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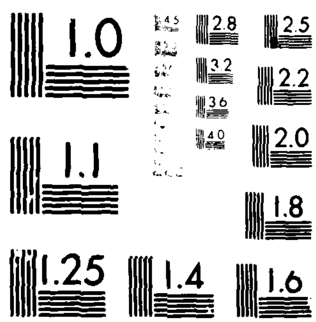

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THESIS

AN EXAMINATION AND EVALUATION
OF THE CONCEPTS OF THE TACTICAL AIR WAR
ANALYSIS GAME (TAWAG).
AN ANALYTICAL 'QUICK GAME MODEL' FOR THE
ASSESSMENT OF LONG-RANGE AIR ARMAMENTS
POLICY OPTIONS (TO BE PUBLISHED).

by

Detlef Erhard/Leger

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An Examination and Evaluation of the Concepts of the Tactical Air War Analysis Game (TAWAG) An Analytical "Quick Game Model" for the Assessment of Long-Range Air Armaments Policy Options (to be published)

by

Detlef Erhard Leger
Major, Federal Republic of Germany Army
Ing.(grad.), Army Academy for Mechanical Engineering, Darmstadt, 1969

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______________________ Second Reader

______________________ Chairman, Department of Operations Research

______________________ Dean of Information and Policy Sciences

ABSTACT

The purpose of this thesis is to investigate and review the basic model structure of a tactical air-ground combat model, the "Tactical Air War Analysis Game" (TAWAG). Existing theater level air/ground combat models (both descriptive and prescriptive) are reviewed and compared with TAWAG on the basis of a number of criteria. Whenever possible an effort is made to draw connections to existing models or point out the differences. The goal of this effort is to present a clear conceptual picture of the TAWAG model and to compare its "heuristic programming" optimization methodology with those used in other models currently used in the USA.

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ABBREVIATIONS

OCA	=	Offensive Counter Air
SAS	=	SAM-Suppression
SAM	=	Surface-to-Air Missile
ABA	=	Air Base Attack
OAS	=	Offensive Air Support
CAS	=	Close Air Support
INT	=	Interdiction
DCA	=	Defensive Counter Air
ABD	=	Air Base Defense
BFD	=	Battlefield Defense
EBA	=	Escort/ABA
EIN	=	Escort/INT
a/c	=	Aircraft
TCM	=	Tactical Cruise Missile
FEBA	=	Forward Edge of the Battlefield
MRCA	=	Multi Role Combat Aircraft

LIST OF SYMBOLS

I. INPUT DATA

I.1 GENERAL

N_T = number of cycles per conflict of duration $T = N_T \Delta t$

Δt = duration of one conflict cycle [time units]

I.2 AIR WAR MODEL

$\left.\begin{array}{l} x_i^A = x_{i,1}^A \\ y_i^A = y_{i,1}^A \end{array}\right\}$ = initial inventories of air systems of type i of sides X and Y

$n_i^{X(Y)}$ = number of runways (bases) for aircraft system i of side X(Y)/number of i-type TCM-units deployed by sides X(Y)

$M_i^{X(Y)}$ = number of air missions that the i-type air system of side X(Y) can perform

$P_{Si}^{X(Y),\mu}$ = basic single sortie survival probability of i-type air system of side X(Y) on mission μ

$P_{Sik}^{X(Y),GF}$ = probability that an i-type ground attack sortie of side X(Y) survives an attack by a k-type fighter of the opponent

$P_{Sik}^{X(Y),FE}$ = probability that an i-type (defense) fighter of side X(Y) survives a duel with a k-type escort fighter of the opponent

$P_{Sik}^{X(Y),EF}$ = probability that an i-type escort fighter of side X(Y) survives a duel with a k-type (defense) fighter of the opponent

$P_{Kik}^{X(Y)}$ = conditional probability that an air system i of side X(Y) is killed on the ground by a k-type ABA-sortie of the opponent

$P_{Bi}^{X(Y)}$ = probability that an i-type system of side X(Y) is on the ground when ABA-attack arrives

$f_{i,\mu}^{X(Y)}(S_k^{X(Y),1})$ = SAS-pay-off function = probability of the enemy's SAM system performance against the i-type air system of X(Y) not being reduced due to SAS by $S_k^{X(Y),1}$ sorties of type k

$g_{i,\mu}^{X(Y)}(H_{ki}^{Y(X)})$ = probability that an i-type air system of side X(Y) can take off for mission μ after an attack on its base by $H_{ki}^{Y(X)}$ k-type sorties of the opponent

$h_{i,\mu}^{X(Y)}(H_{ki}^{Y(X)})$ = ABA-suppression function = probability that ground support organization is at its full capability after an attack on its base by $H_{ki}^{Y(X)}$ k-type sorties of the opponent

$L_i^{X(Y)}$ = i-type (non-recoverable) air system launch rate of side X(Y)

$G_i^{X(Y),\mu}$ = sortie generation rate for (recoverable) air system i of side X(Y) and mission μ (sorties per Δt)

$R_S^{X(Y)}$ = SAM-system regeneration rate of side X(Y)

$T_i^{X(Y)}$ = runway regeneration rate for air systems i of side X(Y)

$Q_i^{X(Y)}$ = i-type ground support organization regeneration rate

I.3 LAND-WAR MODEL

n_S^j = total number of segments of the j-th sector (j = 1,...,J)

$D_{j,s}$ = coordinate of the left boundary of the s-th segment of the j-th sector (j = 1,...,J and s = 1,...,N_s^j)

$Q_{j,s}$ = terrain-trafficability type of the s-th segment of the j-th sector (= 1 for "normal" terrain, = 2 for fortified zones, and = 3 for minefields)

x_k^0, y_k^0 = initial (TO&E) numbers of the k-th X and Y weapon-system types (k = 1,...,$n_{X(Y)}^G$)

$s_{X(Y)}^k$ = firepower score of the k-th X(Y) weapon-system type

$n_j^{CX(Y)}$ = initial number of X(Y) combat units (divisions) in the j-th sector; Y is assumed to try to maintain this number of divisions effective in the sector

n_j^{REX} = total number of time that X replacements are scheduled to enter X's rear region of the j-th sector (n_j^{REX} = 0 means that no X replacements, i.e., divisions, are scheduled)

$n_{j,k}^{RX}$ = number of X divisions that enter as replacements in the j-th sector forthe k-th time (j = 1,...,J and k = 1,...,n_j^{REX}) (n_j^{REX} = 0 means that no X replacements are scheduled)

n_j^{RY} = number of (second echelon) reserve units (divisions) that are initially contained in the Y rear region of the j-th sector

$I_{XC}^{j,0}$ = set of indices of the X divisions (combat units) that are initially contained in the combat region of the j-th sector; there are n_j^{CX} elements in this set

$I_{XSA}^{j,k}$ = indices of the X reserve units that enter the staging area of the X rear region of j-th sector at the k-th time that replacements are so scheduled (n_j^{REX} = 0 means that no input is required)

$T_{XE}^{j,k}$ = time at which X replacements are scheduled to enter the j-th sector for the k-th time; the corresponding time step is computed by $\nu_{E*}^{k} = [T_{XE}^{j,k}/\Delta t+0.5]$

$f_{BP}^{AX(Y)}$ = breakpoint for an attacking X(Y) unit (fraction of initial combat capability at which an attacking X(Y) unit "breaks off" its attack and assumes a defensive posture)

$f_{BP}^{DX(Y)}$ = breakpoint for a defending X(Y) unit (fraction of initial combat capability at which a defending X(Y) unit "breaks off" its defense and starts to withdraw)

f_{CNU}^{Y} = call point for Y replacements (fraction of initial combat capability at which a call is made for a new unit to replace an attacking Y unit)

f_{RU}^{Y} = replacement point for an attacking Y unit (fraction of initial combat capability at which a Y unit is replaced if a replacement is available at the front)

$M_{X}^{l,j,\nu}$ = mission (function) of the l-th X division commander in the combat region of the j-th battlefield sector at time t_{ν}

$M_{Y}^{j,\nu}$ = mission (function) of the j-th Y sector commander at time t_{ν}

$e_{j,\nu}$ = $e_{j,\nu}(M_{X}^{j,\nu}M_{Y}^{j,\nu})$ = engagement type as determined by Table VIII

$f_{X(Y)}(r^{G},e)$ = fractional casualty rate for an X(Y) unit in an engagement of type e as a function of the attacker/defender force ratio r^{G} (see Fig. 13 for typical such curves)

$\frac{\Delta A}{\Delta D}[T_E, (A/D)_0]$ = overall attacker/defender exchange ratio as a function of (1) the duration of the engagement T_E, and (2) the initial (at the beginning of the engagement) attacker/defender force ratio $(A/D)_0$ (special attrition-process option)

$F(\rho;\tau)$ = opposed-movement-rate function, which depends on the attacker/defender force ratio (incorporating the disruptive effects of CAS) ρ; here the tactical situation vector τ is a parameter given by $\tau = (e,Q)$, i.e., the tactical situation is described by the engagement type and the local terrain trafficability (see Fig. 15 for typical such curves)

$K^i_{X(Y)}$ = combat-capability value for FEBA movement of the disruptive, noncasualty producing effects of one CAS sortie by an X(Y) air system of type i $(i = 1,\ldots,n^A_{X(Y)})$

$v_0^{X(Y)}$ = unopposed and noninterdicted movement rate of X(Y) reserve units in transit from the staging area of the X(Y) rear region to the front line

$w_i^{X(Y)}$ = weighting factor for sortie by the i-th X(Y) air system in determining the degradation of the unopposed movement rate of Y(X) reserve units due to enemy air interdiction

$g_{X(Y)}(S_\nu^{Y(X),4})$ = function that determines the fractional reduction of the unopposed and noninterdicted movement rate of X(Y) reserve units due to interdiction sorties in the time interval $[t_\nu, t_{\nu+1})$

$\alpha_i^{X(Y)}$ = combat-capability value of Y(X) ground-force targets destroyed by one successful X(Y) CAS sortie of type i

$\beta_i^{X(Y)}$ = combat-capability value of Y(X) ground-force targets destroyed by one successful X(Y) INT sortie of type i

$T_A^{X(Y)}$ = time required for an X(Y) unit to assemble in staging area after it is ordered to move to the front line from the rear region

$D_R^{X(Y)}$ = distance that an X(Y) unit must travel from the staging area of the rear region to the front line

II. ADDITIONAL SYMBOLS

$R_{X(Y)}^{A(G)}$ = Air (ground) resources (air war systems) available to side x and y

$\nu = 1,\ldots,N_T$ = sequence of rounds to be played

$\sigma_K^{X(Y)}$ = air war strategy of x and y

$\phi_{i,\nu}^{X(Y),\nu}$ = fraction of the number $X_{i,\nu}^A$ and $Y_{i,\nu}^A$ of air system of type i in the inventories of X and Y being allocated to mission μ at time t_ν.

$X_{i,\nu}^{A,\mu}$, $Y_{i,\nu}^{A,\mu}$ = fraction of air war system of type i of X and Y allocated to mission μ

$X_{i,\nu}^{S,\mu}$, $Y_{i,\nu}^{S,\mu}$ = number of systems of type i allocated to mission ν surviving in interval $[t_\nu, t_{\nu+1}]$

μ = type of mission

d_{j,N_T}^K = expected FEBA - movement of sector j for the remainder of the conflict ($\nu = k_1,\ldots,N_T$)

F_K^{ν} = incremental FEBA movement in time-interval ν

$P_{Gi,\nu}^{X(Y)}$ = probability that air system i survives on the ground on $t_\nu \leq t \leq t_{\nu+1}$

$P_{Ti,\nu}^{X(Y),\mu}$ = probability that air system i allocated to mission μ can take off in $[t_\nu, t_{\nu+1}]$

$P_{Di,\nu}^{X(Y),\mu}$ = probability that air system i allocated to mission μ detects target

$P_{Ei,\nu}^{X(Y),\mu}$ = probability that a mission is effective in time interval $[t_\nu, t_{\nu+1}]$

$P_{Ui,\nu}^{X(Y),\mu}$ = probability that air system i assigned to mission μ survives in time interval $[t_\nu, t_{\nu+1}]$

$P_{Gi,\nu}^{X(Y),\mu}$ = ground survivability of an attacked system

$S_{i,\nu}^{X,\mu}$, $S_{i,\nu}^{Y,\mu}$ = number of effective sorties available at time t_ν from air system i for mission μ

$M_i^{X(Y)}$ = number of i-type units deployed

$S_{i,\nu}^{X(Y),E}$ = number of fighters escorting ground attack aircraft

$S_{K,\nu}^{Y(X),F}$ = number of enemy fighters defending against ground attack raids

$S_{i,\nu}^{X(Y),G}$ = number of ground attack air systems

$\eta^{X(Y)}$ = ratio of the number of fighters escorting the ground attack aircraft and the number of enemy fighters defending against ground attack raids

$\gamma^{X(Y)}$ = ratio of the number of AD fighters not engaged by escorts and the number of ground attack aircraft

$\delta^{X(Y)}$ = fraction of air defense fighters which attack enemy attack aircraft

$D_{Ki,}^{Y(X)}$ = average number of air systems of type K attacking i-type air systems on the ground in $[t_\nu, t_{\nu+1}]$

$r_{i,\nu*}(S_{K,\nu}^{Y(X)})$ = reduction factor of the sortie production capability of the ground support organization at time $t_{\nu*} > t_\nu$

$H_{Ki}^{Y(X)}$ = number of K-type enemy sorties attacking per base or runway used by i-type air system

I. INTRODUCTION

Since its introduction in World War I air power has always played an important role in the outcome of ground combat. Since WW II the improvements that aircraft have undergone will make modern warfare more dynamic than ever before, and the allocation of air power will play an ever increasing role in influencing ground combat outcomes. Thus, it is important for the military analyst to help in determining the "best" use of tactical air power.

Starting from the basic premise that tactical air's primary role is in (direct and indirect) support of the ground battle, several models have been developed to allocate weapon systems (of hypothetical general purpose air force alternatives) to the basic tactical air missions in such a way as to optimize a ground war objective or MOE, i.e., FEBA movement. Most earlier attempts to optimize the time-sequential allocation of aircraft to missions have used a system evaluation criteria such as number of tons of ordnance delivered on FEBA by aircarft [Refs. 3,4 and 11]. Such a criteria, however, is a surrogate MOE and does not really quantify the outcome of the ground war.

It is well known that tactical allocations have a significant influence on campaign outcomes and often have proved historically to be frequently much more important than characteristics of equipment or the size of the forces. In military history we find quite a few occasions when inferior forces defeated superior ones due to wise employment of the former and unwise employment of the latter. This, combined with the role air power plays in a battle, makes it absolutely necessary for the operations research analyst to have a means of helping to determine the "best" use of tactical airpower. However, in modelling air war strategies and trying to optimize them, one should not forget that it is the outcome of the ground-war which determines the battle outcome.

In the past, many investigations of optimal air-war strategies (i.e., assignments of tactical aircraft to missions) have not explicitly considered the evolution of the ground battle in their evaluations of air war strateegies [Ref. 11]. TAWAG seeks to determine an "optimal" air allocation strategy within the context of ground-war objectives and to reflect its influence on the outcome of the land battle. Thus it will provide quantitative information to decision makers concerning long-range planning for operational design of future aircraft and eventually being able to investigate trade-offs between air and ground systems.

In Section II.B a comparison of existing models, their criteria and their weaknesses is given. It will provide the reader with an overview of existing efforts in modelling air/ground campaigns and lead the way to understanding this model. Without detailed and thorough understanding of what goes on inside a model, the analyst is likely be left with nothing more than a "black box." Therefore this thesis describes and evaluates

TAWAG. Further investigations could try to test the model's behavior by doing some sample calculations. Wherever possible an effort will be made to draw connections to existing models or point out the differences.

The first part of this effort consists of a literature survey in order to give the reader some background on attrition modeling and on existing models in the USA. In the second part follows the model description with its two basic submodels - the air-war model and the ground-war model - and a description of the optimization methodology.

In the last part, final comments are given on the model in comparison with the other models described in the literature survey.

II. BACKGROUND AND LITERATURE SURVEY

In this chapter a brief review is first given on combat modelling. This effort outlines the two approaches used in the United States for modelling combat attrition in large-scale combat operations:

- detailed Lanchester-type models of attrition in tactical engagements
- aggregated-force casualty-assessment models of attrition in tactical engagements (called firepower-score approach)

TAWAG uses the first approach for modelling the attrition of tactical aircraft in the air war and the latter approach to represent attrition in the ground war. The second part gives a short summary of existing air/ground combat models of both optimizing and nonoptimizing types. Special consideration is given to the optimization methodology and how the models encompass the different air combat missions. The third part summarizes the limitations of the existing models.

A. BACKGROUND INFORMATION ON COMBAT MODELLING

For general-purpose forces and conventional war, combat models may be considered to fall into two categories:

(1) those that treat specific combat actions, e.g., a small unit infantry firefight, tank engagement or air-to-air duel in detail;

(2) those that analyze large confrontations of force aggregations in a campaign.

Detailed models that simulate two-sided engagements represent the highest degree of realism. But since the combat interactions as well as the influences of the terrain (line-of-sight), etc., are microscopically analyzed and studied, the sheer immensity of the volume of the details for the assessment of combat outcomes is so overwhelming that not even large scale computers are today able to handle large scale operations by detailed Monte Carlo techniques. This technique is limited to the first category (ASARS II- platoon, DYNTACS, CARMONETTE - company, battalion). Analytical models (e.g., differential equations) are also used for small unit campaigns.

For large-scale campaigns two approaches are used: analytical models (DIVOPS, VECTOR II) and firepower-score models (ATLAS, GACAM, TCM). Both types are much more abstract than Monte Carlo simulations. The approaches mainly used in the United States for modelling combat attrition (see also Taylor [Ref. 20]) are

- detailed Lanchester-type models of attrition in tactical engagements,
- aggregated-force casualty-assessment models based on the use of index numbers to quantify military capabilities.

The first approach dates back to 1914 when F.W. Lanchester[1] hypothesized that combat between two opposing homogeneous forces

[1] F.W. Lanchester: Aircraft in Warfare: The Dawn of the Fourth Arm - No. V, The Principle of Concentration, Engineering 98 (1914). Reprinted on pp. 2138-2148 in: The World of Mathematics (Newman, Ed.), Simon and Schuster, New York 1956).

could be modelled by

$$\frac{dX}{dt} = -aY \quad \text{with } X(0) = X_o,$$

$$\frac{dY}{dt} = -bX \quad \text{with } Y(0) = Y_o,$$

where X(t) and Y(t) denote the numbers of X and Y combatants, and a and b are constants that are today called Lanchester attrition-rate coefficients. These attrition-rate coefficients represent the fire effectiveness of individuals in each of the two homogeneous forces. More recent extensions of Lanchester's original work and enrichments can be found in References 17 and 20. Also not free of shortcomings, Taylor[2] has pointed out that

> "Lanchester's simple differential equation models are quite reasonable. They yield results that are in consonance with military judgement."

TAWAG uses this approach in the air-war submodel for modelling the attrition of tactical aircraft.

The second approach, often called the firepower-score approach is basically a technique for aggregating heterogeneous forces into a single homogeneous force on each side. It develops one number (referred to as the firepower index) to represent the "combat potential" of a unit. In large-scale ground-combat models, firepower indices are used as a surrogate for unit strength. They are then in general used to

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[2] J.G. Taylor: Lanchester-Type Models of Warfare, Chapter 2, p. 110, July 1977 (to be published).

(1) dtermine engagement outcomes,

(2) assess casualties,

(3) determine FEBA movement.

Although much criticized [Ref. 19] Stockfish [19, p. 128] also claims that no satisfactory simple technique for aggregating modern conventional forces currently exists. Therefore, since conventional forces must be aggregated in many analyses, and until a better alternative is developed, firepower scores will continue to be used. TAWAG uses this approach to represent attrition in the ground war.

B. REVIEW OF AIR/GROUND COMBAT MODELS

The review considers tactical air campaign models for determining a "good" allocation of a/c to missions. These range from player participation to game theoretic optimization approaches. The optimization models while they do determine the optimal strategies for aircraft allocations, require simplifications and abstractions in model development for reasons of mathematical tractability in the determination of optimal strategies. The fixed strategy models on the other hand are higher resolution simulations but are too complex to allow for optimization.

Earlier attempts on modelling air/ground combat are those by A. Mengel [Ref. 16], Fulkerson and Johnson [Ref. 10] and Berkowitz and Dresher [Refs. 3 and 4]. The conclusions concerning aircraft allocation range from splitting between ABA (period 1 to N in a finite n period game) and CAS (period N+1 until n) [Ref. 10] or, depending on pure or mixed strategies, splitting between AD and ABA in the case p/q < 2.7 and splitting between CAS, AD and ABA using the pure strategy or allocating the entire force to AD, ABA or CAS randomly [Ref. 4] in the mixed case when p/q > 2.7 where

p = Blue air strength

q = Red air strength.

Other models included in this review for comparison are the LULEJIAN model, DYGAM, OPTSA and TAC CONTENDER as optimizing models, and VECTOR, TAGS, TALLY, MINICOM, ATLAS and IDAGAM as non-optimizing approaches.

An important problem associated with a tactical air campaign is that of allocating the tactical air force among the various air tasks in a competitive environment.

Table I shows the missions treated by each of the above mentioned models as well as by TAWAG and MAMOTAC. Although TAWAG encompasses the largest set of missions of all models and thus allows more flexibility and perhaps a higher level of real world detail, this might as well result in higher complexity and increased computation time.

Of the optimizing models, DYGAM and OPTSA consider only three basic missions. Except for TAC CONTENDER, TAWAG is the only model that differentiates between the defensive counter air mission of airbase defense (ABD) and battlefield defense (BFD). Except for LULEJIAN, VECTOR and TAGS, the latter two

MISSIONS REPRESENTED IN TAWAG AND OTHER MODELS

Role / Mission	OCA		OAS		DCA					
					*				*	
Model	SAS	ABA	CAS	INT	ABD	BFD	EBA	EIN	(INTER-CEPT)	OTHERS
LULEJIAN	X	X	X	Log.	X		X	X	X	
DYGAM		X	X						X	
OPTSA		X	X						X	
TAC CONTENDER		X	X		X	X				
VECTOR	X	X	X				X	X	X	X
TAGS	X	X	X	Log.			X	X	X	
TALLY		X	X	Mob					X	X
MINICOM		X							X	
TAWAG	X	X	X	X	X	X	X	X	←	
MAMOTAC	X	X	X	X	X					

* only TAC CONTENDER and TAWAG split the intercept missions in ABA and BFD

Table I

non-optimizing models, none accounts for fighter escort and defense suppression missions beside TAWAG.

The distinction between ABD and BFD seems to be more real because it is most likely that BFD will be preassigned to CAP stations near the front lines and ABD similarly deployed in rear ares (see also Ref. 11, p. 7).

1. Model Descriptions and Comparison

a. LULEJIAN Model [Ref. 26]

The model, which discribes activities involving Army units and Air Force tactical aircraft in mid-intensity warfare of the theater level, was developed for use in making net assessments, and in generating information for use in tradeoffs among weapon systems. Rather than using firepower-score-approach to determine attrition and FEBA movement, the model is based upon a concept of trading space fo survivability. As a result of each side contacting and locating one another's forces for direct and support fire, which includes that provided by air, losses are sustained by both sides. The daily movement of the FEBA is associated with the actual attrition of opposing forces relative to their acceptable levels.

The Lulejian Model consists of a set of aggregate submodels combined in a framework designed to allow determination of enforceable outcomes. The five basic submodels are: a Logistics and Interdiction Model, a Tactical Air Model, a Resource Allocation Algorithm, a Ground Force Management Model and a Ground Combat Assessment Model.

An aircraft strategy is specified in the model by assigning a cardinal number to each of five missions considered and thus indicating its priority. The user is required to provide a matrix specifying the order of preference of available a/c types for each mission. The model translates then, for a given force of a/c, the 18 possible strategies into a decision vector. The set of decision vectors is, however, quite sparse compared to that which would result if the TAC CONTENDER, DYGAM or OPTSA (sections B.1 d, b and c respectively) approaches were applied to the case of multiple a/c types.

The Lulejian Model employs a resource allocation algorithm that attempts to determine the approximate game value and approximately optimal strategies for both players at each stage of the game. The basic approach is to decompose the problem into the solution of many one-move games. It is very similar to the "successive sweep" method employed by TAC CONTENDER (see B.Ld, also [Ref. 27]).

b. DYGAM

The DYnamic GAMes solver was developed at Control Analysis Corporation [Ref. 25] to solve general multi-stage games. The general solution technique used is based upon dynamic programming. The main computational limitation of dynamic programming is the dimension of the state space. In order to solve a multi-stage game with dynamic programming, it is necessary to determine (or estimate) the optimal payoff for each state and stage of the game. The approach taken is to approximate these payoff functions with polynomials. Thus it is only necessary to estimate the polynomial coefficients for each period, rather than the actual payoff function. However, this approximation approach may give inaccurate answers due to errors in the approximation. Hence, the polynomials are not used directly to provide the payoff estimate for each initial state, rather, they are used as guideposts for a forward evaluation. The refined estimate of the game value is obtained by a Monte Carlo approach which traces out a number of possible paths through the various stages of the game, each of which results in a different total payoff. These results are averaged to yield the forward value, and are also used to estimate the standard deviation of possible outcomes.

DYGAM (like OPTSA) addresses the behavioral game.[1] Although it obtains only approximate answers, computational experience shows that they were very close to the optimal answers produced by OPTSA. In addition DYGAM is not limited to 3 stages as OPTSA. It has been used to analyze 90-stage games.

The ability to estimate the standard deviation of possible outcomes is a useful feature. Thus, the game value in a behavioral game is the expected total payoff resulting when both players use their optimal probabilistic strategies.

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[1]A behavioral game is an extended game (one which uses expected payoff) which considers only behavioral strategies, where a behavioral strategy is an adaptive one that tells a player what actions to take based on the current system state.

c. OPTSA [Ref. 6]

The OPTimal Sortie Allocation models have been developed at the Institute for Defense Analysis (IDA) for computing percentage assignments of general-purpose aircraft to missions by period, where assessments of occurrences during the war are performed for a specified number of sub-periods within each period. The overall model is a zero-sum, two person, sequential game. It only addresses three missions (see Table I). Aircraft are of four types: general-purpose, special-purpose CAS, special purpose ABA and special-purpose INT. The model optimally assigns only MRCA. The assignments of the special-purpose aircrafts are fixed.

Aircraft assignments are performed within the context of a ground/air war. Three types of ground units on each side, differentiated only by firepower per unit are considered. Total firepower, consisting of firepower contributed by ground units plus firepower contributed by CAS, is used in forming force ratios for calculating casualties to ground forces and FEBA movement. The model exists in two different versions, OPTSA I and OPTSA II.

The OPTSA I model includes nine non-adaptive games, a game type in which the time-phased aircraft assignments of each side do not depend upon the previous history of the time-phased assignments of the opposite side.

There are three measures of effectiveness, each for three specified times during a fixed-duration war:

(1) FEBA position,

(2) Cumulative Blue-minus-Red ground-plus-air firepower,

(3) Cumulative Blue-minus-Red air firepower.

According to the authors, the set of all possible time-phased assignments considered in the model is called the set of strategies. If the solution on both sides is pure (does not involve randomizing over the set of strategies or, equivalently, places probability of unity on one of the deterministic strategies), then the related adaptive game, where knowledge of moves made by both sides is used as the game progresses, has the same solution and value. The OPTSA II model involves a behavioral game. The measure of effectiveness of the behavioral game may be any of the nine measures of OPTSA I. The behavioral game involves randomization over the actions at each stage, where the probability distribution over which the randomization is made depends upon the actions selected at previous stages.

In Reference 6, p. 982, the developers state "The OPTSA models solve an aggregated problem exactly." Their quest for optimality was motivated by their considerable doubt that the solutions to games obtained by other procedures are optimal for the games being played. Thus OPTSA is the only model that employs a mathematically rigorous optimization technique which guarantees finding an optimal solution to a staged game on the mutual allocation of tactical air resources.

OPTSA's limitation to three stages, however, must be seen to be serious when compared to other models.

d. TAC CONTENDER

TAC CONTENDER was developed by the Office of the Assistant Chief of Staff, Studies and Analysis, U.S. Air Force (AF/SA) [Ref. 27]. It is an "air only" computer model which simulates the interactions between two opposing tactical air forces in a campaign of extended duration. The interactions between ground forces are not modeled, but the impact of "air" on the ground mission is accounted for by recording the cumulative amount of ordnance delivered in CAS.

The war is modelled as a multi-stage, two-person, zero-sum game. The model attempts to optimize the strategies used by both sides and thus measure the capabilities of the fighting forces by estimating what outcome of the war could be, if both sides fought optimally. The payoff function is:

$$M = \sum_{i=1}^{n} \text{(Blue Tons on day i)} - \text{(Red Tons on day i)}$$
$$+ \text{(Total Blue Potential Value on day n+1)}$$
$$- \text{(Total Red Potential Value on day n+1)}$$

TAC CONTENDER attempts to determine a pair of allocation strategies which guarantee each side certain "minimal outcomes". If a side were to choose any other allocation it could do no better and would probably do worse.

Although the current version of TAC CONTENDER can address several types of aircraft (up to 10), it basically considers only one type of aircraft for each side in its optimization machinery (by converting the different aircraft types to "equivalent" ones).

Of the two general approaches to solve sequential and differential games - dynamic programming and the variational approach - the model uses the latter by identifying a set of necessary conditions that solutions in pure strategies must satisfy if they are to be solutions of the game G = (U,V,K) (U and V being the strategy sets available to Blue and Red respectively and K the payoff function for Blue), and then employing a discrete analogue of a technique known as the "successive sweep method" normally used for optimal control problems with differential equations defining the state of variables.

2. Non-Optimizing Models

a. VECTOR-O [Ref. 29,30]

The model, which describes activities involving Army units and Air Force tactical aircraft in mid-intensity warfare at the theater level, was developed for use in estimating net assessments performing force deployment studies, and in generating information for use in tradeoffs among weapon systems. Rather than using the firepower score/force ratio concept to determine attrition and FEBA movement, VECTOR-O uses Lanchester-type equations that are based on physically oriented submodels of those combat processes whose parameters are more readily measurable.

The first steps of air-war calculations include bookkeeping functions such as determining the number of sorties flown on each mission per day and grouping aircraft for flying their missions. Then attack aircraft (possibly with escorts) are followed on various attack missions through a sequence of events such as air-to-air combat, target area ground-to-air attrition, delivery of ordnance on a/c targets. Following these computations, results are cumulated over all groups and missions to give expected number of destroyed a/c and expected damage to ground targets. Before any air-war effects are computed for a given battle day, tactical rules - as inputs by the user - are employed to assign available a/c to missions.

The model of Air-to-Air Combat assumes that an escorted attack group on mission m is detected by its opponent's warning and control system and that interceptors can be vectored to the penetrators with a user input probability. Then a given sequence of events is followed which concern the interactions of interceptors, escorts and attack aircraft.

For the Ground-to-Air Attrition Red and Blue are assumed each to possess two kinds of ground-based air defense weapon sites - longe range and short range - uniformly deployed within each sector for a given side. Following air-to-air combat, an aircraft group is assumed to pass through a portion of these randomly located defenses en route to their target, to encounter any air defenses defending the target, and finally to pass through a portion again while returning to friendly territory.

As for TALLY and MINICOM, the approach for attrition calculations used for VECTOR puts a large burden on the user because almost all activities are controlled by user-specified tactical decisions. If detailed submodels and analyses are available to generate realistic values for all of the required quantities, the general approach may be reasonably satisfactory. However, it is often the case that at least some of the required quantities are functionally dependent on the precise numbers and mixture of aircraft types used in a particular situation, and these functional dependencies are often obscured, *e.g.*, the air-to-air attrition rate in VECTOR not being a function of the number of aircraft assigned to air defense or the FEBA movement rates depending on the activity being performed (advance, pursuit, etc.) for each of the maneuver forces at the FEBA but not being a function of the air-ground interactions.

b. TAGS [Ref. 9,11]

TAGS-V (Theater Air-Ground Study) is a highly aggregated computer model that relies on simplified abstractions of elements that are treated in greater detail in more comprehensive models of theater warfare. The model simulates ground and air activities on a day-to-day basis. The ground units in TAGS-V are defined in terms of homogeneous division equivalents, i.e., no distinction is made between armoured or mechanized divisions and infantry divisions. The FEBA moves as a unit and is viewed as the average movement of the entire theater front.

The airforces consist of three aggregated aircraft types - fighters, attack a/c, and bombers - that may be allocated among six different missions: AD, ABA, INT, CAS, SAS and ES. The CAS mission has two primary effects. First, it produces casualties among ground combat personnel, and, second, it influences the FEBA movement. Counter SAM (SAS) missions are designed to destroy area-deployed SAMs in a rollback operation that clears corridors for subsequent deep penetrations by aircraft on interdiction, counter airfield, and escort missions.

The ground forces defend against the offensive air strikes by means of antiaircraft fire and SAMs. The ground combat is quite simplified. Casualties resulting from conventional ground combat are based on planning factors roughly derived from statistical records of World War II and Korean ground actions, and are related to the ratio of combat strengths of the forces in combat. Additional casualties are inflicted by CAS sorties.

The TAGS-V input parameters fall into three categories:

(1) force size and effectiveness factors,

(2) aircraft allocation by mission,

(3) a set of historically-based empirical factors that characterize casualties and movement of ground warfare.

Any of the model parameters can be changed as a function of the value of any other during the course of the conflict. As a special feature any model parameters can be specified as having an uncertain value.

c. TALLY [Refs. 31,32]

TALLY is an air battle computer model developed at the Rand Corporation specifically for use in a study of the effects of various allocations of NATO tactical air resources on a conventional land battle in the Central Region of Europe.

The basic functions of the TALLY program are

(1) the allocation of sorties to five missions: ABA,"mobility interdiction" MI, CAS, RCN, AD;

(2) computation of expected aircraft losses both on the ground as a result of ABA attacks and in the air as a result of either air-to-air or surface-to-air defenses;

(3) the computation of the expected number of weapon-carrying CAS and/or MI sorties which survive defenses and arrive over their targets.

This latter class of sorties can be used as input values to a ground battle model such as the Rand Theater Operations Tactical Evaluation Model (TOTEM).

The current TALLY program assumes a four-cycle day, beginning at midnight, where cycles 1 and 4 are night cycles and 2 and 3 are day cycles. The user has to define and specify many values, e.g., number of cycles per day (optional), airbases, including the types of a/c at the base, beginning a/c inventory at each base, sortie generation rate disruption factor, air-to-air attrition rates. The expected number of a/c located on a base at the time a base is subject to an airbase attack is computed within the program based upon the input sortie rates of the aircraft on the base, their abort rate, sortie length, the disruption factor specified, and the number of daylight hours.

If TALLY is used in conjunction with TOTEM, the function of TALLY is to provide sorties of specified aircraft type, weapon type, and delivery mode in attacks against specific targets in the ground war simulation. The TOTEM model is then used to determine the impact of the tactical air sorties on the course of the ground war.

C. LIMITATIONS OF EXISTING MODELS

1. Optimizing Models

LULEJIAN [Refs. 11,26]	1.	weakness of optimization methodology
	2.	limitation in the number of decision vectors considered
	3.	user-specification of priorities concerning missions might cause conflicts with A/C characteristics
	4.	no distinction between ABD and BFD.
DYGAM [Refs. 11,25]	1.	treats only one A/C type
	2.	use of simplistic attrition equations
	3.	provides no bounds
	4.	no distinction between ABD and BFD
	5.	no escort or suppression missions considered.

OPTSA [Refs. 3,11]	1. long computer running-time caused by insistence on exact optimality 2. limited number of stages 3. no distinction between ABD and BFD 4. no escort or suppression missions considered.
TAC CONTENDER [Refs. 11,27]	1. incorrect treatment of expected values 2. weakness of optimization methodology 3. treatment of multiple A/C types is questionable 4. neither escort nor suppression missions considered 5. inconsistencies in attrition relationships.

2. Non-Optimizing Models (Limitation Itself)

VECTOR [Refs. 11,29 30]	1. ABD and BFD not individually treated.
TAGS [Refs. 9,11]	1. ABD and BFD not individually treated 2. shelters not destroyed.
TALLY [Refs. 11,31, 32]	1. ABD and BFD not individually treated 2. neither escort nor suppression mission considered 3. weakness of attrition methodology (e.g., air-to-air attrition rate not a function of number of AD A/C.
MINICOM [Ref. 11]	1. limited treatment of air-to-air attrition 2. CAS not considered in detail 3. BFD and ABD not treated individually.
ATLAS [Ref. 15]	1. parameters are difficult to determine and to analyze

	2.	highly aggregated and low resolution.
IDAGAM [Ref. 2]	1.	deterministic model
	2.	no relation between rate of advance and CAS effectiveness
	3.	suppression not considered
	4.	more detail than can be supported by realistic data.

SUMMARY OF AIR/GROUND MODELS I

MODEL	CLASS	SCOPE	MOE OR OUTPUT	A/C MISSIONS ENCOMPASSED	ATTRITION METHODOLOGY	OPTIMIZATION MACHINERY
LULEJIAN	STAGED GAME	AIR/GROUND THEATER LEVEL	FEBA MOVEMENT, CASUALTIES	AD,SUPP INT CAS ES	EXP SUBMODELS	DECOMPOSES PROBLEM INTO MANY 1 MOVE GAMES (APPR) "SUCCESSIVE SWEEP" METHOD
DYGAM	STAGED GAME (BEHAVIORAL)	SOLVES AIR CAMPAIGN OF BERKOVITZ AND DRESHER	ESTIMATE OF STANDARD DEVIA-TION OF POSSIBLE OUTCOMES	CAS ABA AD	BERKOVITZ AND DRESHER RELATIONS	DYNAMIC PROGRAMMING (APPROXIMATION)
OPTSA	STAGED GAME I-NONADAPTIVE II-BEHAVIORAL LIMITED TO 3 STAGES	AIR/GROUND 3 TYPES OF GROUND UNITS, 4 TYPES A/C OPTIMIZES FOR 1 A/C ONLY	FEBA POSITION CUMULATIVE BLUE-RED GROUND AND AIR FP, CUMULA-TIVE B-R AIR FP	CAS ABA AD	EXP	EXHAUSTIVE SEARCH GUARANTEES OPTIMALITY
TAC CONTENDER	STAGED GAME	AIR ONLY 10 TYPE A/C, 1 ONLY FOR EACH SIDE IN OPT. MACHINERY	NO A/C SURVIVING CUMULATIVE DIFF. IN TONS OF ORDNANCE DELIVERED IN CAS	CAS ABA AD BD	SUBMODELS EXP	LAGRANGIAN-LIKE METHOD NECESSARY CONDITIONS (DISCRETE ANALOG OF "SUCCESSIVE SWEEP" METHOD)

Table II

SUMMARY OF AIR/GROUND MODELS II

MODEL	SCOPE	OUTPUTS OR PURPOSE	A/C MISSIONS CONSIDERED	ATTRITION METHODOLOGY	METHOD
VECTOR	AIR/GROUND THEATER, MID-INTENSITY	FORCE DEPLOYMENT STUDIES, WPNS TRADEOFF	CAS, AD ABA, SUPD ES	USER SPECIFIED FUNCTIONS	BREAK UNITS INTO SMALLER ELEMENTS OR SYSTEMS, MODELS ACTUAL PHYSICAL PROCESSES AND USES NO FP OR FR
TAGS	AIR/GROUND THEATER 3 TYPE A/C	FEBA MOVEMENT DIVISION LOSSES A/C LOSSES A/C SURVIVORS	CAS, AD ABA, SUPP INT ES	EXP	HIGHLY AGGREGATED SIMULATION
TALLY	AIR ONLY INPUT FOR TOTEM	E (A/C LOST) E (# A/C ON TRGT IN CAS OR INT) INPUT TO GROUND MODEL TOTEM	CAS, AD ABA, RCN INT	ATTRITION RATES SUPPLIED BY USER	EXPECTED VALUE SIMULATION
MINICOM	AIR ONLY	MAINTAINS STATE OF AIR FORCE	ABA INCPT	USER SPECIFIED	EXPECTED VALUE COMPUTER/ HUMAN INTERFACE
ATLAS	AIR/GROUND, LOGISTICS THEATER 3 TYPE A/C	FEBA, EVAL. FORCE DEPLOYMENT	ABA, AD CAS, SUPP INT, RCN INTCP		FAST COMPUTER PROGRAM
IDAGAM	AIR/GROUND THEATER	FEBA	ALL		DETERMINISTIC FIXED TIME STEP

Table III

III. OVERALL MODEL STRUCTURE OF THE TAWAG MODEL

Since ATLAS was developed in the U.S.A. about 15 years ago and has subsequently become the most widely used theater level model, efforts by model developers to add more realism and to assess weapons systems in a wider context to assure their marginal effectiveness and their compatability with operational and strategic objectives lead to a "complexity crisis."

The use of such models, in particular large scale theater level war games, proved that their immediate potential for the assessment of lông range objectives for armaments planning to be rather limited. High level aggregated analytical or deterministic (closed) simulation models seem to be better suited for this purpose. They provide the capability to process large numbers of force structures and environments. This is due to the uncertainties associated with the forecast of long range developments.

Furthermore the assessment of force structure and armaments alternatives on the basis of mutually optimal employment concepts, (see Berkovitz and Dresher [4], Galiano and Miercot [11]), which have proved to often have a greater impact on the outcome of battles as equipment and force size, make it necessary to develop models which reflect the interdependencies of size, structure, equipment and employment of military forces.

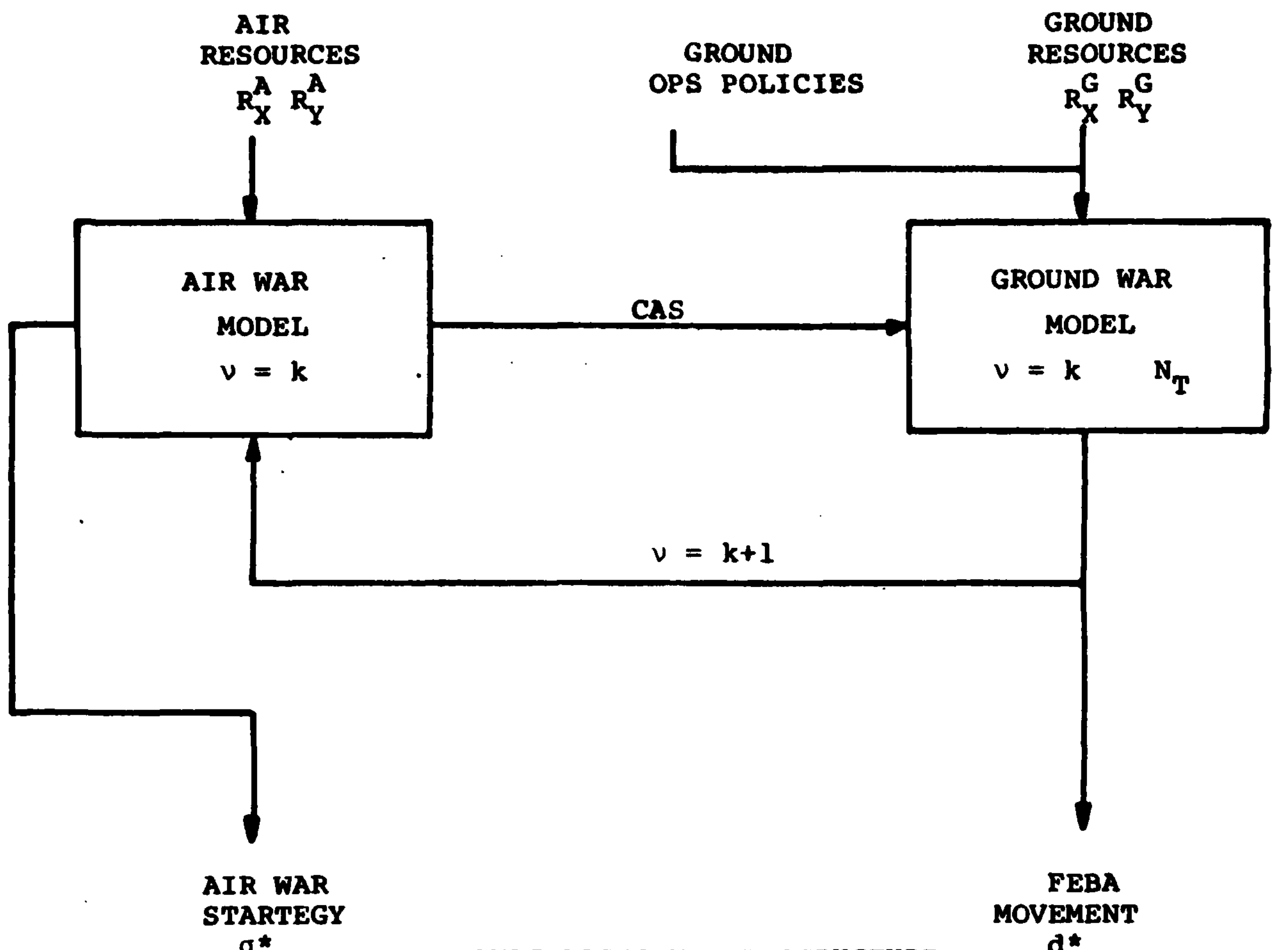
The Tactial Air War Analysis Game (TAWAG) by R. Huber and J. Taylor accounts for these considerations. It is to be considered as an element of a "hierarchically structured compound gaming approach, in which "Quick Games" are used as cursory tools to check the principal viability of innovative structures, technologies and operational concepts, and as screening devices to reduce system variability and establish dominant alternatives for subsequent feasibility analysis by means of "Research Games."

A. THE MODEL

TAWAG is an anlytical "Quick Game Model" to aid decision makers in the assessment of long-range air armaments policy options within the context of high-intensity conventional war in Central Europe.

It may be thought of as consisting of two related time sequential combat models, each of which, in turn, feeds the other. These sub-models are air combat and ground combat. The basic model structure is shown in the following figure 1. The main inputs to both models are related to the combat resources R available to each side, e.g., air combat systems (A), ground combat units (G), their initial deployment, and their performance in the form of:

- sortie survivability ($P_{Si}^{X(Y),\mu}$, $P_{Si}^{X(Y),GF}$, $P_{Si}^{X(Y),EF}$, $P_{Si}^{X(Y),FE}$),
- ground survival probability ($P_{Kik}^{X(Y)}$),
- SAS-payoff function ($f_{i,\mu}^{X(Y)}(S_k^{X(Y),l})$),
- suppression effects ($h_{i,\mu}^{X(Y)}(H_{ki}^{Y(X)})$),



TAWAG BASIC MODEL STRUCTURE

Figure 1

- air system launch rate ($L_i^{X(Y)}$),

- sortie generation rate ($G_i^{X(Y),\mu}$),

- SAM-system regeneration rate ($R_S^{X(Y)}$),

- runway regeneration rate ($T_i^{X(Y)}$),

- ground support organization regeneration rate ($Q_i^{X(Y)}$),

- firepower score of k^{th} weapon system ($s_{X(Y)}^k$),

- breakpoints for attacking or defending units ($f_{Bp}^{AX(Y)}, f_{Bp}^{DX(Y)}$),

- replacement point for an attacking unit (f_{RU}^Y),

- casualty rate ($f_{X(Y)}(r^G,e)$), and

- opposed-movement-rate function ($F(\rho,\tau)$.

The modelling of the attrition of tactical aircraft in the air war is based on Lanchester-type attrition models. For the representation of attrition in the ground war the firepower-score approach is used.

TAWAG considers the conflict as a sequence of $\nu = 1, \ldots, N_T$ stages at the beginning of each of which the attacker and defender decide upon their employment strategies σ_k^X and σ_k^Y for a round K such that the expected penetration d of the attacker's land forces into the defender's territory during the remainder of the conflict ($\nu = k_1,\ldots,N_T$) is as small as possible from the defender's viewpoint and as large as possible from the attackers viewpoint.

For each given pair of opposing forces $R_X(R_X^A, R_X^G)$ and $R_Y(R_Y^A, R_Y^G)$, the model determines the air war strategy σ^* using "heuristic programming" which reflects the mutually best employment policies for the air resources R_X^A and R_Y^A in terms of their direct and/or indicrect support of the land battle with the air resource mix alternatives $R_{X\ell}^A = R_\ell(x_1^A, x_2^A, \ldots, x_i^A, x_{n_X^A}^A)$ and $R_{Y\ell}^A$ the air resource mix preferred by X is obtained from

$$R_{Xg}^A \succ R_{Xh}^A \qquad d^*(R_{Xg}^A) \prec d^*(R_{Xh}^A)$$

and the mutually "best" policy for air resource planning is approximated from

$$D = \min_{\ell_X \varepsilon R_X^A} \max_{\ell_Y \varepsilon R_Y^A} d_\ell^*(R_{X1}^A, R_{Y1}^A, R_X^G, R_Y^G)$$

where $\succ$ stands for preferred, meaning that air resource mix alternative R_{Xg}^A is preferred over R_{Xh}^A iff the resulting penetration d* of the attacker's land forces is smaller using R_{Xh}^A.

B. THE OPTIMIZATION METHODOLOGY

1. The Optimization Problem

There exist several air-ground combat models which use optimization techniques to find optional solutions [Ref. 11]. But the optimization models, or normative models, while determining the optimal strategies for aircraft allocations, require simplifications and abstractions in model development for reasons of mathematical tractability in the determination of optimal strategies. The fixed strategy or descriptive models on the other hand are higher resolution simulations but are too complex to allow for optimization.

The state-of-the-art for determining optimal combat strategies is still fairly primitive and can only consider relative small problems (in the sense of only a few state variables). Most tactical air campaign models which attempt to follow an "optimal" allocation policy which maximizes the force's effectiveness use an approach based on game theory or differential games. Such approaches generate time-dependent strategies for both sides that attempt to achieve a saddle point solution or mutually-enforceable outcome, in the sense that neither side can achieve a better result by using a different strategy against the other side's optimal strategy. Depending upon the representation of time by either a continuous or a discrete variable, the optimization problem is called a differential game or a multi-stage game, respectively.

Solving multi-stage or N-staged games presents a severe methodological challenge. In order to obtain solutions within reasonable computer running times (or at all), a number of simplifying assumptions and approximations are usually employed. Some analysts feel that the resulting lack of real-world detail in the model greatly limits its usefulness. On the other hand, models employing prespecified, heuristically-based, or player-specified strategies can reflect more operational detail, since they are not encumbered by optimization theory.

Another problem with using optimization relates to the dimension of the strategy set. For example, the allocation of a multi-role aircraft to the eight tactical missions as specified in TAWAG, see Table VI, with a min allocation fraction of 1/8 (to permit simultaneous operations) results in 6435 decision vectors σ_ν at each point in time, and since the dimension of the strategy set increases exponentially with the number of decision points or rounds of the game, a 4-round conflict would require more than 1.7×10^{15} allocations to be considered for the side operating the multi-purpose system. Even if one would consider only two different aircraft types, a penetrator for the offensive roles and an intercepter/escort-fighter for the defensive missions, a four-round conflict would result in 230 decision vectors with the strategy set comprising about $3 \cdot 10^{12}$ possible strategies. Therefore, the existing two-sided air campaign models optimizing allocation strategies, as TAC CONTENDER, OPTSA, DYGAM, LULEJIAN, MAMOTAC and ATACAM, are, except for ATACAM, either restricted with respect to the number of missions they consider or the number of rounds the air war may last or both.

Since the authors of TAWAG felt that the explicit consideration of the eight essential tactical air missions, including the distinction between close air support and (mobility and firepower) interdiction, which is not considered by any of the models mentioned before (DYGAM, OPTSA, TAC CONTENDER, VECTOR) [see also Ref. 11], was mandatory for its purposes, they found that none of the solution methodologies of the other optimization models could be employed in TAWAG.

2. Approximation Of the Optimal Air War Strategy

The optimization methodology in TAWAG decomposes the N-staged strategy selection game into N sequential zero-sum games for each of which the decision vector σ_ν ($\nu = 1,\dots,N$) is determined under the assumption that subsequent to t_ν no air resources are committed by either side, i.e., the conflict continues as a pure land war for $t_{\nu+1},\dots,N$. This methodology does account for deferred air/ground effects (e.g., of mobility interdiction, regeneration of resources). It also maximizes the more immediate gains to be obtained in the land battle from the employment of tactical air resources rather than the average gains over longer periods.

At the beginning of a round ν, i.e., interval $[t_\nu,t_{\nu+1}]$ there are $x^A_{i,\nu}$ and $y^A_{i,\nu}$ of air systems of type i in the inventories of X and Y. Then $0 \leq \phi^{X(Y),\mu}_{i,\nu} \leq 1$ is defined as the fraction of the numbers $x^A_{i,\nu}$ and $y^A_{i,\nu}$ being allocated to mission μ at time t_ν.

Also TAWAG defines the air war strategy σ as a rule for determining the sequence of decisions $\sigma_1,\sigma_2,\dots,\sigma_\nu,\dots,\sigma_{N_T}$ concerning the allocation of air resources $i = 1,\dots,n_{X(Y)}$ of the opponents X and Y to the air mission $\mu = 1,\dots,M$ resulting in

$$\sigma_\nu^X = \sigma_\nu(\phi_{1,\nu}^{X,1},\ldots,\phi_{1,\nu}^{X,\mu},\ldots,\phi_{1,\nu}^{X,M};\ldots$$
$$\phi_{i,\nu}^{X,1},\ldots,\phi_{i,\nu}^{X,\mu},\ldots,\phi_{i,\nu}^{X,M};\ldots$$
$$\phi_{n_X,\nu}^{X,1},\ldots,\phi_{n_X,\nu}^{X,\mu},\ldots,\phi_{n_X,\nu}^{X,M})$$

σ_ν^Y respectively.

Each strategy or decision $\sigma_\nu^{X(Y)}$ will result in a certain number $s_\nu^{X(Y)}$, of effective sorties in mission μ at time t_ν under the assumption that all available systems are allocated, i.e.

$$\sum_\mu \phi_{i,\nu}^{X(Y),\mu} = 1, \quad \forall i,\nu.$$

Another prerequisite is that $\phi_{i,\nu}^{X(Y)} > 0$ has to be an integer multiple of $f_{i\,min}^{X(Y)}$, where

$$f_{i\,min}^{X(Y)} = \begin{cases} \dfrac{1}{2M_i^{X(Y)}} & \text{for} \quad M_i^{X(Y)} = 2 \\[2ex] \dfrac{1}{M_i^{X(Y)}} & \text{otherwise,} \end{cases}$$

with $M_i^{X(Y)}$ the number of different missions that air system i of side X(Y) may perform.

With the eight missions TAWAG considers and the different capabilities of the aircraft (see Table IV) this results in:

i	Air system i	$M_i^{X(Y)}$	$f_{i\ min}$
1	CAS	1	1
2	Interceptor	4	1/4
3	Penetrator	3	1/3
4	SAS	1	1
5	Multi-Role	8	1/8
6	TCM	2	1/4

Table IV

Then, given the effective offensive air support sorties $S_k^{X,CAS}$, $S_k^{X,INT}$, $S_k^{Y,CAS}$, $S_k^{Y,INT}$ at time t_k, the ground war model determines, for each combat sector j, the FEBA-movement d_{j,N_T}^k to be expected for the remainder of the conflict ($\nu = k_1,\ldots,N_T$) under the assumption that there is no offensive air support available for $\nu > k$, thus

$$d_{j,N_T}^k = \sum_{\nu=k}^{N_T} F_k^{\nu}(S_k^{X,CAS},S_k^{X,INT},S_k^{Y,CAS},S_k^{Y,INT};\hat{\sigma}_1,\ldots,\hat{\sigma}_{k-1})$$

$$\forall\ \sigma_{k,j}$$

where F_k^{ν} is the incremental FEBA-movement in time interval ν and $\hat{\sigma}_1,\ldots,\hat{\sigma}_{k-1}$ the hitherto selected air war strategies of the opposing tactical air forces.

FEBA-movement is defined in terms of the penetration distance into the territory of the defender X (see Figure 2).

FEBA MOVEMENT

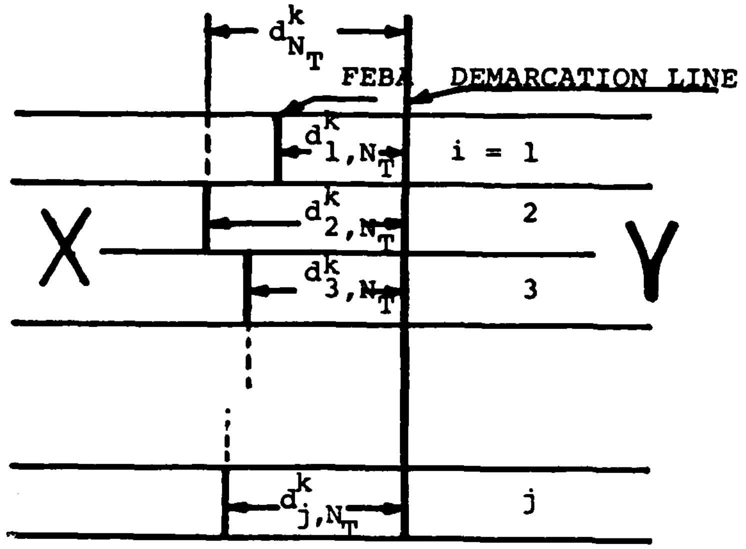


Figure 2

When the defender X considers the worst case to be expected as to ground losses in anyone of the combat sectors j the model has to determine for each strategy $\sigma_k(\sigma^X_k, \sigma^Y_k)$

$$d^k_{N_T} = \max_j d^k_{j,N_T}, \quad \forall \sigma_k$$

Since the attacker Y attempts to maximize the penetration distance into the defender's territory, while the defender X attempts to minimize that same distance, this principle results in

$$\hat{d}^k_{N_T} = \min_{\sigma^X_k} \max_{\sigma^Y_k} d^k_{N_T} , \quad \forall k$$

The mutually "optimal" air war strategy σ^*_k at time t_k is then approximated from

$$\sigma^*_k \sim \hat{\sigma}_k(\hat{d}^k_{N_T})$$

assuming that both sides have complete information on their mutual states and the available strategies.

The optimal air campaign strategies pursued by the *opponents throughout the conflict* are thus approximated by

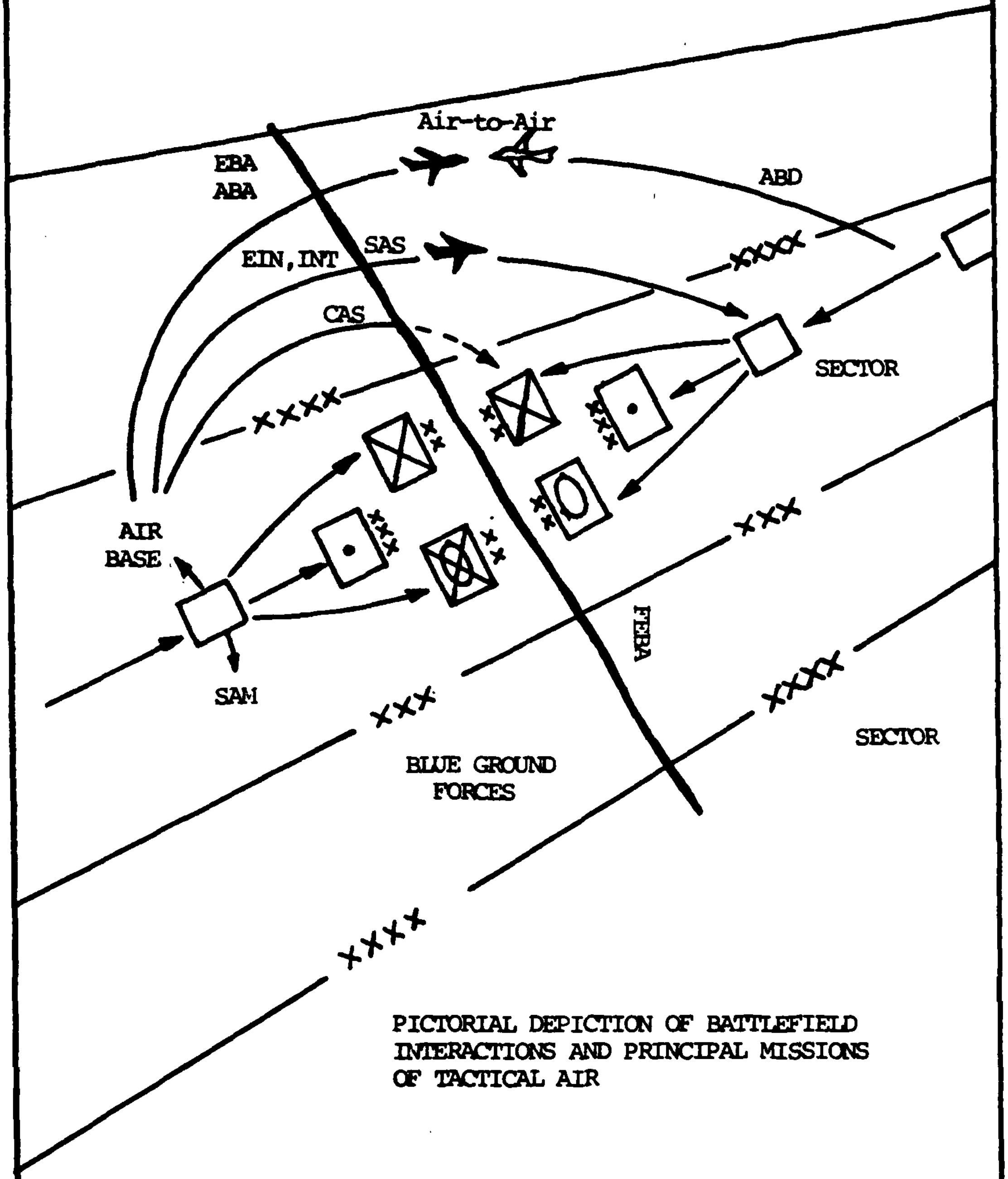
$$\sigma^* \equiv \hat{\sigma}(\hat{\sigma}_1,\hat{\sigma}_2,\ldots,\hat{\sigma}_\nu,\ldots\hat{\sigma}_{N_T})$$

for which the FEBA-displacement results in

$$d^* \sim d^{N_T}_{N_T}$$

C. AIR COMBAT

The principal air-to-air and air-to-ground interactions of an air campaign are indicated in Figure 3 in pictorial form. Over the many days of a war, the air forces on either side can be used for the attack of the opponent's air bases or attack



PICTORIAL DEPICTION OF BATTLEFIELD INTERACTIONS AND PRINCIPAL MISSIONS OF TACTICAL AIR

Figure 3

of the opponent's ground forces. This is called the offensive role which splits in

air superiority - OCA - role

and

close air support - OAS - role.

Or they may be deployed on defensive missions to

intercept the opponent's attack aircraft - DCA - role

Distinguishing between these roles, TAWAG considers the following air missions:

- <u>OFFENSIVE COUNTER AIR-ROLE (OCA)</u>
  1. SAM - suppression (SAS)
  2. Air Base Attack (ABA)
- <u>OFFENSIVE AIR SUPPORT-ROLE (OAS)</u>
  3. Close Air Support (CAS)
  4. Interdiction (INT)
- <u>DEFENSIVE COUNTER AIR-ROLE (DCA)</u>
  5. Air Base Defense (ABD)
  6. Battlefield Defense (BFD)
  7. Escort/ABA (EBA)
  8. Escort/INT (EIN)

Table V

BFD missions provide the air cover for the land forces during buildup phases in staging areas and during transit into the combat region. They are directed at INT- and EIN-missions. The interactions between the air systems as well as between the air systems and the ground forces are conceptualized in Figure 4.

ROLE Mission μ / Air System i		OCA		OAS		DCA			
		SAS	ABA	CAS	INT	ABD	BFD	EBA	EIN
		1	2	3	4	5	6	7	8
CAS	1			*					
Interceptor	2					*	*	P[1)]	P[1)]
Penetrator	3		*	*	*				
SAS	4	*							
Multi-Role	5	*	*	*	*	*	*	*	*
TCM	6		*		*				

[1)] Interceptors provide escort to Penetrators only.

Mission Capability of Air Systems

Table VI

The allocation of air systems to certain missions depends on roles the air system can fulfill. Generally only three special purpose A/C (CAS, INT, ABA) and one general purpose A/C are played.

TAWAG considers six notational air systems of which five are specialized and thus restricted to certain missions. One

TAWAG INTERACTIONS

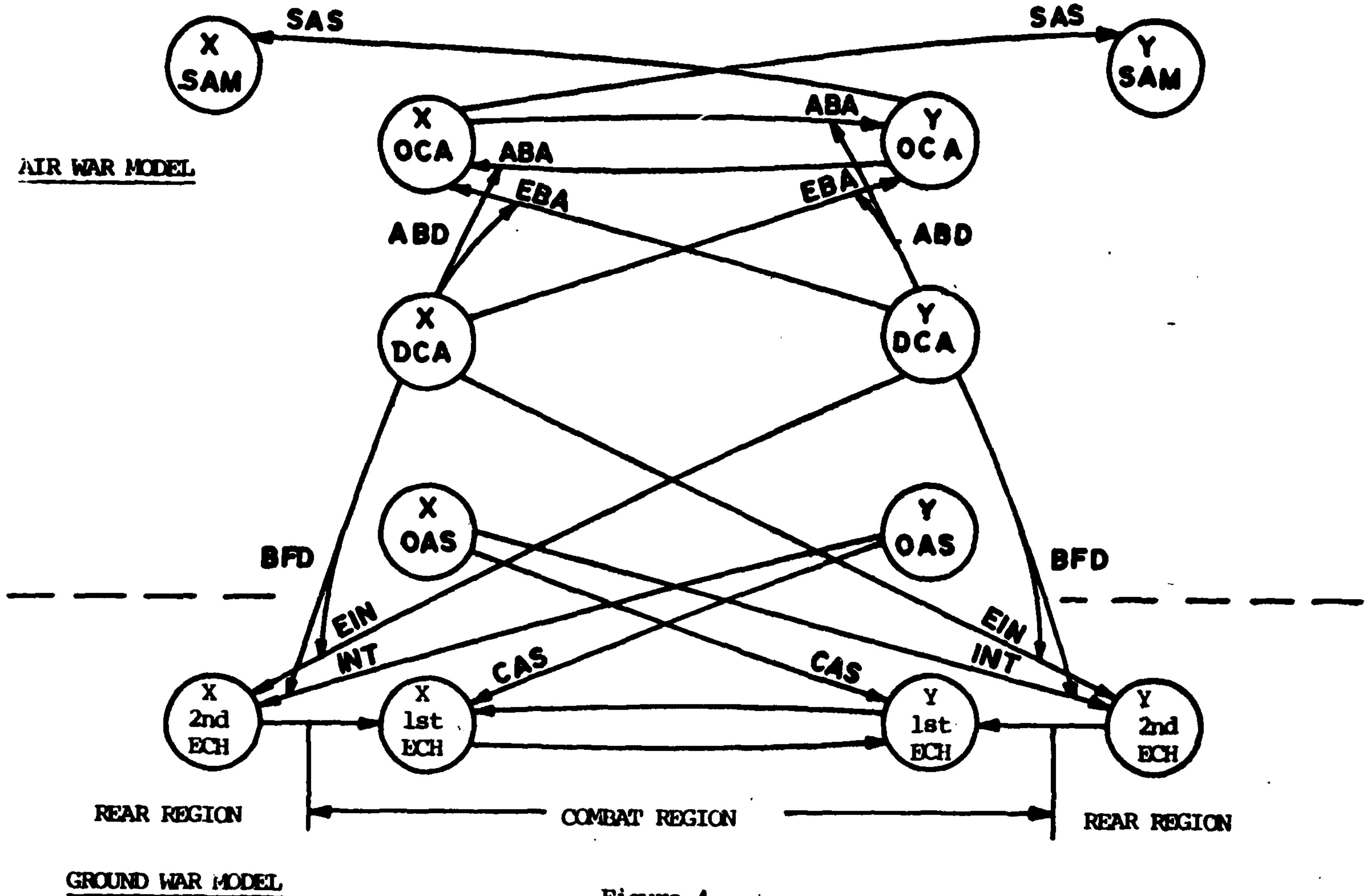


Figure 4

is a multi-role system like the German MRCA. The special purpose system also includes the forthcoming TCM (see Table VI).

1. <u>The Air Combat Model</u>

Before allocating air systems i to missions μ and, as a prerequisite to the approximation of optimal air war strategies, the air combat model determines the instantaneous states of the opposing air forces $\underline{x}_{\nu}^{A}$ and $\underline{y}_{\nu}^{A}$ and calculates the number of effective sorties $S_{i,}^{X(Y),}$ available at time t_{ν} from air system i for mission μ.

The main events that influence the outcome of an air mission can be thought of as follows (see Figure 5):

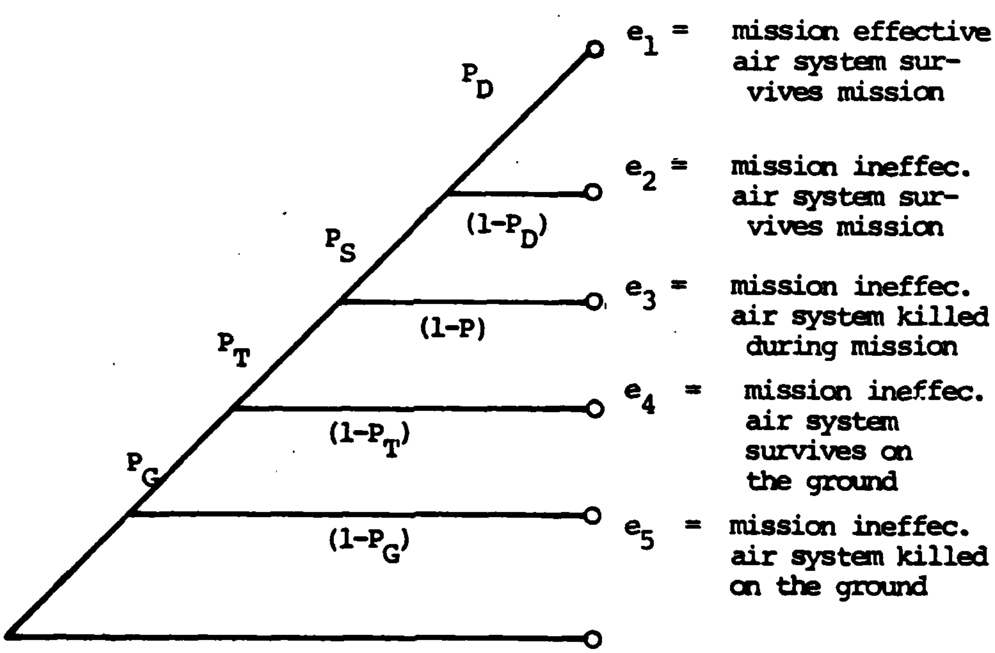


Figure 5

The probabilities are defined as:

$P_{Gi,\nu}^{X(Y),\mu}$ = probability that air system i survives on the ground in $t_\nu \leq t < t_{\nu+1}$; (III.1.b(1))

$P_{Ti,\nu}^{X(Y),\mu}$ = probability that an air system i allocated to mission μ can take off in $[t_\nu, t_{\nu+1})$; (III.1.b(2))

$P_{Si,\nu}^{X(Y),\mu}$ = probability that air system i survives mission μ in $[t_\nu, t_{\nu+1})$; (III.1.a.(2))

$P_{Di,\nu}^{X(Y),\mu}$ = probability that air system i allocated to mission μ detects target (user specified)

and are determined by the model as outlined in the subsequent sections.

From Figure 5 it can be seen that the probability of an effective mission in the time interval $[t_\nu, t_{\nu+1})$, $P_{Ei,\nu}^{X(Y),\mu}$ is equal to the probability that event e_1 occurs and can be calculated with the above defined probabilities as

$$P_{Ei,\nu}^{X(Y),\mu} = P_{Gi,\nu}^{X(Y),\mu} P_{Ti,\nu}^{X(Y),\mu} P_{Si,\nu}^{X(Y),\mu} P_{Di,\nu}^{X(Y),\mu}$$

The probabilities are calculated in subsequent sections. Furthermore, the probability that an air system i assigned to mission μ survives in time interval $[t_\nu, t_{\nu+1})$ is given as the probability that the events e_1 or e_2 or e_4 occur and results in

$$P_{Ui,\nu}^{X(Y),\mu} = P(e_1) + P(e_2) + P(e_4)$$

$$= P_{Gi,\nu}^{X(Y),\mu}[P_{Ti,\nu}^{X(Y),\mu} P_{Si,\nu}^{X(Y),\mu} + (1 - P_{Ti,\nu}^{X(Y),\mu})]$$

with

$G_{i,\nu}^{X(Y),\mu}$ = user input sortie generation rate for system i and mission μ in time interval $[t_\nu, t_{\nu+1})$,

and

$$z = [G_{i,\nu}^{X(Y),\mu}].$$

Also,

$Q_{i,\nu}^{X(Y),\mu}$ $\equiv$ fraction of the numbers $x_{i,\nu}^{A}$ and $y_{i,\nu}^{A}$ of air systems of type i in the inventories of X and Y being allocated to mission at time t_ν

This gives:

$$x_{i,\nu}^{A,\mu} = \left\{ \begin{array}{l} \text{\# of Blue type i} \\ \text{a/c assigned to} \\ \text{mission } \mu \text{ at time } t_\nu \end{array} \right\} = x_{i,\nu}^{A}\ \phi_{i,\nu}^{x,\mu}$$

$$y_{i,\nu}^{A,\mu} = \left\{ \begin{array}{l} \text{\# of Red type i a/c} \\ \text{assigned to mission} \\ \mu \text{ at time } t\nu \end{array} \right\} = y_{i,\nu}^{A}\ \phi_{i,\nu}^{y,\mu}$$

which makes it possible to calculate

$$x_{i,\nu}^{S,\mu} = \left\{ \begin{array}{l} \text{number of air systems of type i} \\ \text{allocated to mission } \mu \text{ surviving} \\ \text{in interval } [t_\nu, t_{\nu+1}) \end{array} \right\} = \left\{ \begin{array}{l} \text{final state} \\ \text{of a cycle} \end{array} \right\}$$

for Blue as:

$$\left\{\begin{array}{l}\text{\# of air systems i}\\ \text{assigned to mission}\\ \mu\text{ surviving in}\\ \text{interval }[t_\nu,t_{\nu+1})\end{array}\right\} = \left\{\begin{array}{l}\text{\# of Blue type}\\ \text{i a/c assigned}\\ \text{to mission }\mu\\ \text{at time }t_\nu\end{array}\right\} \left[\left\{\begin{array}{l}\text{Prob. that air}\\ \text{system i assigned}\\ \text{to mission }\mu\\ \text{survives in}\\ [t_\nu,t_{\nu+1})\end{array}\right\}\right.$$

$$\left\{\begin{array}{l}\text{sortie generation}\\ \text{rate for system i}\\ \text{and mission }\mu\end{array}\right\} + \left\{1 - \left\{\begin{array}{l}\text{sortie generation}\\ \text{rate for system i}\\ \text{and mission }\mu\\ \text{in }[t_\nu,t_{\nu+1})\end{array}\right\}\right\}$$

$$\left.\left\{\begin{array}{l}\text{Prob. that air}\\ \text{system i survives}\\ \text{on the ground in}\\ [t_\nu,t_{\nu+1})\end{array}\right\}\right] \quad \text{for } G^{X,\mu}_{i,\nu} \le 1$$

and for $G^{X,\mu}_{i,\nu} > 1$:

$$x^{S,\mu}_{i,\nu} = x^{A,\mu}_{i,\nu}[(G^{X,\mu}_{i,\nu} - z)P^{X,\mu}_{Ui,\nu} + \sum_{\lambda=1}^{z} (P^{X,\mu}_{Ui,\nu})^{\lambda}]$$

and for Red:

$$y^{S,\mu}_{i,\nu} = \begin{cases} y^{A,\mu}_{i,}[P^{Y,\mu}_{Ui,\nu}\ G^{Y,\mu}_{i,\nu} + (1 - G^{Y,\mu}_{i,\nu})P^{Y,\mu}_{Gi,\nu}] & \text{for } G^{Y,\mu}_{i,\nu} \le 1 \\ y^{A,\mu}_{i,\nu}[G^{Y,\mu}_{i,\nu} - z)P^{Y,\mu}_{Ui,\nu} + \sum_{\lambda=1}^{z}(P^{Y,\mu}_{Ui,\nu})^{\lambda}] & \text{otherwise.} \end{cases}$$

These equations are only valid for recoverable air systems. For expendable system (TCM) they are different.

This finally leads to the determination of the vectors

$$\underline{x}_{\nu}^{A} = (x_{1,\nu}^{A}, x_{2,\nu}^{A}, \ldots x_{i,\nu}^{A}, \ldots, x_{n_x^A}^{A})$$

$$\underline{y}_{\nu}^{A} = (y_{1,\nu}^{A}, y_{2,\nu}^{A}, \ldots y_{i,\nu}^{A}, \ldots, y_{n_y^A}^{A})$$

where

$$x_{i,\nu+1}^{A} = \left\{ \begin{array}{l} \text{\# of Blue type i} \\ \text{a/c available at} \\ \text{time } t_{\nu+1} \end{array} \right\} = \sum_{\substack{\text{all} \\ \text{missions}}} \left\{ \begin{array}{l} \text{\# of air systems of} \\ \text{type i allocated to} \\ \text{mission } \mu \text{ at time} \\ t_{\nu} \text{ which survive in} \\ [t_{\nu}, t_{\nu+1}) \end{array} \right\}$$

$$= \sum_{\mu} x_{i,\nu}^{S,\mu}$$

and

$$y_{i,\nu+1}^{A} = \sum_{\mu} y_{i,\nu}^{S,\mu}$$

The <u>number of effective sorties</u> $S_{i,\nu}^{X(Y),\mu}$ available at time t_{ν} from air system i for mission μ depends on the number of type i a/c assigned to a mission μ at time t_{ν}, the probability that a mission μ is effective in time interval $[t_{\nu}, t_{\nu+1})$ and the sortie generation rate for system i and mission μ and results in

$$S^{X,\mu}_{i,\nu} = \begin{cases} x^{A,\mu}_{i,\nu}\, P^{X,\mu}_{Ei,\nu}\, G^{X,\mu}_{i,\nu} & \text{for } G^{X,\mu}_{i,\nu} \leq 1 \\ x^{A,\mu}_{i,\nu}[(G^{X,\mu}_{i,\nu} - z)\, P^{X,\mu}_{Ei,\nu} + \sum_{\lambda=1}^{z} (P^{X,\mu}_{Ei,\nu})^{\lambda}] & \text{otherwise} \end{cases}$$

$$S^{Y,\mu}_{i,\nu} = \begin{cases} y^{A,\mu}_{i,\nu}\, P^{Y,\mu}_{Ei,\nu}\, G^{Y,\mu}_{i,\nu} & \text{for } G^{Y,\mu}_{i,\nu} \leq 1 \\ y^{A,\mu}_{i,\nu}[(G^{Y,\mu}_{i,\nu} - z)\, P^{Y,\mu}_{Ei,\nu} + \sum_{\lambda=1}^{z} (P^{Y,\mu}_{Ei,\nu})^{\lambda}] & \text{otherwise} \end{cases}$$

These equations determine the number of effective sorties in each role, which result from the strategy set of the two opponents. Especially interesting are the number $S^{Y,CAS}_{i,\nu}$ and $S^{Y,INT}_{i,\nu}$ which are used in the Ground War Model to determine the air induced losses and eventually contribute to the FEBA-movement.

a. Mission Survivability

During its "airborne" phase of a mission an air system can encounter several incidents that could influence its survivability.

Aircraft that are allocated to ABA and INT or to escort missions EBA and EIN may suffer attrition due to area deployed SAM defense en route to their targets. The surviving aircraft can be engaged by enemy fighters (ABD, BFD) in the target area or, if surviving this part of the mission, encounter ground-based air defense systems in the target area.

The model accounts for these events by treating the previous defined mission survivability $P^{X(Y),\mu}_{Si,\nu}$ as a function of:

Probability that air system i survives mission μ in $[t_\nu, t_{\nu+1})$ = f { SAM suppression strategies; the Defense Counter Air Strategies, i.e., ABD versus ABA and EBA, BFD versus INT;

single sortie survival probability when no SAS and/or DCA actions during mission }

The last is the basic survival probability $P_{Si}^{X(Y),\mu}$ which is the single-sortie survival probability when no SAS- and/or DCA actions precede or counter the respective sorties on mission μ. It reflects the effectiveness of unopposed ground-based air defense systems.

(1) <u>SAM-Suppression Strategies</u>

SAM suppression, as an Offensive Counter-Air method, can be played either as SAM-attack preceding ABA and/or INT-mission or in terms of ECM accompanying ABA- and INT-missions. The result of SAS is an increase in the basic single-sortie survival probabilities for the subsequent ground attack and escort sorties.

The SAS pay-off function $f_{i,\mu}^{X(Y)}(S_k^{X(Y),1})$ is a monotonically decreasing function reflecting the relative reduction of the SAM's sortie attrition capability against i-type air systems on mission μ as the number $S_k^{X(Y),1}$ of k-type sorties allocated to SAS (μ = 1) increase

$$f_{i,\mu}^{X(Y)}(S_k^{X(Y),1}) \Longrightarrow \begin{cases} 1.0 & \text{for } S_k^{X(Y),1} = 0 \\ 0 & \text{for } S_k^{X(Y),1} = 800 \end{cases}$$

which is an input.

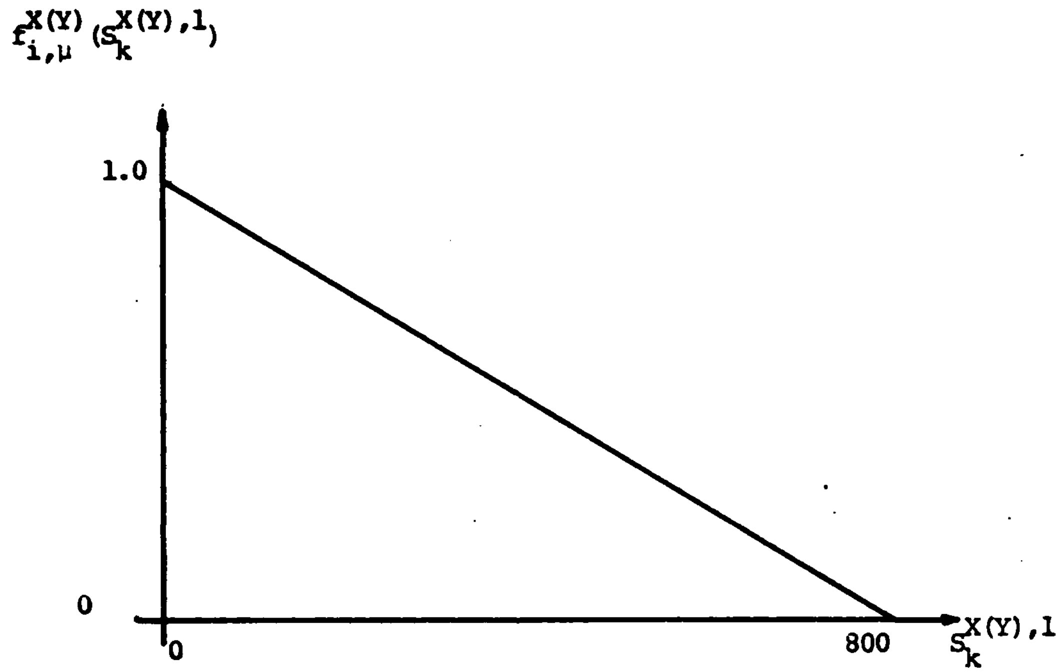


Figure 6

In this way $f_{i,\mu}^{X(Y)}(S_k^{X(Y),1})$ is interpreted as a probability for the SAM's performance reduction. To account for different deployment concepts (i.e., area or belt-type) and SAS tactics other SAS-payoff functions may be specified by the user. The single sortie survival probability of the i-type system on mission μ subsequent to or accompanied by SAS from $S^{X(Y),1} = \sum_k S_k^{X(Y),1}$ sorties is obtained by

$$P_{Si}^{X(Y),\mu}(S^{X(Y),1}) = 1 - (1 - P_{Si}^{X(Y),\mu}) \prod_k f_{i,\mu}^{X(Y)}(S_k^{X(Y),1})$$

which reflects the full suppression effects.

A suppressed and/or damaged SAM system may eventually regenerate to its original performance. Therefore the user can specify a SAM-system generation rate $R_S^{X(Y)}$ which is assumed to be constant for sides X and Y equal 1 for ECM. This means that SAM-systems return to full effectiveness as soon as ECM-SAS ceases. In the case of an attack of SAM positions prior to ABA- and INT-missions $R_S^{X(Y),1}$ is assumed to be 0.5. If the SAM-system has not yet fully regenerated before subsequent SAS missions take place, then the model accounts for this by modifying the basic single sortie survivability.

Considering the damage done by SAS, the model evaluates the single sortie survival probability at time $t_{\nu*} > t_\nu$, i.e., after a SAS by $S_\nu^{X(Y),1}$ sorties at time t_ν as

$$P_{Si}^{X(Y)}(S_\nu^{X(Y),1}, t_{\nu*}) = \begin{cases} P_{Si}^{X(Y),\mu}(S_\nu^{X(Y),1}) + R_S^{Y(X)} k_\nu \{P_{Si}^{X(Y),\mu} \\ \quad (S_\nu^{X(Y),1}) - P_{Si}^{X(Y),\mu}\} \text{for } (t_{\nu*} - t_\nu) < \dfrac{\Delta t}{R_S^{Y(X)}} \\ P_{Si}^{X(Y),\mu} \qquad \text{otherwise} \end{cases}$$

where $k_\nu = 1, \ldots, (t_{\nu*} - t_\nu)/\Delta t$ is the number of time intervals of length Δt elapsed since the last SAS effort.

(2) Air-to-Air Combat Encountered by DCA

Another modification of the basic sortie survival probability is due to the possibility that ABA- and INT-missions, whether accompanied by fighter escort sorties (EGA, EIN) or not, might encounter enemy fighters (ABD, BFD) in the target area after they sustained losses from not suppressed SAM sites.

The number of surviving ABA, EBA or INT, EIN air systems en route to their targets which can be engaged in a dogfight by enemy defense fighters depend on the number of effective sorties $S_{i,}^{X,\prime}$, for = 2, 4, 7 and 8 and the effectiveness of the enemy's SAM system as calculated in the previous section.

The modelling of air-to-air combat assumes that an escorted attack group of attackers (ABA, INT) on mission μ is detected by its opponent's warning and control system and that interceptors (ABD, BFD-fighters) engage the attackers in the following events:

- Escorts are assigned to attack interceptors in one-on-one engagements. Excess escorts are idle; excess interceptors are not attacked.
- Each attacked interceptor counterattacks against the attacking escort and does not engage an attack aircraft.
- Interceptors which were not engaged by escorts engage attack aircraft. Ground attack aircraft don't figh- back. Excess interceptors are idle, excess attack aircraft are not engaged.

- All attack aircraft engaged by interceptors jettison their ordnance and abort their mission in order to evade the interceptors.

Given the probabilities (user specified)

$P_{Sik}^{X(Y),GF}$ = probability that an i-type ground attack sortie of side X(Y) survives an attack by one k-type ABD- or BFD-fighter sortie of the opponent,

$P_{Sik}^{X(Y),FE}$ = probability that an i-type ABD- or BFD-fighter of side X(Y) survives a dogfight with a k-type escort fighter of the opponent,

$P_{Sik}^{X(Y),EF}$ = probability that an i-type EBA- or EIN-fighter of side X(Y) survives a dogfight with a k-type ABD- or BFD-fighter of the opponent,

where

G denounces a ground attack fighter on ABA ($\mu = 2$)- or INT ($\mu = 4$) mission,

E their escorts - EBA ($\mu = 7$) and EIN ($\mu = 8$) and F their fighter opponents on ABD ($\mu = 5$)- and BFD ($\mu = 6$) missions.

The probabilities of survival in these DCA-encounters are determined as

$$\hat{P}_{Si,\nu}^{X(Y),G} = \begin{cases} 1 & , \text{ for } \eta^{X(Y)} \geq 1 \\ (P_{Sik}^{X(Y),GF})^{\gamma^{X(Y)}} & , \text{ for } \eta^{X(Y)} < 1, \gamma^{X(Y)} \geq 1 \\ 1-\gamma^{X(Y)}(1-P_{Sik}^{X(Y),GF}), & \text{ for } \eta^{X(Y)} < 1, \gamma^{X(Y)} < 1 \end{cases}$$

$$\hat{P}^{X(Y),E}_{Si,\nu} = \begin{cases} 1 - \frac{1}{\eta^{X(Y)}}(1 - P^{X(Y),EF}_{Sik}), & \text{for } \eta^{X(Y)} \geq 1 \\ P^{X(Y),EF}_{Sik}, & \text{for } \eta^{X(Y)} < 1 \end{cases}$$

$$\hat{P}^{X(Y),F}_{Si,\nu} = \begin{cases} (P^{X(Y),FE}_{Sik})^{\eta^{Y(X)}} & \text{for } \eta^{Y(X)} \geq 1 \\ \delta^{X(Y)} + (1-\delta^{X(Y)})P^{X(Y),FE}_{Sik}, & \text{for } \eta^{Y(X)} < 1 \end{cases}$$

With η being the ratio of the number of fighters escorting the ground attack aircraft versus the number of enemy fighters defending against the ground attack raids,

$$\eta^{X(Y)} = \frac{S^{X(Y),E}_{i,\nu}}{S^{Y(Y),F}_{k}} ,$$

$$\gamma^{X(Y)} = \frac{\text{number of air defense fighters not engaged by escorts}}{\text{number of ground attack aircraft}}$$

$$\delta^{X(Y)} = \frac{\left\{\begin{matrix}\text{number of air defense}\\ \text{fighters}\end{matrix}\right\} - \left\{\begin{matrix}\text{number of fighters}\\ \text{escorting the ground}\\ \text{attack a/c}\end{matrix}\right\}}{\text{number of air defense fighters}}$$

= fraction of air defense fighters which attack enemy ground attack aircraft.

The thus derived probabilities modify the probability $P^{X(Y),\mu}_{Si,\nu}$ that air system i survives the mission μ such that

$$P_{Si,\nu}^{X(Y),\mu} = \hat{P}_{Si,\nu}^{X(Y),\mu} \; P_{Si}^{X(Y)}(S^{X(Y),1}, t^*)$$

which determines the number of effective ABA- and INT sorties $S_{i,\nu}^{X(Y),2}$ and $S_{i,\nu}^{X(Y),4}$ and the number of surviving air systems $X_{i,\nu}^{S,\mu}$, $Y_{i,\nu}^{S,\mu}$.

(3) Modifications to the Air-to-Air Combat

The following section briefly presents modifications to the original model.

In the original model all attack aircraft engaged by interceptors jettison their ordnance and abort their mission in order to evade the interceptors.

The modification enables the attack aircraft to counterattack the interceptors after dropping their ordnance. This situation will only occur when there are excess interceptors not engaged with escort fighters. Given the probability

$P_{Sik}^{X(Y),FG}$ = probability that an i-type ABD- or BFD-fighter of side X(Y) survives a dogfight with a k-type ground attack a/c of the opponent

the probabilities of survival in the DCA-encounters as calculated in section 1.a.(2) have to be modified.

The existing model doesn't take into consideration that the ground attack a/c (bomber) could engage in a dogfight with enemy fighters (interceptors).

An extension of the model might take a look at this possibility in the following way:

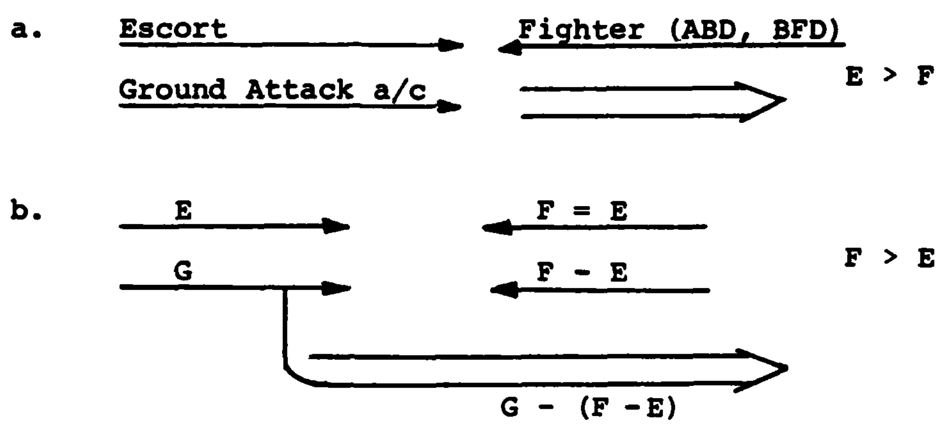


Figure 7

a. If the # of Escort a/c > # F (ABD, BFD) [$n \geq 1$] - then all bombers will go through.

b. If the # Escort < # F then part of the ABD or BFD fighters, (F -E) attack the bomber. The bombers drop their ordnance and engage in a dogfight with the remaining fighters.

   The maximum number of attacked bombers depends on the difference (F -E).

b1) If (F - E) > G all ground attack a/c are attacked and the survivability is calculated as

$$\hat{P}^{X(Y),G}_{Si,\nu} = (P^{X(Y),GF}_{Sik})^{\gamma^{X(Y)}} , \quad \eta^{X(Y)} < 1,$$
$$\gamma^{X(Y)} \geq 1.$$

b2) If (F - E) < G then some of the ground attack a/c (F-E-G) will finish their mission with the basis survivability $P^{X(Y),\mu}_{Si}$ and the rest engage with the ABD or BFD fighter with the probability of survival

$$\hat{P}^{X(Y),G}_{Si,\nu} = 1 - \gamma^{X(Y)}(1 - P^{X(Y),GF}_{Sik})$$
$$\eta^{X(Y)} < 1,$$
$$\gamma^{X(Y)} < 1.$$

The modification however changes the survival probability of the defense fighters due to the dogfight with ground attack a/c in the following way:

a.1. If there are more escort fighters than defense fighters, i.e.,

$$\eta = \frac{S^{X(Y),E}_{i,\nu}}{S^{Y(X),F}_{i,\nu}} \geq 1 ,$$

there is no change because all defense fighters are engaged with the escort fighters.

If $\eta > 1$, there are two cases:

b.1.1. There are more defense fighters, not engaged in a dogfight with the escort fighters left than there are ground attack fighters, i.e.,

$$\gamma = \frac{S_{i,\nu}^{Y(X),F} - S_{i,\nu}^{X(Y),E}}{S_{i,\nu}^{X(Y),G}} \geq 1 \quad \text{or} \quad (F - E) > G$$

this amounts to that a fraction of the difference F-E-G of defense fighters are in a dogfight with ground attack a/c, and the fraction of defense fighters neither engaged with escort fighters nor with ground attack a/c is idle and their $\hat{P}_{Si,\nu}^{X(Y),F} = 1$.

b.1.2. The fraction of defense fighters not engaged with escort fighters is outnumbered by ground attack a/c, i.e., $\gamma < 1$ or $(F - E) < G$, then all the remaining defense fighters are in a dogfight with the ground attack a/c and survive with probability $P_{Si,k}^{X(Y),GF}$

b. Air-To-Ground Attrition Due To Air Base Attack

The TAWAG model considers two types of air base attacks the collateral damage of which may lead to a temporary suppression and degredation of ground support activities.

The first mode is General Aba where the air systems allocated to ABA are uniformly distributed over all enemy systems, $i = 1,\ldots,n_{X(Y)}^{A}$.

The second mode, Concentrated Aba allocates all ABA-sorties to one enemy air system. In both cases, the sorties may be directed to either of the enemy's air systems on the ground or at the takeoff and landing facilities.

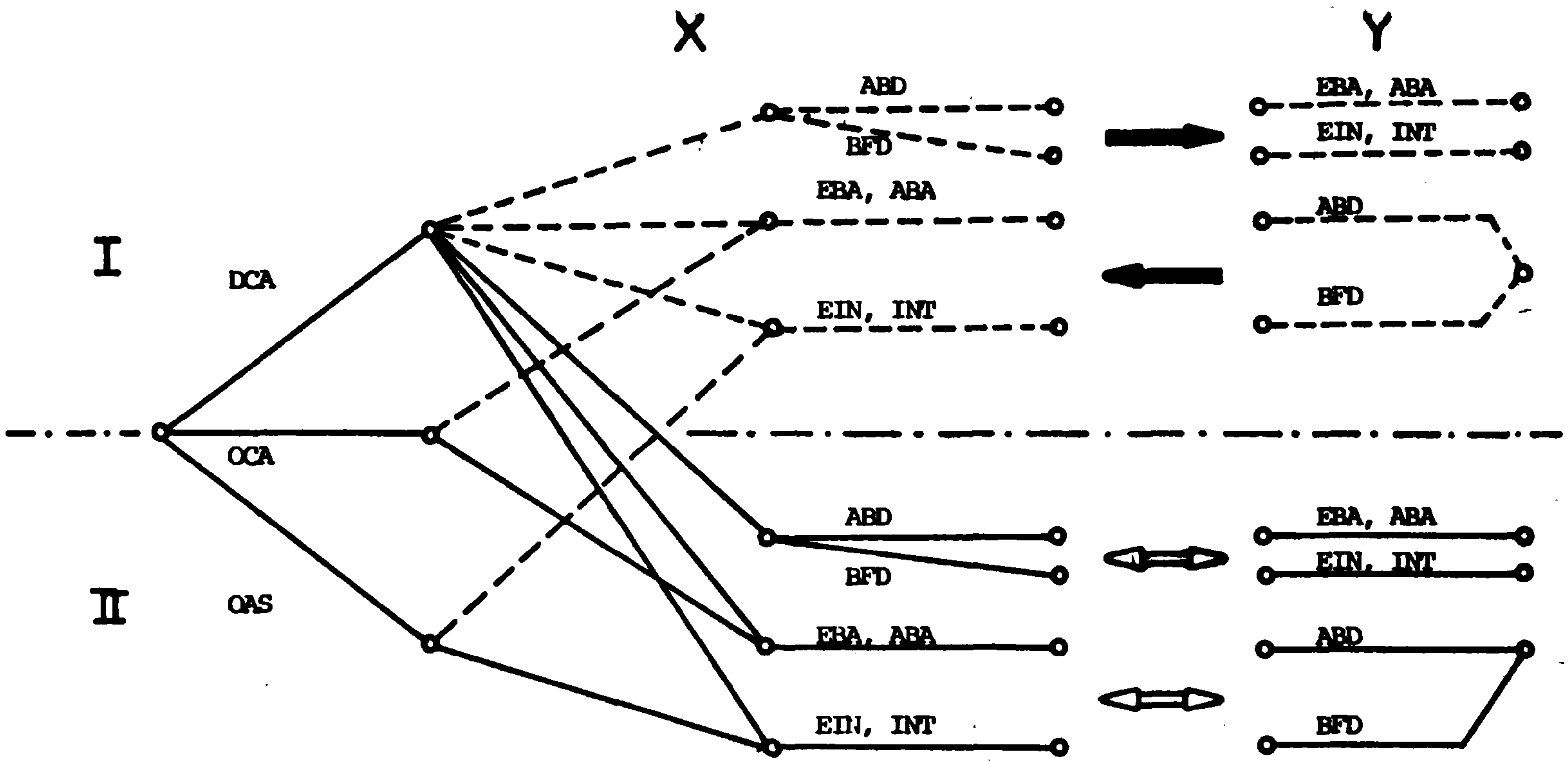


Figure 8

Model I: no dogfight between EGA, ABA ground attack a/c and ABD, BFD fighters.

Model II; after releasing their ordnance ground attack a/c engage with ABD and BFD fighters

(1) Ground Survivability

In III.C.1 two probabilities were defined and used in subsequent sections but not evaluated yet. In the chain of events in Figure 5 the events e_4 and e_5 relate to the possibility that air systems are killed on the ground or that the runway is damaged before they can take off for a mission due to the enemy's attacking the air base.

The probability $P_{Gi,}^{X(Y)}$ that air system i survives on the ground in $[t_\nu, t_{\nu+1})$ is calculated as follows:

$$\begin{array}{l}\text{probability}\\ \text{that air system}\\ \text{i survives on}\\ \text{the ground in}\\ [t_\nu, t_{\nu+1})\end{array} = f\left\{\begin{array}{l}\text{probability}\\ \text{that i-type}\\ \text{air system is}\\ \text{on the ground}\\ \text{when ABA occurs}\end{array} ; \begin{array}{l}\text{average number}\\ \text{of air systems}\\ \text{of type k attacking}\\ \text{i-type air systems}\\ \text{on the ground in}\\ [t_\nu, t_{\nu+1})\end{array} ; \begin{array}{l}\text{probability that}\\ \text{i-type air system}\\ \text{is killed on the}\\ \text{ground by an}\\ \text{attacking k-type}\\ \text{sortie}\end{array}\right\}$$

The conditional probability $P_{Kik}^{X(Y)}$ that an i-type system is killed on the ground by an attacking k-type sortie, and the probability $P_{Si}^{X(Y)}$ that the attacked system i is on the ground when the attack occurs are user specified inputs.

The average number of air systems of type k attacking i-type air systems on the ground in $[t_\nu, t_{\nu+1})$ is different in the two modes: General ABA and Concentrated ABA.

It is calculated as

(a) General ABA

$$D^{Y}_{ki,\nu} = \frac{\dfrac{x^{A}_{i,\nu}}{\sum\limits_{i} x^{A}_{i,\nu}} S^{Y,2}_{k,\nu}}{P^{X}_{Bi}\; x^{A}_{i,\nu}} \qquad \forall k$$

$$D^{X}_{ki,\nu} = \frac{\dfrac{y^{A}_{i,\nu}}{\sum\limits_{i} y^{A}_{i,\nu}} S^{X,2}_{k,\nu}}{P^{Y}_{Bi}\; y^{A}_{i,\nu}} \qquad \forall k$$

(b) Concentrated ABA

$$D^{Y}_{ki,\nu} = \begin{cases} \dfrac{S^{Y,2}_{k,\nu}}{P^{X}_{Bi}\; x^{A}_{i,\nu}}, & \text{for } i = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$D^{X}_{ki,\nu} = \begin{cases} \dfrac{S^{X,2}_{k,\nu}}{P^{Y}_{Bi}\; y^{A}_{i,\nu}}, & \text{for } i = 1 \\ 0 & \text{otherwise} \end{cases}$$

The ground survival probability then results in:

$$P^{X(Y),\mu}_{Gi,\nu} = P^{X(Y)}_{Bi} \prod_{k} P^{X(Y)}_{Gik} + (1 - P^{X(Y)}_{Bi})$$

where

$$P_{Gik}^{X(Y)} = \begin{cases} \left(1 - P_{Kik}^{X(Y)}\right)^{D_{Ki,\nu}^{Y(X)}}, & \text{for} \quad D_{ki,\nu}^{Y(X)} > 1 \\ 1 - P_{Kik}^{X(Y)} \, D_{ki,\nu}^{Y(X)}, & \text{for} \quad D_{ki,\nu}^{Y(X)} \leq 1 \end{cases}$$

(2) Runway Availability

The probability $P_{Ti,\nu}^{X(Y),\mu}$ that an air system i allocated to mission μ can take off is a function of the number of k-type enemy sorties attacking per base or runway used by i-type air systems, and the runway repair capability of the air base organization after an ABA attack.

The runway repair capability can be expressed in terms of a user specified rate $T_i^{X(Y)}$, at which the probability for take-off increases (back to 1) per time interval Δt (0.7 for i = i; 0.5 for i = 2,...,5).

Similar to the SAS-payoff function the monotonically decreasing function $g_{i,\mu}^{X(Y)}(H_{ki}^{Y(X)})$ reflects the reduction in runway availability as the number $H_{ki}^{Y(X)}$ of k-type enemy sorties attacking per base or runway increases.

$$g_{i,\mu}^{X(Y)} \text{ is defined as } \begin{cases} g_{i,\mu}^{X(Y)}(0) = 1 \\ g_{i,\mu}^{X(Y)}(\infty) = 0 \end{cases}$$

and

$$g_{i,\mu}^{X(Y)}(H_{ki}^{X(Y)}) = 0 \quad \text{for} \quad H_{ki}^{X(Y)} = 0.$$

For the two ABA modes the calculation of $H_{ki,\nu}^{Y(X)}$ results in

$$H_{ki,\nu}^{Y(X)} = \frac{S_{k,\nu}^{Y(X),2}}{\sum_{i} n_i^{X(Y)}} \qquad \text{(General ABA)}$$

$$H_{ki,\nu}^{Y(X)} = \begin{cases} \dfrac{S_{k,\nu}^{Y(X),2}}{n_i^{X(Y)}} , & \text{for } i = 1 \\ 0 , & \text{otherwise} \end{cases} \qquad \text{(Concentrated ABA)}$$

with $n_i^{X(Y)}$ denoting the number of bases or runways used by i-type air systems.

The probability for take off then results in

$$P_{Ti,\nu*}^{X(Y),}(S_{k,\nu}^{Y(X),2}) = \begin{cases} g_{i,\mu}^{X(Y)}(H_{ki,\nu}^{Y(X)}) + k_\nu T_i^{X(Y)}\{1 - g_{i,\mu}^{X(Y)}(H_{ki,\nu}^{Y(X)})\} , \\ \qquad \text{for } (t_{\nu*} - t_\nu) < \dfrac{\Delta t}{T_i^{X(Y)}} \\ 1 \qquad \text{otherwise} \end{cases}$$

where $t_{\nu*} > t_\nu$ and t_ν denotes the time at which the last ABA occurred. Also $k_\nu = 1,\ldots,(t_{\nu*} - t_\nu)/\Delta t$ expresses the number of time intervals of length Δt elapsed since the last ABA-attack at t_ν.

The model also allows for considerations that subsequent ABA missions take place at a time $t_{\nu*}$ such that $(t_{\nu*} - t_\nu) < \frac{\Delta t}{T_i^{X(Y)}}$ (i.e., runway has not yet been fully repaired), by calculating the combined effects as $P_{Ti,\nu*}^{X(Y)}(S_{k,\nu}^{Y(X),2})$ related to the previous and the new ABA.

(3) Suppression Effects

Collateral damage from ABA-attacks, which might temporarily reduce the sortie production capability of the ground support organization is specified similar to the defense suppression by the ABA-suppression function $h_{i,\mu}^{X(Y)}(H_{ki}^{Y(X)})$ as the probability that the ground support organization is at its full capacity after an attack on its base by $H_{ki}^{Y(X)}$ k-type sorties of the opponent.

It results in a sortie generation rate

$$G_{i,\nu*}^{X(Y),\mu} = G_i^{X(Y),\mu} \, r_{i,\nu*}(S_{k,\nu}^{Y(X)})$$

where $G_i^{X(Y),\mu}$ is the i-type system's sortie generation rate per time interval $\Delta t = (t_{\nu+1} - t_\nu)(\nu = 1,\ldots,N_T)$ in an OCA-free environment and the second expression is the reduction factor at time $t_{\nu*} > t_\nu$ which yields

$$r_{i,\nu*}(S_{k,\nu}^{Y(X)}) = \begin{cases} h_{i,\mu}^{X(Y)}(H_{ki,\nu}^{Y(X)}) + Q_i^{X(Y)}\ k_\nu\{1-h_{i,\mu}^{X(Y)}(H_{ki,\nu}^{Y(X)})\} \\ \qquad \text{for } (t_{\nu*} - t_\nu) < \dfrac{\Delta t}{Q_i^{X(Y)}} \\ 1 \qquad \text{otherwise} \end{cases}$$

$Q_i^{X(Y)}$ being the regeneration rate of the i-type ground support organization and k_ν as defined before.

D. GROUND COMBAT - THE GROUND WAR MODEL

In the ground war model the firepower-score approach is used to determine the FEBA-movement resulting from the interactions between the opposing ground forces and from the effective offensive air support sorties of the opposing air forces. The firepower-score approach is an <u>index number</u> method for aggregating the heterogeneous forces of each side into a single equivalent homogeneous force. The term firepower-score is usually taken to mean the value or capability of an individual weapon or weapon system where the term firepower index identifies the index number for a unit's capability as the summed firepower scores (or military capability of some aggregation of diverse weapons). Mathematically

$$I_X = \sum_{i=1}^{n} S_i x_i$$

where

S_i = firepower score of ith system of the X force

x_i = number of effective weapons in unit

I_X = firepower index of the unit.

Firepower indices are used to

- determine engagement outcomes
- assess casulaties
- determine FEBA-movement.

This is done by three component processes to be described later:

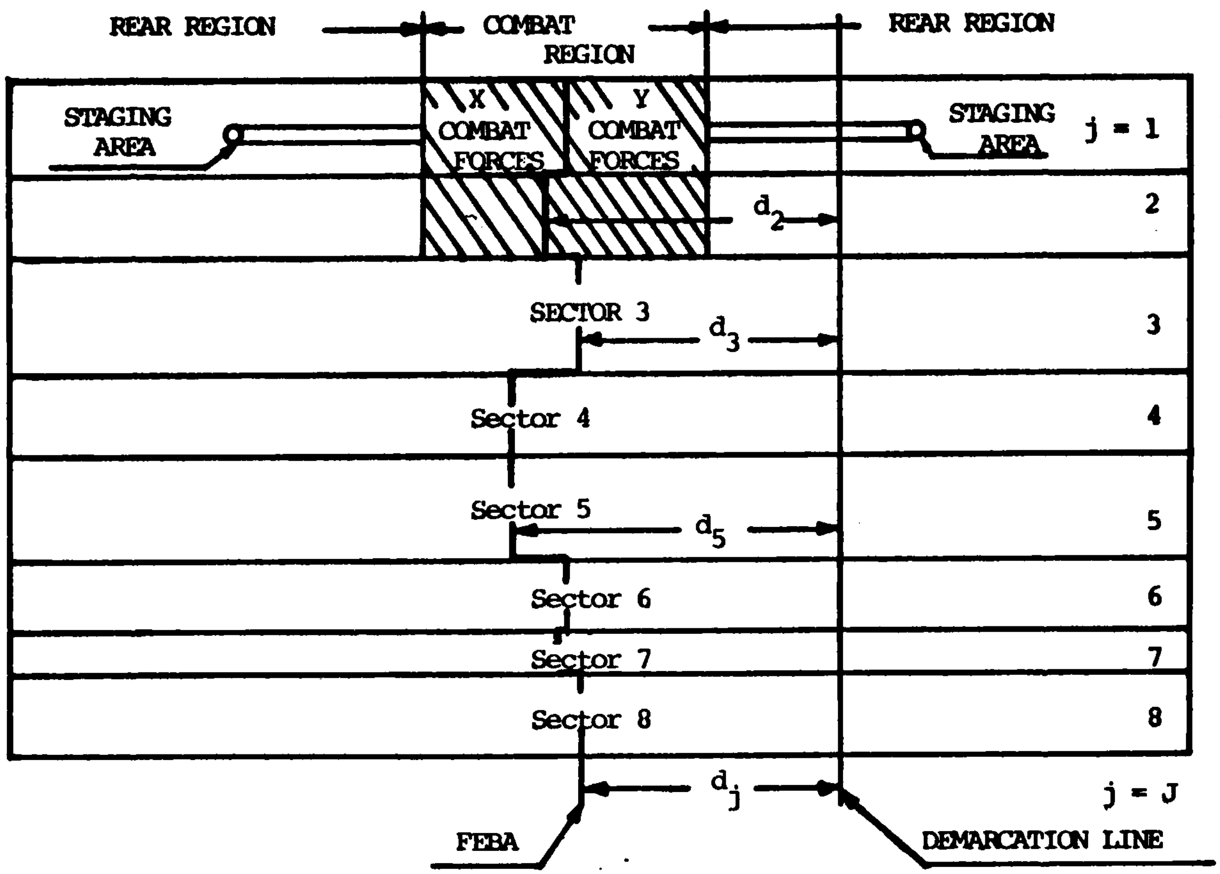
(1) attrition

(2) movement

(3) command and control (C^2).

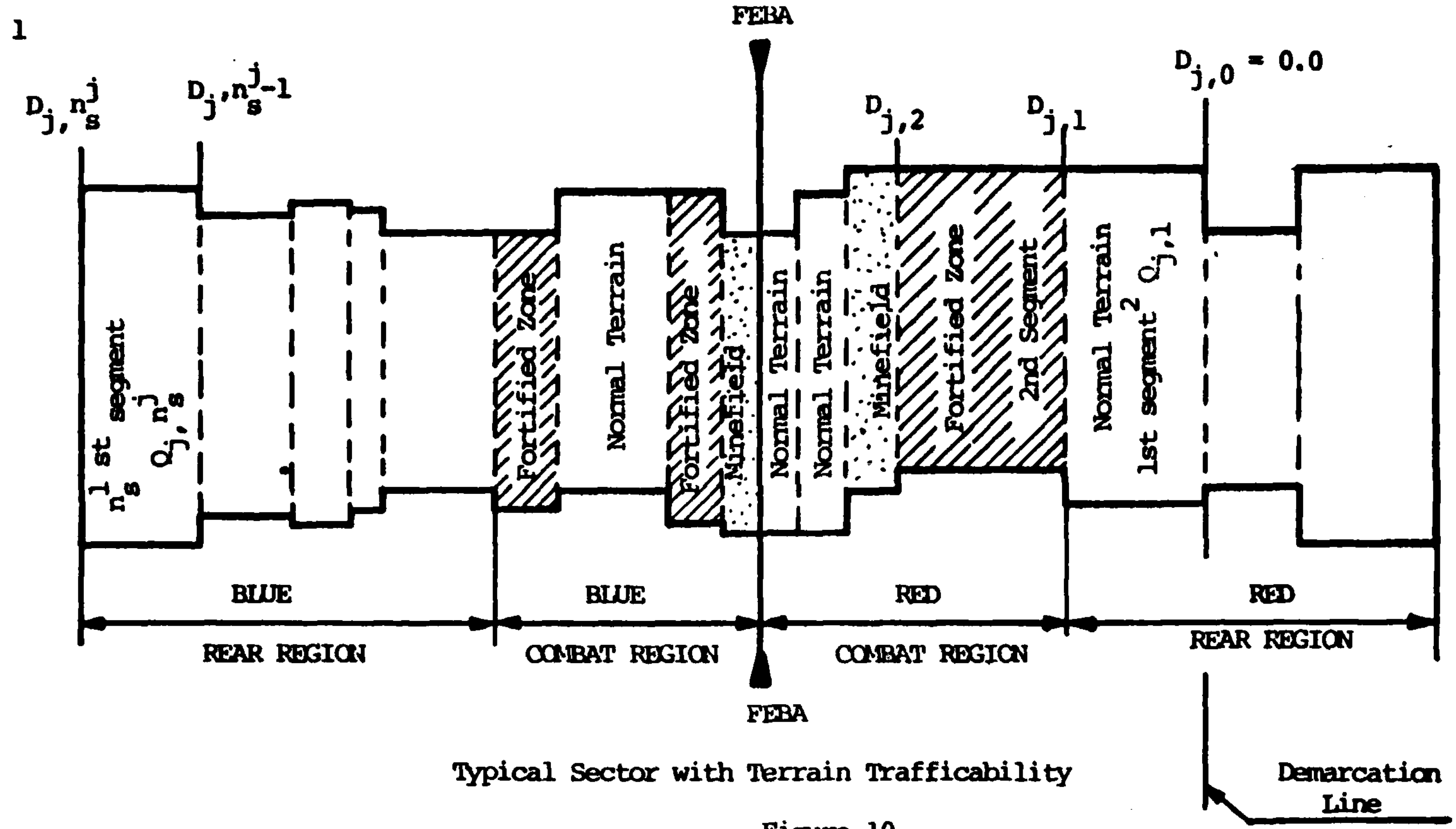
1. The Battlefield

The battlefield is divided into sectors. The number of sectors and the width of each sector can be varied by the user depending on the terrain, natural and artificial obstacles (e.g., forests, rivers) etc. Each sector contains a combat region and a rear region. Ground combat takes place only in the combat region of the sector. The combat forces are separated by the FEBA the initial position of which is given by the demarcation line (see Figure 9). To represent terrain, each sector is divided into terrain-trafficability segments (see Figure 10). Within each such segment, the terrain trafficability is assumed to be homogeneous. There are three types of terrain: normal, fortified zones, and minefields which spread throughout the sectors starting at the demarcation ne Thus, each sector can have its own "terrain features."



THEATER STRUCTURE AND BATTLEFIELD GEOMETRY

Figure 9



Typical Sector with Terrain Trafficability

Figure 10

$^1D_{j,s}$ denotes the coordinate of the left boundary of the s-th segment of the j-th sector

N_s^j denotes the total number of segments into which the j-th sector is divided

$^2\ Q_{j,n_s^j}$ denotes the terrain trafficability for the s-th segment of the j-th sector

2. Aggregation

The combat capability of the ground force which is considered to consist of homogeneous units (divisions) is quantified in terms of a firepower index. The initial (TO&E) stregnth is given by

$$c_X^0 = \sum_{k=1}^{n_X^G} s_X^k \, x_k^0$$

$$c_Y^0 = \sum_{k=1}^{n_Y^G} s_Y^k \, y_k^0$$

where $s_{X(Y)}$ denotes the firepower score of the k-type subunit or (sub) system of X and Y ($k = 1,\ldots,n_{X(Y)}^G$), and x_k^0 and y_k^0 their initial numbers. As an example how the firepower indices are computed, blue and red division are aggregated as follows (Table VII). The resulting firepower index for a Blue Tank Division and a Red Motorized Division would come out as 61680 and 40940 respectively based on the above firepower scores. But it should be mentioned that this is only an example and the input for the firepower scores and the number of weapon systems in the different divisions must be provided by the user. The ground war model provides a different replacement policy for each side.

Y replaces its initially available n_j^{CY} units in total when their combat capability drops below a certain level by n_j^{RY} reserve units from the rear region of a sector. Thus, in

	k	s_X^k	s_Y^k	x_R^0	y_k^0
	weapon system	firepower score Blue	firepower score Red	#/Div Blue (arm.)	#/Div Red (mot.)
1	Rifle	1	12	3600	2800
2	APC (MG)	6	18	180	100
3	ATW k	3	3	900	350
4	m	20	11	180	220
5	h	30	20	460	330
6	Tank	60	75	360	250
7	Art k	45	32	100	180
8	h	120	60	65	20
9	Heli-copter	60	-	50	-

Weapon System Combat Capability

Table VII

each sector j, the number of Y combat units never exceeds n_j^{CY} and the combat capabilities of all units are always identical. The units in each sector j behave as one unit as far as replacements are concerned.

X is assumed to commit reserves as soon as they arrive in the rear area. Therefore, the units comprising the X combat forces in each sector may not be identical with respect to their fighting strength because of different lengths of time in combat. Thus, X units need to be tracked individually rather than merely counted as for Y.

3. Combat Processes

Combat is modeled by three processes:

- attrition
- movement
- command and control.

The attrition and movement processes are represented by a system of three first-order, difference equations for $x^G_{j,\nu}$, $y^G_{j,\nu}$, and $d_{j,\nu}$, where

$x^G_{j,\nu}$ denotes the combat capability of all X ground forces in the combat region of the j-th sector at time t_ν,

$y^G_{j,\nu}$ denotes the firepower index of all Y ground forces in the combat region of the j-th sector at time t_ν, and

$d_{j,\nu}$ denotes FEBA position in the j-th sector at time t_ν.

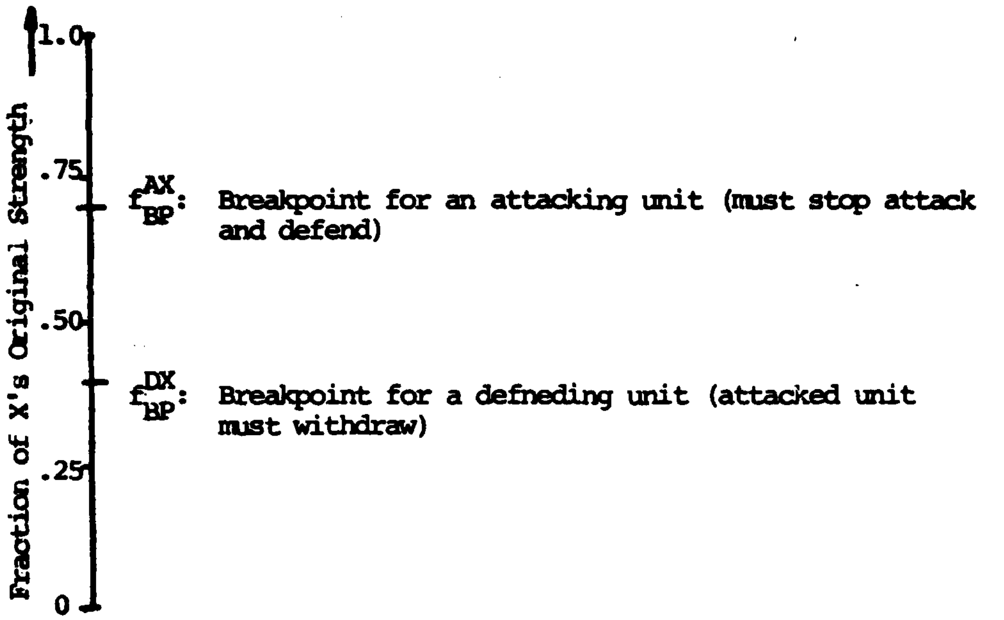
The command and control (C^2) process is represented by a logic which simulates tactical decisions by the unit commanders resulting in

(1) arrival of new units

(2) departure of old units

(3) tactical behavior of units.

Under tactical behavior the behavior of units at varying levels of losses (unit breakpoints) is meant. Examples for the tactical behavior of X and Y units is depicted in Figures 11 and 12.



Unit Breakpoints for X-Force

Figure 11

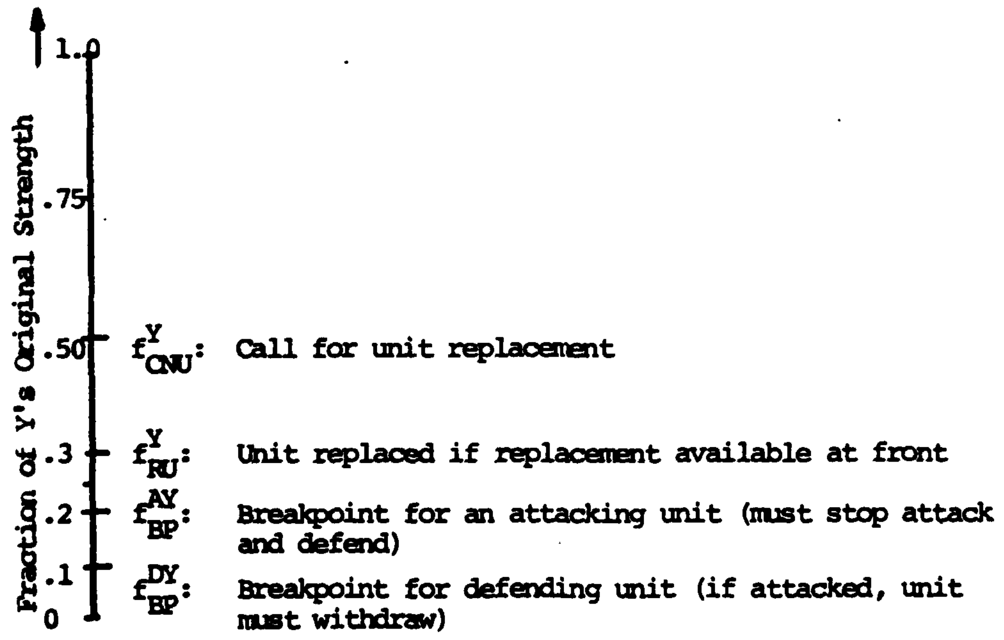


Figure 12

a. The Attrition Process

The model considers two types of possible losses: ground-induced and air-induced. The ground-induced losses of combat capability are developed from "ATLAS-like" casualty curves (see Figure 13, and [Ref. 15]). The air-induced losses result from the opponent's close air support activities in the combat region. In the rear region enemy air attack (interdiction) is assumed to be the only source of attrition. Losses in the rear region need therefore to be treated differently.

(1) Combat Region

For the X-force the combat capability $x^G_{\nu+1}$ at time $t_{\nu+1}$ is equal to the combat capability at the previous time t_ν reduced by the losses suffered in the time interval $[t_\nu, t_{\nu+1})$ of length Δt, i.e.,

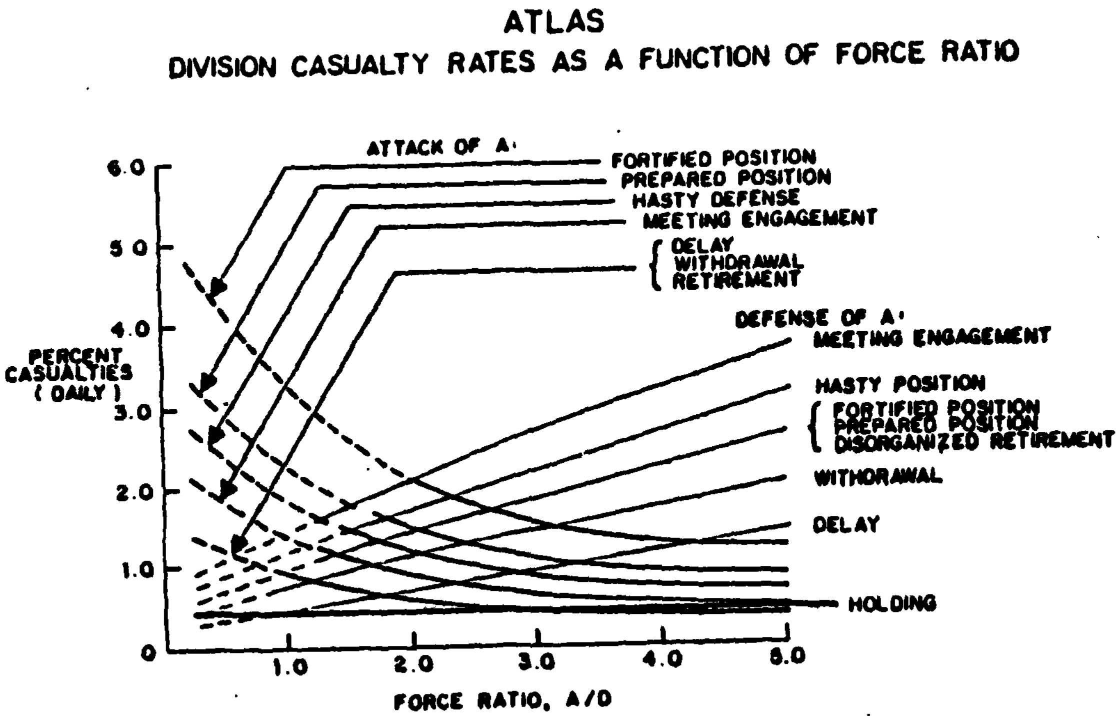
$$X^G_{\nu+1} = X^G_\nu - \left\{\begin{matrix}\text{ground-induced}\\ \text{losses of X}\end{matrix}\right\} - \left\{\begin{matrix}\text{air-induced}\\ \text{losses of X}\end{matrix}\right\}$$

The ground-induced losses are a function of (1) the attacker/defender (A/D) force ratio (i.e., A/D - ratio of firepower indices for all ground forces in the combat region of a sector, and (2) the engagement type. They result in

$$\begin{matrix}\text{ground-induced}\\ \text{losses of X}\end{matrix} = f_X(r^G_\nu, e_\nu)\ (\Delta t)\ X^G_\nu$$

where

$f_X(r^G_\nu, e_\nu)$ denotes X's fractional loss in combat capability per day from enemy ground-force activities (from ATLAS casualty curves (see Figure 13)),



Typical casualty-rate curves used in ATLAS

Figure 13

r^G denotes the attacker/defender force ratio at time t_ν, x^G_ν/y^G_ν or y^G_ν/x^G_ν,

e_ν denotes the engagement type at t_ν, and

Δt denotes the length of the assessment period.

The engagement type is determined by the mission of the opposing commanders in a sector (Table VIII). HENCE

$$e_\nu = e_\nu(M^\nu_X, M^\nu_Y)$$

It should be mentioned that the method for computing force ratios is rather simple and does not reflect real actions. Since no attrition occurs to airpower in the ground model, that contribution to value should not appear in the force ratio for the purpose of having it "attrited." The force ratio is computed in ATLAS and TAWAG as follows:

$$\text{Force Ratio A/D} = \frac{\text{Red's Total Ground Value in Sector} + \text{Red's Total CAS Value in Sector}}{\text{Blue's Total Ground Value in Sector} + \text{Blue's Total CAS Value in Sector}}$$

The air-induced losses of X's combat capability which result from the opponent's Y close air support (CAS) activities in time interval $[t_\nu, t_{\nu+1})$ are computed as

$$\text{air-induced losses of X} = \frac{1}{J} \sum_{i=1}^{n^A_Y} \alpha^Y_i \, S^{Y,3}_{i,\nu}$$

X mission \ Y mission	X position type \ Y position type	Attack	Defend			Delay	Withdrawal
		-	Hasty	Prepared	Fortified	-	-
Attack	-	Meeting Engagement	X Attack of HD	X Attack of PD	X Attack of FD	Delaying Action	Withdrawal
Defend	Hasty	Y Attack of HD	HA Static	HA	HA	HA	-
	Prepared	Y Attack of PD	HA	HA	HA	HA	-
	Fortified	Y Attack of FD	HA	HA	HA	HA	-
Delay	-	Delaying Action	HA	HA	HA	HA	-
Withdrawal	-	Withdrawal	-	-	-	-	-

NOTE: In the above figure, HA = Holding Action.

Engagement-Type Determination According to Mission and Type Defensive Position of Each of the Two Opposing Forces

Table VIII

and reflect one part of the air activities' contribution in support of the ground battle.

The air-induced losses are a function of the number of effective CAS sorties, $S_{i,\nu}^{Y,3}$ generated by the i-th Y air-system during a time step, the value α_i^Y of X's, destroyed ground-force targets by one effective Y CAS sortie and the number of sectors J into which the entire battlefield has been divided, under the assumption that all Y CAS sorties are uniformly distributed over the sectors. The number α_i^Y and J are user inputs whereas $S_{i,\nu}^{Y,3}$ depends on the air war strategies chosen by the opponents.

Then, the ground-combat-capability at a time $t_{\nu+1}$ is computed for X as

$$x_{\nu+1}^G = x_\nu^G - f_X(r_\nu^G, e_\nu)x_\nu^G(\Delta t) - \frac{1}{J}\sum_{i=1}^{n_Y^A} \alpha_i^Y S_{i,\nu}^{Y,3}$$

for Y accordingly. Since the combat takes place in the sectors only, the above equation becomes

$$x_{j,\nu+1}^G = x_{j,\nu}^G - f_X(r_{j,\nu}^G, e_{j,\nu})x_{j,\nu}^G(\Delta t) - \frac{1}{J}\sum_{i=1}^{n_Y^A} \alpha_i^Y S_{i,\nu}^{Y,3}$$

with initial condition for X.

$$x_{j,0}^G = c_X^0\, n_j^{CX} \quad ; \text{ for Y accordingly.}$$

Due to the different replacement policies the losses of the X and Y forces need to be counted differently. Since all the Y ground-combat forces in the combat region of the battlefield sector are treated as one unit, no distribution of the overall Y losses to the component Y divisions is necessary, whereas for the X force, under the assumption that all losses are distributed uniformly to the x divisions within the combat region of a secotr, the

$$c_X^{\ell,\nu+1} = c_X^{\ell,\nu}\left(\frac{x^G_{j,\nu+1}}{x^G_{j,\nu}}\right) \quad \text{for all} \quad \ell \in I_{XS}^{j,\nu}$$

where $I_{XS}^{j,\nu}$ denotes the indices of the X units that are effective in the combat region of a sector at time t_ν.

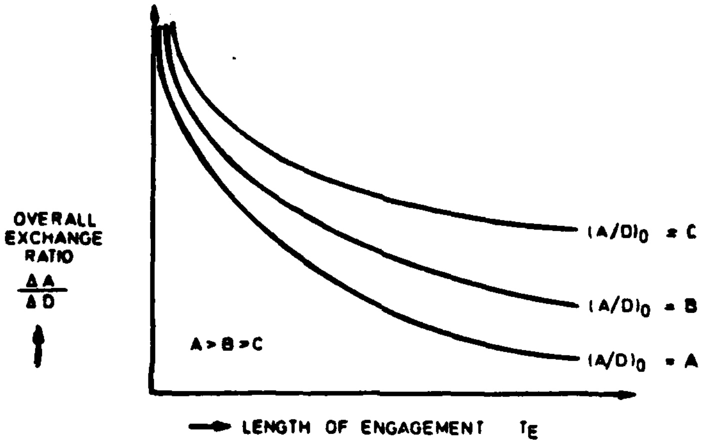
The ground war model also offers a special option for the computation of losses. It reflects the tactical considerations of some new possible NATO defense strategies. Instead of evaluating the exchange ratio with the fire-power indices, the overall exchange ratio is computed from curves depicting its dependence on the length of the engagement $T_E \leq \Delta t$, which is "broken off" by the defender. It reflects the idea, that with increasing time the attackers of fortified positions become more and more aware of the defenders' positions and therefore their fire becomes increasingly more effective. The ground induced losses for both sides become with this option

(1) $\binom{\text{Y's ground-induced}}{\text{casualties}} = f_Y(r^G_{j,\nu}, e_{j,\nu})\, Y^G_{j,\nu}\, T_E$

using ATLAS casualty rate curves

(2) $\binom{\text{X's ground-induced}}{\text{casualties}} = \binom{\text{Y's ground-induced}}{\text{casualties}} \binom{\text{Overall ex-}}{\text{change ratio}}$

from curves like Figure 14.



Dependence of the Overall Exchange Ratio on the Length of Engagement

Figure 14

(2) <u>Rear Region</u>

The attrition process in the rear region differs from that in the combat zone insofar that only air-induced losses from enemy INT-mission account for losses to ground force units, and only when they are assembled in staging areas or when they are in transit from their staging areas

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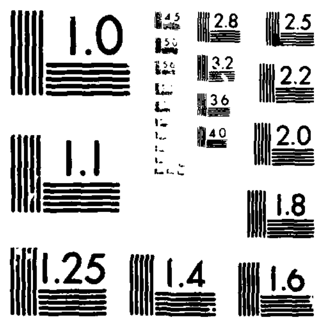


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

to the front. Vulnerability in both cases is assumed to be the same and is defined by the value $\beta_i^{X(Y)}$ of ground force units destroyed by one X(Y) INT-sortie.

TAWAG leaves the user two options. Interdiction being distributed equally among all sectors (general INT), or concentrated in one sector (concentrated *INT*).

For general INT and concentrated INT the combat capabilities are

$$y_{j,\nu+1}^{GSA} = y_{i,\nu}^{GSA} - \frac{1}{J} \sum_{i=1}^{n_X^A} \beta_i^X s_{i,\nu}^{X,4}$$

and

$$y_{jc,\nu+1}^{GSA} = y_{jc,\nu}^{GSA} - \sum_{i=1}^{n_X^A} \beta_i^X s_{i,\nu}^{X,4}$$

respectively. The equation for the X ground forces are similar with only $x_{j,\nu}^{GSA}$ computed as

$$\sum_{l \varepsilon I_{XSA}^{j,\nu}} c_X^{l,\nu}$$

due to the different replacement policy.

b. The Movement Process

(1) Combat Region

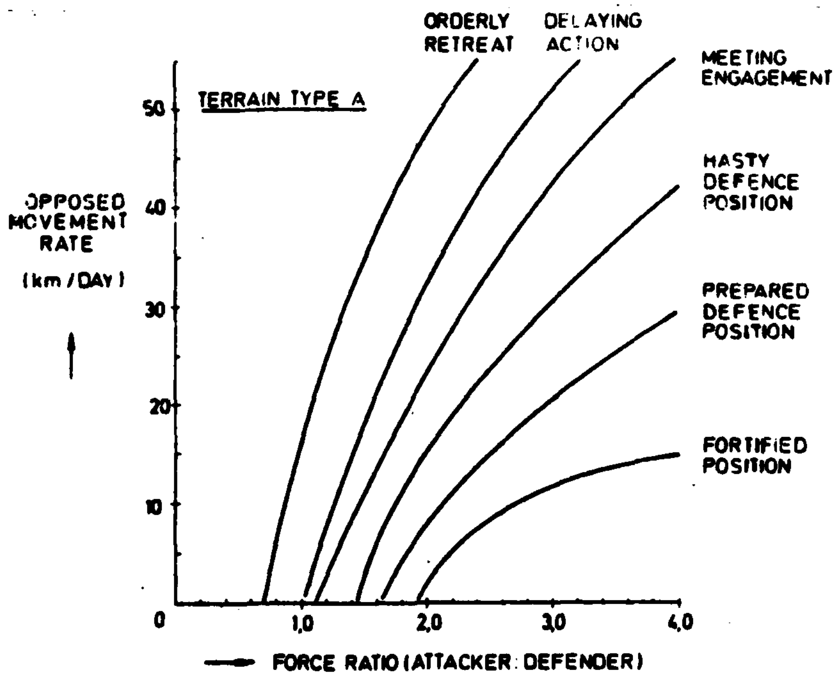
The attackers rate of advance in each sector is determined as a function of defender posture, terrain, and the prevailing force ratio of attacker effectiveness to defender effectiveness.

The ground war model determines FEBA movement by the FEBA-movement equation as follows:

$$d_{\nu+1} = d_\nu + F(\rho_\nu;\tau_\nu)\ (\Delta t)$$

where the expression $F(\rho_\nu;\tau_\nu)$ (Δt) is the change in FEBA position in the sector at time t_ν. Several factors not usually accounted for are included in the above equation:

$F(\rho_\nu;\tau_\nu)$ denotes the opposed movement rate (see Figure 15), values are to be taken from that figure.



Opposed Movement Rate as a Function of the Force Ratio (from [RG])

Figure 15

The curves are derived from Rex Goad [Ref. 33]. ρ_ν denotes the attacker/defender force ratio (including the disruptive effects of CAS), and $\tau_\nu = (e_\nu, Q_\nu)$ is a vector and denotes the tactical situation at time t_ν with e_ν being the engagement type at time t_ν, and Q_ν the terrain trafficability at time t_ν. The FEBA-movement in a given sector is computed accordingly.

To account for disruptive, even noncasualty producing effects of a CAS sortie the A/D force ratio is computed differently compared to the method used so far by embedding a "combat capability value" K_X^i of those effects, i.e.,

$$\rho_\nu = \frac{x_\nu^G + \frac{1}{J} \sum_{i=1}^{n_X^A} K_X^i \, S_{i,\nu}^{X,3}}{y_\nu^G + \frac{1}{J} \sum_{i=1}^{n_X^A} K_Y^i \, S_{i,\nu}^{Y,3}} \quad \text{when X attacks, for Y similarly.}$$

(2) Rear Region

The movement rate of reserve forces which move from the staging area to the front is simulated by

$$v_{j,\nu}^{X(Y)} = v_0^{X(Y)} \, g_{X(Y)}(S_\nu^{Y(X),4})$$

where

$$S_\nu^{Y(X),4} = \begin{cases} \sum_{i=1}^{n^A_{Y(X)}} w_i^{Y(X)} \, S_{i,\nu}^{Y(X),4} & \text{for concentrated INT} \\ \frac{1}{J} \text{ times the above} & \text{for general INT} \end{cases}$$

The value for the parameters for the reduction of unopposed movement rates of reserves from INT must be specified by the user. The authors suggest values of 20 kilometers/hour for both v_o^X and v_o^Y and 1.0 for $w_i^{X(Y)}$.

The functions $g_X(S_\nu^{Y,4})$ and $g_Y(S_\nu^{X,4})$ are montonically decreasing functions that reduce X's and Y's movement rate v_o due to enemy INT. They are proposed as

$$g_X(S_\nu^{Y,4}) = \begin{cases} (500 - S_\nu^{Y,4})/500 & \text{for} \quad 0 \le S_\nu^{Y,4} \le 500 \\ 0 & \text{for} \quad 500 \le S_\nu^{Y,4} \end{cases}$$

and for $g_Y(S_\nu^{X,4})$ similarly.

c. Command and Control (C^2) Process

(1) <u>Combat Region</u>

To simulate the tactical decisions of the section commanders, the ground-war model takes into account the ground-combat capability of the ground forces in the given sector, the A/D force ratio of the opposing forces in that sector, the FEBA-position in the sector, all at time t_ν and evaluates the mission of the section commander as a closed-loop control

$$M_X^{j,\nu} = M_X^{j,\nu}(x_{j,\nu}^G, r_{j,\nu}^G, d_{j,\nu}, t_\nu)$$

and

$$M_Y^{j,\nu} \text{ similarly,}$$

and according to Figures 11 and 12.

It should be noted that for the Y force in a sector the mission $M_Y^{j,\nu}$ = stop and defend is considered if the combat capability $y_{j,\nu}^G$ is less or equal to the fraction of $f_{BP}^{AY}\ y_{j,0}^G$ but greater than $f_{BD}^{DY}\ y_{j,0}^G$. Similarly

$$M_Y^{j,\nu} = \text{withdraw}$$

will be executed ifthe combat capability of a defending unit in a sector falls below the fraction of the original strength determined by the breakpoint for a defending unit.

For the X forces the calculations are similar, but the missions of the divisions, $M_X^{l,j,\nu}$, in a sector must sometimes be considered separately, since different X divisions may be in the combat region for different lengths of time, thus

$$M_X^{l,j,\nu} = M_X^{l,j,\nu}(c_X^{l,\nu}, r_{j,\nu}^G, d_{j,\nu}, t_\nu)$$

Similar to the before mentioned decision factors, the mission stop and defend or withdraw of the units are evaluated corresponding to the fractional strength of the unit. The following missions are implied by the authors:

$$M_X^{l,j,\nu} = \begin{cases} \text{DEFEND} & \text{for } f_{BD}^{DX} < f_X^{l,\nu} \leq 1.0 \\ \text{WITHDRAW} & \text{for } 0 \leq f_X^{l,\nu} \leq f_{BP}^{DX} \end{cases}$$

where

$$f_X^{l,\nu} = \frac{c_X^{l,\nu}}{c_X^0}$$

$$M_Y^{j,\nu} = \begin{cases} \text{ATTACK} & \text{for } f_{BD}^{AY} < f_Y^{j,\nu} \leq 1.0 \\ \text{DEFEND} & \text{for } f_{BP}^{DY} < f_Y^{j,\nu} \leq f_{BP}^{AY} \\ \text{WITHDRAW} & \text{for } 0 \leq f_Y^{j,\nu} \leq f_{BP}^{DY} \end{cases}$$

where

$$f_Y^{j,\nu} = \frac{y_{j,\nu}^G}{y_{j,0}^G}$$

The engagement type $e_{j,\nu}$ is determined from Table VIII, given the pair of missions $(M_X^{l,j,\nu}, M_Y^{j,\nu})$.

The model has a built-in termination rule which becomes effective if $M_X^{l,j,\nu}$ = WITHDRAW for all $l \in I_{XS}^{j,\nu}$ in any sector.

(2) <u>Rear Region</u>

The C^2 process in the rear region deals with the transfer of units from the rear region to the combat region requiring that

(a) a unit must first remain in the staging area for a given time to assemble T_A,

(b) a unit must then move a given distance from the staging area to the front line D_R.

During assembling and marching to the front line, a unit is exposed to enemy INT which causes delay.

Other assumptions inherent in the model are:

(a) replacement units are scheduled to arrive as functions of time only,

(b) when X reserve divisions arrive at the rear region, they are immediately dispatched to the front,

(c) as many divisions as possible are dispatched to the front whenever $y^G_{j,\nu} \leq f^Y_{CNU}\, y^G_{j,0}$ (see Figure 12),

(d) units disperse and new units are called into the staging area if Y units in the rear region are attrited to a fraction f^Y_{CNU} of their original strength.

IV. FINAL COMMENTS

TAWAG was developed to investigate questions relating to force structure planning, e.g., offensive versus defensive systems, direct or indirect offensive air support systems, multi-purpose versus special purpose systems. By providing the reader with an a priori evaluation of the model, the goal was to provide the reader with some feeling as how state-of-the-art TAWAG is. The ultimate evaluation would be to compare the predictions given by the model with real combat data. However this is currently not feasible for any combat model. The next step would be to exercise the model to gain insights into the complex random process of air/ground combat, in particular as to a/c allocations influence ground combat, and insights into the balance between achieving air superiority on the one hand, and providing close air support on the other, and the transition between emphasis on one and then the other component in an optimal strategy. Although this would be highly desirable, it was beyond the scope of this thesis to do that. Such work is highly recommended for the future.

The optimization methodology used in TAWAG is a heuristic one and can be labeled as an approximate optimization. The advantage of so-called approximate game solvers like TAC CONTENDER and DYGAM are that they can treat much bigger staged games in reasonable running times than rigorous game solvers like OPTSA. Where the former treat up to 90 stages, the latter permits only

3. The term strategy used in the context may be misleading as the model does not determine an optimal strategy for allocating a/c to missions but determines the sequence of decisions σ concerning the allocation of the air resources to missions.

TAWAG's biggest advantage over the other 4 described optimizing models is that not only multiple aircraft types can be played but that it makes also a wider selection of missions available than any other model. Further it is the only model besides TAC CONTENDER that distinguishes battlefield defense from air base defense. These two factors make it possible to investigate not only long-range planning concerning operational design of future aircraft but also to investigate trade-offs between air and ground systems. The analyst however should be aware of, as Stockfish [Ref. 19] points out, that model outputs must never be taken as predictive of the actual outcome of combat but as a means of establishing trends, relative comparisons, and insights into the dynamic combat. Even when using the most sophisticated and detailed model, the final decisions have to be made by reasoned military judgement.

BIBLIOGRAPHY

1. Anderson, L.B.: Decision Modelling in Large Scale Conflict Simulations. Pp. 231-256 in: Operationsanalytische Spiele fpr die Verteidigung/Operational Research Games for Defense (Huber, Niemeyer, Hofmann Eds.), Oldenbourg Verlag München-Wien, 1979.

2. Anderson, L.B., Bracken, J., Healy, J.G., Hutzler, M.J., Kerlin, E.P.: IDA Ground-Air Model I(IDAGAM I), Volume 1: Comprehensive Description. Report R-199, Institute for Defense Analyses, Arlington, Virginia, October 1974.

3. Berkovitz, L.D., Dresher, M.: Allocation of Two Types of Aircraft in Tactical Air War: A Game-Theroetic Analysis. Opns. Res. 8 (1960), pp. 694-706.

4. Berkovitz, L.B. and Dresher, M.: A Game Theory Analysis of Tactical Air War. Opns. Res. 7 (1959), pp. 599-620.

5. Bonder, S.: An Overview of Land Battle Modelling in the U.S. in Proceedings of the Thirteenth Annual U.S. Army Operations Research Symposium, Fort Lee, Virginia, 1974, pp. 73-88.

6. Bracken, J., Falk, J.E., Karr, A.F.: Two Models for Optimal Allocation of Aircraft Sorties. Opns. Res. 23 (1975), pp. 979-995.

7. Bracken, J., Falk, J.E., Miercot, F.A.: A Strategic Weapons Exchange Allocation Model. Opns. Res. 6 (1977), pp. 968-976.

8. Brackney, H.: The Dynamics of Military Combat. Opns. Res. 7 (1959), pp. 30-44.

9. Emerson, D.E.: The New TAGS Theater Air Ground Warfare Model (Incorporating Major Ground-Combat Revisions): A Description and Operating Instructions. R-1576-PR, The RAND Corporation, Santa Monica, California, September 1974.

10. Fulkerson, D.R., Johnson, S.M.: A Tactical Air Game. Opns. Res. 5 (1957), pp. 704-712.

11. Galiano, R.I., Miercort, F.A.: Results of a Survey of Tactical Air Campaign Models. Ketron-Report, ACDA/ PAB - 249, Nov. 1974.

12. General Research Corporation Report OAD-CR-60, Conceptual Design for the Army in the Field Alternative Force Evaluation CONAF Evaluation Model IV Part II Input Derivation, by Louer, P.E., and others, P. 1-30 and 69-90, December 1974.

13. General Research Corporation, "A Hierarchy of Combat Analysis Models," Jan. 1973.

14. Hembold, R.: Decision in Battle: Breakpoint Hypothesis and Engagement Termination Data. RAND Corporation Report R-772-Pr.

15. Kerlin, E.P., Cole, R.H.: ATLAS: A Tactical, Logistical and Air Simulation. Research Analysis Corporation, Military Gaming Department. Technical Paper RAC-TP-338.

16. Mengel, A.S.: Optimum Tactics in an Air Superiority Campaign. RM-1068, The RAND Corporation, Santa Monica, California, April 1953.

17. Morse, P.M., Kimball, G.E.: Methods of Operations Research. The M.I.T. Press, Cambridge, Mass. 1951.

18. Shupack, S.L.: An Examination of the Conceptual Basis of the Attrition Processes in the Institute for Defense Analyses Ground-Air Model (IDAGAM). M.S. Thesis in Operations Research, Naval Postgraduate School, Monterey, California, March 1979 (AD B0355 539L).

19. Stockfisch, J.A.: Models, Data and War: A Critique of the Study of Conventional Forces. The RAND Corporation, Santa Monica, California, R-1526-PR, March 1975.

20. Taylor, J.G.: Attrition Modelling, pp. 139-189 in: Operationsanalytische Spiele für die Verteidigung (Huber, Niemeyer, Hofmann Eds.), Oldenbourg Verlag, München 1979.

21. Taylor, J.G.: Recent Developments in the Lanchester Theory of Combat, pp. 773-806 in: Operational Research '78, Proceedings of the Eighth IFORS International Conference on Operational Research (Haley Ed.) North-Holland 1979.

22. Taylor, J.G., and Parry, S.H., "Force Ratio Considerations for Some Lanchester Type Models of Warfare," Operations Research, v. 23, no. 3, p. 522-533, May-June 1975.

23. Taylor, J.G.: Lanchester-Type Models of Warfare, Chapter 2, p. 110, July 1977 (to be published).

24. Lanchester, F.W.: Aircraft in Warefare: The Dawn of the Fourther Arm - No. V, The Principle of Concentration. Engineering 98 (1914). Reprinted on pp. 2138-2148 in: The World of Mathematics (Newman Ed.), Simon and Schuster, New York, 1956.

25. Landsdowne, Z.F. et al.: Development of an Algorithm to solve Multi-Stage Games. Control Analysis Corporation, Palo Alto, California, May 1973.

26. Lulejan and Associates: Prototype of a Theater-Level-Tactical Combat Model, Vol. 2: Model Logic and Equations. Weapons System Evaluation Group, WSEG Report 227, February 1974.

27. Assistant Chief of Staff, Studies and Analysis, U.S. Air Force: Methodology for Use in Measuring the Effectiveness of General Purpose Forces (An Algorithm for Approximating the Game-Theoretic Value of N-staged Games). Saber Grand (Alpha), March 1971.

28. Falk, J.E.: Remarks on Sequential Two-Person Zero-Sum Games and TAC-CONTENDER, T-292, Institute for Management Science and Engineering. The George Washington University, October 1973.

29. Vector Research Inc.: Report WSEG-2-FR 73-1, VECTOR-0. The Battle Model Prototype: Summary and Description. CPU: Seth Bonder, August 1973.

30. Bonder, S.: Vector-0, The Battle Model Prototype: Volume 2, A User's Guide. Weapons System Evaluation Group, Report 222, December 1973.

31. Harris, K.: Air Battle Simulation in the Tactical Air Allocation Study. WN-7654-PR. RAND Corporation, October 1971.

32. Kozacaka, F.: User's Guide to the TALLY Air Battle Model. WN-8727. RAND Corporation, July 1974.

33. Goad, R.: The Modelling of Movement in Tactical Games. Operationsanalytische Spiele für die Verterdigung, p. 199. (Huber, Niemeyer, Hofman Eds.), Oldenbourg Verlag, München-Wien, 1979.

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