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SPATIAL CORRELATION FUNCTION FOR A FREQUENCY BAND FOR VERTICAL --ETC(U)

AUG 65 R L SHAFFER

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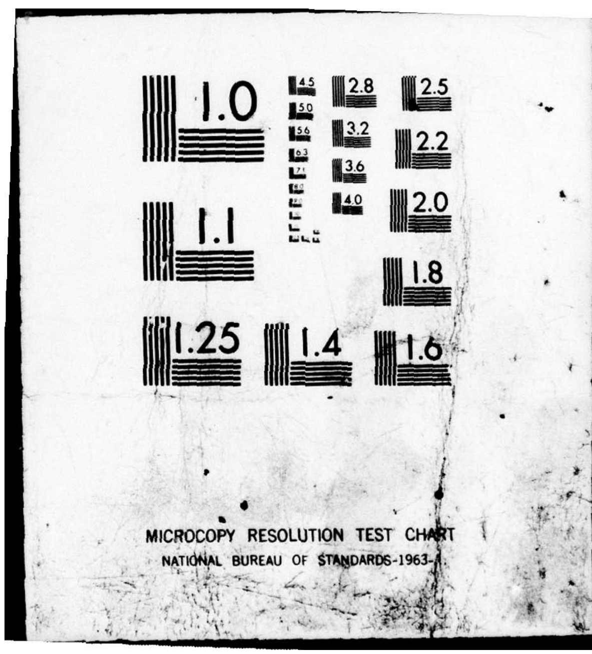
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(6) SPATIAL CORRELATION FUNCTION FOR A FREQUENCY BAND FOR VERTICAL RECEIVERS AND DIRECTIONAL NOISE.

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~~By~~ (10) R. L. Shaffer

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INTRODUCTION

The expression for the single frequency spatial correlation function reported ~~in reference 1 as equation 2 of Section II B~~ has been modified by Mr. B. Cron ~~(Reference (2))~~ to comply with the revised general expression given ~~in reference 3.~~ The expression for the single frequency spatial correlation function with directional noise $g(\alpha) = \cos\alpha$ for vertical receivers as given by reference 2 is:

$$\rho(d,T) = 2\left[\frac{\sin kd}{kd} + \frac{(\cos kd - 1)}{(kd)^2}\right]\cos\omega T$$

$$+ 2\left[\frac{\sin kd}{(kd)^2} - \frac{\cos kd}{kd}\right]\sin\omega T \qquad (1)$$

The definitions of the terms in this expression and those in subsequent equations are found in the glossary at the end of the memorandum. Most of these definitions come from references 1 and 3.

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In this memorandum the expression for the spatial correlation for a frequency band was obtained by forming the product of the single frequency spatial correlation function and the power spectrum and then integrating this product over a bandwidth. This expression for the spatial correlation for a frequency band was checked by letting b approach unity and obtaining equation (1) again.

DEVIATION OF THE SPATIAL CORRELATION FOR A FREQUENCY BAND

The expression for the spatial correlation for a frequency band, $\rho(d, \tau, b)$, as outlined in the introduction is:

$$\rho(d,\tau,b) = \int_{f_1}^{f_2} \left\{ 2 \left[\frac{\sin \beta d}{\beta d} + \frac{(\cos \beta d - 1)}{(\beta d)^2} \right] \cos \omega\tau + 2 \left[\frac{\sin \beta d}{(\beta d)^2} - \frac{\cos \beta d}{\beta d} \right] \sin \omega\tau \right\} \frac{df}{f_2 - f_1} \tag{2}$$

In equation (2) $\rho(d, \tau, b)$ can be thought of as being composed of two parts $\rho_1(d, \tau, b)$ and $\rho_2(d, \tau, b)$ where $\rho_1(d, \tau, b)$ and $\rho_2(d, \tau, b)$ are given by

$$\rho_1(d,\tau,b) = \int_{f_1}^{f_2} 2 \left[\frac{\sin \beta d}{\beta d} + \frac{(\cos \beta d - 1)}{(\beta d)^2} \right] \frac{\cos \omega\tau \, df}{f_2 - f_1} \tag{3}$$

$$\rho_2(d,\tau,b) = \int_{f_1}^{f_2} 2 \left[\frac{\sin \beta d}{(\beta d)^2} - \frac{\cos \beta d}{(\beta d)} \right] \frac{\sin \omega\tau \, df}{f_2 - f_1} \tag{4}$$

Equation (3) has been evaluated in reference 1 and is given by:

$$\rho_1(d,\tau,b) = \frac{1}{2\pi x(1 - 1/b)} \left\{ 2 \left[\frac{\cos(2\pi x/b) - 1}{2\pi x/b} \right] \left[\cos(2\pi x y/b) \right] \right.$$

$$-2\left[\frac{\cos 2\pi b x - 1}{2\pi b x}\right]\left[\cos 2\pi x b \psi\right] + 2\psi\left[\mathrm{Si}(2\pi b x \psi) - \mathrm{Si}(2\pi x \psi / b)\right]$$

$$-\psi\left\{\mathrm{Si}\left[(1+\psi)2\pi b x\right] - \mathrm{Si}\left[(1-\psi)2\pi b x\right] - \mathrm{Si}\left[(1+\psi)2\pi x/b\right]\right.$$

$$\left. + \mathrm{Si}\left[(1-\psi)2\pi x/b\right]\right\} \quad (5)$$

which now leaves the solution of the integral in equation 4 to be evaluated.

In evaluating the integral of equation (4) the following notation is used

$$I_1 = \int_{f_1}^{f_2} \frac{2\sin\beta d}{(\beta d)^2}\,\frac{\sin\omega T}{f_2 - f_1}\,df \quad (6)$$

$$I_2 = \int_{f_1}^{f_2} \frac{\cos\beta d}{\beta d}\,\frac{\sin\omega T}{f_2 - f_1}\,df \quad (7)$$

and

$$\rho_2(d, T, b) = I_1 - I_2 \quad (8)$$

By letting $u = kd = 2\pi f d/c$; then $du = 2\pi d/c\,df$ and $\omega T = u\,c/d\,T = \psi u$.
Equation (6) can now be written as

$$I_1 = \frac{1}{2\pi x(b - 1/b)} \int_{u_1}^{u_2} \frac{2\sin u}{u^2}\,\sin\psi u\,du \quad (9)$$

where $u_1 = 2\pi x/b$ and $u_2 = 2\pi bx$

Using trigonometric formulas, equation (9) becomes

$$I_1 = \frac{1}{2\pi x(b - 1/b)} \int_{u_1}^{u_2} \left[\frac{\cos u(1-\psi)}{u^2} - \frac{\cos u(1+\psi)}{u^2}\right] du \quad (10)$$

The values of the integrals are given by

$$\int_{u_1}^{u_2} \frac{\cos[u(1-\psi)]}{u^2}\,du = -1\,\frac{\cos[(1-\psi)u]}{u}\Bigg|_{u=u_1}^{u=u_2} - (1-\psi)\int_{u_1}^{u_2}\frac{\sin(1-\psi)\,du}{u}$$

$$= -\frac{\cos[(1-\psi)2\pi x b]}{2\pi x b} + \frac{\cos[(1-\psi)2\pi x/b]}{2\pi x/b}$$

$$-(1-\psi)\{Si[(1-\psi)2\pi b x] - Si[(1-\psi)2\pi x/b]$$

(11)

and $$-\int_{u_1}^{u_2}\frac{\cos u(1+\psi)\,du}{u^2} = \frac{\cos[2\pi b x(1+\psi)]}{2\pi b x} - \frac{\cos[2\pi x/b\,(1+\psi)]}{2\pi x/b}$$

$$+(1+\psi)\{Si[(1+\psi)2\pi x b] - Si[(1+\psi)2\pi x/b]\} \quad (12)$$

Using the same substitutions used in arriving at equation (9), equation (7) becomes

$$I_2 = \frac{1}{2\pi x(b-1/b)}\int_{u_1}^{u_2}\frac{2\cos u}{u}\sin\psi u\,du \quad (13)$$

Using trigonometric formulas equation (13) becomes

$$I_2 = \frac{1}{2\pi x(b-1/b)}\int_{u_1}^{u_2}\left[\frac{\sin[u(1+\psi)]}{u}\,du + \frac{\sin[u(\psi-1)]}{u}\,du\right. \quad (14)$$

Integrating this gives

$$I_2 = \frac{1}{2\pi x(b-1/b)}\Big\{Si[(1+\psi)2\pi b x] - Si[(1+\psi)2\pi x/b]$$

$$- Si[(1-\psi)2\pi x b] + Si[(1-\psi)2\pi x/b]$$

(15)

From equations (11), (12), and (15) equation (8) becomes

$$\rho_2(d,\tau,b) = \frac{1}{2\pi x(b-1/b)}\left\{\frac{\cos[2\pi xb(1+\psi)]}{2\pi xb} - \frac{\cos[2\pi x/b(1+\psi)]}{2\pi x/b} - \frac{\cos[(1-\psi)2\pi xb]}{2\pi xb}\right.$$

$$+ \frac{\cos[(1-\psi)2\pi x/b]}{2\pi x/b} + (1+\psi)\left\{Si[(1+\psi)2\pi xb] - Si[(1+\psi)2\pi x/b]\right\}$$

$$- (1-\psi)\left\{Si[(1-\psi)2\pi bx] - Si[(1-\psi)2\pi x/b]\right\}$$

$$- Si[(1+\psi)2\pi bx] + Si[(1+\psi)2\pi bx]$$

$$\left. - Si[(1-\psi)2\pi x/b] + Si[(1-\psi)2\pi xb]\right\}$$

(16)

Regrouping terms and using appropriate trigonometric formulas, equation (16) can be rewritten as

$$\rho_2(d\,\tau\,b) = \frac{1}{2\pi x(b-1/b)}\left\{-\frac{2\sin 2\pi xb\sin 2\pi xb\psi}{2\pi xb}\right.$$

$$+ \frac{2\sin 2\pi x/b\cdot\sin 2\pi x\psi/b}{2\pi x/b} + \psi\left\{Si[(1-\psi)2\pi xb]\right.$$

$$- Si[(1-\psi)2\pi x/b] + Si[(1+\psi)2\pi xb]$$

$$\left.\left. - Si[(1+\psi)2\pi x/b]\right\}\right\}$$

(17)

The expression for $\rho_1(d, \tau, b)$ can be regrouped and written as:

$$\rho_1(d,\tau,b) = \frac{1}{2\pi x(b-1/b)} \Bigg\{ \frac{2\cos 2\pi x/b \cos 2\pi x\psi/b}{2\pi x/b} - \frac{2\cos 2\pi x b \cos 2\pi x b\psi}{2\pi x b}$$

$$- \frac{2\cos 2\pi x\psi/b}{2\pi x/b} + \frac{2\cos 2\pi x\psi b}{2\pi x b} + 2\psi\left[Si(2\pi b x\psi) - Si(2\pi x\psi/b)\right]$$

$$- \psi\Big\{Si\left[(1+\psi)2\pi b x\right] - Si\left[(1-\psi)2\pi b x\right] - Si\left[(1+\psi)2\pi x/b\right]$$

$$+ Si\left[(1-\psi)2\pi x/b\right]\Big\}\Bigg\}$$

(18)

Equations (17) and (18) can be combined to give

$$\rho(d,\tau,b) = \rho_1(d,\tau,b) + \rho_2(d,\tau,b)$$

$$= \frac{1}{2\pi x(b-1/b)} \Bigg\{ 2\frac{\cos\left[2\pi x/b(1-\psi)\right]}{2\pi x/b} - 2\frac{\cos\left[2\pi x b(1-\psi)\right]}{2\pi x b}$$

$$+ \frac{2\cos 2\pi x\psi b}{2\pi x b} - \frac{2\cos 2\pi x\psi/b}{2\pi x/b} + 2\psi\Big\{Si(2\pi b x\psi) - Si(2\pi x\psi/b)$$

$$+ Si\left[(1-\psi)2\pi b x\right] - Si\left[(1-\psi)2\pi x/b\right]\Big\}\Bigg\}$$

(19)

An alternate form of equation (19) is

$$\rho(d,T,b) = \frac{1}{2\pi x(b-1/b)} \left\{ \frac{2 \sin \pi x b \cdot \sin[\pi x b(1-2\psi)]}{\pi x b} - \frac{2 \sin \pi x/b \cdot \sin[\pi x/b(1-2\psi)]}{\pi x/b} + \psi \left\{ Si\, 2\pi b x \psi - Si\, 2\pi x \psi/b + Si\,[(1-\psi)\, 2\pi b x] - Si\,[(1-\psi)\, 2\pi x/b] \right\} \right\} \tag{20}$$

EVALUATION OF THE SPATIAL CORRELATION FOR A FREQUENCY BAND AS b APPROACHES UNITY

As a check on the validity of the spatial correlation for a frequency band, b was allowed to approach unity in equation (19). By letting $2\pi x = A$, $2\pi x(1-\psi) = B$, and $2\pi x\psi = C$ equation (19) becomes

$$\rho(d,T,b) = \frac{2}{A^2(b-1/b)} \left\{ \frac{\cos B/b}{b^{-1}} - \frac{\cos Bb}{b} + \frac{\cos Cb}{b} - \frac{\cos C/b}{b^{-1}} + C\,[Si\, Cb - Si\, C/b] - B\,[Si\, Bb - Si\, B/b] + A\,[Si\, Bb - Si\, B/b] \right. \tag{21}$$

By expressing the cosine and sine integral in the following series (References (4) and (5))

$$\cos Z = 1 + \sum_{n=0}^{\infty} \frac{(-1)^{n+1} Z^{2n+2}}{(2n+2)!} \tag{22}$$

$$Si\, Z = \sum_{n=0}^{\infty} \frac{(-1)^{n} Z^{2n+1}}{(2n+1)(2n+1)!} \tag{23}$$

equation (21) becomes

$$P(d,T,b) = \frac{2}{A^2(b-1/b)} \sum_{n=0}^{\infty} \Bigg\{ (-1)\left[\frac{(-1)^n}{(2n+2)!}\right]\left[C^{2n+2} - B^{2n+2}\right]\left[b^{2n+1} - b^{-2n-1}\right] + \frac{(-1)^n}{(2n+1)(2n+1)!}\left[C^{2n+2} - B^{2n+2}\right]\left[b^{2n+1} - b^{-2n-1}\right] + A\left[B^{2n+1}\left(b^{2n+1} - b^{-2n-1}\right)\right]\Bigg\} \tag{24}$$

Applying L' Hospital's rule and letting b approach unity

$$P(d,T,b) = 1/A^2 \sum_{n=0}^{\infty} \Bigg\{ \left[\frac{(-1)^{n+1}}{(2n+2)!}\right]\left[C^{2n+2} - B^{2n+2}\right]\left[2(2n+1)\right] + (-1)\left[\frac{(-1)^{n+1}(2n+2)\,2(2n+1)}{(2n+1)(2n+2)(2n+1)!}\right]\left[C^{2n+2} - B^{2n+2}\right] + \frac{(-1)^n A}{(2n+1)!}\left[2B^{2n+1}\right]\Bigg\} \tag{25}$$

which can be rewritten as

$$\rho(d,T,b) = \frac{2}{A^2} \sum_{n=0}^{\infty} \left\{ (2n+1)\left[\operatorname{Cos} C - \operatorname{Cos} B\right] - (2n+2)\left[\operatorname{Cos} C - \operatorname{Cos} B\right] \right\}$$

$$+ 2/A \sin B \tag{26}$$

Substituting the appropriate values of A, B, and C equation (26) becomes

$$\rho(d,T,b) = \frac{2}{(2\pi X)^2} \left\{ \operatorname{Cos}\left[2\pi X(1-Y)\right] - \operatorname{Cos}\left[2\pi X Y\right] \right\}$$

$$+ \frac{2}{2\pi X} \left\{ \sin\left[2\pi X(1-Y)\right] \right\} \tag{27}$$

Equation (27) can be written in the form

$$\rho(d,T,b) = 2\left\{ \frac{\cos(bd-\omega T)}{(bd)^2} - \frac{\cos\omega T}{(bd)^2} + \frac{\sin(bd-\omega T)}{bd} \right. \tag{28}$$

Using the proper trigonometric formulas, equation (28) becomes

$$\rho(d,T,b) = 2\left[\frac{\cos bd - 1}{(bd)^2} + \frac{\sin bd}{bd}\right]\cos\omega T$$

$$+ 2\left[\frac{\sin bd}{(bd)^2} - \frac{\cos bd}{bd}\right]\sin\omega T \tag{29}$$

Equation (29) is the desired result. Thus, equation (19) has been verified by letting b approach unity with the expected result of obtaining equation (1) again.

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Research Physicist

GLOSSARY OF TERMS

α is the angle between the line connecting the receiver and noise source and the vertical line passing through the center of the circular area of noise sources

d is the distance between receivers

τ is the electrical time delay

= $2\pi/$ wavenumber

$\lambda = c/f$ wavelength

$\omega = 2\pi f$ angular frequency

c is velocity of sound

$b = \sqrt{f_2/f_1}$

$x = d/\lambda_g$

$\lambda_g = c/\sqrt{f_1 f_2}$ geometric wavelength

$\psi = \tau/\, d/c$

$\sqrt{f_1 f_2}$ is geometric mean frequency of flat bandwidth f_1 to f_2

$Si(x) = \int_0^x \frac{\sin u \, du}{u}$ = sine integral

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