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CONDITIONALLY DISTRIBUTION-FREE TEST FOR CENSORED BIVARIATE OBS--ETC(U)
JUL 79 W J PADGETT , L J WEI F49620-79-C-0140

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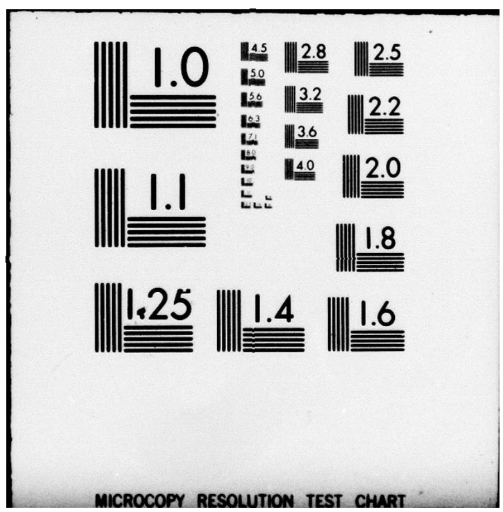
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(19) REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: (18) AFOSR TR-79-1037
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): (6) CONDITIONALLY DISTRIBUTION-FREE TEST FOR CENSORED BIVARIATE OBSERVATIONS
5. TYPE OF REPORT & PERIOD COVERED: (9) Interim rept.
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): (10) W. J. Padgett and L. J. Wei (14) TR-41
8. CONTRACT OR GRANT NUMBER(s): (15) F49620-79-C-0140
9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of South Carolina, Department of Mathematics, Computer Science, & Statistics, Columbia, S.C. 29208
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: (16) 61102F 2304/A5 (17) A5
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB, Washington, D.C. 20332
12. REPORT DATE: July 1979
13. NUMBER OF PAGES: 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

LEVEL

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

(11) Jul 79

(12) 12

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DDC RECEIVED NOV. 30 1979 A

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Marshall and Olkin bivariate exponential model; Right censorship; Permutation test.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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AFOSR-TR- 79-1037

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Statistics Technical Report No. 41
62C10

July, 1979

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*Research supported by the United States Air Force Office of Scientific Research under Contract No. F49620-79-C-0140.

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A conditionally distribution-free test is proposed for testing the symmetry of a bivariate distribution function with observations which are subject to arbitrary right censorship. In a numerical study, this new test is shown to be more powerful than the sign test under Marshall and Olkin's (1967) bivariate exponential model.

Key words and phrases: Right censorship; Marshall and Olkin bivariate exponential model; permutation test.

1. INTRODUCTION

In many situations, the comparisons between two treatments based on paired observations which are censored in one or both variates may arise. For example, Hammond (1964) has used a matched pair analysis to study smoking in relation to mortality in the United States. Batchelor and Hackett (1970) gave a comparison of the survival times between HL-A closely matched and poorly matched skin allografts on the same badly burned patients. Also, in life testing it may be desirable to compare the life times of two components in a system.

Suppose that

$$(X_1^0, Y_1^0)' , (X_2^0, Y_2^0)' , \cdots , (X_n^0, Y_n^0)' \tag{1.1}$$

are independent, identically distributed random vectors, having $H^0(s,t)$ as their distribution function (d.f.) and having $F^0(s)$ and $G^0(t)$ as their marginal d.f.'s, respectively, where ' denotes vector transpose. The null hypothesis, which is to be tested is

$$H_o: \; H^0(s,t) = H^0(t,s) , \quad \text{for} \quad (s,t)' \in R^2 .$$

Since X_i^0 and Y_j^0 may be censored from the right by variables U_i and V_j, respectively, (1.1) cannot always be observed. The observations available to the experimenter actually consist of the minima

$$\begin{aligned} X_1 &= \min(X_1^0, U_1) , \cdots , X_n = \min(X_n^0, U_n) , \\ Y_1 &= \min(Y_1^0, V_1) , \cdots , Y_n = \min(Y_n^0, V_n) , \end{aligned} \tag{1.2}$$

and two random sequences $\{\delta_1, \cdots, \delta_n\}$ and $\{\varepsilon_1, \cdots, \varepsilon_n\}$, where

$$\delta_i = \begin{cases} 1, & \text{if } X_i = X_i^0 , \\ 0, & \text{if } X_i < X_i^0 , \end{cases}$$

$$\epsilon_j = \begin{cases} 1, & \text{if } Y_j = Y_j^0 , \\ 0, & \text{if } Y_j < Y_j^0 . \end{cases}$$

For $i \neq j$, the censoring variables U_i and V_j are assumed to be independent random variables with a common d.f. J. To have the same censoring mechanism for both variates is quite common in paired studies. It is also assumed that $(U_i,V_i)'$ and $(X_i^0,Y_i^0)'$ are independent, $i = 1,2,\cdots,n$.

In the parametric case, the procedure for testing H_o is rather complicated and no useful results have been obtained. However, Holt and Prentice (1974) used the proportional hazards model (Cox (1972)) to analyze the data by Batchelor and Hackett (1970), and Wei (1979) proposed an asymptotically distribution-free test for H_o based on paired observations which are subject to arbitrary right censorship. Since the sample size in paired studies is frequently small, a distribution-free test is highly desirable. Although the sign test is a <u>conditionally</u> distribution-free test for testing H_o, it is rather inefficient when there are too many censored pairs in the data. As an extreme case, for the data $(3^+,4)'$, $(6,5^+)$, $(2^+,4)$, $(9,7^+)$, $(8^+,6^+)$, where "+" denotes censoring, the sign test leads to no conclusion about the null hypothesis H_o.

In this article, a conditionally distribution-free test for H_o is presented in Section 2. In a numerical study, it is shown that the new test is more powerful than the sign test under Marshall and Olkin's bivariate exponential model (Marshall and Olkin (1967)).

We note that all the results of this article can be easily extended to the case of arbitrarily restricted observations (Mantel (1967)).

2. THE TEST STATISTIC

Let $Z_i = X_i$ and $Z_{n+i} = Y_i$, $i = 1, \cdots, n$. We will say that Z_i is definitely greater than Z_j if $Z_i > Z_j$ and Z_j is observed, and Z_i is definitely less than Z_j if $Z_i < Z_j$ and Z_i is observed. Now, let $\xi_i(\eta_i)$, $i = 1,2, \cdots, n$, be the number of the remaining $(2n-1)$ Z's than which $X_i(Y_i)$ is definitely greater minus the number than which it is definitely less. The original observations (1.2) are then replaced by $(\xi_1,\eta_1)', \cdots, (\xi_n,\eta_n)'$. Under H_o and the assumption of an equal censoring mechanism for both variates, all the arrangements of the form

$$(R_1,R_2)', \cdots, (R_{2n-1},R_{2n})'$$

are equally likely, where $(R_{2i-1},R_{2i}) = (\xi_i,\eta_i)$ or (η_i,ξ_i) , $i = 1, \cdots, n$.

The statistic proposed here for testing H_o is

$$W = \sum_{i=1}^{n} R_{2i-1} .$$

Small or large values of $\sum_{i=1}^{n} \xi_i$ lead to the rejection of H_o . Note that $\sum_{i=1}^{n} \xi_i$ is Gehan's (1965) two-sample statistic. Mantel (1967) gave a simple routine to calculate ξ_i and η_i . It should also be noted that scores other than Gehan's (ξ_i,η_i) can be utilized.

The drawback to any permutation test such as W is the usually long and tedious calculations required when the sample size n is large. Fortunately, an asymptotically distribution-free test is available for large sample cases (Wei (1979)). In the rest of this article, we concentrate on the small-sample performance of the W test.

3. THE POWER STUDY

In this section, we study a special alternative hypothesis H_1 : $F^0(s) \leq G^0(s)$ for all s and $F^0(s') < G^0(s')$ for some s' , and compare the W test with the sign test under Marshall and Olkin's bivariate exponential model. The survival function of this model with parameters λ_1, λ_2, and λ_{12} can be written as:

$$P\{X^0 \geq s, Y^0 \geq t\} = \exp[-\lambda_1 s - \lambda_2 t - \lambda_{12} \max(s,t)] . \tag{3.1}$$

The two marginal means are

$$EX^0 = 1/(\lambda_1 + \lambda_{12}) \quad \text{and} \quad EY^0 = 1/(\lambda_2 + \lambda_{12}) .$$

Under this model, the hypotheses to be tested become $H_o : \lambda_1 = \lambda_2$ against $H_1 : \lambda_1 < \lambda_2$.

Three censoring schemes are considered in this comparison:

(A) $J(s)$ is a uniform distribution over $(0, EX^0)$;

(B) $J(s)$ is a uniform distribution over $(0, 2EX^0)$; and

(C) $J(s)$ is a uniform distribution over $(0, 4EX^0)$.

In this numerical study, the censoring variables U_i and V_i are assumed to be independent.

For the sign test and the W test, the proportions of times in the 1,000 Monte Carlo samples generated that H_o was rejected at the $\alpha = .05$ level were calculated for samples of sizes $n = 10$ and 15 from (3.1) with various values of λ_1, λ_2, and λ_{12}. Tables 1 and 2 give the results. As we expected, the W test is uniformly more powerful than the sign test. In addition to the drawback which was illustrated by an example in Section 1, another disadvantage of the sign test is the actual probability of Type I error is far below the specified α value. For example, when $n = 10$, under the severe censorship (A), the empirical levels of the sign test are only .01 as compared with the nominal value $\alpha = .05$.

Table 1. Proportion of times in 1,000 samples of size $n = 10$ that H_o was rejected at $\alpha = .05$ by sign test and W test.

			Censoring Scheme					
			(A)		(B)		(C)	
λ_{12}	λ_1	λ_2	W	Sign	W	Sign	W	Sign
.1	.1	1.0	.85	.60	.91	.73	.92	.76
	.2		.58	.34	.69	.45	.74	.49
	.3		.38	.17	.46	.24	.51	.29
	.4		.25	.09	.32	.14	.36	.18
	.5		.21	.06	.25	.11	.26	.14
	.6		.15	.05	.18	.07	.19	.09
	.7		.09	.02	.11	.05	.13	.06
	.8		.06	.02	.07	.03	.08	.04
	.9		.05	.02	.06	.02	.05	.03
	1.0		.04	.01	.04	.02	.04	.03
.5	.1	1.0	.44	.16	.57	.34	.64	.47
	.2		.33	.09	.42	.20	.48	.30
	.3		.23	.06	.28	.13	.34	.20
	.4		.18	.04	.21	.09	.25	.13
	.5		.15	.02	.18	.06	.19	.09
	.6		.12	.01	.13	.04	.14	.05
	.7		.07	.01	.09	.03	.10	.04
	.8		.06	.01	.07	.02	.07	.03
	.9		.06	.01	.05	.02	.06	.02
	1.0		.04	.01	.05	.01	.04	.02

Table 2. Proportion of times in 1,000 samples of size $n = 15$ that H_o was rejected at $\alpha = .05$ by sign test and W test.

			Censoring Scheme					
			(A)		(B)		(C)	
λ_{12}	λ_1	λ_2	W	Sign	W	Sign	W	Sign
.1	.2	1.0	.79	.55	.86	.66	.88	.74
	.4		.38	.18	.47	.25	.51	.32
	.6		.17	.06	.21	.10	.23	.13
	.8		.09	.03	.10	.04	.11	.05
	1.0		.04	.01	.04	.02	.04	.02
.5	.2	1.0	.48	.25	.60	.42	.67	.50
	.4		.27	.10	.32	.17	.37	.22
	.6		.13	.04	.16	.06	.19	.09
	.8		.07	.01	.09	.03	.09	.04
	1.0		.05	.01	.05	.02	.05	.03

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