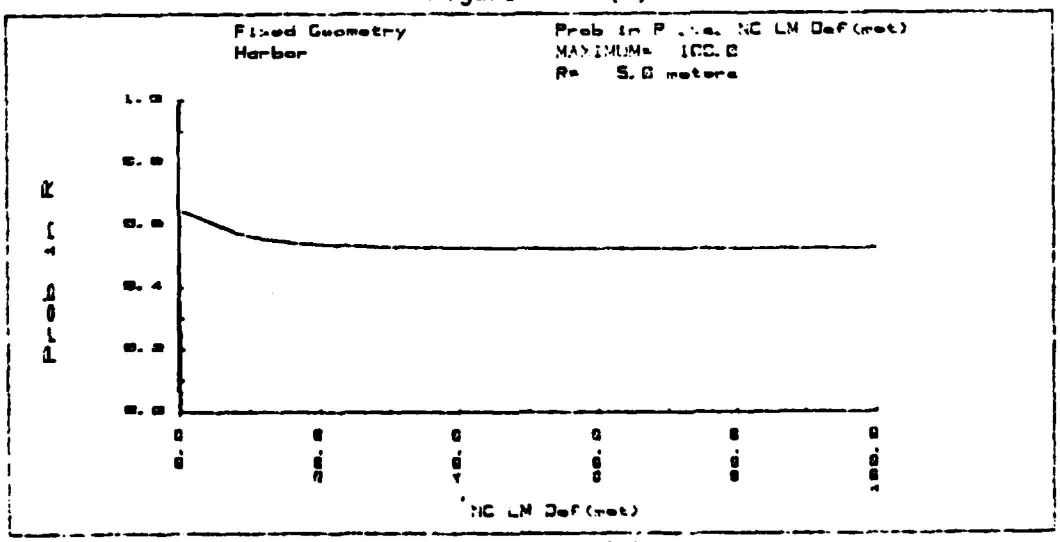

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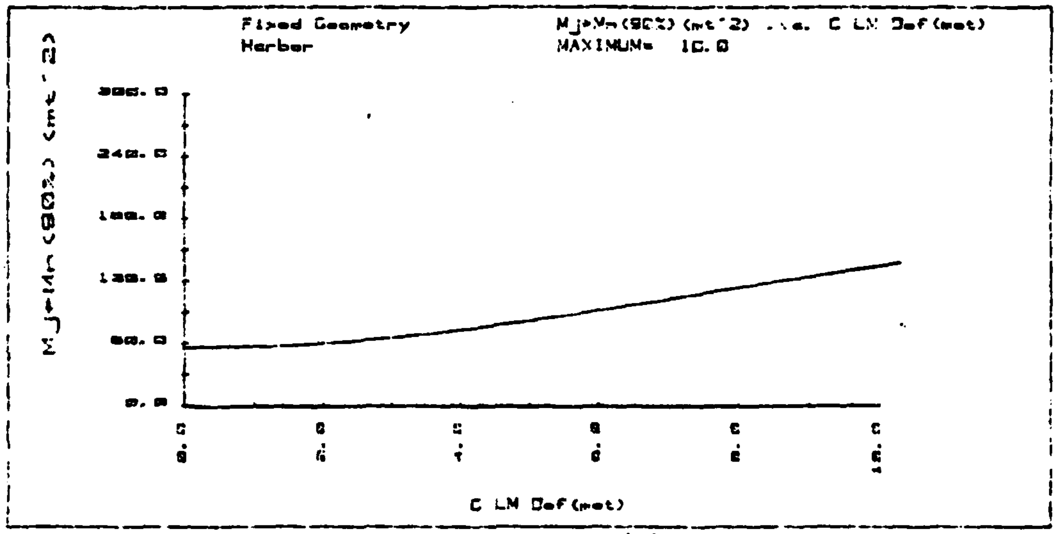

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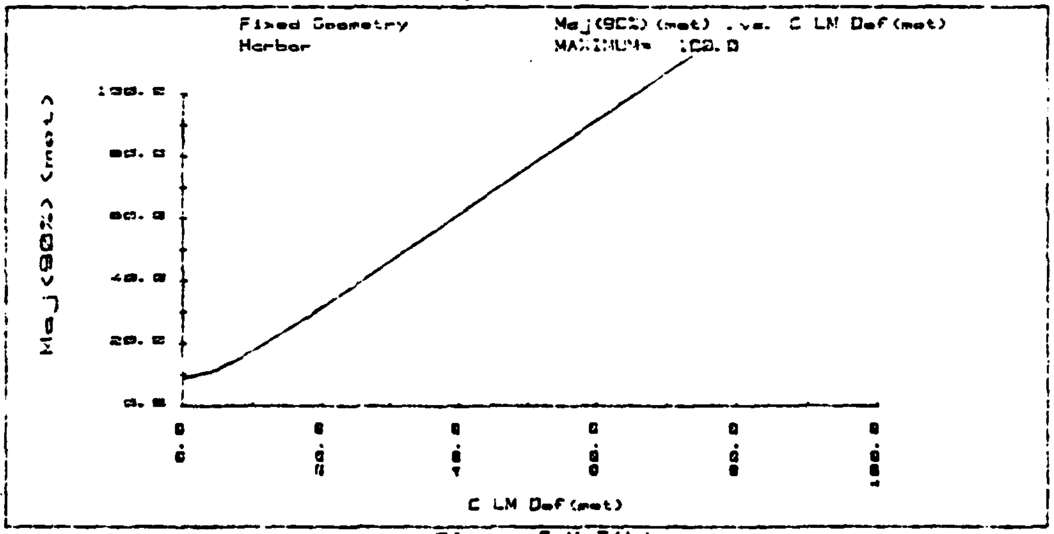

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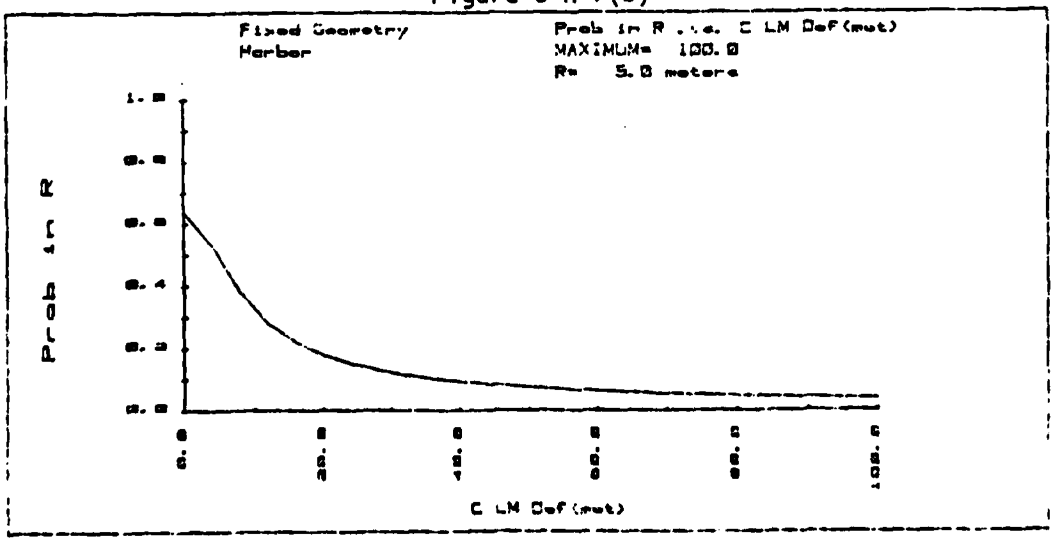

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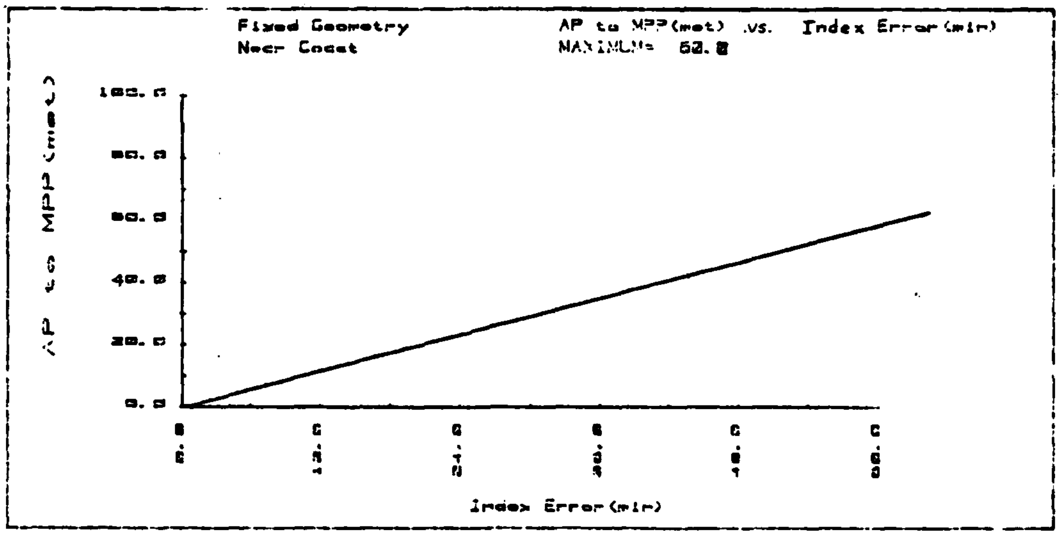

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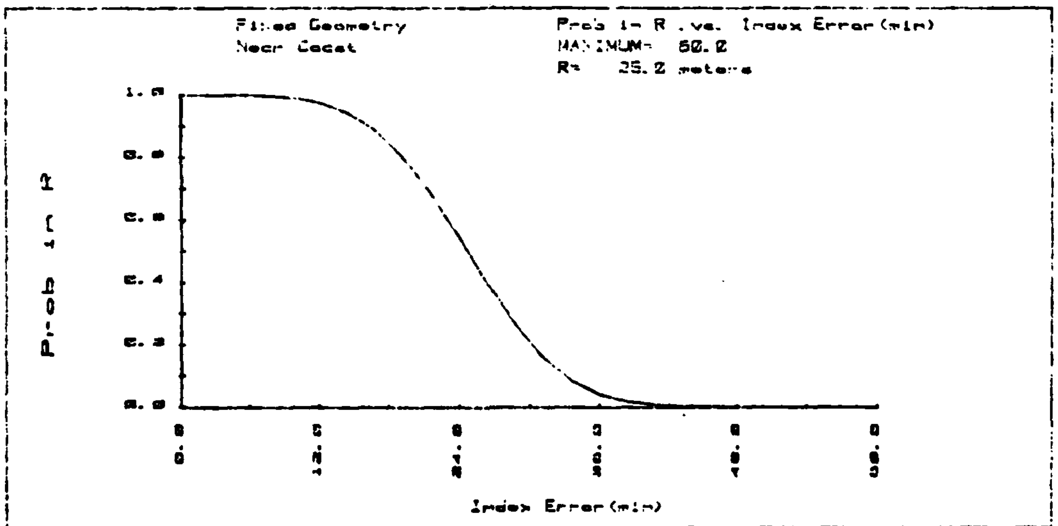

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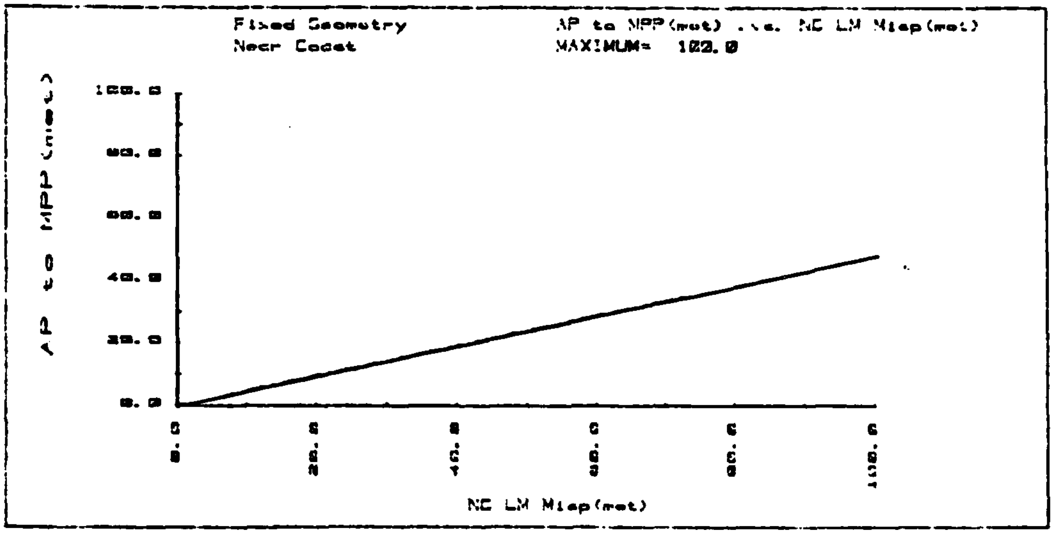

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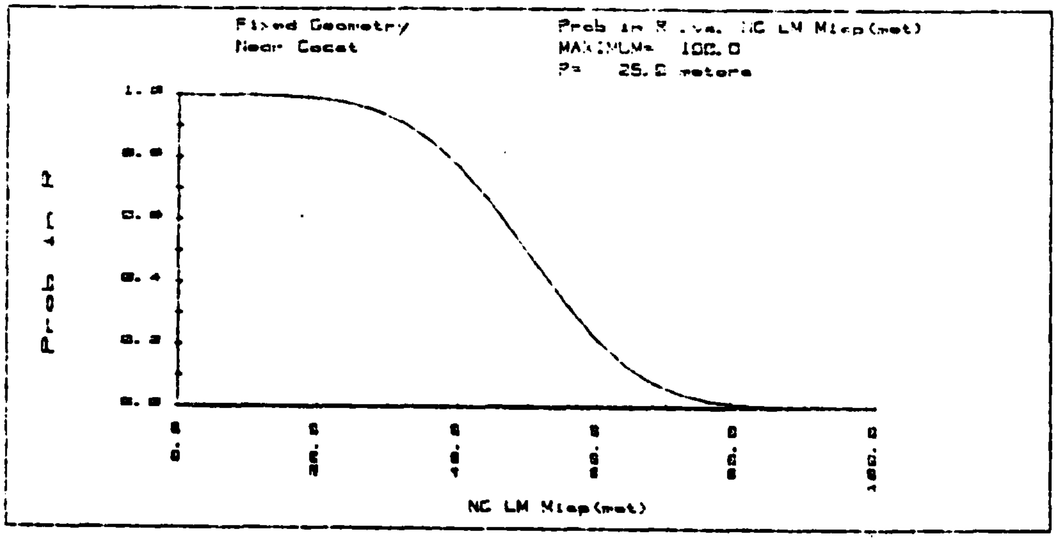

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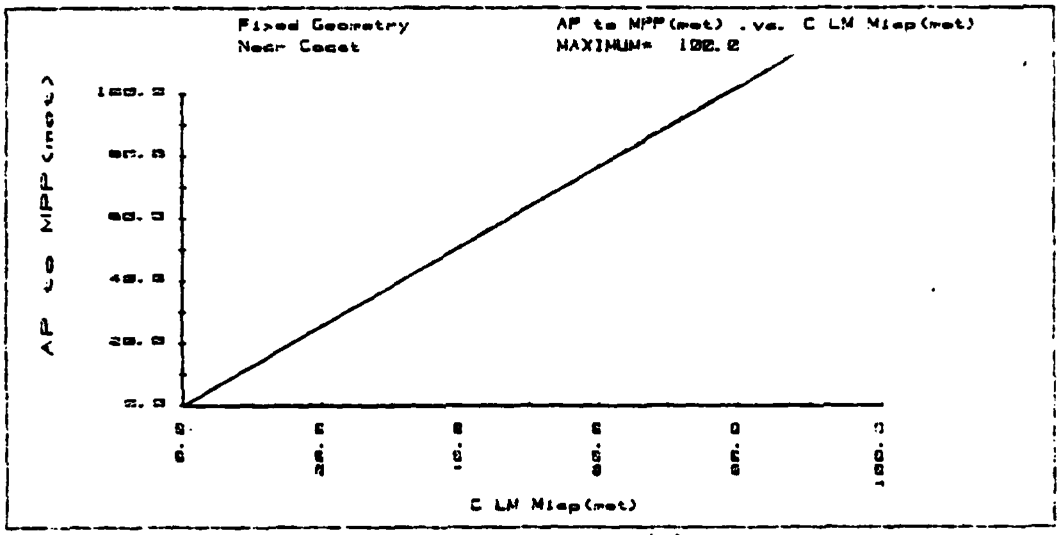

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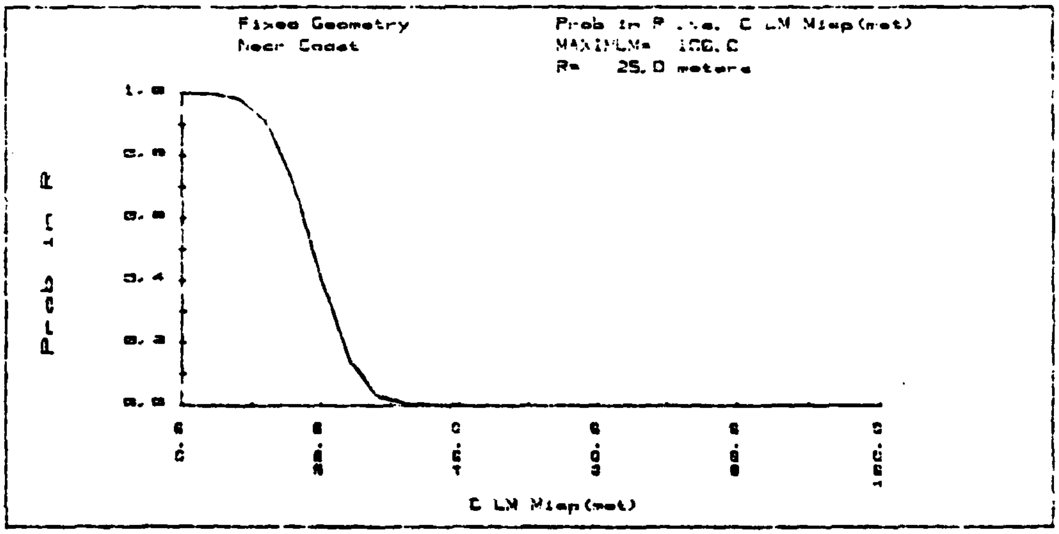

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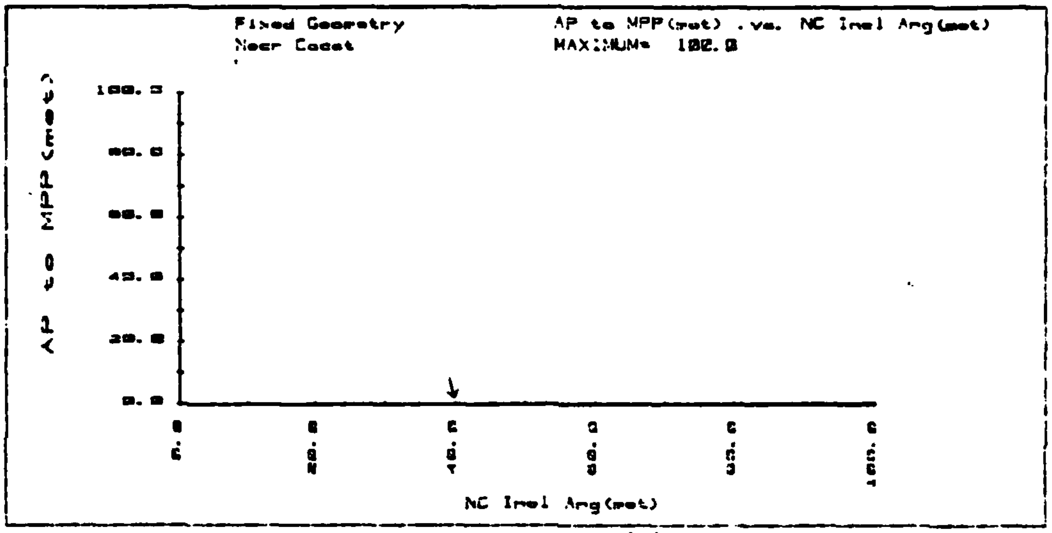

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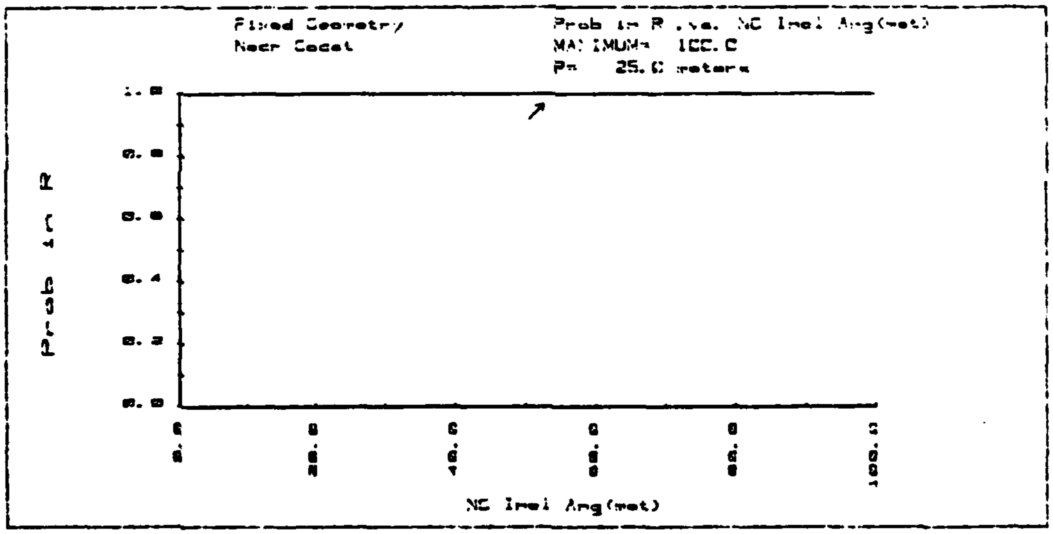

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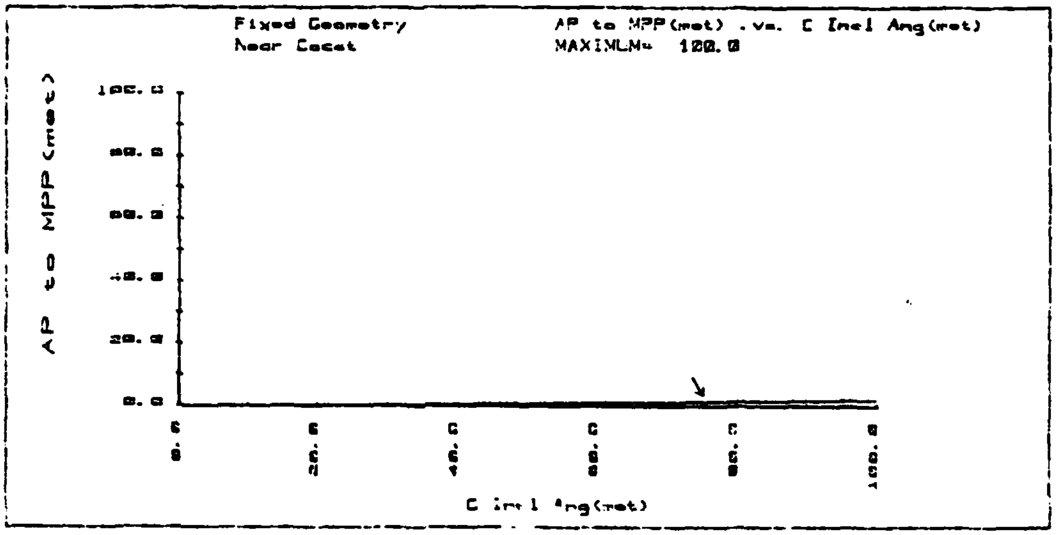

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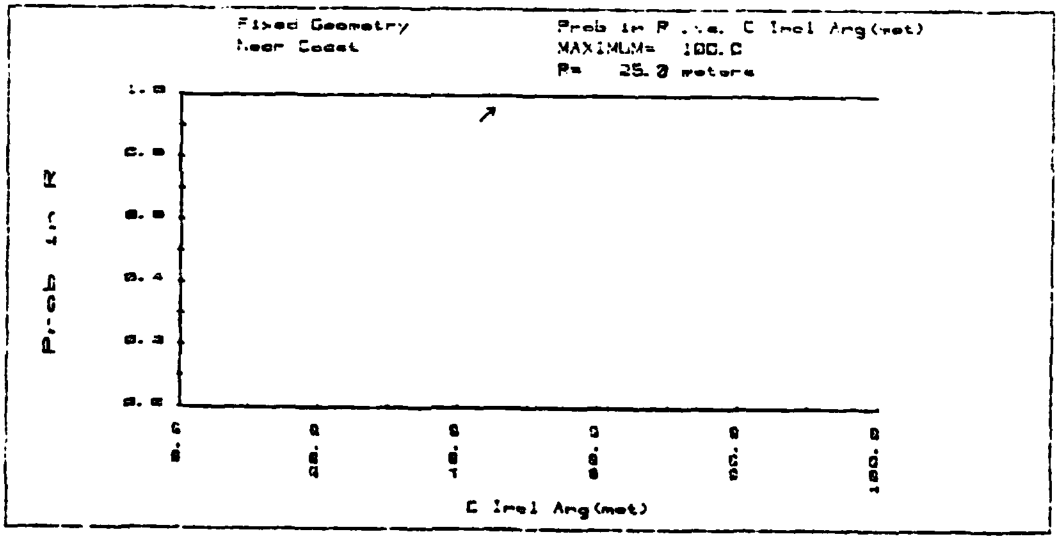

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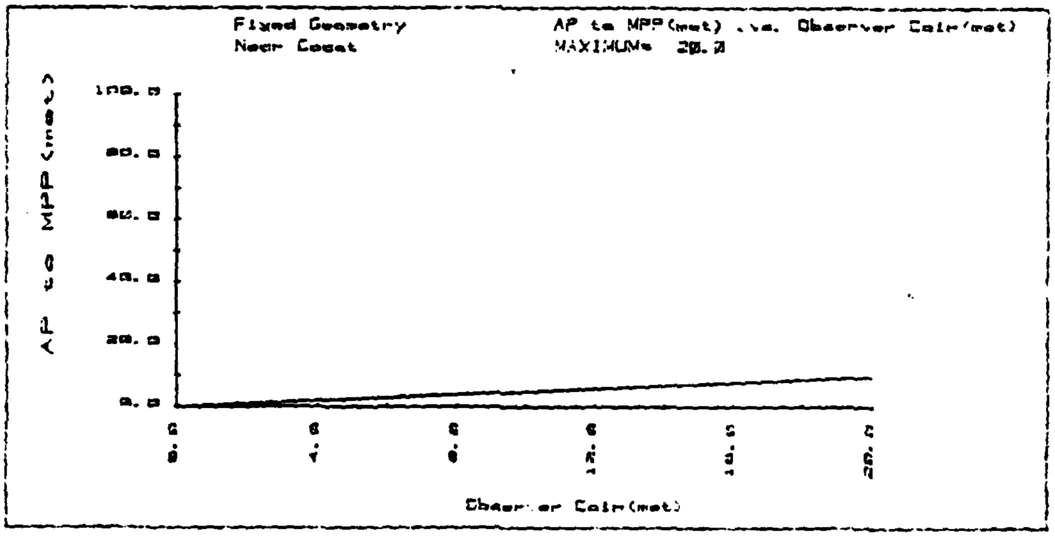

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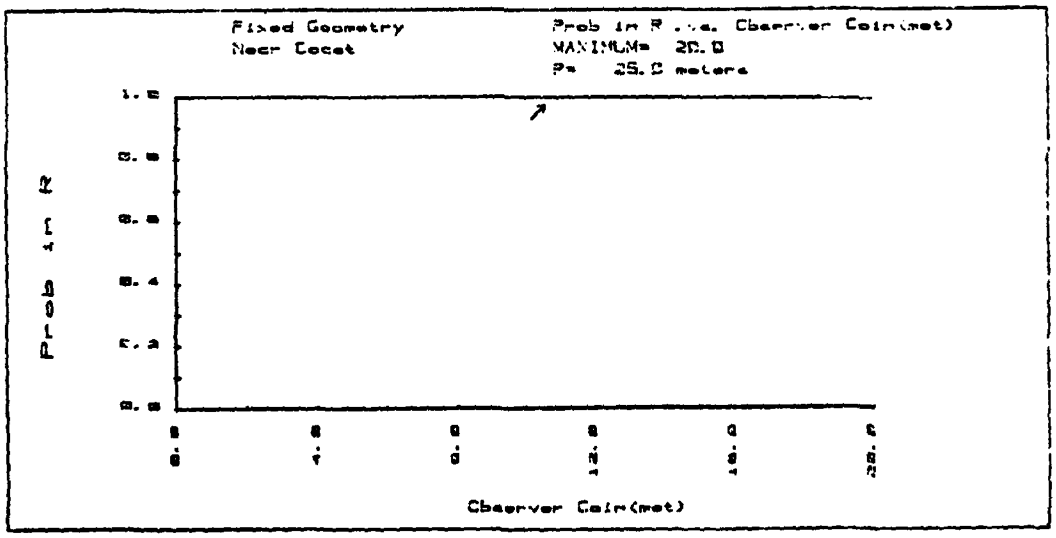

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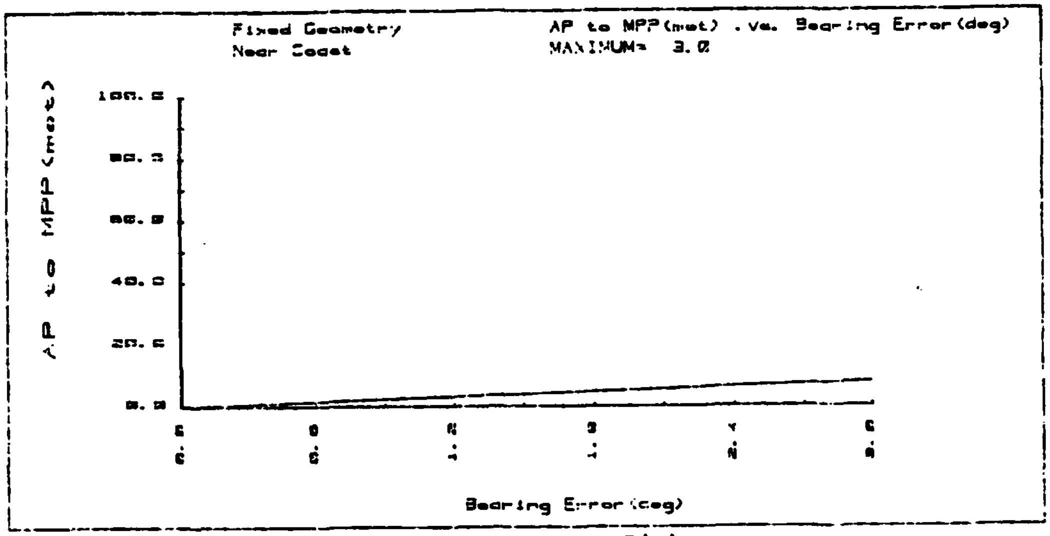

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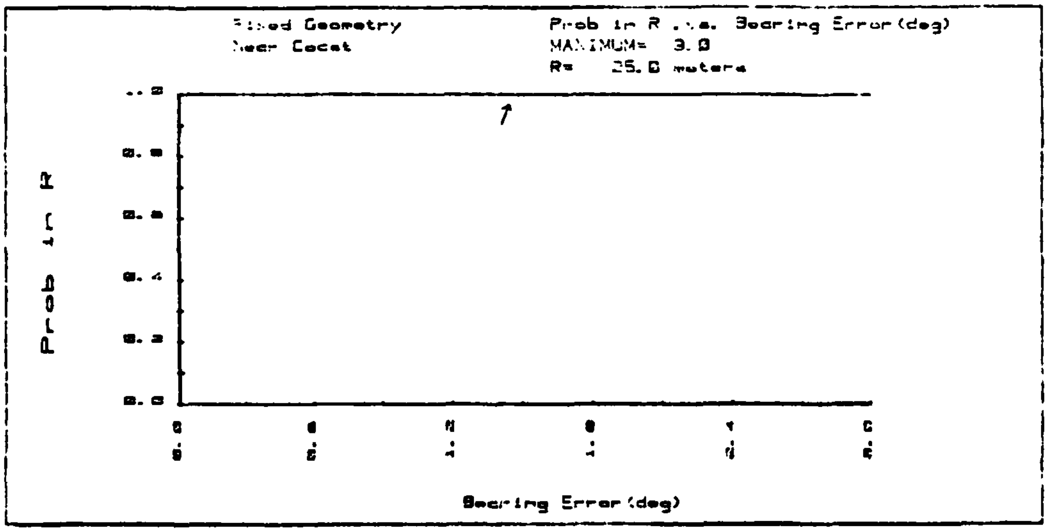

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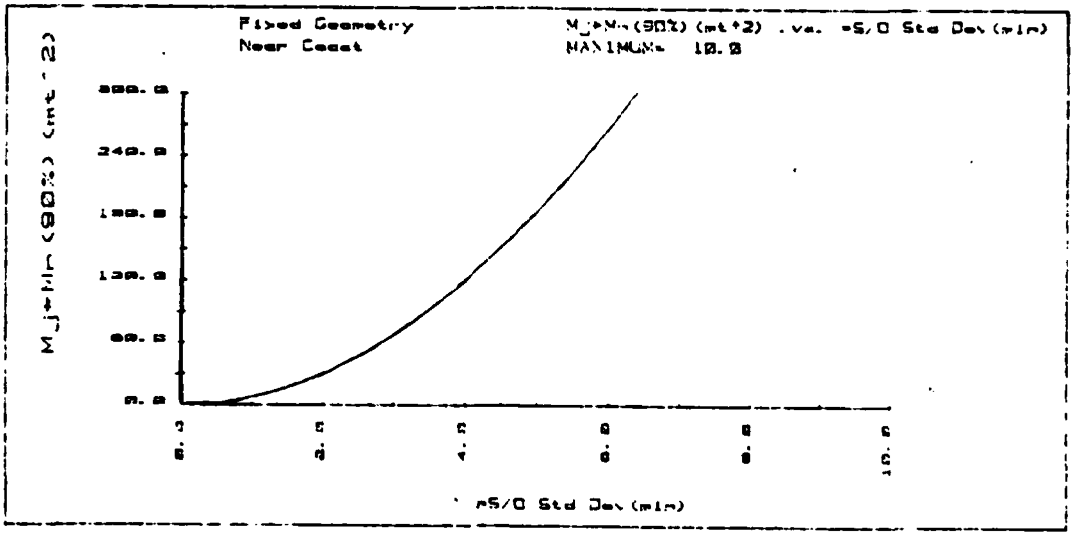

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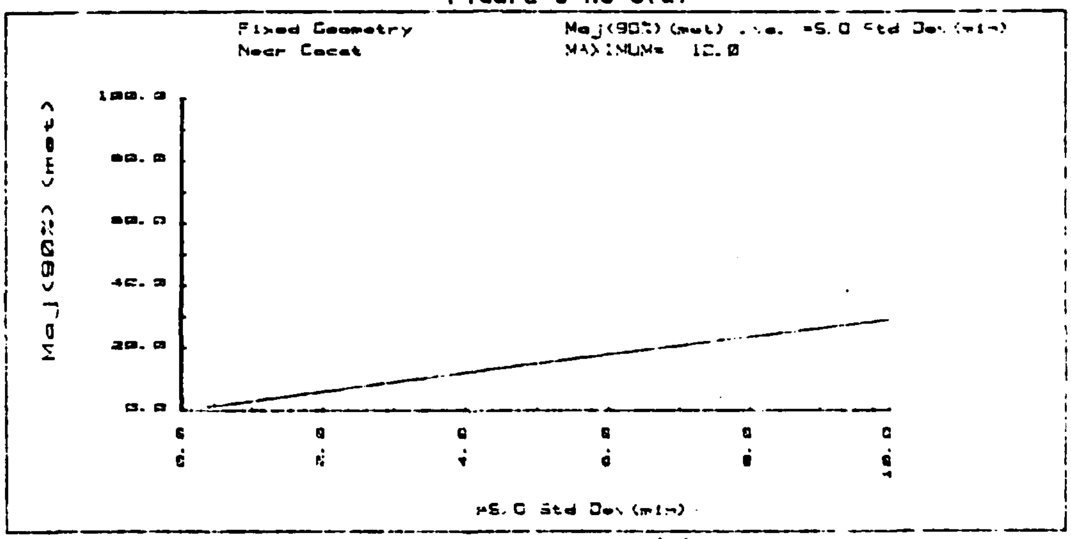

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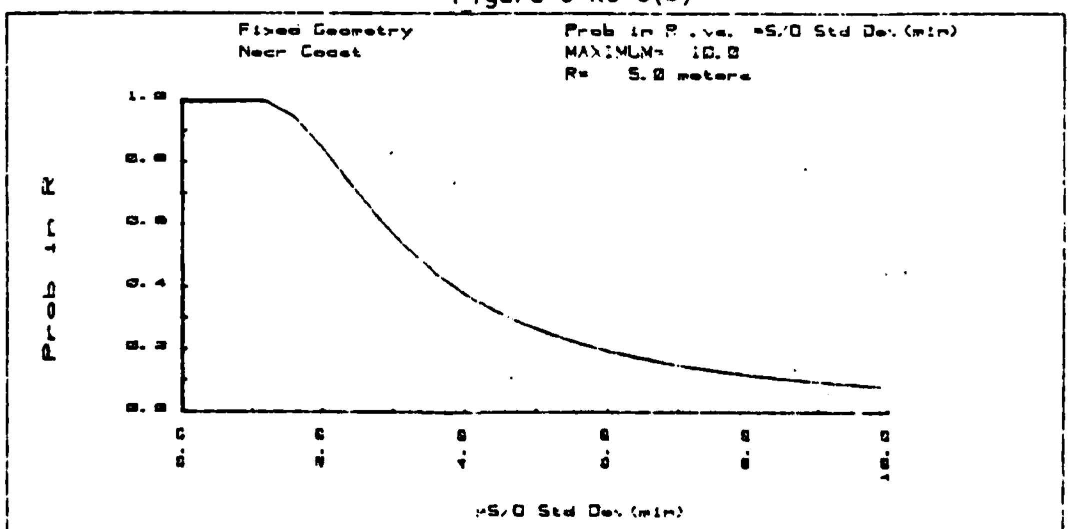

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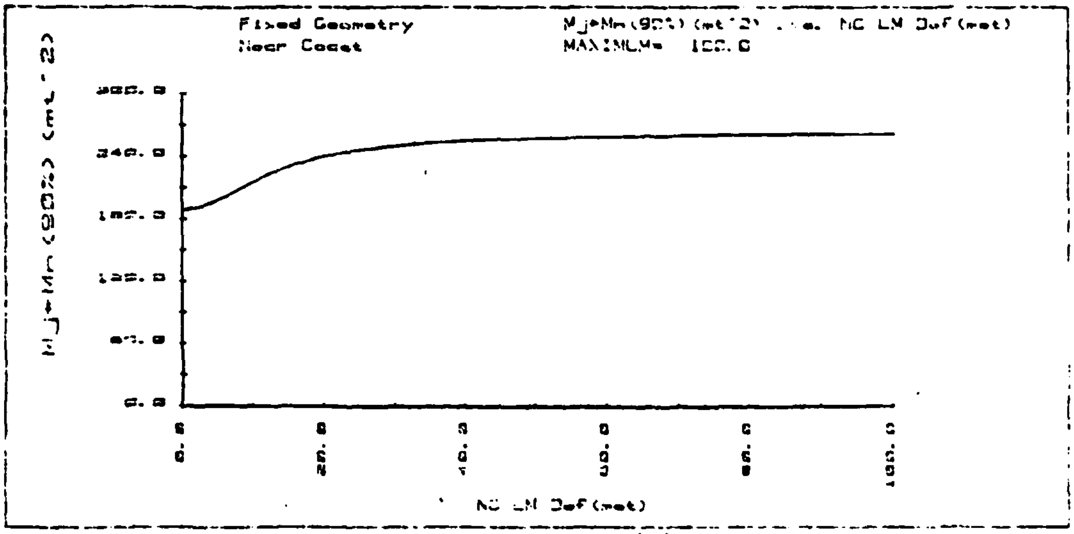

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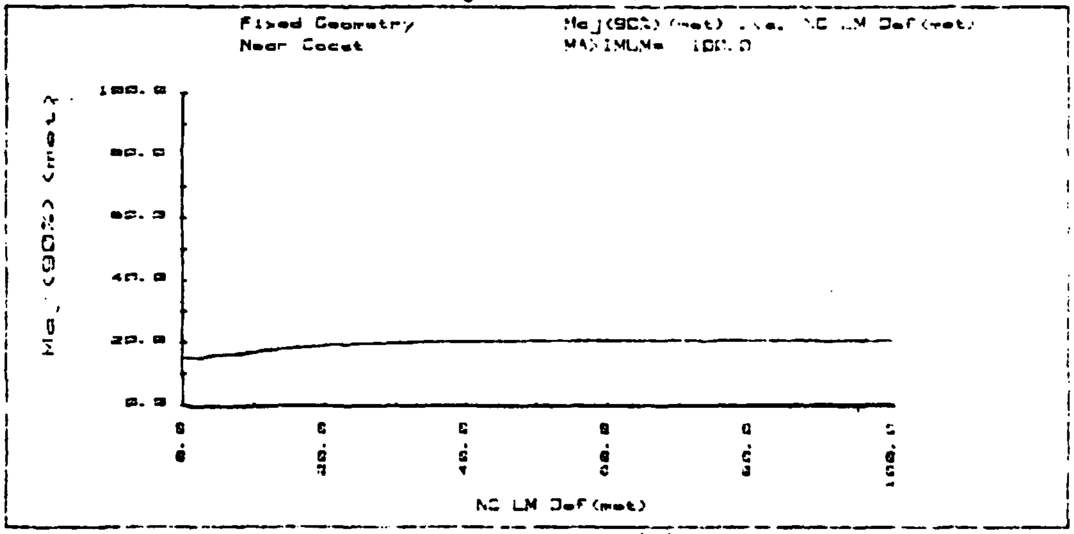

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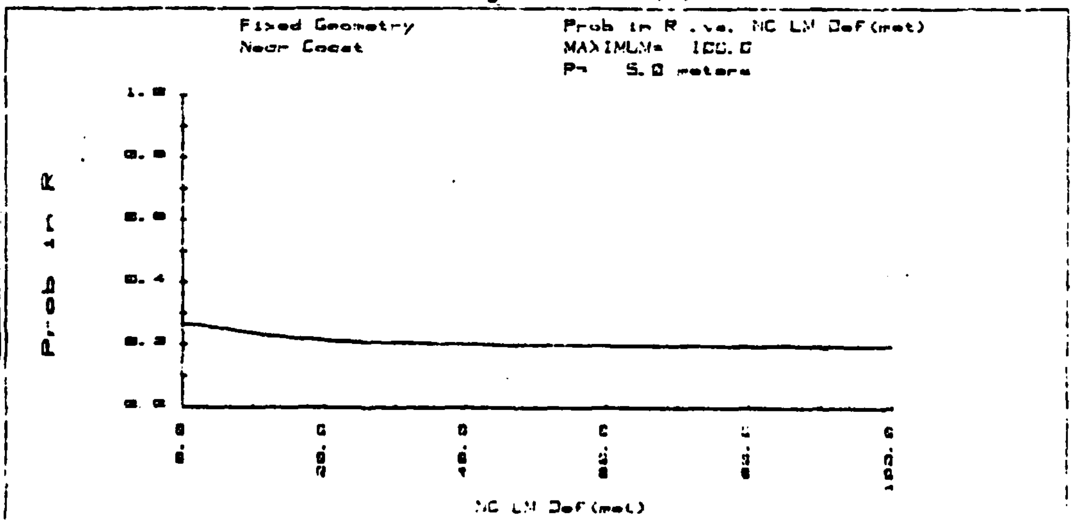

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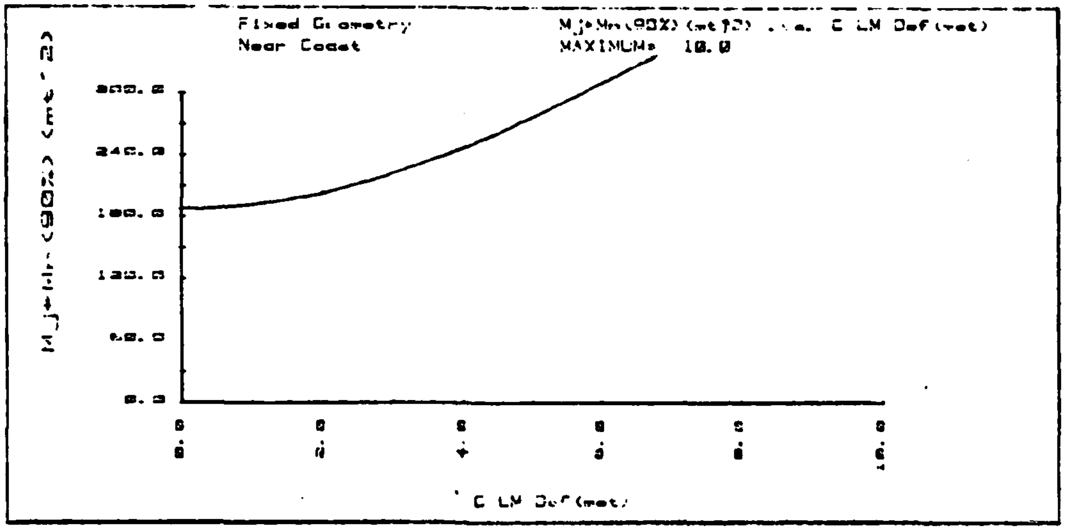

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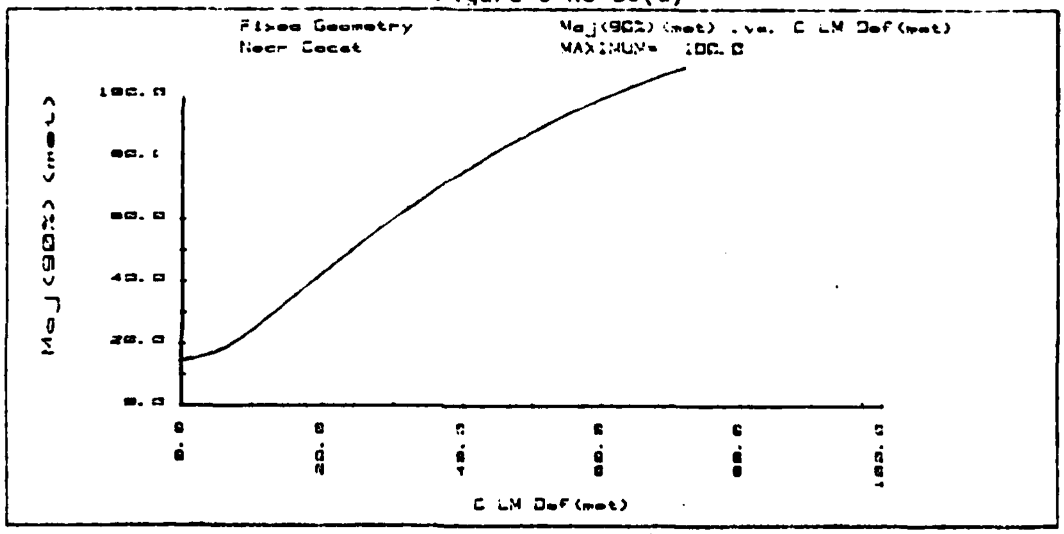

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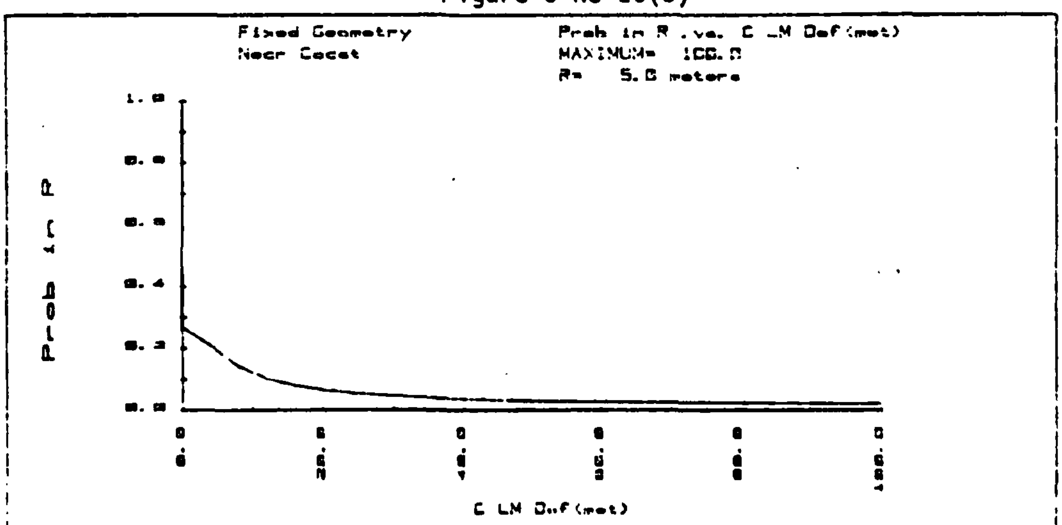

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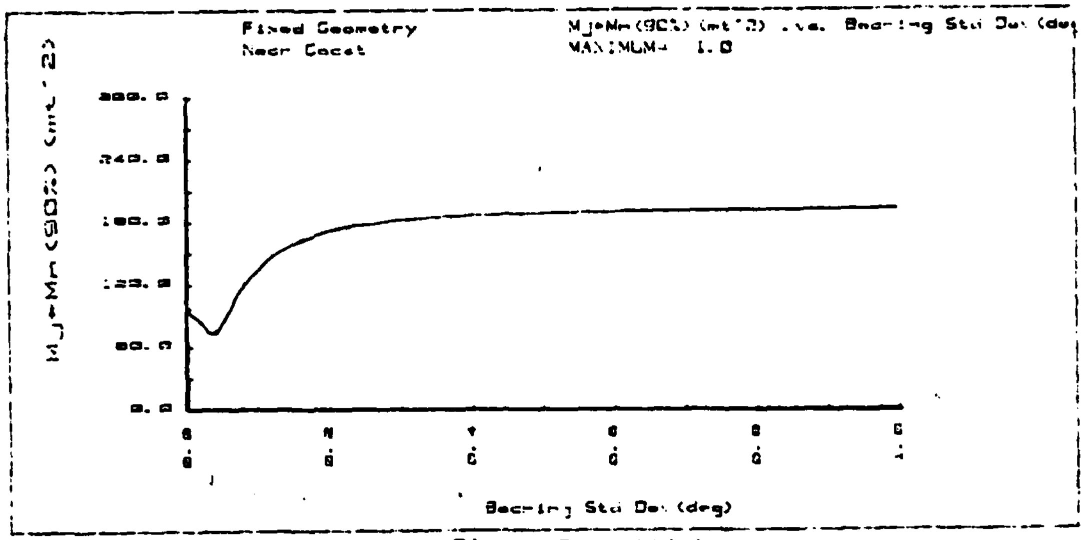

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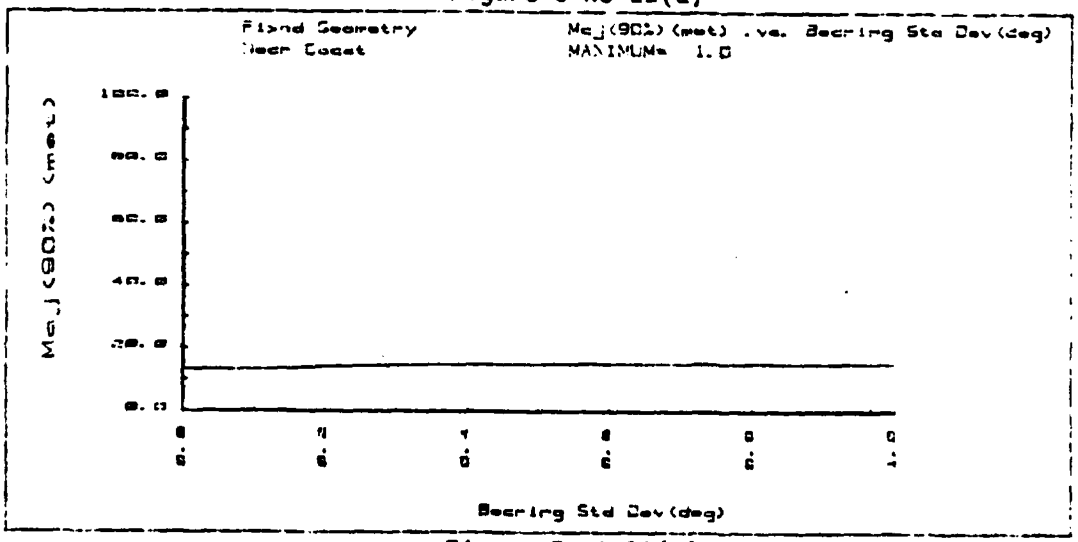

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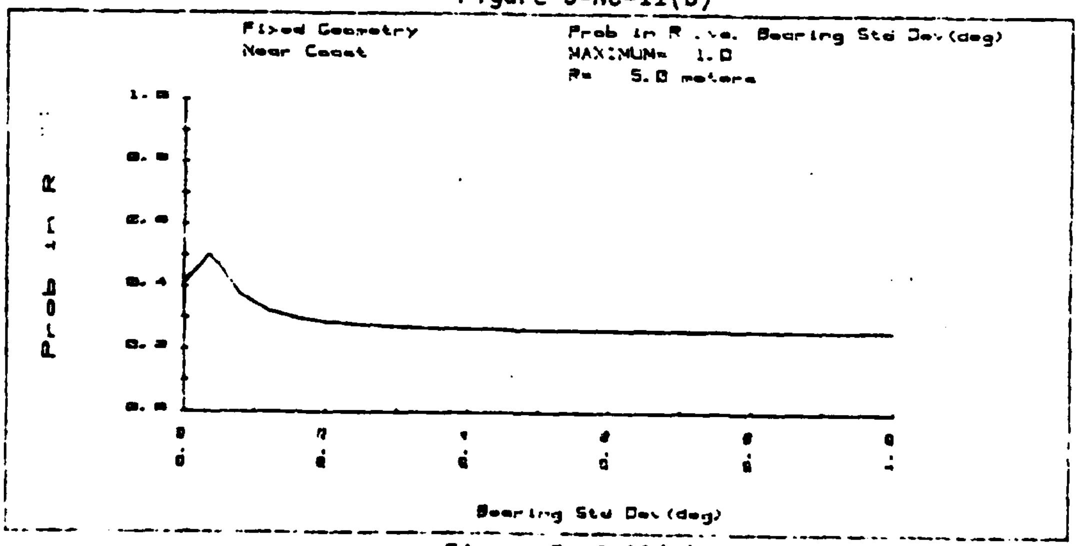

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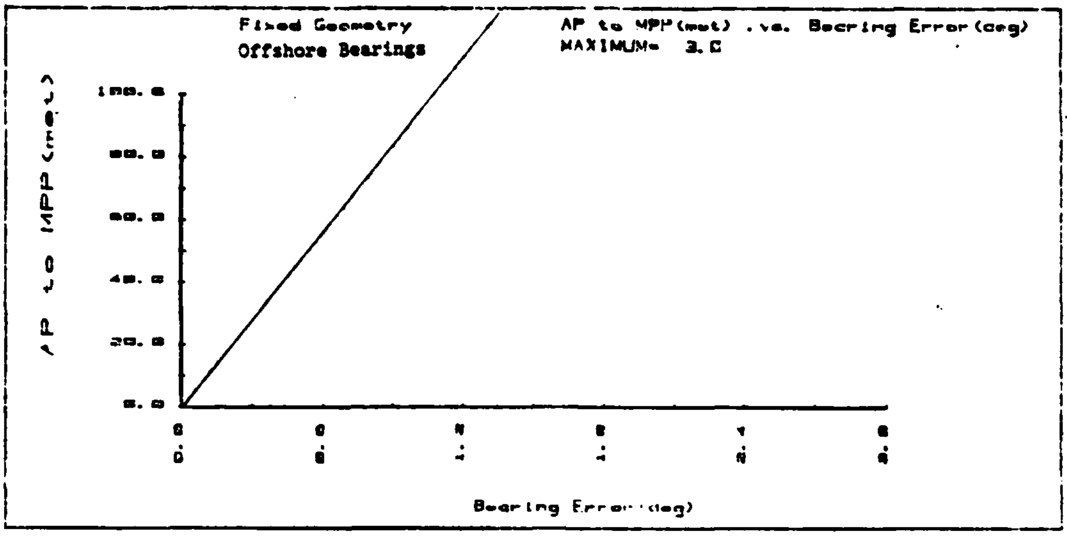
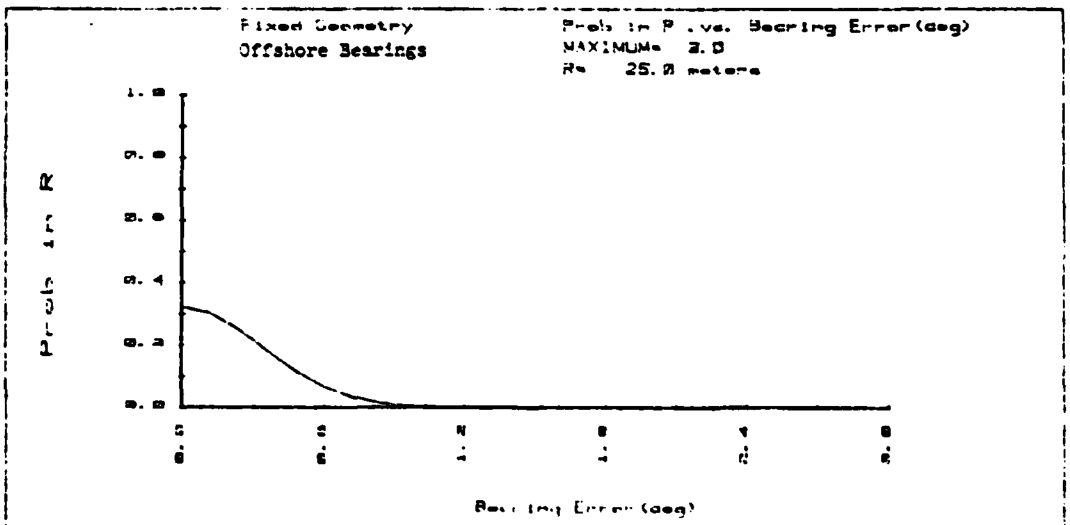


Figure 5-OS-1(a)



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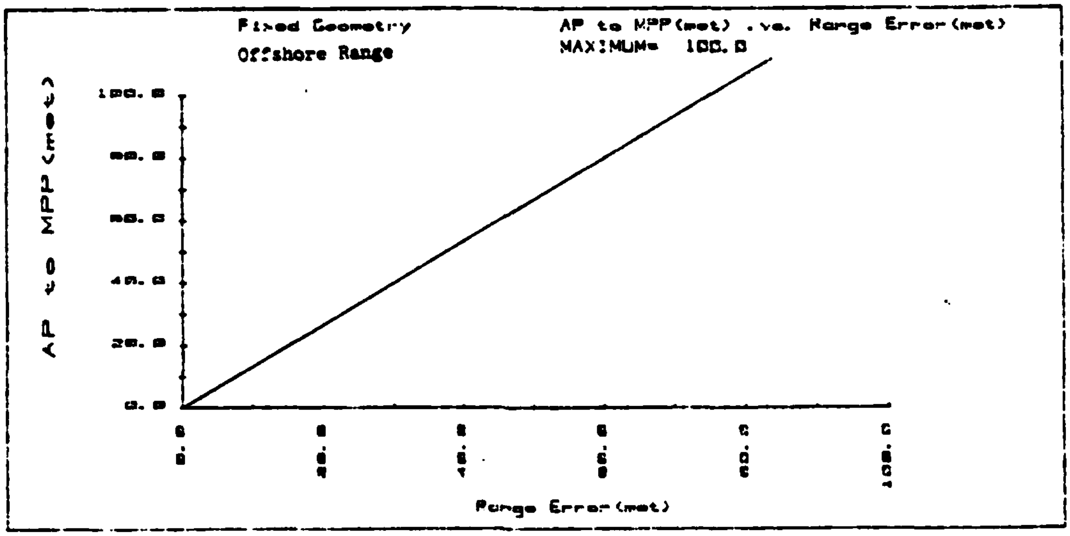

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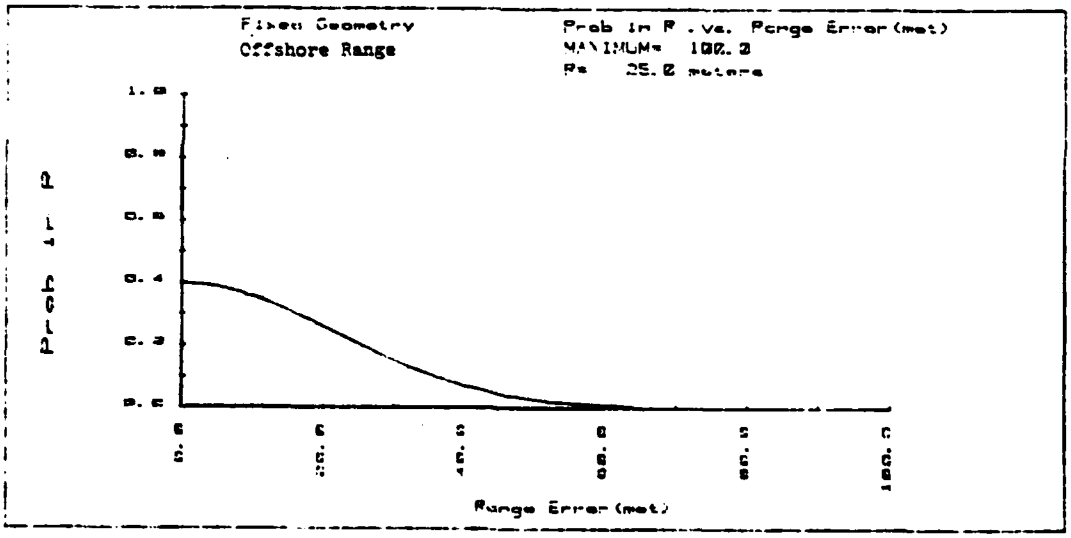

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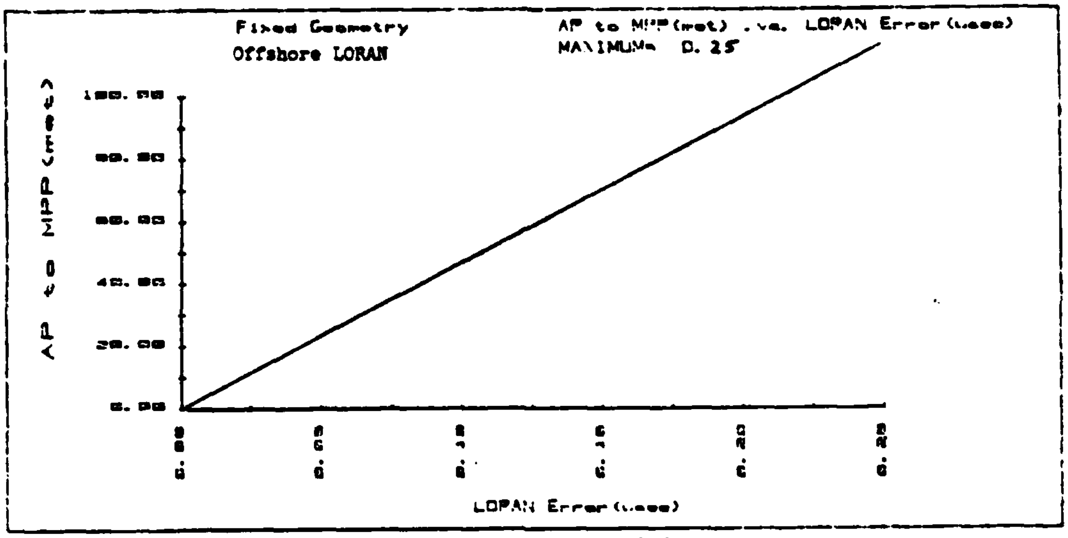

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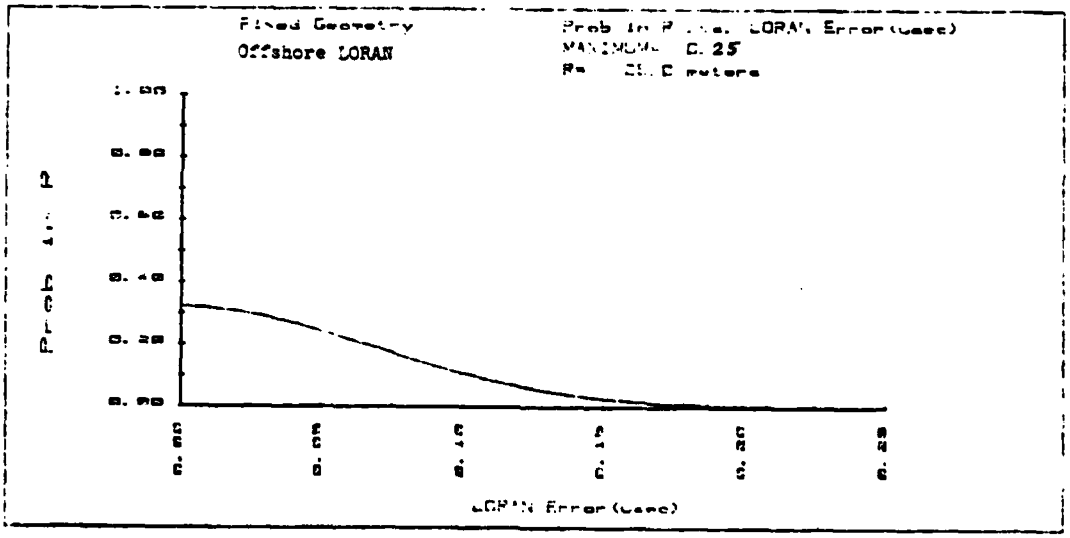

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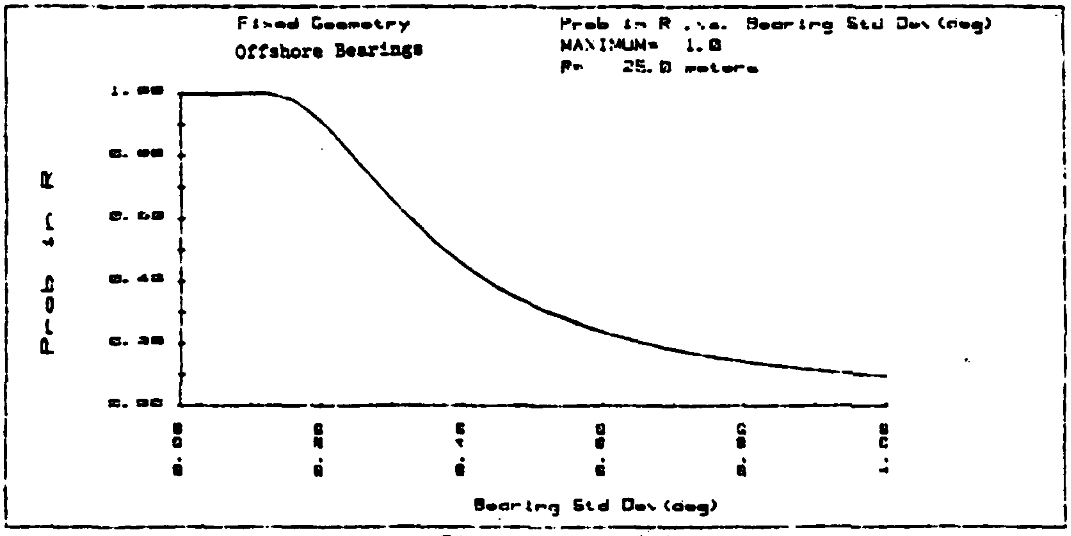

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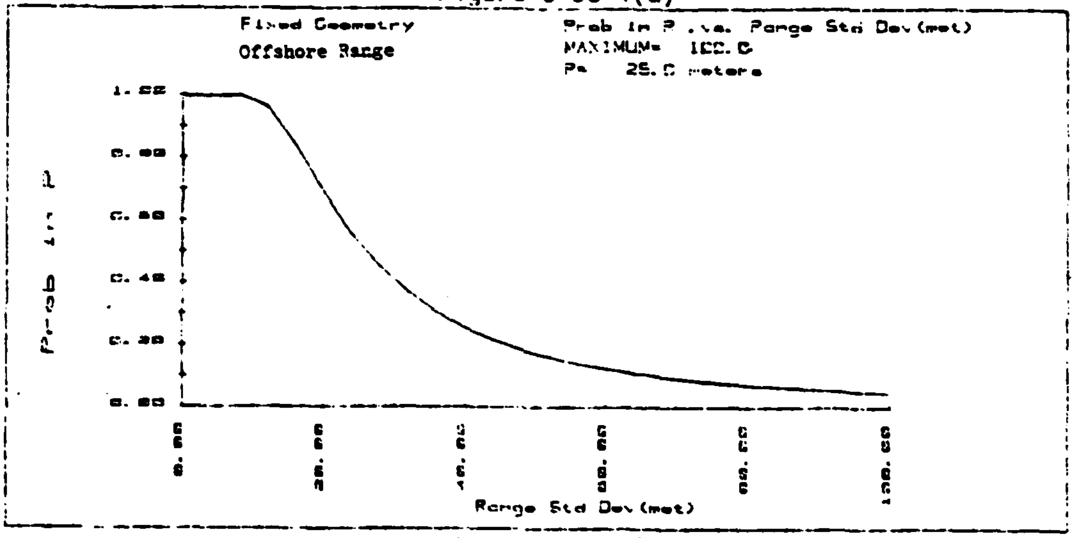

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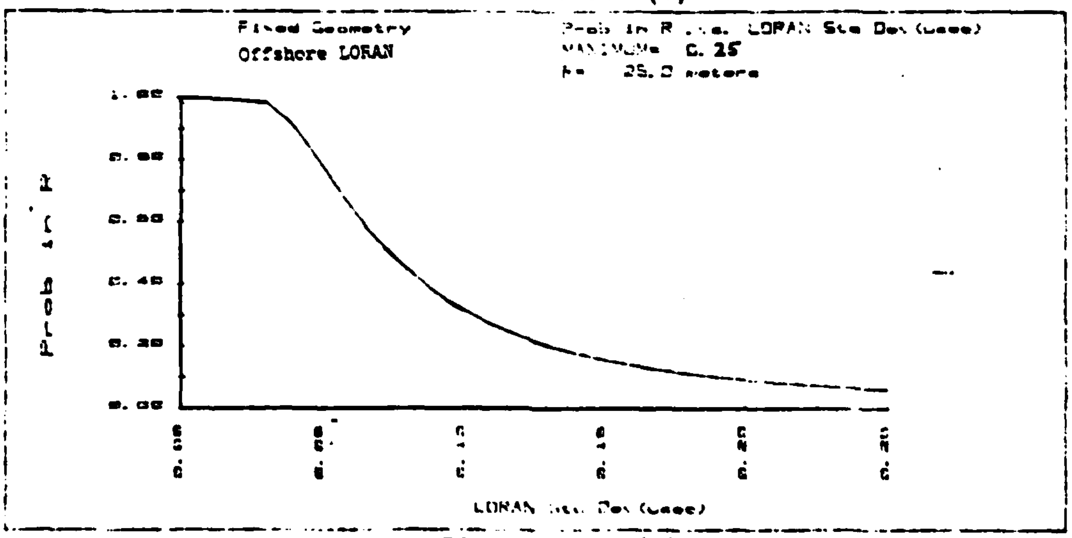
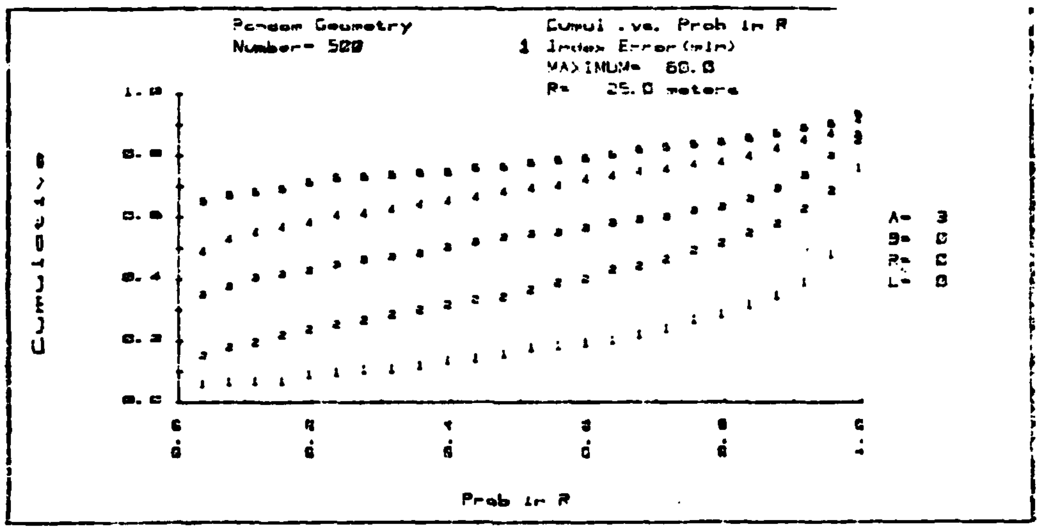


Figure 5-0S-4(c)



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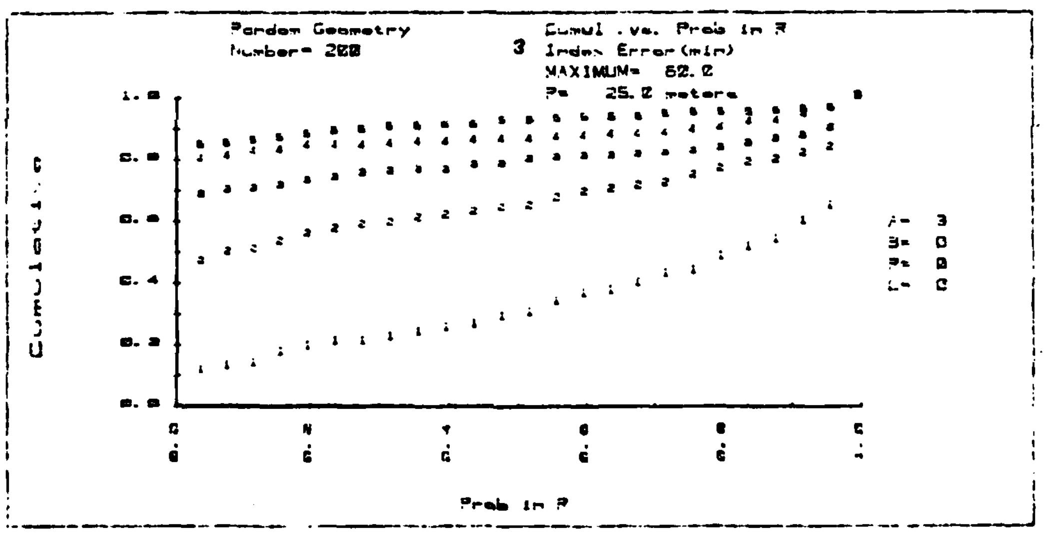

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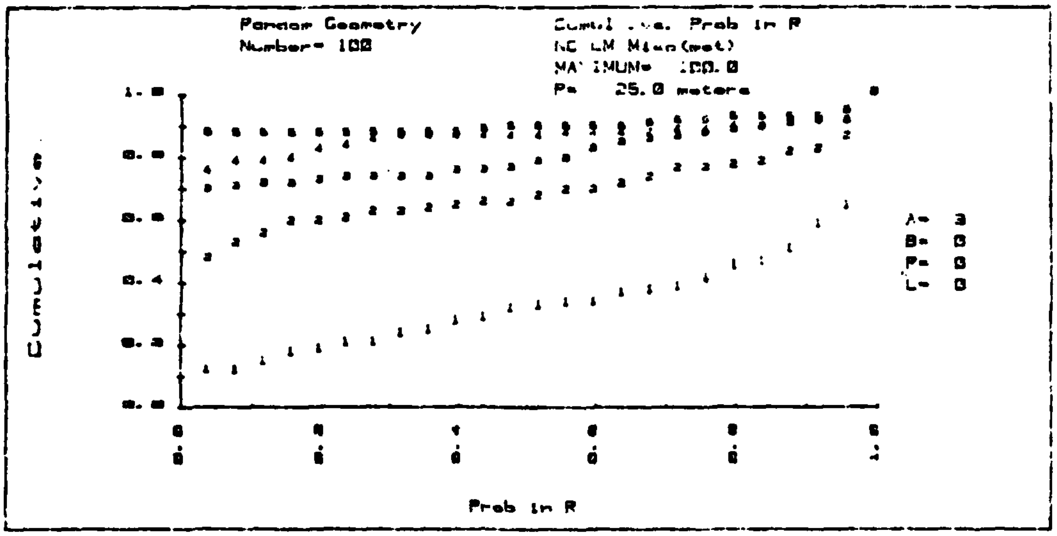

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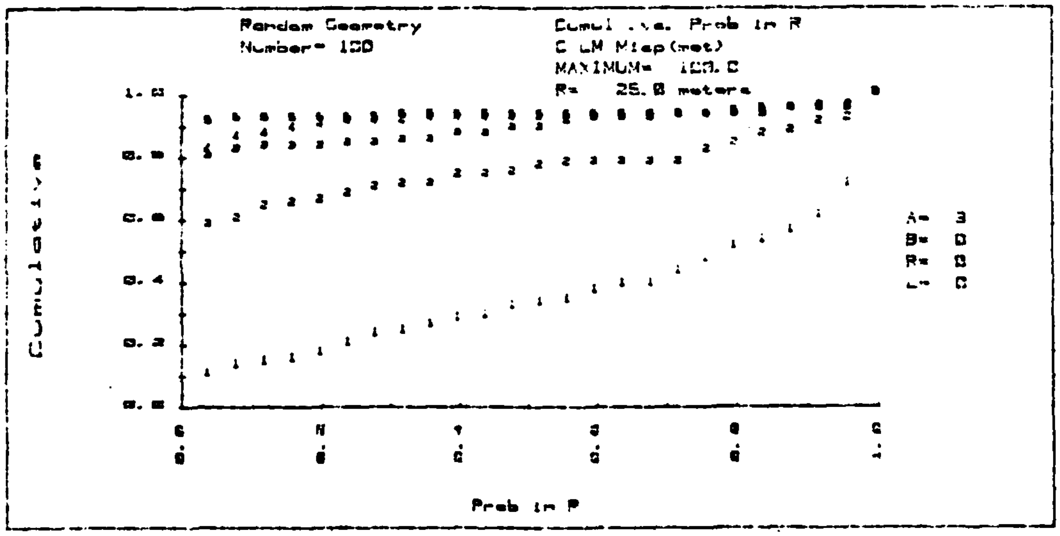


Figure 5-R-2(b)

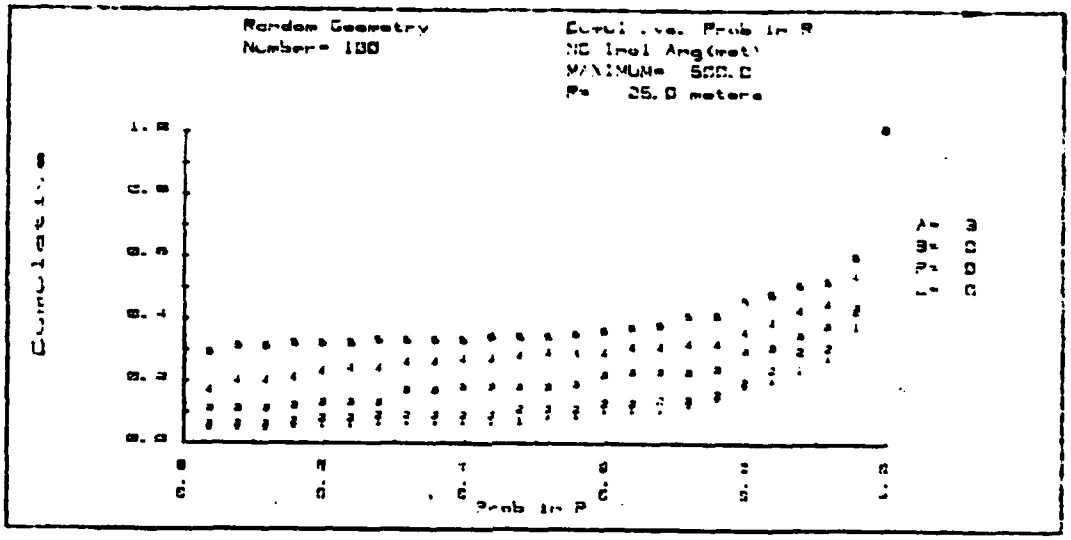
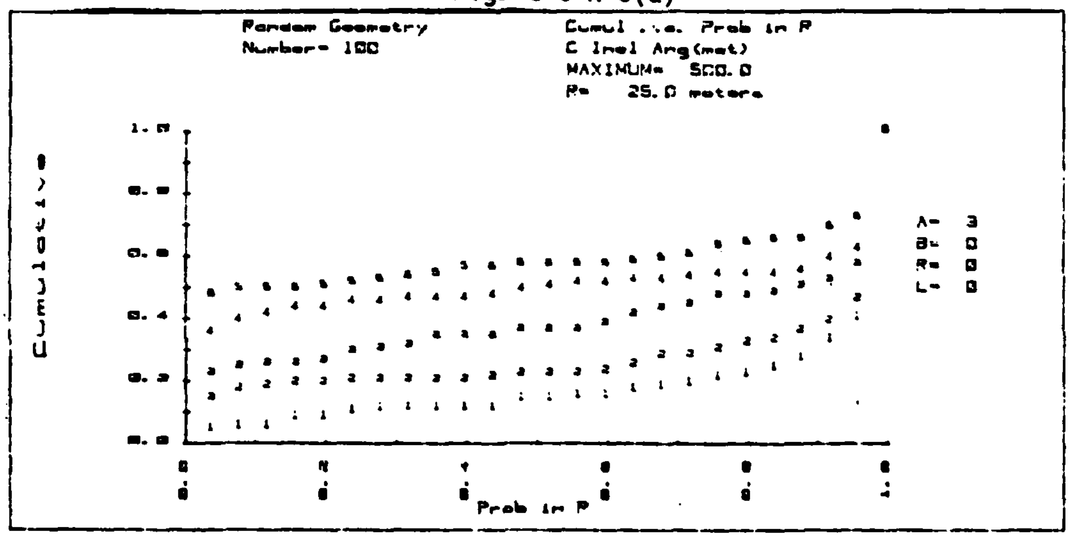


Figure 5-R-3(a)



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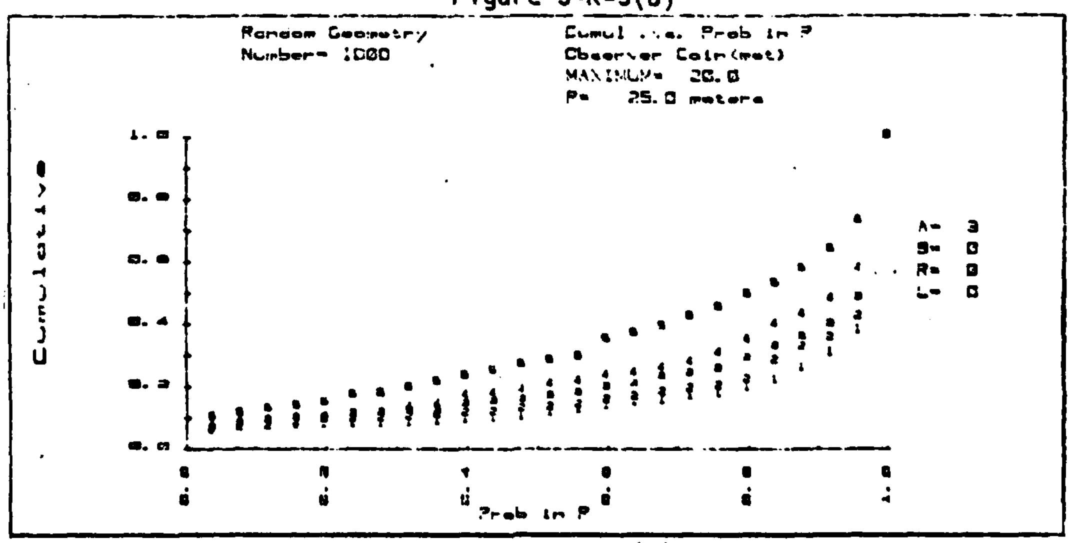


Figure 5-R-3(c)

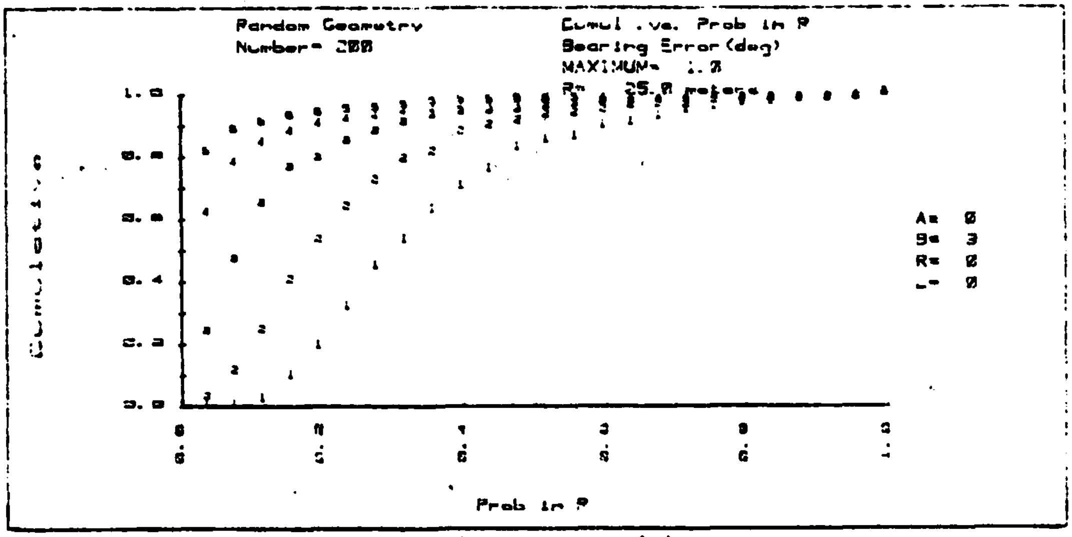
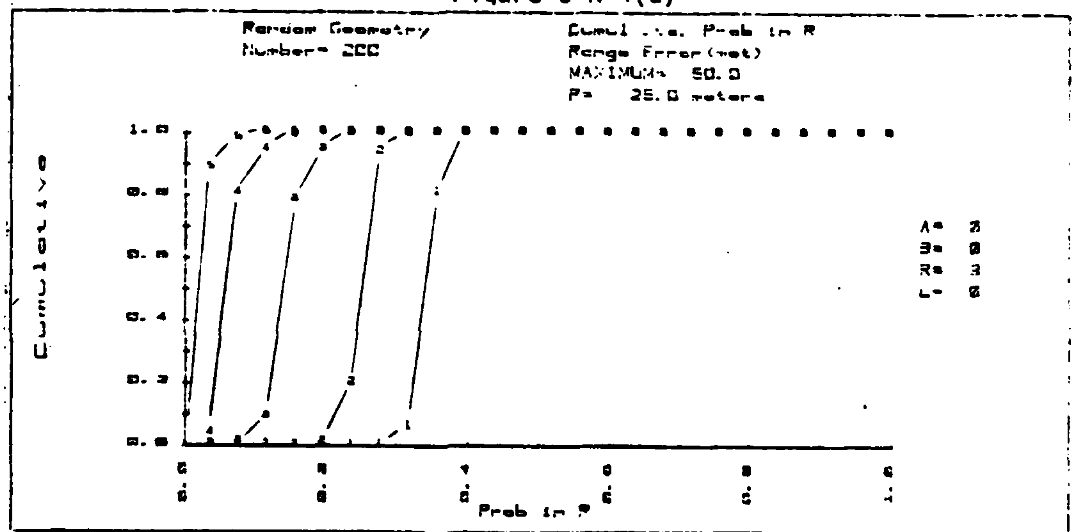


Figure 5-R-4(a)



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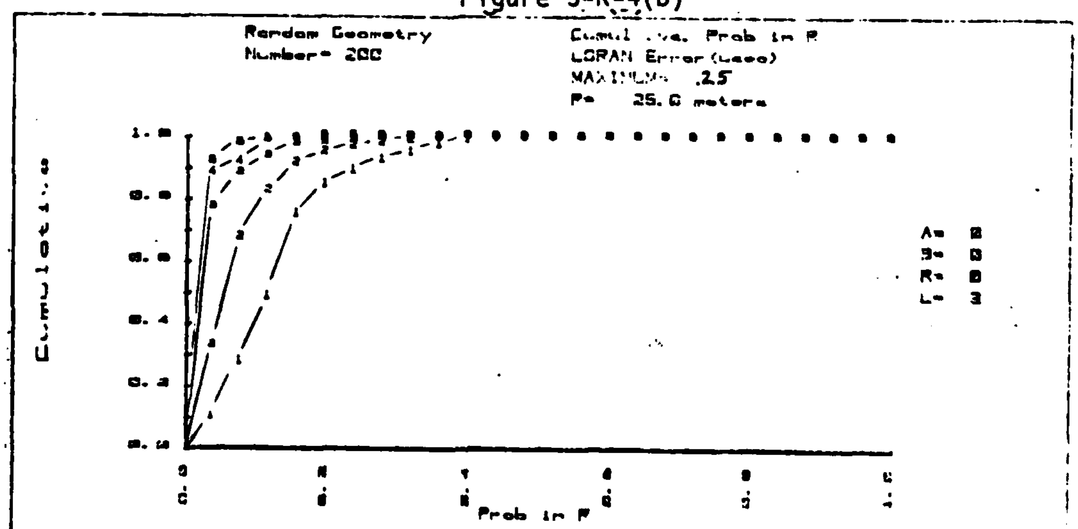

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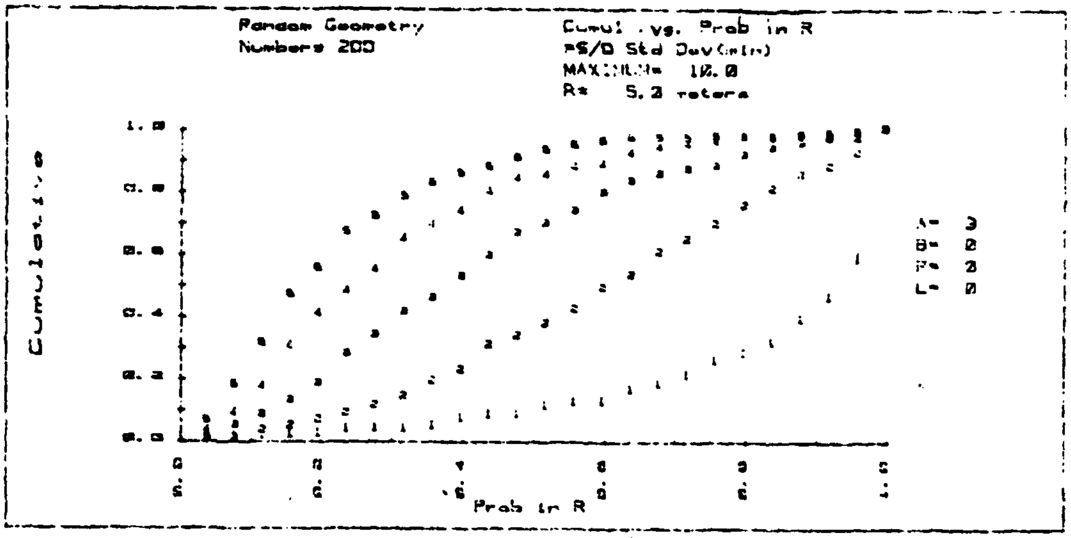
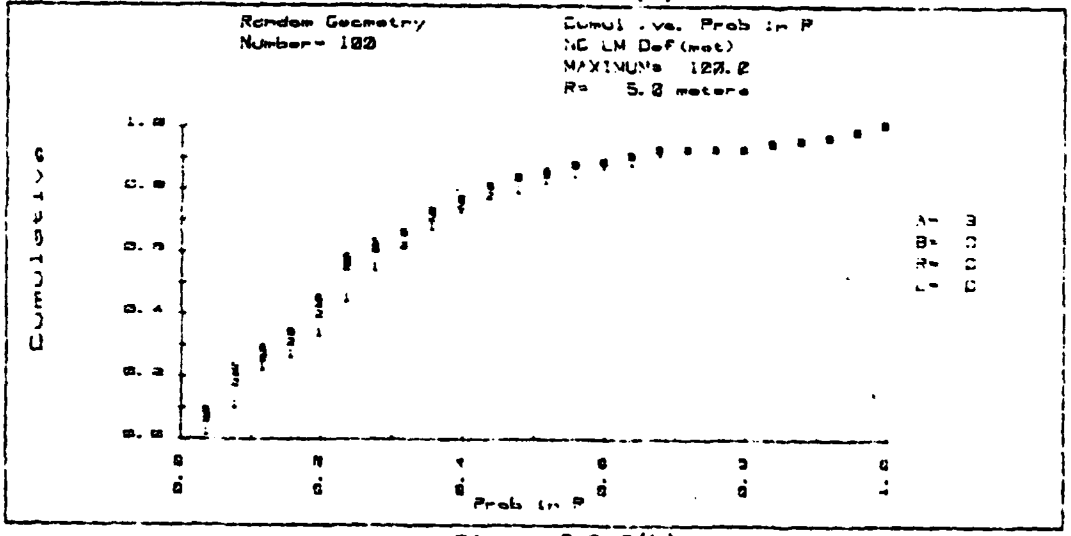


Figure 5-R-5(a)



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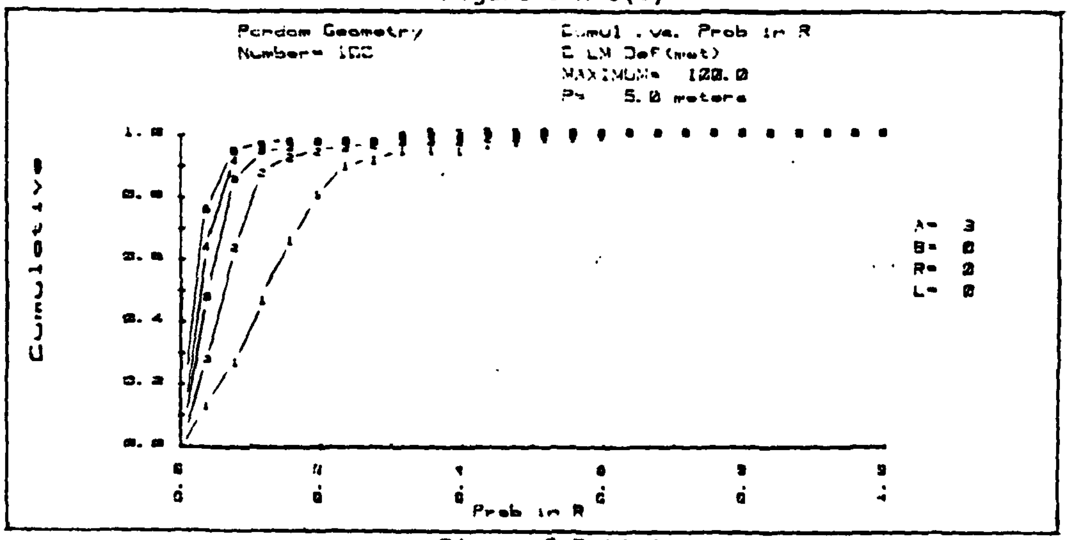

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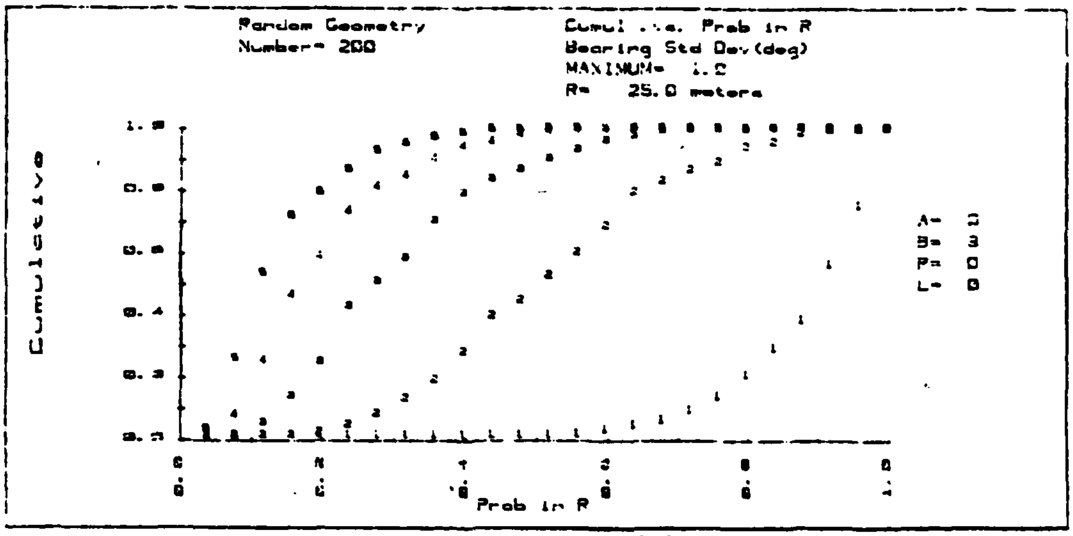

Figure 5-R-6(a)

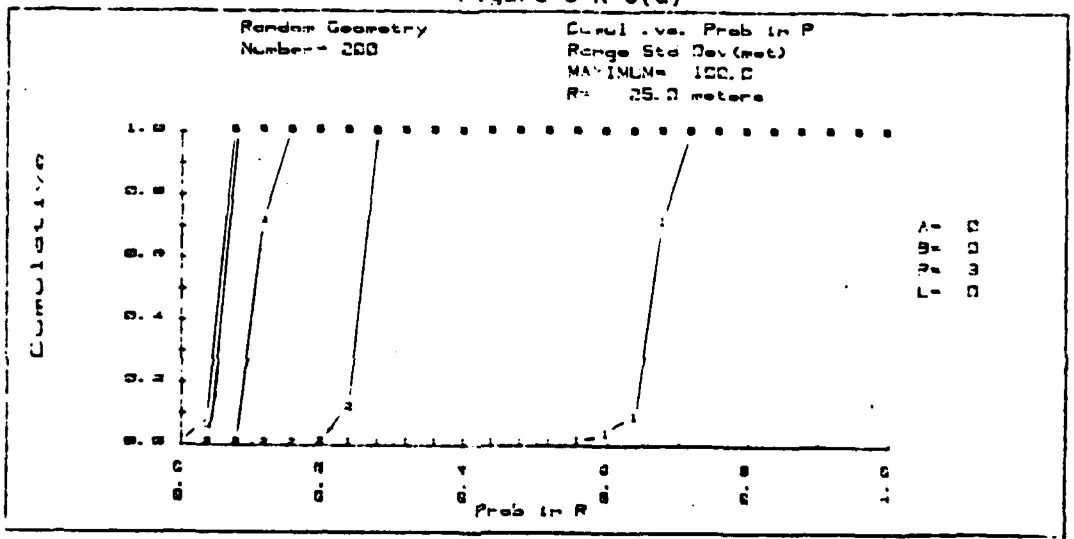

Figure 5-R-6(b)

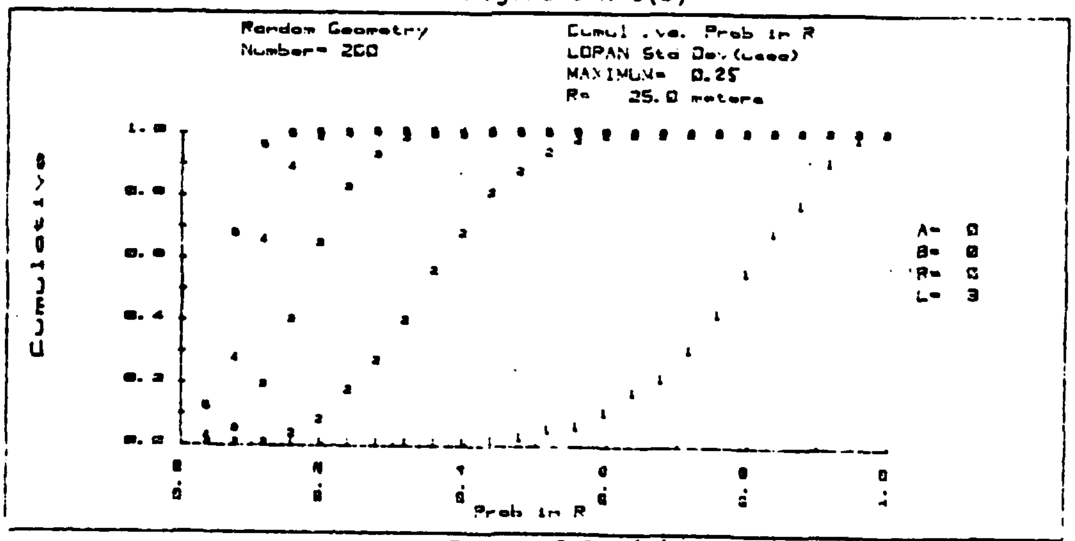

Figure 5-R-6(c)

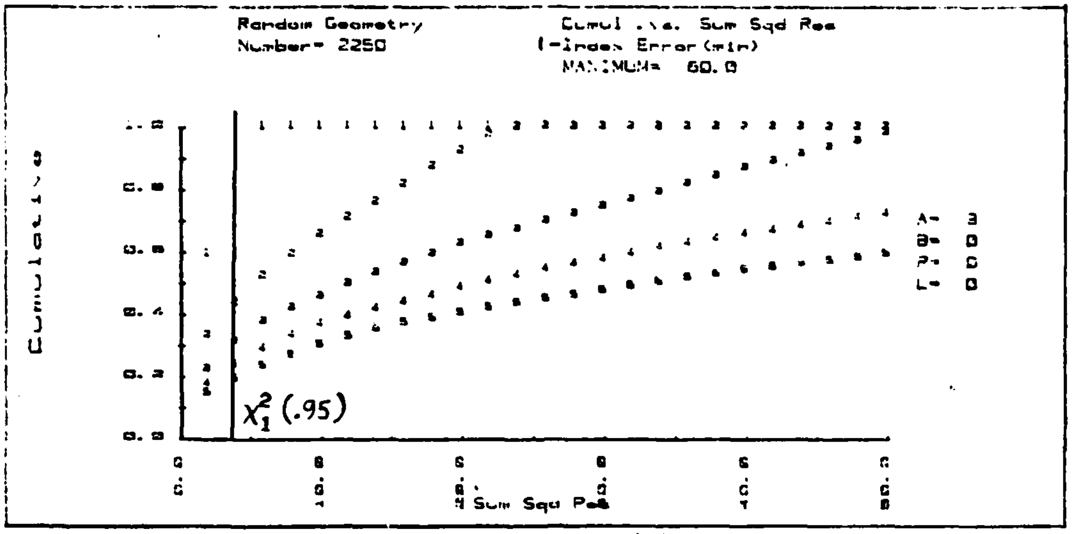

Figure 5-B-1(a)

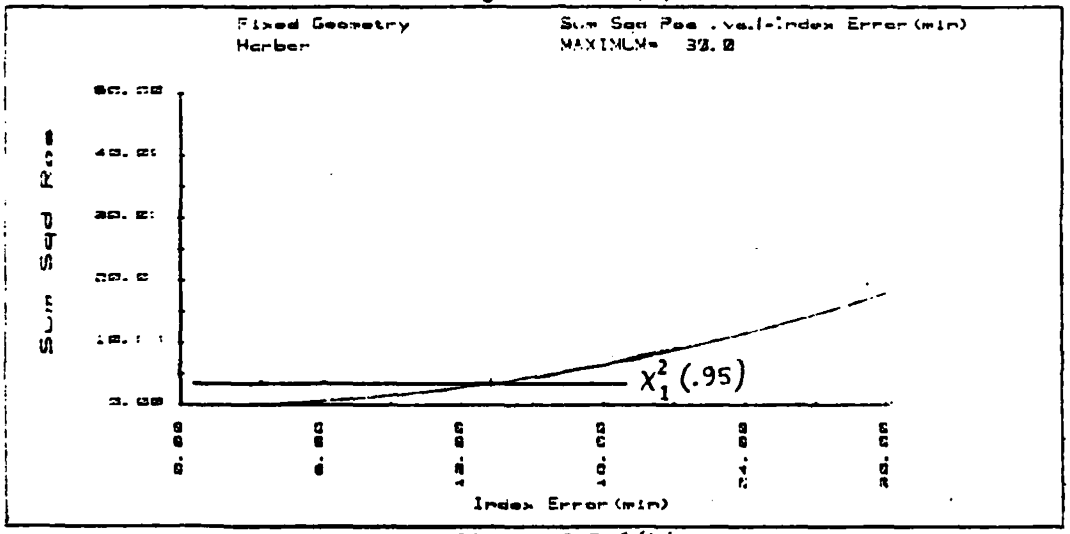

Figure 5-B-1(b)

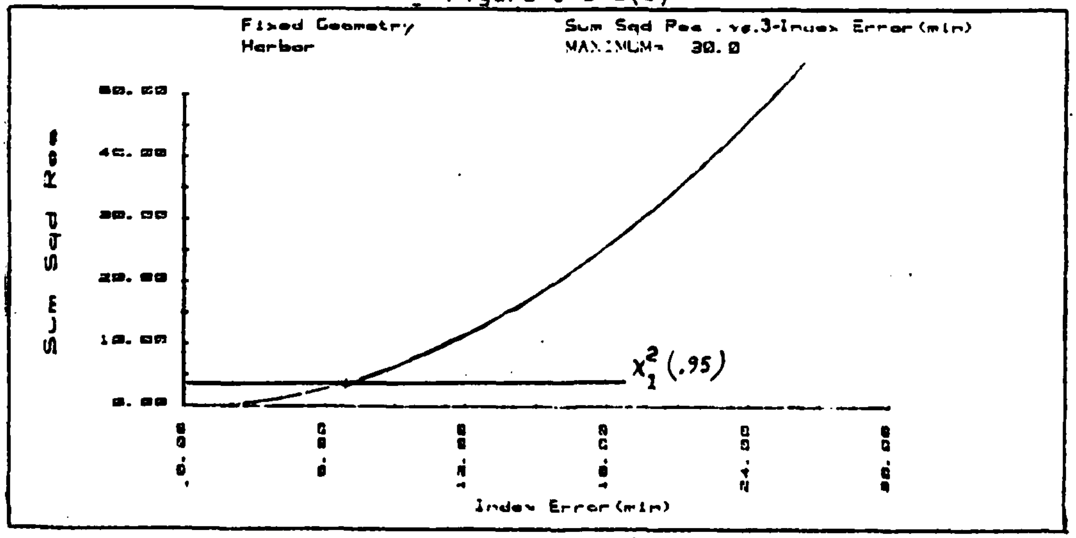

Figure 5-B-1(c)

6.0 CONCLUSIONS

The following list summarizes results obtained by work on the ESM. The conclusions are based on assumptions made throughout this report and, therefore, must not be considered without knowledge of the assumptions:

a. A computer routine has been written which considers random errors, systematic errors, and blunders in a mathematical simulation of the Coast Guard aid to navigation positioning process.

b. Error ranking by average effect on position error has been accomplished (section 5.3). The following error sources, as specified in table 2.1, have the most significant effect on position error:

1. Index Error
2. Landmark Misplacement Error
3. Sextant Observer Standard Deviation
4. Bearing, Range, and LORAN Standard Deviation

c. Calculation of the probability of sinker release within a target circle (P-in-R) is useful in error analysis of the positioning process.

d. Unless the precision of bearing, range, and LORAN measurements is significantly better than that used in this report, their use to improve confidence of sextant fixes is of little consequence. However, independent checks may be useful in identifying blunders.

e. Landmarks common to more than one measurement, on the average, can cause larger position error than landmarks used for only one measurement.

f. A high confidence outlier detection procedure seems tractable.

g. The visual definition of a landmark can contribute significantly to position error under certain circumstances.

h. Observer Coincidence and Inclined Angle errors seldom add significantly to position error.

i. Gyrocompass, radar, and LORAN measurements can be used to position with high confidence only if their measurement s.d.'s are significantly less than those simulated.

j. Landmark Misplacement errors must exceed geodetic survey standards to have a significant effect on position error.

7.0 RECOMMENDATIONS

The following list results from work done on the Positioning/Error Model.

a. The ESM should be considered for use in development of standards and requirements for aid positioning.

b. The ESM should be used to study specific areas of concern. Examples are requirements on sextant quality and survey quality.

c. Further investigation into the value of P-in-R as a planning tool should be performed. Numerical stability and time considerations should be examined thoroughly. The inverse of P-in-R, that is, the radius of a circle required to contain some desired probability mass (R-for-P), should be studied for possible usefulness.

d. The outlier detection scheme briefly introduced should be thoroughly examined for practical use in field procedures.

e. The position error magnitude calculated in this report should be compared to those errors associated with sinker drop and the watch circle of the buoy.

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APPENDIX A

PROGRAM DESCRIPTION AND FLOWCHART

Program Description

The main routine of the ESM is called POSER. This routine contains nearly all the input/output function of the model. POSER calls the subroutine FIXED to define a particular "fix geometry" and the subroutine RANDOM to set up for generation of the sample "Random geometries." The FIXED routine automatically calculates and stores the elements of a parameter matrix. The parameter matrix contains all the information necessary to iteratively study any particular fix geometry. POSER calls DEFAULT to establish the random error parameters of each line of position within the parameter matrix. In the Random Geometry method, POSER calls GENERATE each time a new fix geometry is needed for study. In either method POSER calls GRADIENT to calculate the gradient vector magnitudes and directions of each line of position and stores them within the parameter matrix. POSER then uses ERROR to branch to one of the error input subroutines which are numbered 1 through 18.

The error input subroutines complete the parameter matrix formulation and provide the errors that perturb the system.

POSER then constructs the A matrix of equation 2-1 by calling PARTIALS. The W matrix follows when POSER uses WEIGHTS. The systematic error perturbation is then entered when POSER uses BIAS. The system is now perturbed and ready for a least squares adjustment which is performed through the matrix transforms of equation 2-1 in REDUCE. The completed transformation is then analyzed and the error effect parameters are stored by ZOUTPUT. At this point two different paths are possible depending on what geometry method is being used. If a fixed geometry is being studied, POSER calls TABULATE to plot and print the selected error effect versus the present value of the modeled error. In the random geometry method TABULATE records the result of this particular fix geometry then calls OUTLIERS which searches the results for outliers and computes statistics of the error effect sample. If OUTLIERS finds a large error effect (more than two standard deviations different from the mean of previous values of this parameter), it prints the parameter matrix of that particular fix for future study. POSER then returns to GENERATE to continue the Monte Carlo process. Once all geometries have been studied and the error effects tabulated, POSER calls PLOTOUT and PRINTOUT to provide the distributions to the user. The user has a paper-tape printout of his input parameters, a plot of the results, and a tabulation of the results at iterated error values. In addition, the random geometry user has a list of outliers to study for indications of bad or good geometries.

The format used in describing each subroutine or subfunction in the Error Sensitivity Model includes:

a. A definition of each variable used in the equations of the subroutine (unless they have been defined previously in this appendix).

b. The equations used in the subroutine

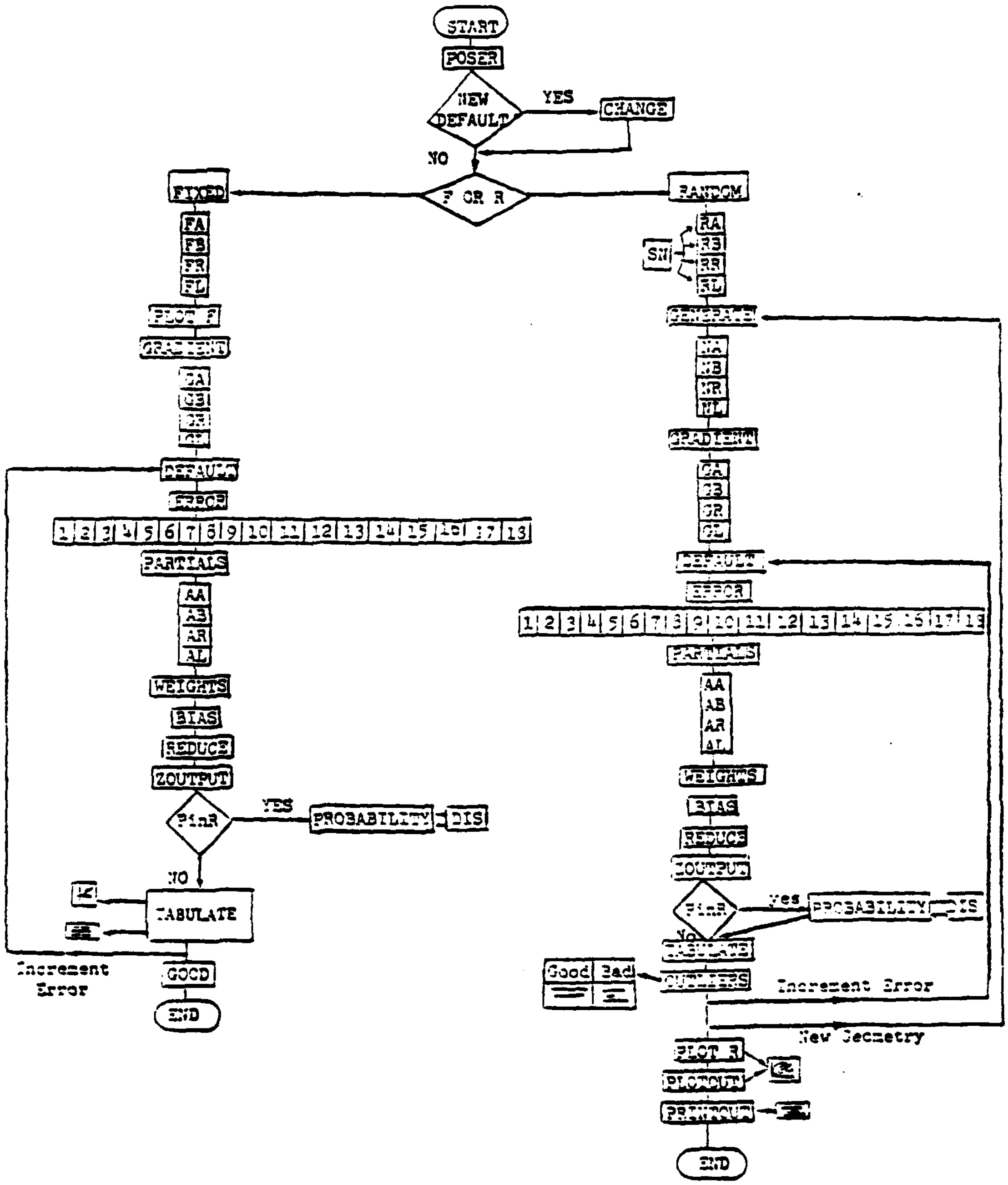

FIGURE A-1

ERROR SENSITIVITY FLOWCHART

c. A brief explanation of the purpose of the subroutine or subfunction, including any circumstances or conditions important in the development of the routine.

Figure A-1 is a flowchart of these routines as they are applied in the programmed sequence.

POSER

Definitions of Variables: N/A

Equations: N/A

Remarks: POSER is the main input/output (I/O) routine whose overall purpose is space allocation, data storage, and program execution.

CHANGE

Definitions of Variables: D is the s.d. storage array
d_i = s.d. of i^{th} measurement type

$$\text{if } i = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \end{cases} \text{ then } d_i = \begin{cases} \text{Sextant measurement s.d.} \\ \text{Gyrocompass measurement s.d.} \\ \text{Radar measurement s.d.} \\ \text{LORAN measurement s.d.} \end{cases}$$

Equations: N/A

Remarks: The ESM assumes measurement s.d. default values for the sextant, gyrocompass, radar, and LORAN receiver of: 5 minutes, 0.5 degrees, 30 meters, and 0.1 μsec, respectively. In the case where other default values are desired, the user instructs POSER to call CHANGE to alter the necessary d_i values.

ERROR

Definitions of Variables: rø = Number assigned to error to be studied

rø =
1 Index Error
2 Non-Common Landmark Misplacement
3 Common Landmark Misplacement
4 Range Error
5 Bearing Error
6 LORAN Error
7 Non-Common Inclined Angle
8 Common Inclined Angle
9 Observer Coincidence
10 =S/O Standard Deviation
11 ≠S/O Standard Deviation
12 Non-Common Landmark Definition
13 Common Landmark Definition
14 Bearing Standard Deviation
15 Range Standard Deviation
16 LORAN Standard Deviation
17 Vacant
18 Vacant

Equations: N/A

Remarks: ERROR transfers execution to the subroutine that corresponds to the error being studied, and returns.

WEIGHTS

Definitions of Variables: P is the (n x 10) parameter matrix
n is number of lines of position

p_{ij} = j^{th} parameter of i^{th} line of position; (i = 1 to n)

$$\text{If } \; p_{i1} = \left\{\begin{array}{l}1\\2\\3\\4\end{array}\right. \text{ then measurement type is } \left\{\begin{array}{l}\text{sextant}\\ \text{gyrocompass}\\ \text{radar}\\ \text{LORAN}\end{array}\right.$$

$$\text{If } \; p_{i1} = \left\{\begin{array}{l}1\\ \\2\\3\\4\end{array}\right. \text{ then } p_{i2} = \left\{\begin{array}{l}\text{x coordinate of left landmark of}\\ \text{horizontal angle}\\ \text{x coordinate of landmark used for bearing}\\ \text{x coordinate of landmark used for range}\\ \text{direction of LORAN gradient vector}\end{array}\right.$$

$$\text{If } \; p_{i1} = \left\{\begin{array}{l}1\\ \\2\\3\\4\end{array}\right. \text{ then } p_{i3} = \left\{\begin{array}{l}\text{y coordinate of left landmark of}\\ \text{horizontal angle}\\ \text{y coordinate of landmark used for bearing}\\ \text{y coordinate of landmark used for range}\\ \text{magnitude of LORAN gradient vector}\end{array}\right.$$

p_{i4} = x coordinate of right landmark of horizontal angle (zero for other measurement types)

p_{i5} = y coordinate of right landmark of horizontal angle (zero for other measurement types)

p_{i6} = measurement which determines the i^{th} line of position of the unperturbed system

p_{i7} = measurement s.d. of the i^{th} measurement

p_{i8} = systematic error for i^{th} measurement

p_{i9} = gradient magnitude of i^{th} line of position

p_{i10} = positive gradient direction (that direction normal to the line of position that will increase the measurement)

W is the (n x n) weighting matrix

w_{ii} = weighting factor for i^{th} line of position (all off diagonal elements are zero)

Equations:

$$\text{If } p_{i1} = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \end{cases} \quad \text{then } w_{ii} = \begin{cases} \left(\dfrac{10800}{p_{i7}\pi}\right)^2 \\ \left(\dfrac{180}{p_{i7}\pi}\right)^2 \\ \left(\dfrac{1}{p_{i7}}\right)^2 \\ \left(\dfrac{1}{p_{i7}}\right)^2 \end{cases} \quad \text{with } p_{i7} \text{ in} \begin{cases} \text{minutes} \\ \text{degrees} \\ \text{meters} \\ \mu\text{secs} \end{cases}$$

Remarks: The measurement s.d.s are used to form the weighting matrix.

BIAS

Definitions of Variables: L is the (n x 1) observation vector

l_i = systematic error perturbation of i^{th} line of position

Equations:

$$\text{if } p_{i1} = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \end{cases} \quad \text{then } l_i = \begin{cases} -\dfrac{p_{i8}\pi}{10800} \\ -\dfrac{p_{i8}\pi}{180} \\ -p_{i8} \\ -p_{i8} \end{cases}$$

Remarks: The systematic error is transferred from the parameter matrix to the observation matrix with units conversion.

REDUCE (reference 3)

Definitions of Variables: A is the (n x 2) matrix of partial derivatives

$$a_{i1} = \frac{\partial m_i}{\partial x} \qquad a_{i2} = \frac{\partial m_i}{\partial y}$$

m_i = variable which represents the measurement which determines the i_{th} line of position

V is the (2x2) matrix of eigenvectors

t = angle between x-axis and major semi-axis of confidence ellipse

$$V = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}$$

Equations:

$$X = -(A^TWA)^{-1} A^TWL = \begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix} \text{ from AP to MPP}$$

$N = (A^TWA)^{-1} = (2x2)$ covariance matrix of MPP (not normally diagonal in the North oriented reference system)

$R = AX + L = (nx1)$ residual matrix

$Q = V^TNV = \begin{bmatrix}\sigma_{x'}^2 & 0 \\ 0 & \sigma_{y'}^2\end{bmatrix}$ rotated covariance matrix of MPP (diagonal)

Remarks: The matrix operations in this subroutine form the basis of the Error Sensitivity Model. The diagonal elements of the Q matrix are the two variances of the bivariate probability distribution which describes the position uncertainty. The X matrix contains the x and y displacement of the MPP from the unperturbed AP. All output parameters are computed from the elements of X, Q, and the residual matrix, R.

ZOUTPUT

Definitions of Variables: Z is the (1 x 7) row vector of the error effect parameters.

Equations:

$$z_1 = \text{AP-to-MPP} = \sqrt{\Delta x^2 + \Delta y^2}$$
$$z_2 = \text{2-drms} = 2\sqrt{\sigma_{x'}^2 + \sigma_{y'}^2}$$
$$z_3 = \text{Maj*Min (90\%)} = (2.15)^2\, \sigma_{x'}\, \sigma_{y'}$$
$$z_4 = \text{Maj (90\%)} = 2.15\sqrt{\max(\sigma_{x'}^2, \sigma_{y'}^2)}$$
$$z_5 = \text{P-in-R} = \text{(see appendix C)}$$
$$z_6 = \text{Sum Sqd Res} = \sum_{i=1}^{n} r_i^2 w_{ii}$$
$$z_7 = \text{Min/Maj} = \sqrt{\min(\sigma_{x'}^2, \sigma_{y'}^2)/\max(\sigma_{x'}^2, \sigma_{y'}^2)}$$

Remarks: The output parameters are transferred from Q, X, R to the error effect parameter matrix.

TABULATE

Definitions of Variables: F is the (6 x 26) error effect parameter tabulation matrix

p is a number 1-7 (Z_1 - Z_7, respectively) which indicates the error effect parameter to be tabulated for plotting and printout

f_{ij} is the jth error effect increment bin for the ith modeled error value

Remarks: In the fixed geometry method, TABULATE plots and prints the calculated error effect parameter versus the modeled error for each iteration. In the random geometry method, the value of the error effect parameter is assessed and the count incremented in the appropriate bin for later frequency distribution calculations.

PLOTOUT

Definitions of Variables: N/A

Equations: N/A

Remarks: PLOTOUT calculates the frequency distributions and offers the user the option of either a frequency distribution or cumulative frequency distribution output.

PRINTOUT

Definitions of Variables:

O = 6x7x4 error effect parameter statistics array

o_{ij3} = mean of j^{th} error effect parameter for the i^{th} modeled error value

o_{ij4} = standard deviation of the j^{th} error effect parameter for the i^{th} modeled error value

o_{ij1} and o_{ij2} are the sum and sum-squared values used to caculate the means and s.d.'s

Equations: N/A

Remarks: PRINTOUT lists the frequency distributions of each output parameter and the array of error effect parameter statistics.

SN (reference 13)

Definitions of Variables: rnd(1) is a uniformly generated random number between 0 and 1.

Equations: $SN = \sqrt{-2 \ln(\text{rnd}(1))} \cos(2\pi \, \text{rnd}(1))$

Remarks: SN is a subfunction which transforms a uniformly distributed random number into a normally distributed random number with a mean of 0 and a standard deviation of 1. It is used in the random geometry method for generating normally distributed angles, ranges, and LORAN gradients.

DIS

Definitions of Variables:

$\sigma_{x'}$ = standard deviation in x' direction

$\sigma_{y'}$ = standard deviation in y' direction

Equations:

$$DIS = \frac{1}{2\pi\,\sigma_{x'}\sigma_{y'}}\left[e^{-\frac{1}{2}\left(\frac{x'^2}{\sigma_{x'}^2}+\frac{y'^2}{\sigma_{y'}^2}\right)}\right]$$

Remarks: This subfunction defines the bivariate normal distribution which is integrated in two dimensions over the target circle (appendix C) to calculate P-in-R.

DEFAULT

Definitions of Variables: N/A

Equations: If $p_{i1} = j$ then $d_j = p_{i7}$

Remarks: DEFAULT transfers the default measurement standard deviations from the storage array (D) to the active parameter matrix (P) before each iterative increase in the modeled error.

GENERATE

Definitions of Variables: M is the (n x 3 x 2) array of geometry sample set statistics.

$i = i^{th}$ line of position

$$\text{if } j = \begin{cases}1\\2\\3\end{cases} \text{ then } \quad m_{ij}1 = \begin{cases}\text{mean of ranges to left landmarks}\\ \text{mean of ranges to right landmarks}\\ \text{mean of angles between landmarks}\end{cases} \quad m_{ij}2 = \begin{cases}\text{s.d. of ranges to left landmarks}\\ \text{s.d. of ranges to right landmarks}\\ \text{s.d. of angles between landmarks}\end{cases}$$

Equations: N/A

Remarks: GENERATE calls subroutines NA, NB, NR, NC as necessary to create a new geometry for the next iteration of the Monte Carlo method.

NA

Definitions of Variables:

SN is the normally distributed random number with mean = 0, s.d. = 1.

Equations:

$r_{iL} = m_{i11} + m_{i12}$ SN = range to left landmark of i^{th} measurement

$r_{iR} = m_{i21} + m_{i22}$ SN = range to right landmark of i^{th} measurement

$a_i = m_{i31} + m_{i32}$ SN = i^{th} angular measurement

Remarks: NA generates the ranges to and angle between two landmarks for a sextant measurement. Additional logic and equations in NA establish parameter matrix elements for the landmarks generated. See appendix E for the sample statistics used in the ESM.

NB

Definitions of Variables: b_i is the bearing measurement for i^{th} line of position.

Equations:

$$b_{i+1}-b_i = 60\text{rnd}(1) + 30 = \mathbf{U}\left[30,90\right]$$

Remarks: This subroutine generates gyrocompass measurements with crossing angles uniformly distributed between 30 and 90 degrees. The ranges to the landmarks being simulated are generated as in NA.

NR

Definitions of Variables: r_i is the range measurement for the i^{th} line of position.

Equations:

$r_i = m_{i11} + m_{i12}$ SN = range measurement for the i^{th} line of position.

$$b_{i+1}-b_i = 60\text{rnd}(1) + 30 = \mathbf{U}\left[30,90\right]$$

Remarks: The ranges are generated as in NA and the crossing angles the same as in NB.

NL

Definitions of Variables: m_{i11} = mean LORAN gradient in meters/μsec
m_{i12} = s.d. of LORAN gradients in meters/μsec

Equations:

$p_{i3} = m_{i11} + m_{i12}$ SN = gradient magnitude of i^{th} LORAN line of position

Remarks: The gradient magnitudes are generated from a LORAN gradient distribution simulation (appendix E). The crossing angles of LORAN line of positions are generated uniformly between 30 and 90 degrees as within NB and NR.

GRADIENT

Definitions of Variables: N/A

Equations: N/A

Remarks: GRADIENT is a branching subroutine which calls GA, GB, GR, or GL to calculate the p_{i9} and p_{i10} parameter matrix elements which are the gradient magnitude and positive gradient direction, respectively.

GA

Definitions of Variables: r6 and r7 are dummy test variables
atn = arc tangent

Equations:

$$p_{i9} = \left[\frac{(p_{i2}^2 + p_{i3}^2)(p_{i4}^2 + p_{i5}^2)}{(p_{i2}-p_{i4})^2 + (p_{i3}-p_{i5})^2}\right]^{\frac{1}{2}} 0.00029 = \text{gradient of ith line of position}$$

$$\frac{p_{i3}(p_{i4}^2+p_{i5}^2) - p_{i5}(p_{i2}^2+p_{i3}^2)}{2(p_{i4}p_{i3} - p_{i2}p_{i5})} = r6$$

$$\frac{p_{i2}(p_{i4}^2+p_{i5}^2) - p_{i4}(p_{i2}^2+p_{i3}^2)}{2(p_{i2}p_{i5} - p_{i4}p_{i3})} = r7$$

$$\text{if } r6 \begin{cases} =0 \text{ and } r7 > 0 \\ =0 \text{ and } r7 < 0 \\ > 0 \\ < 0 \end{cases} \text{ then } p_{i10} = \begin{cases} 0 \\ 180 \\ 90-\text{atn}(r7/r6) \\ 270-\text{atn}(r7/r6) \end{cases}$$

Remarks: GA calculates the gradient, p_{i9}, and the positive gradient direction, p_{i10}, for each line of position determined by resection and places them in the active parameter matrix for later use.

GB

Definitions of Variables: N/A

Equations:

$$(p_{i2}^2 + p_{i3}^2)^{\frac{1}{2}} \quad 0.017453 = p_{i9}$$

$$p_{i10} = p_{i6} - 90 \text{ (but less than 360 degrees)}$$

Remarks: GB calculates the gradient and positive gradient direction for all bearing measurements.

GR

Definitions of Variables: N/A

Equations:

$$p_{i9} = 1$$

$$\text{if } p_{i2} = \begin{cases} 0 \text{ and } p_{i3} > 0 \\ 0 \text{ and } p_{i3} < 0 \\ <0 \\ >0 \end{cases} \text{ then } p_{i10} = \begin{cases} 180 \\ 0 \\ 90 - \text{atn}(p_{i3}/p_{i2}) \\ 270 - \text{atn}(p_{i3}/p_{i2}) \end{cases}$$

Remarks: GR calculates the gradient and positive gradient direction for all range measurements.

GL

Definitions of Variables: N/A

Equations: N/A

Remarks: GL transfers the input gradient and positive gradient direction of LORAN measurements from p_{i2} and p_{i3} to p_{i9} and p_{i10}, respectively.

PARTIALS

Definitions of Variables: N/A

Equations: N/A

Remarks: PARTIALS uses the equati ns derived in appendix G of reference 3 to establish all elements of the A matrix. It calls an AA, AB, AR, and AL for angles, bearings, ranges, and LORAN measurements, respectively.

OUTLIERS

Definitions of Variables: N/A

Equations: N/A

Remarks: OUTLIERS calculates the mean and s.d. of the random geometry error effect parameters for each increment in the modeled error. If (after 30 geometries have been sampled) any error effect parameter exceeds two standard deviations in either direction from the mean, the BAD or the GOOD subroutines are called to print the respective parameter matrix for additional examination.

PROBABILITY

Definitions of Variables: N/A

Equations: N/A

Remarks: PROBABILITY is discussed in appendix C. PROBABILITY calculates the P-in-R error effect parameter.

1 (Index Error)

Definitions of Variables: d = number of measurements in error
q = counter from 1 to d
m = maximum modeled error

$$\text{if} \begin{cases} \text{Fixed geometry method} \\ \\ \text{Random geometry method} \end{cases} \quad \text{then } e = \begin{cases} \text{25 equally spaced modeled error values between zero and m} \\ \\ \text{5 equally spaced modeled error values between zero and m} \end{cases}$$

Equations:

$$\text{if } q \leq d; \text{ then } p_{i8} = e$$

Remarks: Angular measurement error are iteratively entered into the parameter matrix.

2 (Non-Common Landmark Misplacement)

Definitions of Variables: N/A

Equations:

$$p_{i8} = \frac{e\ 10800}{\pi\left(p_{i2}^{2} + p_{i3}^{2}\right)^{\frac{1}{2}}}$$

Remarks: See appendix D.

3 (Common Landmark Misplacement)

Definitions of Variables: O is number of landmarks used for more than one measurement (see appendix B)

Equations:

$$\text{if } i = \begin{cases} 1 \\ \\ o+1 \end{cases} \quad \text{then } p_{i8} = \begin{cases} \dfrac{-e\ 10800}{\pi (p_{i4}^2 + p_{i5}^2)^{\frac{1}{2}}} \\ \dfrac{e\ 10800}{\pi (p_{i2}^2 + p_{i3}^2)^{\frac{1}{2}}} \end{cases}$$

Remarks: See appendix D.

4 (Range Error)
5 (Bearing Error)
6 (LORAN Error)

Definitions of Variables: d = number of measurements in error
q = counter 1 to d

Equations:

if $q \leq d$; then $p_{i8} = e$

Remarks: Errors are entered directly into the parameter matrix.

7 (Non-Common Inclined Angle Error)

Definitions of Variables: N/A

Equations:

$$p_{18} = 60 \left[\cos^{-1}\left\{ \cos(p_{16}) \cos\left(\frac{e180}{\pi (p_{12}^2 + p_{13}^2)^{1/2}} \right) \right\} - p_{16} \right]$$

Remarks: The left landmark of the first sextant measurement is elevated (e) meters and the resulting error in the angular measurement is (p_{18}) in minutes.

8 (Common Inclined Angle Error)

Definitions of Variables: N/A

Equations: Same as 7.

Remarks: The error is the same as for Non-Common Inclined Angle Error except that it affects two angles instead of one.

9 (Observer Coincidence)

Definitions of Variables: p_{19} = gradient of first line of position

Equations:

$$p_{18} = e/p_{19}$$

Remarks: This error simulates the displacement of one observer in the direction of maximum change in angular measurement (the positive gradient direction). Studying error effect by using maximums is discussed in appendix D.

10 (S/O Standard Deviation)

Definitions of Variables: d = number of measurements in error
q = counter 1 to d

Equations:

$$\text{if } q \begin{cases} \leq d \\ > d \end{cases} \text{ then } p_{i7} = \begin{cases} e \\ \text{default value of S/O Standard Deviation} \end{cases}$$

Remarks: Sextant measurement s.d.'s are transferred to the parameter matrix.

11 ($\neq$ S/O Standard Deviation)

Definitions of Variables: i = 1 to n
n = number of measurements

Equations:

$$\text{if } i = \begin{cases} \text{even number} \\ \text{odd number} \end{cases} \text{ then } p_{i7} = \begin{cases} \text{modeled error value, } e \\ \text{default S/O Standard Deviation value} \end{cases}$$

Remarks: Standard deviation of alternate angular measurements are assigned a value of e and the remaining measurements the default s.d. value. This logic can be used to study the effect of assuming equal s.d.'s for all measurements when in fact they may not be equal.

12 (Non-Common Landmark Definition)

Definitions of Variables: d_1 = default s.d. of first measurement

Equations:

$$p_{17} = \left[d_1^2 + \left(\frac{e\ 10800}{\pi (p_{12}^2 + p_{13}^2)^{\frac{1}{2}}} \right)^2 \right]^{\frac{1}{2}} = \text{s.d. of the perturbed measurement in minutes}$$

Remarks: An uncertainty in a landmark's position adds uncertainty to the measurements made using the landmark. The s.d. of the position of the landmark along a line perpendicular to the line of sight from the observer (which represents the uncertainty) is weighted by the distance from the observer to the landmark. The resulting s.d. is provided by the equation above. (See appendix D.)

13 (Common Landmark Definition)

Definitions of Variables: N/A

Equations: Same as 12.

Remarks: This random error adds to measurement s.d. in the same way as Non-Common Landmark uncertainty except that more than one angle is effected. If a common landmark has an uncertainty in its position perpendicular to the observer's line of sight, then all measurements using that landmark are effected. If both landmarks used in a horizontal angle measurement are poorly defined, then both contribute to uncertainty in the measurement.

14 (Bearing Standard Deviation)
15 (Range Standard Deviation)
16 (LORAN Standard Deviation)

Definitions of Variables: q = counter from 1 to d
d = number of lines affected

Equations:

if $q \leq d$; then $p_{i7} = e$

Remarks: The measurement s.d.'s are transferred into the parameter matrix, replacing the default s.d. values iteratively through the domain of the modeled error.

17 (Vacant)
18 (Vacant)

GOOD

Definitions of Variables: N/A

Equations: N/A

Remarks: The subroutine OUTLIERS calls GOOD if an error effect parameter calculated for one of the random fix geometries is two s.d.'s less than the mean of all previously calculated error effect parameters of that type. GOOD prints parameter matrix and error effect parameter values for further study.

BAD

Definitions of Variables: N/A

Equations: N/A

Remarks: BAD performs the same function as GOOD except that the error effect parameter must be more than two s.d.'s greater than the mean of all previously calculated error effect parameters of that type.

FIXED
FA
FB
FR
FL

Definitions of Variables: N/A

Equations: N/A

Remarks: FIXED is called by POSER to accept user definition of a fix geometry. It accepts ranges and true bearings to all landmarks which are part of the unperturbed system. It also accepts LORAN measurement information. FIXED calls FA, FB, FR, and FL as necessary to define the fix.

RANDOM
RA
RB
RR
RL

Definitions of Variables: G = number of random geometries to be generated

Equations: N/A

Remarks: RANDOM is called by POSER to accept user input of the statistics for generation of random geometries. The statistics used in the ESM are derived from a sample set of fix geometries from the USCGC REDWOOD (WLM 685). Any other sample statistics require changes to RANDOM. (See appendix E.) RA, RB, RR, and RL are called as necessary to establish statistics for generation of angular, bearing, range, and LORAN measurements, respectively.

PLOTR
PLOTF

Definitions of Variables:

Z$ = a string array of output parameter names
E$ = a string array of error names
k_p = maximum scale value of the p^{th} error effect parameter

Equations: N/A

Remarks: PLOTR and PLOTF create the axis on which the error effect parameters are plotted as a function of the modeled error.

APPENDIX B

ERROR SENSITIVITY MODEL OPERATING INSTRUCTIONS

Prospective "operators" of the ESM should first become familiar with the modeled errors and the error effect parameters before attempting to operate the routine. Because no data entry error feedback mechanism was built into the routine, the instructions should be strictly followed.

Listed below is the equipment needed to operate the routine. Familiarity with the operation of the HP-9825A is assumed throughout this appendix.

a. Hewlett-Packard desk-top calculator HP-9825A (Option 002, 23K memory)
b. Read Only Memory (ROM) String Advanced Programming HP-92210A
c. ROM HP-9872A Plotter General I/O-Extended I/O HP-98216A
d. ROM Matrix HP-98211A
e. Plotter/Printer HP-7245A or HP-9872A
f. Printer HP-9866B
g. Interface HP-IB98034A

INSTRUCTIONS	DISPLAY	KEYSTROKE
1. Insert tape cartridge		
2. Turn on calculator		RUN
3. The calculator displays	#A,#B,#R,#L	
a. ENTER #A (The number of sextant measurements in the fix geometry)		CONTINUE
b. The calculator displays	B?	
c. ENTER #B (The number of gyrocompass measurements in the fix geometry)		CONTINUE
d. The calculator displays	R?	
e. ENTER #R (The number of radar measurements in the fix geometry)		CONTINUE
f. The calculator displays	L?	
g. ENTER #L (The number of LORAN measurements in the fix geometry)		CONTINUE

INSTRUCTIONS	DISPLAY	KEYSTROKE
4. If the number of sextant measurements is greater than one, the calculator displays	# COMMON LM's	
a. ENTER # COMMON LM's (The number of landmarks used for more than one sextant measurement - the model is limited in this simulation, the first landmark defined is always a non-common landmark, then all common landmarks are defined, then the final landmark is always a non-common landmark)		CONTINUE
5. The calculator displays	ANY NEW DEFAULT STANDARD DEVIATIONS?	
a. ENTER y or n (n denotes the default measurement s.d.'s are adequate. y signifies that the default measurement s.d.'s are inadequate and new ones are desired.) The calculator displays	DEF. VALS.?	
(1) Enter desired sextant default value (minutes)		CONTINUE
(2) The calculator displays	D(2)	
(3) Enter desired gyrocompass default value (degrees)		CONTINUE
(4) The calculator displays	D(3)	
(5) Enter desired radar default value (meters)		CONTINUE
(6) The calculator displays	D(4)	
(7) Enter desired LORAN default value (μsec)		CONTINUE
6. The calculator displays	OUTPUT PARAMETER NUMBER?	
a. ENTER one of the following numbers for error effect output parameter: 1 - AP-to-MPP 2 - 2-drms 3 - Maj*Min (90%) 4 - Maj (90%) 5 - P-in-R 6 - Sum Sqd Res 7 - Min/Maj		CONTINUE

INSTRUCTIONS	DISPLAY	KEYSTROKE
7. If output parameter number 5 (P-in-R) was selected, the calculator will display	TARGET CIRCLE RADIUS?	
a. ENTER the radius of the target circle (meters)		CONTINUE
8. The calculator will display	NEW OUTPUT SCALE NEEDED?	
a. ENTER y or n (n means that the dimensions of plotting grid are sufficient to display the range of the output parameter selected, y means that the dimensions are not sufficient.) The calculator will display	K(P)?	
b. ENTER the desired range of the output parameter. (Trial and error may be needed on this step if the user finds the graphical output inadequate.) The default ranges for the error effect parameters are: 1 - AP-to-MPP = 100 meters 2 - 2-drms = 100 meters 3 - Maj*Min (90%) = 300 meters squared 4 - Maj (90%) = 100 meters 5 - P-in-R = 1.0 6 - Sum Sqd Res = 25 7 - Min/Maj = 1.0		CONTINUE
9. The calculator will display	ERROR STUDIED?	
a. ENTER the number of the modeled error to be studied: 1 - Index Error 2 - NC LM Misplacement 3 - C LM Misplacement 4 - Range Error 5 - Bearing Error 6 - LORAN Error 7 - NC Inclined Angle 8 - C Inclined Angle 9 - Observer Coincidence 10 - = S/O Standard Deviation 11 - # S/O Standard Deviation 12 - NC LM Definition 13 - C LM Definition 14 - Bearing s.d. 15 - Range s.d. 16 - LORAN s.d. 17 - Not used 18 - Not used		CONTINUE

INSTRUCTIONS	DISPLAY	KEYSTROKE
10. The calculator will display	# TIMES IT OCCURRED?	
a. ENTER the number of measurements to which this error is being subjected to. (Consecutive, starting with first measurement to which the error can be subjected)		CONTINUE
11. The calculator will display	MAXIMUM OF ERROR STUDIED?	
a. ENTER the maximum of the error studied (respective units)		CONTINUE
12. The calculator will display	FIXED GEOMETRY STUDIED?	
a. ENTER y or n (y means that the fixed geometry method of error analysis is desired. n means that the random geometry method of error analysis is desired.) If n is pressed, the calculator will display	# AND SEED?	
b. ENTER # and SEED (# represents the number of sample geometries desired for study of the error effect and SEED is a nine-digit fraction between zero and one to start the random number generator)		CONTINUE
c. If y is pressed, the calculator will display	GEOMETRY NAME?	CONTINUE
d. Enter the desired geometry name (This will label output graphs)		CONTINUE
e. The calculator will display	R&B TO LEFT OBJECT	CONTINUE
f. ENTER the range then the true bearing to the left landmark of the first sextant measurement simulation	r3?	CONTINUE
g. The calculator will display	R&B TO RIGHT OBJECT?	CONTINUE
h. ENTER the range then the true bearing to the right landmark of the first sextant measurement simulation	r3?	CONTINUE
i. Repeat f, g, and h for all sextant measurements		CONTINUE

INSTRUCTIONS	DISPLAY	KEYSTROKE
j. The calculator will display	R&B TO OBJECT (B)	CONTINUE
k. ENTER the range then the true bearing to landmark used in gyrocompass measurement simulation	r3?	CONTINUE
l. Repeat k for all gyrocompass measurement simulations		CONTINUE
m. The calculator will display	R&B TO OBJECT (R)	CONTINUE
n. ENTER range then the true and bearing to landmark used in radar measurement simulation	r3?	CONTINUE
o. Repeat n for all radar measurement simulations		CONTINUE
p. The calculator will display	GRAD DIR AND MAGNITUDE? (L)	CONTINUE
q. ENTER the LORAN gradient then the magnitude	P (I,3)?	CONTINUE
r. Repeat q for all LORAN measurement simulations		CONTINUE
13. The calculator will display	PAPER IN PLOTTER?	
a. Press CONTINUE when the plotter is set to plot		CONTINUE
14. The calculator will plot the desired grid and error effect parameter in the fixed geometry case		
15. The calculator will begin the time-consuming study of a random sample of geometries in the random geometry method. Following completion, the calculator will display	CUMULATIVE?	
a. ENTER y if a cumulative frequency plot are required		
b. ENTER n if frequency distribution is required		CONTINUE

INSTRUCTIONS	DISPLAY	KEYSTROKE
16. The calculator will display	PAPER IN PLOTTER?	
a. Press CONTINUE when the plotter is set to plot		CONTINUE
17. The calculator will print out all plotted values and label the rows and columns with appropriate headings for future reference		
18. The calculator prints	END OF RUN	

APPENDIX C

NUMERICAL INTEGRATION BY GAUSSIAN QUADRATURE
(Reference 14)

Frequent use of the P-in-R parameter in the ESM dictates the necessity for a mathematical description of the method. A brief discussion of some mathematical aspects of the numerical method is made in this appendix leaving the more complicated explanations to reference 14.

Many numerical methods for integration of known functions call for equally spaced points within the region of integration at which the integrand is evaluated. Gaussian Quadrature employs points of unequal spacing within the region and is the preferred scheme for numerical integration for this application. The selected points are zeros of orthogonal polynomials. The degree of the polynomials is determined by the desired accuracy of the integration.

The concept is to evaluate an integral by selection of the formula:

$$\int_b^a y(x)\, dx \approx \sum_{i=1}^{n} A_i y(x_i) \qquad \text{(C-1)}$$

Where: x_i = unequally spaced zeros of orthogonal (Legendre Polynomials) polynomials.
A_i = weighting values determined by orthogonal polynomials.

The details of calculating the x_i and A_i values are found in reference 14. For ESM application, the sixteen-point Gaussian Quadrature is adequate. The sixteen-point method requires evaluation of the integrand at 256 points within a two-dimensional region of integration. When multiplied by the model error value and geometry iterations, a huge number of calculations are required consuming much computer time. However, by comparison, the Gaussian Quadrature scheme, for equivalent accuracy, is at least twice as fast as any other scheme.

Errors occur when the integrand changes rapidly over the region of integration. In the ESM this error appeared when the s.d.'s, which define the bivariate probability density function, were less than 20% of the radius of the circular region of integration. If the P-in-R parameter proves to be a candidate as a measure of success in aid positioning, more study will be needed to define the tradeoff between speed and accuracy.

The calculation of P-in-R within the ESM is performed by the subroutine PROBABILITY. PROBABILITY evaluates the integral,

$$\int_R\int DIS(x,y)\, dx\, dy \approx \sum_{i=1}^{16} \sum_{j=1}^{16} A_i A_j\, DIS(x_i, y_j) \qquad \text{(C-2)}$$

Where: The weighting values A_i are the same as described above and the (x_i, y_i) points of integration are located within the region of interest (reference 14). R is the radius of the circular region of integration.

The calculations were verified by comparison with table Q-6-c in appendix Q of Bowditch (see reference 2).

The numerically evaluated integrals were within 0.1% of being equal to the tabulated values.

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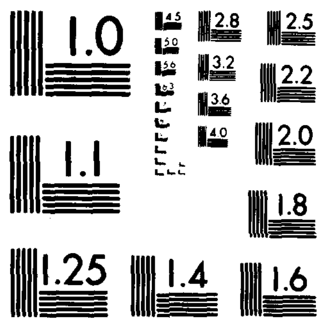

MICROCOPY RESOLUTION TEST CHART

APPENDIX D

CONSEQUENCES OF STUDYING ERRORS ONLY IN THE DIRECTION OF MAXIMUM EFFECT

Within the ESM, the following systematic modeled errors could have been studied as displacement vectors:

a. Non-common landmark misplacement

b. Common landmark misplacement

c. Observer coincidence

Furthermore, landmark definition could have been studied using bivariate probability distributions.

However, studying each of the above modeled errors as vectors or bivariate probability distributions would add another dimension as well as mathematical complications to the error modeling task. The discussion in this appendix verifies the legitimacy of studying the component of the displacement vector in the direction of maximum affect and verifies the use of a univariate probability distribution in studying Landmark Definition.

The position P is determined by resection with L as one of the reference landmarks. The effect landmark displacement has on the position P is determined by the effect it has on the lines of position which were determined using L as a landmark. The effect displacement has on the lines of position is determined through use of the gradient equation which is (reference 3, appendix B):

$$D = G\, d\theta \qquad \text{(D-1)}$$

Where: D = Distance LOP is displaced
G = Gradient of the line of position
$d\theta$ = Small angular change in measurement

For all practical situations the gradient remains constant in the region of interest. That is, when the displacement magnitude is small compared to distances between landmarks and P. The angular measurement change due to landmark displacement is therefore the only important quantity.

Figure D-1 depicts a representative landmark displacement error vector showing its effect on an angular measurement. The symbology in the figure is as follows:

E = Magnitude of displacement vector
P = Position of observer
R = Range from observer to landmark
L = Landmarks position without error
L_1 = Landmark position with projected displacement
L_2 = Landmark position with vector displacement
E_p = Magnitude of projected displacement vector
α = Angle of displacement vector with line perpendicular to line of sight $(\overline{PL})$
$d\theta_1$ = Angular error due to projected landmark displacement
$d\theta_2$ = Angular error due to displacement vector

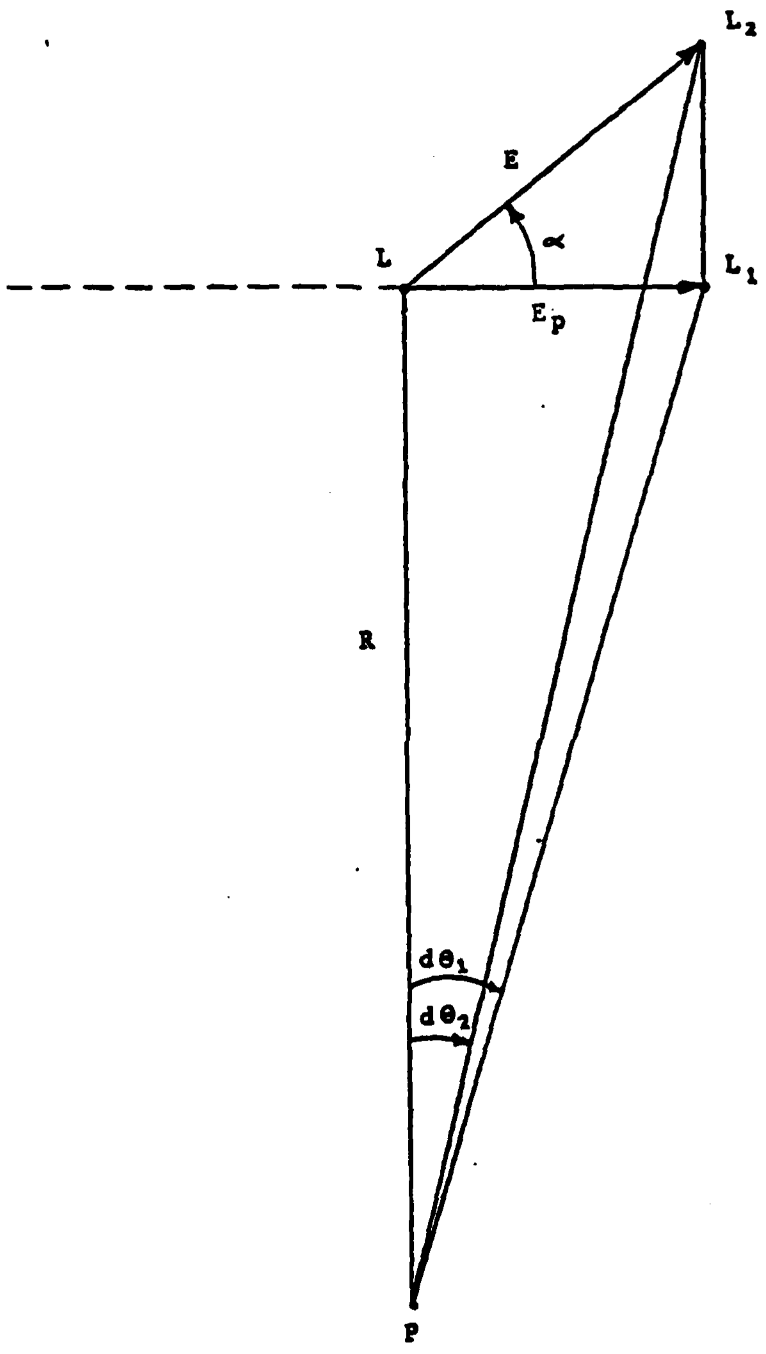

FIGURE D-1

DISPLACEMENT VECTOR DIAGRAM

It can be seen from figure D-1, that studying displacement perpendicular to the line of sight $(\overline{PL})$ is a close approximation to studying the displacement vector when considering the angular change $d\Theta$. A condition for this approximation is that E be much smaller than R, which is the case for most displacement of landmarks. The need for this condition is shown as follows:

when $E << R$

$$\sin d\Theta_1 = \frac{E\cos\alpha}{(E^2\cos^2\alpha + R^2)^{1/2}} \approx \frac{E\cos\alpha}{R} \qquad \text{(D-2)}$$

By the law of sines

$$\sin d\Theta_2 = \frac{E\sin(180-(90+\alpha+d\Theta_2))}{R} \approx \frac{E\cos\alpha}{R} \qquad \text{(D-3)}$$

Therefore, $\sin d\Theta_1 \approx \sin d\,\Theta_2$, and $d\Theta_1 \approx d\Theta_2$ and it is legitimate to study the projection of the displacement vector upon the line perpendicular to the line of sight. It is now possible to extend the results of studying the projected displacement error to results concerning the displacement error vectors.

The direction of the displacement vector is uniformly distributed in all directions from the landmarks true location. The distribution which represents the magnitude, E, is unknown and unimportant for only the effect of E on P is modeled in the ESM. For all values of E, α is uniformly distributed. For the first quadrant the probability density function for α is:

$$f(\alpha) = \frac{2}{\pi}\,\mathbf{U}(0, \frac{\pi}{2}) \text{ (Similiarly for all other quadrants)} \qquad \text{(D-4)}$$

The projected value of E is $E_p = E\cos\alpha$

Averaging both sides of the equation over the respective intervals will yield the expected value of E_p when the error E and directions are known.

$$\langle E_p\rangle = E\,\langle\cos\alpha\rangle = \frac{2\,E}{\pi}\int_0^{\pi/2}\cos\alpha\;d\alpha = \frac{2E}{\pi} \qquad \text{(D-5)}$$

This result indicates that studies within the ESM which involve any of the three systematic modeled errors listed at the start of this appendix represent conservative estimates using maximum error effect. A better measure of the average effect due to a displacement vector of magnitude E can be calculated by using a modeled error of magnitude $\frac{2\,E}{\pi}$. This gives results which differ by a factor of $\frac{2}{\pi}$ for those errors which propagate linearly. A similar argument can be made for landmark definition as it can be considered a displacement of the observed landmark coordinates from the true (horizontal control) coordinates.

APPENDIX E

DISTRIBUTIONS FOR RANDOM GEOMETRY METHOD

The statistics used in generating random geometries were calculated from historical data on fixes taken by the crew of USCGC REDWOOD (WLM 685). The following procedures were followed:

a. SANDS forms were researched.

b. A sample ($n \approx 100$) of each of the following variates was taken from the data:

(1) Range to non-common landmarks
(2) Range to common landmarks
(3) Angles between landmarks
(4) Gradients
(5) Line of position crossing angles

c. The mean and s.d. of the first three variates were calculated and the distributions were tested for normality.

The information compiled was used to derived the following results:

a. The distribution of ranges to common and non-common landmarks were not significantly different.

b. The ranges had a mean of 4000 meters and a s.d. of 2000 meters, and were distributed as a truncated normal distribution (truncated at zero meters). The effect of this truncation is insignificant in the generation of fix geometries.

c. The angles had a mean of 60^{o} and a s.d. of 35^{o} and were distributed as a truncated normal distribution (truncated at 0^{o}). The effect of this truncation is insignificant in the generation of fix geometries.

d. The gradients generated by the Monte Carlo Routine were distributed identically with those of the sample set.

e. For a fix with n lines of position the n-1 smallest crossing angles are uniformly distributed between zero and $180^{o}/n$.

The statistics derived from the sample allow random generation of fix geometries for the sextant, gyrocompass, and radar in the following way:

a. Sextant - The routine generates a simulated landmark with a range distributed as per sample statistics and with a bearing uniformly distributed around the horizon. An angle is generated using sample statistics and is followed by another range. The three variates together simulate the geometry of one sextant angle. Other simulated measurements of the same fix are similarly generated. If two angles use a common landmark, the right landmark of the first measurement is used as the left landmark of the second.

b. Gyrocompass - The routine generates a simulated landmark with a range distributed as per sample statistics and with a bearing uniformly distributed around the horizon. Other simulated gyrocompass measurements of the same fix are generated with a crossing angle uniformly distributed between 30^{0} and 90^{0} of the previous simulated measurement of the geometry.

c. Radar - Generates a simulated landmark in same manner as with gyrocompass simulations.

d. LORAN - The routine generates crossing angle in the same manner as for both the gyrocompass and radar routines. As a first approximation, LORAN gradients are generated with a truncated normal distribution with a mean of 300 meters/μsec and a standard deviation of 150 meters/μsec (truncated at 150 meters/μsec). The effect of this truncation will slightly raise the mean gradient by discarding all values less than 150 meters/μsec and regenerating the gradient needed. This distribution was created by studying a random sample of LORAN grids on Mercator projection navigation charts (sample size n = 100).

APPENDIX F

DISCUSSION OF OUTLIER DETECTION

A method to determine whether or not a given set of measurements conform to normal expectations is discussed in this appendix. Before any method can be successfully employed as a part of a calculator-based positioning system, the following items will require further consideration.

a. Assumed measurement variances
b. Conditions unique to specific aid locations
c. Combinatorial analysis of fix situations
d. Independence of measurements
e. Post blunder detection procedures

A discussion in reference 11 provided the basis for the outlier detection method presented here.

The computed values of measurements for the desired location are assumed to be the parent population means of each measurement. Ideally, assuming no systematic error, the measurements are normally distributed about the parent population means. The differences (l_i) in the measurements from the parent population means are normally distributed variates with a mean of zero and a variance of σ_i^2, where σ_i is the measurement s.d. The error detection method employs the x^2 probability distribution which is introduced as follows. The sum of the squares of n independent random variables having standard normal distributions has the x_n^2 distribution where n denotes the degrees of freedom. The variates l_i/σ_i form a standard normal distribution (reference 11).

Thus:

$$\sum_{i=1}^{n} \left(\frac{l_i}{\sigma_i}\right)^2 = x_n^2 \qquad \text{(F-1)}$$

The sum of n normalized squared errors are x_n^2 distributed. This is equivalent to the x^2 "goodness of fit" test with no parameters being estimated by the sample measurements. With the x^2 method, any set of the measurements can be checked for agreement with the computed values at some desired confidence level, α.

For any subset of m measurements, the test is,

$$\sum_{i=1}^{m} \left(\frac{l_i}{\sigma_i}\right)^2 \geq x_m^2(\alpha) \qquad \text{(F-2)}$$

The confidence level chosen is dependent upon the geometry of the fix and criticality of the aid (reference 1) being positioned.

If the sum exceeds the χ^2_m, inconsistency of the measurements relative to the assumed measurement variance is indicated and one or more of the following possibilities exist concerning the measurement subset,

a. The position determined by the measurement subset is not the desired position.

b. Blunders and/or uncompensated for systematic errors exist in one or more of the measurements.

c. A chance $(1-\alpha)$ outlier situation occurred.

In case a, subsequent maneuvering should allow improvement. In the event of case b, the measurements in question should be investigated. Possibilities include individual checks of each measurement, measuring instruments, and of signal sources for accuracy. An example of the χ^2 method is as follows: a vessel is maneuvered by "marking two measurements" and the error in a third measurement is checked against some prespecified χ^2 value. If the observation error exceeds χ^2_3 at some prespecified confidence level, α, an investigation of the all three measurements is in order.

A mathematically more complicated test of the residuals of a set of n measurements can be performed using a similar procedures. In this procedure, the measurements are made, a most probable position is determined from the measurements, and the χ^2 test is performed on the weighted residuals. The formula is,

$$\sum_{i=1}^{n} \left(\frac{r_i}{\sigma_i}\right)^2 > \chi^2_{n-2}(\alpha) \qquad \text{(F-3)}$$

Where the n-2 is the number of degrees of freedom resulting from the loss of two degrees in the estimation of the MPP from the measurement set.

If the test indicates inconsistency, the following possibilities exist:

a. The measurement geometry is functionally inconsistent.

b. One or more measurements are in error and exceed the confidence limits imposed by the assumed σ.

Case a requires redundant precomputation or, if that fails, a new geometry. Case b requires investigation of the measurement set (all measurements). Possibilities here also include study of measuring instruments and signal sources for accuracy. Further statistical tests can be performed on subsets of the measurement set. If four or more lines are used, the sum of the normalized squared residuals of each three-measurement subset with n-3 degrees of freedom can be compared to the normalized squared residual of the nth measurement with reference to the MPP determined by the n-1 measurement subset. The normalized squared residual of the nth measurement has one degree of freedom. The test is for $n(>3)$ measurements:

$$\frac{\left(\frac{r_n}{\sigma_n}\right)^2}{\sum_{i=1}^{n-1}\left(\frac{r_i}{\sigma_i}\right)^2} = \frac{\chi_1^2}{\chi_{n-3}^2} = \frac{1}{n-3}\ F\ (1, n-3, \alpha) \qquad \text{(F-4)}$$

Where: $F(1, n-3, \alpha)$ is the F distribution with 1 degree of freedom in the numerator and n-3 degrees of freedom in the denominator.

This calculation would be repeated for each subset of (n-1) measurements with the largest of the ratios tested for significance against the desired F-statistic. A significant result would indicate that the corresponding nth measurement is likely in error.

APPENDIX G

ERROR SENSITIVITY MODEL LISTING

```
0: gto "POSER"
1: "SN":rad;√(-2ln(rnd(1)))cos(2πrnd(1))→p1;deg;ret p1
2: "DIS":exp(-.5(p1p1/p3p3+p2p2/p4p4))/2πp3p4→p5;ret p5
3: "DEFAULT":for I=1 to N;for J=1 to 4;if P[I,1]=J;D[J]→P[I,7]
4: next J;next I;ret
5: "GENERATE":0→r4→r5→r6
6: for I=1 to N
7: gsb "NA";if P[I,1]>1;gsb "NB";if P[I,1]>2;gsb "NR";if P[I,1]>3;gsb "NL"
8: next I;ret
9: "NA":
10: if O#0 and I>1 and I<=O+1;jmp 3
11: M[I,1,1]+'SN'M[I,1,2]→r1;if r1<0 or r1>16000;gto +0
12: rnd(1)*360→r2;r1sin(r2)→P[I,2];r1cos(r2)→P[I,3];jmp 2
13: P[I-1,4]→P[I,2];P[I-1,5]→P[I,3]
14: M[I,2,1]+'SN'M[I,2,2]→r1;if r1<0 or r1>16000;gto +0
15: M[I,3,1]+'SN'M[I,3,2]→r3→P[I,6];if r3<0 or r3>180;gto +0
16: r2+r3→r2;r1sin(r2)→P[I,4];r1cos(r2)→P[I,5]
17: if r2>360;r2-360→r2
18: ret
19: "NB":1+r4→r4
20: M[I,1,1]+'SN'M[I,1,2]→r1;if r1<0 or r1>16000;gto +0
21: if r4<2;rnd(1)*360→r2→P[I,6];jmp 2
22: r2+rnd(1)*60+30→r2→P[I,6]
23: if r2>360;r2-360→r2
24: r1sin(r2)→P[I,2];r1cos(r2)→P[I,3];ret
25: "NR":1+r5→r5
26: M[I,1,1]+'SN'M[I,1,2]→r1→P[I,6];if r1<0 or r1>16000;gto +0
27: if r5<2;rnd(1)*360→r2;jmp 2
28: r2+rnd(1)*60+30→r2
29: if r2>360;r2-360→r2
30: r1sin(r2)→P[I,2];r1cos(r2)→P[I,3];ret
31: "NL":1+r6→r6
32: if r6<2;rnd(1)*360→r2;jmp 2
33: r2+rnd(1)*60+30→r2
34: if r2>360;r2-360→r2
35: r2→P[I,2]
36: M[I,1,1]+'SN'M[I,1,2]→r1;if r1<150 or r1>1500;gto +0
37: r1→P[I,3];0→P[I,6];ret
38: "GRADIENT":
39: for I=1 to N
40: P[I,1]→r1;P[I,2]→r2;P[I,3]→r3;P[I,4]→r4;P[I,5]→r5
41: gsb "GA";if r1>1;gsb "GB";if r1>2;gsb "GR";if r1>3;gsb "GL"
42: if r1=1 and P[I,9]>100;sfg 3;gsb "GENERATE"
43: if flg3;cfg 3;gto -4
44: next I;ret
45: "GA":
46: √((r2r2+r3r3)(r4r4+r5r5)).00029/√((r2-r4)↑2+(r3-r5)↑2)→P[I,9]
47: (r3(r4r4+r5r5)-r5(r2r2+r3r3))/2(r4r3-r2r5)→r6
48: (r2(r4r4+r5r5)-r4(r2r2+r3r3))/2(r2r5-r4r3)→r7
49: if r6=0 and r7>0;0→P[I,10];jmp 4
*6627
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50: if r6=0 and r7<0;180→P[I,10];jmp 3
51: if r6>0;90-atn(r7/r6)→P[I,10];jmp 2
52: if r6<0;270-atn(r7/r6)→P[I,10]
53: ret
54: "GB":
55: r(r2r2+r3r3).017453→P[I,9]
56: P[I,6]-90→P[I,10];if P[I,10]<0;P[I,10]+360→P[I,10]
57: ret
58: "GR":
59: 1→P[I,9]
60: if r2=0 and r3>0;180→P[I,10];jmp 4
61: if r2=0 and r3<0;0→P[I,10];jmp 3
62: if r2<0;90-atn(r3/r2)→P[I,10];jmp 2
63: if r2>0;270-atn(r3/r2)→P[I,10]
64: ret
65: "GL":P[I,3]→P[I,9];P[I,2]→P[I,10];ret
66: "PARTIALS":
67: for I=1 to N
68: -P[I,3]→B[2I-1];-P[I,5]→B[2I]
69: -P[I,2]→C[2I-1];-P[I,4]→C[2I]
70: r(B[2I-1]↑2+C[2I-1]↑2)→S[2I-1];r(B[2I]↑2+C[2I]↑2)→S[2I]
71: next I
72: for L=1 to N
73: gsb "AA";if P[L,1]>1;gsb "AB";if P[L,1]>2;gsb "AR";if P[L,1]>3;gsb "AL"
74: next L;ret
75: "AA":
76: 2L-1→I;2L→J
77: (C[I]/S[I])(1/S[J]-cos(P[L,6])/S[I])→A[L,1]
78: A[L,1]+(C[J]/S[J])(1/S[I]-cos(P[L,6])/S[J])→A[L,1]
79: -A[L,1]/sin(P[L,6])→A[L,1]
80: (B[I]/S[I])(1/S[J]-cos(P[L,6])/S[I])→A[L,2]
81: A[L,2]+(B[J]/S[J])(1/S[I]-cos(P[L,6])/S[J])→A[L,2]
82: -A[L,2]/sin(P[L,6])→A[L,2]
83: ret
84: "AB":
85: if P[L,3]=0;0→A[L,1];jmp 2
86: -cos(P[L,6])↑2/P[L,3]→A[L,1]
87: if P[L,2]=0;0→A[L,2];jmp 2
88: sin(P[L,6])↑2/P[L,2]→A[L,2]
89: ret
90: "AR":
91: if P[L,6]=0;0→A[L,1];jmp 2
92: -P[L,2]/P[L,6]→A[L,1]
93: if P[L,6]=0;0→A[L,2];jmp 2
94: -P[L,3]/P[L,6]→A[L,2]
95: ret
96: "AL":
97: sin(P[L,2])/P[L,3]→A[L,1]
98: cos(P[L,2])/P[L,3]→A[L,2]
99: ret
*7663
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```
100: "OUTLIERS":
101: for J=1 to 7
102: pJ+O[p8,J,1]→O[p8,J,1];pJ↑2+O[p8,J,2]→O[p8,J,2]
103: next J;if p9<30;ret
104: for J=1 to 7
105: O[p8,J,1]/p9→O[p8,J,3]
106: r((O[p8,J,2]/p9-O[p8,J,3]↑2)p9/(p9-1))→O[p8,J,4]
107: if J=5 or J=7;jmp 3
108: if pJ>O[p8,J,3]+2O[p8,J,4] and flg1=0;gsb "BAD"
109: if pJ<O[p8,J,3]-2O[p8,J,4] and flg1=0 and p8#0;gsb "GOOD"
110: next J;ret
111: "PROBABILITY":sfg 14;-p4cos(p1)-p5sin(p1)→p15;p4sin(p1)-p5cos(p1)→p5
112: if 2r(p2p2+p3p3)+r(p5p5+p15↑2)<p6;ret 1
113: if r(p5p5+p15↑2)-2r(p2p2+p3p3)>p6;ret 0
114: for I=1 to 16;0→p9;p6I[I,1]+p15→p10;r(p6p6-(p10-p15)↑2)→p11
115: for J=1 to 16;I[J,2]'DIS'(p10,p11I[J,1]+p5,p2,p3)+p9→p9
116: next J;p9I[I,2]p11+p14→p14;next I;p14p6→p14;cfg 14;ret p14
117: "1":
118: 0→Q
119: for I=1 to N
120: if P[I,1]=1;Q+1→Q;if Q<=D;E→P[I,8]
121: next I;ret
122: "2":
123: E/r(P[1,2]↑2+P[1,3]↑2)*10800/π→P[1,8]
124: ret
125: "3":
126: for I=1 to N
127: if P[I,1]=1 and I=1;-E/r(P[I,4]↑2+P[I,5]↑2)*(10800/π)→P[I,8]
128: if P[I,1]=1 and I=O+1;E/r(P[I,2]↑2+P[I,3]↑2)*(10800/π)→P[I,8]
129: next I;ret
130: "4":
131: 0→Q
132: for I=1 to N
133: if P[I,1]=3;Q+1→Q;if Q<=D;E→P[I,8]
134: next I;ret
135: "5":
136: 0→Q
137: for I=1 to N
138: if P[I,1]=2;Q+1→Q;if Q<=D;E→P[I,8]
139: next I;ret
140: "6":
141: 0→Q
142: for I=1 to N
143: if P[I,1]=4;Q+1→Q;if Q<=D;E→P[I,8]
144: next I;ret
145: "7":
146: (acs(cos(P[1,6])cos(E180/r(P[1,2]↑2+P[1,3]↑2)π))-P[1,6])60→P[1,8]
147: ret
148: "8":
149: for I=1 to N
*23721
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150: if P[I,1]#1;jmp 5
151: if I#1;jmp 2
152: (acs(cos(P[I,6])cos(E180/√(P[I,4]↑2+P[I,5]↑2)π))-P[I,6])60→P[I,8]
153: if I#O+1;jmp 2
154: (acs(cos(P[I,6])cos(E180/√(P[I,2]↑2+P[I,3]↑2)π))-P[I,6])60→P[I,8]
155: next I;ret
156: "9":
157: E/P[1,9]→P[1,8]
158: ret
159: "10":
160: 0→Q
161: for I=1 to N
162: if P[I,1]=1;Q+1→Q;if Q<=D;E→P[I,7]
163: next I;ret
164: "11":
165: for I=1 to N
166: if P[I,1]=1 and int(I/2)=0;E→P[I,7]
167: next I;ret
168: "12":
169: √(P[1,7]↑2+(E10800/√(P[1,2]↑2+P[1,3]↑2)π)↑2)→P[1,7]
170: ret
171: "13":
172: for I=1 to N
173: if P[I,1]#1;jmp 6
174: if I=1;√(P[I,7]↑2+(E10800/√(P[I,4]↑2+P[I,5]↑2)π)↑2)→P[I,7];jmp 5
175: if I=O+1;√(P[I,7]↑2+(E10800/√(P[I,2]↑2+P[I,3]↑2)π)↑2)→P[I,7];jmp 4
176: if I>O+1;jmp 3
177: √(P[I,7]↑2+(E10800/√(P[I,2]↑2+P[I,3]↑2)π)↑2)→P[I,7]
178: √(P[I,7]↑2+(E10800/√(P[I,4]↑2+P[I,5]↑2)π)↑2)→P[I,7]
179: next I;ret
180: "14":
181: 0→Q
182: for I=1 to N
183: if P[I,1]=2;Q+1→Q;if Q<=D;E→P[I,7]
184: next I;ret
185: "15":
186: 0→Q
187: for I=1 to N
188: if P[I,1]=3;Q+1→Q;if Q<=D;E→P[I,7]
189: next I;ret
190: "16":
191: 0→Q
192: for I=1 to N
193: if P[I,1]=4;Q+1→Q;if Q<=D;E→P[I,7]
194: next I
195: ret
196: "GOOD":wrt 6;sfg 1;if flg0#1;wrt 6.2
197: wrt 6.3;wrt 6.4
198: for I=1 to N
199: for J=1 to 10;P[I,J]→rJ;next J
*23540
```

```
200: wrt 6.5,r1,r2,r3,r4,r5,r6,r7,r8,r9,r10
201: next I
202: wrt 6;wrt 6.6;wrt 6.7
203: wrt 6.5,E-C,Z[1],Z[2],Z[3],Z[4],Z[5],Z[6],Z[7]
204: ret
205: "BAD":wrt 6;sfg 1;if flg0#1;wrt 6.1
206: wrt 6.3;wrt 6.4
207: for I=1 to N
208: for L=1 to 10;P[I,L]→rL;next L
209: wrt 6.5,r1,r2,r3,r4,r5,r6,r7,r8,r9,r10
210: next I
211: wrt 6;wrt 6.6;wrt 6.7
212: wrt 6.5,E-C,Z[1],Z[2],Z[3],Z[4],Z[5],Z[6],Z[7];ret
213: "FIXED":sfg 0
214: "FA":
215: for I=1 to A
216: 1→P[I,1]
217: enp "R&B to LO",r2,r3;r3→r4
218: r2sin(r3)→P[I,2];r2cos(r3)→P[I,3]
219: enp "R&B to RO",r2,r3;r3-r4+360→P[I,6]
220: if P[I,6]>360;P[I,6]-360→P[I,6]
221: r2sin(r3)→P[I,4];r2cos(r3)→P[I,5]
222: next I
223: "FB":
224: for I=A+1 to A+B
225: 2→P[I,1]
226: enp "R&B to Object(B)",r2,r3;r3→P[I,6]
227: r2sin(r3)→P[I,2];r2cos(r3)→P[I,3]
228: next I
229: "FR":
230: for I=A+B+1 to A+B+R
231: 3→P[I,1]
232: enp "R&B to Object(R)",r2,r3;r2→P[I,6]
233: r2sin(r3)→P[I,2];r2cos(r3)→P[I,3]
234: next I
235: "FL":
236: for I=A+B+R+1 to N
237: 4→P[I,1]
238: enp "Grad dir. and mag.(L)",P[I,2],P[I,3];0→P[I,6]
239: next I
240: ret
241: "RANDOM":
242: enp "# and Seed?",G,r3;wait rnd(-r3)
243: "RA":
244: for I=1 to A
245: 1→P[I,1]
246: if O#0 and I>1 and I<=O+1;jmp 2
247: 4000→M[I,1,1];2000→M[I,1,2]
248: 4000→M[I,2,1];2000→M[I,2,2]
249: 60→M[I,3,1];35→M[I,3,2]
*30526
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250: next I
251: "RB":
252: for I=A+1 to A+B
253: 2→P[I,1]
254: 4000→M[I,1,1];2000→M[I,1,2]
255: next I
256: "RR":
257: for I=A+B+1 to A+B+R
258: 3→P[I,1]
259: 4000→M[I,1,1];2000→M[I,1,2]
260: next I
261: "RL":
262: for I=A+B+R+1 to N
263: 4→P[I,1]
264: 300→M[I,1,1];150→M[I,1,2]
265: next I
266: ret
267: "PLOTR":
268: wrt 705,"IP000,000,6000,3000"
269: scl 0,9,0,6;xax 0,0,0,9;yax 0,0,0,6;xax 6,0,0,9;yax 9,0,0,6
270: csiz 2,1.7,2/3;plt 4,.2;lbl Z$[P]
271: fxd 0;plt 7.75,3.5;lbl "A= ";lbl A;plt 7.75,3.25;lbl "B= ";lbl B
272: plt 7.75,3;lbl "R= ";lbl R;plt 7.75,2.75;lbl "L= ";lbl N-(A+B+R)
273: plt 2,5.75;lbl "Random Geometry";plt 2,5.5;lbl "Number=";lbl G;fxd 1
274: plt 4.75,5.75;if cap(A$)="Y";lbl "Cumul .vs. ";jmp 2
275: lbl "Fract .vs. "
276: lbl Z$[P]
277: plt 4.75,5.5;lbl E$[r0]
278: plt 4.75,5.25;lbl "MAXIMUM= ";lbl M
279: if flg2;plt 4.75,5;lbl "R=  ";lbl F;lbl " meters"
280: csiz 2,1,2/3,90;plt .3,2;if cap(A$)="Y";lbl "Cumulative";jmp 2
281: lbl "Fraction"
282: csiz 2,1,2/3
283: wrt 705,"IP1000,500 ,5000,2300"
284: scl 0,K[P],0,1
285: xax 0,K[P]/10,0,K[P],2;yax 0,.1,0,1,2
286: csiz 2,1.7,2/3;ret
287: "PLOTF":
288: wrt 705,"IP000,000,6000,3000"
289: scl 0,9,0,6
290: scl 0,9,0,6;xax 0,0,0,9;yax 0,0,0,6;xax 6,0,0,9;yax 9,0,0,6
291: csiz 2,1.7,2/3;plt 4,.2;lbl E$[r0]
292: plt 2,5.75;lbl "Fixed Geometry";plt 2,5.5;lbl N$
293: plt 4.75,5.75;lbl Z$[P];lbl " .vs. ";lbl E$[r0]
294: plt 4.75,5.5;lbl "MAXIMUM= ";lbl M
295: if flg2;plt 4.75,5.25;lbl "R=  ";lbl F;lbl " meters"
296: csiz 2,1,2/3,90;plt .4,2;lbl Z$[P]
297: wrt 705,"IP1000, 700,5000,2500"
298: scl 0,M,0,K[P]
299: fxd 2;xax 0,M/10,0,M,2;yax 0,K[P]/10,0,K[P],2
*25468
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```
300: ret
301: "CHANGE":enp "Def Vals?A,B,R,L",D[1],D[2],D[3],D[4];ret
302: "POSER":trk 0
303: fmt 1,"Bad Outlier",/
304: fmt 2,"Good Outlier",/
305: fmt 3,4x,"TYPE      XL        YL        XR        YR        MEAS    SIGMA",z
306: fmt 4,"    BIAS     GRAD       PGD"
307: fmt 5,10f8.2
308: fmt 6,"       E          Z        2RMS    AREA       MAJ     P IN R ",z
309: fmt 7,"    RES     RATIO"
310: fmt 8,4x,"BIN        0        .2M        .4M       .6M       .8M       M",/
311: fmt 9,"  MEANS       0        .2M        .4M       .6M       .8M       M",/
312: enp "#A,#B,#R,#L",A,B,R,L;if A>=2;enp "# Common  LM's?",O
313: A+B+R+L→N
314: dim A[N,2],B[2N],C[2N],D[4],E[2,N],F[0:5,26],H[2,2],I[16,2],K[7],L[N]
315: dim M[N,3,2],N[2,2],O[0:5,7,4],P[N,10],Q[2,2],R[N],S[2N],T[2,N]
316: dim V[2,2],W[N,N],X[2],Y[2],Z[7],A$[1],E$[20,20],N$[15],Z$[7,20]
317: ldf 1,E$;ldf 2,Z$;ldf 3,I[*]
318: 100→K[1];100→K[2];300→K[3];100→K[4];1→K[5];50→K[6];1→K[7]
319: 5→D[1];.5→D[2];30→D[3];.1→D[4]
320: enp "Any New Default Std. Dev.?y or n",A$;if cap(A$)="Y";gsb "CHANGE"
321: enp "Output Parameter Number?",P;prt Z$[P]
322: if P=5;enp "Target Circle Radius?(meters)",F;sfg 2
323: enp "New Output Scale Needed?y or n",A$
324: if cap(A$)="Y";enp "New Output Scale Value?",K[P]
325: enp "Error studied?",r0;enp "# Times it Occurred?",D;prt E$[r0]
326: enp "Maximum of Errors studied?",M
327: enp "Fixed Geometry Method?y or n",A$
328: gsb "RANDOM";if cap(A$)="Y";enp "Fixed Geometry Name?",N$;gsb "FIXED"
329: if flg0;fxd 1;dsp "Paper in Plotter?";stp ;gsb "PLOTF"
330: if flg0;M/25→C;wrt 6.6;wrt 6.7;jmp 4
331: M/5→C
332: for K=1 to G
333: gsb "GENERATE"
334: gsb "GRADIENT"
335: cfg 1
336: for E=0 to M by C
337: gsb "DEFAULT"
338: "ERROR":
339: gto +1;if r0>5;gto +3;if r0>10;gto +5;if r0>14;gto +7
340: gsb "1";if r0>1;gsb "2";if r0>2;gsb "3";if r0>3;gsb "4";if r0>4;gsb "5"
341: jmp 6
342: gsb "6";if r0>6;gsb "7";if r0>7;gsb "8";if r0>8;gsb "9";if r0>9;gsb "10"
343: jmp 4
344: gsb "11";if r0>11;gsb "12";if r0>12;gsb "13";if r0>13;gsb "14"
345: jmp 2
346: gsb "15";if r0>15;gsb "16";if r0>16;gsb "17";if r0>17;gsb "18"
347: gsb "PARTIALS"
348: "WEIGHTS":
349: for I=1 to N
*10638
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```
if P[I,7]=0;10↑6→W[I,I];jmp 5
if P[I,1]=1;(10800/P[I,7]π)↑2→W[I,I]
if P[I,1]=2;(180/P[I,7]π)↑2→W[I,I]
if P[I,1]=3;1/P[I,7]↑2→W[I,I]
if P[I,1]=4;1/P[I,7]↑2→W[I,I]
next I
"BIAS":
for I=1 to N
if P[I,1]=1;-P[I,8]π/10800→L[I]
if P[I,1]=2;-P[I,8]π/180→L[I]
if P[I,1]>=3;-P[I,8]→L[I]
next I
"REDUCE":
trn A→T;mat TW→E;mat EA→V;mat EL→X;inv V→N;mat NX→Y;smpy -1Y→X
mat AX→R
ara R+L→R
if N[1,1]-N[2,2]=0 and N[1,2]<0;135→T;jmp 6
if N[1,1]-N[2,2]=0 and N[1,2]=0;0→T;jmp 5
if N[1,1]-N[2,2]=0 and N[1,2]>0;45→T;jmp 4
.5atn(2N[1,2]/(N[1,1]-N[2,2]))→T
if N[1,1]-N[2,2]<0;T+90→T
if T<0;T+180→T
cos(T)→V[1,1]→V[2,2];sin(T)→V[2,1];-V[2,1]→V[1,2]
trn V→Q;mat QN→H;mat HV→Q
"ZOUTPUT":
√(X[1]↑2+X[2]↑2)→Z[1]
2√(Q[1,1]+Q[2,2])→Z[2]
2.15↑2√(Q[1,1]Q[2,2])→Z[3]
2.15√max(Q[1,1],Q[2,2])→Z[4]
√max(Q[1,1],Q[2,2])→r11;√min(Q[1,1],Q[2,2])→r12
if flg2;'PROBABILITY'(T,r11,r12,X[1],X[2],F)→Z[5]
0→Z[6];for I=1 to N;R[I]↑2W[I,I]+Z[6]→Z[6];next I
min(r11,r12)/max(r11,r12)→Z[7]
if flg0;jmp 5
cll 'OUTLIERS'(Z[1],Z[2],Z[3],Z[4],Z[5],Z[6],Z[7],E/C,K)
"TABULATE":
int(Z[P]*25/K[P]+1)→r6;if r6>25;26→r6
F[E/C,r6]+1→F[E/C,r6]
if flg0;plt E,Z[P];wrt 6.5,E,Z[1],Z[2],Z[3],Z[4],Z[5],Z[6],Z[7]
next E
if flg0;prt "End of Run";gsb "GOOD"
if flg0;end
next K
"PLOTOUT":
ent "Cumulative?",A$
fxd 1;dsp "Paper in Plotter?";stp ;gsb "PLOTR"
fxd 0
for I=1 to 26;for J=0 to 5;F[J,I]/G→F[J,I];next J;next I
if cap(A$)="Y";jmp 3
for J=1 to 5
*21936
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400: for I=1 to 25;plt (I-.5)*K[P]/25,F[J,I],3;lbl J;next I;next J;jmp 3
401: for J=1 to 5;for I=1 to 24;0→r1;for H=1 to I;F[J,H]+r1→r1;next H
402: plt (I-.5)*K[P]/25,r1,3;lbl J;next I;plt .98,1,3;lbl J;next J
403: "PRINTOUT":
404: wrt 6;wrt 6.8;wrt 6
405: for I=1 to 26
406: wrt 6.5,(I-.5)K[P]/25,F[0,I],F[1,I],F[2,I],F[3,I],F[4,I],F[5,I]
407: next I
408: wrt 6;wrt 6
409: for J=3 to 4
410: if J=3;wrt 6.9;jmp 2
411: wrt 6.5,"STD DEV      0      .2M      .4M      .6M      .8M      M"
412: wrt 6
413: for I=1 to 7
414: wrt 6.5,I,O[0,I,J],O[1,I,J],O[2,I,J],O[3,I,J],O[4,I,J],O[5,I,J]
415: next I;wrt 6;next J;end
*14322
```

E$	Z$
Index Error(min)	AP to MPP(met)
NC LM Misp(met)	2DRMS(met)
C LM Misp(met)	Mj*Mn(90%)(mt↑2)
Range Error(met)	Maj(90%)(met)
Bearing Error(deg)	Prob in R
LORAN Error(usec)	Sum Sqd Res
NC Incl Ang(met)	Min/Maj
C Incl Ang(met)	
Observer Coin(met)	
=S/O Std Dev(min)	
#S/O Std Dev(min)	
NC LM Def(met)	
C LM Def(met)	
Bearing Std Dev(deg)	
Range Std Dev(met)	
LORAN Std Dev(usec)	

I[*]

-0.989400930	0.027152460
-0.944575020	0.062253520
-0.865631200	0.095158510
-0.755404410	0.124628970
-0.617876240	0.149595990
-0.458016780	0.169156520
-0.281603550	0.182603420
-0.095012510	0.189450610
0.095012510	0.189450610
0.281603550	0.182603420
0.458016780	0.169156520
0.617876240	0.149595990
0.755404410	0.124628970
0.865631200	0.095158510
0.944575020	0.062253520
0.989400930	0.027152460