

AD-A069 999 CINCINNATI UNIV OH DEPT OF AEROSPACE ENGINEERING AND--ETC F/6 20/4
A NEW APPROACH FOR SOLVING THE VORTICITY AND CONTINUITY EQUATIO--ETC(U)
JAN 79 A HAMED, S ABDALLAH F49620-78-C-0041

UNCLASSIFIED

AFOSR-TR-79-0630

NL

1 OF 1
AD A069999



END
DATE
FILMED
7-79
DDC

19 REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER AFOSR TR. 79-0630	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) A NEW APPROACH FOR SOLVING THE VORTICITY AND CONTINUITY EQUATIONS IN TURBOMACHINERY DUCTS.		5. TYPE OF REPORT & PERIOD COVERED INTERIM report	
7. AUTHOR(s) A/HAMED S/ABDALLAH		6. PERFORMING ORG. REPORT NUMBER 79-0046	8. CONTRACT OR GRANT NUMBER(s) F49620-78-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS UNIVERSITY OF CINCINNATI DEPARTMENT OF AEROSPACE ENGINEERING & APPLIED MECHANICS CINCINNATI, OHIO 45221		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2307A4 31102F A41	
11. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA BLDG 410 BOLLING AIR FORCE BASE, D C 20332		12. REPORT DATE 11 Jan 79	13. NUMBER OF PAGES 8
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) LEVEL 1		15. SECURITY CLASS. (of this report) UNCLASSIFIED	
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE			
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES Aerospace Sciences Meeting 17th, New Orleans, LA 15-17 Jan 79			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) VORTICITY EQUATION NUMERICAL FLOW SOLUTIONS CONTINUITY EQUATION AXISYMMETRIC FLOW FIELD STREAMLIKE FUNCTION DIRICHLET BOUNDARY CONDITIONS			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Repeated solutions to the continuity and vorticity equations are frequently required in computations of three dimensional flows in turbomachinery passages. When the two equations are nonhomogeneous previous formulations resulted in two second order differential equations. A new approach is presented here, which is applicable in a generalized two dimensional domain or axisymmetric field. It is based on the definition of a streamlike function which is used to transform these nonhomogeneous first order partial differential equations to a single second order equation with Dirichlet boundary conditions over the solid boundaries.			

A069999
DDC FILE COPY

DD FORM 1 JAN 73 1473

410 677

UNCLASSIFIED (cont)

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DDC
RECEIVED
JUN 18 1979

Some applications are presented to show how this new approach can be used to save computer time in numerical flow solutions.

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DDC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	<input type="checkbox"/>
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or special
A	

UNCLASSIFIED

~~AFOSR-TR-79-0630~~
AFOSR-TR- 79 - 0630



~~79-0046~~
79-0046

**A New Approach for Solving the Vorticity and
Continuity Equations in Turbomachinery Ducts**

A. Hamed and S. Abdallah, *University of
Cincinnati, Ohio*

**17th AEROSPACE SCIENCES
MEETING**

New Orleans, La./January 15-17, 1979

~~AFOSR-TR-79-0630~~
Approved for public release;
Distribution unlimited.

A NEW APPROACH FOR SOLVING THE VORTICITY AND CONTINUITY

EQUATIONS IN TURBOMACHINERY DUCTS[†]

A. Hamed* and S. Abdallah**

University of Cincinnati
Cincinnati, Ohio 45221

Abstract

Repeated solutions to the continuity and vorticity equations are frequently required in computations of three dimensional flows in turbomachinery passages. When the two equations are nonhomogeneous previous formulations resulted in two second order differential equations. A new approach is presented here, which is applicable in a generalized two dimensional domain or axisymmetric field. It is based on the definition of a streamlike function which is used to transform these nonhomogeneous first order partial differential equations to a single second order equation with Dirichlet boundary conditions over the solid boundaries. Some applications are presented to show how this new approach can be used to save computer time in numerical flow solutions.

Nomenclature

C_1 to C_6	arbitrary constants in Eqs. (11) to (16).
j	parameter equal to zero for two dimensional flow, and to one for axisymmetric flow
S	source/sink term in Eq. (1)
u	velocity component in x-direction
v	velocity component in y-direction
x_r	reference value of the x coordinate in the x_1 formulation
y_r	reference value of the y coordinate in the x_2 formulation
σ_1 and σ_2	the nonhomogeneous terms in the second order equations for the streamlike functions, Eqs. (5), (6), (9) and (10)
x_1 and x_2	streamlike functions defined in Eqs. (3), (4), (7) and (8)
ω	flow vorticity, Eq. (2)

Introduction

The simultaneous solution of the two first order partial differential equations representing the conservation of mass and the vorticity, is required in many flow studies.¹⁻⁵ The authors interest in this problem is connected to their internal nonviscous subsonic flow investigations^{1,2} in the various turbomachinery passages. The need for the solution to the outlined problem is also encountered in other diversified flow fields such as external two dimensional transonic flows³ and internal viscous flows.^{4,5} In three dimensional flow fields, the equation of conservation of mass includes a derivative of the third velocity component. Therefore, except for two dimensional incompressible flow or for irrotational flow, the two equations are generally nonhomogeneous. When at least one of the equations is homogeneous, the traditional formulation of flow problems has been in terms of a potential function or a stream function. In the absence of a source term in the continuity equation, the stream function is introduced into the rotationality equation to obtain the second order partial differential equation for the stream function. On the other hand, the irrotational flow is usually studied in terms of the potential function which is used into the continuity equation to obtain the governing second order partial differential equation. There has been no single unified approach, however, for the mathematical formulation of the problem when both the continuity and the rotationality equations are nonhomogeneous. In references 4, 5 and 6, two different mathematical approaches were used for the solution of the continuity and rotationality equations. Rubin and Khosla⁴ formulated the problem in terms of a stream function and a potential function, resulting in two second order partial differential equations in these functions. The problems of handling the Neumann boundary conditions for the potential function over irregular boundaries are well known.⁷ There are additional problems involved in the numerical solution

[†] This research work was sponsored by AFOSR ~~Contract~~ ^{contract} F49620-78-^c0041, Air Force Office of Scientific Research, Bolling Air Force Base, D.C.

* Associate Professor, Department of Aerospace Engineering and Applied Mechanics, University of Cincinnati, Cincinnati, Ohio. Member AIAA.

** Graduate Research Assistant, Department of Aerospace Engineering and Applied Mechanics, University of Cincinnati, Cincinnati, Ohio.

using finite difference methods, when the potential equation has a nonhomogeneous term, with this type of boundary conditions over all the boundaries of the solution domain.^{2,8,9} A different approach was followed in references 5 and 6. Cross differentiation was used to obtain two second order partial differential equations in the two velocity components. In this case, additional boundary conditions are required which are obtained from the original two first order equations.

The authors are proposing in this paper a new unified approach for the mathematical formulation of the nonhomogeneous continuity and rotationality equations. The approach is based on the definition of a new dependent variable which will be referred to as "a streamlike function". The introduction of this new dependent variable transforms the two first order nonhomogeneous partial differential equations to a single second order equation in the new variable. This formulation is clearly superior therefore to those used in references 4, 5 and 6, which resulted in two simultaneous second order equations. The advantage of the new formulation in terms of computer time savings are obvious, since only half the number of the difference equations need to be solved. Another advantage of the formulation is that the boundary conditions for the streamlike function are of the Dirichlet type over the solid boundaries. This can particularly be very helpful in the numerical solution of flow problems with irregular boundaries, using finite difference methods. Some applications of the streamlike function formulation to the flow in turbo-machine passages will be presented to show the diversity and flexibility of this new approach.

Mathematical Formulation

The continuity and rotationality equations can generally be written in the following form:

$$\frac{\partial}{\partial x} (x^j u) + \frac{\partial}{\partial y} (x^j v) = x^j S(x, y) \quad (1)$$

$$\frac{\partial}{\partial y} (u) - \frac{\partial}{\partial x} (v) = -\omega(x, y) \quad (2)$$

Where u , v are the velocity components in the x and y directions respectively, S is the source/sink term, and ω is the vorticity in the direction normal to plane x, y . The value of j is zero for two dimensional flow and j is one for the axisymmetric case.

The Streamlike Function

A new streamlike function is introduced, such that the continuity equation is automatically satisfied. The velocity components are defined in terms of the streamlike function and the source term as

follows:

$$u = \frac{1}{x^j} \frac{\partial \chi_1}{\partial y} + \frac{1}{x^j} \int_{x_r}^x x^j S(x, y) dx \quad (3)$$

and

$$v = -\frac{1}{x^j} \frac{\partial \chi_1}{\partial x} \quad (4)$$

Where the subscript r refers to a chosen reference value.

When equations (3) and (4) are substituted into equation (2), we obtain:

$$\frac{\partial^2 \chi_1}{\partial x^2} + \frac{\partial^2 \chi_1}{\partial y^2} - \frac{j}{x} \frac{\partial \chi_1}{\partial x} = \sigma_1(x, y) \quad (5)$$

where

$$\sigma_1(x, y) = x^j \omega(x, y) + \frac{\partial}{\partial y} \int_{x_r}^x x^j S(x, y) dx \quad (6)$$

The deviation from the standard definition of the stream function in this case is in the x -velocity component u , given by equation (3). Another streamlike function, χ_2 , that satisfies equation (1) automatically, could also be defined as follows:

$$u = \frac{1}{x^j} \frac{\partial \chi_2}{\partial y} \quad (7)$$

and

$$v = -\frac{1}{x^j} \frac{\partial \chi_2}{\partial x} + \frac{1}{x^j} \int_{y_r}^y x^j S(x, y) dy \quad (8)$$

When this definition is substituted into equation (2), the following equation is obtained for the streamlike function χ_2 :

$$\frac{\partial^2 \chi_2}{\partial x^2} + \frac{\partial^2 \chi_2}{\partial y^2} - \frac{j}{x} \frac{\partial \chi_2}{\partial x} = -\sigma_2(x, y) \quad (9)$$

where

$$\sigma_2(x, y) = x^j \omega(x, y) - x^j \frac{\partial}{\partial x} \int_{y_r}^y S(x, y) dy \quad (10)$$

In this case, the deviation from the traditional stream function appears in equation (8), which defines v , the velocity component in the y direction.

It can be seen from equations (6) and (10) that the nonhomogeneous terms σ_1 and σ_2 in the resulting second order equations are not only dependent on the vorticity ω , but are also dependent on the source term S in the original continuity equation, and on the choice of the reference coordinate x_r or y_r .

The Boundary Conditions

Two types of boundary conditions will be discussed, namely that involving a specified velocity component in the direction perpendicular to a boundary, and the other involving a specified velocity component in the direction tangent to a boundary.

Specified Normal Velocity:

This type of boundary condition is encountered when the volume flux rate is specified over a given portion of the boundary. Zero normal velocity components are usually associated with stationary impermeable solid walls.

i. $u = f(y)$ on $x = \text{constant}$, can be expressed in terms of x_1 as

$$x_1 = \int x^j f(y) dy - \iint_{x_r}^x x^j S(x,y) dx dy + C_1 \quad (11)$$

and can be expressed in terms of x_2 as

$$x_2 = \int x^j f(y) dy + C_2 \quad (12)$$

ii. $v = g(x)$ on $y = \text{constant}$, can be expressed in terms of x_1 as

$$x_1 = \int x^j g(x) dx + C_3 \quad (13)$$

and can be expressed in terms of x_2 as

$$x_2 = - \int x^j g(x) dx + \iint_{y_r}^y x^j S(x,y) dy dx + C_4 \quad (14)$$

iii. More generally, when the normal velocity component V_n is specified over a general irregular boundary ℓ , the boundary conditions, in terms of the streamlike function, x_1 , can be expressed as:

$$x_1 = \int x^j V_n d\ell - \iint_{x_r}^x x^j S(x,y) dx dy + C_5 \quad (15)$$

and in terms of the streamlike function x_2 as

$$x_2 = \int x^j V_n d\ell + \iint_{y_r}^y x^j S(x,y) dy dx + C_6 \quad (16)$$

where C_1 to C_6 in equations (11) to (16) represent arbitrary constants.

It is clear from equations (11) through (16) that, Dirichlet type boundary conditions for the streamlike function, result when the flow velocity normal to the boundary is specified. As expected, the line integral of the volume flux

rates contributes to the variation in the streamlike function over a boundary. In addition, the area integrals of the source terms $S(x,y)$ in equations (11), (14), (15) and (16) can account for streamlike function variation over a boundary, through which there is no flux.

Specified Tangential Velocity:

In this case the boundary conditions are of the Neumann type when expressed in terms of the streamlike function.

i. $v = g(y)$ on $x = \text{constant}$, can be expressed in terms of x_1 as

$$\frac{\partial x_1}{\partial x} = - x^j g(y) \quad (18)$$

and can be expressed in terms of x_2 as

$$\frac{\partial x_2}{\partial x} = - x^j g(y) + \int_{y_r}^y x^j S(x,y) dy \quad (19)$$

ii. $u = f(x)$ on $y = \text{constant}$, can be expressed in terms of x_1 as

$$\frac{\partial x_1}{\partial y} = x^j f(x) - \int_{x_r}^x x^j S(x,y) dx \quad (20)$$

and can be expressed in terms of x_2 as

$$\frac{\partial x_2}{\partial y} = x^j f(x) \quad (21)$$

This type of boundary conditions will not be formulated over a general boundary shape, since it is generally associated with the boundary conditions at infinity. Equations (18) through (21) are sufficient for this purpose.

Numerical Methods

The rest of this paper is mainly intended for showing how the new streamlike function formulation can be used in conjunction with already existing numerical methods for second order equations to obtain solutions to flow problems. A brief review of the available numerical methods for solving nonhomogeneous second order differential equations is appropriate therefore at this point. Before proceeding with this review however, we will describe very briefly the numerical methods which are available for solving the two first order equations directly for the velocity components. These methods are based on writing the finite difference form of these equations at staggered grid points rather than at the same grid point. This idea was first introduced by Gates and Von Rosenberg¹⁰ who used centered difference schemes for expressing the first order derivatives of the flux components in

potential flow. They developed a direct numerical method and various implicit line iterative methods for solving the resulting sets of equations in the flux components. Using different grid sizes, they evaluated the accuracy and the approximate total number of arithmetic operations involved in each of their methods of solution in a given regular domain. Martin and Lomax¹¹ employed the same difference scheme over a staggered grid in their solution of the nonhomogeneous continuity and rotationality equations. In their procedure, they manipulated the resulting finite difference equations, to obtain a set of algebraic equations in only one of the velocity components which they solved using the cyclic reduction method.

Relaxation methods have been used for a long time and are still in great use for the solution of elliptic equations.¹² Recently, several investigators became interested, however, in the development of fast direct methods for solving the centered difference form of Poisson's equation. The first two of these investigations which were reported by Buneman¹³ and Hockney¹⁴ use the cyclic reduction method, and the finite Fourier transform method, respectively, for the numerical solution of Poisson equations with Dirichlet boundary conditions. Several other investigators^{12,13} have contributed since then to reducing the limitations to the application of these two methods. The improvements included generalizing Buneman's cyclic reduction method to Neumann and periodic boundary conditions on regular boundaries and to be used with arbitrary number of grids by Sweet.¹⁵ Hockney's finite Fourier transform method was applied to problems with Dirichlet boundary conditions on irregular boundaries, and with Neumann boundary conditions on regular boundaries by Buzbee et al.¹⁶ In cases involving complicated boundary conditions, the relaxation methods have remained to be the alternative method of solution.^{17,18} Although these methods are generally much simpler to program, it has been found that the boundary conditions have to be carefully handled in the case of nonhomogeneous elliptic equations with Neumann boundary conditions, if a convergent solution is to be obtained.^{2,8,9}

Applications

Two example problems are presented to illustrate the use of the streamlike function in the solution of flow problems in turbomachinery passages. The first set of results simulate the secondary flow in the planes perpendicular to the through flow direction for the nonviscous rotational flow in a curved duct. The numerical procedure for determining this three dimensional flow field was described in reference 1; here we only present the results, using the present formulation for a given vorticity and source distribution. The case under investigation represents a source distribution with linear variation in the

horizontal direction, with a mean value of zero and maximum and minimum values of +1 and -1 at the walls. The vortex distribution changes linearly in the vertical direction with a maximum value of 1.5 at the bottom wall and a minimum value of 0.5 at the upper wall. The results for this case are presented in Figures 1 and 2 for the first and second streamlike function formulations, χ_1 and χ_2 , respectively.

When x_r is taken outside the solution domain, the χ_1 contours are symmetric about the vertical centerline as shown in Fig. 1 and do not change whether x_r is taken to the right or to the left hand side. With $u = 0$ over the boundaries $x = \text{constant}$ and $v = 0$ over the boundaries $y = \text{constant}$, equations (11) and (13) give constant values of χ_1 over all the solid boundaries as shown in Fig. 1. Furthermore, the streamlike lines, $\chi_1 = \text{constant}$, constitute closed contours inside the solution domain in this case. In the second formulation, χ_2 is also constant over the two solid boundaries $x = \text{constant}$ and the χ_2 contours are symmetric about the vertical centerline. The streamlike function χ_2 is not necessarily constant however over the two boundaries, $y = \text{constant}$. When y_r is chosen outside of the solution domain, χ_2 remains constant over the wall facing the reference line, but varies along the opposite wall. This means that the streamlike lines will intersect that boundary opposite to the side where y_r is chosen. This is seen in Fig. 2, in which y_r was chosen on the lower side. Although the streamlike lines are dependent on the formulation and the choice of the reference line, the actual flow velocities are independent of these choices. The arrows in Fig. 3 show the magnitude and direction of the resulting secondary flow. While the previous results were for a domain with regular boundary, the rest of the applications are for a flow problem with an irregular boundary.

The second set of results are presented for the nonviscous incompressible irrotational flow in the cross-sectional planes of a radial inflow turbine scroll. In this case, the vorticity ω in the rotationality equation is equal to zero everywhere in the flow field, and the source term S , in the continuity equation is dependent on the through flow velocity profile variation.² The boundary conditions for this case consist of specified normal velocity component, over all the boundaries of the solution domain shown in Fig. 4. The velocities normal to the solid boundary ABCD and to the axis of symmetry EA are equal to zero, and the velocity normal to the scroll exit DE is uniform and different from zero. In reference 2 this problem was formulated in the traditional way, using the potential function, which resulted in a Poisson equation with Neumann boundary conditions. Because of the irregular boundary shape, the numerical solution was obtained using relaxation methods. A large number of iterations were

required before the solution converged in spite of the special care in handling the boundary conditions. The same problem is reformulated here in terms of the streamlike functions with the results shown in Figs. 5 through 8 for the case with the uniform source distribution.² In all cases, the value of the streamlike function was taken equal to zero at the corner point A. Both Figs. 5 and 6 represent the results for the first streamlike function formulation, χ_1 . The difference between the resulting streamlike lines in the two figures is due to the different choices of the reference x-coordinate, x_r . In the first case, x_r was taken external to the solution domain to the right hand side, and in the second case, x_r was taken external to the solution domain to the left hand side. According to equation (15), when x_r is on the right hand side, the streamlike function χ_1 remains constant over the part BC of the curved boundary as shown in Fig. 5, and when x_r is to the right, χ_1 remains constant over the part AB of the curved boundary as shown in Fig. 6. In both cases, the streamlike function χ_1 remains constant over the straight boundary portions CD and AE according to equation (13) since the flow velocity, v , is equal to zero.

Figures 7 and 8 represent the results obtained using the second streamlike function formulation, χ_2 , with y_r taken above the solution domain in the first case and below the solution domain in the second case. It is interesting to notice that when y_r is taken below AE, the right hand side of equation (10) becomes identically equal to zero. As in the previous formulation, the streamlike function, χ_2 , remains constant over the solid boundaries to the side on which y_r is chosen outside the domain.

Discussion

It has been shown through the results presented that the choice of the type of streamlike function formulation and the reference line alter not only the boundary conditions, but also the nonhomogeneous term in the resulting differential equation. This choice can be used to obtain the boundary condition which is easiest to handle. This can usually be accomplished by placing the reference line on the side of the most irregular solid boundary to have constant value of the streamlike function over it. The numerical solution itself can be sensitive to these choices, as the authors found in the case of the scroll where the flow velocities are very small in the part opposite the exit neck. Taking the reference line to the right, with the χ_1 formulation, was found to result in the closest spaced streamlike lines in this region. In all the results presented here, the reference lines were generally chosen external to the solution domain for the purpose of demonstration. The formulation itself

does not place any restriction, however, on placing the reference line inside the solution domain. The choices for the location of the reference line in any situation are infinite and should be determined by the user depending on the type of his problem.

The other aspect to be discussed is the consequence of using this new formulation on the computer time for the numerical solutions. Unlike the previous formulations, in references 4, 5 and 6, which result in two second order equations, there is only a single second order equation to be solved for the streamlike function. This fact by itself will result in computer time savings of not less than fifty percent, whether relaxation or direct methods are used in solving the resulting finite difference equations. The time savings will be more than that in comparison to the stream function and potential function formulation of reference 4, in which the boundary conditions are of the Neumann type over all the solid boundaries for the potential function. This is true whether relaxation methods or the fast direct methods are used in the numerical solution. The fast direct methods could not be applied up till now to flow problems with irregular solid boundary when any of the approaches in references 4, 5, 6, 11, or 19 are used, since boundary conditions of the Neumann or mixed type are involved. On the other hand the fast direct methods can be applied to the same flow problems with irregular solid boundaries, when they are formulated in terms of the new streamlike function. This is possible since the boundary conditions are of the Dirichlet type over these irregular boundaries in the new formulation. In fact, the second set of results for the turbine scroll problem, illustrates another very important application of the present formulation. The streamlike function can be introduced in the various problems which are traditionally formulated in terms of potential functions. This way the boundary conditions are converted from Neumann type in terms of the potential function to Dirichlet type in terms of the streamlike function. The new formulation presented here, can therefore be used to extend the applicability of the fast direct methods for solving elliptic equations, to a new class of problems, with irregular boundaries.

References

1. Hamed, A. and Abdallah, S., "Three Dimensional Rotational Flow in Highly Curved Ducts Due to Inlet Vorticity," AIAA Paper 78-146, 1978.
2. Hamed, A., Abdallah, S. and Tabakoff, W., "Flow Study in the Cross Sectional Planes of a Turbine Scroll," AIAA Paper 77-714, 1977.
3. Martin, E.D., "A Split-Recoupled-Semi-direct Computational Technique Applied

to Transonic Flow Over Lifting Airfoils," AIAA Paper 78-11, 1978.

4. Rubin, S.G., Khosla, P.K. and Saari, S., "Laminar Flow in Rectangular Channels, Part II: Numerical Solution for a Square Channel," Numerical/Laboratory Computer Methods in Fluid Mechanics, ASME 1976, pp. 42-51.
5. Caretto, L.S., Curr, R.M. and Spalding, D.B., "Two Numerical Methods for Three-Dimensional Boundary Layers," Computer Methods in Applied Mechanics and Engineering, 1972, pp. 39-57.
6. Johnson, G.M. and Wirz, H.J., "The Computation of Nonpotential Transonic Flows by Relaxation Methods," VKI, 1976.
7. Taylor, T.D., "Numerical Methods for Predicting Subsonic, Transonic, and Supersonic Flow," AGARD AG-187, 1974.
8. Briley, W.R., "Numerical Method for Predicting Three-Dimensional Steady Viscous Flow in Ducts," Journal of Computational Physics, Vol. 14, 1974, pp. 1-20.
9. Miyakoda, K., "Contribution to the Numerical Weather Prediction Computation with Finite Difference," Japanese Journal of Geophysics, Vol. 3, 1962, pp. 160-167.
10. Gates, W.J. and Von Rosenberg, D.U., "An Improved Method of Numerical Solution for Flux Components in Potential Flow," Chemical Engineering Science, Vol. 25, 1970, pp. 535-547.
11. Martin, E.D. and Lomax, H., "Variants and Extensions of a Fast Direct Numerical Cauchy-Riemann Solver with Illustrative Applications," NASA TN D-7934, 1977.
12. Lomax, H. and Steger, J.L., "Relaxation Methods in Fluid Mechanics," Annual Review of Fluid Mechanics, Vol. 7, 1975, pp. 63-88.
13. Buneman, O., "A Compact Non-Iterative Poisson Solver," SUIPR Report No. 294, May 1969.
14. Hockney, R.W., "The Potential Calculation and Some Applications," Methods in Computational Physics, Vol. 9, 1970, pp. 135-211.
15. Sweet, R.A., "A Cyclic Reduction Algorithm for Solving Block Tridiagonal Systems of Arbitrary Dimension," SIAM Journal on Numerical Analysis, Vol. 14, No. 4, September 1977, pp. 706-720.
16. Buzbee, B.L., Dorr, F.W., George, J.A. and Golub, G.H., "The Direct Solution of the Discrete Poisson Equation on Irregular Regions," SIAM Journal on Numerical Analysis, Vol. 8, No. 4, December 1971, pp. 722-736.
17. Katsanis, T. and McNally, W., "Revised Fortran Program for Calculating Velocities and Streamlines on the Hub-Shroud Midchannel Stream Surface of an Axial-, Radial-, or Mixed-Flow Turbomachine or Annular Duct," NASA TN D-8430, 1977.
18. Katsanis, T., "Fortran Program for Calculating Velocities on a Blade-to-Blade Stream Surface of a Turbomachine," NASA TN D-5427, 1969.
19. Martin, E.D., "Fast Methods Incorporating Direct Elliptic Solvers for Non-linear Applications in Fluid Dynamics," NASA TM 78426, 1977.

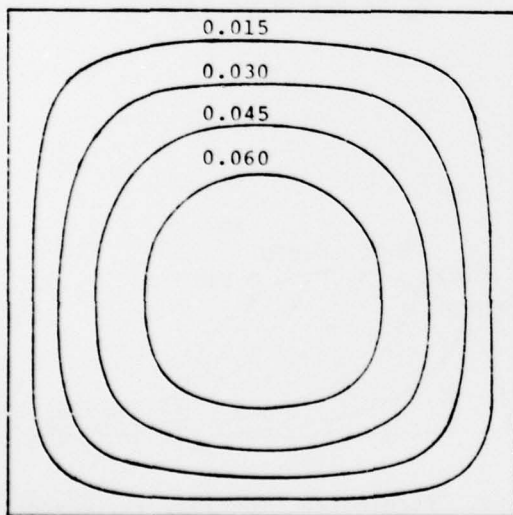


Fig. 1. Streamlike Function, x_1 , Contours.

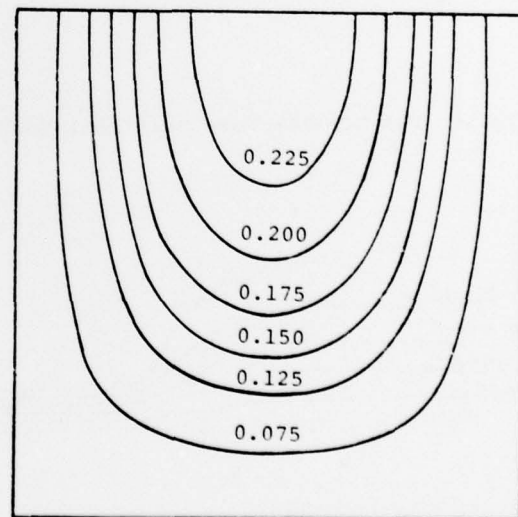


Fig. 2. Streamlike Function, x_2 , Contours.

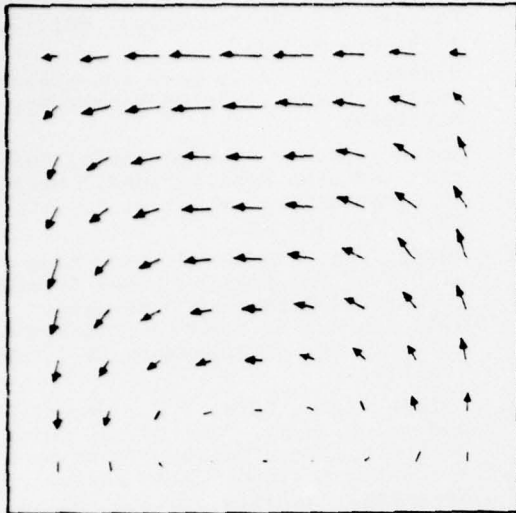


Fig. 3. Secondary Flow Velocity Vectors.

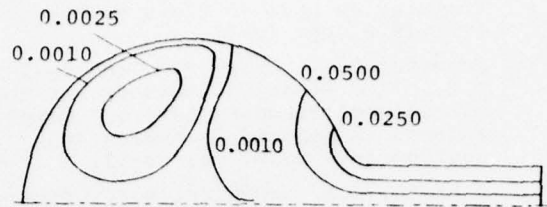


Fig. 6. Streamlike Function, χ_1 , Contours x_r on the Left Hand Side.

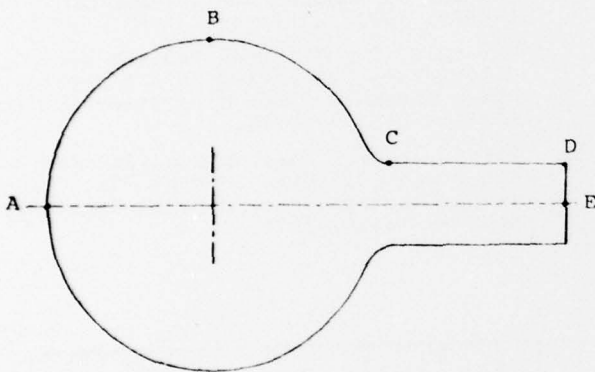


Fig. 4. The Scroll-Cross Sectional Geometry.

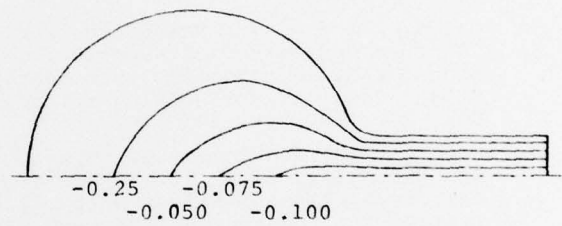


Fig. 7. Streamlike Function, χ_2 , Contours y_r on the Upper Side.

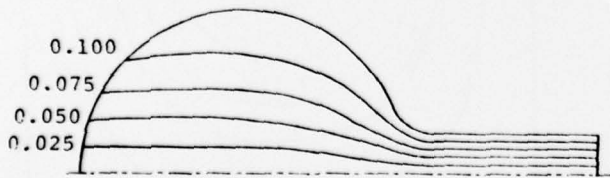


Fig. 5. Streamlike Function, χ_1 , Contours x_r on the Right Hand Side.

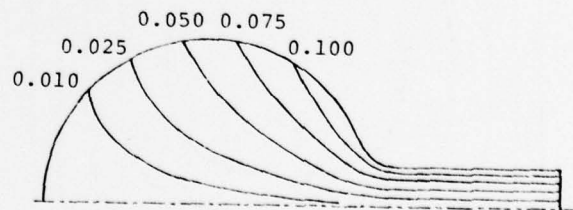


Fig. 8. Streamlike Function, χ_2 , Contours y_r on the Lower Side.