

AD-A088 308

CORNELL UNIV ITHACA N Y SCHOOL OF OPERATIONS RESEARC--ETC F/6 12/2
INCOMPLETE BLOCK DESIGNS FOR COMPARING TREATMENTS WITH A CONTRO--ETC(U)
APR 80 R E BECHHOFFER, A C TAMHANE

DAA629-80-C-0036

UNCLASSIFIED

TR-453

NL

| OF |
AD
A088 308

END
DATE
FILMED
9-80
DTIC



1.0



1.25



1.5



1.8



2.0



2.2



2.5



2.8



3.2



3.6



4.0



4.5



5.0

MADE IN U.S.A. BY N. L. L. CO. 1967

NEW YORK, N.Y. 10011

LEVEL

(12)

**SCHOOL
OF
OPERATIONS RESEARCH
AND
INDUSTRIAL ENGINEERING**

AD A088308

**DTIC
ELECTE
AUG 27 1980**



**COLLEGE OF ENGINEERING
CORNELL UNIVERSITY
ITHACA, NEW YORK 14853**

This document has been approved
for public release and sale; its
distribution is unlimited.

FILE COPY

(12)

SCHOOL OF OPERATIONS RESEARCH
AND INDUSTRIAL ENGINEERING
CORNELL UNIVERSITY
ITHACA, NEW YORK

TECHNICAL REPORT NO. 453

April 1980

INCOMPLETE BLOCK DESIGNS FOR COMPARING
TREATMENTS WITH A CONTROL (VI): CONJECTURED
MINIMAL COMPLETE CLASS OF GENERATOR DESIGNS
FOR $p = 5, k = 4$ and $p = 6, k = 4$

by

Robert E. Bechhofer
Cornell University

Ajit C. Tamhane
Northwestern University

Stephen W. Mykytyn
Cornell University

Research supported by
U.S. Army Research Office-Durham Contract DAAG-29-80-C-0036,
Office of Naval Research Contract N00014-75-C-0586
at Cornell University

and

NSF Grant ENG-77-06112
at Northwestern University

Approved for Public Release; Distribution Unlimited

TABLE OF CONTENTS

	<u>Page</u>
Abstract	
1. Introduction	1
2. Conjectured minimal complete class of generator designs	3
3. Concluding remarks	9
4. Acknowledgment	10
Appendix	
Proofs of C-inadmissibility of certain designs	11
A.1 Proof for $p = 5, k = 4$ of C-inadmissibility of D_8 for all b	11
A.2 Proof for $p = 6, k = 4$ of C-inadmissibility of D_8 for all b	12
A.3 Proof for $p = 6, k = 4$ of C-inadmissibility of D_9 for all b	13
A.4 Proof for $p = 6, k = 4$ of C-inadmissibility of D_{10} for all b	14
A.5 Proof for $p = 6, k = 4$ that D_6 can only appear in admissible designs of the form $D_6 \cup fD_7$ with $f \geq 1$.	15
References	16

Accession For	
NTIS GEM&I	☒
DDC TAB	☐
Unannounced	☐
Justification	
By _____	
Distribution/ _____	
Availability Codes	
Dist	Avail and/or special
A	

ABSTRACT

The present paper continues the study of balanced treatment incomplete block (BTIB) designs initiated in [1]-[5]. This class of designs was proposed for the problem of comparing simultaneously $p \geq 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p+1$. In [2]-[5] we gave lists of generator designs, the conjectured minimal complete class of generator designs, a catalog of admissible designs, and tables of optimal designs for $p = 2(1)6, k = 2$; $p = 3(1)6, k = 3$; and $p = 4, k = 4$. This present paper gives our conjectured minimal complete class of generator designs for $p = 5, k = 4$ and $p = 6, k = 4$ based on a generalized notion of C-inadmissibility. At this time we have made no further computations based on these classes of designs. Interested researchers are encouraged to supplement these classes if they are not already minimal complete.

Key words and phrases: Multiple comparison with a control, balanced treatment incomplete block (BTIB) designs, admissible designs, S-inadmissible designs, C-inadmissible designs, minimal complete class of generator designs, optimal designs.

1. INTRODUCTION

The present paper continues the study of balanced treatment incomplete block (BTIB) designs initiated in [1]-[5]. This class of designs was proposed for the problem of comparing simultaneously $p \geq 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p+1$. In [2]-[5] we gave lists of generator designs, the conjectured minimal complete class of generator designs, a catalog of admissible designs, and tables of optimal designs for $p = 2(1)6, k = 2$; $p = 3(1)6, k = 3$; and $p = 4, k = 4$.

In this paper we give our conjectured minimal complete class of generator designs for two additional cases: $p = 5, k = 4$ and $p = 6, k = 4$. While studying the case $p = 6, k = 4$ we encountered a phenomenon which did not arise with any of the cases considered previously (or with the other case considered in the present paper): for the case $p = 6, k = 4$ we found three generator designs which are neither S-inadmissible nor C-inadmissible nor equivalent to another generator design (or union of replications of other generator designs), and yet they can be eliminated from our conjectured minimal complete class without sustaining any statistical loss. This happens here because these three generator designs have the property that if any BTIB design containing these designs is admissible then there is an equivalent BTIB design which does not contain these designs. To account for this possibility we give below as Definition 2.1 a generalization of the notion of C-inadmissibility; using this new definition we are thus able to cut the number of generator designs in our conjectured minimal complete class for $p = 6, k = 4$ from ten to seven. (See Table 2.2.)

At the present time we have made no further computations based on our conjectured minimal complete class of generator designs for these two new cases. Our reason for not doing so stems from the fact that we are not as confident as we were for the cases studied in [2]-[5] that we have indeed obtained the minimal complete classes for the two new cases. The number of generator designs increases rapidly as p and k increase, and in general the number of generator designs in the minimal complete class also increases. This is shown by Table 1.1 below which is based on available information in [2]-[5] and the present paper. As noted in [3]-[5] and above, many of these numbers are as yet unverified.

Table 1.1
Number of Generator Designs
in the Conjectured Minimal Complete Class

p	k		
	2	3	4
2	2	-	-
3	2	2	-
4	2	5	4
5	2	6	7
6	2	5	7

We have decided to record the results that we have obtained thus far concerning these classes of generator designs in the hope that other researchers

in the combinatorial design area will become interested in the problem of constructing minimal complete classes of generator designs for the cases that we have studied, and perhaps also for other cases. In fact they may be able to propose a method or methods of constructing such minimal complete classes. Hopefully, the results that we have obtained will provide a point of departure for such studies.

Remark 1.1: The concept of a minimal complete class of generator designs may, at first glance, appear to be just an esthetic or mathematical nicety; there are, however, very important practical reasons for identifying these classes. When we set out to prepare a catalog of admissible designs for a given (p,k,b) or to determine the optimal design for a given (p,k,b) and d/σ (as in [2]-[5]) we do so using a computer search which enumerates all possible BTIB designs. The cost of such a search became prohibitive in terms of time (and hence cost) if the number of generator designs in the minimal complete class is "too large"; just noting that the number of generator designs in the conjectured minimal complete class can be cut by one, may make such a computation affordable when b is large.

2. CONJECTURED MINIMAL COMPLETE CLASS OF GENERATOR DESIGNS

We first give a generalization of our earlier definition of $\mathcal{C}$ -inadmissibility (see Definition 2.5 in [3]) which is useful for further restricting the number of generator designs in our conjectured minimal complete class; this will prove helpful for the $p = 6, k = 4$ case (and probably also for additional cases not considered heretofore) but was not necessary for the (p,k) cases considered in [2]-[5], or for the $p = 6, k = 3$ case.

Definition 2.1: Suppose that for given (p,k) we have $n \geq 2$ generator designs D_i ($1 \leq i \leq n$) no two of which are equivalent, and none of which is equivalent to the union of two or more generator designs. Suppose further that no D_i ($1 \leq i \leq n$) is S-inadmissible. Consider a design $D = \bigcup_{i=1}^n f_i D_i$ and an arbitrary design $D' = \bigcup_{i=1}^n g_i D_i$. If given $D \cup D'$ one can find a design $D'' = \bigcup_{i=1}^n h_i D_i$ such that D is not included in D'' and $D \cup D'$ is either inadmissible wrt D'' or equivalent to D'' , then we say that D is C-inadmissible wrt the set $\{D_1, D_2, \dots, D_n\}$.

As in Remark 2.6 of [3], if a design $D_i \in \{D_1, D_2, \dots, D_n\}$ is C-inadmissible, then we shall say that D_i is C-inadmissible wrt $\{D_j \mid (j \neq i, 1 \leq j \leq n)\}$. Thus if D_i is itself inadmissible (which is not guaranteed by Definition 2.1), and furthermore is C-inadmissible, i.e., every design containing D_i is either inadmissible or equivalent to another design not containing D_i , then the minimal complete class must not contain D_i . This leads us to the following definition.

Definition 2.2: If the set $\{D_1, \dots, D_n\}$ contains all generator designs for given (p,k) , and if $\{D_{i_1}, \dots, D_{i_m}\}$ with $m \leq n$ is the subset which contains all generator designs which are nonequivalent, non S-inadmissible, and none of which is C-inadmissible for all b , then the latter set will be referred to as the minimal complete class of generator designs for given (p,k) .

Our conjectured minimal complete class of generator designs for $p = 5$, $k = 4$ and $p = 6$, $k = 4$ is given in Tables 2.1 and 2.2, respectively. In the Appendix we show that D_8 in Table 2.1 is C-inadmissible wrt $\{D_1, D_2, \dots, D_7\}$ for all b , and that D_8, D_9 and D_{10} in Table 2.2 are

Table 2.1

Conjectured Minimal Complete Class of Generator Designs for $p = 5$, $k = 4$

Label	Design	b_i	$\lambda_0^{(i)}$	$\lambda_1^{(i)}$
D_1	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 5 \end{Bmatrix}$	5	3	0
D_2	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{Bmatrix}$	5	4	0
D_3	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 2 & 0 & 0 & 3 \\ 2 & 2 & 3 & 4 & 1 & 3 & 4 \\ 3 & 5 & 4 & 5 & 4 & 5 & 5 \end{Bmatrix}$	7	4	2
D_4	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 3 & 1 & 1 & 3 & 3 & 0 \\ 4 & 5 & 3 & 5 & 4 & 5 & 2 \end{Bmatrix}$	7	5	1
D_5	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 \\ 2 & 2 & 2 & 3 & 3 & 4 & 3 & 3 & 4 & 4 \\ 3 & 4 & 5 & 4 & 5 & 5 & 4 & 5 & 5 & 5 \end{Bmatrix}$	10	6	3
D_6	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 & 4 & 2 & 3 & 4 & 3 & 3 \\ 2 & 3 & 4 & 5 & 5 & 5 & 4 & 5 & 4 & 5 \end{Bmatrix}$	10	7	2
D_7	$\begin{Bmatrix} 1 & 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 3 & 3 \\ 3 & 3 & 4 & 4 & 4 \\ 4 & 5 & 5 & 5 & 5 \end{Bmatrix}$	5	0	3
D_8	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 4 \\ 2 & 3 & 4 & 5 & 3 & 4 & 5 & 4 & 5 & 5 \end{Bmatrix}$	10	8	1

Table 2.2

Conjectured Minimal Complete Class of Generator Designs for $p = 6, k = 4$

Label	Design	b_i	$\lambda_0^{(i)}$	$\lambda_1^{(i)}$
D_1	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{Bmatrix}$	6	4	0
D_2	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 2 & 3 & 2 & 3 & 4 \\ 2 & 4 & 3 & 5 & 3 & 4 & 5 \\ 5 & 6 & 4 & 6 & 6 & 5 & 6 \end{Bmatrix}$	7	2	2
D_3	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 & 2 & 4 \\ 4 & 6 & 5 & 6 & 3 & 6 & 5 \end{Bmatrix}$	7	4	1
D_4	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 4 \\ 2 & 2 & 3 & 3 & 4 & 3 & 3 & 4 & 5 & 5 \\ 5 & 6 & 4 & 6 & 5 & 4 & 5 & 6 & 6 & 6 \end{Bmatrix}$	10	5	2
D_5	$\begin{Bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 3 \\ 1 & 2 & 2 & 2 & 2 & 2 & 3 & 4 & 3 & 3 & 4 \\ 3 & 5 & 3 & 3 & 4 & 4 & 5 & 5 & 4 & 4 & 5 \\ 4 & 6 & 5 & 6 & 5 & 6 & 6 & 6 & 5 & 6 & 6 \end{Bmatrix}$	11	1	4
D_6	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 & 2 & 2 & 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 3 & 4 & 3 & 3 & 5 & 5 & 4 & 5 \\ 4 & 5 & 6 & 5 & 6 & 4 & 6 & 6 & 6 & 5 & 6 \end{Bmatrix}$	11	3	1
D_7	$\begin{Bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 3 \\ 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 4 & 3 & 3 & 3 & 4 & 4 \\ 3 & 3 & 3 & 4 & 4 & 5 & 4 & 4 & 5 & 5 & 4 & 4 & 5 & 5 & 5 \\ 4 & 5 & 6 & 5 & 6 & 6 & 5 & 6 & 6 & 6 & 5 & 6 & 6 & 6 & 6 \end{Bmatrix}$	15	0	6
D_8	$\begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{Bmatrix}$	6	3	0

Table 2.2 (continued)

Label	Design	b_i	$\lambda_0^{(i)}$	$\lambda_1^{(i)}$
D_9	$\left\{ \begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 & 3 & 5 \\ 2 & 5 & 6 & 3 & 4 & 5 & 6 & 5 & 6 & 4 & 6 \end{array} \right\}$	11	7	1
D_{10}	$\left\{ \begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 4 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 & 3 & 4 & 5 & 6 & 4 & 5 & 6 & 5 & 6 & 6 \end{array} \right\}$	15	10	1

C-inadmissible wrt $\{D_1, D_2, \dots, D_7\}$ for all b ; we also show that D_6 can only appear in admissible designs of the form $D_6 \cup fD_7$ with $f \geq 1$. We mention that designs D_8, D_9, D_{10} in Table 2.2 are C-inadmissible in the new sense but not in the old sense, i.e., according to Definition 2.5 in [3].

Equivalent designs and S-inadmissible designs are not given in these tables.

For example, for $p = 5, k = 4$ design

$$\left\{ \begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 \\ 1 & 1 & 3 & 0 & 0 & 2 & 2 & 2 & 3 & 4 & 3 & 3 & 4 & 4 \\ 3 & 4 & 4 & 2 & 5 & 3 & 4 & 5 & 5 & 5 & 4 & 5 & 5 & 5 \end{array} \right\}$$

with $b = 14, \lambda_0 = 9, \lambda_1 = 3$ is a generator design but it is equivalent to $D_3 \cup D_4$ in Table 2.1, while the design

$$\left\{ \begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 3 & 0 & 0 & 0 & 0 & 2 & 2 \\ 3 & 4 & 2 & 2 & 0 & 0 & 3 & 4 \\ 5 & 5 & 3 & 5 & 1 & 4 & 4 & 5 \end{array} \right\}$$

with $b = 8, \lambda_0 = 4, \lambda_1 = 2$ is also a generator design but it is S-inadmissible wrt D_3 in Table 2.1.

We point out that C-inadmissible generator designs which are not S-inadmissible are given in these tables (separated from the minimal complete class of generator designs by a double line) because, if our conjecture concerning the minimal complete class for some (p, k) is in fact incorrect,

i.e., if we have failed to include some generator design(s) in the minimal complete class, then it is possible that a design which is C-inadmissible with respect to the present conjectured minimal complete class may not be C-inadmissible wrt the new minimal complete class. In that situation such a C-inadmissible design must be included in the new minimal complete class. This possibility does not arise with equivalent or with S-inadmissible designs; they can be deleted without loss, and hence are not included in the tables.

We also point out that not only may some designs be missing from these tables but also one or more designs which are presently listed may not appear on a final list. For example, if the design

$$D_3^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 4 & 3 & 4 \\ 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 4 & 3 & 3 & 5 & 5 & 4 & 5 \\ 3 & 4 & 5 & 4 & 5 & 6 & 5 & 6 & 6 & 4 & 6 & 6 & 6 & 5 & 6 \end{pmatrix}$$

with $b = 15$, $\lambda_0 = 6$, $\lambda_1 = 3$ had been given instead of D_3 in Table 2.2, then none of the designs $\{D_1, D_2, D_3^*, D_4, \dots, D_{10}\}$ would have been S-inadmissible; however, the addition of D_3 to the set eliminates D_3^* since D_3^* is S-inadmissible with respect to $D_2 \cup D_3$.

3. CONCLUDING REMARKS

We would be grateful if researchers who obtain results which throw additional light on these problems would communicate their findings to us. If the minimal complete class of generator designs can be established for

the two cases under consideration in the present paper (or if our conjectured minimal complete classes of generator designs prove to be incorrect for the cases studied in [2]-[5]), then, if feasible, we will compute the optimal designs for these cases (or improved optimal designs for the cases studied earlier).

4. ACKNOWLEDGMENT

The authors are indebted to Mr. Carl Emont who brought to our attention several of the generator designs exhibited in the present paper.

APPENDIX

Proofs of C-inadmissibility of certain designs

A.1 Proof for $p = 5, k = 4$ of C-inadmissibility of D_3 for all b

First note that D_8 is inadmissible wrt D_5 but not S-inadmissible. We now consider unions of D_8 with all generator designs. We note that* $D_8 \cup D_1 (15,11,1)$ is S-inadmissible wrt $D_2 \cup D_6 (15,11,2)$, $D_8 \cup D_3 (17,12,3)$ is equivalent to $D_4 \cup D_6$, $D_8 \cup D_5 (20,14,4)$ is equivalent to $2D_6$, $D_8 \cup D_7 (15,8,4)$ is S-inadmissible wrt $2D_3 (14,8,4)$. Thus it only remains to consider unions of D_8 with D_2, D_4 and D_6 . We shall show that every $D = f_2 D_2 \cup f_4 D_4 \cup f_6 D_6 \cup f_8 D_8$ with $f_8 > 0$ is inadmissible wrt (or equivalent to) some design D' not containing D_8 ; $D' = f_2 D_2 \cup f_4 D_4 \cup f_3 D_5 \cup f_6 D_6$ is the required design. Thus $b = b' = 5f_2 + 7f_4 + 10f_6 + 10f_8$, $\lambda_0 = 4f_2 + 5f_4 + 7f_6 + 8f_8$, $\lambda_1 = f_4 + 2f_6 + f_8$, $\lambda'_0 = 4f_2 + 5f_4 + 7f_6 + 6f_8$, $\lambda'_1 = f_4 + 2f_6 + 3f_8$. Hence, $\rho < \rho'$ iff

$$\frac{f_4 + 2f_6 + f_8}{4f_2 + 6f_4 + 9f_6 + 9f_8} < \frac{f_4 + 2f_6 + 3f_8}{4f_2 + 6f_4 + 9f_6 + 9f_8}$$

which is always true for $f_8 > 0$. Also, $\tau^2 \geq \tau'^2$ iff

$$\frac{4f_2 + 6f_4 + 9f_6 + 9f_8}{(4f_2 + 5f_4 + 7f_6 + 8f_8)(4f_2 + 10f_4 + 17f_6 + 13f_8)} \geq$$

$$\frac{4f_2 + 6f_4 + 9f_6 + 9f_8}{(4f_2 + 5f_4 + 7f_6 + 6f_8)(4f_2 + 10f_4 + 17f_6 + 21f_8)}$$

which is always true. This completes the proof of the C-inadmissibility of D_3 .

* In this Appendix we use the notation $D(b, \lambda_0, \lambda_1)$ to indicate that design D has parameter values $(b, \lambda_0, \lambda_1)$.

A.2 Proof for $p = 6, k = 4$ of C-inadmissibility of D_8 for all b

First note that D_8 is inadmissible wrt D_1 but not S-inadmissible. We now consider unions of D_8 with all generator designs. We note that $D_8 \cup D_2 (13,5,2)$ is S-inadmissible wrt $D_4 (10,5,2)$, $D_8 \cup D_3 (13,7,1)$ is S-inadmissible wrt $D_9 (11,7,1)$, $D_8 \cup D_4 (16,8,2)$ is S-inadmissible wrt $2D_3 (14,8,2)$, $D_8 \cup D_5 (17,4,4)$ is S-inadmissible wrt $2D_2 (14,4,4)$, $D_8 \cup D_6 (17,6,1)$ is S-inadmissible wrt $D_1 \cup D_2 (13,6,2)$, $D_8 \cup D_7 (21,3,6)$ is S-inadmissible wrt $D_2 \cup D_5 (18,3,6)$, and $D_8 \cup D_9 (17,10,1)$ is S-inadmissible wrt $D_{10} (15,10,1)$. Thus it only remains to consider unions of D_8 with D_1 and D_{10} . Note that $f_1 D_1 \cup f_8 D_8$ is inadmissible wrt $(f_1 + f_8) D_1$ for $f_8 > 0$ but not S-inadmissible. Also $3D_8 (18,9,0)$ is S-inadmissible wrt $D_3 \cup D_4 (17,9,3)$. Therefore we must show that every $D = f_1 D_1 \cup f_8 D_8 \cup f_{10} D_{10}$ with $f_8 = 1$ or 2 and $f_{10} > 0$ is inadmissible wrt some design D' not containing D_8, D_9 or D_{10} . $D' = (f_1 + f_8) D_1 \cup 2f_{10} D_3$ is the desired design. Thus $b = 6f_1 + 6f_8 + 15f_{10} < b' = 6f_1 + 6f_8 + 14f_{10}$, $\lambda_0 = 4f_1 + 3f_8 + 10f_{10}$, $\lambda_1 = f_{10}$, $\lambda_0' = 4f_1 + 4f_8 + 8f_{10}$, $\lambda_1' = 2f_{10}$. Hence $\rho < \rho'$ iff

$$\frac{f_{10}}{4f_1 + 3f_8 + 11f_{10}} < \frac{2f_{10}}{4f_1 + 4f_8 + 10f_{10}}$$

which is always true. Also $\tau^2 \geq \tau'^2$ iff

$$\frac{4f_1 + 3f_8 + 11f_{10}}{(4f_1 + 3f_8 + 10f_{10})(4f_1 + 3f_8 + 16f_{10})} \geq \frac{4f_1 + 4f_8 + 10f_{10}}{(4f_1 + 4f_8 + 9f_{10})(4f_1 + 4f_8 + 20f_{10})}$$

which is satisfied for $f_8 = 1, 2$. This completes the proof.

A.3 Proof for $p = 6, k = 4$ of C-inadmissibility of D_9 for all b

First note that D_9 is inadmissible wrt D_4 but not S-inadmissible.

We now consider unions of D_9 with all generator designs. We note that $D_9 \cup D_2$ (18,9,3) is S-inadmissible wrt $D_3 \cup D_4$ (17,9,3), $D_9 \cup D_4$ (21,12,3) is equivalent to $3D_3$, $D_9 \cup D_5$ (22,8,5) is S-inadmissible wrt $2D_2 \cup D_3$ (21,8,5), $D_9 \cup D_6$ (22,10,2) is S-inadmissible wrt $D_1 \cup D_2 \cup D_3$ (20,10,3) and $D_9 \cup D_7$ (26,7,7) is S-inadmissible wrt $D_2 \cup D_3 \cup D_5$ (25,7,7). We also note that $2D_9$ (22,14,2) is equivalent to $D_3 \cup D_{10}$ and $5D_9$ (55,35,5) is equivalent to $D_4 \cup 3D_{10}$. It is not necessary to consider unions of D_9 with D_8 since D_8 was shown to be C-inadmissible in Section A.2.

Thus it only remains to consider unions of D_9 with D_1, D_3 and D_{10} with $f_9 = 1$. Therefore we must show that every $D = f_1 D_1 \cup f_3 D_3 \cup D_9 \cup f_{10} D_{10}$ is inadmissible wrt or equivalent to some design not containing D_8, D_9 or D_{10} . $D' = f_1 D_1 \cup f_{10} D_2 \cup (f_3 + f_{10}) D_3 \cup D_4$ is the desired design. Thus $b = 6f_1 + 7f_3 + 15f_{10} + 11 > b' = 6f_1 + 7f_3 + 14f_{10} + 10$, $\lambda_0 = 4f_1 + 4f_3 + 10f_{10} + 7$, $\lambda_1 = f_3 + f_{10} + 1$, $\lambda'_0 = 4f_1 + 4f_3 + 6f_{10} + 5$, $\lambda'_1 = f_3 + 3f_{10} + 2$. Hence $\rho < \rho'$ iff

$$\frac{f_3 + f_{10} + 1}{4f_1 + 5f_3 + 11f_{10} + 8} < \frac{f_3 + 3f_{10} + 2}{4f_1 + 5f_3 + 9f_{10} + 7}$$

which is always true. Also $\tau^2 > \tau'^2$ iff

$$\frac{4f_1 + 5f_3 + 11f_{10} + 8}{(4f_1 + 4f_3 + 10f_{10} + 7)(4f_1 + 10f_3 + 16f_{10} + 13)} > \frac{4f_1 + 5f_3 + 9f_{10} + 7}{(4f_1 + 4f_3 + 6f_{10} + 5)(4f_1 + 10f_3 + 24f_{10} + 17)}$$

which can be shown to be always true. This completes the proof of the C-inadmissibility of D_9 .

A.4 Proof for $p = 6, k = 4$ of C-inadmissibility of D_{10} for all b

First we note that D_{10} is inadmissible wrt $D_2 \cup D_3$ but not S-inadmissible. We now consider unions of D_{10} with all generator designs (except D_8 and D_9 because these were shown to be C-inadmissible in A.2 and A.3). We note that $D_{10} \cup D_2 (22,12,3)$ is S-inadmissible wrt $3D_3 (21,12,3)$, $D_{10} \cup D_5 (26,11,5)$ is S-inadmissible wrt $D_2 \cup D_3 \cup D_4 (24,11,5)$, $D_{10} \cup D_6 (26,13,2)$ is S-inadmissible wrt $D_1 \cup D_3 \cup D_4 (23,13,3)$ and $D_{10} \cup D_7 (30,10,7)$ is S-inadmissible wrt $D_3 \cup D_4 \cup D_5 (28,10,7)$. Thus it only remains to consider unions of D_{10} with D_1, D_3 and D_4 . Note that $D_{10} \cup 2D_4 (35,20,5)$ is equivalent to $5D_3$. Therefore we must show that every $D = f_1 D_1 \cup f_3 D_3 \cup f_4 D_4 \cup f_{10} D_{10}$ with $f_{10} > 0$ is inadmissible wrt or equivalent to a design D' not containing D_8, D_9 or D_{10} when $f_4 = 0$ or 1. $D' = f_1 D_1 \cup f_{10} D_2 \cup (f_3 + f_{10}) D_3 \cup f_4 D_4$ is the desired design. Thus $b = 6f_1 + 7f_3 + 10f_4 + 15f_{10} > b' = 6f_1 + 7f_3 + 10f_4 + 14f_{10}$, $\lambda_0 = 4f_1 + 4f_3 + 5f_4 + 10f_{10}$, $\lambda_1 = f_3 + 2f_4 + f_{10}$, $\lambda'_0 = 4f_1 + 4f_3 + 5f_4 + 6f_{10}$, $\lambda'_1 = f_3 + 2f_4 + 3f_{10}$. Hence $\rho < \rho'$ iff

$$\frac{f_3 + 2f_4 + f_{10}}{4f_1 + 5f_3 + 7f_4 + 11f_{10}} < \frac{f_3 + 2f_4 + 3f_{10}}{4f_1 + 5f_3 + 7f_4 + 9f_{10}}$$

which is always true. Also $\tau^2 > \tau'^2$ iff

$$\frac{4f_1 + 5f_3 + 7f_4 + 11f_{10}}{(4f_1 + 4f_3 + 5f_4 + 10f_{10})(4f_1 + 10f_3 + 17f_4 + 16f_{10})} >$$

$$\frac{4f_1 + 5f_3 + 7f_4 + 9f_{10}}{(4f_1 + 4f_3 + 5f_4 + 6f_{10})(4f_1 + 10f_3 + 17f_4 + 24f_{10})}$$

which is true for $f_4 = 0, 1$. This completes the proof of C-inadmissibility of D_{10} .

A.5 Proof for $p = 6, k = 4$ that D_6 can only appear in admissible designs of the form $D_6 \cup fD_7$ with $f \geq 1$.

First we note that D_6 is inadmissible wrt D_5 but not S-inadmissible. We now consider unions of D_6 with the generator designs D_i ($1 \leq i \leq 6$). We note that $D_6 \cup D_1 (17, 7, 1)$ is S-inadmissible wrt $D_2 \cup D_4 (17, 7, 4)$, $D_6 \cup D_2 (18, 5, 3)$ is S-inadmissible wrt $D_3 \cup D_5 (18, 5, 5)$, $D_6 \cup D_3 (18, 7, 2)$ is S-inadmissible wrt $D_2 \cup D_4 (17, 7, 4)$, $D_6 \cup D_4 (21, 8, 3)$ is S-inadmissible wrt $2D_2 \cup D_3 (21, 8, 5)$, $D_6 \cup D_5 (22, 4, 5)$ is S-inadmissible wrt $D_1 \cup D_7 (21, 4, 6)$ and $D_6 \cup D_6 (22, 6, 2)$ is S-inadmissible wrt $3D_2 (21, 6, 6)$. Thus it follows that D_6 can appear in admissible designs only of the form $D_6 \cup fD_7$ with $f \geq 1$. (We have verified that $D_6 \cup fD_7$ is indeed admissible at least for $1 \leq f \leq 6$.)

REFERENCES

- [1] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (I): General theory. Accepted for publication in Technometrics.
- [2] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (II): Optimal designs for $p = 2(1)6$, $k = 2$ and $p = 3$, $k = 3$. Technical Report No. 425, School of Operations Research and Industrial Engineering, Cornell University, May 1979.
- [3] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (III): Optimal designs for $p = 4$, $k = 3$ and $p = 5$, $k = 3$. Technical Report No. 436, School of Operations Research and Industrial Engineering, Cornell University, October 1979.
- [4] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (IV): Optimal designs for $p = 4$, $k = 4$. Technical Report No. 440, School of Operations Research and Industrial Engineering, Cornell University, January 1980.
- [5] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (V): Optimal designs for $p = 6$, $k = 3$. Technical Report No. 441, School of Operations Research and Industrial Engineering, Cornell University, April 1980.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER #453	2. GOVT ACCESSION NO. AD-AC88 308	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Incomplete Blocks Designs for Comparing Treatments with a Control (VI): Minimal Complete Class of Generator Designs for $p = 5, k = 4$ and $p = 6, k = 4$		5. TYPE OF REPORT & PERIOD COVERED Technical Report
7. AUTHOR(s) Robert E. Bechhofer and Ajit C. Tamhane		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS School of Operations Research and Industrial Engineering, College of Engineering, Cornell University, Ithaca NY 14853		8. CONTRACT OR GRANT NUMBER(s) DAAC29-77-C-0003 N00014-75-C-0586 NSF ENG 77-06112
11. CONTROLLING OFFICE NAME AND ADDRESS National Science Foundation Washington, D.C. 20550		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Sponsoring Military Activities: US Army Research Office, P.O. Box 12211, Research Triangle Park, NC 27709, and Statistics and Probability Program, Office of Naval Research, Arlington, VA 22217		12. REPORT DATE April 1980
		13. NUMBER OF PAGES 16
		15. SECURITY CLASS. (of this report) Unclassified
		18a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution Unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Multiple comparisons with a control, balanced treatment incomplete block (BTIB) designs, admissible designs, S-inadmissible designs, C-inadmissible designs, minimal complete class of generator designs, optimal designs		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) On Reverse Side		

The present paper continues the study of balanced treatment incomplete block (BTIB) designs initiated in [1]-[5]. This class of designs was proposed for the problem of comparing simultaneously $p > 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p+1$. In [2]-[5], we gave lists of generator designs, the conjectured minimal complete class of generator designs, a catalog of admissible designs, and tables of optimal designs for $p = 2(1)6, k = 2$; $p = 3(1)6, k = 3$; and $p = 4, k = 4$. This present paper gives our conjectured minimal complete class of generator designs for $p = 5, k = 4$ and $p = 6, k = 4$ based on a generalized notion of C-inadmissibility. At this time we have made no further computations based on these classes of designs. Interested researchers are encouraged to supplement these classes if they are not already minimal complete. _____

- [1] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (I): General theory. Accepted for publication in Technometrics.
- [2] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (II): Optimal designs for $p = 2(1)6, k = 2$ and $p = 3, k = 3$. Technical Report No. 425, School of Operations Research and Industrial Engineering, Cornell University, May 1979.
- [3] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (III): Optimal designs for $p = 4, k = 3$ and $p = 5, k = 3$. Technical Report No. 436, School of Operations Research and Industrial Engineering, Cornell University, October 1979.
- [4] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (IV): Optimal designs for $p = 4, k = 4$. Technical Report No. 440, School of Operations Research and Industrial Engineering, Cornell University, January 1980.
- [5] Bechhofer, R.E. and Tamhane, A.C. Incomplete block designs for comparing treatments with a control (V): Optimal designs for $p = 6, k = 3$. Technical Report No. 441, School of Operations Research and Industrial Engineering, Cornell University, April 1980.

Unclassified