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INVESTIGATION OF MRAS IDENTIFICATION TECHNIQUES FOR THE HUMAN O--ETC(U)
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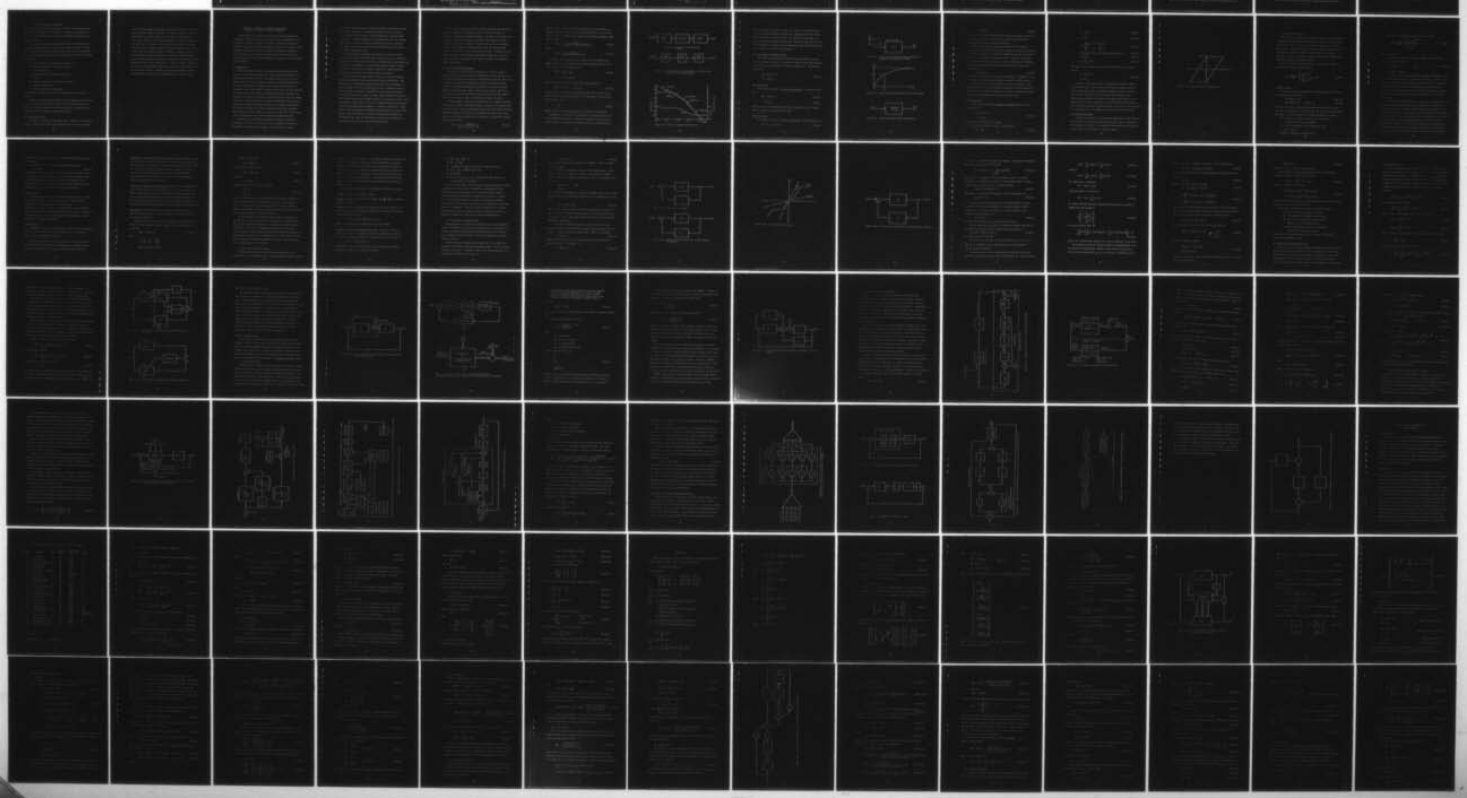

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INVESTIGATION OF MRAS IDENTIFICATION TECHNIQUES FOR THE HUMAN OPERATOR PROBLEM

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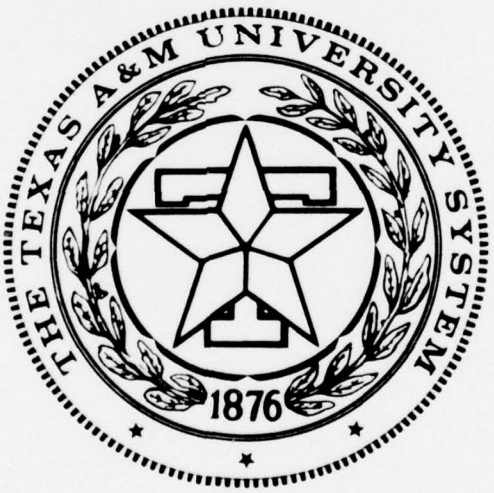

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INVESTIGATION OF MRAS IDENTIFICATION TECHNIQUES

FOR THE HUMAN OPERATOR PROBLEM

Prepared by

Telecommunications and Control Systems Laboratory
Texas A&M University
College Station, Texas 77843
Bruce K. Colburn, Project Leader

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FINAL REPORT

September 15, 1978

Grant No. 77-3295
Air Force Office of Scientific Research
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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: AFOSR-TR-79-0670	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): Investigation of MRAS Identification Techniques For the Human Operator Problem	5. TYPE OF REPORT & PERIOD COVERED: FINAL rept. 1 Mar 1977 - 30 Jun 1978
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Bruce K. Colburn	8. CONTRACT OR GRANT NUMBER(s): AFOSR-77-3295
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Texas A&M Research Foundation, Faculty Exchange Box H, College Station, Texas 77843	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F, 2313D9
11. CONTROLLING OFFICE NAME AND ADDRESS: AFOSR (NL), USAF Bolling AFB, D.C. 20332	12. REPORT DATE: 15 September 1978
	13. NUMBER OF PAGES: 275
14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)* 282 p.	15. SECURITY CLASS. *(of this report)*: Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

MRAS, Identification, Human Operator, Nonlinear Identification, Adaptive Observers

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report summarizes, develops, analyzes, and compares MRAS identifiers for each of the three classes: linear time-invariant, linear time-varying, and nonlinear time-invariant systems. Design guidelines are presented as to practical implementation considerations such as identifier stability, computer computational burden, tracking accuracy and noise effects. Results are provided to demonstrate the good accuracy achievable with each of the three classes of MRAS identifiers.

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ACKNOWLEDGEMENT

The research for this report was performed under Grant No. 77-3295 awarded to the Texas A&M Research Foundation, College Station, Texas by the Air Force Office of Scientific Research. The Project Leader for this work was Assistant Professor Bruce K. Colburn, of the Telecommunication and Control System Laboratory and Department of Electrical Engineering, Texas A&M University. This work was initiated originally by the Biometrics Division, USAFSAM, Brooks AFB, Texas, under the auspices of the 1976 USAF-ASEE Summer Faculty Research Program.

SUMMARY

The use of identification techniques using the model-reference adaptive system (MRAS) approach is investigated. The problems of identifying systems of the type represented by the human operator in a compensatory tracking task are investigated by considering the three cases of linear time-invariant, linear time-varying, and nonlinear time-invariant systems. Identifiers applicable to each of the three cases are summarized as to existing methods, new methods developed, analysis of identifier stability characteristics performed, and design guidelines for implementing them in practice are advanced. Experimental results are provided to indicate the accuracy of the identifiers in the presence of measurement noise for each of the three classes of identifiers investigated. Results indicate the good results possible using some of the MRAS identifiers, especially as regards the time-varying and nonlinear cases.

CHAPTER 1. INTRODUCTION

A. History and Preliminaries

The identification of significant parameters in pilot transfer function models is an area that has been of considerable interest to aerospace researchers since before World War II. The problem is one of cost and safety versus accuracy. Many flight scenarios exist where great danger is involved if a pilot were to be present, yet the pilot's dynamic response characteristics during such stress manuevers are precisely what are required to determine a flying vehicle's ability to function properly under adverse conditions. To this end, pilot dynamic models are sought which can be substituted for an actual pilot in a dangerous flight simulation.

The field of identification is long and diverse, encompassing both statistics and engineering. A large variety of off line techniques have been developed over the year, including least squares [1], weighted least squares [2], generalized least squares [3,4], instrument variable [5,6], frequency domain [7], cross correlation [8,9], stochastic approximation [10-12], Bayesian [13], Kalman Filtering [14-16], and the Kleinman optimal control model [17-19]. General surveys covering this field include [20-30].

To obtain pilot models, tracking tasks are utilized to place subjects in comparable flight scenarios. Both humans and monkeys are utilized in simple tasks, as it has been shown that highly trained monkeys exhibit dynamics similar to that of pilots. The usual scenario is that of the single axis compensator tracking problem, shown in Figure I-1 [31]. Here a subject is to visualize an error e and keep it "small" by adjusting a "joy stick" output y.

Special problems which arise demanding modeling of pilot response in place of pilots are stress conditions such as 1) nuclear irradiation

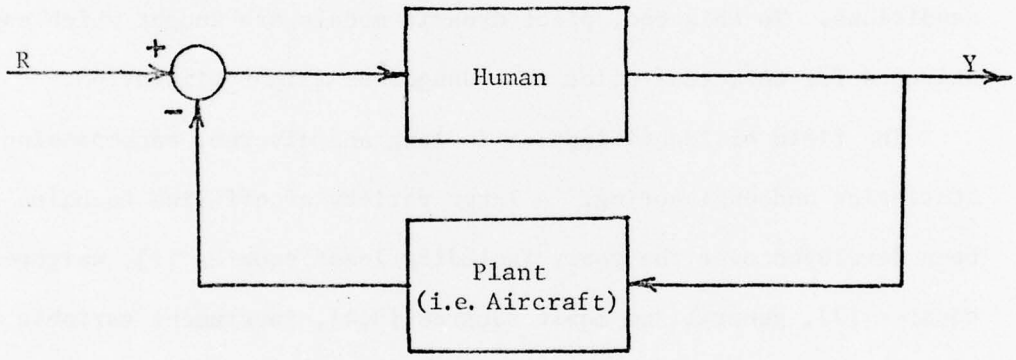


Figure I-1(a). Single Axis Compensatory Tracking System.

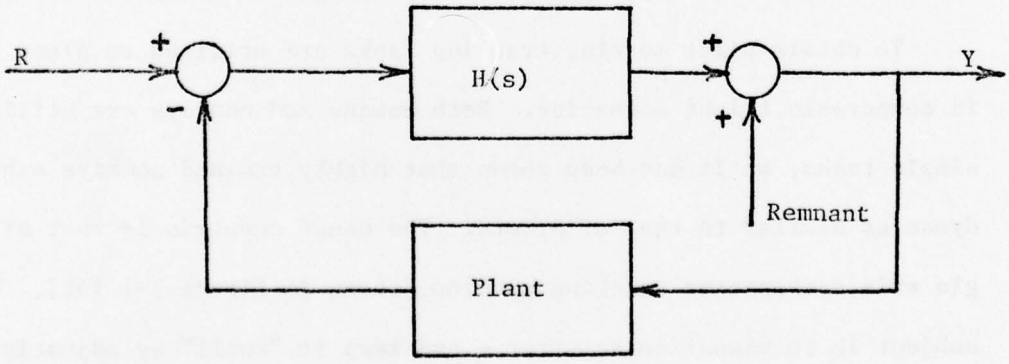


Figure I-1(b). Standard Linear Model for Figure II-1(a).

representing a nuclear blast, 2) high-g cornering manuevers, 3) hypoxia, 4) effects of increased task rate, 5) response due to environmental inputs such as temperature and vibration, and 6) effects of psychological variables such as training, motivation, and task difficulty. The resulting information derived from such studies is useful for design of new high performance aircraft and their control systems (CCV and DFBW systems).

Previous work in this area has principally relied on either frequency response transfer function models [32-34] or optimal control models [17, 18,35]. The former relies on a describing function sine-wave driven approach [36-39], and the latter on the assumption that a trained pilot operates to minimize tracking errors in a least-squares sense. The latter work relies on Kalman filtering [40], off-line maximum likelihood [41], and least-squares identification approaches. The work reported herein is predicated upon a different concept, the model reference adaptive system (MRAS) approach. This method, not previously applied to human operator work, is conceptually suited to this goal because the human is inherently adaptive [42-46] and the MRAS concept has built into its' formulation the ability to adaptively track time-varying human performance parameters [47-49]. This adaptation can manifest itself in the form of task allocation [50], changing ability to process sensory inputs [51], and altered reaction time and response [19]. Therefore, the MRAS approach offers a natural mathematical vehicle for formulating the human modeling problem.

Such an approach would complement the Optimal Control Model work on model development and least squares cost functional formulation. The MRAS approach, however, offers a more natural setting for the problem (i.e. human operator characteristics are identified as they occur without need for previous restrictions as to time, invariance and optimization form such as

least-squares, etc.), and also allows for adaptation, nonstructured cost functional optimization (e.g. other than least squares tracking error), and straightforward inclusion of nonlinear model phenomenon. The ultimate goal of this work is to use MRAS methods to identify general parameterized human operator models, from pilot-generated operating records, that are markedly superior to least squares and Optimal Control Model approaches.

Results of such analyses are useful in providing quantitative and qualitative design information on flying qualities, pilot/vehicle integration, cockpit information display, and stability and control characteristics. Many useful results have been made available from the quasi-linear modeling approach of the past [33], but it is felt that in developing new model concepts, including nonlinear, time-varying, multi-loop, and adaptive, then new analysis methodologies are warranted for investigative purposes.

Because of the complex nature of the human operator identification problem and the embryonic stage of MRAS identifier development, it was deemed to be a multiple stage task to the practical implementation of MRAS identifiers to the human operator problem, namely 1) develop tuning and optimization procedures for MRAS, 2) determine model structures suited to MRAS, and 3) implement the identifiers on operator task data. The philosophy behind this is that other identification techniques have been improved and optimized over many years of use and are about as good as they will ever be, while MRAS is relatively new and offers features beyond other methods, such as 1) the tracking error e can be guaranteed to match $\lim_{t\to\infty} e(t) = 0$, even in the presence of noise (E{e} then), 2) the problem formulation lends itself to meaningful parameter identification, 3) nonlinearities can be simply incorporated. For the preceding reasons, it was deemed necessary to concentrate the bulk of the present effort on MRAS identifier design and

analysis for general use, with particular emphasis on the solution of practical problems which relate to the human operator problem.

Although the main thrust of this work was motivated by the pilot model problem, there are a considerable number of other Air Force areas where such work is of direct benefit. These include a) determining the parameterization of wing rock due to high angle of attack, b) on-line response of new AF high performance control configured vehicles for finer weapons aiming, c) model determination for closed-loop flying qualities calculation, d) foreign technology weapons system assessment, e) anti missile, cruise missels, and MIRV pursuit-evasion problem, f) trajectory control for the new class of missiles (Missile X), and g) human capability in television guided bomb systems (Maverick) and other man-in-a-loop-inside-another-loop.

The practical implementation problems investigated include

(a) parameter bias in the presence of noise

(b) tuning methods for selecting designer controlled identification parameters

(c) convergence rate prediction and control, and the concomitant effect on efficiency

(d) determination of stability boundaries

(e) robustness of identifiers in handling time-varying models

(f) algorithm modification and tuning for identification of both *memory* and *single-valued* functions

(g) frequency richness of the input and its

effect on model identifiability

These topics have only been addressed in a cursory and superficial manner in the past, resulting in *ad-hoc* methods. The present work provides an analytic framework, the end result of which is maximum user flexibility.

B. Thrust of Work

Because the use of MRAS in human operator work is new, preliminary general identification work needed to be done first to be able to insure that validated MRAS identifiers could function in realistic environments. To this end, new work was undertaken to provide answers to the engineering problems of

1) noise effects on accuracy and convergence rate
2) effects of frequency richness on parameter tracking accuracy
3) identification algorithm development and analysis
4) computer computational requirements (storage, computation time, etc.)
5) on-line model order determination.

To show the work-to-date and expected direction, these topics will be covered.

Therefore, identified human operator models is NOT the goal of the work reported herein. Hence the thrust of the work will deal with technical aspects of stability, modeling, model structure, etc. of identification algorithms, and not on human results using I/O data.

C. Report Organization

The report is divided into four main areas. In Chapter 2 the general identification problem is developed mathematically, the linear, nonlinear

and time-varying human operator models are reviewed, and the various MRAS and related identifiers are defined. The difference between identifiers, which fall into the two classes of series-parallel and parallel, is quantified. Chapter 3 covers the MRAS synthesis and analysis linear time-invariant systems. Chapter 4 deals with nonlinear identifiers, model structure, and the critical subject of parameter identifiability. Chapter 5 deals with time-varying parameter identifiers. Chapter 6 covers implementation problems from an empirical and approximate analysis approach, so as to address practical situations in which the theory fails to provide substantive answers about identification. Chapter 7 provides simulation results and design guidelines based on the analytical development analysis of the previous chapters. Chapter 8 summarizes the findings, provides conclusion from the results, and offers suggestions for future work.

CHAPTER 2. MATHEMATICAL TECHNIQUES USED IN THE SYNTHESIS AND ANALYSIS OF MRAS IDENTIFIERS

In this chapter the background material and relevant mathematical preliminaries necessary for an understanding of the algorithms which follow will be developed. Concepts which will be employed include general nonlinear system stability, hyperstability, positive real functions, parallel and series-parallel model configurations, and frequency richness. It is necessary to present these concepts because of the need for functional analysis to understand the mathematical subtleties of the identifiers to be presented.

A. Preliminary

The work performed in system identification covered three distinct phases: linear time-invariant systems, linear time-varying systems, and nonlinear time-invariant systems. The question may have arisen to the reader, "Why would anyone continue to perform a functional investigation of systems identification techniques since a high level of effort has occurred for over thirty years?" The answer lies in the fact that although much identification work has been done by statisticians, mathematicians, engineers, and scientists, the bulk of such work has been under the assumption that the system consists of a linear, time-invariant plant structure. Although the majority of physical processes can be roughly characterized by a linearized model, there comes a time when higher-order, secondary model effects are sought, and it is to this issue that part of the present study is addressed, the rest dealing with general MRAS identifiers.

Many different techniques for parameter identification have been developed over the years, each based on a set of mathematical modeling assumptions concerning noise, biasedness, plant linearity, *a priori*

knowledge of noise statistics, correlated residuals, identifying an open-loop plant operating in a closed-loop (the human operator problem is such a case), and many others. From such information, identification procedures such as least squares, maximum likelihood, etc. have evolved. With respect to linear time-invariant identification, most techniques have failure modes, and for time-varying and nonlinear systems results can become progressively worse.

One relatively new technique (developed in the last five years) offers the possibility of much improved accuracy and greater flexibility in practical implementation (i.e. noise, nonlinearities, time-varying terms, stability). This technique is called model-reference adaptive systems (MRAS) and is predicted on the simple concept that a model observer is adjusted until the plant output matches that of the model.

Most of the past identification techniques have involved off-line methods which identify constant coefficient linear system models. The MRAS concept is a significant departure from traditional approaches in a number of ways, (1) a scalar metric $\lim_{t\to\infty} E\{e\} = 0$ as opposed to scalar or vector summation or integral criterion, (2) using Lyapunov and related stability theories it is possible to insure global stability of the parameter identification, (3) the MRAS technique is inherently an on-line sequential approach, allowing for its use in the identification of nonstationary systems, (4) unknown system parameters are treated as "gains" of a model, with the end result being the matching of model parameters to those of the system. Stability capability, coupled with significant previous work with MRAS control, makes the MRAS concept attractive to investigate as regards its utility for human operator identification.

This report, therefore, is limited to MRAS identification techniques, their similarities and differences, and the further development of analytical implementation design procedures. To effect this, linear time-invariant, linear time-varying, and nonlinear MRAS concepts are first surveyed and reviewed, followed by new algorithms and design techniques. In this way, an extension of MRAS methods towards the human operator problem can be rationally and objectively determined. It should be noted that it is because of the newness of the MRAS approach that it has not been employed for the present problem, not necessarily its inability to handle the problem.

B. System Identification Concept

To begin the results of the work performed, it will be useful to review briefly the concept of dynamic system identification and why it is a non-trivial problem. Two classes of identifiers are *off-line* and *on-line*. A technique processing input-output data during normal operation and provides a continuously updated real-time model is an on-line approach. Off-line methods generally employ stored operating records and special test inputs to aid in system identification; computational time requirements are relaxed here. This is illustrated in Figure II-1 [52].

The most common method of system identification is employed regularly by electronics people, namely Bode plotting. Using the very special input of a sinusoid to a linear, time-invariant plant, the input-output magnitude and phase response of a system may be obtained, from which system poles and zeros (and hence parameters) may be extracted. To show this, consider the plant [53] given by

$$H(s) = \frac{10(3s+1)}{10s^4 + 111s^3 + 111s^2 + 10s} \qquad \text{(II-1.B)}$$

By subjecting the plant to sinusoidal signals of varying frequency, a magnitude-phase plot of H vs. ω can be obtained, where $s = j\omega$, as shown in Figure II-2. The plots can be analyzed from frequency response knowledge to obtain

$$H(j\omega) \simeq \frac{(3j\omega+1)}{(j\omega)(10j\omega+1)(j\omega+1)(.1j\omega+1)} \qquad \text{(II-2.B)}$$

and hence

$$H(s) = \frac{(3s+1)}{(s)(10s+1)(s+1)(.1s+1)} \qquad \text{(II-3.B)}$$

However, not all systems can be subjected to a sinusoidal input. Suppose a step input is applied to an unknown plant H(s) known to be of first order, as shown in Figure II-3:

$$\frac{y}{u}(s) = H(s) = \frac{a}{s+a} \qquad \text{(II-4.B)}$$

$$u = 1/s \quad \text{(step input)}$$

Then it is known that the resulting output time-response is

$$y(t) = 1 - e^{-at} \quad ; \quad y(o) = 0 \qquad \text{(II-5.B)}$$

If the system response y(t) is recorded for $u(s) = 1/s$, then "a" can be determined by calculating the time $t = t_a$ at which $y(t_a) = .632$, as shown in Figure II-4. Then

$$a = \frac{1}{t_a} \qquad \text{(II-6.B)}$$

Similar results hold for higher-order systems with step inputs [54].

Unfortunately, if the order of the system is unknown, the input can not be preselected (or is not a simple test signal), or the system has multiple inputs and outputs (multivariable structure), then determining

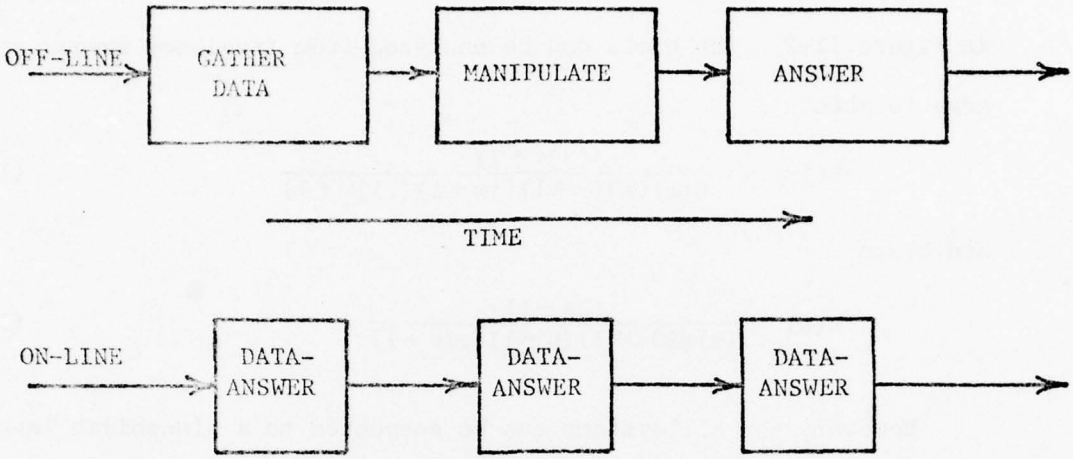


Figure II-1. Illustration of the principle of off-line versus on-line system identification.

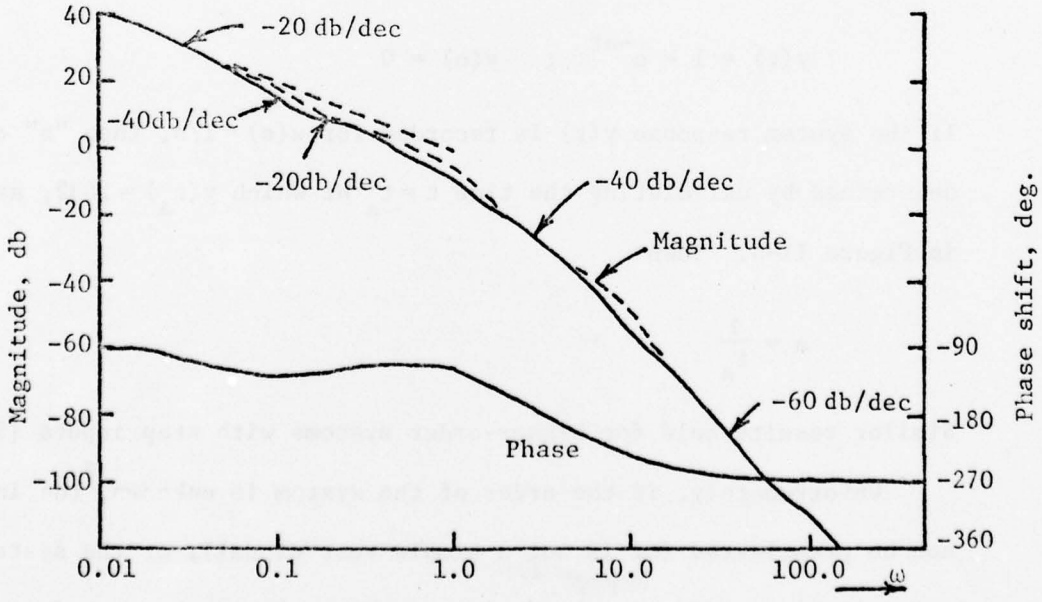


Figure II-2. Frequency Response Identification

the system parameters is not so easy. For such cases it is more appropriate to develop numerical techniques for calculating system parameters from input-output measurements only. The main work of this report is concerned with developing, analyzing, and calculating identification rules and associated tools (model order determination, input nature requirements, etc.) needed for practical implementation.

C. Three Classes of Plant Structure

To clarify the key differences between the three different classes of plant structures alluded to previously, consider an nth order dynamical system with input u and output y as given by Figure II-5. The system is described by

$$\dot{\underline{x}}_p = \underline{f}(\underline{x}_p, u, t)$$
$$y = g(\underline{x}_p, t) \qquad \text{(II-1.C)}$$

Definition II-1.

If the system (II-1.C) is linear, time-invariant, a model description is

$$\dot{\underline{x}}_p = A\,\underline{x}_p + B\,u \qquad \text{(II-2.C)}$$

$$y = C^T\,\underline{x}_p \qquad \text{(II-3.C)}$$

where A is an n x n constant matrix, B and C constant n-vectors. Superposition and homogeneity hold in this case.

Definition II-2.

If the system (II-1.C) is linear, time-varying, a model description is

$$\dot{\underline{x}}_p = A(t)\,\underline{x}_p + B(t)\,u \qquad \text{(II-4.C)}$$

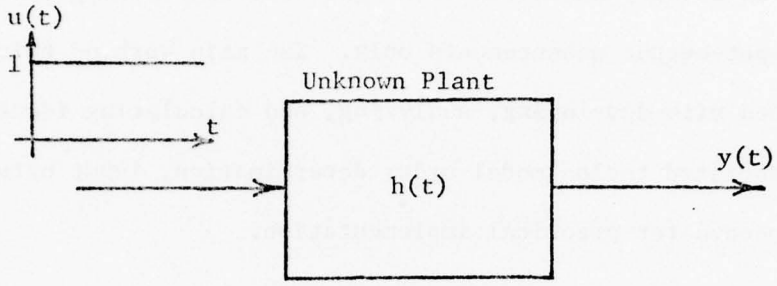


Figure II-3. General Impulse Response Representation of a Single-Input Single-Output System.

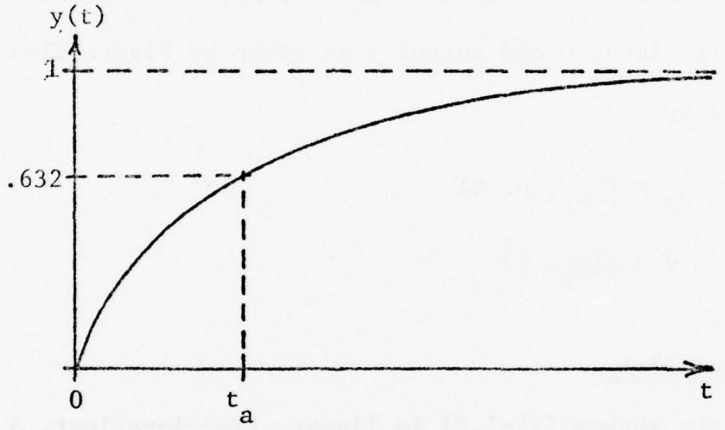


Figure II-4. First-Order System Response Due to a Step Input.

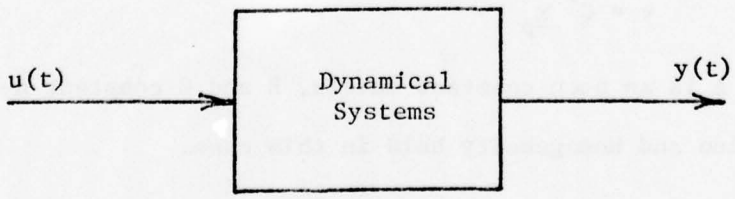


Figure II-5. General Dynamical System Configuration.

$$y = C^T(t)\, \underline{x}_p \qquad \text{(II-5.C)}$$

where A is an n x n matrix with possibly each element being a function of time (i.e. $a_{12} = t + 2$), and B(t) and C(t) time-variable n-vectors. Superposition and homogeneity still hold here.

It should be noted that the time-invariant case is actually a special case of a time-varying system, but because time-invariant systems are used so extensively in science and engineering the two are treated in practice as two distinct modeling approaches. Unique to time-varying systems is the *rate-of-change* of A, B, and C. Questions of determining stability of systems of the form (II-4.C), (II-5.C) arise since the simple idea of

$$|\lambda I - A(t)| = 0 \qquad \text{(II-6.C)}$$

has no meaning in the strict sense [55,56], although for $\frac{1}{n}||\dot{A}(t)|| < \dot{\eta}$, η a design constant, meaningful approximate solutions may be obtained.

The third class of systems addressed is that of nonlinear, time-invariant plants. Here, certain plant terms involve squares, magnitudes, saturation terms, hysterisis effects, etc. in order to define the plant. Superposition and homogeneity do not hold in such cases.

Definition II-3.

If the system (II-1.C) is nonlinear time-invariant, then a model description is

$$\begin{aligned} \dot{\underline{x}}_p &= \underline{f}(\underline{x}_p, u) \\ y &= g(\underline{x}_p) \end{aligned} \qquad \text{(II-7.C)}$$

where $\underline{f}$ is an n-vector and g a scalar.

Some typical examples of $\underline{f}$ and g functions are

$$f_i = |x_p| \qquad \text{(II-8.C)}$$

$$f_i = x_p^2 + u \tag{II-9.C}$$

$$f_i = x_p^2 \tag{II-10.C}$$

$$f_i = \begin{cases} K_1 x_p & \text{if} \quad |x_p| \leq C \\ K_2 & \text{if} \quad |x_p| > C \end{cases} \tag{II-11.C}$$

f_i shown in Figure II-6 (hysterisis), as well as (II-12.C)

$$g = K' x_p^2 \tag{II-13.C}$$

$$g = \sqrt{x_p} \quad , \text{ etc.} \tag{II-14.C}$$

The linear, time invariant case is then a special case of this class, wherein

$$\begin{aligned} \underline{f} &= A\, \underline{x}_p + B\, u \\ g &= C\, \underline{x}_p \end{aligned} \tag{II-15.C}$$

In general, the human operator is a multimodal, adaptive, learning control system [44]. The learning occurs because the human changes its control performance due to sensing the environment in which it operates and altering its characteristics to improve the performance. The adaptation is manifested in predictive and correlating capabilities, in adjusting sensory input information by ignoring immediately unneccessary quantities, and altering the control response to meet a new "cost criterion" as a test progresses. An example of this adaptation is shown in [43].

D. Stability Techniques

In this section some of the mathematical tools used for MRAS identifier determination are presented, along with appropriate analysis definitions. This material is included so as to provide a background to the theoretical results which will be developed in ensuing chapters.

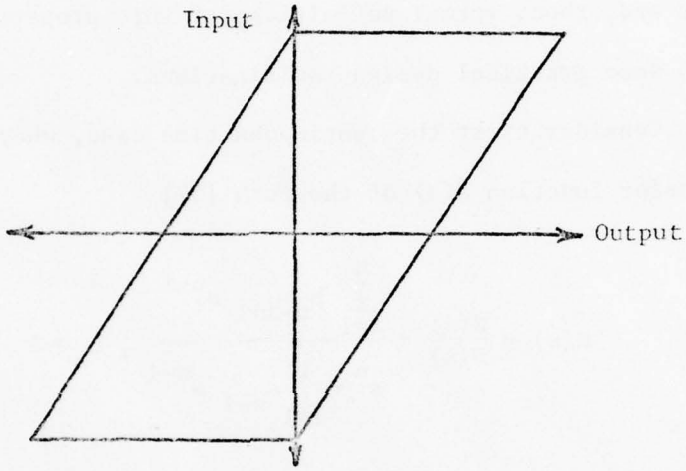


Figure II-6. Typical Hysterisis Nonlinearity.

1. Positive Real Functions

The concept of a positive real function is central to the development and analysis of asymptotically hyperstable identifiers, both continuous and discrete. This is because, as in circuit theory where it evolved from, stability is based on whether a system absorbs or generates power, representing a positive or negative real "impedance" requirement. To this end, then, formal definitions of this property will be stated, along with some practical design ramifications.

Consider first the continuous time case, wherein one is given a transfer function H(s) of the form [57]

$$H(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=1}^{n} b_{n-i+1} s^{n-i}}{s^n + \sum_{i=1}^{n} a_{n-i} s^{n-i}} \quad , \; b_n = 1 \qquad \text{(II-1.D)}$$

Definition II-4.

A continuous-time function H(s) of the complex variable $s = \sigma + j\omega$ is strictly positive real (SPR) if

(1) $H(\sigma)$ is real (II-2.D)

(2) $\mathrm{Re}\{H(s)\} \geq 0 \quad ; \; \mathrm{Re}\{s\} > 0$ (II-3.D)

The term real refers to the realness of H(s) for real values of s and positive to the positivity of the real part for values of s with positive real parts. From the definition, the following guidelines result:

(1) If H(s) is rational, no poles or zeros may lie in the open right half plane (ORHP).

(2) If the coefficients of N(s) and D(s) are real, $H(\sigma)$ is real.

(3) $\left[\text{Order } \{N(s)\} - \text{order } \{D(s)\}\right] \in [-1,1]$.

For discrete time systems, one must refer to a G(z),

$$G(z) = \frac{N(z)}{D(z)} = \frac{\sum_{i=1}^{m} b_{n-i+1} z^{-(n-i)}}{1 + \sum_{i=1}^{n} a_i z^{-i}} \qquad \text{(II-4.D)}$$

Definition II-5.

A discrete-time function G(z) of the complex variable $z = \gamma + j$ is strictly positive real if

$$\mathrm{Re}\{G(z)\} > 0 \qquad |z| = 1 \qquad \text{(II-5.D)}$$

2. Lyapunov Stability

There are two key analysis techniques developed by Lyapunov. These are the First Method and the Second (or Direct) Method. The First approach deals with determining the local stability nature of a nonlinear dynamic system and the Second with sufficient conditions to insure stability without actually solving in closed-form the dynamic equations. It is the Second Method which has been demonstrated to be of value in MRAS work.

The Second Method of Lyapunov, also referred to as Lyapunov's Direct Method, is a mathematical technique for determining the stability of ordinary differential equations without requiring the solution of the differential equation. It is based on the dissertation of A. M. Lyapunov written in 1892. It has its greatest value in application to non-linear and/or time-varying differential equations where classical control results for stability are generally not applicable. It is based on the energy concept of n-dimensional systems and requires the determination of a scalar function satisfying certain rules. The basic definitions

and theorems applicable to the model-reference adaptive problem will now be introduced.

For the general nth order non-linear automonmous system

$$\dot{\underline{x}} = \underline{f}(\underline{x}) \qquad \text{(II-6.D)}$$

with equilibrium state $\underline{x}_e$, there are different kinds of stability which apply to $\underline{x}_e$. The basic stability concept formulated by Lyapunov relates to the bounding of a system trajectory $\underline{x}(t)$. The Euclidian norm, denoted by $||\ \ ||$, is the basic rule used to define "closeness" to the equilibrium state. The definition of Lyapunov stability is:

Definition II-6.

An equilibrium state, $\underline{x}_e$, of an autonomous dynamic system is stable in the sense of Lyapunov, if for $\varepsilon > 0$ there is a $\delta > 0$ such that if $||\underline{x}_o - \underline{x}_e|| < \delta$, then $||\underline{x}(t) - \underline{x}_e|| < \varepsilon$ for all $t > t_o$.
In the above $\underline{x}_o$ refers to $\underline{x}(t_o)$. This implies that a system trajectory initially restricted to some region will never leave another well-defined region. However, of far greater importance in model-reference systems is the case of asypmtotic stability:

Definition II-7.

An equilibrium state $\underline{x}_e$ is asymptotically stable if (1) the equilibrium state is Lyapunov stable, and (2) there exists a $\delta_p > 0$ such that if $||\underline{x}_o - \underline{x}_e|| < \delta_p$ then $\lim_{t\to\infty} ||\underline{x}(t) - \underline{x}_e|| = 0$.

These stability concepts are local in nature in that they are only good for "small" regions about an equilibrium state. Lyapunov theorems will now be stated which can be used to determine global stability.

Theorem II-1 - Lyapunov's First Stability Theorem. An equilibrium state, $\underline{x}_e$, is stable in the sense of Lyapunov if (1) there exists a scalar function $V(\underline{x})$ which is continuous and has continuous first partial derivatives in some region R about $\underline{x}_e$, (2) $V(\underline{x})$ is positive definite (p.d.) in the region R, and (3) the time derivative of the scalar function $\dot{V}(\underline{x})$ evaluated along the trajectories of the system under investigation, is negative semi-definite in the region R.

Theorem II-2 - Lyapunov's Second Stability Theorem. An equilibrium state $\underline{x}_e$ is asymptotically stable if (1) there exists a scalar function $V(\underline{x})$ which is continuous and has continuous first partial derivatives in some region R about the equilibrium state, (2) $V(\underline{x})$ is positive definite in the region R, and (3) the time derivative of the scalar function $\dot{V}(\underline{x})$ evaluated along the system trajectory is negative definite in the region R.

Theorem II-3. An equilibrium state $\underline{x}_e$ is asymptotically stable if (1) the equilibrium state is Lyapunov stable, and (2) the curve $\dot{V} = 0$ is not a system trajectory.

Theorems II-2 and II-3 are the two most important theorems which are employed in model-reference adaptive design. The time derivative of V is found from

$$\dot{V}(\underline{x}) = \nabla V(\underline{x}) \cdot \underline{f}(\underline{x}) \qquad \text{(II-7.D)}$$

where

$$\nabla = \left[\frac{\partial}{\partial x_1} \quad \frac{\partial}{\partial x_2} \cdots \frac{\partial}{\partial x_n}\right]$$

$\underline{f}(\underline{x})$ defined by (II-6.D).

Given an unknown plant

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u} \tag{II-8.D}$$

it is desired to form a model

$$\dot{\underline{x}}_m = A_m \underline{x}_m + B_m \underline{u} \tag{II-9.D}$$

such that

$$\underline{e} = \underline{x}_m - \underline{x}_p \tag{II-10.D}$$

approaches zero as $t \to \infty$ in such a way that

$$\begin{aligned} A_m &\to A_p \\ B_m &\to B_p \end{aligned} \tag{II-11.D}$$

This is called the observer problem and in general depends upon the output structure and canonical realizations of A_p, B_p. The equilibrium state is then $\underline{e} = \underline{0}$, $\Psi = A_m - A_p = 0$, $\underline{\phi} = B_m - B_p = 0$. The scalar function $V(\underline{x})$ is almost always selected as a quadratic function so as to facilitate ease of the determination of sign definiteness.

The shortcomings of the Lyapunov approach applied to MRAS system are two-fold: (1) the selection of the V function and identifier gains a_{ij}^m, b_{ij}^m are arbitrary and depend upon a combination of skill, patience, and luck on the part of the designer so as to insure $\dot{V}$ is n.d., and (2) the Lyapunov stability theorems provide only sufficient conditions for determining the stability of an equilibrium state. This means if a V cannot be found which satisfies the stability theorems, it does not follow that the equilibrium state is unstable.

3. M-K-Y Lemma and Popov's Method

The Meyer-Kalman-Yakubovich (MKY) Lemma is an outgrowth of the studies into the use of Lyapunov theory in determining sufficient conditions

for stability of control systems. The approach originated with Yakubovich [58], was extended to the problem of Lurĕ by Kalman [59], and further extended by Meyer [60]. It provides the link between the existence of the solution to a set of vector-matrix relations and the stability conditions in the frequency domain (such as the Popov Criterion) through a special form of a Lyapunov function. As with the Popov Criterion and Lyapunov theory, the MKY Lemma provides only sufficient conditions for stability [61].

Before stating the MKY Lemma, the following definitions are given.

Definition II-8. E^n is Euclidian n-space.

Definition II-9. If A is a real n x n matrix, $A(s) = [sI - A]$ is the characteristic matrix of A.

Definition 10. The subspaces of E^n generated by vectors b, Ab, $\cdots$ are denoted by [A, b]. The orthogonal complement of [A, b] in E^n is denoted by $[A, b]^o$, and, in general

$$[A, b]^o = \{x \in E^n: \quad x^T A^k b = 0, 1, 2, \cdots\}.$$

Using these definitions, the MKY Lemma is as follows [60]:

Theorem II-4 - Meyer-Kalman-Yakubovich Lemma. Let A_m be a real n x n matrix all of whose characteristic roots have negative real parts; let τ be a real nonnegative number and let $\underline{d}$, $\underline{k}$ be two real n-vectors. If

$$T(s) = \tau + 2\underline{k}^T A_m(s)^{-1} \underline{d}$$

is a positive real function then there exist two n x n real symmetric matrices Q, C, and a real n-vector $\underline{q}$ such that

1) $A_m^T Q + Q A_m = -\underline{q}\underline{q}^T - C.$

2) $Q\underline{d} - \underline{k} = \sqrt{\tau}\underline{q}$.

3) C is positive semi-definite and Q is positive definite.

4) $\{x \in E^n: \ \underline{x}^T C \underline{x} = 0\} \bigcap [A^T, q]^o = \{0\}.$

5) $\underline{q} \notin [A, d]^o$

6) If $j\omega$, ω real, is a zero of $-\underline{q}^T A_m(s)^{-1}\underline{d} + \sqrt{\tau}$, then it is a zero of $\underline{d}^T A(-s)^{-1} C\ A(s)^{-1}\underline{d}$.

The main contribution of the MKY Lemma in MRAS identification problems is two-fold: (1) it provides a mechanism for determining the sign-definiteness of a Lyapunov-like scalar V function, without which it would be difficult or impossible to ascertain stability for a class of V functions, and (2) it makes it possible to eliminate the need for measurement of all the plant states and instead requires only the system output to be available. This second point is important as it is often the case that not all of the plant states are available and noise considerations exclude the possibility of repeated differentiation.

4. Popov's Hyperstability Approach

Hyperstability is a condition of stability for a class of dynamical systems. Developed by V.M. Popov [62], the approach is a generalization of some well-known nonlinear stability phenomena for the case referred to as "absolute stability" [63,64]. All are an outgrowth of the linear systems condition.

Consider the block diagram given in Figure II-7. For simplicity the system is autonomous (unforced and time-invariant). G(s) is a Laplace transfer function and F a constant. Using linear systems concepts, the system $\frac{y}{r}(s)$ is asymptotically stable if

$$1 + F\,G(s) = 0 \qquad \text{(II-12.D)}$$

has all roots $R_e\{s_i\} < 0$. In general, F can take on a range of values, $F_{min} \leq F \leq F_{max}$.

This idea can be further refined to the generalized Luré problem [65,66], as shown in Figure II-8. Here a nonlinearity F(y) appears (actually the theory holds for F(y,t) also). If F is a continuous function, and

$$F(y)y > 0 \qquad y \neq 0$$

$$F(o) = 0$$

then Popov's Method [67,68] can be used to determine ranges for the sector nonlinearity F(y) for the system (Fig. II-8) to be asymptotically stable, i.e.

$$\lambda_1 y^2 < F(y)y < \lambda_2 y^2 \qquad \text{(II-13.D)}$$

This concept is shown in Figure II-9, which demonstrates the fact that the graph of F is entirely contained in the sector bounded by the lines $\sigma = \lambda_1 y$ and $\sigma = \lambda_2 y$. This method allows for static F but does not admit dynamical relations in F.

A more general problem than above is that of allowing F to be dynamical in nature and G(s) becomes a general nth order system of state equations. This is depicted in Figure II-10. Block G is described

$$\dot{\underline{x}} = A\underline{x} + B\,u \qquad \text{(II-14.D)}$$

where $\underline{x}$ is an n-vector, u is a scalar, and A and B are n x n and n x 1 constant matrices. The output y is

$$y = C^T\underline{x} + D\,u \qquad \text{(II-15.D)}$$

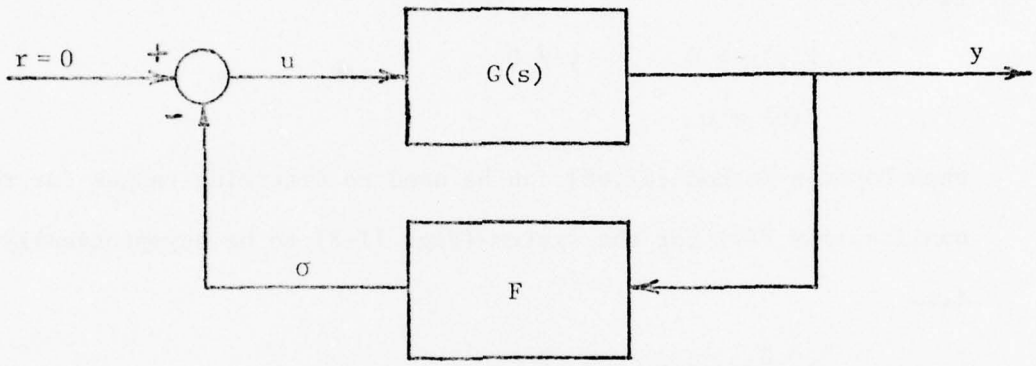


Figure II-7. Basic Case, F = constant.

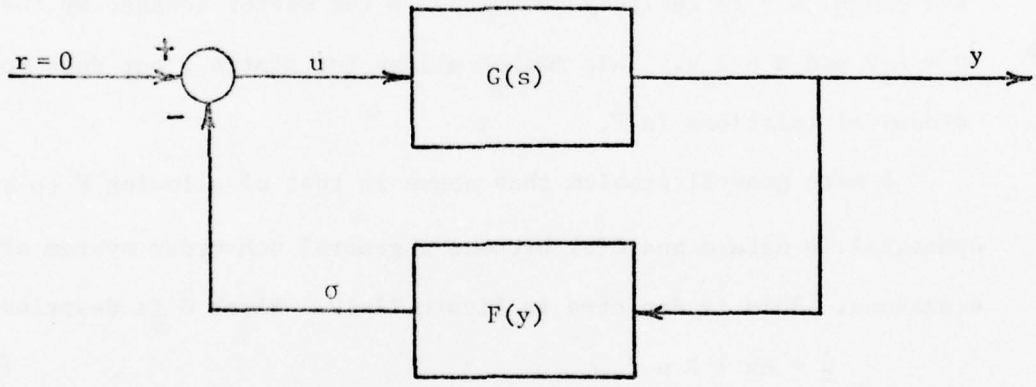


Figure II-8. Generalized Luré Problem, F(y) A Single-Valued Nonlinearity.

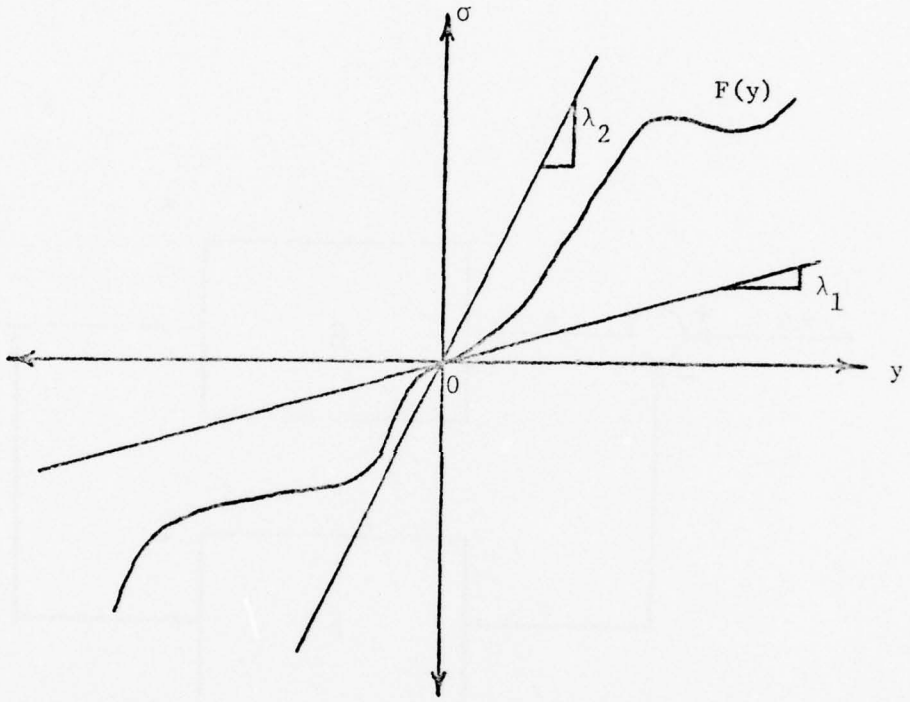


Figure II-9. Sector Nonlinearity Bounds.

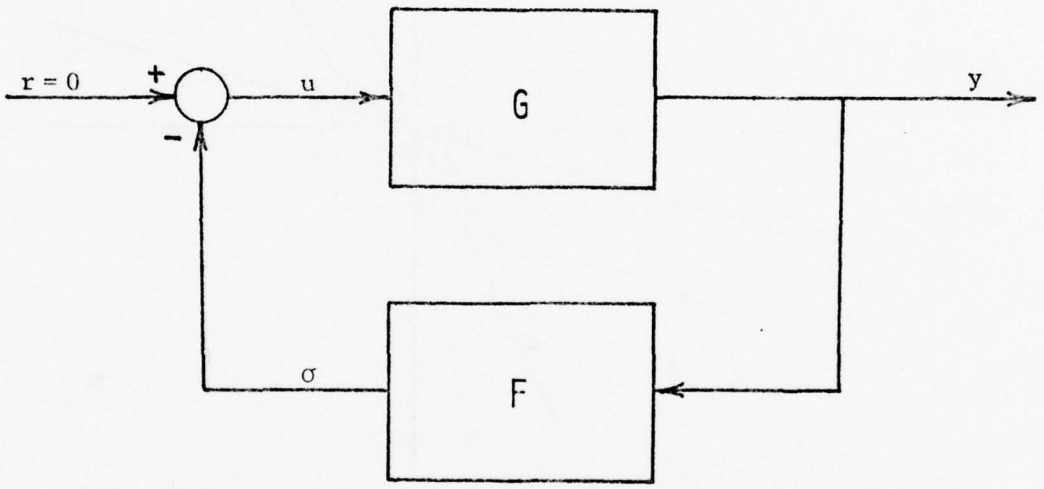


Figure II-10. General System Configuration For Hyperstability Condition.

C a 1 x n constant vector and D a scalar constant. The function F satisfies the condition that, for any $T > 0$, [62]

$$\int_0^T y(t)\,\sigma(t)\,dt \leq \delta \left[\sup_{0 \leq t \leq T} ||\underline{x}(t)||\right] \qquad \text{(II-16.D)}$$

where $\delta \geq 0$ is independent of T but may depend on $\underline{x}(o)$. The system (II-14.D) is hyperstable with respect to $u = 0$ if there exists a constant K such that any solution $\underline{x}(t)$ of (II-14.D) satisfies

$$||\underline{x}(t)|| \leq K[||\underline{x}(o)|| + \delta] \qquad \text{(II-17.D)}$$

The system is asymptotically hyperstable if it is hyperstable and

$$\lim_{t \to \infty} \underline{x}(t) = 0. \qquad \text{(II-18.D)}$$

The condition (II-16.D), the heart of the hyperstability concept, represents a "time average" instead of a sector requirement. This is needed because y and σ are time-varying. It seems reasonable that stability should be kept even if

$$u(t)\,\sigma(t) \not> 0 \qquad \text{(II-19.D)}$$

for short time increments, but is satisfied in the mean.

For the dynamical system case just described, necessary and sufficient conditions for the system to be asymptotically hyperstable are:

(1) $$Z(s) = C^T(sI - A)^{-1}B + D \qquad \text{(II-20.D)}$$

be strictly positive real, and

(2) all roots s_i of $Z(s) = 0$ are such that $\mathrm{Re}\{s_i\} < 0 \quad i=1,2,\cdots,n$

This set of conditions will be very important in the development of some of the MRAS identifiers.

For the discrete time case, hyperstability conditions are similar, except integrations are replaced with infinite summations. Given the plant

$$x_p(k) = \sum_{i=1}^{n} a_i \, x_p(k-i) + \sum_{j=1}^{m} b_j \, u(k-j) \qquad \text{(II-21.D)}$$

and model

$$x_m(k) = \sum_{i=1}^{n} \hat{a}_i \, x_m(k-i) + \sum_{j=1}^{m} \hat{b}_j \, u(k-j) \qquad \text{(II-22.D)}$$

the output error is defined as

$$e(k) = x_m(k) - x_p(k) \qquad \text{(II-23.D)}$$

A filtered version of (II-23.D) is

$$\varepsilon(k) = e(k) + \sum_{i=1}^{n} c_j \, e(k-i) \qquad \text{(II-24.D)}$$

The system (II-21.D)-(II-24.D), along with identifier terms $\hat{a}_i$ and $\hat{b}_i$ is asymptotically hyperstable if

$$\text{Re}\left\{ \frac{1 + \sum_{i=1}^{n} c_i z^{-i}}{1 - \sum_{i=1}^{n} a_i z^{-i}} \right\} \qquad \text{(II-25.D)}$$

is strictly positive real, and

$$\sum_{k=0}^{\infty} \left[\varepsilon(k) \left\{ \sum_{i=1}^{n} [a_i - \hat{a}_i(k)] x_m(k-i) + \sum_{j=1}^{m} [b_j - \hat{b}_j(k)] u(k-j) \right\} \right] > -\gamma^2 \qquad \text{(II-26.D)}$$

where γ is a constant which depends on the initial conditions of the plant.

The connection between the Lyapunov approach and hyperstability can be seen from the following example [69,70]. Using (II-8.D)-(II-10.D) as a *control* case with known model A_m, B_m, it is desired to determine $\dot{A}_p$, $\dot{B}_p$ to

insure $\underline{e} \to 0$ as $t \to \infty$ (asymptotic tracking). It can be shown that

$$\dot{\underline{e}} = A_m \underline{e} + [A_m - A_p]\underline{x}_p + [B_m - B_p]\underline{u} \qquad \text{(II-27.D)}$$

Using (II-27.D) together with a linear compensator which generates the vector $\underline{v}$

$$\underline{v} = D\underline{e} \qquad \text{(II-28.D)}$$

where D is a square matrix. Defining

$$-w = [A_m - A_p]\underline{x}_p + [B_m - B_p]\underline{x} \qquad \text{(II-29.D)}$$

then

$$\begin{aligned} \underline{w} = &\left[\int_o^t \Phi(v(\tau),t)dt - A_m + A_p(t_o)\right]\underline{x}_p(t) \\ &+ \left[\int_o^t \underline{\Psi}(v(\tau),t)dt - B_m + B_p(t_o)\right]u(t) \end{aligned} \qquad \text{(II-30.D)}$$

Equations (II-27.D)-(II-30.D) define a nonlinear, time-varying feedback system, with (II-30.D) the nonlinear, time-varying part and the others a linear part with input $(-\underline{w})$ and output (v) as characterized by

$$\underline{v}(s) = D(sI - A_m)^{-1}(-\underline{w}(s)) \qquad \text{(II-31.D)}$$

To use hyperstability, Φ and Ψ must be selected such that

$$\eta(0,t) = \int_o^t \underline{v}^T \underline{w}\, dt \geq -\gamma_o^2 \quad \forall \quad t \geq 0, \quad \gamma_o^2 < \infty \qquad \text{(II-32.D)}$$

Non-unique solutions include

$$\begin{aligned} \dot{\Phi}(\underline{v}(t),t) &= \dot{A}_p = R\underline{v}\,\underline{x}_p^T \\ \dot{\underline{\Psi}} &= \dot{B}_p = \tilde{R}\,\underline{v}\,\underline{u}^T \end{aligned} \qquad \text{(II-33.D)}$$

R, $\tilde{R}$ are p.d. matrices. Using the MKY Lemma [71,72], D must be a solution of the Lyapunov equation

$$A_m^T D + D A_m = -C \qquad \text{(II-34.D)}$$

where C is a p.d., symmetric arbitrary matrix.

An "equivalent" Lyapunov Function could have been formed which resulted in identical laws (II-33.D). This is

$$V = \underline{e}^T D\underline{e} + 2 \int_o^t \underline{v}^T \underline{w}\, dt + 2\gamma_o^2 \qquad \text{(II-35.D)}$$

where $\underline{w}$ is given by (II-30.D) and

$$\gamma_o^2 = \frac{1}{2} \operatorname{tr}[A_m - A_p(t_o)]^T F_A^{-1} [A_m - A_p(t_o)] + \frac{1}{2} \operatorname{tr}[B_m - B_p(t_o)]^T F_B^{-1} [B_m - B_p(t_o)] \qquad \text{(II-36.D)}$$

The key difference is the use of $\underline{v} = D\underline{e}$ with $\underline{v}$ formed to assure convergence of Φ, Ψ with no magnitude restrictions.

In summary, a hyperstable system may be described in part as:

(1) It is stable (or asymptotically stable)

(2) BIBO (bounded input-bounded output) stable

(3) The parallel combination of two hyperstable blocks is also a hyperstable block

(4) The feedback combination of two hyperstable blocks is also a hyperstable block

This reduces the stability problem to one of adding and feeding back small local hyperstable blocks.

E. The Two MRAS Identifier Structures

There are two key problem formulations possible in systems identification, the equation error formulation and the response error formulation. Both approaches occur and have good points and drawbacks for each of the identification cases of a) time-invariant parameter identification, b) time-varying parameter identification, and c) nonlinear, time-invariant

system identification.

The first method is the equation error method. The phrase comes from statistical time-series work and refers to creating an error signal from plant state information. In MRAS work this method is referred to as series-parallel MRAS. This is due to the both parallel calculations occuring as well as the tandem (or series) calculations required. Given a SISO identification system as shown in Figure II-11, for a plant defined by

$$y_p^{(n)} = \sum_{i=1}^{n} a_i \, y_p^{(i-1)} + \sum_{i=1}^{m} b_j \, u^{(i-1)} \qquad \text{(II-1.E)}$$

a reference model is defined by

$$y_m^{(n)} = \sum_{i=1}^{n} \hat{a}_i \, y_p^{(i-1)} + \sum_{i=1}^{m} b_i \, u^{(i-1)} \qquad \text{(II-2.E)}$$

where $\hat{}$ refers to an estimate. For simplicity, define

$$\hat{A}(s) = - \sum_{i=1}^{n} \hat{a}_i \, s^{i-1} \qquad \text{(II-3.E)}$$

$$\hat{B}(s) = \sum_{i=1}^{m} \hat{b}_i \, s^{i-1} \qquad \text{(II-4.E)}$$

which are shown in Figure II-11. An equation error, ε, is defined as

$$\varepsilon = y_p^{(n)} - y_m^{(n)} \qquad \text{(II-5.E)}$$

which can be written in the normal form

$$\varepsilon = y_p^{(n)} - \left\{ \sum_{i=1}^{n} \hat{a}_i \, y_p^{(i-1)} + \sum_{i=1}^{m} b_i \, u^{(i-1)} \right\} \qquad \text{(II-6.E)}$$

Note that this error only has plant state and input information. If $\hat{a}_i = a_i$, $\hat{b}_i = b_i$ then $\epsilon = 0$ and (II-6.E) merely restates (II-1.E). The equation error is therefore a self error, only depending on plant states.

This approach is what is used by the classical least-squares method of identification. Drawbacks include 1) the fact that if x_p is corrupted by noise $\omega(t)$, then the parameter estimates $\hat{a}_i$, $\hat{b}_i$ will be biased, and 2) most identifiers using the equation error formulation are <u>not</u> guaranteed to yield convergence to the correct answer (a class of MRAS equation error methods will be shown to converge, however).

The second approach to be presented is the output error method, shown in Figure II-12. As can be seen, the name is appropriate because two related outputs (plant and model) are compared and an error signal formed. In MRAS this is referred to as the "parallel" approach since the model and plant operate simultaneously in parallel with one another.

The plant is defined by (SISO case for simplicity)

$$y_p^{(n)} = \sum_{i=1}^{n} a_i \, y_p^{(i-1)} + \sum_{i=1}^{m} b_i \, u^{(i-1)} \qquad \text{(II-7.E)}$$

with corresponding model

$$y_p^{(n)} = \sum_{i=1}^{n} \hat{a}_i \, y_m^{(i-1)} + \sum_{i=1} \hat{b}_i \, u^{(i-1)} \qquad \text{(II-8.E)}$$

from which an error signal is formed

$$e = x_m - x_p \qquad \text{(II-9.E)}$$

Note that this case, distinct from the previous equation error method, has the input u passing through a filter (the model) separate from the plant. As before, however, the tracking concept is similar, i.e. if $e = 0 \; \forall \; t > T$,

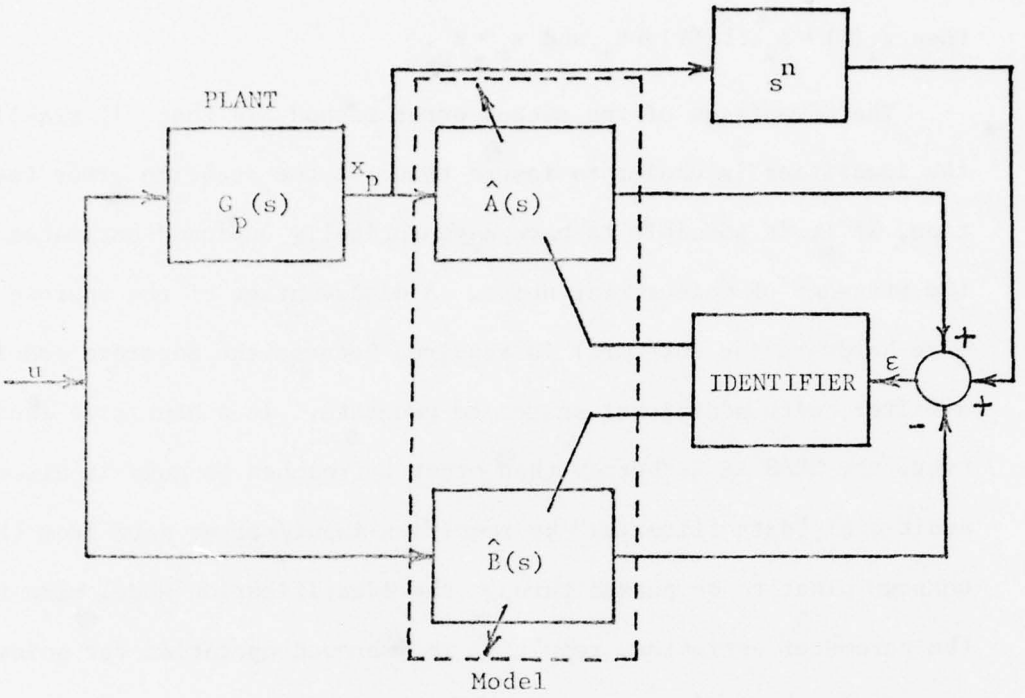


Figure II-11. Equation Error ("Series-Parallel") MRAS Identifier Structure.

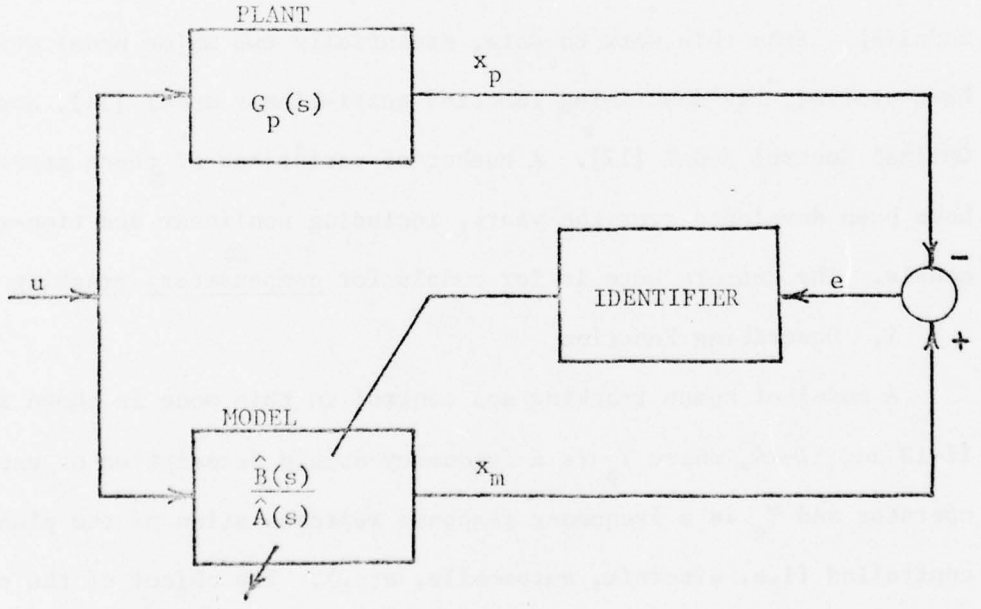


Figure II-12. Output Error ("Parallel") MRAS Identifier Structure.

then $\hat{a}_i(t) = a_i$, $\hat{b}_i(t) = b_i$ and $x_m = x_p$.

The advantages of the output error method are that 1) stability of the identifier is easier to insure than for the equation error formulation, 2) it is possible to have asymptotically unbiased estimates in the presence of measurement noise. A disadvantage of the approach is that more hardware (or software) is required because the separate model is a filter, with additional states to generate. In a heuristic sense, however, the MRAS RE is better than other approaches because it allows for additional "data filtering" by requiring input/output data from the unknown plant to be passed through the identification model made up of the parameter estimates, resulting in improved operation for noisy measurements, unlike equation error-type formulations.

F. Human Operator Models

Over the past 30 years much work has been done on human operator modeling. From this work to date, essentially two major model structures have evolved, the describing function quasi-linear model [32], and the Optimal Control Model [17]. A number of variations of these approaches have been developed over the years, including nonlinear and time-varying models. The concern here is for models for compensatory tracking tasks.

1. Describing Function

A model of human tracking and control in this mode is shown in Figure II-13 and II-14, where Y_p is a frequency-domain description of the pilot/operator and Y_c is a frequency response representation of the plant to be controlled (i.e. aircraft, automobile, etc.). The object of the operator is to control Y_c using e as an available signal. Y_p is developed by the human to meet the *Primary Rule-of-Thumb for Frequency-Domain Synthesis* [73]:

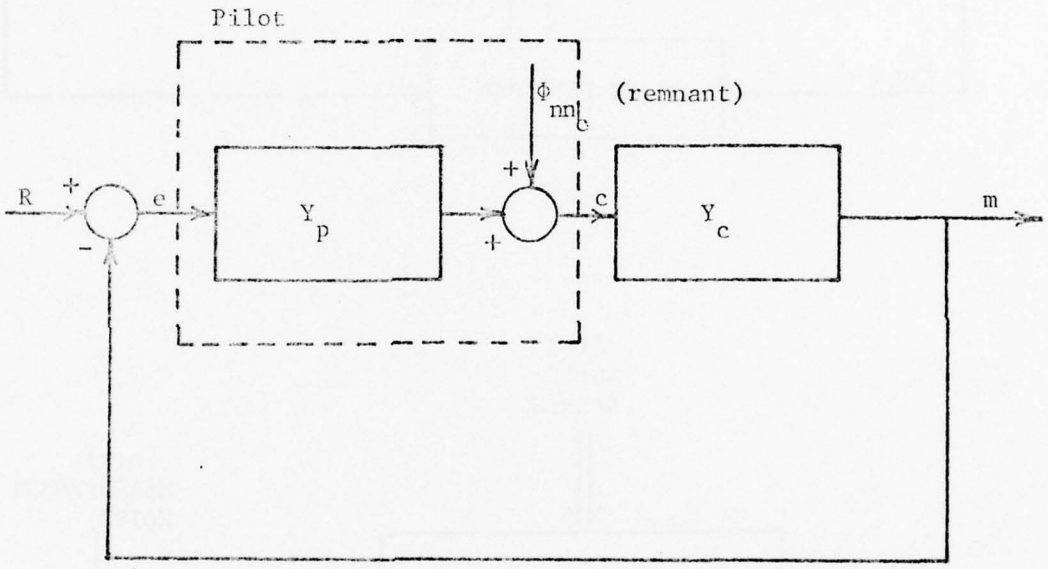
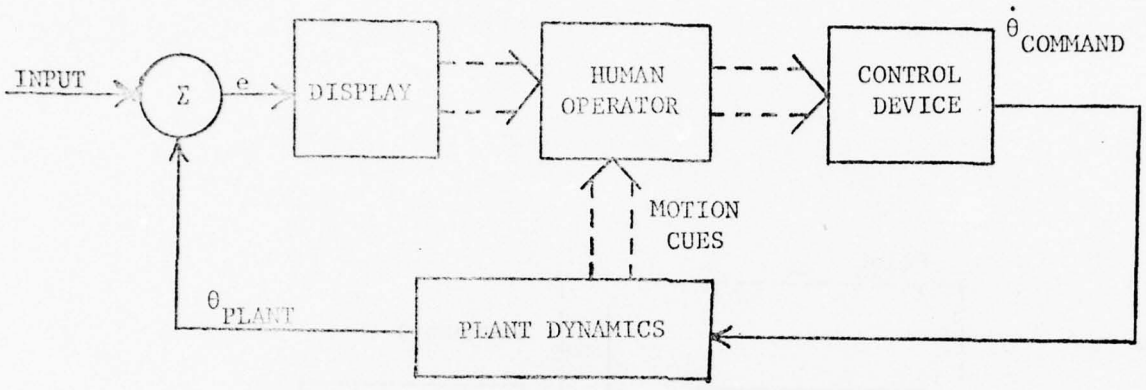
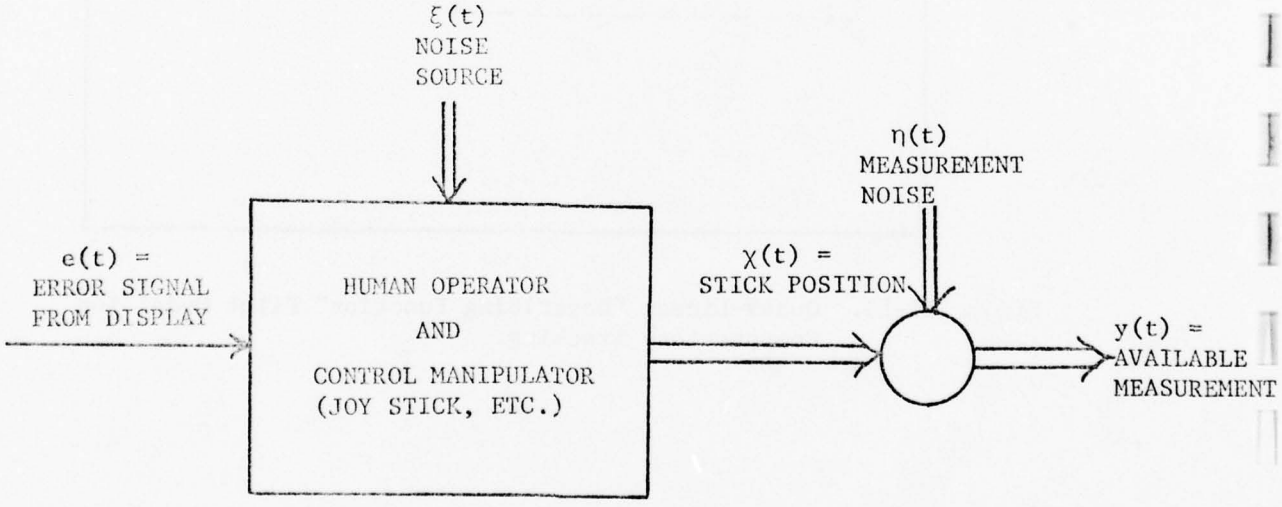


Figure II-13. Quasi-Linear "Describing Function" Pilot Model for Compensatory Tracking.



(a)



(b)

Figure II-14(a),(b). Human Operator Tracking Configuration
(a) Basic System Breakdown, (b) Sources of Error Due to Human Response

"At frequencies just within and beyond the input bandwidth, seek a region of -20 dB/decade slope for the amplitude ratio and adjust the loop gain so as to put the unity-amplitude crossover frequency near the higher edge of this region, while maintaining adequate stability margins".

Defining

$$\left|Y_{OL}\right| = \left|Y_p\ Y_c\right| \qquad \text{(II-1.F)}$$

then $\left|Y_{OL}\right|$ lies close to -20 dB/decade in the region of crossover frequency ω_c.

A typical parameterization of Y_p is

$$Y_p = K_p \frac{(T_L\ j\omega + 1)}{(T_I\ j\omega + 1)}\ e^{-j\omega\tau_e} \qquad \text{(II-2.F)}$$

where

K_p - static gain

T_L - lead time constant

T_I - lag time constant

τ_e - effective transport lag

Typical structures for Y_c are

$$K_c$$

$$\frac{K_c}{j\omega} \qquad \text{(II-3.F)}$$

$$\frac{K_c}{j\omega(Tj\omega + 1)}$$

Numerical values of τ_e depend on human response capability and system latency, with minimums of $\simeq$ .05 seconds and maximums a few tenths of a second. Typical trained aircraft pilot values are $\tau_e \simeq .2$ seconds.

Shown in Figure II-13 is Φ_{nmc}, a noise power remnant. A remnant is the portion of the output of Y_p which is not linearly correlated with the system input. With Φ_{nn} the closed-loop remnant power,

$$\Phi_{nn} = \frac{\Phi_{nmc}}{|1 + Y_p Y_c|^2} \qquad \text{(II-4.F)}$$

If there is an input remnant at R, denoted Φ_{ii}, then

$$\Phi_{cc} = \Phi_m + \left|\frac{Y_p}{1 + Y_p Y_c}\right|^2 \Phi_{ii} \qquad \text{(II-5.F)}$$

where Φ_{cc} is the total control output spectrum. It has been shown that the linear describing function can lead to large remnants [39]. Overall, such a model as in Figure II-12 is reasonable when stress effects are minimal, the input R is not predictable [37], and nonlinear effects are minimal.

Variations on this structure result for the pursuit mode, where a time-varying input (enemy aircraft motion, evasive manuever, etc. is desired to be closely tracked. The general model is shown in Figure II-15, where Y_{p_i} represents an open-loop feedforward response to the input, Y_{p_m} an output feedback sensing, and Y_{p_e} and Y_c as before. The term n_c represents internal loop disturbance or uncertainty, which is always present. Some compensatory tracking tasks with quasi-predictable inputs require Y_{p_i} but no Y_{p_m} [39], as consistent with successive organization concepts in [74].

Other possible modes for the quasi-linear model use include preview control as in automobiles as well as aircraft scenarios [75,76]. Variations on this to include only future data for tracking is given by [77].

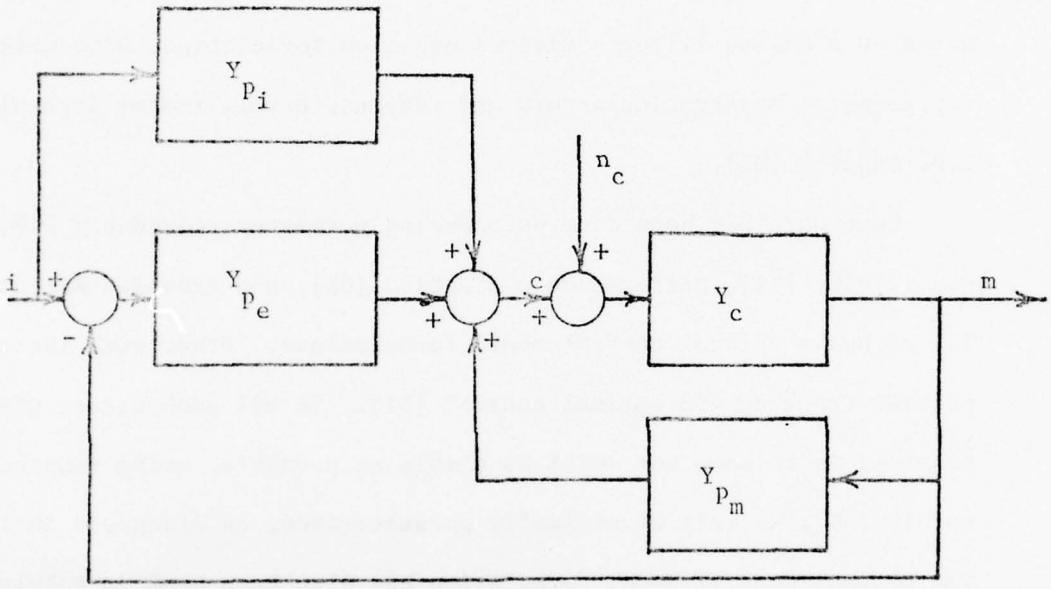


Figure II-15. Pursuit Tracking Model Employing "Describing Function" Approach.

2. Optimal Control-Theoretic Model

This modeling approach treats a human operator as an optimal estimator-predictor-controller. It was developed under the assumption that a highly trained and motivated human acts to minimize a quadratic cost functional as used in optimal control [17,18,78,79]. A block diagram is shown in Figures II-16, II-17 where the "estimator" and "gains" are based on a Kalman filter - Riccati equation formulation, with noises representing observation errors and response errors (motor irregularities, i.e. muscle) [81].

Some work has been done on reducing parameter redundancy [19], model sensitivity [81], performance evaluation [82], and extended Kalman filtering of human optimal control model formulations. Other work includes preview tracking via optimal control [51]. In all such cases, one of the problems is to keep the model as simple as possible, using reduced order models [85], as well as minimally parameterized, as discussed in [19]. The Optimal Control Model formulation has also been used in solving attention allocation tasks with good results [50].

The human's control characteristics are a cascade of the operations 1) time delay, 2) equalization, and 3) nueromuscular and actuator dynamics. In Figure II-16, it is presumed that the human's equalization is performed under an optimal controller assumption. He is displayed one or more outputs $\underline{y} = [y_1 \quad y_2 \cdots y_m]^T$ and manipulates a single control variable u.

The vehicle dynamics (i.e. aircraft flight dynamics) are assumed known,

$$\dot{\underline{x}} = A\underline{x} + \underline{b}u + E\underline{\omega} \qquad \text{(II-6.F)}$$

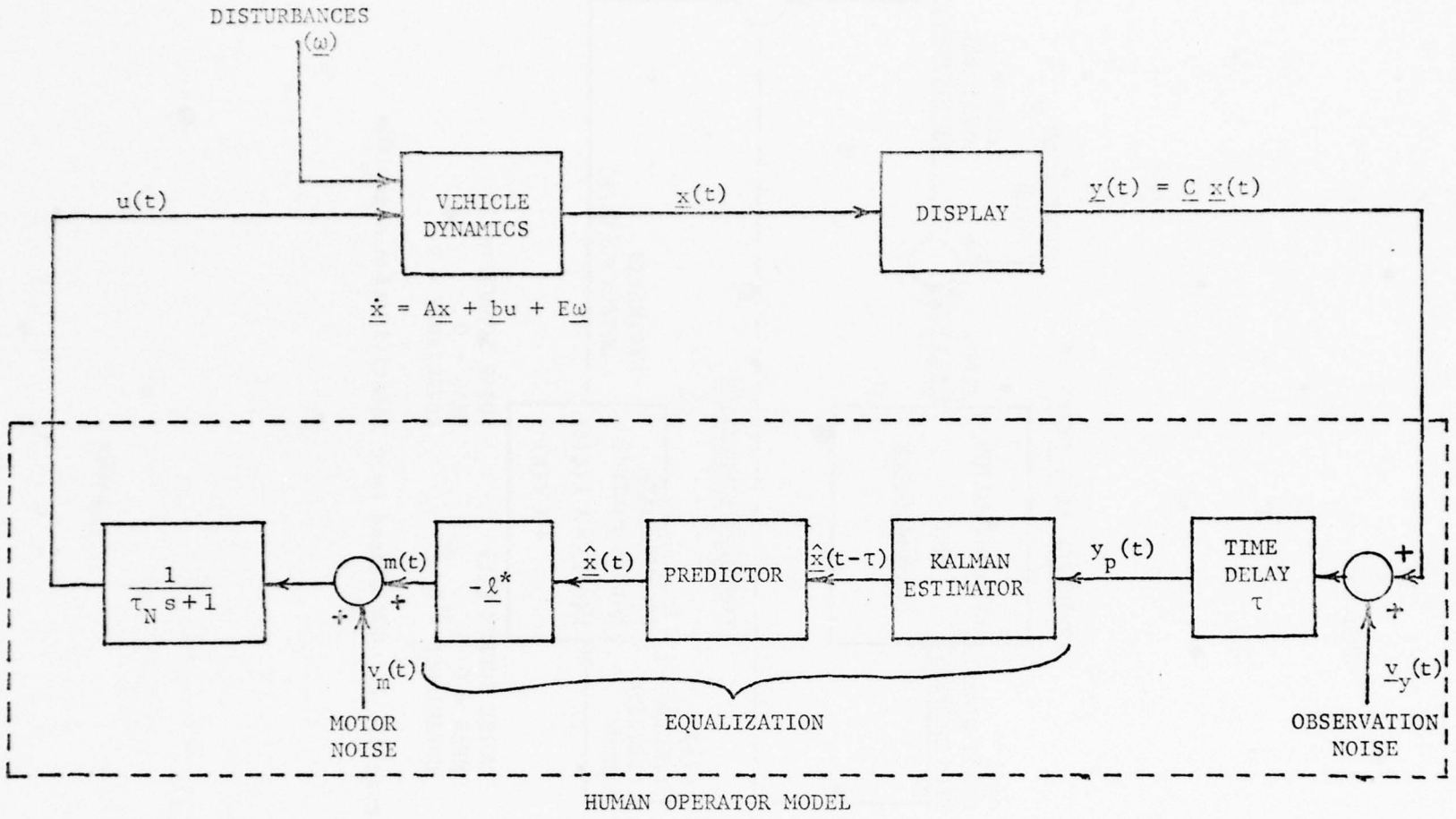


Figure II-16. Control Theoretic Model of Optimal Human Behavior.

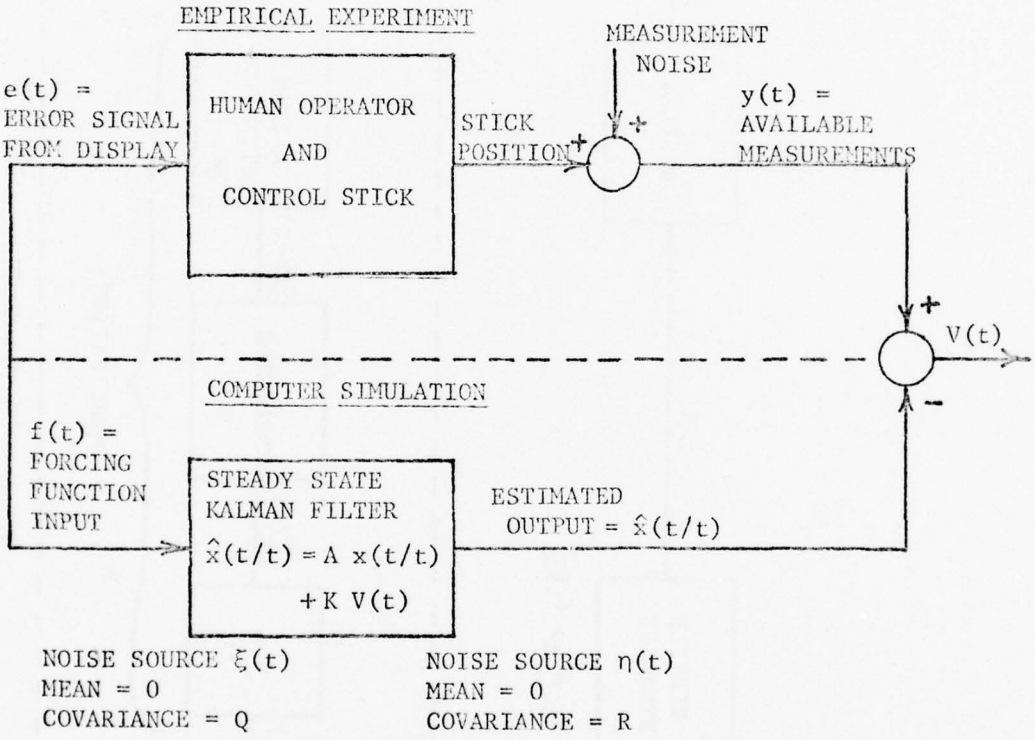


Figure II-17. OCM Closed Loop Identification Algorithm

$\underline{x}$ is an n-vector (representing vehicle states), u is the human's system input, $\underline{\omega}$ is a zero mean, normal white noise representing random external disturbances (turbulence, wind gusts, etc.), $E\{\underline{\omega}(t)\ \underline{\omega}^T(\sigma)\} = W_1\delta(t-\sigma)$. The output is

$$\underline{y} = C\ \underline{x} + \underline{d}\ u \tag{II-7.F}$$

with y an r-vector, and A, B, C, D known. $\underline{v}_y$ is an observation noise, with [84]

$$E\{v_{y_i}(t)\ v_{y_i}(\sigma)\} = V_{y_i}\ \delta(t-\sigma) \qquad i=1,2,\cdots,r \tag{II-8.F}$$

Due to human perception, $\underline{y}(t)$ is seen as the *perceived* delayed output (by factor τ)

$$\underline{y}_p(t) = \underline{y}(t-\tau) + \underline{v}_y(t-\tau) \tag{II-9.F}$$

Using $\underline{y}_p$, the human generates a command u_c, but since he doesn't know exactly u, then an error u_m is added

$$u = u_c + u_m \tag{II-10.F}$$

$$\dot{u}_m + \gamma\ u_m = \gamma\ v_m \tag{II-11.F}$$

$$E\{v_m\} = 0 \qquad E\{v_m^2\} = V_m \tag{II-12.F}$$

The human is assumed as an optimal estimator-controller, so

$$\tau_N\ \dot{u}_c + u_c = -\underline{\ell}^*\ \hat{\underline{x}}(t) - \ell_m^*\ \hat{u}_m \tag{II-13.F}$$

where τ_N is the neuromotor time constant, $\hat{\underline{x}}(t)$ the optimal estimate of $\underline{x}(t)$ given measurements $\underline{y}_p$, and $\hat{u}_m$ is the optimal estimate of u_m.

$$\tau_N = \lambda_{n+1}^{-1} \ ; \ \ell_i^* = \tau_N\ \lambda_i \qquad i=1,2,\cdots,n \tag{II-14.F}$$

$$\underline{\lambda} = [\lambda_1 \quad \lambda_2 \cdots\cdots \lambda_{n+1}]^T\ , \tag{II-15.F}$$

$$\underline{\lambda} = b_o^T\ K_o/g \tag{II-16.F}$$

$$A_o^T K_o + K_o A_o + C_o^T Q C_o - K_o b_o b_o^T K_o/g = 0 \qquad \text{(II-17.F)}$$

$$Q_o = \text{diag}[q_1 \quad q_2 \cdots q_r], \; \underline{b}_o = [0 \quad 0 \cdots 0 \quad 1]^T$$

$$A_o = \left[\begin{array}{c|c} A & b \\ \hline 0 & 0 \end{array}\right] \qquad C_o = [C \,\vdots\, \underline{d}]$$

$$\ell_m^* = \tau_N[(\gamma I - A_o + b_o \underline{\lambda})^{-1} b_o]^T (C_o^T Q \underline{d} + K_o \tilde{\underline{b}})/g \qquad \text{(II-18.F)}$$

$$\tilde{\underline{b}} = [\underline{b}, \; 0]^T \qquad \text{(II-19.F)}$$

With $\gamma \simeq \frac{1}{\tau_N}$, typical numbers are .07 sec $< \gamma \leq$ .12 sec. So

$$\tau_N \dot{u} + u = m + v_m \qquad \text{(II-20.F)}$$

$$m = -\underline{\ell}^* \hat{\underline{x}} \qquad \text{(II-21.F)}$$

The optimal operator assumption means the human performs Kalman filtering to generate $\hat{x}(t-\tau)$, from

$$\dot{\underline{\chi}}(t-\tau) = A_1 \hat{\underline{\chi}}(t-\tau) + \sum C_o^T V_y^{-1} \cdot$$

$$[\underline{y}_p(t) - C_o \hat{\underline{\chi}}(t-\tau)] + \underline{b}_o \tau_N^{-1} m(t-\tau) \qquad \text{(II-22.F)}$$

where

$$\hat{\underline{\chi}}(t) = [\hat{\underline{x}}(t), \; \hat{u}(t)]^T \qquad \text{(II-23.F)}$$

with $\sum$ the p.d. error covariance matrix

$$0 = A_1 \sum + \sum A_1^T + W - \sum C_o^T V_y^{-1} C_o \sum \qquad \text{(II-24.F)}$$

$$A_1 = \begin{bmatrix} A & b \\ 0 & -\tau_N^{-1} \end{bmatrix} \qquad W = \begin{bmatrix} EW_1E^T & 0 \\ 0 & V_m \tau_N^{-2} \end{bmatrix} \qquad \text{(II-25.F)}$$

A predictor uses $\underline{p} = \hat{\underline{\chi}}(t-\tau)$ to determine $\hat{\underline{\chi}}(t)$

$$\hat{\underline{\chi}} = \underline{\xi}(t) + e^{A_1 \tau} [\underline{p} - \underline{\xi}(t-\tau)] \tag{II-26.F}$$

$$\dot{\underline{\xi}} = A_1 \underline{\xi} + \underline{b}_o \tau_N^{-1} m \tag{II-27.F}$$

Typical parameter ranges are $\tau_N \simeq .1$ seconds, $(V_y)_{ii} \simeq \pi(.01)\ E\{y_i^2\}$, $V_m \simeq \pi(.003)\ E\{m^2\}$. The expected values are

$$E\{y_i^2\} = (C_o \times C_o^T)_{ii} \qquad i=1,2,\cdots,r \tag{II-28.F}$$

$$E\{u^2\} = X_{n+1,n+1} \tag{II-29.F}$$

with X found by solving

$$E\{\underline{\chi}(t)\ \underline{\chi}^T(t)\} = e^{A_1 \tau} \sum e^{A_1^T \tau} + \int_o^\tau e^{A_1 \sigma} W e^{A_1^T \sigma}\ d\sigma$$
$$+ \int_o^\infty e^{\bar{A} \sigma} e^{A_1 \tau} \sum C_o^T V_y^{-1} C_o \sum e^{A_1^T \tau} e^{\bar{A}^T \sigma}\ d\sigma \tag{II-30.F}$$

$$\bar{A} = A_o - \underline{b}_o \underline{\lambda} \tag{II-31.F}$$

$$E\{x_i^2\} = X_{ii} \qquad i=1,2,\cdots,n \tag{II-32.F}$$

Using the above, V_y and V_m can be adjusted until the assumed values match those from (II-28.F) and (II-29.F).

3. Nonlinear and Time-Varying Operator Models

Although the bulk of human operators are based on a quasi-linear assumption, there are situations in which such an assumption is not reasonable. Nonlinearities due to "indifference thresholds", rate saturation, and relay effects produce harmonics which cannot be accounted for by linear theory alone. A characteristic of such cases is that the more pronounced the nonlinearity, the greater the remnant [85].

Nonstationary response is exhibited in some cases, due to the learning and adaptation capability of man. This is especially true in tracking, with noticeable changes in gains and time delays [86-88]. Other work has shown the effect on the gain and phase response of a controller as a function of time due to changes in the nature of the tracking signal [89], clearly indicating the possible time-varying nature of the human operator. These ideas are illustrated graphically in Figure II-18.

First, a Goodyear nonlinear model [90] which considered the effect of dither is shown in Figure II-19. Here threshold is shown as a dead-zone, task overload as rate saturation, and dither habit as an active "adaptive" control probing signal.

Second, a sampled-data model [91] based on the intermittent attention allocation capability of the human is considered, where the model is of an information theory structure. This model, shown in Figure II-20, includes quantization, interrupted output sensing, prediction, estimation, and short-term moving average memory.

A third model, incorporating adaptive sampling, is shown in Figure II-21 [92]. The key features of this model are that 1) adaptation occurs based on human prediction capability, and 2) a variable sampling period controlled by external stimulus occurs.

Typically, time-varying models will be assumed of a linear form with the parameters allowed to vary with time. The idea behind this is that by allowing for nonstationarity, previously treated as quasi-linear time-invariance, that a better fit to the data results. For continuous time such an input-output model is

$$H(s) = \left[K_p(t) \frac{(\tau_L(t)\ s+1)}{(\tau_I(t)\ s+1)} \right] \left\{ \frac{e^{-\tau s}}{\tau_N\ s+1} \right\} \qquad \text{(II-33.F)}$$

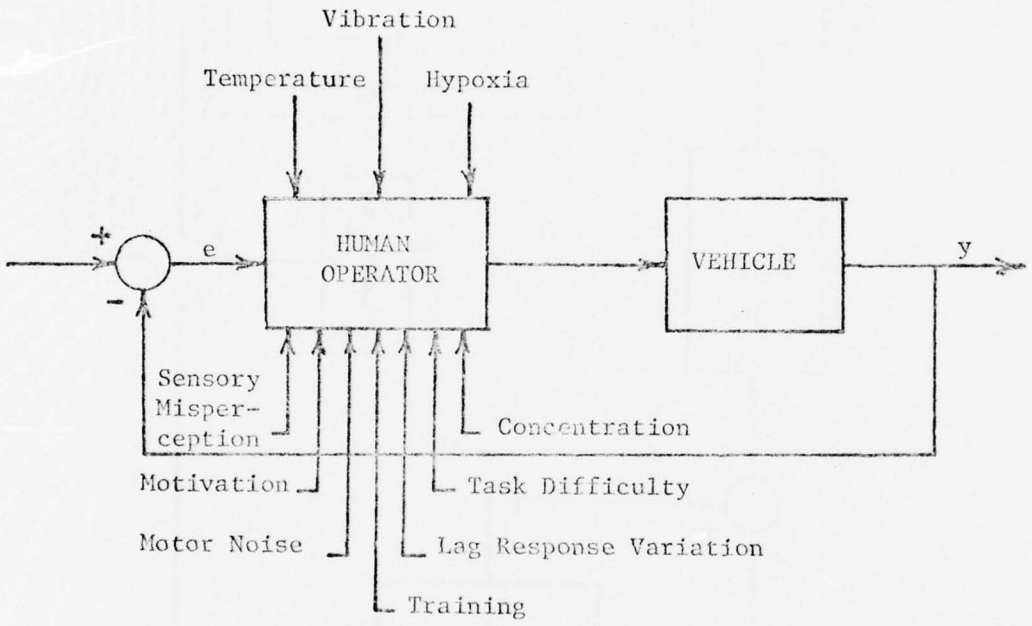


Figure II-18. External Influences Causing Time-Varying Operator Effects In a Compensatory Control Problem.

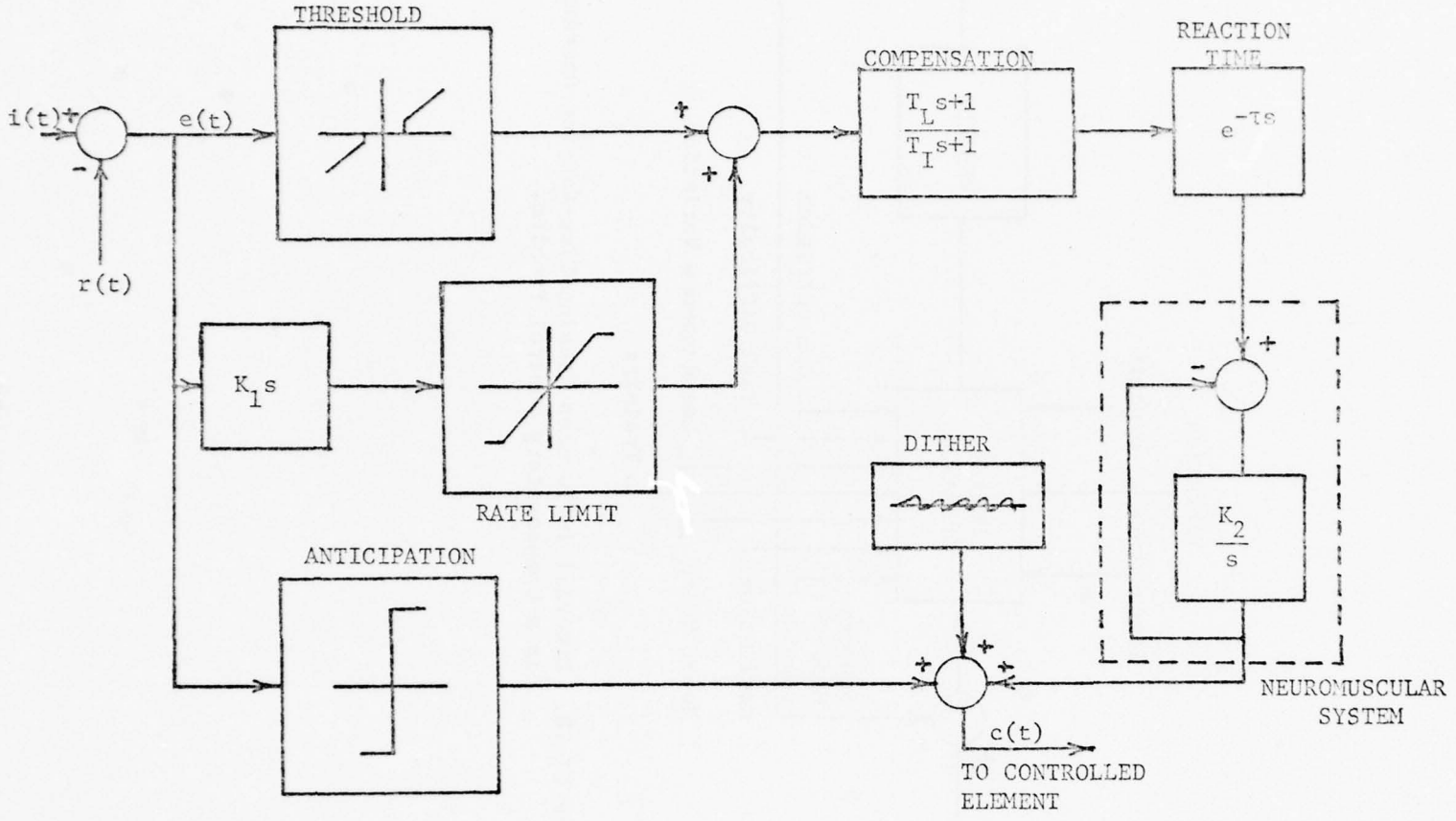


Figure II-19. Goodyear Nonlinear Model.

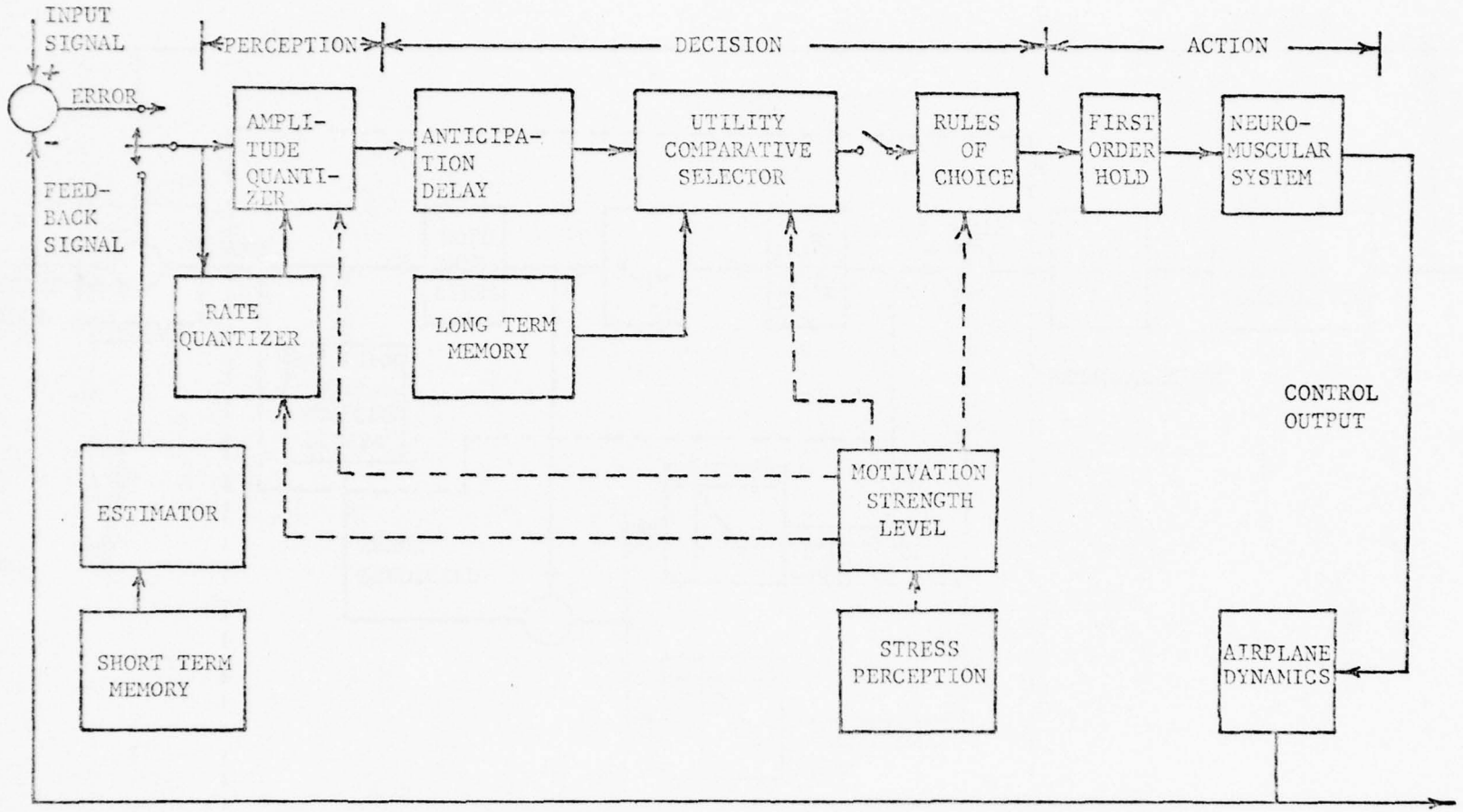


Figure II-20. Sampled Data Information Transfer Model

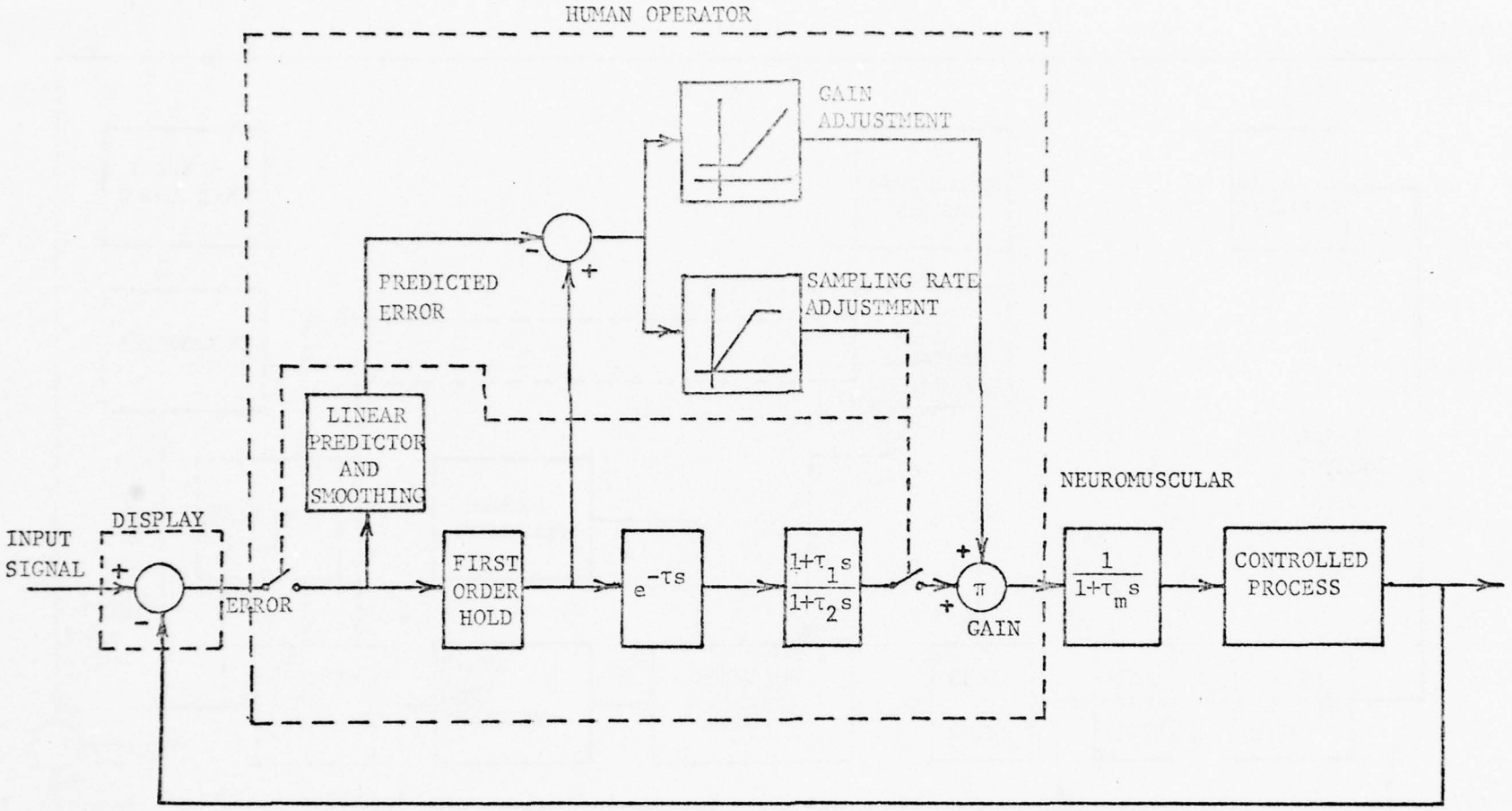


Figure II-21. Adaptive Sampled-data Model

where

τ_N - neuromuscular time constant

τ - human reaction time delay

τ_L - lead compensation time constant

τ_I - lag compensation

K_p - gain

The { } expression represent human response limitations that degrade performance and represent the variable compensating dynamic terms $K_p(t)$, $\tau_L(t)$ and $\tau_I(t)$. For discrete-data modeling, a similar case results,

$$G(z) = \frac{\left(b_m(k) + b_{m-1}(k)z^{-1} + b_{m-2}(k)z^{-2} + \cdots + b_o(k)z^{-m}\right)z^{-d(k)}}{\left(1 + a_{n-1}(k)z^{-1} + a_{n-2}(k)z^{-2} + \cdots + a_o(k)z^{-n}\right)} \qquad \text{(II-34.F)}$$

where d is the delay of dT time units, and b_i, a_j are functions of the time counter k, $t = kT$, $k=0,1,2,\cdots$. Typically $m < n$, and $n \leq 3$, so just as with the continuous case higher order linear discrete models do *not* improve the accuracy; this is due to the relatively large noise remnant.

A model by Sheridan [89] accounts for time-variability of the human during a transient tracking mode. The model developed was based on the concept of the human as a modulation system changing the amplitude and phase characteristics of the human to different input frequencies. In particular the operator input was

$$r(t) = \sum_{i=1}^{5} A_i \sin \omega_i t \qquad \text{(II-35.F)}$$

and the operator output

$$c(t) = \sum_{i=1}^{5} B_i(t) \sin[\omega_i t + \theta_i(t)] \qquad \text{(II-36.F)}$$

The problem was to determine B_i and θ_i as a function of time. This system is shown in Figure II-22.

The human does not necessarily act as a continuous controller, so work has been done to develop discrete models. The discrete-time concept has support on psychological grounds [93]. Models developed include those by Bekey [94], Figure II-23, Raovlt [95], Figure II-24, Lemay and Wescott [96], Figure II-25, and Navas [97], Figure II-26.

One final time-varying parameter modeling approach fits no particular model, but is a concept. Using an extended Kalman Filter approach, it is presumed that the parameter variation can be modelled by the stochastic difference equation

$$\underline{\phi}(k+1) = \underline{\phi}(k) + \underline{\omega}(k) \qquad \text{(II-37.F)}$$

where $\underline{\phi}$ is the parameter vector and $\underline{\omega}$ a gaussian zero-mean, white noise. This noise increases the uncertainty in the Kalman measurement $\hat{\underline{y}}(k+1/k)$ and hence causes the predicted covariance matrix $P(k+1/k)$ to be biased and hence not "believe" all the old data $\{y(k), y(k-1), \cdots\}$. Such an approach allows for unknown parameter tracking without need of *a priori* knowledge of the rates of change of the parameters. Results demonstrate the potential power of this approach [98].

G. Open Loop Versus Closed-Loop Identification

In identification work a distinction is made between open-loop and closed-loop identification. This is because it is possible to obtain incorrect solutions if the proper conditions are met. To see this, consider Figure II-27, where ε_1 is a noise source and G_i are z-transform functions, r is the input and y the output. If G_2 is known and $\varepsilon_1 = 0$, then measurement of e and y will yield G_1 [99] (this case is representative of the

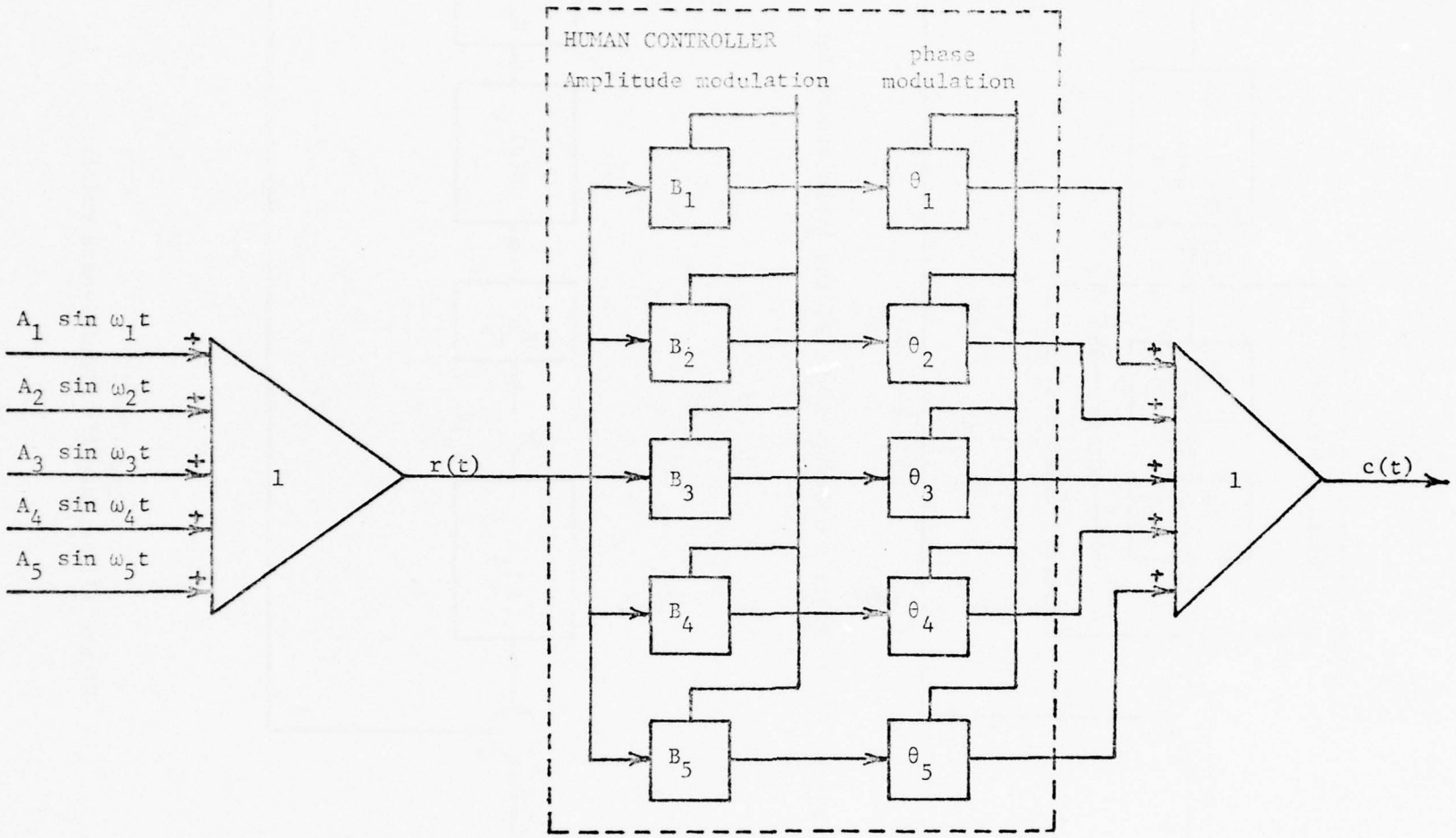


Figure II-22. A time-variable linear, continuous model of the human controller as proposed by Sheridan.

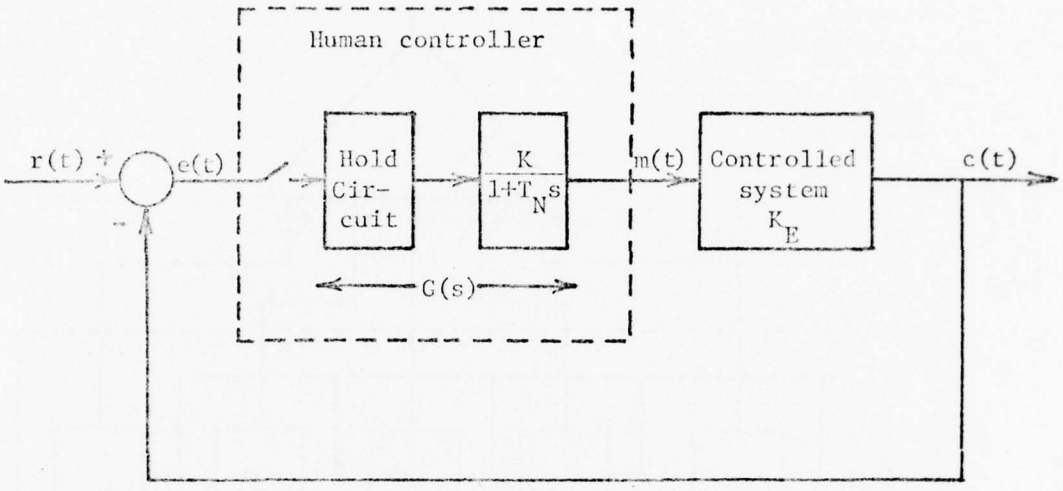


Figure II-23. Bekey's discrete model of the human controller.

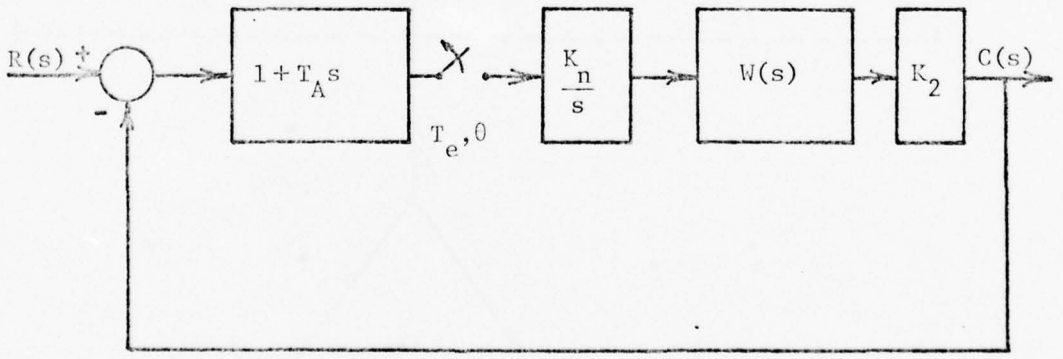


Figure II-24. Raoult's sampled-data model.

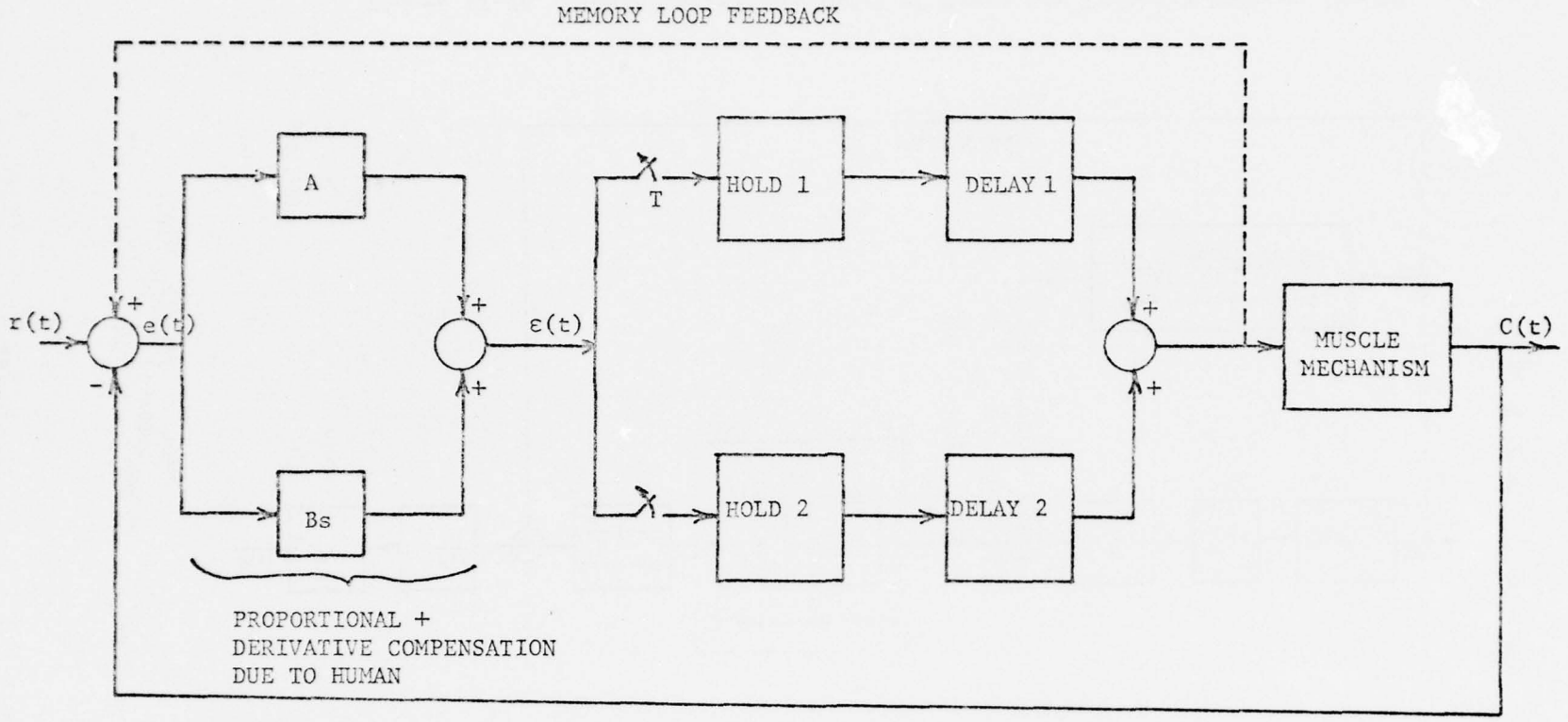


Figure II-25. Lemay and Westcott Discrete Model of Human Showing P-D Prediction Effects.

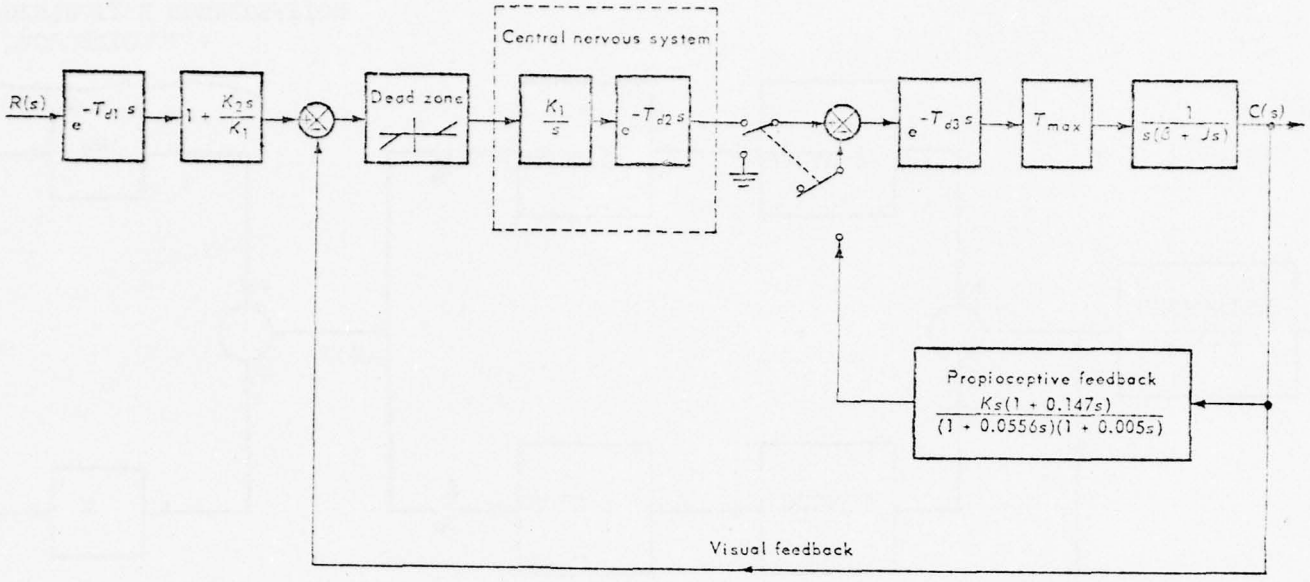


Figure II-26. Human Controller Model of Navas for Tracking Unknown Inputs

human operator problem). If ε_1 is correlated with e, then calculations of $\hat{G}_1$ can lead to errors so $\hat{G}_1 \neq G_1$ [22]. This latter case lead to the use of bootstrapping [100,101] and instrument-variables [5] to approximately uncorrelate e and y, and hence avoid parameter biasing.

In practice the human operator problem has ε_1 and e correlated, since ε_1 and G_2 are human remnant and noise filters respectively. The degree of noise depends on the model and the type of structure, i.e. linear, nonlinear, time-varying, etc. These problems will be addressed at the appropriate time as regards simulation results.

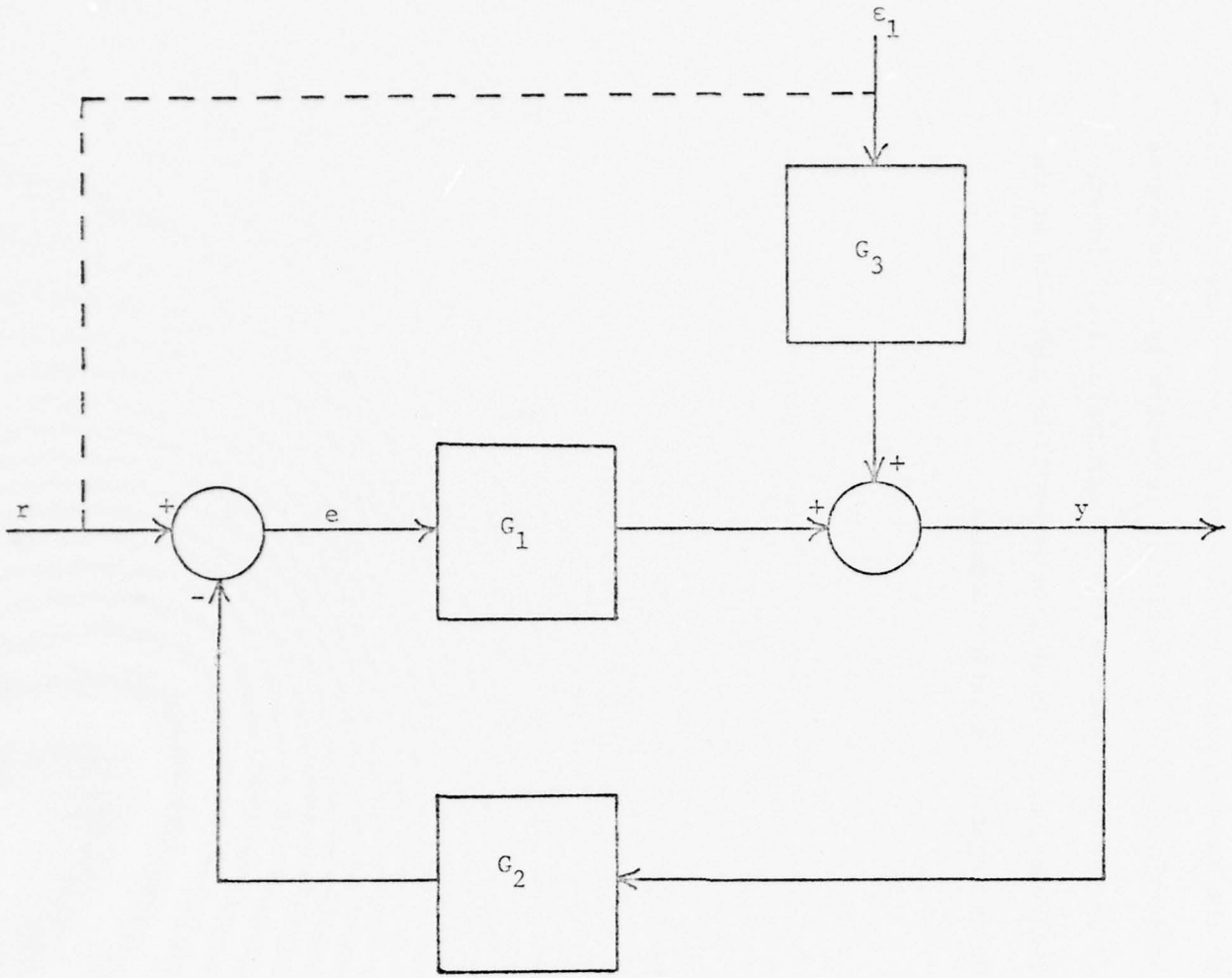


Figure II-27. System Considered For Closed-Loop Identification With Injected Noise.

CHAPTER 3. LINEAR, TIME-INVARIANT MRAS IDENTIFIERS

A. Preliminary

A number of identification techniques have been developed in the literature based on the MRAS concept for unknown plants which are assumed of known order, linear, time-invariant, and deterministic. Their basic development and structure form will be presented in this section to organize the diverse work in this arena. This then will provide a guide to what the methods have in common with traditional statistical techniques, how they differ, and possible design and implementation guidelines for using the identifiers in practice.

B. Methods

In this section, each of a series of identifiers will be briefly developed to clearly present the identification algorithm, key assumptions involved in its development, and implementation techniques. This will allow for straightforward comparison and review of the many diverse approaches which have been developed by a variety of people. Both equation error ("series-parallel") and output error ("parallel") methods are explored.

For comparison purposes, Table III-1 is given which presents a summary of all the methods which will follow. The "type" refers to whether the identifier method is continuous-time (C), or discrete-time (D). The "error structure" refers to whether the method employs the equation error (EE), generalized equation error (GEE), or response error (RE) formulation. The "input/output structure" relates whether a particular method handles only single-input/single-output (SISO), or the multivariable case (MIMO). The column "exact?" refers to whether the algorithms is an approximate (N)

Table III-1. Linear MRAS Type Identification Techniques

Number	Method	Type*	Error Structure	Input/Output Structure	Exact
1	Lion [102]	C	GEE	SISO	Y
2	Pazdera [103]	C	RE	MIMO	Y
3	Van de Linde [105]	C	RE	MIMO	N
4	Carroll [106]	C	GEE	SISO	Y
5	Lüders [107]	C	EE	SISO	Y
6	Kudva, Narendra [108]	C	EE	SISO	Y
7	Hang [109]	C	RE	SISO,MIMO	Y,N
8	Hirsch [111]	D	RE	SISO	Y
9	Lüders, Narendra Continuous [112]	C	EE	SISO	Y
10	Kudva [113]	D	EE	MIMO	Y
11	Kudva, Narendra [114]	D	EE	MIMO	Y
12	Kudva [114,115]	D	GEE	SISO	Y
13	Landau [116]	D	RE	SISO	Y
14	Valavani I [118,119]	C	EE	MIMO	Y
15	Carroll [120]	C	GEE	SISO	Y
16	Hang [121]	C	RE	SISO	Y
17	Molnar [123]	C	EE	SISO	Y
18	Kraft [124]	C	EE	SISO	Y
19	Akashi [125,126]	D	EE	SISO	Y,N
20	Tomizuka [127]	D	RE	SISO	Y(theory) N(practice)
21	Johnson [128,129]	D	RE	SISO	N
22	Pandya [130,131]	D	EE	SISO	N
23	Least-Squares [132-133]	D	EE	SISO	N
24	Kreisselmeier [134]	C	EE	SISO	Y

*C = Continuous D = Discrete

or exact approach (Y) as regards asymptotic stability.

1. Lion [102]

This method employs a SISO formulation for a linear continuous-time system of the form

$$y_p^{(n)} + \sum_{i=0}^{n-1} a_i\, y_p^{(i)} = \sum_{j=0}^{m} b_j\, u^{(j)} \qquad \text{(III-1.B)}$$

It is assumed the plant input and output are u and y_p. A tracking error is defined by

$$\varepsilon = y_p^{(n)} - y_m^{(n)} \qquad \text{(III-2.B)}$$

where a model is patterned after the plant as

$$y_m^{(n)} = -\sum_{i=0}^{n-1} \alpha_i\, y_p^{(i)} - \sum_{j=0}^{m} \beta_j\, u^{(j)} \qquad \text{(III-3.B)}$$

The error ε then becomes

$$\varepsilon = y_p^{(n)} + \sum_{i=0}^{n-1} \alpha_i\, y_p^{(i)} + \sum_{j=0}^{m} \beta_j\, u^{(j)} \qquad \text{(III-4.B)}$$

Pseudo-states and inputs y_{p_k} and u_j can be defined as

$$y_{p_k} = M_k\, y_p \qquad \text{(III-5.B)}$$

$$u_j = M_j\, u \qquad \text{(III-6.B)}$$

$$M_i = H(s)\,(s+c)^i \qquad \text{(III-7.B)}$$

where H(s) is an arbitrary stable filter and $c > 0$, constant. Additional equation errors can then be defined from

$$\varepsilon_k = y_{n+k} + \sum_{\ell=0}^{n-1} \alpha_\ell\, y_{\ell+k} + \sum_{j=0}^{m} \beta_j\, u_{j+k} \qquad \text{(III-8.B)}$$

$$k = 0, 1, \cdots, m+n-1$$

or

$$\varepsilon_i = \omega_i \underline{Z} \qquad i = 0,1,\cdots (n+m+1) \tag{III-9.B}$$

where

$$\underline{\omega}_i = [y_{p_i} \; y_{p_{i+1}} \cdots y_{p_{(n-1+i)}} \; u_i \; u_{i+1} \cdots u_{m+1}] \tag{III-10.B}$$

$$\underline{Z}^T = [(\alpha_o - a_o), (\alpha_1 - a_1), \cdots (\alpha_{n-1} - a_{n-1}), \; (\beta_o - b_o), (\beta_1 - b_1), \cdots (\beta_m - b_m)] \tag{III-11.B}$$

Defining

$$\underline{\varepsilon}^T = [\varepsilon_o \; \varepsilon_1 \cdots \varepsilon_{n+m+1}] \tag{III-12.B}$$

then

$$\underline{\varepsilon} = W(t) \; \underline{Z} \tag{III-13.B}$$

The parameter adjustment law for $\underline{\phi}^T = [\underline{\alpha}, \underline{\beta}]$ is

$$\dot{\underline{\phi}} = - k \, W^T(t) \; \underline{\varepsilon} \tag{III-14.B}$$

To relate $\underline{\alpha}$, $\underline{\beta}$ to $\underline{a}$ and $\underline{b}$, comparison of (III-4.B) for $\varepsilon = 0$ is necessary. By equating appropriate filtering terms of y_p and u, $n+m+1$ linear algebraic relations between $(\underline{\alpha}, \underline{\beta})$ and $(\underline{a}, \underline{b})$ results.

2. Pazdera [103]

Given the unknown plant

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{III-15.B}$$

where A and B are unknown matrices, an identifier model can be formulated as

$$\dot{\hat{\underline{x}}} = \hat{A}\underline{x} + \hat{B}\underline{u} + \underline{v} \tag{III-16.B}$$

where $\hat{A}$ and $\hat{B}$ are continuously adjusted estimates of A and B, and $\underline{v}$ is an auxiliary input vector satisfying $\underline{v} = \underline{0} \to \infty$. A workable implementation

of this approach is

$$\dot{\hat{A}} = E^{-1} P \underline{e}\, \underline{x} \qquad \text{(III-17.B)}$$

$$\dot{\hat{B}} = F^{-1} P \underline{e}\, \underline{u} \qquad \text{(III-18.B)}$$

$$\underline{v} = - D \underline{e}$$

where E, F, P are constant real symmetric positive-definite matrices, $\underline{e} = \underline{x} - \hat{\underline{x}}$, D is an arbitrary matrix whose eigenvalues all have negative real parts, and u is sufficiently frequency rich [104]. So long as, given an arbitrary p.d. matrix Q, that

$$PD + D^T P = -2Q \qquad \text{(III-20.B)}$$

then as $t \to \infty$, $\hat{A} \to A$, $\hat{B} \to B$. The shortcoming of this approach for the human operator problem is that it requires full state measurement $\underline{x}$ for implementation.

3. Van de Linde [105]

This approach uses the method of subsection 2 as a basis, but allows for an increased convergence rate by developing an algebraic condition on solutions for times when $\underline{e} \simeq 0$. It is based on the concept that an extraneous null exists if $\underline{e} = \dot{\underline{e}} = 0$ but $\hat{A} \neq A$ and $\hat{B} \neq B$.

Suppose the plant-identifier system has attained an extraneous null,

$$\dot{\underline{e}} = \dot{\underline{x}} - \dot{\hat{\underline{x}}} = 0 \qquad \text{(III-21.B)}$$

and hence

$$A\underline{x} + B\underline{u} = \dot{\hat{\underline{x}}} = H \underline{z} \qquad \text{(III-22.B)}$$

$H = (A,B)$ and $\underline{Z}^T = (\underline{x}^T, \underline{u}^T)$. At time t_j when (III-21.B) is satisfied, then store $\hat{\underline{x}}(t_j)$ and $\underline{z}(t_j)$. Continue adjusting $\hat{A}$ and $\hat{B}$ and store values for $j=1,2,\cdots$. A total of p data sets are required for p parameters to be identified. Then defining

$$\Omega = [\underline{Z}(t_1)\ \underline{Z}(t_2)\ \cdots\ \underline{Z}(t_p)] \qquad \text{(III-23.B)}$$

From (III-22.B), then

$$\hat{\dot{\underline{x}}} = H\Omega \qquad \text{(II-24.B)}$$

from which

$$H = (A,B) = \hat{\dot{\underline{x}}}\ \Omega^{-1} \qquad \text{(III-25.B)}$$

In terms of application to human operator work, this approach suffers from the need for all states to be available. The matrix inversion procedure does offer a pseudo on-line approach to parameter identification. Results demonstrate that improved convergence rate can result, but the overall approach is not deemed directly amenable to iterative use.

4. Carroll [106]

A single-input, single output plant with unknown dynamics is assumed to be transformable into the form

$$\dot{\underline{x}} = (A_o + \Delta A_o)\underline{x} + B_1 r \qquad \text{(III-26.B)}$$

Only one output is available,

$$y = x_1 \qquad \text{(III-27.B)}$$

with

$$A_o = \begin{bmatrix} -a_{n-1} & 1 & 0 & \cdots \\ -a_{n-2} & 0 & 1 & \cdots \\ & & \ddots & 1 \\ -a_o & \cdots & \cdots & 0 \end{bmatrix} \qquad B_1 = \begin{bmatrix} b_{n-1}' \\ b_{n-2}' \\ \vdots \\ b_o \end{bmatrix} \qquad \text{(III-28.B)}$$

r = external plant input

ΔA_o is defined from an error equation

$$\dot{e} = K_o e + (G_1 C - \Delta A_o)\underline{x} + \Delta B\, r + H\underline{u} \qquad \text{(III-29.B)}$$

$$G = [g_{n-1} \quad g_{n-2} \cdots g_o]^T \qquad \text{(III-30.B)}$$

$$H = \text{diag}[0 \quad h_{n-2} \quad h_{n-3} \cdots h_o] \qquad \text{(III-31.B)}$$

$\underline{u}$ = generated set of signals

$$K_o = \begin{bmatrix} -k_{n-1} & 1 & 0 & \cdots & 0 \\ -k_{n-2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -k_o & 0 & 0 & \cdots & 0 \end{bmatrix} \qquad \text{(III-32.B)}$$

Since the a_i's are arbitrary, define auxiliary variables v_i as

$$\sum_{j=0}^{n-1} a_j\, v_i^{(j)} = x_1^{(i)} \qquad \text{(III-33.B)}$$

$$v_{n-1} = x_1 \qquad \text{(III-34.B)}$$

$$\sum_{j=0}^{n-1} a_j\, v_i^{(j)} = r^{(i-n)} \qquad \text{(III-35.B)}$$

$$v_{2n-1} = r \qquad \text{(III-36.B)}$$

A parameter error vector $\underline{\phi}$ is defined as

$$\phi_i = \begin{cases} \alpha_i - a_i\, \alpha_{n-1} & i=0,1,\cdots,n-2 \\ \alpha_{n-1} & i = n-1 \\ \beta_{i-n} & i=n,\ n+1,\ \cdots\ m+n \end{cases} \qquad \text{(III-37.B)}$$

Auxiliary inputs $\underline{u}$ are defined according to

$$f_x + f_r + \sum_{j=0}^{n-2} h_j\, u_j^{(j)} = 0 \qquad \text{(III-38.B)}$$

where f_x, f_r are functions of $x^{(i)}$, $\dot{\phi}_k$, $v_k^{(i)}$, $r^{(\ell)}$. Through state variable manipulation, the j_i's may be generated as tabulated in Table III-2. Using

TABLE III-2.

Notation for generation of v_i in normal form. In $v_i(j)$, j denotes the state variable. Example: for n = 4, v_1 is defined by

$$v_1 + a_2\ddot{v}_1 + a_1\dot{v}_1 + a_0 v_0 = \dot{x}_1$$

and generated by

$$\begin{bmatrix} \dot{v}_1(1) \\ \dot{v}_1(2) \\ \dot{v}_1(3) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} v_1(1) \\ v_1(2) \\ v_1(3) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -a_1 \end{bmatrix} x_1$$

n = 2, m = 0
$$u_0 = \dot{\phi}_2 v_2 + \dot{\phi}_0 v_0$$

n = 2, m = 1
$$u_0 = \dot{\phi}_0 v_0 + \dot{\phi}_2 v_2$$

n = 3, m = 0
$$\begin{aligned} u_0 &= \dot{\phi}_1 v_1(2) + a_1\dot{\phi}_1 v_1(1) + \dot{\phi}_0 v_0(2) + a_1\dot{\phi}_0 v_0(1) \\ &\quad + a_1\dot{\phi}_3 v_3(1) + \dot{\phi}_3 v_3(2) \\ &= \dot{\phi}_1(v_1(2) + a_1 v_1(1)) + \dot{\phi}_0(v_0(2) + a_1 v_0(1)) \\ &\quad + \dot{\phi}_3(v_3(2) + a_1 v_3(1)) \\ u_1 &= \dot{\phi}_0(v_0(1) + \dot{\phi}_1 v_1(1) + \dot{\phi}_3 v_3(1)) \end{aligned}$$

n = 3, m = 1 i
$$\begin{aligned} u_0 &= \dot{\phi}_0(v_0(2) + a_1 v_0(1)) + \dot{\phi}_1(v_1(2) + a_1 v_1(1)) \\ &\quad + \dot{\phi}_3(v_3(2) + a_1 v_3(1)) + \dot{\phi}_4(v_4(2) + a_1 v_4(1)) \end{aligned}$$

$$u_1 = \sum_{\substack{i=0 \\ i\neq 2}}^{4} \dot{\phi}_i v_i(1)$$

n = 3, m = 2
same as n=3 m=1

n = 4, m = 0
$$u_0 = \sum_{i=0}^{2} \sum_{j=1}^{3} \dot{\phi}_i a_j v_i(j) + \dot{\phi}_4 \sum_{j=1}^{3} a_j v_4(j)$$

$$u_1 = \sum_{i=0}^{2} \sum_{j=1}^{3} \dot{\phi}_i \, a_{j+1} v_1(j) + \dot{\phi}_4 \sum_{j=1}^{2} a_{j+1} v_4(j)$$

$$u_2 = \sum_{i=0}^{2} \dot{\phi}_i v_i(1) + \dot{\phi}_4 v_4(1)$$

n = 4, m = 1

$$u_0 = \sum_{\substack{i=0 \\ i \neq 3}}^{5} \sum_{j=1}^{3} \dot{\phi}_i a_j v_i(j)$$

$$u_1 = \sum_{\substack{i=0 \\ i \neq 3}}^{5} \sum_{j=1}^{2} \dot{\phi}_i \, a_{j+1} v_i(j)$$

$$u_2 = \sum_{\substack{i=0 \\ i \neq 3}}^{5} \dot{\phi}_i v_i(1)$$

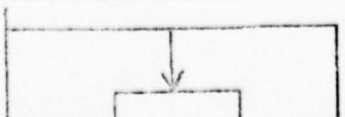
n = 4, m = 2

$$u_0 = \sum_{\substack{i=0 \\ i \neq 3}}^{6} \sum_{j=1}^{3} \dot{\phi}_i \, a_j v_i(j)$$

$$u_1 = \sum_{\substack{i=0 \\ i \neq 3}}^{6} \sum_{j=1}^{2} \dot{\phi}_i \, a_{j+1} v_i(j)$$

$$u_2 = \sum_{\substack{i=0 \\ i \neq 3}}^{6} \dot{\phi}_i v_i(1)$$

n = 4, m = 3 same as n=4 m=2



an appropriate Lyapunov function, $\dot{\phi}_i$ is defined as

$$\dot{\phi}_i = -\frac{m_s}{m_i} v_i e_1 , \quad e_1 = z_1 - x_1 \tag{III-39.B}$$

with an adaptive observer defined as

$$\dot{\underline{z}} = K_o \underline{z} + GC x_1 + D r + H\underline{u} \tag{III-40.B}$$

Adjusting elements of G and ΔB, the input-output description of the system is assured if r is sufficiently exciting.

5. Lüders [107]

This is a continuous-type response equation error-type formulation for SISO systems. It has been shown that for any arbitrary set of constant λ_i, $i = 2, 3, \cdots n$, $\lambda_i < 0$ that a SISO system can always be represented as

$$\dot{\underline{x}}_p = \begin{bmatrix} a_1 & 1 & 1 & 1 \cdots & 1 \\ a_2 & -\lambda_2 & & & \\ \vdots & & -\lambda_3 & & \\ a_n & & & & -\lambda_n \end{bmatrix} \underline{x}_p + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} u \tag{III-41.B}$$

where a_i and b_i are the 2n system parameters being sought. An observer is developed as

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \vdots \\ \dot{\hat{x}}_n \end{bmatrix} = \begin{bmatrix} \alpha_1 & 1 & 1 \cdots & 1 \\ \alpha_2 & -\lambda_2 & & \\ \vdots & & -\lambda_3 \cdots & \\ \alpha_n & & & -\lambda_n \end{bmatrix} \begin{bmatrix} x_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \vdots \\ \hat{x}_n \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} u + \begin{bmatrix} -\lambda_1 e \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix} \tag{III-42.B}$$

where $e = \hat{x}_1 - x_{p_1}$ (III-43.B)

$$\dot{v}_i + \lambda_i v_i = x_{p_1} \qquad i=2,3,\cdots,n \qquad \text{(III-44.B)}$$

$$\dot{s}_i + \lambda_i s_i = u$$

$$\omega_i = -(c_i v_i^2 + d_i s_i^2)e \qquad \text{(III-45.B)}$$

and c_i, d_i are arbitrary constants > 0. The parameters $\underline{\alpha}$ and $\underline{\beta}$ are defined as

$$\dot{\underline{\phi}} = \begin{bmatrix} -c_1 e_1 x_{p_1} \\ -c_2 e_1 \varphi^{-1}\left\{\frac{x_{p_1}}{s+\lambda_2}\right\} \\ -c_3 e_1 \varphi^{-1}\left\{\frac{x_{p_1}}{s+\lambda_3}\right\} \\ \vdots \\ -c_n e_1 \varphi^{-1}\left\{\frac{x_{p_1}}{s+\lambda_n}\right\} \\ -d_2 e_1 u \\ -d_2 e_1 \varphi^{-1}\left\{\frac{u}{s+\lambda_2}\right\} \\ -d_3 e_1 \varphi^{-1}\left\{\frac{u}{s+\lambda_3}\right\} \\ \vdots \\ -d_n e_1 \varphi^{-1}\left\{\frac{u}{s+\lambda_n}\right\} \end{bmatrix} \qquad \text{(III-46.B)}$$

with $\underline{\phi}^T = (\alpha_1, \alpha_2, \cdots \alpha_n, \beta_1, \beta_2, \cdots \beta_n)$. From this formulation, the plant transfer function becomes

$$\frac{y}{u}(s) = \frac{\beta_1 + \sum_{i=2}^{n} \frac{\beta_i}{s+\lambda_i}}{s - \alpha_1 - \sum_{i=2}^{n} \frac{\alpha_i}{s+\lambda_i}} \qquad \text{(III-47.B)}$$

The identifier configuration is shown in Figure III-1.

6. Kudva, Narendra [108]

This method involves continuous time parameter estimation using the Meyer-Kalman-Yacubovitch (MKY) Lemma. Use of this makes it possible to employ only input-output data with no differentiations required. The unknown plant is of the form

$$\dot{\underline{x}}_s = A_s \underline{x}_s + b_s u \qquad \text{(III-48.B)}$$

$$y = h_s^T \underline{x}_s \qquad \text{(III-49.B)}$$

where u is a scalar input, $h_s = [1 \ 0 \cdots 0]$, and A_s and b_s are unknown but of the form

$$A_s = \left[\begin{array}{c|c} -\underline{a} & \begin{array}{c} I \\ \hline 0 \end{array} \end{array}\right] \text{ , } b_s \text{ n-vector} \qquad \text{(III-50.B)}$$

It is required that the triple $\{A_s, b_s, h_s\}$ is controllable and observable.

A model observer is set up as

$$\dot{\underline{z}} = K\underline{z} + [\underline{k} - \hat{\underline{a}}(t)]x_1 + \hat{\underline{b}}(t)u + \underline{w} + \underline{r} \qquad \text{(III-51.B)}$$

$$\hat{y} = h^T \underline{z} = z_1 \qquad \text{(III-52.B)}$$

where

$$K = \left[\begin{array}{c|c} -\underline{k} & \begin{array}{c} I \\ \hline 0 \end{array} \end{array}\right] \qquad \text{(III-53.B)}$$

is a stable matrix. The error vectors are

$$\underline{\alpha} = \underline{a} - \hat{\underline{a}} \qquad \underline{\beta} = \underline{b} - \hat{\underline{b}}, \quad \underline{e} = \underline{z} - \underline{x} \qquad \text{(III-54.B)}$$

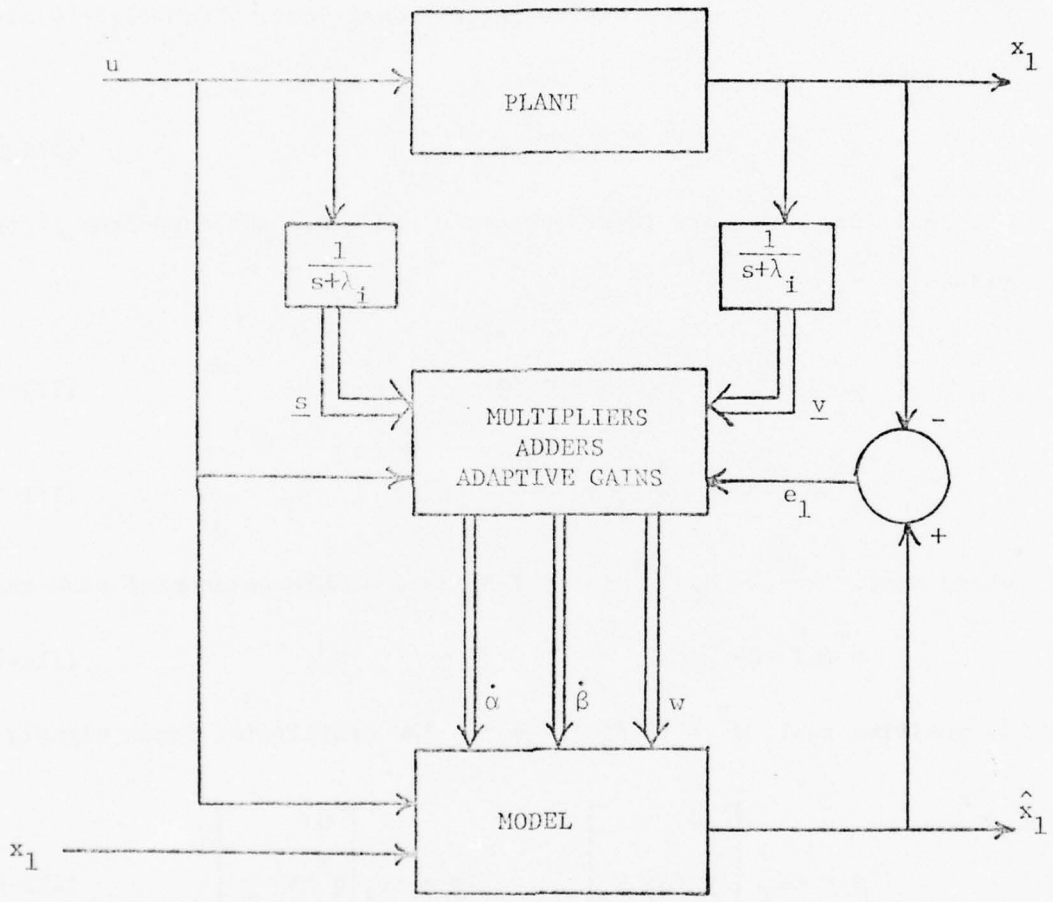


Figure III-1. Block Diagram of the Lüders, Narendra Adaptive Observer [107].

Using Lyapunov theory and the MKY Lemma, the parameter identification laws become

$$\dot{\underline{\alpha}} = -\Gamma e_1 \underline{v}$$
$$\dot{\underline{\beta}} = -\Lambda e_1 \underline{q} \qquad \text{(III-55.B)}$$

where Γ, Λ are diagonal positive-definite matrices. From (III-54.B) and (III-55.B),

$$\dot{\hat{\underline{a}}} = \Gamma e_1 \underline{v} \qquad \dot{\hat{\underline{b}}} = \Lambda e_1 \underline{q} \qquad \text{(III-56.B)}$$

The auxiliary state and input vectors $\underline{q}$ and $\underline{v}$ are obtained from filtered values of x_1 and u,

$$\underline{v}^T = \frac{[s^{n-1}, s^{n-2}, \cdots, 1]}{D(s)} \; x_1(s) \qquad \text{(III-57.B)}$$

$$\underline{q}^T = \frac{[s^{n-1}, s^{n-2}, \cdots, 1]}{D(s)} \; u(s) \qquad \text{(III-58.B)}$$

where $D(s) = s^{n-1} + d_2 s^{n-2} + \cdots + d_n$ is a stable polynomial such that

$$h^T(sI - K)^{-1}\underline{d} \qquad \text{(III-59.B)}$$

is positive real, $\underline{d}^T = (1 \; d_2 \; \cdots \; d_n)$. The stabilizing input signals

$$\underline{w} = -e_1 \begin{bmatrix} 0 \\ \underline{v}\,\Gamma A_2\,\underline{v} \\ \vdots \\ \underline{v}^T \Gamma A_n \,\underline{v} \end{bmatrix} \qquad \underline{r} = -e_1 \begin{bmatrix} 0 \\ \underline{q}^T \Lambda A_2\,\underline{q} \\ \vdots \\ \underline{q}^T \Lambda A_n \,\underline{q} \end{bmatrix} \qquad \text{(III-60.B)}$$

$$A_m = \begin{bmatrix} 0 & -d_m & -d_{m+1} & \cdots & -d_n & 0 & 0 & 0 \cdots & 0 \\ 0 & 0 & -d_m & \cdots & -d_{n-1} & -d_n & 0 & 0 \cdots & 0 \\ \vdots & & & & \vdots & & & & \\ 0 & 0 & 0 & -d_m & & \cdot & \cdot & \cdot & -d_n \\ 0 & 1 & d_2 & \cdots -d_{m-1} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & d_2 & -d_{m-2} & -d_{m-1} & 0 & \cdot \cdot \cdot & 0 \\ \vdots & & & & & & & & \\ 0 & 0 & \cdots & 1 & d_2 & \cdots & & & d_{m-1} \end{bmatrix} \qquad \text{(III-61.B)}$$

If the input is sufficiently rich (n distinct frequencies or more), then as $t \to \infty$, $\underline{\alpha} \to \underline{0}$, $\underline{\beta} \to \underline{0}$.

7. Hang [109]

Using the response error technique, an MRAS identifier can be developed which requires no signal differentiations for implementation for multi-input-single output systems. Given the system

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u}$$

$$y_p = C \underline{x}_p \qquad \text{(unknown) plant (III-62.B)}$$

$$\dot{\underline{x}}_m = A_m \underline{x}_m + B_m \underline{u}$$

$$y_m = C \underline{x}_m \qquad \text{(adjustable) model (III-63.B)}$$

$$\underline{e} = \underline{y}_p - \underline{y}_m \qquad \text{System Error (III-64.B)}$$

$$v_1 = D \underline{e} \qquad \text{Filtered Error (III-65.B)}$$

it is desired to construct A_m, B_m with only x_{p_1} and x_{m_1} needed (for realizability purposes). To determine D, A_m, and B_m a theorem using Popov's

Hyperstability Theory is needed.

Theorem

Sufficient and partially necessary conditions in order that the parameter adjustments are asymptotically hyperstable are

a) $Z_1(s)\ C\ (sI - A_p)^{-1}$ (III-66.B)

is strictly positive real, where $Z_1(s) = \sum_{i=0}^{\ell} z_i\ s^i$

b) the A_m and B_m terms are

$$\dot{a}_{m_j} = -[\alpha_j\ v_{f_1}\ y_{m_j} + \alpha_j\ \frac{d}{dt}(\alpha_j\ v_1\ y_{m_j})] \qquad \text{(III-67.B)}$$

$$\dot{b}_{m_j} = \beta_j\ v_{f_1}\ u_j + \delta_j\ \frac{d}{dt}(\beta_j\ v_1\ u_j)$$

$$\text{with } A_p = \begin{bmatrix} 0 & 1 & 0 \cdots \\ 0 & 0 & 1 \cdots \\ -a_o & -a_1 & \cdots -a_{n-1} \end{bmatrix} \qquad B_p = \begin{bmatrix} 0 & 0 & \cdots 0 \\ b_o & b_1 & \cdots b_{n-1} \end{bmatrix}$$

$$\text{and } \dot{u}_i = u_{i+1}, \quad i = 1, 2, \cdots, n-1$$

The adjustment gains α_j and $\beta_j > 0$, α_j and $\delta_j \geq 0$. The states y_{m_j} are obtained by passing y_{m_1} through a filter $\frac{1}{N_f(s)}$, an nth order state-variable filter and

$$e_{f_1} = \theta_{p_f} - x_{m_1}$$

$$\theta_{p_f} = \frac{1}{N_f(s)}\ x_{p_1} \qquad \text{(III-68.B)}$$

$$v_{f_1}(s) = z_1(s)\ e_{f_1}(s)$$

It occurs that $Z_1(s)/D_p(s)$ must be positive real and hence $(n-1) \leq \ell \leq (n+1)$. It should be noted that by using successive outputs of $\frac{1}{N_f(s)}$ it is possible to synthesize v_{f_1} without differentiating, as suggested by (III-68.B).

This approach is very similar in concept to the original adaptive *control* technique of [110], except that use of Hyperstability theory allows for one measurable output only. By using the stable response error approach (as opposed to a generalized equation error technique), asymptotically unbiased estimates result if x_{p_1} is corrupted by a zero-mean noise. The GEE results in biased estimates, although use of instrument variable techniques [5] can be combined to approximately remove the bias.

8. Mirsch [111]

This is a SISO method for discrete systems using an output error formulation. The plant structure is given by the ARMA process

$$x_p(k) = \sum_{i=1}^{n} a_i \, x_p(k-i) + \sum_{i=0}^{m} b_i \, u(k-i) \qquad \text{(III-69.B)}$$

where a_i, b_i are the unknown plant parameters, x_p is the output and u the input. The adjustable model is given by

$$x_m(k) = \sum_{i=1}^{n} \hat{a}_i(k) \, x_m(k-i) + \sum_{i=0}^{m} \hat{b}_i(k) \, u(k-i) \qquad \text{(III-70.B)}$$

A measure of the parameter fit is given by the tracking error

$$e(k) = x_m(k) - x_p(k) \qquad \text{(III-71.B)}$$

The plant parameter identifier equations can be developed using Popov's Hyperstability Theory to be

$$\hat{a}_i(k) = \hat{a}_i(k-1) - \alpha_i \, \nu(k) \, x_m(k-i) \qquad i = 1,2,\cdots,n \qquad \text{(III-72.B)}$$

$$\hat{b}_j(k) = \hat{b}_j(k-1) - \gamma_j \, \nu(k) \, u(k-j) \qquad j = 0, 1, \cdots, m \qquad \text{(III-73.B)}$$

where

$$v(k) = \frac{\sum_{i=1}^{n} \hat{a}_i(k-i)\, x_m(k-i) + \sum_{i=0}^{m} \hat{b}_i(k-i)\, u(k-i) - x_p(k) + \sum_{i=1}^{\ell} c_i\, e(k-i)}{\left[1 + \sum_{i=1}^{n} \alpha_i\, x_m^2(k-i) + \sum_{i=0}^{m} \gamma_i\, u^2(k-i)\right]}$$

α_i, γ_j weighting constants > 0. The design parameters c_i and p are selected to satisfy the hyperstability constraint that the transfer function

$$H(z) = \frac{1 + \sum_{i=1}^{p} c_i\, z^{-i}}{1 - \sum_{i=1}^{n} a_i\, z^{-i}} \qquad \text{(III-75.B)}$$

is SPR. Selection of the c_i are hindered as a_i are unknown. Design procedures to skirt this issue are presented later.

9. Lüders, Narendra Continuous [112]

This approach is a SISO approach for continuous-time systems. It is assumed the system to be identified is completely observable and may therefore be represented (for SISO) as

$$\begin{bmatrix} \dot{y} \\ \hline \dot{z} \end{bmatrix} = \left[\begin{array}{c|c} \underline{a} & \underline{r}^T \\ \hline & F \end{array}\right] \begin{bmatrix} y \\ \hline z \end{bmatrix} + \begin{bmatrix} \underline{b} \\ \hline \end{bmatrix} u \qquad \text{(III-76.B)}$$

where $\underline{a}$ and $\underline{b}$ are the 2n parameters to be identified, u the scalar input, y the scalar output, and (r^T, F) observable. The adaptive observer is

$$\dot{\hat{\underline{w}}} = F^T \hat{\underline{w}} + \underline{r}u \qquad \text{(III-77.B)}$$

$$\begin{bmatrix} \dot{\hat{y}} \\ \hline \dot{\hat{x}} \end{bmatrix} = \left[\begin{array}{c|c} \hat{\underline{a}}^T & \\ \hline \underline{r} & F^T \end{array}\right] \begin{bmatrix} y \\ \hline \hat{x} \end{bmatrix} + \begin{bmatrix} 1 \\ \hline 0 \end{bmatrix} (\hat{\underline{b}}^T \hat{\underline{\omega}} - \lambda_1 e_1) \qquad \text{(III-78.B)}$$

with

$$e_1 = \hat{y} - y \tag{III-79.B}$$

where

$$\begin{aligned}
\hat{\underline{\omega}}^T &= [\hat{\omega}_2 \quad \hat{\omega}_3 \cdots \quad \hat{\omega}_n] \\
\hat{\underline{a}}^T &= [\hat{a}_1 \quad \hat{a}_2 \cdots \quad \hat{a}_n] \\
\hat{\underline{b}}^T &= [\hat{b}_1 \quad \hat{b}_2 \cdots \quad \hat{b}_n] \\
\hat{\underline{\omega}}^T &= (u, \quad \hat{\underline{\omega}}^T)
\end{aligned} \tag{III-80.B}$$

and $\hat{\underline{a}}$ and $\hat{\underline{b}}$ are estimates of $\underline{a}$ and $\underline{b}$. F and $\underline{r}$ are designer selected. For the special case F = diagonal $(-\lambda_2, -\lambda_3, \cdots, -\lambda_n)$, λ_i distinct, $\underline{r}^T = [1, 1, \cdots, 1]$, then

$$\frac{y}{u} = \frac{b_1 + \sum_{i=2}^{n} \frac{b_i}{s+\lambda_i}}{s - a_1 - \sum_{i=2}^{n} \frac{a_i}{s+\lambda_i}} \tag{III-81.B}$$

By selection of an appropriate Lyapunov function, parameter identifier equations insuring convergence are

$$\begin{aligned}
\dot{\hat{a}}_1 &= -\gamma_1 x_1 e_1 \\
\dot{\hat{a}}_i &= -\gamma_1 \hat{x}_i e_1 \qquad i = 2,3,\cdots,n \\
\dot{\hat{b}}_1 &= -\delta_1 u\, e_1 \\
\dot{\hat{b}}_i &= -\delta_i \hat{\omega}_i e_1 \qquad i = 2,3,\cdots,n
\end{aligned} \tag{III-82.B, III-83.B}$$

where γ_i, $\delta_i > 0$ are constants selected by the designer for speed of convergence.

10. Kudva [113]

This is a MIMO approach for discrete-time identification. It employs the equation error formulation. The plant is described as

$$\underline{x}_p(k+1) = A(k)\ \underline{x}_p(k) + B(k)\ \underline{u}(k) \qquad \text{(III-84.B)}$$

The identification model used to identify A and B is

$$\underline{x}_m(k+1) = \hat{A}(k+1)\underline{x}_p(k) + \hat{B}(k+1)\underline{u}(k) \qquad \text{(III-85.B)}$$

Employing an appropriate Lyapunov function, closed-form expressions for the parameter estimates $\hat{A}$, $\hat{B}$ can be determined as

$$[\hat{A}(k+1) \vdots \hat{B}(k+1)] = [\hat{A}(k) \vdots \hat{B}(k)] - \frac{\alpha\ P\underline{e}(k)\ \underline{y}^T(k-1)}{\lambda_{max}\ \underline{y}^T(k-1)\ \underline{y}(k-1)} \qquad \text{(III-86.B)}$$

$$e(k) = x_m(k) - x_p(k) \qquad \text{(III-87.B)}$$

P is an arbitrary symmetric, positive-definite weighting matrix,

$$\lambda_{max} = \sup_i\{\lambda_i:\ |\lambda I - P| = 0\} \qquad \text{(III-88.B)}$$

$$\underline{y}^T(k) = [\underline{x}_p^T(k) \vdots \underline{u}^T(k)] \qquad \text{(III-89.B)}$$

Because of the MIMO formulation, plus (III-89.B), it is clear that all states must be available for measurement. This is a severe constraint in practice and is one reason SISO identifiers are more regularly investigated: they are easier to justify from an implementation viewpoint.

11. Kudva, Narendra [114]

This is a MIMO identifier of the equation error form. This method requires all states to be available for measurement. The plant is given by (III-84.B) and it is assumed all plant states may be measured. The reference model is given by

$$\underline{x}_m(k+1) = \hat{A}(k+1)\underline{x}(k) + \hat{B}(k+1)\underline{u}(k) + C\ \underline{e}(k) \qquad \text{(III-90.B)}$$

with

$$\underline{e}(k) = \underline{x}_m(k) - \underline{x}_p(k) \qquad \text{(III-91.B)}$$

C an arbitrary constant matrix with $|\lambda_i| < 1$ for $|\lambda I - C| = 0$. Using a Lyapunov function similar to that for Method 10, the parameter identification terms $\hat{A}$, $\hat{B}$ can be given by

$$[\hat{A}(k+1)\ \hat{B}(k+1)] = [\hat{A}(k)\ \hat{B}(k)] - \frac{\alpha\ P\left[\underline{e}(k) - C\,\underline{e}(k-1)\right]\underline{y}^T(k-1)}{\lambda_{max}\ \underline{y}^T(k-1)\ \underline{y}(k-1)} \qquad \text{(III-92.B)}$$

$0 < \alpha < 2$ and P, λ_{max} as before. It is clear that (III-92.B) includes autoregressive terms not present in (III-86.B), so as to weight previous errors and therefore aid in convergence of $\hat{A}$, $\hat{B}$, and $\underline{e}$.

12. Kudva [114,115]

This is an equation error formulation for discrete SISO systems and requires only a scalar I/O measurement sequence. An nth order SISO plant {y,u} is parameterized such that

$$\frac{y}{u}(z) = \frac{b_1 + \underline{\bar{b}}^T(zI - F)^{-1}\ \underline{\bar{r}}}{z - \underline{\bar{a}}^T(zI - F)^{-1}\ \underline{\bar{r}} - a_1} \qquad \text{(III-93.B)}$$

where u and y are the system input and output, $(\underline{\bar{r}}^T, F)$ is any completely observable (n-1) x (n-1) pair, and b_1, $\underline{\bar{b}}^T$, a_1, $\underline{\bar{a}}^T$ are the plant parameters to be identified. To estimate the parameters, an observer is established as

$$\hat{x}_{m_1}(k+1) = \hat{a}_1(k+1)\ x_{p_1}(k) + \underline{\hat{\bar{a}}}^T(k+1)\ \underline{\bar{x}}_m(k)$$

$$+ \hat{b}_1(k+1)\ u(k) + \underline{\hat{\bar{b}}}^T(k+1)\ \underline{\hat{\bar{f}}}(k) + \lambda_1[\hat{x}_{m_1}(k) - x_{p_1}(k)] \qquad \text{(III-94.B)}$$

$$\underline{\bar{x}}_m(k+1) = F\,\underline{\bar{x}}_m(k) + \bar{r}\,x_{p_1}(k) \qquad \text{(III-95.B)}$$

$$\underline{\hat{\bar{f}}}(k+1) = F\,\underline{\hat{\bar{f}}}(k) + \bar{r}\,u(k) \qquad \text{(III-96.B)}$$

$$|\lambda_1| < 1, \text{ constant}$$

Defining the 2n parameter vector $\underline{\phi}$ as

$$\underline{\phi}^T = [a_1, \underline{\bar{a}}^T, b_1, \underline{\bar{b}}^T] \qquad \text{(III-97.B)}$$

and the adjustment erroe $e_1(k)$ as

$$e_1(k) = x_{m_1}(k) - y(k)$$

then the parameter update equation can be developed using a Lyapunov function as

$$\underline{\hat{\phi}}(k+1) = \underline{\hat{\phi}}(k) - \frac{\alpha\, P[e_1(k) - \lambda_1 e_1(k-1)]\,\underline{y}(k-1)}{\lambda_{max}\,\underline{y}^T(k-1)\,\underline{y}(k-1)} \qquad \text{(III-98.B)}$$

where α, P, λ_{max}, $\underline{y}$ are defined previously. The observer structure is shown in Figure III-2.

13. Landau [116]

This method is a discrete SISO approach employing the output error (or "parallel") MRAS concept. Stability is insured using Popov's Hyperstability Theory. The plant is given by (III-69.B), with a general stacked parameter vector $\underline{\phi}$ given by

$$\underline{\phi}^T = [a_1 \;\; a_2 \;\cdots\; a_n \;\; b_o \;\; b_1 \;\cdots\; b_m] \qquad \text{(III-99.B)}$$

and a generalized input/output vector by $\underline{q}^T(k-1) = [x_p(k-2) \cdots x_p(k-n)\; u(k-1) \ldots u(k-m)]$ so that (III-69.B) may also be expressed as

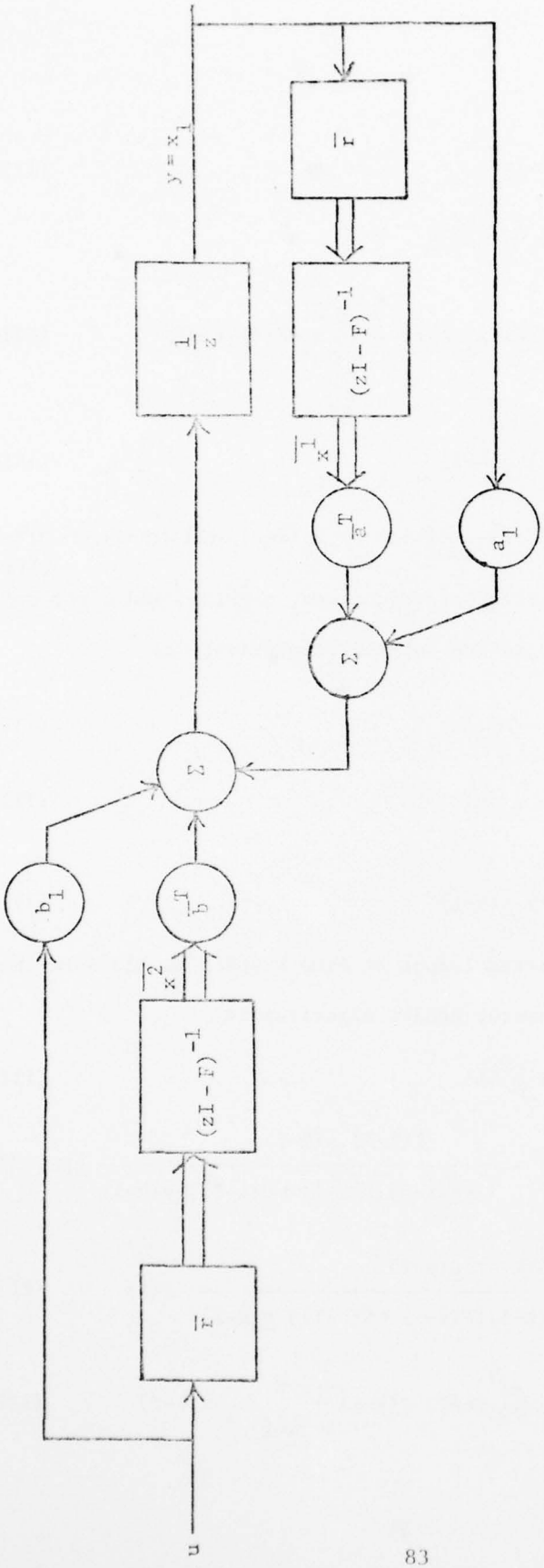


Figure III-2. Non-minimal realization for a single-input single-output plant.

$$x_p(k) = \underline{\phi}^T \underline{q}(k-1) \qquad \text{(III-100.B)}$$

The MRAS adjustable ARMA model is

$$x_m(k) = \sum_{i=1}^{n} \hat{a}_i(k-1) x_m(k-i) + \sum_{i=0}^{m} \hat{b}_i(k-i) u(k-i) \qquad \text{(III-101(a).B)}$$

or

$$x_m(k) = \hat{\underline{\phi}}^T(k-1)\ \underline{s}(k-1) \qquad \text{(III-101(b).B)}$$

$$\underline{s}^T(k-1) = [x_m(k-1)\quad x_m(k-2) \cdots x_m(k-n)\ u(k)\ u(k-1) \cdots u(k-m)] \qquad \text{(III-101(c).B)}$$

To determine the parameter update equations, *a priori* and *a posteriori* plant-model tracking errors are defined respectively as

$$e^\circ(k) = x_p(k) - x^\circ_m(k) \qquad \text{(III-103.B)}$$

$$e(k) = x_p(k) - x_m(k) \qquad \text{(III-104.B)}$$

where

$$x^\circ_m(k) = \hat{\underline{\phi}}^T(k-1)\ \underline{s}(k-1) \qquad \text{(III-105.B)}$$

which is the model estimated output at time k based on old data through instant (k-1). The parameter update algorithm is

$$\hat{\underline{\phi}}(k) = \hat{\underline{\phi}}^I(k) + \hat{\underline{\phi}}^P(k) \qquad \text{(III-106.B)}$$

$$\hat{\underline{\phi}}^I(k) = \hat{\underline{\phi}}(k-1) + \frac{F(k-1)\ \underline{s}(k-1)}{1 + \underline{s}^T(k-1)[F(k-1) + G(k-1)]\ \underline{s}(k-1)}\ \varepsilon(k) \qquad \text{(III-107.B)}$$

$$\hat{\underline{\phi}}^P(k) = \frac{G(k-1)\ \underline{s}(k-1)}{1 + \underline{s}^T(k-1)[F(k-1) + G(k-1)]\ \underline{s}(k-1)}\ \varepsilon(k) \qquad \text{(III-108.B)}$$

$$\varepsilon(k) = x_p(k) - \hat{\underline{\phi}}^{I^T}(k-1)\ \underline{s}(k-1) + \sum_{i=1}^{n} c_i\ e(k-i) \qquad \text{(III-109.B)}$$

$$F(k) = F(k-1) - \frac{F(k-1)\ \underline{s}(k-1)\ \underline{s}^T(k-1)\ F(k-1)}{1+\underline{s}^T(k-1)\ F(k-1)\ \underline{s}(k-1)} \qquad \text{(III-110.B)}$$

$F(o)$ p.d.

$$G(k) \geq -\frac{1}{2} F(k) \qquad \text{(III-111.B)}$$

and the *a posteriori* weighting coefficients c_i satisfy the constraint that

$$H(z) = \frac{1 + \sum_{i=1}^{n} c_i\ z^{-i}}{1 - \sum_{i=1}^{n} a_i\ z^{-i}} - \frac{1}{2} \qquad \text{(III-112.B)}$$

where z is the usual z-transform operator, is strictly positive real [117]. This last constraint raises physical realizability questions. Because the c_i are designer-selected values to determine $\hat{a}_i$ because a_i are unknown, then this last condition requires the answer (a_i) to yield c_i, to yield $\hat{a}_i = a_i$. This problem is addressed in a later chapter with regard to design-controlled c_i selection techniques.

The special case $G(k) \triangleq 0$ yields the result analogous to least-squares type algorithms,

$$\underline{\hat{\phi}}(k) = \underline{\hat{\phi}}(k-1) + \frac{F(k-1)\ \underline{s}(k-1)}{1+\underline{s}^T(k-1)\ F(k-1)\ \underline{s}(k-1)}\ \varepsilon(k) \qquad \text{(III-113.B)}$$

where $\varepsilon(k)$ now simplifies to

$$\varepsilon(k) = x_p(k) - x_m^{\circ}(k) + \sum_{i=1}^{n} c_i\ e(k-i) \qquad \text{(III-114.B)}$$

14. Valavani [118,119]

This method is a MIMO, equation error formulation of a multivariable identifier. All states are assumed available for measurement. The plant

is described by

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u} \qquad \text{(III-115.B)}$$

where $\underline{x}_p$ is an n-vector of measurable states, $\underline{u}$ is an r-vector of the measurable inputs, and A_p and B_p are the unknown plant parameter matrices. The MRAS model is of the equation error formulation,

$$\dot{\underline{x}}_m = A_m(t)\underline{x}_p + B_m(t)\,\underline{u} + K\underline{e} \qquad \text{(III-116.B)}$$

where

A_m, B_m are the adjustable matrices, K is an n x n stable matrix, and $\underline{e}$ is given by

$$\underline{e} = \underline{x}_m - \underline{x}_p \qquad \text{(III-117.B)}$$

Using an appropriate Lyapunov function, model identifier terms which insure asymptotic stability are

$$\dot{A}_m = - A^{-1} Q\underline{e}\,\underline{x}_p^T \qquad \text{(III-118.B)}$$

$$\dot{B}_m = -\Gamma^{-1} Q\underline{e}\,\underline{u}^T \qquad \text{(III-119.B)}$$

with A, Γ are arbitrary n x n p.d., symmetric weighting matrices, and Q is an n x n p.d., symmetric constant matrix satisfying

$$K^T Q + QK = - C \qquad \text{(III-120.B)}$$

where $C = C^T > 0$, constant.

15. Carroll [120]

This method is a SISO approach utilizing a generalized equation error formulation. The plant is of the form

$$\dot{\underline{x}}_p = A_p \underline{x}_p + \underline{b}\,u$$

$$y = C\,\underline{x}_p = x_{p_1} \qquad \text{(III-121.B)}$$

where the 2n parameters $\underline{a}$ and $\underline{b}$ are defined as

$$\frac{y}{u}(s) = \frac{\sum_{i=1}^{n} b_i s^{n-i}}{s^n + \sum_{i=1}^{n} a_i s^{n-i}} \qquad \text{(III-122.B)}$$

Based on a generalization of the method in [102], 2n independent error equations are defined as

$$\underline{\varepsilon} = W(t)\ \underline{\phi} + \underline{\omega}_o \qquad \text{(III-123.B)}$$

where

$$\underline{\phi}^T = [\alpha_1 \quad \alpha_2 \cdots \alpha_n \quad \beta_1 \quad \beta_2 \cdots \beta_n] \qquad \text{(III-124.B)}$$

is the model parameter vector with which the a_i and b_i will be determined,

$$\varepsilon^T = [\varepsilon_1 \quad \varepsilon_2 \cdots \varepsilon_{2n}] \qquad \text{(III-125.B)}$$

is a *generalized equation error* used in the identification phase ($\underline{\varepsilon} \to \underline{0}$ as $t \to \infty$), and

$$W(t) = [\underline{w}_1 \quad \underline{w}_2 \cdots \underline{w}_n] \qquad \text{(III-126.B)}$$

$$\underline{w}_i^T = [v_{n-1+i} \quad v_{n-2+i} \cdots v_i \quad q_{n-1+i} \quad q_{n-2+i} \cdots q_i] \qquad \text{(III-127.B)}$$
$$i = 1,2,\cdots 3n$$

$$\underline{w}_o^T = [v_{n+1} \quad v_{n+2} \cdots v_{3n}] \qquad \text{(III-128.B)}$$

Equation (III-123.B) defines the model for this case, with α_i, β_i the parameters to be identified, analogous to a_i and b_i. The signals v_i, q_i are filtered versions of the input u and output $y(= x_{p_1})$ of the plant,

$$\dot{\underline{v}} = \Lambda\ \underline{v} + \underline{\sigma}\ y \qquad \text{(III-129.B)}$$

$$\dot{\underline{q}} = \Lambda\ \underline{q} + \underline{\sigma}\ u \qquad \text{(III-130.B)}$$

$$v_{3n} = \dot{v}_{3n-1} \qquad \text{(III-131.B)}$$

where A is $(3n-1) \times (3n-1)$ such that

$$A = \begin{bmatrix} 0 & 1 & 0 \cdots & 0 \\ 0 & 0 & 1 \cdots & 0 \\ \vdots & & & \\ \vdots & \underline{\lambda}^T & & \end{bmatrix} \qquad \underline{\sigma}^T = [0 \;\; 0 \cdots 0 \;\; 1] \quad \text{(III-132.B)}$$

$$\underline{\lambda}^T = [\lambda_m \quad \lambda_{m-1} \cdots \lambda_1] \quad \text{(III-133.B)}$$

where $(A, \underline{\lambda})$ is controllable and A is stable. The adaptation terms are

$$\dot{\underline{\phi}} = -G\, W(t)\, \underline{\varepsilon} \quad \text{(III-134.B)}$$

where G is a p.d. gain matrix. From (III-124.B) and (III-134.B), the plant becomes

$$\frac{y}{u}(s) = \frac{-\sum_{i=1}^{n} \beta_i s^{n-i}}{s^n + \sum_{i=1}^{n} \alpha_i s^{n-i}} \quad \text{(III-135.B)}$$

16. Hang [121]

This is a SISO method utilizing the output error formulation. Stability and asymptotically unbiased parameter estimates are assured through the use of Popov's Hyperstability Theory. The plant to be identified is of the form in (III-122.B), and it is assumed that $y = x_{p_1}$ and u are the only available plant signals. Assuming a phase variable form,

$$\underline{x}_p^T = [x_{p_1} \quad \dot{x}_{p_1} \cdots x_{p_1}^{(n-1)}] \qquad \underline{u}^T = [u \quad \dot{u} \cdots u^{(n-1)}] \quad \text{(III-136.B)}$$

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u}$$

$$y = C \underline{x}_p \quad \text{(III-137.B)}$$

$$A_p = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ -a_1 & -a_2 & \cdots & & -a_n \end{bmatrix} \qquad B_p = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \vdots & & \\ b_1 & b_2 & \cdots\ b_n \end{bmatrix} \tag{III-138.B}$$

$$C = [1 \quad 0 \cdots 0] \tag{III-139.B}$$

As is done in others work [122], state-variable filters are utilized to generate approximate derivatives of x_{p_1} and u, so that

$$S_f = \frac{x_{p_{f_1}}}{x_{p_1}} = \frac{u_{f_1}}{u_f} = \frac{d_o}{s^n + \sum_{i=1}^{n} d_{n-i}\, s^{n-i}} \tag{III-140.B}$$

from which derivatives of $x_{p_{f_1}}$ can be picked off, as shown in Figure III-3. The d_i are selected so that the passband of (III-140.B) is much greater than frequency ranges of x_{p_1} of interest. More will be said about this later in Chapter 6.

Using S_f, a modified plant formulation results

$$\dot{\underline{x}}_{p_f} = A_p \underline{x}_{p_f} + B_p \underline{u}_f$$

$$y_{p_f} = C \underline{x}_{p_f} \tag{III-141.B}$$

$$\dot{\underline{x}}_{m_f} = A_m(t) \underline{x}_{m_f} + B_m(t) \underline{u}_f$$

$$y_{m_f} = C \underline{x}_{m_f} \tag{III-142.B}$$

The tracking error is

$$e = y_{m_f} - y_{p_f} \tag{III-143.B}$$

To insure stability, define

$$v_{f_1}(s) = Z_1(s)\, e(s) \tag{III-144.B}$$

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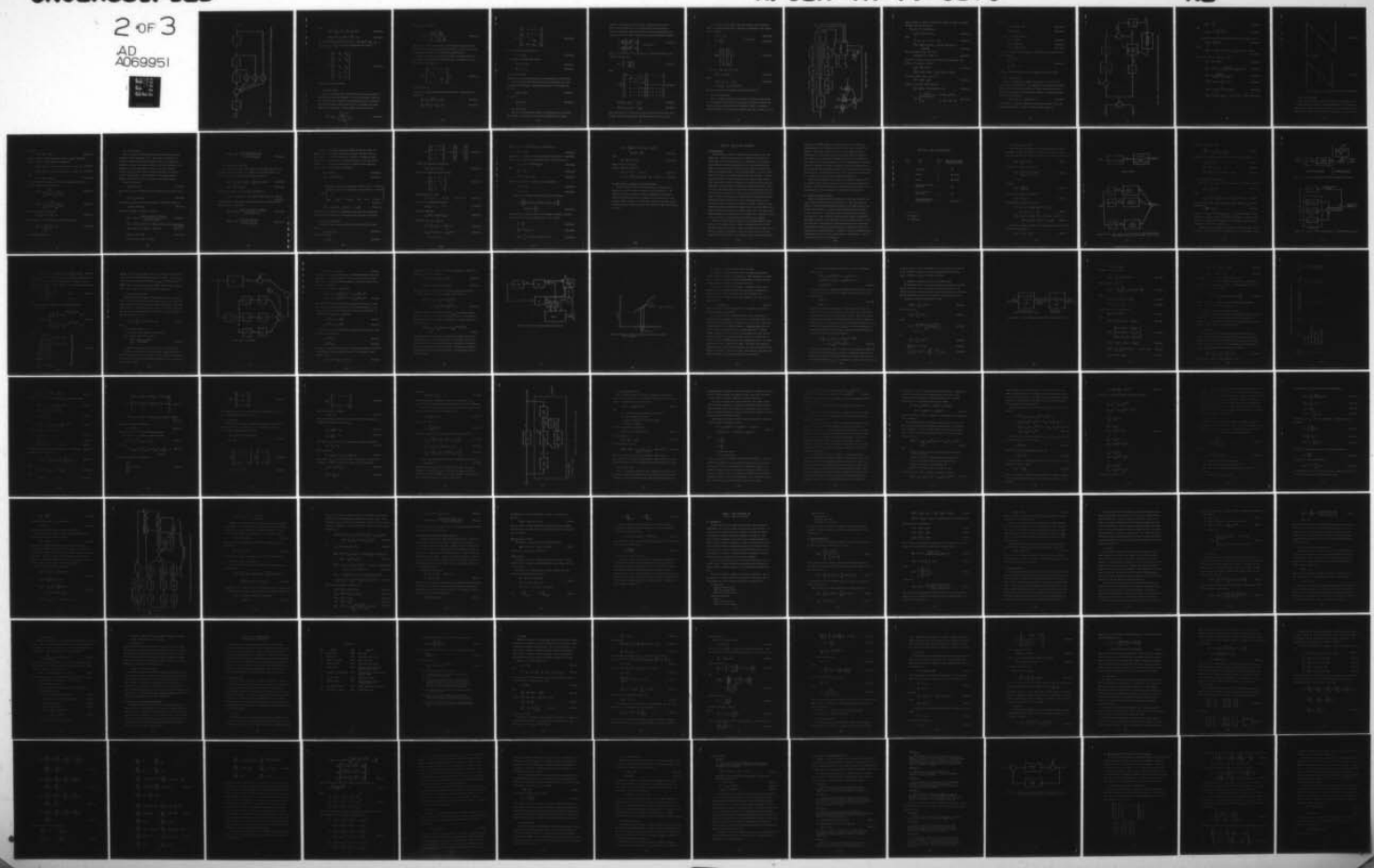


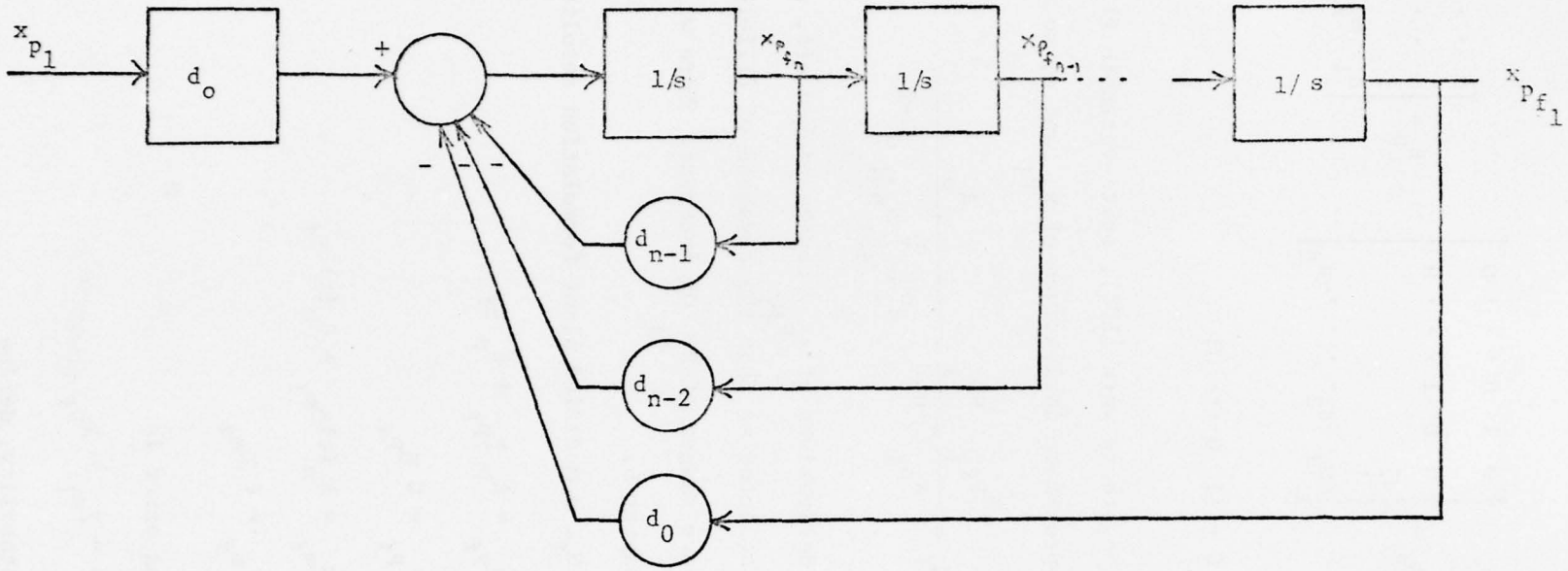


Figure III-3. Phase Variable Cananical Form for One Form of State Variable Filter (SVF) Implementation.

where

$$Z_1(s) = \sum_{i=0}^{\ell} z_i s^i \; ; \; (n-1) \leq \ell \leq (n+1) \qquad \text{(III-145.B)}$$

$$\mathrm{Re}\{Z_1(s)\, C(sI - A_p)^{-1} G\} > 0 \quad \forall \quad s = j\omega \qquad \text{(III-146.B)}$$

is strictly positive real, $G = [0\ 0\ \cdots\ 1]$. Defining $\underline{\phi}^T = [a_{m_1}\ a_{m_2}\ \cdots\ a_{m_n}\ b_{m_1}\ b_{m_2}\ \cdots\ b_{m_n}]$, the parameter adjustment rule becomes

$$\dot{\underline{\phi}} = \begin{bmatrix} -\alpha_1 & v_{f_1} & x_{m_{f_1}} \\ -\alpha_2 & v_{f_1} & x_{m_{f_2}} \\ \vdots & & \\ -\alpha_n & v_{f_1} & x_{m_{f_n}} \\ \beta_1 & v_{f_1} & u_{f_1} \\ \beta_n & v_{f_1} & u_{f_n} \end{bmatrix} \qquad \text{(III-147.B)}$$

where α_i, $\beta_i > 0$ are constants.

17. Molnar [123]

This is a continuous time, SISO identifier based on the results of Lion (Method 1) and Lüders (Method 5). Its main improvements appear to be 1) simpler state variable filter implementation, and 2) elimination of auxiliary feedback signals (which $\to 0$ as $t \to \infty$) plaguing many linear observers. The plant has a linear transfer function of the form

$$\frac{x_p}{u} = G_p(s) = \frac{\sum_{i=1}^{n} b_i s^{i-1}}{s^n + \sum_{i=1}^{n} a_i s^{i-1}} \qquad \text{(III-148.B)}$$

with a model of the form

$$G_m(s) = \frac{\hat{\beta}_n + \sum_{i=1}^{n-1} \frac{\hat{\beta}_i}{s+\lambda_i}}{s + \hat{\alpha}_n + {}_n + \sum_{i=1}^{n-1} \frac{\hat{\alpha}_i}{s+\lambda_i}} \qquad \text{(III-149.B)}$$

where $\lambda_i > 0$, constant, and are known by the designer, and α_i, β_i are the adaptation parameters. The transformation from $\alpha_i \to a_i$ and $\beta_i \to b_i$ is found by equating like powers of s in (III-148.B) and (III-149.B).

State variable filters are employed to generate "pseudo-states" and "pseudo-inputs", based on the scalar input-output measurements u and y_p. The pseudo-input generator is

$$\dot{\hat{\underline{\omega}}} = \begin{bmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & -\lambda_2 & 1 & \cdots & 0 \\ & 0 & \ddots & & \\ & & & -\lambda_{n-1} & 1 \\ 0 & & & 0 & 0 \end{bmatrix} \underline{r} \qquad \text{(III-150.B)}$$

$$\underline{r}^T = [\hat{\omega}_1 \quad \hat{\omega}_2 \cdots u]$$

with $\hat{\omega}_1, \hat{\omega}_2, \cdots \hat{\omega}_{n-1}$ the additional "input" signals. The pseudo-state generator is

$$\dot{\hat{\underline{v}}} = \hat{M}\, \underline{x}_m + h[\hat{\underline{\beta}}^T \underline{r} - \lambda_n \hat{v}_n] \qquad \text{(III-151.B)}$$

$$\underline{x}_m^T = [\hat{v}_1 \quad \hat{v}_2 \cdots \hat{v}_{n-1} \quad x_p] \qquad \text{(III-152.B)}$$

$$\hat{M} = \begin{bmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & -\lambda_2 & 1 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & -\lambda_{n-1} & 1 \\ -\hat{\alpha}_1 & -\hat{\alpha}_2 & \cdots & -\hat{\alpha}_{n-1} & -\hat{\alpha}_n \end{bmatrix} \qquad \text{(III-153.B)}$$

With the tracking error defined as

$$e = x_p - \hat{v}_n \qquad \text{(III-154.B)}$$

the parameter adjustment terms become

$$\dot{\hat{\alpha}}_i = -\alpha_i \, e \, x_{m_i} \qquad \text{(III-155.B)}$$

$$\dot{\hat{\beta}}_i = \gamma_i \, e \, r_i \qquad \text{(III-156.B)}$$

$\alpha_i, \gamma_i > 0$, constant.

Although developed for SISO systems, it can be extended to the MIMO case.

Using (III-155.B) and (III-156.B), $\hat{\alpha}_i$, $\hat{\beta}_i$ are developed, which then form $G_m(s)$ in (III-149.B). Using Lyapunov methods, it is assured that, as $t \to \infty$,

$$G_m(s) \to G_p(s) \qquad \text{(III-157.B)}$$

so at "$t = \infty$"

$$\frac{x_p}{u} \triangleq G_m(s) \qquad \text{(III-158.B)}$$

18. Kraft [124]

This is a SISO generalized equation error formulation for continuous-time systems. Given a plant of the form in (III-148.B), this method

identifies the parameter vectors $\underline{a}$ and $\underline{b}$. Using filtered versions of y and u (as in Method 16), a set of generalized states and equation errors are formulated, which are then used to develop an arbitrarly fast convergence rate identifier. Define filtered states and filtered inputs by

$$y_i = \mathcal{L}^{-1}\left\{\left(\frac{d}{s+d}\right)^{i-1} y(s)\right\} \qquad i=2,3,\cdots,3n \qquad \text{(III-159.B)}$$

$$r_i = \mathcal{L}^{-1}\left\{\left(\frac{d}{s+d}\right)^{i-1} u(s)\right\}$$

where d is a constant, $d > 0$, $y_1 = y$, and $r_1 = u$. Transformed states $\underline{z}_i$ are related to y_i and r_i by

$$\underline{z}_i = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} \underline{y}_i \\ \underline{r}_i \end{bmatrix} \qquad \text{(III-161.B)}$$

where

$$T = \frac{1}{d^n} \begin{bmatrix} d^{n-1} & 0 & \cdots & 0 & 0 & \\ 0 & d^{n-2} & 0 \cdots & 0 & 0 & \\ 0 & 0 & d^{n-3} & 0 & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & \cdots & d^2 & 0 & 0 \\ 0 & 0 & \cdots & 0 & d & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & \cdots & \cdots & & 1 \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 \cdots & 1 & -3 & & 3 & -1 \\ 0 \cdots & 0 & 1 & & -2 & 1 \\ 0 \cdots & 0 & 0 & & 1 & -1 \\ 0 \cdots & 0 & 0 & & 0 & 1 \end{bmatrix} \qquad \text{(III-162.B)}$$

$$\underline{y}_i^T = [y_{i+1} \quad y_{i+2} \cdots y_{i+n}] \qquad \text{(III-163.B)}$$

$$\underline{r}_i^T = [r_{i+1} \quad r_{i+2} \cdots r_{i+n}] \qquad \text{(III-164.B)}$$

The second matrix of T is such that each row consists of the binomial coefficients, with alternating signs, starting with unity in the t_{ii} entry.

Using the two sets of (3n-1) first-order filters, (3n-1) additional states and filtered inputs occur. From this, 2n independent error signals can be generated

$$E_i = y_i - \hat{y}_i \qquad \text{(III-165.B)}$$

$$\hat{y}_i = \underline{z}_i^T \hat{\underline{\phi}} \qquad i = 1,2,\cdots,2n \qquad \text{(III-166.B)}$$

or

$$\underline{E}(t) = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_{2n} \end{bmatrix} - \begin{bmatrix} \underline{z}_1^T \\ \underline{z}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \underline{z}_{2n}^T \end{bmatrix} \hat{\underline{\phi}} \qquad \text{(III-167.B)}$$

The parameter adaptation law is then

$$\dot{\hat{\underline{\phi}}}(t) = G\, Z(t)\, \underline{E}(t) \qquad \text{(III-168.B)}$$

where

$$Z(t) = [\underline{z}_1 \;\; \underline{z}_2 \cdots \underline{z}_{2n}] \qquad \text{(III-169.B)}$$

$$G = [g_{ii}], \quad g_{ii} > 0, \text{ constant}$$

The implementation of this method is shown in Figure III-4.

19. Akashi [125,126]

This is a SISO, equation error formulation for discrete time systems. It is a practical implementation using Lyapunov methods which insures that the parameter error bound can be made arbitrarily small. It inherently allows for output noise $\omega(k)$ as long as upper bounds are available as to its

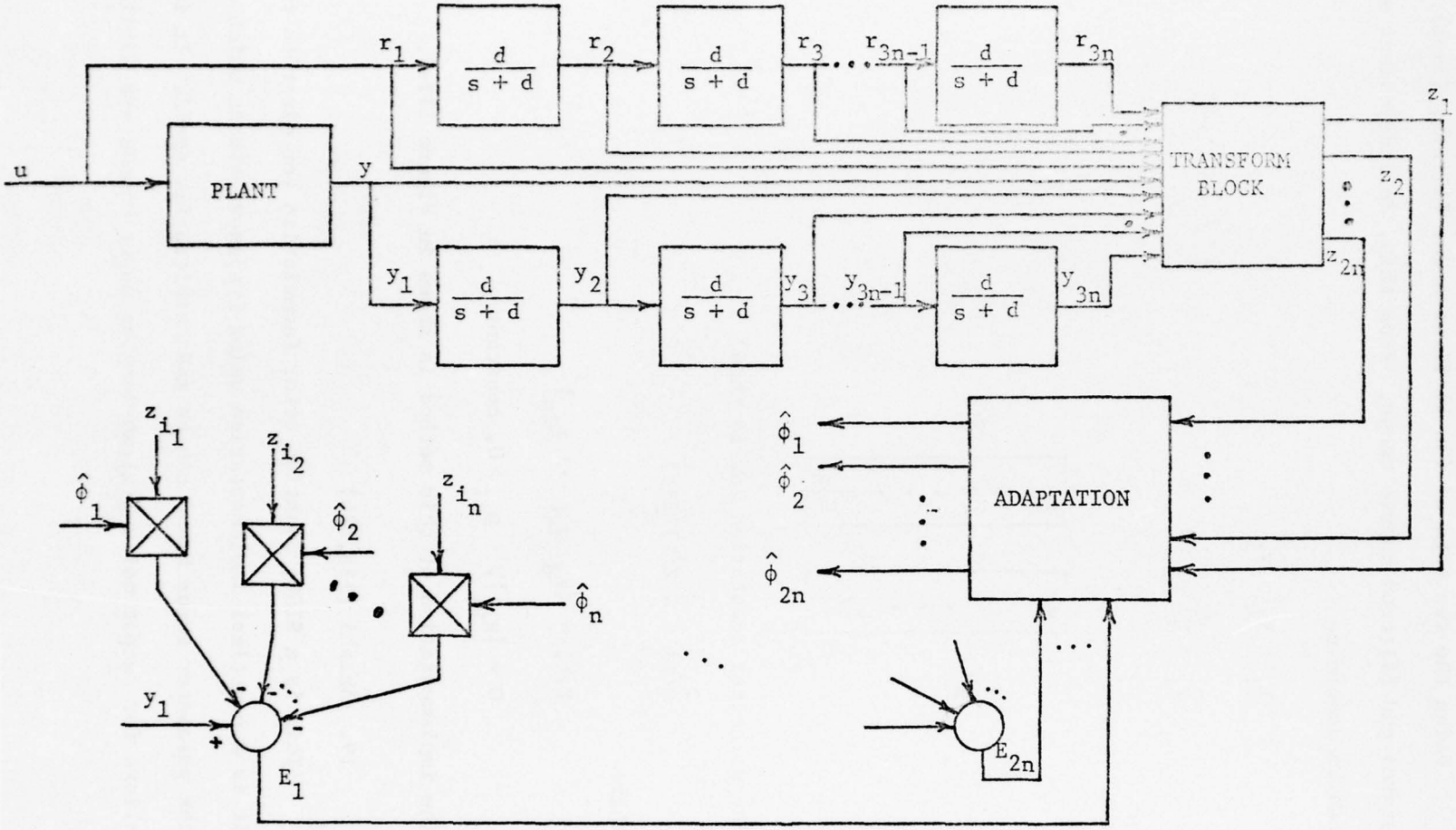


Figure III-4. Kraft Identifier Structure.

noise spectrum. As would be expected, the smaller the upper noise bound, the better the convergence rate.

The plant is of the ARMA form

$$x_p(k) = \underline{\phi}^T \underline{Z}(k-1) + \omega(k) \tag{III-170.B}$$

where

$$\underline{\phi}^T = [a_1 \quad a_2 \cdots a_n \quad b_1 \cdots b_m] \tag{III-171.B}$$

$$\underline{Z}^T(k) = [x_p(k) \; x_p(k-1) \cdots x_p(k-n+1) \; u(k) \; u(k-1) \cdots u(k-m+1)] \tag{III-172.B}$$

and ω is a Gaussian white noise with,

$$E\{\omega(k)/\omega(k-1), \cdots \omega(0)\} = 0 \tag{III-173.B}$$

where σ^2 is a designer selected upper bound on the measurement noise variance. The model is taken as

$$x_m(k) = \hat{\underline{\phi}}^T(k) \; Z(k-1) \tag{III-174.B}$$

$$\hat{\underline{\phi}}^T(k) = [\hat{a}_1(k) \quad \hat{a}_2(k) \cdots \hat{a}_n(k) \quad \hat{b}_1(k) \cdots \hat{b}_m(k)]$$

The scalar tracking error is given by

$$e(k) = x_m(k) - x_p(k) \tag{III-175.B}$$

yielding the parameter update equation

$$\hat{\underline{\phi}}(k) = \hat{\underline{\phi}}(k-1) - \varepsilon(k-1) \; \underline{Z}(k-2) \; e(k-1) \tag{III-176.B}$$

$$\varepsilon(k) = \begin{cases} \dfrac{\alpha \, \Lambda(k)}{\lambda(k-1)[\Lambda(k) + \sigma^2]} & \text{if } e^2(k) \geq 2M^2 + \delta M \\ \ell(k) & \text{if } e^2(k) < 2M^2 + \delta M \end{cases} \tag{III-177.B}$$

where

$$\Lambda(k) = .5^2 e(k) - M^2 \qquad \text{(III-178.B)}$$

$$\max_k |\omega(k)| \leq M \qquad \text{(III-179.B)}$$

M a designer-selected constant,

$$\lambda(k) = \text{tr } Q(k) \qquad \text{(III-180.B)}$$

$$Q(k) = \Omega^T(k)\ \Omega(k) \qquad \text{(III-181.B)}$$

$$\Omega(k) = [\underline{Z}(k)\ \underline{Z}(k-1)] \qquad \text{(III-182.B)}$$

and $\delta > 0$ a designer selected constant. The term $\ell(k)$ is an arbitrary function satisfying

$$\sum_{k_a} \ell(k) = \infty$$

$$\sum_{k_a} \ell^2(k) < \infty \qquad \text{(III-183.B)}$$

and $\sum_{k_a}$ defines all admissable k for the summation (usually $\ell(k) = \frac{1}{k}$).

20. Tomizuka [127]

This is a modification to Landau's MRAS method. It is a discrete, SISO, output error formulation. It is designed to account for deterministic disturbance by creating "rejection filters" which remove the disturbance but have no effect on asymptotic parameter stability. Referring to Figure III-5, it is desired that

$$G_d(z)\ d_1(z) = 0\ ,\quad G_d(z)\ d_2(z) = 0 \qquad \text{(III-184.B)}$$

where $G_d(z)$ is a so-called deterministic disturbance rejector, and d_i are external disturbances. If $d_i(k)$ = constant, possible $G_d(z)$ are

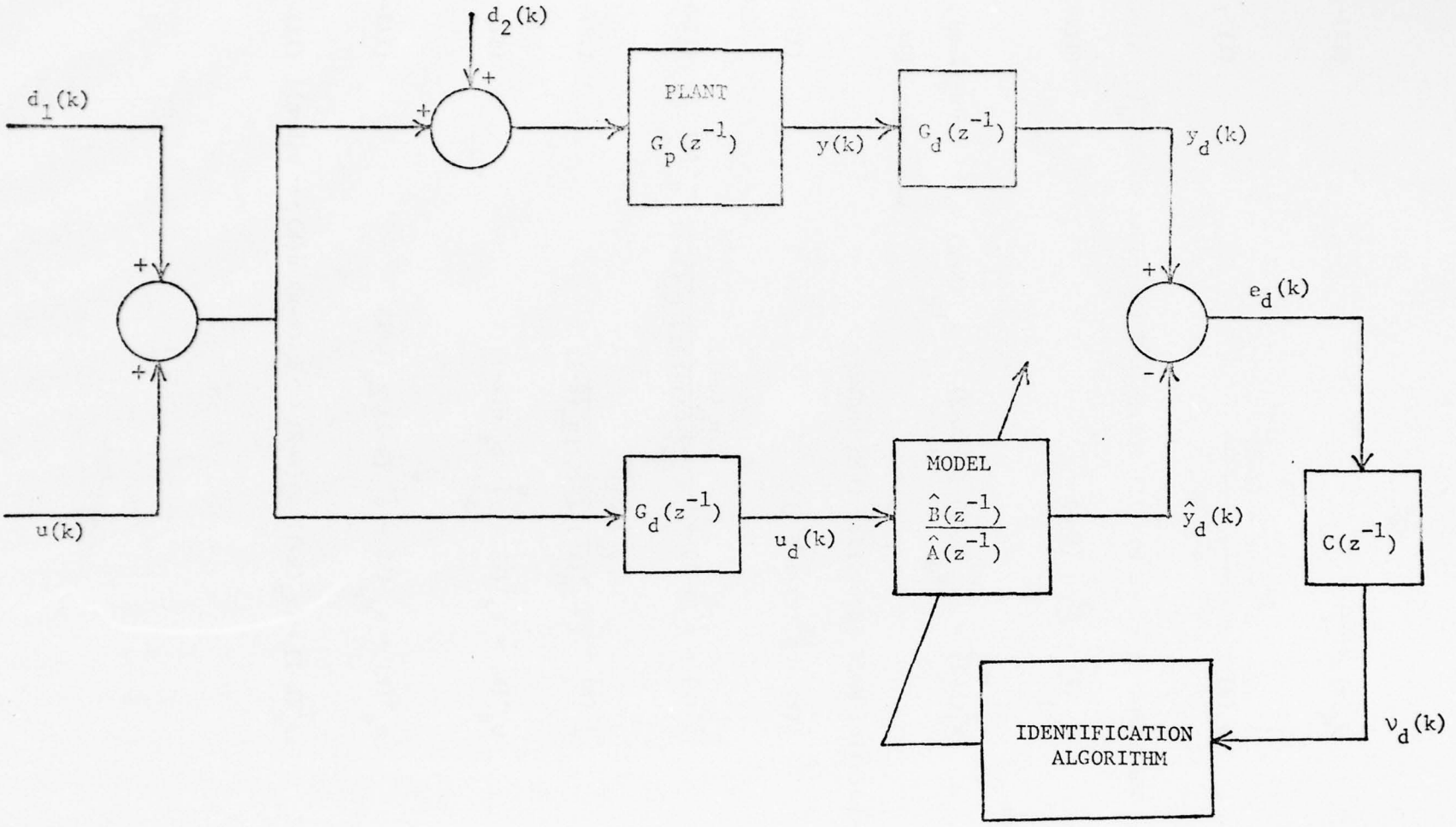


Figure III-5. Parallel MRAS Identifier With Deterministic Disturbance Rejectors.

$$G_d(z) = \frac{z^{-1}}{z} \qquad \text{(III-185.B)}$$

and

$$G_d(z) = \frac{z^2 - hz - (1-h)}{z^2} \qquad \text{(III-186.B)}$$

The model is given by (III-101.B) and the plant with $G_d(z)$ given by

$$x_{p_d}(k) = \underline{\phi}^T \underline{s}_d(k-1) \qquad \text{(III-187.B)}$$

where

$$\underline{s}_d^T(k-1) = [x_{p_d}(k-1)\ x_{p_d}(k-2) \cdots x_{p_d}(k-n)\ u_d(k) \cdots u_d(k-m)] \qquad \text{(III-188.B)}$$

The parallel MRAS identifier then becomes

$$\hat{\underline{\phi}}(k) = \hat{\underline{\phi}}^I(k) + \hat{\underline{\phi}}^P(k) \qquad \text{(III-189.B)}$$

$$\hat{\underline{\phi}}^I(k) = \hat{\underline{\phi}}^I(k-1) + \frac{K\, \underline{s}_m(k-1)}{1 + \underline{s}_m^T(k-1)[K+L]\, \underline{s}_m(k-1)}\, \nu_d^{\circ}(k) \qquad \text{(III-190.B)}$$

$$\underline{\phi}^P(k) = \frac{L\, \underline{s}_m(k-1)}{1 + \underline{s}_m^T(k-1)[K+L)]\, \underline{s}_m(k-1)} \qquad \text{(III-191.B)}$$

$$\nu_d^{\circ}(k) = e_d^{\circ}(k) + \sum_{i=1}^{n} c_i\, e(k-i) \qquad \text{(III-192.B)}$$

$$e_d^{\circ}(k) = x_{p_d}(k) - \hat{\phi}^{I^T}(k-1)\, \underline{s}_m(k-1) \qquad \text{(III-193.B)}$$

$$s_m^T(k-1) = [x_m(k-1)\ x_m(k-2) \cdots x_m(k-n)\ u(k) \cdots u(k-n)] \qquad \text{(III-194.B)}$$

$$K = \begin{bmatrix} k_{a_1} & & & & & & & & \\ & k_{a_2} & & & & & 0 & & \\ & & \ddots & & & & & & \\ & & & k_{a_n} & & & & & \\ & & & & k_{b_o} & & & & \\ & & & & & k_{b_1} & & & \\ & & & 0 & & & \ddots & & \\ & & & & & & & k_{b_m} & \end{bmatrix} \qquad \text{(III-195.B)}$$

$k_{a_i} > 0$, $k_{b_i} > 0$

$$L = \begin{bmatrix} \ell_{a_1} & & & & & & & & \\ & \ell_{a_2} & & & & & 0 & & \\ & & \ddots & & & & & & \\ & & & \ell_{a_n} & & & & & \\ & & & & \ell_{b_o} & & & & \\ & & & & & \ddots & & & \\ & & & 0 & & & & \ell_{b_m} & \end{bmatrix} \qquad \text{(III-196.B)}$$

$\ell_{a_i} > .5\, k_{a_i}$, $\ell_{b_i} > .5\, k_{b_i}$ and the c_i given by (III-112.B).

21. Johnson [128,129]

This is an approximate technique for parameter identification which is a simplification of Landau's hyperstable MRAS method. It represents a SISO approach using the output error formulation. For the plant given by (III-69.B), the update model (III-101.B) is employed. The tracking error

is given by

$$e(k) = x_m(k) - x_p(k) \qquad \text{(III-197.B)}$$

Following Landau's integral adaptation concept, a simple hyperstable, adaptive, recursive filter (SHARF) is developed as

$$\hat{a}_i(k) = \hat{a}_i(k-1) - \alpha(k-1)\ \varepsilon(k-1)\ x_m(k-i-1) \quad i=1,2,\cdots,n \qquad \text{(III-198.B)}$$

$$\hat{b}_j(k) = \hat{b}_i(k-1) - \gamma(k-1)\ \varepsilon(k-1)\ u(k-j-1) \quad j=0,1,\cdots,m \qquad \text{(III-199.B)}$$

where

$$\varepsilon(k) = e(k) + c\ e(k-1) \qquad \text{(III-200.B)}$$

is a first-order moving average of the error, and $\alpha_i(k)$ and $\gamma_i(k)$ are positive weighting terms of the form

$$\alpha_i(k) = \frac{M_{1_i}}{\sum_{i=1}^{n} x_m^2(k-i) + \sum_{j=0}^{m} u^2(k-j)} \qquad \text{(III-201.B)}$$

$$\gamma_j(k) = \frac{M_{2_j}}{\sum_{i=1}^{n} x_m^2(k-i) + \sum_{j=0}^{m} u^2(k-j)} \qquad \text{(III-202.B)}$$

where M_i are constants satisfying

$$0 < M_{1_i} < 2 \quad \text{and} \quad 0 < M_{2_j} < 2 \qquad \text{(III-203.B)}$$

The coefficient c is selected based on the condition that

$$H(z) = \frac{1 + c\ z^{-1}}{1 - \sum_{i=1}^{n} a_i\ z^{-i}} - \frac{1}{2} \qquad \text{(III-204.B)}$$

is strictly positive real.

22. Pandya [130,131]

This is a discrete-time SISO method which is specifically designed to handle the noisy measurement case. The method is referred to as a "bootstrap" method in that process parameter estimates are employed in estimating noise parameters and vice versa. It is an equation-error type formulation with the bootstrapping serving to eliminate correlated residuals and hence tends to remove parameter bias such as occurs with the least squares method. Unfortunately this bootstrapping violates stability conditions and hence the 'system' is only locally stable.

The plant is given by

$$x_p(k+1) = \underline{\phi}^T \underline{s}(k) \tag{III-205.B}$$

where $\underline{s}(k-1)$ is given by (III-172.B), $\underline{\phi}$ by (III-99.B), and the plant output by

$$y_p(k) = x_p(k) + \omega(k) \tag{III-206.B}$$

where $\omega(k)$ is a zero-mean white noise sequence. The model is defined as

$$x_m(k+1) = \hat{\underline{\phi}}^T(k) \underline{s}(k) \tag{III-207.B}$$

The recursive parameter estimator is given by

$$\hat{\underline{\phi}}(k+1) = \hat{\underline{\phi}}(k) + \frac{P(k)\,\hat{\underline{x}}(k)\,[y_p(k+1) - \hat{\underline{\phi}}^T(k)\,\underline{s}(k)]}{1 + \underline{s}^T(k)\,P(k)\,\hat{\underline{x}}(k+1)} \tag{III-208.B}$$

$$\hat{\underline{x}}^T(k) = [\hat{x}(k/k)\ \hat{x}(k-1/k-1) \cdots \hat{x}(k-n+1/k-n+1)\ u(k+1)\ u(k) \cdots u(k-m+1)] \tag{III-209.B}$$

$$\hat{x}(k/k) = \hat{x}(k/k-1) + \gamma(k)[y(k) - \hat{x}(k/k-1)] \tag{III-210.B}$$

$$\hat{x}(k/k-1) = \hat{\underline{\phi}}^T(k)\,\hat{\underline{x}}(k) \tag{III-211.B}$$

$0 < \gamma(k) < 1$ (generally a constant)

$$P(k+1) = P(k) - \frac{P(k)\ \hat{\underline{x}}(k+1)\ \hat{\underline{x}}^T(k+1)\ P(k)}{1 + \underline{s}^T(k)\ P(k)\ \hat{\underline{x}}(k+1)} \qquad \text{(III-212.B)}$$

23. Least Squares [132,133]

This is a discrete-time, SISO, equation error formulation method and is included here for reference as a baseline for the MRAS methods. The plant is given by (III-69.B), with the model related by

$$x_m(k) = \sum_{i=1}^{n} \hat{a}_i(k-i)\ x_p(k-i) + \sum_{j=0}^{m} \hat{b}_j(k-1)\ u(k-j) \qquad \text{(III-213.B)}$$

$$x_m(k) = \hat{\underline{\phi}}^T(k-1)\ \underline{s}(k-1) \qquad \text{(III-214.B)}$$

where $\underline{s}(k-1)$ is defined by (III-172.B), and $\hat{\underline{\phi}}^T(k-1)$ as

$$\hat{\underline{\phi}}^T(k-1) = [\hat{a}_1(k-1)\ \hat{a}_2(k-1) \cdots \hat{a}_n(k-1)\ \hat{b}_o(k-1)\ \hat{b}_1(k-1) \cdots \hat{b}_m(k-1)] \qquad \text{(III-215.B)}$$

Using [132] and the least-squares data fitting concept, the update equations for the parameter estimates are

$$\hat{\underline{\phi}}(k+1) = \hat{\underline{\phi}}(k) + \frac{P(k)\ \underline{s}(k)\ [x_p(k+1) - x_m(k+1)]}{1 + \underline{s}^T(k)\ P(k)\ \underline{s}(k)} \qquad \text{(III-216.B)}$$

$$P(k+1) = P(k) - \frac{P(k)\ \underline{s}(k)\ \underline{s}^T(k)\ P(k)}{1 + \underline{s}^T(k)\ P(k)\ \underline{s}(k)} \qquad \text{(III-217.B)}$$

For operation, initial conditions on $\hat{\underline{\phi}}(k)$ and P(k) are needed. In practice they could be arbitrarily selected values, with $\hat{\underline{\phi}}(o)$ based on "best-guess *a priori* plant information, and P(o) a p.d. diagonal matrix with entries representing the corresponding variances of the inaccuracies in $\hat{\underline{\phi}}(o)$. If sufficient measurement data is available before algorithm startup, initial estimates may be obtained as

$$\hat{\underline{\phi}}(o) = \Psi^{-1}\, \underline{q}(o) \tag{III-218.B}$$

$$P(o) = \Psi(o)^{-1}\, (\Psi^{-1}(o))^{T} \tag{III-219.B}$$

$$\Psi(0) = \begin{bmatrix} x_p(-n-m) & x_p(-n-m-1) & \cdots & x_p(-2n-m) & u(1-n-m) & u(-n-m) & \cdots & u(-n-2m) \\ x_p(1-n-m) & x_p(-n-m) & \cdots & x_p(1-2n-m) & u(2-n-m) & u(1-n-m) & \cdots & u(1-n-2m) \\ \vdots & & & & & & & \\ x_p(0) & x_p(-1) & \cdots & x_p(-n) & u(0) & u(-1) & \cdots & u(-m) \end{bmatrix} \tag{III-220.B}$$

$$\underline{q}^{T}(0) = [x_p(-n-m-1) \quad x_p(-n-m) \cdots x_p(1)] \tag{III-221.B}$$

The notation is such that k=0 corresponds to the arbitrary starting time, by which (n+m+1) input-output data sequences are available for operation.

24. Kreisselmeier [134]

The unknown, time invariant, SISO plant is assumed of the nth order form

$$\dot{\underline{x}}_p = A_p \underline{x}_p + \underline{b}\, u \tag{III-222.B}$$

with scalar output

$$y = \underline{c}^{T}\, \underline{x}_p \tag{III-223.B}$$

$$A = \begin{bmatrix} -a_1 & 0 & 0 \cdots 0 \\ -a_2 & 0 & 1 \cdots 0 \\ \vdots & \vdots & \ddots \; 1 \\ -a_n & 0 & 0 \end{bmatrix} \quad , \quad \underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ \\ b_n \end{bmatrix} \quad , \quad \underline{c} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \text{(III-224.B)}$$

An identifier-observer is defined as

$$\dot{\underline{x}}_m = F \, \underline{x}_m + \underline{g} \, y + \underline{h} \, u \qquad \text{(III-225.B)}$$

where F is a designer controlled matrix,

$$F = \begin{bmatrix} -f_1 & 1 & 0 \cdots \\ -f_2 & 0 & 1 \cdots \\ \vdots & \vdots & \ddots \; 1 \\ -f_n & 0 & 0 \end{bmatrix} \qquad \text{(III-226.B)}$$

and $\underline{g}$ and $\underline{h}$ are such that

$$a_i = f_i - g_i \qquad b_i = h_i \qquad i = 1, 2, \cdots, n \qquad \text{(III-227.B)}$$

F has a set of eigenvalues such that

$$\text{Re}\{\lambda_i\} < -\sigma \quad , \qquad \sigma > 0 \qquad \text{(III-228.B)}$$

It can be shown that

$$\underline{x}_m = \underline{\xi}^T \, \underline{p} + \exp(Ft) \, \underline{x}_m(t_o) \qquad \text{(III-229.B)}$$

where $\underline{\xi}^T = [\xi_1 \quad \xi_2 \cdots \xi_{2n}]$ are

$$\dot{\underline{\xi}}_1 = F^T \, \underline{\xi}_1 + \underline{e}_1 \, y \quad , \quad \underline{\xi}_1(o) = 0 \qquad \text{(III-230.B)}$$

$$\dot{\underline{\xi}}_{i+n} = F^T \, \underline{\xi}_2 + \underline{e}_1 \, u \quad , \quad \underline{\xi}_{i+n}(o) = 0 \qquad \text{(III-231.B)}$$

$$i = 1, 2, \cdots, n$$

and $\underline{e}_i$ is the unit vector $g^T \underline{e}_i = g_i$. The vector $\underline{p}$ is

$$\underline{p}^T = [\underline{g}, \underline{h}] \tag{III-232.B}$$

representing the parameters to be identified, which may then be transformed to the physical-world a_i, b_i using (III-227.B). The model output is

$$y_m = \underline{z}^T \underline{p} + c^T \exp(Ft)\, \underline{x}_m(o) \tag{III-233.B}$$

with

$$\underline{z}^T = c^T \underline{\xi}^T \tag{III-234.B}$$

$$\underline{\xi}^T = [\xi_1 \quad \xi_2 \cdots \xi_{2n}] \tag{III-235.B}$$

The state and observation errors are then, respectively

$$\underline{\varepsilon} = \underline{x}_m - \underline{x}_p \tag{III-236.B}$$

$$\eta = y_m - y_p \tag{III-237.B}$$

To determine the identification rule, the cost functional

$$J = \int_o^t \left[\left\{ \underline{z}^T(\tau)\, \underline{p}(t) + \underline{c}^T \exp(F\tau)\, x_m(0) - y_p(\tau) \right\}^2 \exp\{-q(t-\tau)\} \right] d\tau \tag{III-238.B}$$

is selected to be minimized, where $q > 0$, constant. Reducing J satisfies the dual identifier goals of $(\underline{p} - \hat{\underline{p}}) \to 0$ and $\eta \to 0$. Select

$$\dot{\underline{p}} \propto \frac{\partial J}{\partial \underline{p}} \tag{III-239.B}$$

$$\frac{\partial J}{\partial \underline{p}} = 2\{R(t)\, \underline{p} + r\} \tag{III-240.B}$$

$$\underline{R}(t) = \int_o^t \underline{z}(\tau)\, z^T(\tau) \{\exp\{-q(t-\tau)\}\, d\tau \tag{III-241.B}$$

$$\underline{r}(t) = \int_o^t \left[\underline{z}(\tau)\{\underline{c}^T \exp(F\tau)\ \underline{x}_m(o) - y_p(\tau)\}\right.$$

$$\left.\exp\{-q(t-\tau)\}\right]d\tau \qquad \text{(III-242.B)}$$

Then

$$\dot{\underline{p}} = -G\{\underline{R}(t)\ \underline{p} + \underline{r}(t)\} \qquad \text{(III-243.B)}$$

where G is symmetric and p.d.

with $\underline{R}(t)$ and $\underline{r}(t)$ defined from

$$\dot{\underline{R}} = -q\ \underline{R} + \underline{z}\ \underline{z}^T \qquad \underline{R}(o) = \underline{0} \qquad \text{(III-244.B)}$$

$$\dot{\underline{r}} = -q\ \underline{r} + \underline{z}[\underline{c}^T \exp(Ft)\ \underline{x}_m(o) - y_p] \qquad \underline{r}(o) = \underline{0} \qquad \text{(III-245.B)}$$

C. Other Stability Approaches for MRAS Identifiers

Numerous techniques other than Lyapunov Theory or Hyperstability Theory can be used for insuring stability of an MRAS identifier. One particularly promising approach is that of *contraction mapping* [135], wherein the identification goins are designed to "contract" to the true parameter values [136,137]. This technique has been successfully employed in adaptive control [138], but has not been employed in MRAS identification as of yet.

CHAPTER 4. NONLINEAR MRAS IDENTIFIERS

A. Introduction

The third class of parameter identifiers investigated are the nonlinear type. Although many concepts for nonlinear identification have common bonds to the linear approaches, the differences are significant. Recent general work has centered on the use of such seemingly esoteric topics as Catastrophe Theory [139] and pattern recognition [140] in nonlinear system identification. This is because despite ≃ 50 years of work, little in the way of key modeling breakthroughs have occurred. On-going work into the theory of data requirements, etc. then is still important [141], along with the further development of the identification algorithms themselves. The motivation for the nonlinear identification work reported here stems from certain human operator modeling characteristics which cannot be directly accounted for by linear models. Particular items investigated are effects of transportation lag (related to neuromuscular delays), nonlinearity deadzone effects (due to human perception dropout at low excitation levels), and parameter biasing due to noise. Results from the two previous classes of identifiers suggests that the parallel MRAS structure offers the best workable approach to human operator work. For that reason, methods studied were predominately of the parallel type, although the *analysis and implementation* of series parallel identifiers is also studied.

There are two key approaches possible for handling dynamical systems with one or more nonlinearities. These are the cases when the a) nonlinearity form is known *a priori*, and b) nonlinearity form is unknown *a priori*. The first approach is predicated on when the *structural form* of the nonlinearity is known in advance (e.g. quadratic curve, symmetrical saturation

function, etc.) [109,113,142], or the nonlinearity is one in which the salient parameters to be identified enter linearly (e.g. $y + a_1 y + cy^2 = u$) [102,25]. The second approach is based on a series expansion concept (Taylor Series, sine-cosine, etc.) so as to obtain piecewise continuous curve fitting; this approach can be applied when no *a priori* knowledge of the nonlinearity is available [142-144]. The second approach is of greatest interest, especially as regards the ability to admit memory nonlinearities, since the human operator problem does not necessarily suggest the nonlinearity form in advance. It should be noted that case "a" above can be reformulated as a linear, time-invariant system identification problem.

The classes of nonlinear system models and nonlinear algorithms will be developed in the following sections to clarify the similarities and differences. Analysis of the identifiers will then follow, with the goal being the development of design guidelines for simplifying implementation.

B. Classes of Nonlinear Systems

There are many nonlinearity structural forms which are utilized in nonlinear system parameterization and identification. Since most practical problems, including the human operator problem, do not exhibit simple, well-defined characteristics, a series of general nonlinear models is needed. However, variations on standardized models can yield as good of results as a special model. Some of the more widely used ones include 1) Hammerstein [145-148], 2) Volterra Series [149-154], 3) Uryson Model [155-156], 4) Wiener Model [157-160], 5) Memoryless nonlinearity piecewise series fit [143,161, 162], 6) Memory nonlinearity piecewise series fit [144], and 7) known nonlinearity model structure [109,113]. Certain of these have been employed for modeling purposes in the present study and will be detailed here. They are tabulated in Table IV-1 for convenience.

Table IV-1. Types of Nonlinear Models

Number	Name	Type *	Applicable to Human Operating Modeling
1	Hammerstein	D	Yes
2	Volterra	D	Not Easily
3	Uryson	C	Not Easily
4	Memoryless Piecewise Series Fit	C	Yes
5	Memory Piecewise Series Fit	C	Yes
6	Hammerstein-Discrete Laguerre Function	D	Not Easily

*D = Discrete

C = Continuous

1. Hammerstein [145-148]

The Hammerstein model consists of a nonlinear static gain element operating on a plant input feeding a linear dynamic element. This is shown in Figure IV-1 for discrete time systems. The nonlinear gain P(u) is

$$P(u) = \sum_{i=1}^{\ell} \gamma_i \, u^i(j) \qquad \text{(IV-1.B)}$$

and the linear transfer function G(z)

$$G(z) = \frac{a_o + a_1 z^{-1} + \cdots + a_m z^{-m}}{1 + b_1 z^{-1} + \cdots + b_n z^{-n}} \qquad \text{(IV-2.B)}$$

$$m \leq n$$

Defining

$$G(z) = \frac{N(z^{-1})}{1 + D(z^{-1})} \qquad \text{(IV-3.B)}$$

then the output $x_m(j)$ can be defined as

$$x_m(j) = \underline{\phi}^T \, \underline{q}(j) \qquad \text{(IV-4.B)}$$

with (assuming a_o normalized to $a_o = 1$)

$$\underline{\phi}^T = [\gamma_1, \gamma_1 a_1, \cdots \gamma_1 a_m, \gamma_2 \gamma_2 a_1, \cdots \gamma_\ell, \gamma_\ell a_1 \cdots$$

$$\gamma_\ell a_m, -b_1 \; -b_2 \cdots \; -b_n] \qquad \text{(IV-5.B)}$$

$$\underline{q}^T(j) = [u(j) \cdots u(j-m), \, u^2(j) \cdots u^2(j-m), \cdots, u^\ell(j) \cdots$$

$$u^\ell(j-m) \;\; x_m(j-1) \;\; x_m(j-2) \cdots x_m(j-m)] \qquad \text{(IV-6.B)}$$

For further detail, a stochastic noise model may be added so that the noisy model output becomes

$$y_m(j) = x_m(j) + e_c(j) \qquad \text{(IV-7.B)}$$

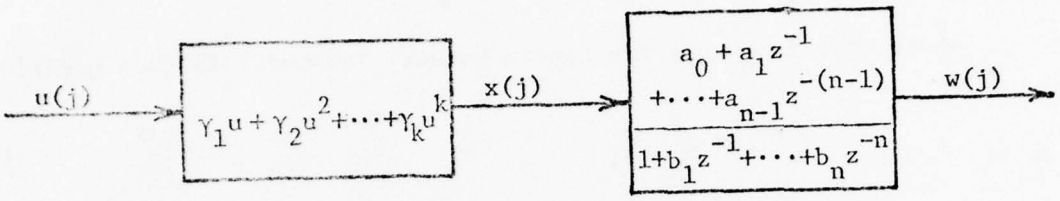


Figure IV-1(a).

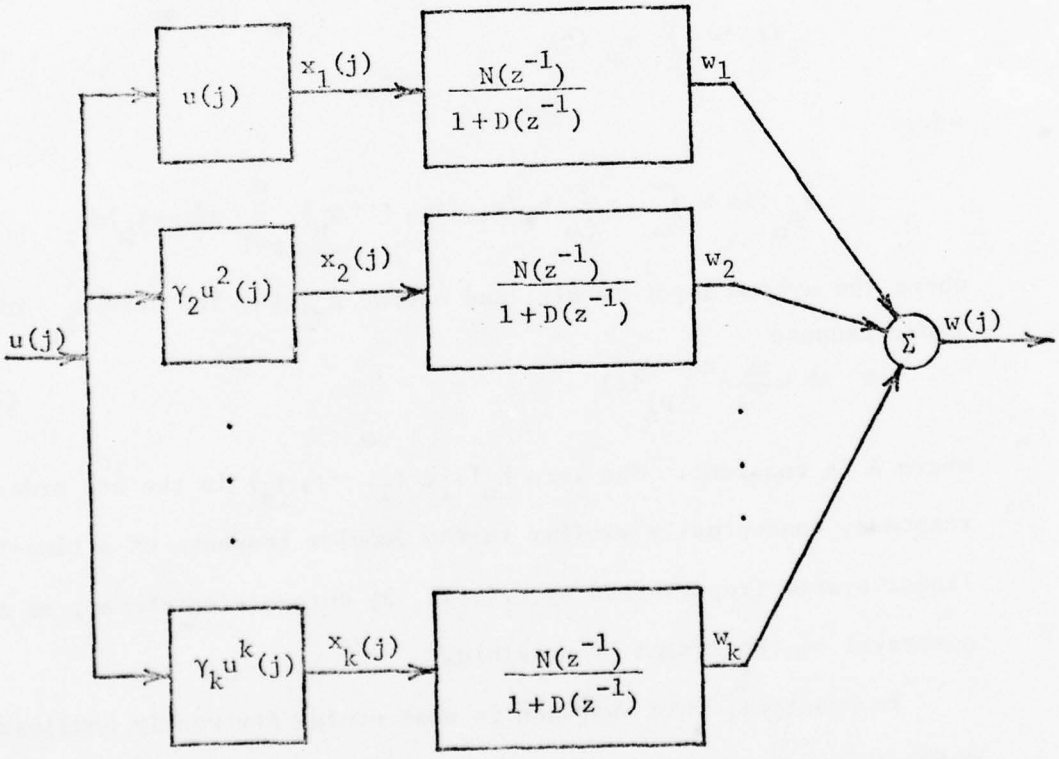


Figure IV-1(b). Rearrangement of Fig. IV-1(a) to convert the nonlinear single-input system into a multi-input "linear" systen.

as shown in Fig. IV-2, where

$$\frac{e_c(j)}{d(j)} = \frac{1}{1+\mu_1 z^{-1} + \cdots + \mu_p z^{-p}} \qquad \text{(IV-8.B)}$$

where d(j) is a gaussian random noise. The model is then modified to

$$y_m(j) = \underline{\phi}_a^T \; \underline{q}_a \qquad \text{(IV-9.B)}$$

$$\underline{\phi}_a^T = [\underline{\phi}^T \vdots -\mu_1 \quad -\mu_2 \cdots \quad -\mu_p] \qquad \text{(IV-10.B)}$$

$$\underline{q}_a^T = [\underline{q}^T \vdots e_c(j-1) \quad e_c(j-2) \cdots e_c(j-p)] \qquad \text{(IV-11.B)}$$

2. Volterra Series [149-154]

The Volterra approach assumes that the response of a nonlinear system is

$$x_p(t) = \sum_{n=1}^{\infty} x_{p_n}(t) \qquad \text{(IV-12.B)}$$

where

$$x_{p_n}(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \cdots \tau_n) \prod_{p=1}^{n} u(t-\tau_p) d\tau_p \qquad \text{(IV-13.B)}$$

where the system input is u(t) and output $x_p(t)$. The term x_{p_n} is of nth order because

$$A\,u \Rightarrow A^n \, x_{p_n}(t) \qquad \text{(IV-14.B)}$$

where A is constant. The term $h_n(\tau_1, \tau_2, \cdots, \tau_n)$ is the nth order impulse response, conceptually similar to the impulse response of a time-invariant linear system (represented by $h_1(\tau_1)$). By determining the h_i, an accurate portrayal of the system is possible.

In practice, this approach is most useful for weakly nonlinear systems, where the response (IV-13.B) can be approximated by a finite number of terms,

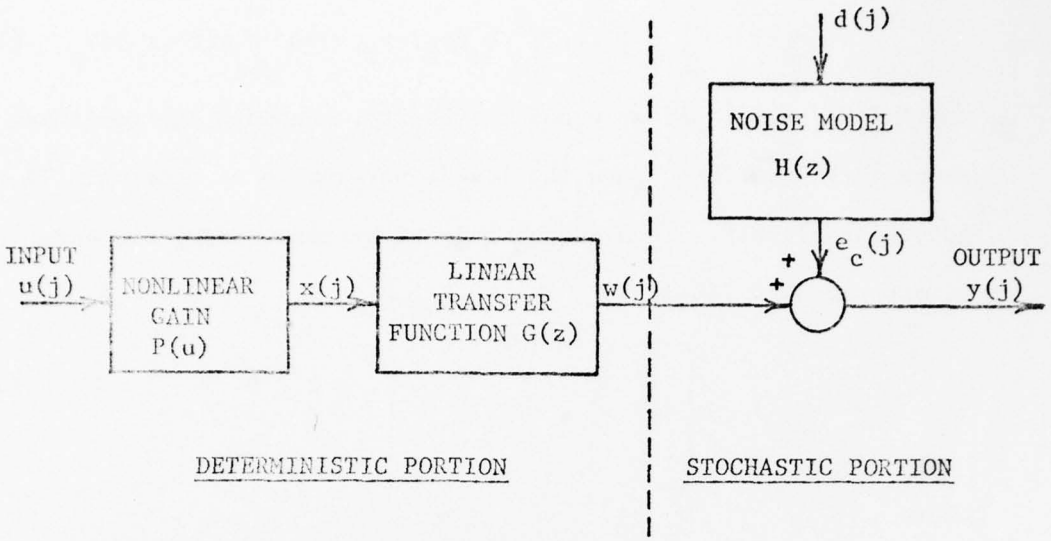


Figure IV-2. Proposed structure for modeling nonlinear noisy system.

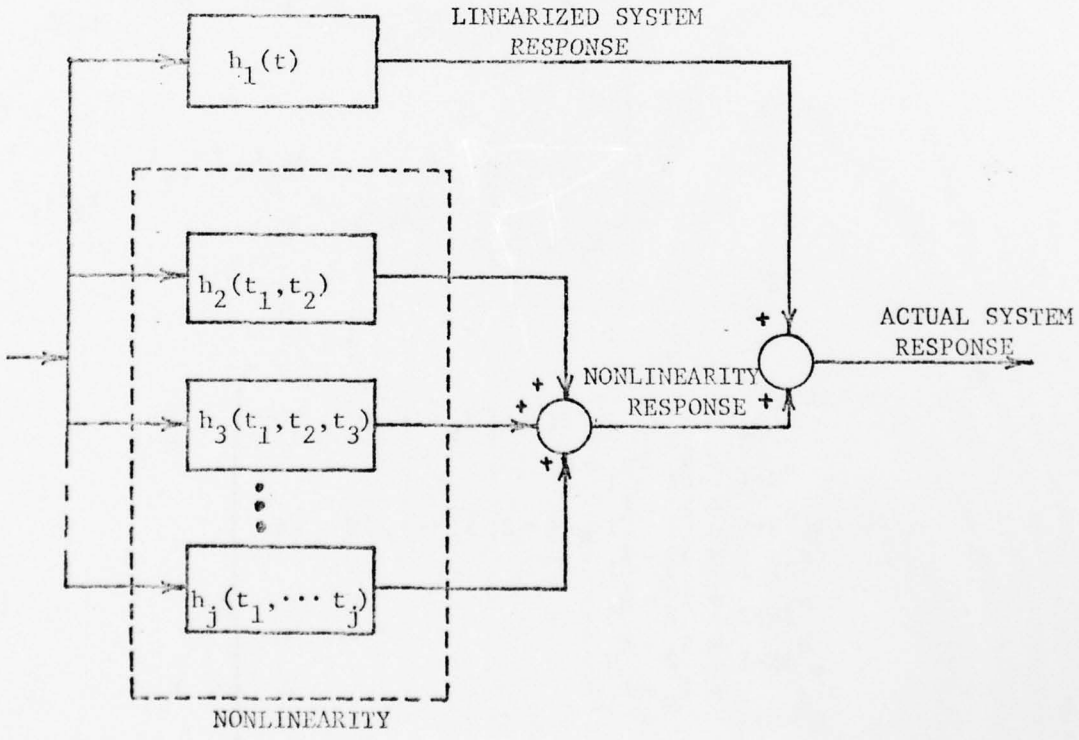


Figure IV-3. Volterra Series Representation of a General Nonlinear System.

$$x_p(t) = \sum_{n=1}^{\ell} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \cdots) \prod_{p=1}^{n} u(t-\tau_p) d\tau_p \qquad \text{(IV-15.B)}$$

where ℓ is small. As an example of $h_i(t)$, consider the nonlinear system shown in Figure IV-3 where the linear portion is of order N. If the general nonlinear system can be modeled by zero memory elements, then the h_i are given by [154]:

$$h_i(t) = \begin{cases} \sum_{i=1}^{N} R_i \, e^{\lambda_i t} & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad \text{(IV-16.B)}$$

where

R_i - residue which depends on the linearized dynamics

λ_i - eigenvalue

$$h_2(t_1,t_2) = \begin{cases} \sum_{k_1=1}^{\omega} \sum_{k_2=1}^{N} A_{k_1 k_2} \, e^{(a_{k_1} t_1 + a_{k_2} t_2)} & t_2 \geq t_1 \\ \sum_{k_1=1}^{\omega} \sum_{k_2=1}^{N} A_{k_1 k_2} \, e^{a_{k_1} t_2 + a_{k_2} t_1} & t_2 < t_1 \end{cases} \qquad \text{(IV-17.B)}$$

where

$$\left.\begin{aligned} \omega &= N^2 + 1 \\ a_i &= \lambda_i \qquad i = 1, 2, \cdots, N \\ a_{N+1} &= \lambda_1 - \lambda_1 = 0 \\ a_{N+i} &= \lambda_1 - \lambda_i \qquad i = 2, 3, \cdots, 2N \\ a_{2N+1} &= \lambda_2 - \lambda_1 \\ a_{2N+2} &= \lambda_2 - \lambda_3 \\ a_{3N-1} &= \lambda_2 - \lambda_N \\ a_{N^2-N+3} &= \lambda_N - \lambda_1 \\ a_{N^2-N+4} &= \lambda_N - \lambda_2 \\ &\vdots \\ a_{N^2+1} &= \lambda_N - \lambda_{N-1} \end{aligned}\right\} \qquad \text{(IV-18.B)}$$

The h_i, $i \geq 3$ are far more complicated. It is clear that for more than a few terms, the parameterization problem can be severe. For h_2, a_{k_1}, a_{k_2}, and N are known from h_1, which leaves only $A_{k_1 k_2}$ of which there are $\frac{N}{2}(3N+1)$ terms. For the human operator problem, typical numbers would be $N = 4$, $\ell = 4$, resulting in a tremendous number of parameters.

3. Uryson Model [155,156]

This nonlinear model includes the Hammerstein systems as a special class. It represents a third level of nonlinear models, the first being linear models, the second Hammerstein, and the third Uryson. The Uryson model consists of several Hammerstein-type models in parallel, each with the same input passed through Hermite polynomials, as shown in Figure IV-4. The model output is given by

$$x_m(t) = \sum_{i=0}^{M} \int_0^\infty h_i(\tau)\, H_i(u(t-\tau))d\tau \qquad \text{(IV-19.B)}$$

where

$M + 1$ = number of terms

h_i = linear impulse response of the ith path

$H_i(u)$ = Hermite polynomial of order i

$$= \sum_{k=0}^{[i/2]} \frac{(-1)^k\, i!\, (2u)^{i-2-k}}{k!(i-2k)!} \qquad \text{(IV-20.B)}$$

4. Memoryless Nonlinearity Piecewise Series Fit [143,161,162]

Consider a nonlinear element N, representing a math operator from $C(-\infty,\infty)$, where $C(-\infty,\infty)$ denotes the linear space of continuous real-valued functions over $(-\infty,\infty)$, and $x(t)$ an input with $x \in (-\infty,\infty)$. N is a *memoryless* nonlinearity if [163]

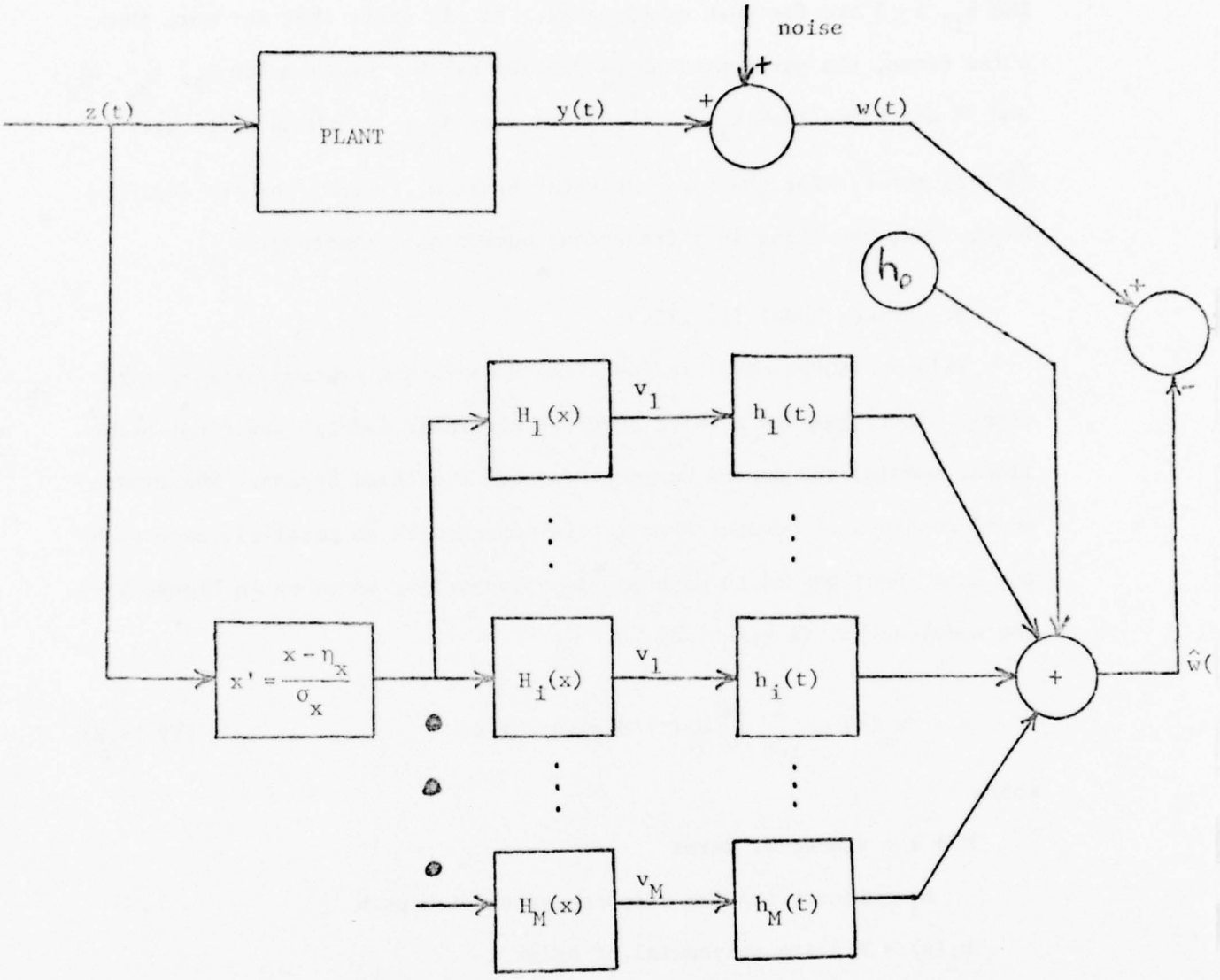


Figure IV-4. Uryson model.

$$N(x(t)) = n[t, x(t)] \tag{IV-21.B}$$

where $n(t, \cdot)$ is a mapping from $R \to R$. N is memoryless because the output depends only on the input value x at time t, and not on past or future values of x. For present modeling purposes, a series of elements of the type N are considered.

A dynamical system is described by

$$x_p^{(n)} + \cdots + f_i\,(x_p^{(j)})\; x_p^{(i)} + \cdots + f_o\,(x_p^{(k)})\; x^{(o)}$$
$$= b_1 u^{(o)} + \cdots + g_i(u^{(h)})\; u^{(i)} + \cdots + g_m\,(u^{(q)})\; u^{(m)} \tag{IV-22.B}$$

where $u^{(o)}$, $x_p^{(o)}$ are the system input and output, the functions f_r and g_s are in general nonlinear functions of the indicated argument, $m < n$. The g_s and f_r must be single-valued, (hence *memoryless*), continuous, and a function of one argument. In practice,

$$f_i\,(x_p^{(j)}) \triangleq f_i\,(x_p^{(i)}) \tag{IV-23.B}$$

$$g_i\,(u^{(h)}) \triangleq g_i\,(u^{(i)}) \tag{IV-24.B}$$

A typical model will possess few actual nonlinear terms f_i, g_s, and many will be constants, i.e.

$$f_i(x^{(i)}) \triangleq a_i \tag{IV-25.B}$$

$$g_i(j^{(j)}) \triangleq b_j \tag{IV-26.B}$$

The practical modeling problem reduces to deciding which terms of (IV-22.B) are actually nonlinear and which are linear. As an example of this, consider a plant

$$\ddot{x}_p + a_2\,\dot{x}_p + f_1(x_p)x_p = b_1 u + b_2\dot{u} \tag{IV-27.B}$$

A model might be assumed (Remember: the actual structure of (IV-27.B) is unknown to the typical modeler) as

$$\ddot{x}_m + f_2(\dot{x}_p)\dot{x}_p + f_1(x_p)x_p = b_1 u + b_2 \dot{u} \qquad \text{(IV-28.B)}$$

or

$$\ddot{x}_m + a_2(\dot{x}_p) + f_1(x_p)x_p = b_1 u + g_2(\dot{u})\dot{u} \qquad \text{(IV-29.B)}$$

This approach can only be employed for nonlinearities without memory.

These nonlinearities can be expanded in power series as

$$f_k(y)\Big|_p \simeq a_{k1_p} + a_{k2_p}(y-y_p) + a_{k3_p}(\dot{y}-\dot{y}_p) + \cdots + a_{kr_p}(y^{(r-2)} - y_p^{(r-2)}) + \cdots \qquad \text{(IV-30.B)}$$

where the expansion is in the pth interval, a_{ki_p} are modeling constants in the p^{th} interval, y is the function argument, and y_p is the expansion point in the p^{th} interval (usually the midpoint). A similar expansion is performed for g_j,

$$g_j(u)\Big|_p \simeq b_{j1_p} + b_{j2_p}(u-u_p) + \cdots + b_{jr_p}(u^{(r-2)} - u_p^{(r-2)}) + \cdots \qquad \text{(IV-31.B)}$$

In practice, usually only the first two terms of (IV-30.B) and (IV.31-B) are employed because a) the approximation employing higher order terms is no more accurate than using linear terms, b) complexity is minimized, and c) less parameters need to be identified. The resulting identifier structure and the expansion interval concept are demonstrated in Figures IV-5(a) and 5(b).

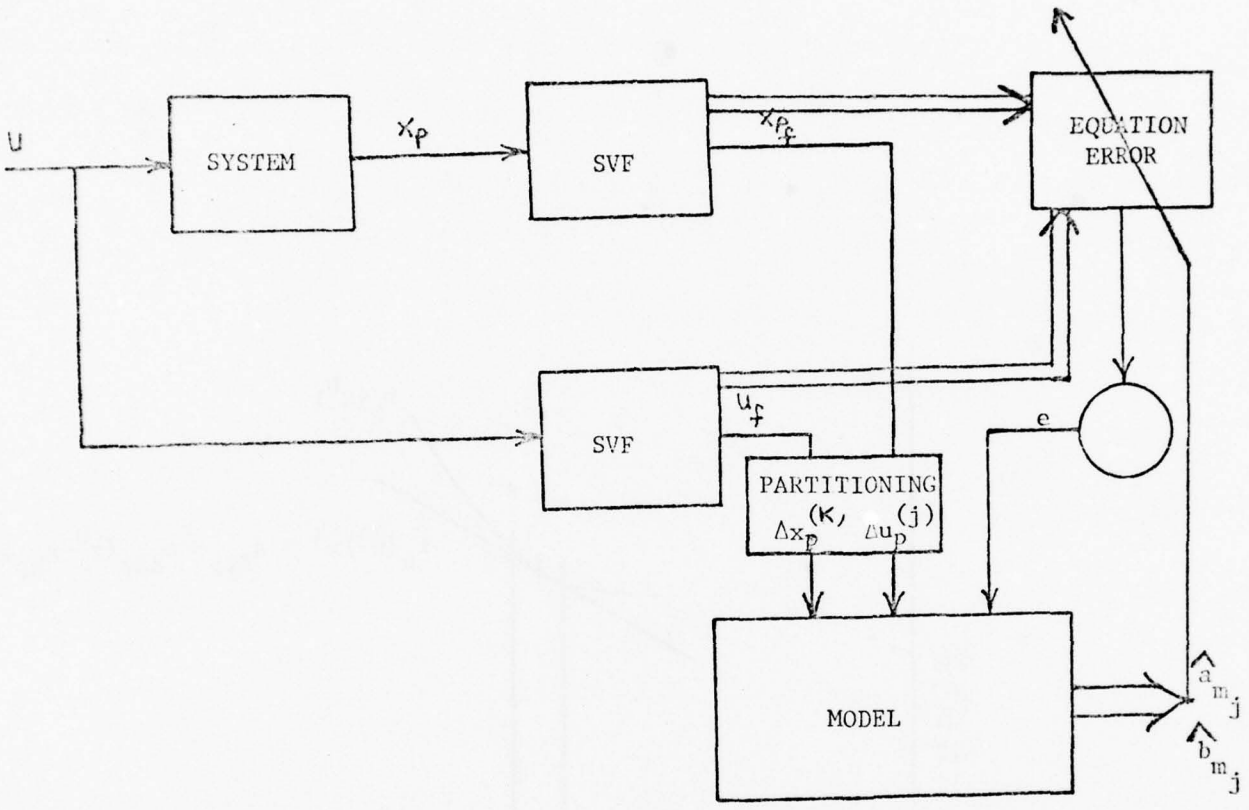


Figure IV-5(e), General Nonlinear Identifier Structure.

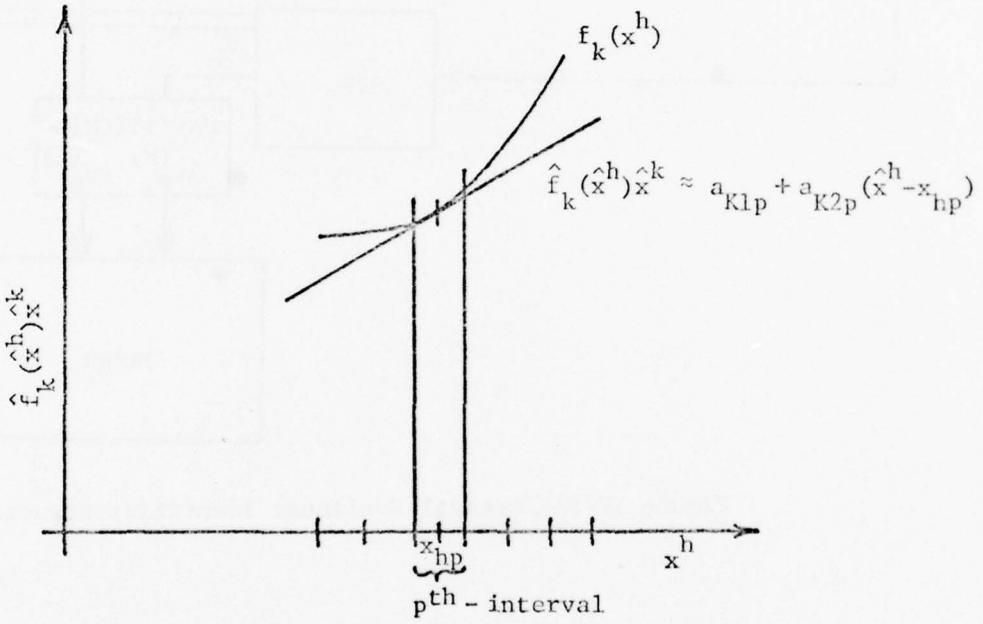


Figure IV-5(b). Linearization of a Nonlinear Function Using Taylor Series Expansion.

5. Memory Nonlinearity Piecewise Series Fit [144]

This modeling approach parallels that of the memoryless piecewise series fit and includes it as a special case. More importantly, it allows for the analytical modeling of memory nonlinearities, noticeable examples of which are gear backlash and relay-with-hysterisis.

A memory nonlinearity is describable in input-output terms only if initial pairs of input-output values, plus the entire time histories of the inputs, are specified. Therefore, the memory nonlinearity is a *functional*, since it yields an output which depends on the input *function*, represented as

$$y(t) = \mathcal{F}[x(t)] \qquad \text{(IV-32.B)}$$

where $x(t)$ is the input *function* and $y(t)$ is the output at t due to the time history $x(t)$ and operator $\mathcal{F}$.

There are two classes of memory elements, *passive* and *active* [1]. A memory element is passive if, over one cycle of a sinusoidal input $x(t)$, the nonlinear characteristic is traversed in such a way that a nonzero area is encircled in the $x(t) - \mathcal{F}[x(t)]$ plane in a counterclockwise direction. Such an element always gives rise to a lagging phase shift. A memory element is active if, over one cycle of a sinusoidal input $x(t)$, the nonlinear characteristic is traversed in such a way that a nonzero area is encircled in the $x(t) - \mathcal{F}[x(t)]$ plane in a clockwise direction. The active memory always yields a leading phase shift. These phase shift phenomena distinguish memory elements from memoryless, because a zero-memory element yields no phase shift. This idea of phase shift comes from describing function theory, which is related to quasilinear pilot modeling.

A general dynamic system may be made up of a series of nonlinear elements, i.e.

$$x^{(n)} + \cdots + f_k(x^{(h)})x^{(k)} + \cdots + F_i(x^{(i)}) + \cdots$$
$$+ f_o(x^{(s)})x^{(o)} = u^{(o)} + \cdots + g_j(u^{(h)})\, u^{(j)} + \cdots$$
$$+ g_m(u^{(g)})\, u^{(m)} \qquad \text{(IV-33.B)}$$

The functions f_i, g_i are memoryless nonlinearities which are of class C_o (continuous function whose first and higher derivatives may not be continuous), and

$$f_i(o) = 0$$
$$g_j(o) = 0 \qquad \text{(IV-34.B)}$$

$F_i(x^{(i)})$ is a multivalued functional, of which only one may occur in (IV-33.B) due to uniqueness conditions. Each of the f_i, g_i, F_k elements may be expanded with piecewise continuous functions. This is accomplished by dividing each argument into intervals bounding the range of operation of the function or functional and a power series expansion effected in each interval. The single valued terms f_i and g_s have the same expansions as in (IV-30.B) and (IV-31.B). The single allowable multivalued functional is expanded in the pth interval according to

$$F_i(\omega)\Big|_p = A_{i1_p} + A_{i2_p}(\omega - \omega_p) + A_{i3_p}(\omega^{(1)} - \omega_p^{(1)})$$
$$+ \cdots + A_{ir_p}(\omega^{(r-2)} - \omega_p^{(r-2)}) + \cdots \qquad \text{(IV-35.B)}$$

where ω_p is the expansion point in the pth interval (usually the midpoint). The reason for the restriction to only one F_i memory functional is due to uniqueness and parameter identifier convergence problems associated with the zero-power constant A_{i1_p}. If a multiplicity of these terms were present,

it would not be possible to distinguish between such terms as they all act as D.C. "inputs" to a model. Note the significant difference between A_{i1_p} in (IV-35.B) and a_{k1_p} in (IV-30.B), (IV-27.B).

6. Hammerstein-Laguerre Discrete Function Model [165]

The following model is developed for discrete-time systems of the form shown in Figure IV-6, where a memoryless nonlinear polynomial gain is followed by a linear discrete impulse response defined via Laguerre functions. The same modeling principle will also hold for continuous-time systems.

The model is defined by the non-linear part

$$f(u(kT)) = \sum_{i=1}^{\ell} \gamma_i \, u^i(kT) \tag{IV-36.B}$$

and the linear part

$$g(kT) = \sum_{i=1}^{N} A_i \, \Theta_i(kT) \tag{IV-37.B}$$

where

$$\Theta_i(z) = \sqrt{1 - e^{-2T}} \left[\frac{(z^{-1} - e^{-T})^{i-1}}{(1 - z^{-1} e^{-T})^i} \right] \tag{IV-38.B}$$

$$i = 1, 2, \cdots$$

or

$$\Theta_i(z) = \sum_{n=0}^{\infty} \theta_i(nT) z^{-n} \tag{IV-39.B}$$

$$\mathcal{Z}\{u(k-1)T\} = z^{-1} \, u(z) \tag{IV-40.B}$$

$$\sum_{n=0}^{\infty} \theta_i(nT) \, \theta_j(nT) = \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases} \qquad i,j \leq n \tag{IV-41.B}$$

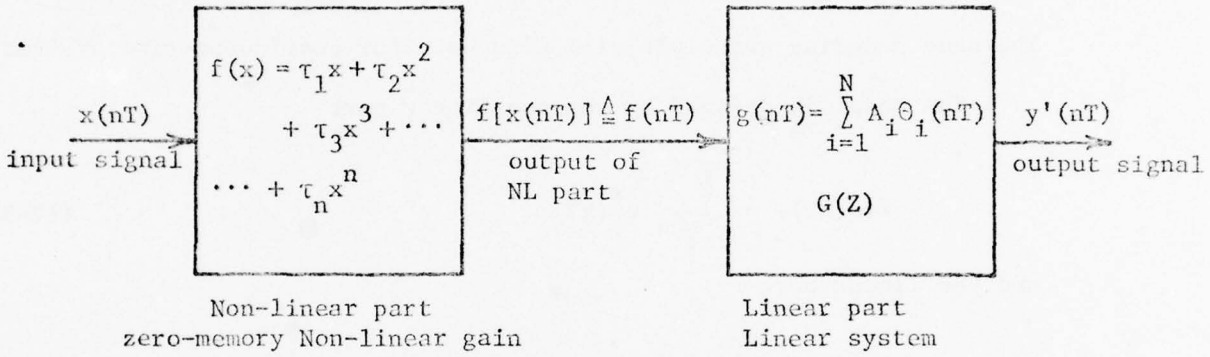


Figure IV-6. Model of the Nonlinear System Using Laguerre Functions.

T = sampling period

The model output is given by

$$x_m(nT) = \sum_{k=0}^{\infty} g(kT)\ f[(n-k)T] \qquad \text{(IV-42.B)}$$

where $f[u(nT)] = f(nT)$, or

$$x_m(nT) = \hat{\underline{Y}}^T \sum_{i=0}^{\infty} \left[\sum_{j=1}^{N} \hat{A}_j\ \theta_j(iT)\ \underline{u}[(n-i)T] \right] \qquad \text{(IV-43.B)}$$

where

$$\underline{u}^T(\cdot) = [u(\cdot)\ u^2(\cdot) \cdots u^{\ell}(\cdot)] \qquad \text{(IV-44.B)}$$

$$\hat{\underline{Y}}^T = [\hat{Y}_1\ \ \hat{Y}_2 \cdots\ \hat{Y}_{\ell}] \qquad \text{(IV-45.B)}$$

One parameterization of (IV-43.B) is

$$\hat{\underline{A}}(pT) = \hat{B}(pT)^{-1}\, \hat{\underline{C}}(pT) \qquad \text{(IV-46.B)}$$

where

$$\hat{\underline{A}}^T(pT) = [\hat{A}_1(pT)\ \hat{A}_2(pT) \cdots\ \hat{A}_N(pT)] \qquad \text{(IV-47.B)}$$

$$\hat{\underline{B}}(pT) = \begin{bmatrix} \hat{B}_1(pT) & \hat{B}_2(pT) & \cdots \hat{B}_N(pT) \\ \hat{B}_2(pT) & \hat{B}_3(pT) & \cdots \hat{B}_{N+1}(pT) \\ \hat{B}_N(pT) & \hat{B}_{N+1}(pT) & \cdots \hat{B}_{2N-1}(pT) \end{bmatrix} \qquad \text{(IV-48.B)}$$

$$\hat{\underline{C}}^T(pT) = [\hat{C}_1(pT)\quad \hat{C}_2(pT) \cdots\ \hat{C}_N(pT)] \qquad \text{(IV-49.B)}$$

$$\hat{B}_j(pT) = \frac{1}{p+1} \sum_{r=0}^{p} u(pT)\ f_j(pT) \qquad j=1,2,\cdots,2N-1 \qquad \text{(IV-50.B)}$$

$$f_j(pT) = f(pT) \cdot \theta_j(pT) \qquad \text{(IV-51.B)}$$

$$\hat{C}_k(pT) = \frac{1}{p+1} \sum_{r=0}^{N} u(rT)\, y_{k-1}(rT) \qquad \text{(IV-52.B)}$$

$$y_{k-1}(rT) = \theta_{k-1}(rT)\, y(rT) \qquad \text{(IV-53.B)}$$

and $y(rT)$ is the actual system output at time $t = rT$.

Using the computed values $\hat{\underline{A}}(pT)$, the $\hat{\gamma}_k$ terms can be determined at $t = pT$ using some optimization criteria. In [165], minimization of the mean square error is suggested,

$$\overline{e^2} = \frac{1}{M} \left\{ \sum_{m=1}^{M} \hat{\underline{\gamma}}^T \sum_{i=0}^{\infty} g(iT)\, \underline{u}[(m-1)T] - y(mT) \right\}^2 \qquad \text{(IV-54.B)}$$

Then at each value of p, $\hat{\underline{\gamma}}(pT)$ can be determined assuming $\underline{A} = \hat{\underline{A}}(pT) =$ constant, thus alternating the $\underline{A}$ and $\underline{\gamma}$ values.

C. Nonlinear MRAS Identification Algorithms

In this section, the MRAS identification algorithms investigated will be presented. Both series-parallel and parallel configurations are represented. For completeness, Table IV-2 is given, tabulating all of the nonlinear methods and their operating conditions.

1. Lion [102]

As an extension of the linear, time-invariant results given in Chapter 3, Lion's method may be applied to nonlinear systems where the parameters to be identified enter linearly and the structure of the nonlinearity is known *a priori*. It is a continuous-time SISO formulation. The general (memoryless) nonlinear plant is of the form

$$y_p^{(n)} + \sum_{i=0}^{\ell} a_i\, f_i(\underline{v}_p) = \sum_{j=0}^{m} b_j\, u^{(j)} \qquad \text{(IV-1.C)}$$

where $\ell \geq n-1$, $f_i(\cdot)$ are nonlinear functions of $\underline{v}$,

TABLE IV-2. NONLINEAR MRAS-TYPE IDENTIFICATION APPROACHES

Number	Method	Type	Knowledge of Non-Linearity Structure Required	Error Structure	I/O Structure
1	Lion [102]	C	Y	GEE	SISO
2	Hang [109]	C	Y	RE	SISO
3	Kudva, Narendra [113]	D	Y	EE	MIMO
4	Sehitoglu [144]	C	N	EE	SISO
5	Tomizuka [142]	C	Y	RE	SISO
6	Discrete Nonlinear	D	Y	RE	SISO

$$\underline{v}_p^T = [y_p \quad y_p^{(1)} \quad y_p^{(2)} \cdots y_p^{(n)}] \tag{IV-2.C}$$

The input is u and the measured output y. A model is defined by

$$y_m^{(n)} + \sum_{i=0}^{\ell} \alpha_i f_i(\underline{v}_p) = -\sum_{j=0}^{m} \beta_j u^{(j)} \tag{IV-3.C}$$

The equation error ε_o is defined as

$$\varepsilon_o = y_p^{(n)} - y_m^{(n)} \tag{IV-4.C}$$

or

$$\varepsilon_o = y_p^{(n)} + \sum_{i=0}^{\ell} \alpha_i f_i(\underline{v}_p) + \sum_{j=0}^{m} \beta_j u^{(j)} \tag{IV-5.C}$$

New states y_{p_k} and u_j are defined as

$$y_{p_k} = M_k y_p \tag{IV-6.C}$$

$$u_j = M_j u \tag{IV-7.C}$$

$$M_j = H(s) (s+c)^j \qquad i = 0,1,\cdots \tag{IV-8.C}$$

where H(s) is an arbitrary stable filter and $c > 0$ is constant. Additional equation errors are defined as

$$\varepsilon_i = y_{p(n+i)} + \sum_{i=0}^{\ell} \alpha_i f_i(\underline{v}_{p_i}) + \sum_{j=0}^{m} \beta_j u_{j+i} \tag{IV-9.C}$$

where

$$\underline{v}_{p_i}^T = [y_{p_i} \quad y_{p_{(i+1)}} \quad \cdots \quad y_{p_{(n+i)}}] \tag{IV-10.C}$$

Then

$$\underline{\varepsilon} = W(t) \underline{z} \tag{IV-11.C}$$

$$W(t) = \begin{bmatrix} f_1(\underline{v}_{p_1}) & f_2(\underline{v}_{p_1}) \cdots f_\ell(\underline{v}_{p_1}) & u_1 & u_2 \cdots u_{m+1} \\ f_1(\underline{v}_{p_2}) & f_2(\underline{v}_{p_2}) \cdots f_\ell(\underline{v}_{p_2}) & u_2 & u_3 \cdots u_{m+2} \\ \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \end{bmatrix} \qquad \text{(IV-12.C)}$$

$$\underline{z}^T = [(\alpha_1 - a_1)\ (\alpha_2 - a_2) \cdots (\alpha_\ell - a_\ell)\ (\beta_o - b_o)\ (\beta_1 - b_1) \cdots (\beta_m - b_m)] \qquad \text{(IV-13.C)}$$

The parameter adjustment equations are

$$\dot{\underline{z}} = -k\, W^T(t)\, \underline{\varepsilon} \qquad \text{(IV-14.C)}$$

$k > 0$ constant (designer selected weighting parameter).

As an example of this concept, consider the plant

$$y_p^{(2)} + a_z\, y_p^{(1)} + a_1\, y_p + a_o\, y_p^3 = u \qquad \text{(IV-15.C)}$$

The ε_i are

$$\varepsilon_i = y_{p_{(i+2)}} + \alpha_z\, y_{p_{(i+1)}} + \alpha_1\, y_{p_{(i)}} + \alpha_o\, y^3_{p_{(i)}} - u_i \qquad \text{(IV-16.C)}$$

$$i = 1, 2, 3$$

The parameter adjustment equations are

$$\begin{bmatrix} \dot{\alpha}_2 \\ \dot{\alpha}_1 \\ \dot{\alpha}_o \end{bmatrix} = -k\, W(t)\, \underline{\varepsilon} \qquad \text{(IV-17.C)}$$

$$W(t) = \begin{bmatrix} \dot{y}_{p_1} & y_{p_1} & y^3_{p_1} \\ \dot{y}_{p_2} & y_{p_2} & y^3_{p_2} \\ \dot{y}_{p_3} & y_{p_3} & y^3_{p_3} \end{bmatrix} \qquad \text{(IV-18.C)}$$

It is important to note that, although the plant is nonlinear, the parameter adjustment equations are linear in $\underline{c}$.

2. Hang [109]

This is a multi-variable, continuous-time formulation utilizing the response error approach. Consider a class of nonlinear systems with *a priori* known nonlinearity plant structure

$$\begin{aligned} \dot{\underline{x}}_p &= A_{pL}\,\underline{x}_p + B_p\,\underline{u} + A_{pN}\,\underline{F}(\underline{x}_p, \underline{u}) \\ \underline{y}_p &= C\,\underline{x}_p \end{aligned} \qquad \text{(IV-19.C)}$$

where

$$A_{p_L} = \begin{bmatrix} 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & & & \\ -a_{1_L} & -a_{2_L} & \cdots & -a_{n_L} \end{bmatrix} \qquad B_p = \begin{bmatrix} 0 & 0 \cdots & 0 \\ \vdots & & \\ b_1 & b_2 \cdots & b_n \end{bmatrix} \qquad \text{(IV-20.C)}$$

$$C = [1 \quad 0 \cdots 0] \qquad \text{(IV-21.C)}$$

$$A_{p_N} = \begin{bmatrix} 0 & 1 \cdots & 0 \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ -a_{1_N} & -a_{2_N} \cdots & -a_{n_N} \end{bmatrix} \qquad \text{(IV-22.C)}$$

The tracking error is defined as

$$\underline{e} = \underline{x}_p - \underline{x}_m \qquad \text{(IV-23.C)}$$

Since generally only one input and one output are available, pseudo-state-variables are generated from u_1 and x_{p_1}

$$x_{p_{f_1}} = \frac{1}{N_f(s)} \quad x_{p_1} \qquad \text{(IV-24.C)}$$

$$u_{f_1} = \frac{1}{N_f(s)} \quad u_1 \qquad \text{(IV-25.C)}$$

where $N_f(s)$ is selected to meet an approximate transport lag condition.

$$e_{f_1} = x_{p_{f_1}} - x_{m_1} \qquad \text{(IV-26.C)}$$

where the model is

$$\dot{\underline{x}}_m = A_{m_L} \underline{x}_m + B_m \underline{u}_f + A_{m_N} \underline{F}(\underline{x}_{p_f}, \underline{u}_f) \qquad \text{(IV-27.C)}$$

The remaining $x_{p_{f_i}}$, u_{f_i} come from (IV-24.C) and (IV-25.C). Using Popov's Hyperstability Theory, a generalized response error is defined as

$$v_{f_1}(s) = Z_1(s)\, e_{f_1}(s) \qquad \text{(IV-28.C)}$$

$$Z_1(s) = \sum_{j=0}^{\ell} z_i\, s^i \qquad \text{(IV-29.C)}$$

such that

$$Z_1(s)\ C(sI - A_{p_L})^{-1} \qquad \text{(IV-30.C)}$$

is strictly positive real, $(n-1) \leq \ell \leq (n+1)$. Since derivatives of $x_{p_{f_1}}$, x_{m_1} are available, no actual differentiations from (IV-28.C) are required.

Because of the form of (IV-19.C), $\underline{F}$ acts as an external input similar to $\underline{u}$, so filtered values of $\underline{F}$ are obtained from

$$F_{f_1} = \frac{1}{N_f(s)}\ F_1(s) \qquad \text{(IV-31.C)}$$

with F_{f_i} available from

$$F_{f_i} = \left[\frac{1}{N_f(s)}\right] F_i(s) \qquad \text{(IV-32.C)}$$

Using these expressions, the parameter identifier expressions are

$$\dot{a}_{m_{L_j}} = -\left[\alpha_j\ v_{f_1}\ x_{m_j} + \beta_j \frac{d}{dt}\ (\alpha_j\ v_{f_1}\ x_{m_j})\right] \qquad \text{(IV-33.C)}$$

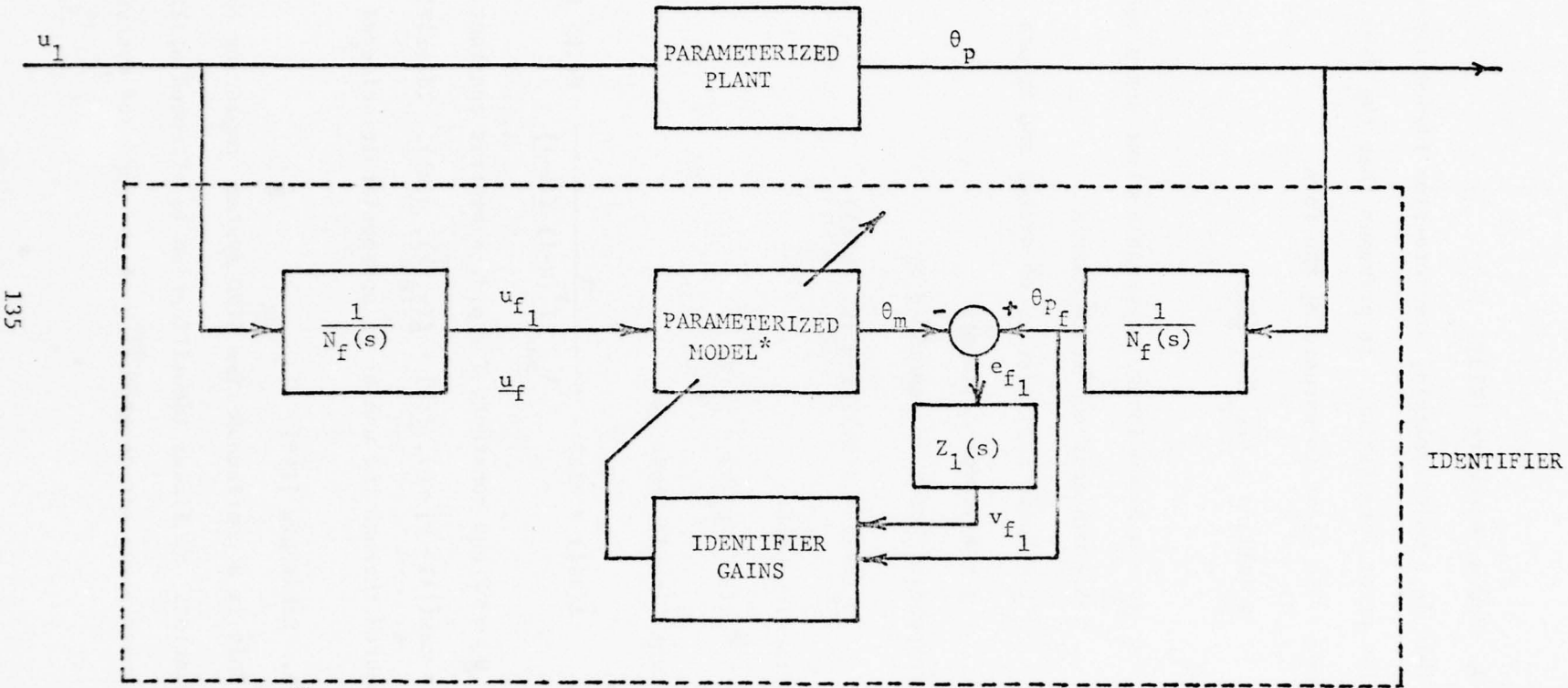
$$\dot{b}_{m_j} = \alpha_j\ v_{f_1}\ u_{f_j} + \delta_j \frac{d}{dt}\ (\alpha_j\ v_{f_1}\ u_{f_1}) \qquad \text{(IV-34.C)}$$

$$\dot{a}_{m_{N_j}} = -\left[\Psi_j\ v_{f_1}\ F_{f_i} + \rho_j \frac{d}{dt}\ (\Psi_j\ v_{f_1}\ F_{f_i})\right] \qquad \text{(IV-35.C)}$$

If the output equation from (IV-19.C) is corrupted by a noise $\underline{\eta}$, i.e.

$$\underline{y}_p = C\ \underline{x}_p + \underline{\eta} \qquad \text{(IV-36.C)}$$

then the estimates $a_{m_{N_j}}$ will be asymptotically biased. This can be minimized by replacing $\underline{F}(\underline{x}_{p_f}, \underline{u}_f)$ in (IV-27.C) by $\underline{F}(\underline{x}_m, \underline{u}_f)$, but the tradeoff is that global stability is no longer assured. This general nonlinear identifier structure is shown in Figure IV-7.



*If linear, $\frac{N_m}{D_m}(s)$

If nonlinear, $F(u_f, \theta_m)$

Figure IV-7 Generalized Hang Identifier Structure.

3. Kudva, Narendra [113]

This is a multivariable, discrete-time identifier employing the equation error formulation. It presumes that the *form* of the nonlinearity is known. The plant is assumed of the form

$$\underline{x}_p(k+1) = A\ \underline{f}(\underline{x}_p(k), \underline{u}(k)) \qquad \text{(IV-37.C)}$$

where

$\underline{x}_p$ - n dimensional, measurable plant state vector

A - nxn constant state matrix

$\underline{f}$ - known nonlinearity of states and inputs

$\underline{u}$ - m vector of inputs

A corresponding model is described by

$$\underline{x}_m(k+1) = A(k+1)\ \underline{f}(\underline{x}_p(k), \underline{u}(k)) \qquad \text{(IV-38.C)}$$

with tracking error

$$\underline{e}(k) = \underline{x}_m(k) - \underline{x}_p(k) \qquad \text{(IV-39.C)}$$

The identification rule is

$$\hat{A}(k+1) = \hat{A}(k) - \frac{\alpha}{\lambda_{max}\ \underline{f}^T(k-1)\ \underline{f}(k-1)}\ P\underline{e}(k)\ \underline{f}^T(k-1) \qquad \text{(IV-40.C)}$$

where $0 < \alpha < 2$ and constant, P a p.d. symmetric constant matrix, $\lambda_{max} = \max_i\{|\lambda I - P| = 0\}$, $\underline{f}(k) = \underline{f}(\underline{x}_p(k), \underline{u}(k))$. Identification convergence is assured through the use of an appropriately selected Lyapunov function.

4. Sehitoglu [144]

This is a continuous-time SISO system employing the equation error formulation. Nonlinear identification is effected utilizing a piecewise continuous power series expansion of the supposed system nonlinearities.

By approximating the nonlinear terms in this way, it is possible to develop an error expression which, when driven to zero, adjusts the coefficients of the piecewise continuous expansion to fit the nonlinearity.

The types of systems which can be addressed by this method include those which have nonlinear, time invariant, and lumped parameter dynamics where the nonlinearities may include up to 1 multivalued, piecewise continuous, memory nonlinearity and as many single-valued, piecewise continuous nonlinearities that the user may want.

Suppose that the system or "plant" to be identified may be described by the nth order differential equation:

$$x^n = -f_{n-1}(x^\ell)x^{n-1} - \cdots - f_k(x^h)x^k - \cdots - F_i(x^i) - \cdots$$
$$-f_o(x^s)x^o + u^o + \cdots + g_j(u^h)j^u + \cdots + g_m(u^g)u^m \qquad \text{(IV-41.C)}$$

where

$$x^n = \frac{d^n x}{dt^n} ,$$

$$u^j = \frac{d^j u}{dt^j} ,$$

u is the plant input,

x is the plant output,

and f_{n-1}, F_i, and g_m are functions of the indicated arguments. The terms $f_k(x^h)x^k$ and $g_m(u^g)u^m$ are "single-valued" functions which are assumed to be unknown, of class C° (continuous but not necessarily smooth), and identically equal to zero at the origin. The function $F_i(x^i)$, of which only one may appear in any system equation, is assumed to be unknown and possibly multivalued. The multivalued nature of this functional need not pass through the origin and may include memory nonlinearities.

From (IV-41.C), it can be seen that: $0 = x^n + f_{n-1}(x^\ell)x^{n-1} + \cdots$

$$+ f_k(x^h)x^k + \cdots + F_i(x^i) + \cdots + f_o(x^s)x^o - u^o - \cdots - g_m(u^g)u^m \qquad \text{(IV-42.C)}$$

For the identification scheme used by this paper, the following restrictions must be observed:

1) $n > m$, or the output must have more available states than the input,

2) the plant input, u^o, and the plant output, x^o, must be measurable,

3) the input, u^o is selectable by the user and must be sufficiently frequency rich [143], and

4) the system may possess at most one multivalued nonlinear function.

The first restriction is used to allow for physical realizability where knowledge of the future is unavailable (pure differentiation). The second restriction results from the identification process itself where the input and the output are required to be present and measurable. The frequency richness criterion imposed by item (3) above is necessary in order to stimulate all the plant states and elicit a response from them. The final requirement is necessary because of uniqueness problems if more than one nonlinearity is present.

In the identification process x^o and u^o are passed through state variable filters which develop approximations to the system's states which are not accessible to the identifier. Hoberock and Kohr [162] employed a "time delay filter" which approximately reconstructs the system's inaccessible states, given that the system is single input-single output. Kohr in [161] expanded on the idea of the "state variable" or "time lag" filter and presents a technique for determining a single nonlinear element in a system by equating it to the remaining known terms of the differential

describing the system. Sehitoglu and Klein [144] discuss the generalized equation error method by which nonlinearities in all states of the dynamic equation describing a system may be identified. From these approximate states an "equation error" may be formed"

$$\hat{E} = \hat{x}^n + \hat{f}_{n-1}(\hat{x}^{\ell})\hat{x}^{n-1} + \cdots + \hat{F}_i(\hat{x}^i) + \cdots + \hat{f}_o(\hat{x}^s)\hat{x}^o$$

$$-\hat{u}^o - \cdots - \hat{g}_j(\hat{u}^h)\hat{u}^j - \cdots - \hat{g}_m(\hat{u}^g)\hat{u}^m \qquad \text{(IV-43.C)}$$

analogous to (IV-42.C), where ^ represents an estimation of the appropriate state or function.

By using Taylor series expansions of each function f, F, or g over the expected operating domain, piecewise approximations to the functions can be obtained. To effect this the domain is broken up into intervals where a first-order approximation would be valid. For a single valued function, f or g,

$$\hat{f}_k(\hat{x}^h)\hat{x}^k = [a_{k1p} + a_{k2p}(\hat{x}^h - x_{hp}) + \cdots + a_{krp}(\hat{x}^h - x_{np})^{(r-2)} + \cdots]\hat{x}^k \qquad \text{(IV-44.C)}$$

where

k denotes that the function is of the kth order in the differential equation

r denotes the power of the approximation expansion term

p denotes the interval number that the function is in

h denotes the function argument index, and

x_{hp} denotes the midpoint value of the pth interval;

for example, for the single allowable multivalued function:

$$\hat{F}_i(\hat{x}^i) = A_{i1p} + A_{i2p}(\hat{x}^i - x_{ip}) + \cdots + A_{irp}(\hat{x}^i - x_{ip})^{(r-2)} + \cdots \qquad \text{(IV-45.C)}$$

where similar notation as (IV-44.C) is used. Here it can be seen why only one multivated function is allowed; since the entire function must be expanded, if more than one multivalued function were expanded in this manner, the constant terms at the beginning of each function would not be distinguishable by the identifier.

Using (IV-44.C) and (IV-45.C) in (IV-43.C) results in the equation error, $\hat{E}$,

$$\hat{E} = \hat{x}^n + [(a_{(n-1)1p} + a_{(n-1)2p}(\hat{x}^{\ell} - x_{\ell p})]\hat{x}^{n-1} + \cdots$$
$$+ [a_{k1p} + a_{k2p}(\hat{x}^h - x_{hp})]\hat{x}^k + \cdots + A_{i1p} + A_{i2p}(\hat{x}^i - x_{ip}) + \cdots$$
$$+ [x^a_{o1p} + a_{o2p}(\hat{x}^s - x_{sp})]\hat{x}^o - u^o - \cdots - [b_{j1p} + b_{j2p}(\hat{u}^h - u_{hp})]\hat{u}^j - \cdots$$
$$- [b_{m1p} + b_{m2p}(\hat{u}^g - u_{gm})]\hat{u}^m \qquad \text{(IV-46.C)}$$

To develop the identifier expressions for a_{k1p}, A_{i1p}, b_{j1p}, etc., it is necessary to define a cost function J,

$$J = \int_o^t f(E)dt \qquad \text{(IV-47.C)}$$

where f(E) is an even valued function of E. Let

$$J = \int_o^t 1/2\ E^2 dt \qquad \text{(IV-48.C)}$$

It is desired that J be optimized for a "best fit" result to occur. For a typical parameter, γ, define

$$\dot{\gamma}_k = -\ \Gamma_k \frac{\partial J}{\partial \gamma_k} \qquad \text{(IV-49.C)}$$

where Γ_k is positive so that γ_k is proportional to the negative gradient of the cost functional with respect to the parameter of interest. From (IV-49.C),

$$\gamma_k = -\Gamma_k \frac{\partial J}{\partial E} \frac{dE}{d\gamma_k} = -\Gamma(E)(\hat{x}^k) \qquad \text{(IV-50.C)}$$

This gives the following parameter adaptation equations:

$$\begin{aligned}
\dot{a}_{(n-1)1p} &= -G_{(n-1)1p}\hat{E}\,\hat{x}^{n-1} \\
\dot{a}_{(n-1)2p} &= -G_{(n-1)2p}\hat{E}(\hat{x}^{\ell} - x_{\ell p})\hat{x}^{n-1} \\
&\vdots \\
\dot{a}_{k1p} &= -G_{k1p}\hat{E}\,\hat{x}^{k} \\
&\vdots \\
\dot{A}_{i1p} &= -G_{i1p}\hat{E} \\
\dot{A}_{i2p} &= -G_{i2p}\hat{E}(\hat{x}^{i} - x_{ip}) \qquad \text{(IV-51.C)} \\
&\vdots \\
\dot{a}_{01p} &= -G_{01p}\hat{E}\,\hat{x}^{o} \\
\dot{a}_{02p} &= -G_{02p}\hat{E}(\hat{x}^{s} - x_{sp})\hat{x}^{o} \\
&\vdots \\
\dot{b}_{j1p} &= -G_{j1p}\hat{E}\,\hat{u}_j \\
\dot{b}_{j2p} &= -G_{j2p}\hat{E}(\hat{u}^{h} - u_{np})\hat{u}^{j} \\
&\vdots \\
\dot{b}_{m1p} &= -G_{m1p}\hat{E}\,\hat{u}^{m} \\
\dot{b}_{m2p} &= -G_{m2p}\hat{E}(\hat{u}^{g} - u_{gp})\hat{u}^{m}
\end{aligned}$$

where H, G are constants > 0.

An important point to note about such nonlinear identifiers is that essentially only one set of dynamics are identified by such an approach. This contrasts with the approach of employing a series of linearized models where all parameters must be re-identified at each discretized interval. Only key nonlinear terms need to be redetermined this way, allowing for a fixed set of linear dynamic terms, more consistant models from interval to interval, and an expected faster convergence rate (since less parameter identification term-by-term coupling).

5. Tomizuka [142]

This is a SISO, continuous-time technique employing the parallel error configuration. The method is exact for a class of nonlinear systems in which a zero-memory nonlinear gain is followed by a linear dynamical system. Given a scalar input u(t), the plant is described by

$$\frac{x_p}{r} = \frac{\sum_{j=0}^{m} b_i s^i}{s^n + \sum_{j=0}^{n-1} a_j s^i} \qquad \text{(IV-52.C)}$$

where

$$r(t) = \sum_{h=1}^{\ell} k_h \, n_h(u(t)) \qquad \text{(IV-53.C)}$$

k_n - unknown plant nonlinearity constants

n_h - known single-valued functions of u(t)

a_i, b_i - unknown linear plant portion constn dynamic parameters

$m \leq n$

ℓ - number of nonlinear gains n_h

For simplicity, the nonlinear system is parameterized as

$$x_p(s) = \sum_{h=1}^{\ell} \frac{F_n(s)}{A(s)} n_h(u(s)) \qquad \text{(IV-54.C)}$$

$$F_n(s) = \sum_{i=0}^{m} f_{hi} s^i \qquad \text{(IV-55.C)}$$

$$f_{n_i} = k_h b_i \qquad \text{(IV-56.C)}$$

$$A(s) = s^n + \sum_{j=0}^{n=1} a_j s^j \qquad \text{(IV-57.C)}$$

where the a_i and f_{h_i} are to be identified. An estimation model is defined as

$$x_m = \sum_{h=1}^{\ell} \frac{\hat{F}_h(s)}{\hat{A}(s)} n_h \qquad \text{(IV-58.C)}$$

$$\hat{F}_h(s) \sum_{i=0}^{m} \hat{f}_{h_i} s^i \qquad \text{(IV-59.C)}$$

$$\hat{A}(s) = s^n + \sum_{j=0}^{n-1} \hat{a}_i s^i \qquad \text{(IV-60.C)}$$

and $\hat{f}_{h_i}$, $\hat{a}_i$ are time varying. The "input" $\hat{n}_h$ is obtained from

$$\hat{n}_h = \frac{n_h(s)}{D(s)} \qquad \text{(IV-61.C)}$$

where D(s) is a state variable filter,

$$D(s) = s^{n-1} + \sum_{i=0}^{n-2} d_i s^i \qquad \text{(IV-62.C)}$$

The filter $\frac{1}{D(s)}$ is selected such that the filter passband is much greater than that of the plant. On a similar basis, define

$$\hat{x}_p(s) = \frac{x_p(s)}{D(s)} \qquad \text{(IV-63.C)}$$

The scalar tracking error, e, is then given by

$$e = \hat{x}_p - x_m \qquad \text{(IV-64.C)}$$

A block diagram of this approach is shown in Figure IV-8.

To insure asymptotic stability, hyperstability conditions require that a compensation block C(s) process e(s),

$$v(s) = C(s)\, e(s) \qquad \text{(IV-65.C)}$$

with the condition $\frac{C(s)}{A(s)}$ be strictly positive real. This creates a physical realizability problem in that A(s) is unknown and the SPR condition yields non-unique solutions for C(s). This is discussed at length in Chapter 6. Note that although (IV-65.C) suggests differentiation, because of (IV-60.C) and (IV-63.C) $\dot{e}$, $\ddot{e}$, etc. are physically available and hence v(t) is available without differentiating e(t).

The identification equations then become

$$\hat{a}_i(t) = -\alpha_{a_i} \int_o^t v(\tau)\, \frac{d^i}{d\tau^i}\,[\hat{x}_p(\tau)]d\tau$$

$$-\beta_{a_i}\, v(t)\, \frac{d^i}{dt^i}[\hat{x}_p(t)] + \hat{a}_i(o) \qquad \text{(IV-66.C)}$$

$$\hat{f}_{h_j}(t) = \alpha_{fh_j} \int_o^t v(\tau)\, \frac{d^j}{d\tau^j}[\hat{n}_h(\tau)]d\tau$$

$$+ \delta_{fh_j}\, v(t)\, \hat{n}_h(t) + \hat{f}_{h_j}(o) \qquad \text{(IV-67.C)}$$

$$\alpha_{a_i} > 0, \quad \beta_{a_i} \geq 0 \qquad i = 0,1,\cdots,n-1$$

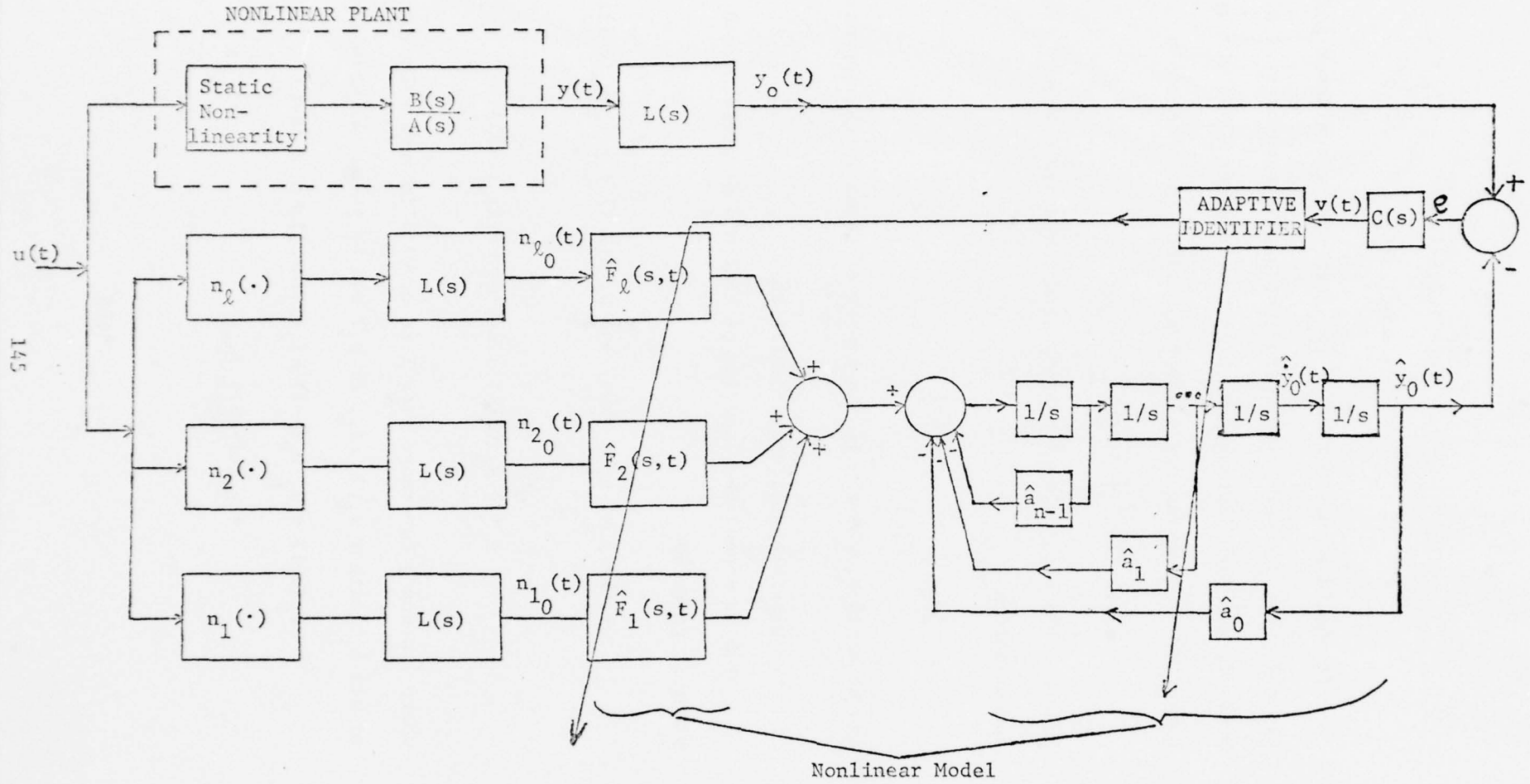


Figure IV-6. Parallel Structure for Continuous-Time Nonlinear MRAS Identifier.

$$\gamma_{f_{h_j}} \geq 0, \quad \delta_{f_{h_j}} \geq 0, \qquad \begin{matrix} h = 1,2,\cdots,\ell \\ j = 0,1,\cdots,m \end{matrix}$$

It should be noted that if the form of the nonlinear gains $n_h(u(t))$ are not known *a priori*, the method outlined cannot be strictly employed. However, an approximate approach is to represent (IV-53.C) by a polynomial or sine-cosine series approximation. As an example, suppose

$$m(t) = 2u^2 + 3|u(t)| \tag{IV-68.C}$$

but that the u^2, $|u|$ nonlinearity structures are unknown. Then an approximation

$$\hat{m}(t) = \sum_{i=1}^{p} k_i \, u^{i-1} \tag{IV-69.C}$$

can be defined, where p is the number of terms in the approximation.

6. Discrete Nonlinear Identifier

This is a nonlinear version of Landau's discrete method [116]. The plant is given by

$$x_p(k+1) = \sum_i^n a_i \, x_p(k-i) + \sum_i^m b_i \, u(k-i) + \sum_i^{n'} f_i(x_p(k-i)x_p(k-i)$$

$$+ \sum_i^{m'} g_i(u(k-i)) \, u(k-i) + F_\ell(x_p(k-\ell)) \tag{IV-70.C}$$

where $\sum_j^\omega$ means j can range from 1 to ω, with integer values of j and that any or all A_j (either a_i, b_i, f_i, or g_i) can be zero. An example of (IV-70.C) is

$$x_p(k+1) = a_1 \, x_p(k-1) + f_2(x_p(k-2)) + a_4 \, x_p(k-4)$$

$$+ g_2(u(k-2)) \tag{IV-71.C}$$

The f_i, g_i are memoryless terms expanded as in (IV-30.B) and (IV-31.B), and the F_ℓ a memory nonlinearity parameterized as in (IV-35.B). Using the notation a_{ij_k} to mean the ith nonlinearity, the jth term of the series expansion, in interval k of the $x_p(k-i)$ term, a model can be approximately parameterized as

$$x_m(k+1) \cong [\hat{a}_1 \cdots \hat{a}_{il_\ell}\ \hat{a}_{i2_\ell} \cdots \hat{a}_k \cdots \hat{A}_{\Lambda 1_d}\ \hat{A}_{\Lambda 2_d} \cdots a_{j1_\omega}\ a_{j2_\omega}]\underline{x}_m(k)$$
$$+ [\hat{b}_1 \cdots \hat{b}_{ql_r}\ \hat{b}_{q2_r} \cdots \hat{b}_s \cdots \hat{b}_{t1_y}\ \hat{b}_{t2_y}]\underline{u}(k) \qquad \text{(IV-72.C)}$$

or

$$x_m(k) = \hat{\underline{p}}^T(k)\ \underline{x}_m(k-1) + \hat{R}(k) \qquad \text{(IV-73.C)}$$

$$\hat{\underline{p}}^T(k) = [\hat{a}_1(k) \cdots \hat{a}_{il_\ell}(k)\ \hat{a}_{i2_\ell}(k) \cdots \hat{A}_{\Lambda 1_d}(k)\ \hat{A}_{\Lambda 2_d}(k) \cdots \hat{a}_{j1_\omega}(k)\ \hat{a}_{2j_\omega}(k)$$
$$\hat{b}_1(k) \cdots \hat{b}_{ql_r}(k)\ \hat{b}_{q2_r}(k)] \qquad \text{(IV-74.C)}$$

$$\underline{x}_m^T(k-1) = [x_p(k-1) \cdots x_p(k-i)\ x_p^2(k-i) \cdots 1\ x_p(k-\Lambda) \cdots x_p(k-j)\ x_p^2(k-j)$$
$$u(k-1) \cdots u(k-q)\ u^2(k-q) \cdots] \qquad \text{(IV-75.C)}$$

$$R(k) = -[a_{i2_\ell}(k)\ \overline{x}_{p_\ell}(k-i)\ x_p(k-i) + \hat{\hat{A}}_{\Lambda 2_d}(k-\Lambda)\ \overline{x}_{p_d}(k-\Lambda)$$
$$+ \hat{a}_{2j_\omega}(k)\ \overline{x}_{p_\omega}(k-j)\ x_{p_\omega}(k-j) + \hat{b}_{q2_r}(k)\ \overline{u}_r(k-q)\ u(k-q)] \qquad \text{(IV-76.C)}$$

The recursive algorithm is then given by

$$x_m(k) = \hat{\underline{p}}^T(k)\ \underline{x}_m(k-1) + \hat{R}(k) \qquad \text{(IV-77.C)}$$

$$x_m^\circ(k) = \hat{\underline{p}}^T(k-1)\ \underline{x}_m(k-1) + \hat{R}(k-1) \qquad \text{(IV-78.C)}$$

$$\varepsilon^\circ(k) = x_p(k) - x_m^\circ(k) \qquad \text{(IV-79.C)}$$

$$\varepsilon(k) = x_p(k) - x_m(k) \qquad \text{(IV-80.C)}$$

$$\hat{\underline{p}}(k) = \hat{\underline{p}}(k-1) + \frac{F(k-1)\ x_m(k-1)}{1 + x_m^T(k-1)\ F(k-1)\ \underline{x}_m(k-1)}\ \nu^\circ(k) \qquad \text{(IV-81.C)}$$

$$\nu^{\circ}(k) = \varepsilon^{\circ}(k) + \sum_{i=1}^{n} c_i \, \varepsilon(k-i) \qquad \text{(IV-82.C)}$$

$$F(k) = F(k-1) - \frac{F(k-1)\, \underline{x}_m(k-1)\, \underline{x}_m^T(k-1)\, F(k-1)}{1 + \underline{x}_m(k-1)\, F(k-1)\, \underline{x}_m(k-1)} \qquad \text{(IV-83.C)}$$

$F(o)$ is an arbitrary p.d. symmetric matrix, and $\underline{x}_m(o)$ and $\underline{p}(o)$ are designer-controlled initial condition guesses.

D. Identifiability of Nonlinear Model Structure

In general the parameter identification problem involves the calculation of unknown plant parameters using an identification algorithm requiring only input/output type data. Although this idea sounds simple, there is the problem of how accurate a model parameterization can be, or whether a given model can possess unique parameters. This is the problem of "identifiability" [166]. To relate this problem to nonlinear identification and eventually the nonlinear human operator problem characterization, appropriate definitions will be stated to clarify the problem and its practical analysis.

Consider the general plant

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, t, \underline{p}) \qquad \underline{x}(t_o) = \underline{x}_o$$

$$\underline{y} = \underline{h}(\underline{x}, \underline{u}, t, \underline{p}) \qquad \text{(IV-1.D)}$$

where $\underline{x}$ is an n-vector, $\underline{f}$ is a nonlinear vector function, $\underline{u}$ is an m-vector of inputs, $\underline{h}$ is a nonlinear vector function ($\underline{h} \neq \underline{f}$), $\underline{p}$ is a q-vector of parameters, and $\underline{y}$ is an r-vector of outputs. If the solution to (IV-1.D) were known, then $\underline{x}$ could be written as

$$\underline{x}(t) = \underline{\Phi}(t, t_o)\, \underline{x}(t_o) \qquad \text{(IV-2.D)}$$

with $\underline{\Phi}(\cdot,\cdot)$ the system state-transition matrix. Let the output be defined by

$$\underline{y}(t,\underline{p}) = \underline{F}(\underline{x}_o, u(\cdot), t, \underline{p}) \qquad \text{(IV-3.D)}$$

where $(\underline{x}_o, u(\cdot))$ is an experiment and the problem reduces from distinguishing between parameter values to distinguishing between corresponding $\underline{y}$ (output) values.

Definition IV-1 [167-169]

The parameter vectors $\underline{p}$ and $\underline{\alpha}$ are *indistinguishable* if

$$\underline{F}(\underline{x}_o, \underline{u}(\cdot), t, \underline{p}) = \underline{F}(\underline{x}_o, \underline{u}(\cdot), t, \underline{\alpha}) \qquad \text{(IV-4.D)}$$

and otherwise $\underline{p}$ and $\underline{\alpha}$ are *distinguishable*.

Definition IV-2

A parameter vector $\underline{p}$ is locally identifiable if $(\underline{p}, \underline{\alpha})$ is distinguishable for $\underline{\alpha}$ contained in an ε-neighborhood of $\underline{p}$, $\underline{\alpha} \in N(\underline{p},\varepsilon)$ for some $\varepsilon > 0$ and $\underline{\alpha} \neq \underline{p}$.

Using (IV-1.D), a linearized set of state equations about the equilibrium state-input pair $(\underline{x}_e, u_e)$ is

$$\begin{aligned} \delta\dot{\underline{x}} &= A(\underline{p},t)\ \delta\underline{x} + B(\underline{p},t)\ \delta\underline{u} \\ \delta\underline{y} &= C(\underline{p},t)\ \delta\underline{x} + D(\underline{p},t)\ \delta\underline{u} \end{aligned} \qquad \text{(IV-5.D)}$$

$$\delta\underline{x} = \underline{x} - \underline{x}_e\ , \quad \delta\underline{u} = \underline{u} - \underline{u}_e\ , \quad \delta\underline{y} = \underline{y} - \underline{y}_e$$

and

$$A = \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{\underline{x}_e, \underline{u}_e} \qquad B = \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\underline{x}_e, \underline{u}_e} \qquad \text{(IV-6.D)}$$

$$C = \left.\frac{\partial \underline{h}}{\partial \underline{x}}\right|_{\underline{x}_e, \underline{u}_e} \qquad D = \left.\frac{\partial \underline{h}}{\partial \underline{u}}\right|_{\underline{x}_e, \underline{u}_e} \tag{IV-7.D}$$

The system (IV-1.D) is identifiable, if at a nominal parameter value $\hat{\underline{p}}$, the following Markov parameter matrix is a one-to-one mapping from $\underline{p}$ to the Markov parameters

$$G = [D^T, (CB)^T, (CAB)^T, \cdots (CA^{2n-1}B^T)] \tag{IV-8.D}$$

If A has constant rank β in a neighborhood of $(\underline{x}_e, \underline{u}_e)$, then the parameterization is locally identifiable if

$$\text{rank}\left\{\frac{\partial G(\underline{p})}{\partial \underline{p}}\right\} = \beta \tag{IV-9.D}$$

for all $\underline{p}$ in some neighborhood of $\underline{p}$.

An important result of the preceding is that local identifiability is independent of the parameter identification method employed, but is instead only an inherent system plant structure problem [170]. Further work on identifiability has shown that the optimal input selection criterion (such as trace of the information matrix) employed for parameter identification can result in parameterized models which are unidentifiable, if the input selection problem is not well posed [171]. This problem is further studied in Chapter 6.

CHAPTER 5. MRAS IDENTIFIERS FOR LINEAR, TIME-VARYING SYSTEMS

A. Introduction

In this chapter four key algorithmic techniques are presented for using MRAS methods in time-varying on-line parameter identification. To compare these MRAS approaches, other traditional modeling and analysis techniques for handling time-varying parameters are presented. The reason "special" algorithms are needed for time-varying parameters, as opposed to time-invariant identifiers, is that for time-invariant identifiers all observations are treated with equal weight regardless of their occurrence in time, whereas time-varying identification requires recent data only (as the old data is "outdated" since significantly different parameter values were employed at earlier time instants).

The four key techniques are 1) limited memory filtering, 2) "forgetting factor, 3) time-varying updating of output error weighting coefficients, and 4) oscillating memory. All are applied to discrete identifiers.

The type of identifier depends on the system requirements. Some of the results of a previous study [173] of these three MRAS models for use in identification of time invariant systems are tabulated below:

Parallel Model
- Asymptotically unbiased
- Moderate convergence rate
- Adaptable for time-varying

Series-Parallel Model
- Biased
- Poor convergence rate
- Adaptable for time varying

Bootstrap Model

Slightly biased

Good convergence rate

Adaptable for time varying

Based upon the above results it was decided to use the parallel model approach in forming an identifier suitable for identification of time varying systems.

B. Problem Formulation

The plant to be identified is assumed to be of known order, linear, discrete, and possibly time-varying of the form

$$\frac{x_p}{u}(z) = \frac{\left[\sum_{j=1}^{m} b_j(k) z^{i}\right] z^{-d}}{\left[1 - \sum_{i=1}^{n} a_i(k) z^{-i}\right]} \qquad \text{(V-1.B)}$$

where $b_j(k)$ and $a_i(k)$ represent possibly time-variable parameters, n is the plant order, and $m \leq n$, d is the time-delay, and $t = kT$. Such a system is in the ARMA form

$$x_p(k) = \sum_{i=1}^{n} a_i(k)\, x_p(k-i) + \sum_{j=0}^{m} b_j(k)\, u(k-j-d) \qquad \text{(V-2.B)}$$

It is assumed n, m, and d are known and the a_i, b_j are to be determined on-line in real-time.

Using an output error MRAS formulation, the estimation model is

$$x_m(k) = \sum_{i=1}^{n} \hat{a}_i(k)\, x_m(k-i) + \sum_{j=0}^{m} \hat{b}_j(k)\, u(k-j-d) \qquad \text{(V-3.B)}$$

or

$$x_m(k) = \hat{\underline{\phi}}^T(k)\, \underline{y}(k-1) \qquad \text{(V-4.B}$$

$$\underline{\hat{\phi}}^T(k) = [\hat{a}_1(k) \quad \hat{a}_2(k) \cdots \hat{a}_n(k) \quad \hat{b}_o(k) \cdots \hat{b}_m(k)] \tag{V-5.B}$$

$$\underline{y}^T(k-1) = [x_m(k-1) \quad x_m(k-2) \cdots x_m(k-n) \quad u(k-d) \cdots u(k-d-m)] \tag{V-6.B}$$

Two output errors are defined as

$$\varepsilon^\circ(k) = x_p(k) = x_m^\circ(k) \tag{V-7.B}$$

$$\varepsilon(k) = x_p(k) - x_m(k) \tag{V-8.B}$$

$$x_m^\circ(k) = \underline{\hat{\phi}}^T(k-1)\ \underline{y}(k-1) \tag{V-9.B}$$

where $\varepsilon^\circ(k)$ is an *a priori* error and $\varepsilon(k)$ is an *a posteriori* error. From Chapter 3, one form of MRAS identification is Landau [116],

$$\underline{\hat{\phi}}(k) = \underline{\hat{\phi}}(k-1) + \frac{F(k-1)\ \underline{y}(k-1)}{1+\underline{y}^T(k-1)\ F(k-1)\ \underline{y}(k-1)}\ e(k) \tag{V-10.B}$$

$$e(k) = \varepsilon^\circ(k) + \sum_{i=1}^{n} c_i\ \varepsilon(k-i) \tag{V-11.B}$$

where

$$H(z) = \frac{1 + \sum_{i=1}^{n} c_i\ z^{-i}}{1 - \sum_{i=1}^{n} a_i(k) z^{-i}} \tag{V-12.B}$$

is SPR, and

$$F(k) = F(k-1) - \frac{F(k-1)\ \underline{y}(k-1)\ \underline{y}^T(k-1)\ F(k-1)}{1 + y^T(k-1)\ F(k-1)\ \underline{y}(k-1)} \tag{V-13.B}$$

This is the so-called "integral" identifier because it is the discretized equivalent of a continuous-time identifier having an integral form. F is a monotonic decreasing matrix

$$F(k+1) < F(k) \qquad \text{(V-14.B)}$$

and is analogous to the P matrix in the least-squares method [132], proportional to a "covariance" matrix.

Such a result is fine for the time invariant case ($a(k) = a(k+1)$), but causes incorrect results for the time-varying case. This is because the estimate $\underline{\hat{\phi}}$ is based upon k sets of observations, and as the (k+1)st observation is processed, the incremental adjustment to $\underline{\hat{\phi}}$ goes as $\frac{1}{(k+1)}$. As $k \to \infty$, new observations therefore have almost no effect on the parameter estimates.

Landau [116] has shown that the previous time-invariant identifier is asymptotically unbiased, so if the measurable output is <u>not</u> x_p but

$$y_p(k) = x_p(k) + \eta(k) \qquad \text{(V-15.B)}$$

where $\eta(k)$ is a zero-mean, gaussian white noise, then as $k \to \infty$, $\underline{\hat{\phi}}(k) \to \underline{\phi}$ independent of η. If noise is included, then everywhere $x_p(k)$ was included, the scalar $y_p(k)$ would be substituted.

C. MRAS Memory Shaping

From Chapter 3 it was determined that the time invariant systems identifier is unsuitable for the identification of time-varying systems because all observations are treated with equal weight regardless of their occurrence in time. Thus it becomes desirable to seek a means of limiting the number of observations used in determining the parameter estimates. However it should be noted that because of possible noise effects, the number of observations should be as large as possible. This, then, results in the dilemma associated with identification of time-varying parameters in a noisy observational environment of finite data but infinite data.

Four basic MRAS identifier approaches for linear time-varying parameter identification were considered. The first two are limited memory filtering [173] and a "forgetting factor" method [174], the effect in either case being to interfere with the normally decreasing gain matrix F (as in (III-110.B)). Method 3 involves varying weighting coefficients c_i in a tracking error signal formed by the identifier (Eqs. (III-109.B) and (III-112.B)) as a function of present data. The fourth method is an oscillating memory approach, related to the first two, only this time data is literally subtracted out at the end of a time period when new parameter histories replace old.

1. W Method

In the limited memory, or "W" approach, the continuously varying plant is modeled as a chronological sequence of time invariant models over $\Delta t = nT$ time increments. Each interval of N samples is called a "cycle". The final parameter estimate of cycle ℓ is used for the initial parameter estimate of cycle $(\ell+1)$. However, the parameter adjustment gains F(k) are re-initialized according to a fixed algorithm at the beginning of each cycle. During the cycle, the parameter adjustment gains decreases with each succeeding iteration. During a cycle, then, the F(k) behaves as in the time-invariant identification case.

To define this method, suppose some previous off-line simulation study has been done to determine how F(j) propagates. Using $F(o) = K_1 I_n$, K_1 a large positive constant propagate F(j) using the time-invariant identifier (V.10.B)-(V-13.B). From this determine the f_{ii} diagonal entry of $[f_{ij}]$ which stays the largest (F is p.d.), denoted f^*. Selecting a K_2 for sizing purposes, and an updating cycle span of N periods based on

upper bound parameter rate estimates, *a priori* knowledge of the plant, etc., the algorithm is

1. $F(o) = K_1 I_n$ (V-1.C)

2. Propagate F(j), j=1,2,$\cdots$,N+1 using (V-13.B)

3. Recalculate $F(N+1) = \dfrac{K_2}{f^*(N+1)} F(N+1)$ (V-2.C)

4. In general

$$F(j) = \begin{cases} \dfrac{K_2}{f^*(\ell'N+1)} F(\ell'N+1) & \text{for } j = \ell'N+1 \\ \\ \{\text{Equation (V-13.B)}\} & \text{for } \ell'N+1 < j \leq \ell'N+N+1 \end{cases} \quad \text{(V-3.C)}$$

$\ell' = 1, 2, \cdots$

2. *λ Method*

The second method, the "forgetting factor", or "λ" approach, is a more direct adaptation of the time-invariant MRAS identifier. The effect is similar to that of the weighted least squares. The parameter identifier gains are not allowed to decrease in the same manner as would occur with the time-invariant approach. The method is based on a cost function of equation error

$$J(k+1) = \sum_{i=1}^{k+1} \lambda^{k-i+1} \left(y_p(i) - \underline{y}^T(i)\, \hat{\underline{\phi}}(k+1) \right)^2 \quad \text{(V-4.C)}$$

where λ is a scalar constant which exponentially weights the data, $|\lambda|<1$. Using the Matrix Inversion Lemma,

$$\hat{\underline{\phi}}(k+1) = \hat{\underline{\phi}}(k) + \frac{F(k)\, \underline{y}(k)\, [y_p(k) - \hat{\underline{\phi}}^T(k)\, \underline{y}(k)]}{\lambda + \underline{y}^T(k)\, F(k)\, \underline{y}(k)} \quad \text{(V-5.C)}$$

$$F(k+1) = \frac{1}{\lambda}\left[F(k) - \frac{F(k)\ \underline{y}(k)\ \underline{y}^T(k)\ F(k)}{\lambda + \underline{y}^T(k)\ F(k)\ \underline{y}(k)}\right] \qquad \text{(V-6.C)}$$

$$0 << \lambda \leq 1$$

The recursions in (V-5.C) and (V-6.C) should be compared to (V-10.B) and (V-13.B). The selection of λ will be given later, but is close to $\lambda = 1$, in which case (V-5.C)-(V-6.C) become (V-10.B) and (V-13.B). Typical results show $.9 < \lambda < .97$ to be best based on tracking response due to output measurement noise [175]. This can be seen from the simulation results of Chapter 7.

3. Time-Varying Weighting Coefficients

The third case uses time-varying values of the c_i weighting terms in (V-11.B) and (V-12.B). There is a designer-realization problem in selecting the c_i anyway, inasmuch as advance knowledge of the a_i are needed to select c_i, but if a_i were known, no identification would be needed. Landau suggests using

$$\hat{a}_i(o) = -c_i$$

where $\hat{a}_i(o)$ is the initial estimate using off-line calculations for the time-invariant a_i case. Updating this idea, the c_i are defined as

$$c_i(k) = -\hat{a}_i(k)$$

This technique would not be expected to perform as well as the other two because they manipulate the relative data weighting as a function of time, but the c_i's merely adjust a modest scalar correction term in $\Delta\underline{\hat{\phi}}$. Although the $c_i(k)$ adjustments are very easy to implement, it is questionable whether it would be worth the added computational burden.

4. Oscillating Memory

The oscillating memory is a rectangular data window method of variable length. The computational effort involved is greater than that of the exponential window method because data outside the window boundary is subtracted off. The cost function employed in this case is

$$J_3 = \sum_{i=k-N(k)+1}^{k+1} [y_p(i) - \hat{\underline{\phi}}^T(k)\ \underline{s}(i)]^2 \qquad \text{(V-7.C)}$$

where $\underline{s}(k)$ is from (III-101(c).B), and N(k) is a variable integer such that the number of samples included in the memory varies with k between specified lengths N_1 and N_2, $N_2 > N_1 > 0$,

$$N(k) = k - (n-1)(N_2 - N_1 + 1) \qquad n = 1, 2, \cdots \qquad \text{(V-8.C)}$$

The oscillating memory is accomplished using the following algorithms:

1. Compute $\hat{\underline{\phi}}(k+i)$, $F(k+i)$ for $i = 1, 2, \cdots N_2$ using (III-101(a).B)-(III-111.B)

2. At $t = (k+N_2)T$, using initial conditions $F'(j) = F(k+N_2)$, $\hat{\underline{\phi}}(j) = \hat{\underline{\phi}}(k+N_2)$ compute

$$\hat{\phi}'(j+1) = \hat{\underline{\phi}}'(j) - g'(j)\ e_p'(j)/d'(j) \qquad \text{(V-9.C)}$$

$$F'(j+1) = (I - \underline{g}'(j)\ \underline{y}(k-M)/(d'(j)]F'(j) \qquad \text{(V-10.C)}$$

$$\underline{g}'(j) = F'(j)\ \underline{y}(k-M) \qquad \text{(V-11.C)}$$

$$d'(j) = 1 - \underline{y}^T(k-M)\underline{g}'(j) \qquad \text{(V-12.C)}$$

$$e_p'(j) = y_p(k-M) - \underline{y}^T(k-M)\ \hat{\underline{\phi}}'(j) + \sum_{i=1}^{n'} c_i\ \varepsilon(k-i-M) \qquad \text{(V-13.C)}$$

for $M = N_2 - j$, $j = 1, 2, \cdots, (N_2 - N_1)$.

3. Redefine $k = k+N_2$, $F(k) = F'(N_2 - N_1)$, $\underline{\hat{\phi}}(k) = \underline{\hat{\phi}}(N_2-N_1)$ and commpute $\underline{\hat{\phi}}(k+i)$, $F(k+i)$ for $i = 1, 2, \cdots (N_2 - N_1)$.

4. Go to Step #2.

Equations (III-101(a).B)-(III-111.B) represent the usual case of equal data weighting ("adding memory") as in the time-invariant case. Equations (V-9.C)-(V-13.C) represent the memory shed recursion so that in a cycle of period $(N_2 - N_1)T$, only the last $(N_2 - N_1)$ pieces of data are carried forward to a new interval, with the other terms having their former (equal) weighting eliminated (T is the sample period).

5. Other Time-Varying Approaches

Other possible approaches include rectangular windowing [99, ch. 7], a form of limited memory filtering [173], extended Kalman Filtering [176], time-invariant-in-interval-time-varying-between-intervals [177], a hunting response [178], and optimal smoothing [197]. All are extensions of originally time-invariant cases. The sparcity of definitive work in time-varying identification can be seen from the fact that the December 1974 Special Issue of the IEEE Transactions on Automatic Control on System Identification [180] had no papers on time-varying work.

D. Applications of Time-Varying Identifiers

Simulation results and analyses of the time-varying identifier algorithms will be given in Chapter 7. In general it will be seen that using time-invariant identifiers for time-varying plants tends to yield constant parameter estimates as $k \to \infty$ which are incorrect. This is essentially due to the equal weighting given all data. As will be shown, it is essential some form of "sliding memory" be employed for effective time-varying plant parameter identification.

CHAPTER 6. DESIGN PROCEDURES FOR PRACTICAL MRAS IMPLEMENTATION

Now that the various algorithms and concepts for each of time-invariant, time-varying and nonlinear identification have been dilineated, some of the theoretical problems of practical interest from the design and implementation viewpoint will be addressed. These topics will include a) convergence rate, b) effect of input frequency richness on convergence rate, c) state-variable filter selection, d) identifier design parameter selection, e) effects of noise on tracking accuracy, f) computational considerations, and g) model order determination. Each of the following sections includes results and summarizes work to date in those areas.

A. Convergence Rate

For convergence rate analysis of MRAS-type identifiers, a number of methods have been developed. The problem is compounded in the presence of measurement and/or input noise. Given in Table VI-1 is a listing of available MRAS identifier convergence rate methodologies, a few of which will now be discussed. The use of such concepts for a priori *design* will be stressed. Although not of direct interest here, a recent paper by Ljung [181] presents a convergence rate approach for a class of discrete, non-MRAS identifiers which provides a deterministic means of analyzing some stochastic identifiers.

1. Kalman

This approach employs the concept of bounding the transient response of the nonlinear identifier equations by a linear equivalent one such that a conservative exponential convergence rate envelope is established. Using

Table VI-1

No.	Method	Year	Approach
1	Kalman [182]	1960	$\dot{V}/V$ Bound
2	Graupe [48]	1972	Hyperplane Intersection
3	Van de Linde [105]	1972	Matrix Inversion
4	Pereiras [183]	1974	Functional Relationship
5	Molnar [123]	1975	Compare V and $\dot{V}$ magnitudes via design parameters
6	Colburn, et al [184-187]	1975	Linearized error characteristic equation (LECE)
7	Carroll [120]	1976	Hyperplane Concept
8	Anderson [188]	1977	Input Frequency Richness + Design Parameters
9	Kim, Lindorf [189]	1977	Matrix Eigenvalue Maximization
10	Kreisselmeier [134]	1977	Exponential Bound

Lyapunov V and $\dot{V}$ functions, the *ratio* $\dot{V}/V$ can be employed as

$$\dot{V}(\underline{x},t) = \left[\frac{\dot{V}(\underline{x},t)}{V(\underline{x},t)}\right] V(\underline{x},t) \qquad \text{(VI-1.A)}$$

If, in a region about the origin ($\underline{x} = 0$) it can be established that

$$\frac{\dot{V}(\underline{x},t)}{V(\underline{x},t)} \leq -k \qquad \text{(VI-2.A)}$$

$k > 0$ constant, then

$$V(\underline{x},t) \leq V(\underline{x}(t_o),t_o)e^{-k(t-t_o)} \qquad \text{(VI-3.A)}$$

The difficulty in using this approach is

a) establishing simple V functions insuring global stability of the identifier

b) the trial and error "guessing" approach required to determine V and $\dot{V}$ that are of such a character that the resulting $\dot{V}/V$ ratio is simple enough for for *a priori* designer interpretation to control the constant "k" by selecting design parameters

c) not all identifiers necessarily can best be depicted by exponential bound

d) the fact that V and $\dot{V}$ requirements alone do not insure global stability of all identifier methods due to frequency richness demands on the input [104].

e) the limited utility shown to date in the control case, where more experience is available than the identifier case.

2. Graupe

Although this approach is not directly amenable to MRAS, the concepts, approach, and methodology are closely related to some of the discrete MRAS identifiers, including [70,113,114,116]. Due to time limitations, a complete analysis could not be effected, but the following is presented to "round out" the convergence study and provide a basic methodology for MRAS applications on convergence rate control through *a priori* design.

Consider the discrete time system

$$x_j = \underline{g}^T \underline{u}_j \tag{VI-4.A}$$

where

$$g^T = [g_1 \quad g_2 \cdots g_N] \ , \quad \underline{u}_j^T = [u_{j-1} \quad u_{j-2} \cdots u_{j-N}] \tag{VI-5.A}$$

and u_j, x_i are the input and output, g_i is the impulse response at time i. The terms g_i are identified by a model

$$\hat{x}_j = \underline{\hat{h}}_j^T \underline{u}_j \tag{VI-6.A}$$

where

$$\underline{\hat{h}}_j^T = [\hat{h}_1^{(j)} \quad \hat{h}_2^{(j)} \quad \cdots \quad \hat{h}_N^{(j)}] \tag{VI-7.A}$$

and $\hat{x}_j$ is the model output. Using [48, p. 136]

$$\underline{\hat{h}}_{j+1} = \underline{\hat{h}}_j + \Delta\underline{\hat{h}}_j \tag{VI-8.A}$$

$$\Delta\underline{\hat{h}}_j = (x_j - \hat{x}_j) \frac{\underline{u}_j}{\underline{u}_j^T \underline{u}_j} \qquad j=1,2,\cdots \tag{VI-9.A}$$

g_i may be determined.

Convergence is determined from a geometric interpretation. Define the N-1 dimensional hyperplane P_{j-1} in N-dimensional parameter space, such that P_{j-1} is normal to $\underline{u}_{j-1}$. From (VI-4.A)

$$\underline{u}_{j-1}^{T}\ \underline{g} = x_{j-1} \tag{VI-10.A}$$

using (VI-9.A)

$$\underline{u}_{j-1}^{T}\ \Delta\hat{\underline{h}}_{j-1} = \underline{u}_{j-1}^{T}\ (\hat{\underline{h}}_{j} - \hat{\underline{h}}_{j-1}) = (x_{j-1} - \hat{x}_{j-1}) \tag{VI-11.A}$$

so

$$\underline{u}_{j-1}^{T}\ \hat{\underline{h}}_{j} = x_{j-1} \tag{VI-12.A}$$

and the points $\underline{g}$ and $\hat{\underline{h}}_j$ are on P_{j-1}. Also $\underline{g}$ and $\hat{h}_{j-1}$ are on P_j. Equation (VI-9.A) shows $\underline{u}_j \,||\, \Delta\hat{\underline{h}}_j$ and $\langle (\underline{g} - \underline{h}_{j+1}),\ \Delta\hat{\underline{h}}_j \rangle = 0$. Defining

$$\underline{e}_j = \underline{g} - \hat{\underline{h}}_j \tag{VI-13.A}$$

and Ψ_j as the angle between $\underline{u}_j$ and $\underline{e}_j$. From (VI-14.A)

$$\underline{e}_{j+1} = \Delta\hat{\underline{h}}_j \tag{VI-15.A}$$

$$\frac{||\underline{e}_{j+1}||^2}{||\underline{e}_j||^2} = \sin^2 \Psi_j = 1 - \cos^2 \Psi_j \tag{VI-16.A}$$

$$||\underline{e}_{j+1}||^2 = ||\underline{e}_{N+1}||^2 \prod_{r=N+1}^{j} (1-\cos^2\Psi_r) \tag{VI-17.A}$$

Since $\cos\Psi_r \leq 1 \quad \forall r$, then (VI-17.A) shows

$$||\underline{e}_j|| \to 0 \text{ as } j \to \infty \tag{VI-18.A}$$

More important, it can be shown [48, p. 138] that the same type proof holds for an error adjustable version of (VI-8.A),

$$\hat{\underline{h}}_{j+1} = \hat{\underline{h}}_j + \alpha(x_j - \hat{x}_j)\ \frac{\underline{u}_j}{-\underline{u}_j^{T}\ \underline{u}_j} \tag{VI-19.A}$$

for $0 < \alpha < 2$. The question now arises as to how to provide the designer a rationale for selecting α to optimize (maximize) the parameter identifier

convergence rate.

Consider the modified process

$$x_j = \underline{u}_j^T \underline{g} + n_j \tag{VI-20.A}$$

with $E\{n_j\} = 0$ and $E\{n_j^2\} < \infty$, where $E\{\cdot\}$ represents expectation. Using (VI-19.A), parameter estimates for $\underline{g}$ in (VI-20.A) may be obtained. Then

$$e_{j+1}^2 = ||\hat{\underline{h}}_{j+1} - \underline{g}||^2 \tag{VI-21.A}$$

and from (VI-19.A)

$$\hat{\underline{h}}_{j+1} - \underline{g} = \left(I - \frac{\alpha \, \underline{u}_j \, \underline{u}_j^T}{||\underline{u}_j||^2} \right) (\underline{h}_j - \underline{g}) + \alpha \frac{n_j \, \underline{u}_j}{||\underline{u}_j||^2} \tag{VI-22.A}$$

Then

$$E\{e_{j+1}^2\} = E\left\{ \left|\left| \left(I - \frac{\alpha \, \underline{u}_j \, \underline{u}_j^T}{||\underline{u}_j||^2} \right) E\{e_j^2\} \right|\right|^2 \right\} + E\left\{ \frac{\alpha^2 \, n_j^2}{||\underline{u}_j||^2} \right\} \tag{VI-23.A}$$

Defining the matrix

$$Z_j = I - \frac{\alpha \, \underline{u}_j \, \underline{u}_j^T}{||\underline{u}_j||^2} \tag{VI-24.A}$$

and noting that $\underline{u}_j^T \, \underline{u}_j = ||\underline{u}_j||^2$

then

$$Z_j^2 = I - \alpha(2 - \alpha) \frac{\underline{u}_j \, \underline{u}_j^T}{||\underline{u}_j||^2} \tag{VI-25.A}$$

If u_i are independent and possess the same stochastic distribution, then

$$E\left\{ \frac{\underline{u}_j \, \underline{u}_j^T}{||\underline{u}_j||^2} \right\} = I \, N^{-1} \tag{VI-26.A}$$

$$E\left\{e_{j+1}^2\right\} = \left[1 - \left(\frac{\alpha(2-\alpha)}{N}\right)\right] E\{e_j^2\} + \alpha^2 \, E\{n_j^{*2}\} \qquad \text{(VI-27.A)}$$

$$n_j^* = \frac{n_j}{||\underline{u}_j||} \qquad \text{(VI-28.A)}$$

$$\lim_{\to\infty} E\{e_j^2\} = \frac{\alpha \, N \, E\{n^{*2}\}}{2 - \alpha} > 0 \qquad \text{(VI-29.A)}$$

For low tracking error,

$$0 < \alpha << 1 \qquad \text{(VI-30.A)}$$

Define

$$\gamma_j = \frac{e_{j+1}^2}{e_j^2} = \left(1 - \frac{\alpha(2-\alpha)}{N}\right) + \alpha^2 \left\{\frac{n^{*2}}{e_j^2}\right\} \qquad \text{(VI-31.A)}$$

The "best" α^* to minimize γ_j satisfies

$$\frac{\partial \gamma_j}{\partial \alpha} = 0 \qquad \text{(VI-32.A)}$$

yielding

$$\alpha^* = \frac{E\{e_j^2\}}{E\{e_j^2\} + N \, E\{n^{*2}\}} \qquad \text{(VI-33.A)}$$

Since γ_j relates directly to convergence rate, the best α^* is from (VI-33.A), which states $\alpha < 1$ but no more, since $E\{e_j^2\}$ is unknown a priori. The design trade-off is:

i) $\alpha < 1$ but close to 1 for rapid convergence

ii) $0 < \alpha << 1$ small tracking error

3. Van de Linde [105]

This method employs extraneous nulls to yield a set of algebraic equations which can be solved, as shown in Eqs. (III-22.B)-(III-25.B),

for H. The number of nulls needed is (n+1) if a single input is employed. After inversion, the V-Lyapunov function employed is reduced to 10^{-3} or smaller, effectively the origin. Observation times vary from 5-10 system time constants, considerably faster than others [11].

4. Pereiras

Using the Kudva and Narendra [108] methods as an example, convergence rate can be affected (see Chapter 3). Given the SISO plant (III-48.B)-(III-49.B), the adaptive observer (III-51.B)-(III-56.B) can be summarized here

$$\dot{\underline{e}} = K\underline{e} + \underline{d}(\underline{\alpha}^T\underline{v} + \underline{\beta}^T\underline{q}) \qquad \text{(VI-34.A)}$$

where (K,d) satisfies the MKY Lemma (see Chapter 2) and are designer selected, and $\underline{\alpha}$, $\underline{\beta}$ are the parameter error vectors (III-54.B), and

$$\dot{\underline{\alpha}} = -\Gamma\, e_1\, \underline{v}$$
$$\dot{\underline{\beta}} = -\Lambda\, e_1\, \underline{q} \qquad \text{(VI-35.A)}$$

Defining

$$\underline{V}(s) = \underline{g}(s)\, Y(s) \qquad \underline{Q}(s) = \underline{g}(s)\, U(s) \qquad \text{(VI-36.A)}$$

where

$$\lim_{s\to\infty} \underline{g}(s)/s = 0 \qquad \text{(VI-37.A)}$$

To analyze the system convergence properties, consider the 3n error vector

$$\underline{\xi}^T = (\underline{e}\ \underline{\alpha}\ \underline{\beta}) \qquad \text{(VI-38.A)}$$

from which it follows that

$$\dot{\underline{\xi}} = A_I\, \underline{\xi} \qquad \text{(VI-39.A)}$$

$$A_I = \left[\begin{array}{ccc|cc} & K & & \underline{d}\,\underline{v}^T & \underline{d}\,\underline{q}^T \\ \hline -\Gamma\,\underline{v} & & 0 & & \\ & & & 0 & \\ -\Lambda\,\underline{q} & & 0 & & \end{array}\right] \qquad \text{(VI-40.A)}$$

If u is periodic in T, then

$$\underline{\xi}[(k+1)T] = C\,\underline{\xi}(kT) \qquad \text{(VI-41.A)}$$

where C is the transition matrix found by integrating

$$\dot{C}(t) = A_I(t)\,C(t), \quad C(o) = I \qquad \text{(VI-42.A)}$$

over the interval $0 \leq t \leq T$.

Defining a cost functional

$$J = \int_0^{t_1} (1+t) \sum_{i=1}^{m} \sum_{j=1}^{m} \left| c_{ij}(t) \right| dt \qquad \text{(VI-43.A)}$$

where $m = n + \bar{n}$, n = order of observer $\bar{n}$ = number of unknown parameters, and c_{ij} is the response of C(t) due to $\underset{+}{\xi}^T(o) = [0 \cdots 0\ 1\ 0 \cdots 0]$ ↑ jth row

An optimization of Λ, Γ from J then yields the "best" answer. An example is given in [183]. The practical value of such an approach is that sensitivity to initial conditions is minimized. In practice, however, minimization of J with respect to Λ, Γ is unwieldy.

5. Molnar

From the adaptive observer given in (III-150.B)-(III-156.B), Molnar [123] suggests considering the terms in the Lyapunov $\dot{V}$ function. A V function is defined as

$$V = \frac{1}{2}\,e^2 + \frac{1}{2} \sum_{i=1}^{n} \frac{(a_i - \hat{a}_i)^2}{\gamma_i} + \frac{1}{2} \sum_{i=1}^{n} \frac{(b_i - \hat{b}_i)^2}{\delta_i} \qquad \text{(VI-44.A)}$$

where the γ_i, δ_i are designer-selected parameters. For the nonstationary case ($\dot{a}_i \neq 0$, $\dot{b}_i \neq 0$), $\dot{V}$ is

$$\dot{V} = -\lambda_n e^2 + \sum_{i=1}^{n} \left[\frac{(a_i - \hat{a}_i)\dot{a}_i}{\gamma_i} + \frac{(b_i - \hat{b}_i)b_i}{\delta_i} \right]$$

$$+ e[-\underline{a}^T \exp(\bar{L}t)\Delta\underline{v}(0) + \underline{b}^T \exp(\bar{L}t)\Delta\underline{w}(0)] \qquad \text{(VI-45.A)}$$

where $\Delta w(0) = w(0) - \hat{w}(0)$ and $\Delta v(0) = v(0) - \hat{v}(0)$. Equation (VI-45.A) indicates that maximum $(\lambda, \delta, \gamma)$ are needed for fast parameter covergence rate, based on the concept of the more negative $\dot{V}$ the better. But since the frequency richness requirement is not considered in this case, this does not necessarily solve the problem, since the $(\lambda, \delta, \gamma)$ can only affect response rates up to the limit of excitation of the various plant modes by the input.

6. LECE Approach

The linearized error characteristic equation (LECE) approach has its roots in the adaptive *control* problem. Essentially, it is a technique for obtaining an approximate characteristic equation of the error dynamics. Unfortunately, the LECE approach has been shown to have great utility for *error* convergence rate determination, but only limited value for *parameter* convergence rate prediction. Both discrete [185] and continuous-time identifier systems [186] have been analyzed with useful design rules available from each.

As an example of the LECE method applied to an adaptive identifier, consider the case given by Colburn in [186]. This is for an extended identifier law based on the Lüders and Narendra method [107].

The LECE is first developed for a second order case and the general nth order result with one input stated. Results are then applied to a nontrivial third order example in order to illustrate the technique of possible

improved rate of convergence of the identification law. Notation and equations for this method are given in (III-41.B)-(III-46.B), except the identifier gains have proportional plus derivative terms added (see [186]).

Using $\underline{e} = \underline{\hat{x}} - \underline{x}$ and a theorem from [107], the scalar error state equation for $\dot{e}_1$ becomes

$$\dot{e}_1 = -\lambda_1 e_1 + x_1 \phi_1 + \left[H \mathcal{L}^{-1}\left\{ (sI-\Lambda)^{-1} x_1(s) \right\} \underline{\Phi} \right]$$
$$+ u \Psi_1 + H \mathcal{L}^{-1}\left\{ (sI-\Lambda)^{-1} u(s) \right\} \underline{\Psi}$$
$$+ H \exp[\Lambda T] \underline{e}(t_o) \qquad \text{(VI-46.A)}$$

$\dot{e}_1$ is made up of a large number of time-varying closed-loop feedback terms which are nonlinear functions of the plant state x_1 and input u, as well as a number of adjustable design parameters. As with previous adaptation approaches, it is desired to use the linearized error equation technique to develop a design approach for determining the selection of the c_i, d_i, λ_i coefficients and how their selection affects the adaptive error transient response.

As has been done for MRAS control cases [184,191], the linearization approach for a second-order case will be presented first and results then extended to the general case. First, define the second-order plant as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_1 & 1 \\ a_2 & -\lambda_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u \qquad \text{(VI-47.A)}$$

and model as

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 & 1 \\ \alpha_2 & -\lambda_2 \end{bmatrix} \begin{bmatrix} x_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} u + \begin{bmatrix} -\lambda_1(\hat{x}-x_1) \\ \omega_2 \end{bmatrix} \qquad \text{(VI-48.A)}$$

The LECE for the last method presented in [241] will be developed, as it will be shown that the first case of it, plus the Lüders and Narendra [107] identifiers are special cases of the method to be presented. The adaptive identifier gains are of the form in (III-46.B).

The error and identification equations can then be written functionally as

$$\dot{e}_1 = f_1(e_1, x_1, \phi_1, \phi_2, u, \Psi_1, \Psi_2, v_2, s_2) \tag{VI-49.A}$$

$$\dot{\phi}_1 = f_2(e_1, x_1, \dot{e}_1, \dot{x}_1, \ddot{e}_1, \ddot{x}_1) \tag{VI-50.A}$$

$$\dot{\phi}_2 = f_3(e_1, v_2, \dot{e}_1, \dot{v}_2, \ddot{e}_1, \ddot{v}_2) \tag{VI-51.A}$$

$$\dot{\Psi}_1 = f_4(e_1, u, \dot{e}_1, \dot{u}, \ddot{e}_1, \ddot{u}) \tag{VI-52.A}$$

$$\dot{\Psi}_2 = f_5(e_1, s_2, \dot{e}_1, \dot{s}_2, \ddot{e}_1, \ddot{s}_2) \tag{VI-53.A}$$

where $f_1 - f_5$ represent the nonlinear functions for $\dot{e}_1$, $\dot{\phi}_1$, etc. Expanding $f_1 - f_5$ in Taylor Series' about nominal operating point (to be defined shortly) and truncating after linear terms yields

$$\Delta\dot{e}_1 = \left.\frac{\partial f_1}{\partial e_1}\right|_o \Delta e_1 + \left.\frac{\partial f_1}{\partial x_1}\right|_o \Delta x_1 + \left.\frac{\partial f_1}{\partial \phi_1}\right|_o \Delta\phi_1 + \left.\frac{\partial f_1}{\partial \phi_2}\right|_o \Delta\phi_1 + \left.\frac{\partial f_1}{\partial \phi_2}\right|_o \Delta\phi_2 +$$

$$+ \left.\frac{\partial f_1}{\partial u}\right|_o \Delta u + \left.\frac{\partial f_1}{\partial \Psi_2}\right|_o \Delta\Psi_1 + \left.\frac{\partial f_1}{\partial \Psi_2}\right|_o \Delta\Psi_2$$

$$+ \left.\frac{\partial f_1}{\partial v_2}\right|_o \Delta v_2 + \left.\frac{\partial f_1}{\partial s_2}\right|_o \Delta s_2 \tag{VI-54.A}$$

$$\Delta\dot{\phi}_1 = \left.\frac{\partial f_2}{\partial e_1}\right|_o \Delta e_1 + \left.\frac{\partial f_2}{\partial x_1}\right|_o \Delta x_1 + \left.\frac{\partial f_2}{\partial \dot{e}_1}\right|_o \Delta\dot{e}_1 + \left.\frac{\partial f_2}{\partial \dot{x}_1}\right|_o \Delta\dot{x}_1$$

$$+ \left.\frac{\partial f_2}{\partial \ddot{e}_1}\right|_o \Delta\ddot{e}_1 + \left.\frac{\partial f_2}{\partial \ddot{x}_1}\right|_o \Delta\ddot{x}_1 \qquad \text{(VI-55.A)}$$

$$\Delta\dot{\phi}_2 = \left.\frac{\partial f_3}{\partial e_1}\right|_o \Delta e_1 + \left.\frac{\partial f_3}{\partial v_2}\right|_o \Delta v_2 + \left.\frac{\partial f_3}{\partial \dot{e}_1}\right|_o \Delta\dot{e}_1 + \left.\frac{\partial f_3}{\partial \dot{v}_2}\right|_o \Delta\dot{v}_2$$

$$+ \left.\frac{\partial f_3}{\partial \ddot{e}_1}\right|_o \Delta\ddot{e}_1 + \left.\frac{\partial f_3}{\partial \ddot{v}_2}\right|_o \Delta\ddot{v}_2 \qquad \text{(VI-56.A)}$$

$$\Delta\dot{\Psi}_1 = \left.\frac{\partial f_4}{\partial e_1}\right|_o \Delta e_1 + \left.\frac{\partial f_4}{\partial u}\right|_o \Delta u + \left.\frac{\partial f_4}{\partial \dot{e}_1}\right|_o \Delta\dot{e}_1 + \left.\frac{\partial f_4}{\partial \dot{u}}\right|_o \Delta\dot{u}$$

$$+ \left.\frac{\partial f_4}{\partial \ddot{e}_1}\right|_o \Delta\ddot{e}_1 + \left.\frac{\partial f_4}{\partial \ddot{u}}\right|_o \Delta\ddot{u} \qquad \text{(VI-57.A)}$$

$$\Delta\dot{\Psi}_2 = \left.\frac{\partial f_5}{\partial e_1}\right|_o \Delta e_1 + \left.\frac{\partial f_5}{\partial s_2}\right|_o \Delta s_2 + \left.\frac{\partial f_5}{\partial \dot{e}_1}\right|_o \Delta\dot{e}_1 + \left.\frac{\partial f_5}{\partial \ddot{e}_1}\right|_o \Delta\ddot{e}_1$$

$$+ \left.\frac{\partial f_5}{\partial \dot{s}_2}\right|_o \Delta\dot{s}_2 \qquad \text{(VI-58.A)}$$

with the partial derivatives defined as

$$\left.\frac{\partial f_1}{\partial e_1}\right|_o = -\lambda_1 \qquad \left.\frac{\partial f_1}{\partial x_1}\right|_o = \phi_1{}^o$$

$$\left.\frac{\partial f_1}{\partial u}\right|_o = \Psi_1{}^o \qquad \left.\frac{\partial f_1}{\partial \phi_1}\right|_o = x_1{}^o \qquad \text{(VI-59.A)}$$

$$\left.\frac{\partial f_1}{\partial \Psi_1}\right|_o = u^o \qquad \left.\frac{\partial f_1}{\partial \Psi_2}\right|_o = v_2{}^o$$

$$\left.\frac{\partial f_1}{\partial v_2}\right|_o = \Psi_2{}^o \qquad \left.\frac{\partial f_1}{\partial s_2}\right|_o = \phi_2{}^o$$

$$\left.\frac{\partial f_2}{\partial e_1}\right|_o = -c_1 x_1{}^o - f_1 \dot{x}_1{}^o - \rho_1 \ddot{x}_1{}^o \quad \left.\frac{\partial f_2}{\partial x_1}\right|_o = -c_1 e^o - f_1 \dot{e}_1{}^o - \rho_1 \ddot{e}_1{}^o$$

$$\left.\frac{\partial f_2}{\partial \dot{e}_1}\right|_o = -f_1 x_1^o - 2\rho_1 \dot{x}_1^o; \quad \left.\frac{\partial f_2}{\partial \dot{x}_1}\right|_o = -f_1 e_1^o - 2\rho_1 \dot{e}_1^o \qquad \text{(VI-60.A)}$$

$$\left.\frac{\partial f_2}{\partial \ddot{e}_1}\right|_o = -\rho_1 x_1^o \quad \left.\frac{\partial f_2}{\partial \ddot{x}_1}\right|_o = -\rho_1 \ddot{e}_1^o$$

$$\left.\frac{\partial f_3}{\partial e_1}\right|_o = -c_2 v_2^o - f_2 \dot{v}_2^o - \rho_2 \ddot{v}_2^o \qquad \left.\frac{\partial f_3}{\partial v_2}\right|_o = -c_2 e_1^o - f_2 \dot{e}_1^o - \rho_2 \ddot{e}_1^o$$

$$\left.\frac{\partial f_3}{\partial \dot{e}_1}\right|_o = -f_2 v_2^o - 2\rho_2 \dot{x}_1^o \qquad \left.\frac{\partial f_3}{\partial \dot{v}_2}\right|_o = -f_2 e_1^o - 2\rho_2 \ddot{e}_1^o \qquad \text{(VI-61.A)}$$

$$\left.\frac{\partial f_3}{\partial \ddot{e}_1}\right|_o = -\rho_2 v_2^o \qquad \left.\frac{\partial f_3}{\partial \ddot{v}_2}\right|_o = -\rho_2 \ddot{e}_1{}^o$$

$$\left.\frac{\partial f_4}{\partial \dot{e}_1}\right|_o = -d_1 u^o - \sigma_1 \ddot{u}^o - g_1 \dot{u}^o \qquad \left.\frac{\partial f_4}{\partial u}\right|_o = -d_1 e_1{}^o - g_1 \dot{e}_1{}^o - \sigma_1 \ddot{e}_1{}^o$$

$$\left.\frac{\partial f_4}{\partial \dot{e}_1}\right|_o = -g_1 \dot{u}^o - 2\sigma_1 \dot{u}^o \qquad \left.\frac{\partial f_4}{\partial \dot{u}}\right|_o = -g_1 e_1^o - 2\sigma_1 e_1^o \qquad \text{(VI-62.A)}$$

$$\left.\frac{\partial f_4}{\partial \ddot{e}_1}\right|_o = -\sigma_1 u^o \qquad \left.\frac{\partial f_4}{\partial \ddot{u}}\right|_o = -\sigma_1 e_1{}^o$$

$$\left.\frac{\partial f_5}{\partial e_1}\right|_o = -d_2 s_2^o - g_2 \dot{s}_2^o - \sigma_2 \ddot{s}_2^o \qquad \left.\frac{\partial f_5}{\partial s_2}\right|_o = -d_2 e_1^o - g_2 \dot{e}_1^o - \sigma_2 \ddot{e}_1^o$$

$$\left.\frac{\partial f_5}{\partial \dot{e}_1}\right|_o = -g_2 s_2^o - 2\sigma_2 \dot{s}_2^o \qquad \left.\frac{\partial f_5}{\partial \dot{s}_2}\right|_o = -g_2 e_1^o - 2\sigma_s \dot{e}_1^o \qquad \text{(VI-63.A)}$$

$$\left.\frac{\partial f_5}{\partial \ddot{e}_1}\right|_o = -\sigma_2 s_2^o \qquad \left.\frac{\partial f_5}{\partial \ddot{s}_2}\right|_o = -\sigma_2 \ddot{e}_1^o$$

The nominal (equilibrium) operating point, denoted by °, is defined to be $x_1 = x_1^o$, $v_2 = v_2^o$, $u = u^o$, $s_2 = s_2^o$, $\phi_1 = \phi_1^o = 0$, $\phi_2 = \phi_2^o = 0$, $\Psi_1 = \Psi_1^o$, $\Psi_2 = \Psi_2^o = 0$ and it is assumed that the constant input u has been applied for a "long" time (so the time derivatives will be zero). It should be noted that this analysis cannot be strictly applied to the identification problem at hand because in (VI-55.A)-(VI-63.A) it is assumed u is constant but a necessary condition for identification was that u possess at least n distinct frequencies. Without making the assumption of constant inputs, meaningful results are not possible. As long as the time derivative terms ($f_1\dot{x}_1$, $\rho_1\ddot{x}_1$, $\rho_1\dot{e}_1$, $\sigma_1\ddot{e}_1$, etc.) are much less than the "steady-state" terms (c_1x_1, c_2v_2, d_1u, g_2s_2, etc.) then the analysis will be meaningful. One way this can be accomplished is if there is a large bias term on u so that the bias tends to swamp the a.c. variation and the a.c. signals are of low enough frequencies so that the time derivatives are small.

Taking the Laplace transforms of (VI-54.A)-(VI-63.A) and substituting the expressions for $\Delta\phi_1(s)$, $\Delta\phi_2(s)$, $\Delta\Psi_1(s)$, $\Delta\Psi_2(s)$ into $\Delta E_1(s)$ results in the LECE for $\Delta E_1(s)$:

$$s\,\Delta E_1(s) = -\lambda_1\,\Delta E_1(s) + x_1^o\left\{\frac{-c_1x_1^o - f_1x_1^o s - \rho_1 x_1^o s^2}{s}\;\Delta E_1(s)\right\}$$
$$+ u^o\left\{\frac{-d_1u^o - g_1u^o s - \sigma_1 u^o s^2}{s}\;\Delta E_1(s)\right\}$$
$$+ v_2^o\left\{\frac{-c_2v_2^o - f_2v_2^o s - \rho_2 v_2^o s^2}{s}\;\Delta E_1(s)\right\}$$
$$+ s_2^o\left\{\frac{-d_2s_2^o - g_2s_2^o s - \sigma_2 s_2^o s^2}{s}\;\Delta E_1(s)\right\} \qquad \text{(VI-64.A)}$$

whose characteristic equation for $\Delta E_1(s)$ can be put in the form

$$1 + \frac{\left(K_1 + K_2 s + K_3 s^2\right)}{s(s+\lambda_1)} = 0 \qquad \text{(VI-65.A)}$$

where

$$K_1 = c_1 {x_1}^{o^2} + c_2 {v_2}^{o^2} + d_1 {u}^{o^2} + d_2 {s_2}^{o^2}$$
$$K_2 = f_1 {x_1}^{o^2} + f_2 {v_2}^{o^2} + g_1 {u}^{o^2} + g_2 {s_2}^{o^2} \qquad \text{(VI-66.A)}$$
$$K_3 = \rho_1 {x_1}^{o^2} + \rho_2 {v_2}^{o^2} + \sigma_1 {u}^{o^2} + \sigma_2 {s_2}^{o^2}$$

The general result for an nth order system with one input is of the same form as (VI-65.A), except K_1, K_2, K_3 are defined as

$$K_1 = c_1 {x_1}^{o^2} + c_2 {v_2}^{o^2} + c_3 {v_3}^{o^2} + \cdots + e_n {v_n}^{o^2}$$
$$+ d_1 {u}^{o^2} + d_2 {s_2}^{o^2} + d_3 {s_3}^{o^2} + \cdots + d_n {s_n}^{o^2}$$

$$K_2 = f_1 {x_1}^{o^2} + f_2 {v_2}^{o^2} + f_3 {v_3}^{o^2} + \cdots + f_n {v_n}^{o^2}$$
$$+ g_1 {u}^{o^2} + g_2 {s_2}^{o^2} + g_3 {s_3}^{o^2} + \cdots + g_n {s_n}^{o^2} \qquad \text{(VI-67.A)}$$

$$K_3 = \rho_1 {x_1}^{o^2} + \rho_2 {v_2}^{o^2} + \rho_3 {v_3}^{o^2} + \cdots + \rho_n {v_n}^{o^2}$$
$$+ \sigma_1 {u}^{o^2} + \sigma_2 {s_2}^{o^2} + \sigma_3 {s_3}^{o^2} + \cdots + \sigma_n {s_n}^{o^2}$$

Note that (VI-65.A) is in the general form $1+k\,P(s) = 0$ for root locus analysis. It can be seen that the design parameter λ_1 required to implement the observer equations acts as a pole in the error response. From Lyapunov stability considerations, $\lambda_1 > 0$ and this couples with (VI-65.A) to insure that the open-loop error pole $-\lambda_1$ is in the open left hand s-plane (OLHP). The pure integration term in (VI-65.A) can be traced back to the integration term in the identification gains. The numerator terms K_1, K_2, K_3 which are a function of the actual state values and identification gains act together to represent zeros of the open-loop error.

Using the generalized identifier gain form, it can be seen the gains are like a proportional-integral-derivative (P-I-D) controller. If $\rho_i = \sigma_i = 0$, $(i=1,2,\cdots,n)$ then the system is a P-I type controller since $K_3 = 0$. If, in addition to $\rho_i = \sigma_i = 0$ $(i=1,2,\cdots,n)$, $f_i = g_i = 0$ $(i=1,2,\cdots,n)$, then $K_2 = K_3 = 0$ and the controller is an I type controller, analogous to the Lüders case (Method 5 of Ch. 3, Section B).

7. Carroll

The problem of parameter convergence rate has been attacked by Carroll from the viewpoint of considering a criterion surface in the parameter space. For example, for typical equation error MRAS formulation, the equation error is [120]

$$\varepsilon_i = \sum_{i=1}^{\nu} v_i(t)\,\Delta p_i \qquad \text{(VI-68.A)}$$

where v_i are filtered state variables, ν is the number of unknown parameters in (III-124.A), and $\Delta\underline{p} = \hat{\underline{p}} - \underline{p}^*$ with $\hat{\underline{p}}$ the identifier estimate and $\underline{p}^*$ the actual (unknown) parameter vector. The right side of (III-68.A) defines a ν-1 dimensional hyperplane in ν-dimensional $\Delta\underline{p}$ space, passing through $\underline{p}^*$ $\forall$ $t \geq t_o$ and rotating about it due to the time-varying "coefficients"

$v_i(t)$. The parameter estimate $\hat{\underline{p}}$ generally approaches this hyperplane at a rate proportional to $||\Delta\underline{p}||$ at a fixed angle controlled by the designer parameter such as G in (III-134.B). Typical runs confirm $\hat{p}$ quickly approaches the *vicinity* of the hyperplane, but because of tiny ε_1 values, further motion to $\Delta\underline{p} = 0$ is small.

To solve the convergence problem, Carroll uses a closed criterion in ν-dimensional parameter space. To accomplish this, ν independent hyperplanes are defined using ν independent equation errors ε_i as in (III-123.B), where less ε_i are used if $\nu \leq 2n$. Defining

$$F(\underline{\varepsilon}) = \underline{\varepsilon}^T\underline{\varepsilon} \tag{VI-69.A}$$

then using the steepest descent concept

$$\dot{\underline{\rho}} = -G \frac{\partial(\underline{\varepsilon}^T\underline{\varepsilon})}{\partial\,\underline{\rho}} \tag{VI-70.A}$$

Using Lyapunov Theory and a theorem of Lasalle, asymptotic stability has been assured. Using such a geometrical interpretation, Kim's [189] optimization represents selecting an optimum direction of hyperplane approach for a given input frequency. Extended P-I and P-I-D versions of observer rules [186,192] change the rate of approach to the plane, but not its direction.

8. Anderson [188]

A series of necessary and sufficient conditions are developed regarding exponential and asymptotic stability for a class of general time-varying differential equations arising in a variety of model reference adaptive identification schemes. Due to lack of project time and money, the practical *design* implications of the work will not be presented here, but are presently under active investigation.

9. Kim, Lindorff [189]

The proposed convergence optimization method of Kim and Lindorff makes use of the concept by Pereiras [183] in this section of the report. Using the idea of an unforced error response vector

$$\underline{\xi} = (\underline{e}, \underline{\alpha}, \underline{\beta})^T \tag{VI-71.A}$$

$$\dot{\underline{\xi}} = A_I(t)\ \underline{\xi} \tag{VI-72.A}$$

the transition matrix C, as in (VI-42.A), is investigated [15]. Since a periodic forcing function is employed, C is periodic with period T, and hence a discretized set of equations using Floquet Theory results. The convergence rate criterion employed is

$$\rho = \max_i |\lambda_i| \quad i = 1, 2, \cdots, n$$

and λ_i are the (time-invariant) eigenvalues of C. By design (asymptotic stability), $|\lambda_i| < 1 \quad \forall\ i = 1,2,\cdots,n$. Using hyperstability [193], time-varying Λ, Γ in (VI-40.A) can be determined. The problem, then, is one of minimizing the maximum eigenvalue of C by changing Λ, Γ. In practice, the optimization is very slow and virtually unwieldy for $n \geq 2$. Further, Carroll [120] has shown this approach is highly sensitive to input frequencies.

10. Kreisselmeier [134]

In [134] Kreisselmeier develops a series of MRAS-type adaptive observers with guaranteed exponential bounds. Along with these, theorems insuring the size of the bounds are developed, the results being useful in a sufficiency sense for the designer to *a priori* select design constants to control the adaptive parameter convergence rate.

Using the particular adaptation mechanism in Chapter 3, equations (III-205.B)-(III-222.B) describe the system nomenclature. The following

theorem results:

Theorem:

The adaptive identifier (III-212.B) and (III-222.B) has the property that, if $\underline{Z}(t)$ is bounded and if there exists constants k and T such that

$$\int_{t}^{t+T} Z(\tau)\, Z^{T}(\tau)\, d\tau \geq kI \quad 0 \qquad \forall\, t \geq 0 \tag{VI-73.A}$$

then the identifier is globally exponentially convergent with exponential rate $\geq e^{-\gamma t}$,

$$\gamma = \min\{\sigma, q, K\} \tag{VI-74.A}$$

$$K = k\, \lambda_{min} \exp(-qT) \tag{VI-75.A}$$

$$\lambda_{min} = \min_{i} \lambda_{i} \quad |\lambda I - G| = 0 \tag{VI-76.A}$$

From an analyzis of the system equations, plus the previous Theorem, σ, q, K are designer controllable, σ from F, q from the cost function relative term weighting (III-217.B), and K from the judicious selection and bounding of G, k, q, and T. The convergence rate bound $e^{-\gamma t}$ can be arbitrarily increased up to σ by selecting $q \geq \sigma$ and $G \geq \{\sigma/k \exp(qT)\}I$ given that u is sufficiently exciting. This also demonstrates that although the adaptive gain parameter weighting matrix influences the identifier convergence rate as shown for other identifiers [108,185,186], the rate is not unbounded, i.e. the law of diminishing returns comes into play here. Similar results for an "open-loop" adaptation system for communication systems have also been developed [194].

It should be noted that additional design rules are available in [134] for other identification laws. For brevity, these are omitted since the guidelines are of a similar nature.

B. Effects of Noise on Tracking Accuracy

The effect of noise, both input as well as output measurement error generally causes parameter tracking inaccuracies. The question arises as to whether any MRAS methods exist to eliminate the noise by structural design, or whether designer-controlled parameter selection can minimize noise effects.

The following design guidelines are given to this end. Although not a full set of analytical guides to the designer, results direct the user in a direction which optimizes an identification scenario.

Guideline 1.

Eliminate or minimize any unmeasurable noises such as input and/or output noise. Barring this, force the random errors to be zero mean (and Gaussian), if possible.

Guideline 2.

Given that noise is present, minimize it for identifier data manipulation by determining the frequency characteristics of the noise and pre-filtering to remove high-frequency effects. Such an approach has proven successful in practice using simple first-order filters [195].

Guideline 3.

Avoid the use of the equation error formulation, either regular equation error or GEE if possible. This is due to the noise biasing effect of having the noisy output appear directly in the observer model, i.e.

$$\underline{x}_m(k+1) = \hat{A}(k+1)\,\underline{x}_p(k) + \hat{B}(k+1)\,\underline{u}(k) \qquad \text{(VI-1.B)}$$

Note how $\underline{x}_m$ depends on the product $\hat{A}\,\underline{x}_p$, but

$$\hat{A}(k+1) = f(\underline{x}_p) \qquad \text{(VI-2.B)}$$

So $\hat{A}$ is in error due to noise and then is multiplied by the same form of error which yields a biasing effect. Output error ("parallel MRAS") formulations generally alleviate such problems.

Guideline 4.

For closed-loop identification [28], using error and output, the signals need to be independent, usually requiring instrument-variable or "bootstrap" techniques.

Guideline 5.

There is usually a trade-off between tracking accuracy due to noise and parameter convergence rate. To counteract this problem, parameters α, p, λ_1 in [113] need to be selected as a compromise. In Section A, subsection 2 of this chapter, a parameter range on α is defined such that convergence rate must be sacrificed for accuracy.

Guideline 6.

The response error MRAS technique yields *asymptotically* unbiased estimates if the output measurement noise is zero mean [196].

Guideline 7.

If an equation error formulation is used, instrument variable methods [5] can be employed to reduce or eliminate noise biasing of parameters, but at the expense of assured global stability [109].

Guideline 8.

Use D.C. blocking filters (i.e. $\frac{s}{s+1}$) to remove D.C. bias levels. Such an approach simplifies algorithm calculation, and merely acts as an additional part of the state-variable filters used in many identifier concepts [5].

Based on the present study, plus a comparison of the works of others, the following results can be stated as regards tracking accuracy due to unmeasurable noise:

Guideline 9.

From Section A, subsection 2 of this chapter, a designer-controlled parameter can be selected to minimize bias.

Guideline 10.

The output error MRAS methods such as Landau [116], Hirsh [111], and Hang [109] are asymptotically unbiased for open-loop identification, although no estimate toward the unbiased condition can be made at present.

Guideline 11.

For closed loop identification as in Figure VI-1, with e and y as measurables, both the equation error and output error methods yield biased estimates if the values of e are correlated. They can be roughly uncorrelated using bootstrap and/or instrument variable methods at the possible expense of stability.

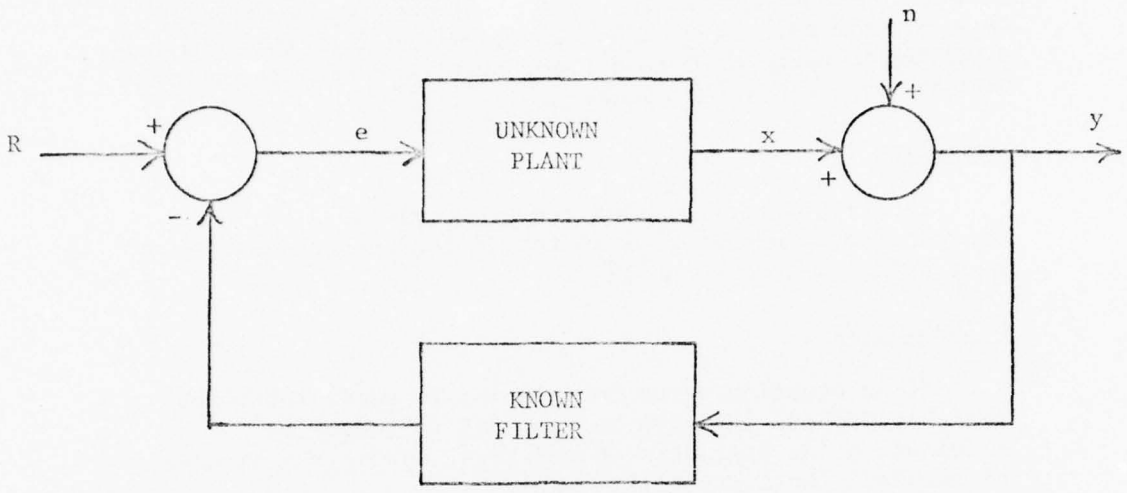


Figure VI-1. General Identification Problem With Injected Noise In A Closed-Loop Configuration Causing Correlated Residuals.

C. Effect of Input "Frequency Richness" On Convergence

It has been clearly demonstrated [5,22,99,102,107,113,197 among others] that for guaranteed parameter convergence, most identifier algorithms require the input to be sufficiently rich in frequencies. In practice when possible, this concept leads to use of white noise inputs, roughly representing an infinity of frequencies. It makes heuristic sense, and has been supported by simulation experience, that the frequencies present effect the parameter convergence rate, i.e. higher frequencies speed up the convergence rate, although there are limits to this correlation.

1. Minimal Frequency Richness Requirements

To see the need for frequency-richness, consider the equation error MRAS method of Lüders and Narendra [107] (see Chapter 3). Given that $\underline{x}_p$ is the plant state vector and $\hat{\underline{x}}$ the model state vector, the state error from equations are $\underline{e} = \hat{\underline{x}} - \underline{x}$, or

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_n \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 1 & 1 & \cdots & 1 \\ 0 & & & & \\ \vdots & & & \Lambda & \\ 0 & & & & \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

$$+ \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} x_1 + \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix} u + \begin{bmatrix} 0 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \omega_n \end{bmatrix} \qquad \text{(VI-1.C)}$$

where ϕ_i, Ψ_i are the parameter errors, u is the input. With ω_i defined by

$$\underline{\omega} = \mathcal{L}^{-1}\left\{(sI-\Lambda)^{-1}\, x_1(s)\right\} \dot{\underline{\phi}} + \mathcal{L}^{-1}\left\{(sI-\Lambda)^{-1}\, u\right\} \dot{\underline{\Psi}} \qquad \text{(VI-2.C)}$$

$$\underline{\phi}^T = (\phi_2 \quad \phi_3 \cdots \phi_n), \quad \underline{\Psi}^T = (\Psi_2 \quad \Psi_3 \cdots \Psi_n) \qquad \text{(V-3.C)}$$

it can be shown [107] that (VI-1.C) above can be reduced to

$$\dot{e}_1 = -\lambda_1 e_1 + x_1\phi_1 + \left\{h_1^T\, \mathcal{L}^{-1}\{(sI-\Lambda)^{-1}\, x_{p_1}(s)\}\right\} \underline{\phi} + u\Psi_1$$
$$+ h_1^T\, \mathcal{L}^{-1}\left\{(sI-\Lambda)^{-1} u(s)\right\} \underline{\Psi} + h_1^T \exp[\Lambda t]\, \underline{e}(t_o) \qquad \text{(VI-4.C)}$$

Using Lyapunov Theory, it can be proven that $e_1 \to 0$ as $t \to \infty$ with the identifier rule given in Section B subsection 5 of Chapter 3. However, for parameter convergence it must be shown that $e_1 \to 0 \Rightarrow \underline{\phi} = \underline{\Psi} = 0$.

To show this, a sufficient condition is that $e_1 \to 0$ assures that $\underline{\phi} = \underline{\Psi} = \underline{0}$. From (VI-4.C) if all terms on the right hand side (RHS) add to zero, then parameter convergence is assured. Since x_{p_1} and u are the plant output and input, then in steady-state x_{p_1} and u are related from (III-47.B) by

$$x_1 = \frac{\left[b_1 + \dfrac{b_2}{s+\lambda_2} + \cdots + \dfrac{b_n}{s+\lambda_n}\right] u}{\left[s - a_1 - \dfrac{a_2}{s+\lambda_2} - \cdots - \dfrac{a_n}{s+\lambda_n}\right]} \qquad \text{(VI-5.C)}$$

Substituting (VI-5.C) into (VI-4.C) results in a homogeneous equation for u,

$$\left\{\left[b_1 + \frac{b_2}{s+\lambda_2} + \cdots\right]\left[\phi_1 + \frac{\phi_2}{s+\lambda_2} + \cdots + \frac{\phi_n}{s+\lambda_n}\right]\right.$$
$$\left.\left[\Psi_1 + \frac{\Psi_2}{s+\lambda_2} + \cdots + \frac{\Psi_n}{s+\lambda_n}\right]\left[s - a_1 - \frac{a_2}{s+\lambda_2} - \cdots\right]\right\} u(s) = 0 \qquad \text{(VI-6.C)}$$

A sufficient condition for (VI-6.C) to imply $\underline{\Psi} = \underline{\phi} = 0$ as $e \to 0$ is that u contain at least n different frequencies.

This does not aid in "selecting" the best frequencies, but it does state that for the nth order system parameterized as in (VI-5.C) that n distinct frequencies must occur to *insure* parameter tracking. It does not say that less than n frequencies could *not* yield asymptotic stability also. Note that this additional condition is required because the Lyapunov V function employed for the parameter law synthesis only yielded a n.s.d. $\dot{V}$ function and hence it was necessary to insure $\dot{V}(\underline{r}) \neq 0$ for $\underline{r} \neq 0$, where $\underline{r}^T = [\underline{a}^T, \underline{b}^T, \underline{e}^T]$.

A similar situation occurs in discrete time synthesis. Using [113] (see Chapter 3) a plant is tracked using (III-85.B) and the parameter adjustment rule (III-86.B). However, stability is once again determined from Lyapunov Theory where a p.d. V(k) yields only a n.s.d. $\Delta V(k)$, so to insure parameter convergence, an additional condition is required, namely that $\Delta V(k) = 0$ or $\underline{e}(k) = 0$, with

$$\underline{e}(k) = \underline{\underline{\phi}}\, \underline{x}_p(k) + \Psi\, \underline{u}(k) \qquad \text{(VI-7.C)}$$

or

$$0 = \underline{\underline{\phi}}\, \underline{x}_p + \Psi\, \underline{u} \qquad \text{(VI-8.C)}$$

It must be assured that (VI-8.C) is true only if $\underline{\underline{\phi}} = \Psi = 0$ and <u>not</u> because u is a solution to it. It can be shown this is true if there exists a finite $k \geq (n+1)$ such that

$$M = [M_1 \,\vdots\, M_2 \,\vdots\, M_3 \,\vdots\, \cdots \,\vdots\, M_m] \qquad \text{(VI-9.C)}$$

is of rank n, where

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$$M_j = \begin{bmatrix} u_j(0) & 0 & 0 & \cdots & 0 \\ u_j(1) & u_j(0) & 0 & \cdots & 0 \\ u_j(2) & u_j(1) & u_j(1) & & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ u_j(k) & u_j(k-1) & u_j(k-2) & & u_j(k-n) \end{bmatrix} \qquad \text{(VI-10.C)}$$

2. Optimal Input Design

Previous work has been on optimal input design for identification techniques such as maximum likelihood and stochastic approximation using optimization of the Fisher Information Matrix [171,198-200] as a design criterion. Both time-domain [198,200,201] and frequency-domain [199,202,203] techniques are employed. All work of this type, of course, assumes that the input is designer-controlled, which is not always the case for the human operator problem.

Consider a model of the form

$$y(z) = G_1(z)\, u(z) + G_2(z)\, \varepsilon(z) \qquad \text{(VI-11.C)}$$

where $\{u(k)\}$ and $\{y(k)\}$ are the input and output sequences and $\{\varepsilon(k)\}$ is a sequence of indendepent Gaussian random variables with identical distributions with covariance Σ. The G_1 and G_2 are linear system transfer functions with $G_2(\infty) = 1$,

$$G_1(z) = \frac{B(z)}{A(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} \qquad \text{(VI-12.C)}$$

$$G_2(z) = \frac{D(z)}{C(z)} = \frac{1 + d_1 z^{-1} + \cdots + d_z z^{-q}}{1 + c_1 z^{-1} + \cdots + c_q z^{-q}} \qquad \text{(VI-13.C)}$$

Hence (VI-11.C) is a form of an ARMA process. It can be shown [99, ch. 6] that the *average* Fisher Information Matrix $\bar{M}$ for such a case is

$$\bar{M} = \bar{M}_1 + \bar{M}_c \tag{VI-14.C}$$

$$\bar{M}_1 = \frac{1}{\Pi} \int_0^{\Pi} \tilde{M}(\omega)\, d\xi(\omega) \tag{VI-15.C}$$

$$d\xi(\omega) = \begin{cases} \frac{1}{2}\, dF(\omega) & \omega = 0\,, \\ dF(\omega) & \omega \varepsilon (0, \Pi) \end{cases} \tag{VI-16.C}$$

$F(\omega)$ is the spectral distribution of the input

$$\tilde{M}(\omega) = \mathrm{Re}\left\{ \frac{1}{\Sigma} \left[\frac{\partial G_1(e^{j\omega})}{\partial \underline{\beta}} \right]^T G_2^{-1}(e^{j\omega}) \cdot G_2^{-1}(e^{-j\omega}) \left[\frac{\partial G_1(e^{-j\omega})}{\partial \underline{\beta}} \right] \right\} \tag{VI-17.C}$$

$$\bar{M}_c = \frac{1}{2\Pi} \int_{-\Pi}^{\Pi} \left\{ \left[\frac{\partial G_2(e^{-j\omega})}{\partial \underline{\beta}} \right]^T G_2^{-1}(e^{j\omega}) \cdot G_2^{-1}(e^{-j\omega}) \left[\frac{\partial G_2(e^{-j\omega})}{\partial \underline{\beta}} \right] \right\} d\omega + \left[\frac{1}{2\Sigma^2} \right] \left[\frac{\partial \Sigma}{\partial \underline{\beta}} \right]^T \left[\frac{\partial \varepsilon}{\partial \beta} \right] \tag{VI-18.C))}$$

$$\underline{\beta}^T = [b_1, b_2, \cdots, b_n, a_1, a_2, \cdots, a_n, d_1, d_2, \cdots, d_q, c_1, c_2, \cdots, c_q, \Sigma\,] \tag{VI-19.C}$$

An input constraint is normally specified to limit the energy demand,

$$P_u = \frac{1}{\Pi} \int_0^{\Pi} d\xi(\omega)\,. \tag{VI-20.C}$$

Various optimization procedures for $\bar{M}$ exist, some of which can yield solutions for unidentifiable systems [171,198]. Defining the *Dispersion Matrix* [199]

$$D = \bar{M}^{-1} \tag{VI-21.C}$$

various input design criteria can then be used. Input design $F_1(\omega)$ is preferred to $F_2(\omega)$ will be denoted as $F_1 > F_2$. Criteria include

$$D(F_1) < D(F_2) \tag{VI-22.C}$$

$$|D(F_1)| < |D(F_2)| \tag{VI-23.C}$$

$$\text{Tr } D(F_1) < T_r\, D(F_2) \tag{VI-24.C}$$

$$\text{Tr } \bar{M}(F_1) < T_r\, \bar{M}(F_2) \tag{VI-25.C}$$

$$\lambda_{max}\, D(F_1) < \lambda_{max}\, D(F_2) \tag{VI-26.C}$$

λ_{max} is the maximum eigenvalue of D. For systems of $n \geq 2$, search routines such as in [199] are required.

This is a fertile area for MRAS input design, however, most human operator problems do not allow input control explicitly. Some results for human operator cases include [204], where a forcing function is needed for human respiratory response work, and pilot aircraft plant identification [205]. Some of the work in this report pertains to nonlinear system identification, and optimal input selection in these cases is needed. Such a general approach is developed in [206], wherein the solution of 4n two-point boundary value problems is required, although the method applies to any nonlinear dynamic system.

3. MRAS Observer Frequency Selection

From Section A of this chapter a frequency optimization technique for certain MRAS equation error observers was developed by Kim [189]. This approach supplies optimum frequencies of the input based on a matrix eigenvalue requirement, similar to some of the design approaches in Section 2. Unfortunately, the optimal design which maximizes the convergence rate is very sensitive to frequency deviations about the optimal, and hence

further work is needed.

D. State Variable Filter Selection

Most practical physical systems have only a limited number of state variable measurements available, and some of these even possess noise. Some of these problems may be alleviated by use of a *state-variable filter* (SVF), a dynamic element which provides the dual goals of a) approximating transport lag, and b) providing estimates of derivatives of the measurable output signals for use as replacements for those unmeasurable derivatives. Such an approach is of most value when SISO type systems are to be identified, wherein the need for derivatives of the system input and output are needed, as well as potential noise filtering. The SVF is used both for approximate as well as exact identification methods using both the equation error and the output error approaches. Since the purpose of the SVF differs slightly for the nonlinear case from the linear case, the SVF use for that case will be addressed separately from that of both the equation error and output error methods.

1. Equation Error SVF Use for Linear Identifiers

Various forms of SVF are used for equation-error approaches. All are used to, in some way, generate "pseudo-states" either involving the input or the output, to be used with some identifier algorithm.

The first SVF form is

$$M_k(s) = H(s)\ (s+c_k)^k \qquad \text{(VI-1.D)}$$

where H(s) is an arbitrary stable filter and c_k is a constant > 0, and $k = 1,2,\cdots,n$ where n is the order of the unknown plant to be identified. Such an approach was used by Lion [102] and Rucker [122], with the simplest

and most convenient form being

$$M_k(s)\ s^k \qquad \text{(VI-2.D)}$$

since identifier parameterization is one to one between model and unknown plant parameters.

The next is the simple form

$$M_k(s) = \frac{1}{s+\lambda_k} \qquad k = 2, 3, \cdots, n \qquad \text{(VI-3.D)}$$

where $\lambda_k > 0$. These are used with the measured input and output signals to generate the additional 2(n-1) state and input terms needed for a general formulation. Approaches utilizing this include Lüders [107] and Kraft [124].

A third general SVF form is

$$M(s) = \frac{d_o}{s^n + \sum_{i=0}^{n-1} d_i s^i} \qquad \text{(VI-4.D)}$$

where the d_i terms are selected such that the filter passband is larger than the frequencies of interest in the identification process. Although written in various state-variable forms, Carroll [106], Carroll [120], and Hang [109] all employ this standard linear SVF formulation.

A fourth general SVF form is

$$\underline{x} = F^T \hat{\underline{x}} + \underline{r}\, z \qquad \text{(VI-5.D)}$$

where $(\underline{r}^T, F)$ is an observable pair, z is either the input or output from which pseudo-states are to be generated, and $\hat{\underline{x}}$ is the generated psuedo-state vector from z. In practice, the dimension of F is (n-1)x(n-1) and $\underline{r}$ is a vector of size (n-1)x1. The theoretical requirements for parameter selection are only that $(\underline{r}^T, F)$ is observable, a nebulous design condition

at best. This generalized SVF form is used by Lüder [112], Kudva [114], and Kreisselmeier [134].

A fifth SVF form is generated by Molnar [123] as

$$\dot{\underline{\hat{\omega}}} = \begin{bmatrix} -\lambda_1 & 1 & 0 \cdots 0 \\ 0 & -\lambda_2 & 1 \cdots 0 \\ & & \ddots \\ 0 & 0 \cdots & -\lambda_{n-1} \quad 1 \\ 0 & 0 \cdots & 0 \quad 0 \end{bmatrix} \underline{x} \qquad \text{(VI-6.D)}$$

$$\underline{x}^T = [\hat{\omega}_1 \quad \hat{\omega}_2 \cdots \hat{\omega}_{n-1} \quad z] \qquad \text{(VI-7.D)}$$

where z is either the input or output, and $\lambda_i > 0$. He uses it to develop pseudo states for an equation error type SISO observer.

In general SVF are required for continuous time systems only because of the need to generate derivative or filtered pseudo-states from a single measurement. In SISO discrete time work, the additional "states" are directly available as delayed versions of the input or output, i.e. if y(k) is the output, y(k-1), y(k-2) ··· is a set of state variables directly available with no formal filtering (just delaying, or "waiting"). This set of delayed states in discrete time is directly analogous to differentiation in continuous time, as can be seen from

$$s \simeq \frac{1 - z^{-1}}{T} \quad [207] \qquad \text{(VI-8.D)}$$

$$s\, y(s) \simeq \left(\frac{1-z^{-1}}{T}\right) y(z) = \frac{y(z) - z^{-1}y(z)}{T} \qquad \text{(VI-9.D)}$$

where

$$\mathcal{Z}^{-1}\{z^{-1}y(z)\} = y[(k-1)T] \qquad \text{(VI-10.D)}$$

or

$$\dot{y} \simeq \frac{y(k) - y(k-1)}{T} \qquad \text{(VI-11.D)}$$

It is clear then that differentiation relates to discrete time-delay.

An exception to the guideline that discrete-time systems do not use SVF is the approach of [114]. In this case additional "states" are generated using the SVF form of (VI-5.D). This approach in using a SVF, is then not a minimal realization form, and was designed to follow the continuous identifier approach in [112]. Another discrete approach using SVF is [208].

Not only direct MRAS approaches use SVF structures, but also others. Examples include [5,122,208-212] among others.

2. Output Error SVF for Linear Identifiers

The output error formulation has been shown to be asymptotically stable under a variety of practical conditions. Along with the SVF, however, is generally the need for an additional "compensator" to insure positive realness of a filter, as in [109,121,193,213-215]. Unfortunately this PRF requirement utilizes information about the (unknown) plant and hence is difficult to enforce exactly, although various guidelines have been developed [116,128,129]. This approach will be illustrated to clarify the difficulty.

Consider the system in Figure VI-2 [121]. The plant can be expressed as

$$\frac{x_p}{u} = \frac{\sum_{j=0}^{n-1} b_{p_j} s^j}{s^n + \sum_{j=0}^{n-1} a_{p_j} s^j} \qquad \text{(VI-12.D)}$$

$$y_p = x_p + \eta \qquad \text{(VI-13.D)}$$

Using identical SVF, u and y_p are filtered to yield the plant

$$\dot{\underline{x}}_{p_f} = A_p \underline{x}_{p_f} + B_p \underline{u}_f \qquad \text{(VI-14.D)}$$

$$y_{p_f} = C_1 \underline{x}_{p_f} \qquad \text{(VI-15.D)}$$

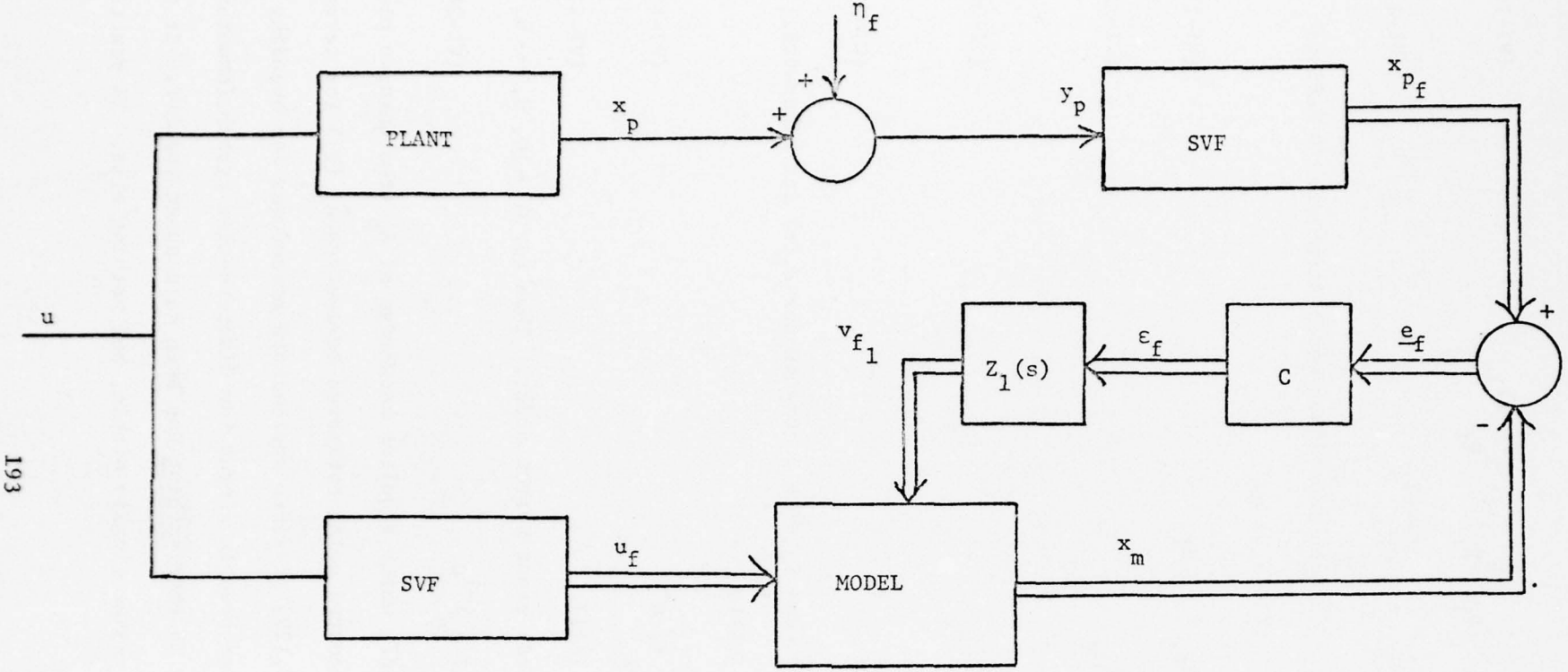


Figure VI-2. Parallel MRAS Identification Employing State Variable Filters (SVF).

$$\underline{x}_{p_f}^T = [x_{p_f} \quad \dot{x}_{p_f} \quad \ddot{x}_{p_f} \quad \cdots \quad x_{p_f}^{(n-1)}] \tag{VI-16.D}$$

$$\underline{u}_f^T = [u_f \quad \dot{u}_f \quad \cdots \quad u_f^{(n-1)}] \tag{VI-17.D}$$

where $x_{p_f} \simeq x_p$ and $u_f \simeq u$. An identifier model structure is defined similar to (VI-14.D), (VI-15.D) as

$$\dot{\underline{x}}_{m_f} = A_m \underline{x}_{m_f} + B_m \underline{u}_f \tag{VI-18.D}$$

$$y_{m_f} = C_1 \underline{x}_{m_f} \tag{VI-19.D}$$

with tracking error

$$\underline{e}_f = \underline{x}_{p_f} - \underline{x}_{m_f} \tag{VI-20.D}$$

Defining

$$\varepsilon_f = C_1 \underline{e}_f \tag{VI-21.D}$$

it is necessary to process ε_f with a compensator $Z_1(s)$ for hyperstability [216] to be assured, where

$$Z_1(s) = \sum_{i=1}^{\ell} z_i s^i \tag{VI-22.D}$$

$$(n-1) < \ell < (n+1) \tag{VI-23.D}$$

and n is the (expected) plant system order. Then for $G^T = [0, 0, \cdots 0, 1]$,

$$Z_1(s)\ C_1(sI-A_p)^{-1}G \tag{VI-24.D}$$

must be a strictly PRF, which requires knowledge of A_p (the unknown plant). Landau [116] has suggested using estimated parameters $\hat{a}_{p_j}(t_o)$ for terms in A_p, and Johnson [128,129] provides approximate procedures for bounding the z_i selection. Similar results accrue for discrete-time system formulations. What is unusual here is that nothing has been said about the SVF. It is required that it be asymptotically stable, but nothing else. In practice,

the passband of the SVF should be wide enough to let through the main u and x_p signals, but reject η, as discussed in [209].

3. Nonlinear System SVF Requirements

With *continuous* nonlinear system identification, the SVF supplies not only approximate derivatives, but also attempts to approximate transport lag [142-144,161,217,218]. This transport lag phenomena has to do with general operator theory on nonlinear functions, namely that passing a signal through a nonlinear operator followed by a transport lag filter is the same as passing the same signal through a transport lag unit first followed by the same nonlinear operator [161]. This operator can be memory or memoryless in nature [144].

To see this, consider a general nonlinear dynamic system

$$\left[s^n + \sum_{i=0}^{n-1} a_i s^i \right] x(s) + f_k(x^{(k)}) = u(s) \qquad \text{(VI-25.D)}$$

where $a_k \neq 0$, $0 \leq k < n$ and integer, u is the input and x the output. If x and all its derivatives are measurable, then the nonlinearity can be computed from

$$f_k(x^{(k)}) = u - (x^{(n)} + a_{n-1} x^{(n-1)} + \cdots + a_{k+1} x^{(k+1)} + a_{k-1} x^{(k-1)} + \cdots + a_o x) \qquad \text{(VI-26.D)}$$

In general, however, only x and u are available with $x^{(j)}$, $j > 0$ unavailable. Using the linear operator L(s)

$$x_c(s) = L(s)\, x(s) \qquad \text{(VI-27.D)}$$

If L(s) has available from it derivatives of x, i.e. $\hat{\dot{x}}$, $\hat{\ddot{x}}$, etc., then

$$s\hat{x}(s) = s\,L(s)\,x = L(s)\,(sx)$$

$$\vdots \qquad \text{(VI-28.D)}$$

$$s^n\hat{x}(s) = s\,L(s)\,x = L(s)\,(s^n x)$$

and $\quad L(s)\,(s^n x) = s^n L(s)x$

Using L(s) on u to yield u_c, then an approximation to $f_k(x^{(k)})$ in (VI-26.D) is

$$\hat{f}_k(\hat{x}^{(k)}) = u_c - (\hat{x}^{(n)} + a_{n-1}\hat{x}^{(n-1)} + \cdots + a_{k+1}\hat{x}^{(k+1)} + a_{k-1}\hat{x}^{(k-1)} + \cdots + a_o\hat{x}) \qquad \text{(VI-29.D)}$$

where the RHS of (VI-29.D) is obtained by operating on the RHS of (VI-26.D) with L(s). This yields

$$\hat{f}_k(\hat{x}^{(k)}) = L(s)\,f_k(x^{(k)}) \qquad \text{VI-30.D)}.$$

For identification to be meaningful, then also

$$\hat{f}_k(\hat{x}^{(k)}) \triangleq f_k(\hat{x}^{(k)}) \qquad \text{(VI-31.D)}$$

Substituting the necessary constraint (VI-31.D) for $\hat{f}$ into (VI-30.D) yields

$$f_k(\hat{x}^{(k)}) = L(s)\,f_k(x^{(k)}) \qquad \text{(VI-32.D)}$$

Using (VI-28.D),

$$f_k(L(s)\,x^{(k)}) = L(s)\,f_k(x^{(k)}) \qquad \text{(VI-33.D)}$$

Since f_k is a nonlinear operator just as L(s) is a linear one, then (VI-33.D) requires L(s) must commute with the general (and as of yet unspecified) nonlinearity. The only known L(s) satisfying this requirement is the transport lag filter,

$$L(s) = e^{-Ts} \qquad \text{(VI-34.D)}$$

Since e^{-Ts} is not realizable in a finite filter form, a phase-variable canonical form filter (which yields $\hat{\dot{x}}$, $\hat{\ddot{x}}$, etc directly) is employed in such a way that the amplitude and phase of L(s) are "close" to that of e^{-Ts} (which has unity gain and a phase shift which varies linearly with frequency,

$$M = \left| e^{-Ts} \right|_{s = j\omega} = 1$$
$$\phi = \angle e^{-Ts} \Big|_{s = j\omega} = -\omega T \qquad \text{(VI-35.D)}$$

Typical guidelines for filter coefficients include ITAE zero-displacement error coefficients [219] as given by Kohr [161], Ricker [217], and Kohr [218]. Any frequency domain fit such that for a given cut-off frequency ω_o, $\angle L(j\omega) \simeq \alpha\omega$ for $\omega << \omega_o$ will do. Examples include Pade, Bessell, Butterworth, etc. [220]. Shown in Table VI-2 are the ITAE ($J = \int_o^t t|e|dt$) coefficients for the normalized transport lag approximation

$$e^{-Ts} \simeq \frac{1}{S^r + q_{r-1} S^{r-1} + \cdots + q_1 S + 1} \qquad \text{(VI-36.D)}$$

where $S = \left(\frac{T}{\omega_o}\right) s$, r is the desired filter order, T is the delay time (want it small), and ω_o is the desired upper cut-off frequency.

Another SVF approach was proposed by Paynter [221] using the hyperbolic relations

$$e^{Ts} = \cosh Ts + \sinh Ts \qquad \text{(VI-37.D)}$$

and replacing sinh and cosh with their infinite product expansions. Krouse has shown its applicability to nonlinear identification and its superiority to the ITAE coefficients. These coefficients are given in Table VI-3 for the normalized filter (VI-36.D). Hence for r = 2, the second order fit is

TABLE VI-2. ITAE FILTER COEFFICIENTS

r	q_1	q_2	q_3	q_4	q_5	q_6	q_7
2	1.4						
3	2.15	1.75					
4	2.7	3.4	2.1				
5	3.4	5.5	5.0	2.8			
6	3.95	7.45	8.60	6.60	3.25		
7	4.56	10.64	15.54	15.08	10.42	4.47	
8	5.15	13.30	22.20	25.75	21.60	12.80	5.20

$$e^{-Ts} \simeq \frac{1}{\left(\frac{Ts}{\omega_o}\right)^2 + 1.571\left(\frac{Ts}{\omega_o}\right) + 1} \qquad \text{(VI-38.D)}$$

A third and final transport lag approximation is the well-known Padé approximation [222]. It is given by

$$e^{-s\tau} \simeq \frac{\alpha_o + \alpha_1(-s\tau) + \alpha_2(-s\tau)^2 + \cdots + \alpha_n(-s\tau)^n}{\alpha_o + \alpha_1(s\tau) + \alpha_2(s\tau)^2 + \cdots + \alpha_n(s\tau)^n} \qquad \text{(VI-39.D)}$$

where the all-pass form is shown. Coefficients α_i for different n are given in Table VI-4.

4. SVF Design Guidelines

Based on the preceding development of L(s), a summary of design procedures for selecting L(s) is in order. These include

1) L(s) must be asymptotically stable.

2) If no noise is present, select the SVF to be of the same order and structure for both the input and output.

3) For L(s) use $\omega_o \geq 10\ \omega_c$, where ω_c is the highest expected system frequency.

4) For L(s) use either ITAE zero displacement error coefficients in Table VI-2 or the Paynter coefficients in Table VI-3.

5) Noise bias effects can be managed by selecting ω_o and T as regards noise bandwidth according to [209].

6) For nonlinear identification, use the coefficients in Table VI-3, with $r \geq n$ for noise immunity.

TABLE VI-3. PAYNTER SVF COEFFICIENTS

r	q_1	q_2	q_3	q_4	q_5	q_6	q_7
2	1.57080						
3	2.14503	1.86478					
4	2.72070	3.33333	2.04052				
5	3.29710	4.89532	4.53948	2.15678			
6	3.87392	7.00129	7.36310	5.75450	2.23919		
7	4.45100	9.24756	12.16091	10.02849	6.97450	2.30061	
8	5.02825	12.00450	17.53256	18.80000	12.83252	8.19756	2.34811

TABLE VI-4. ALL-PASS PADÉ COEFFICIENTS

n	α_o	α_1	α_2	α_3	α_4	α_5	α_6
1	2	1					
2	12	6	1				
3	120	60	12	1			
4	1680	840	180	20	1		
5	30240	15120	3360	420	30	1	
6	665280	332640	75600	10080	840	42	1

E. Identifier Gain Design Parameter Selection

The identifier gain coefficients are the acceleration (or weighting) coefficients used in the identifier algorithms, i.e.

$$\dot{p} = \delta \, ex_p \tag{VI-1.E}$$

$$p(k+1) = p(k) - \alpha \frac{ex_p}{u^2} \tag{VI-2.E}$$

δ, α are the coefficients. Little in the way of exact results are available, so findings to date will be tabulated.

1. For Kudva [113], an approximate analysis approach is given by Colburn [185].

2. For the method in [114], parameter coefficient selection is related to transient response in [26].

3. For Carroll [120], it is shown that the higher the gain the faster the parameter convergence rate, although the *value* of the convergence rate cannot be predicted.

4. Kreisselmeier [134] provides a bound on exponential parameter convergence rate from a selection of the gain adjustment parameters.

5. Sprague [143] suggests a signal averaging process. Consider the plant

$$a_1 x^{(1)} + a_o x = u \tag{VI-3.E}$$

and identifier

$$\dot{\delta}_1 = -G_1 (x^{(1)})^2 \delta_1 - G_1 x^{(1)} x \, \delta_o \tag{VI-4.E}$$

$$\dot{\delta}_o = -G_o (x^{(1)})^2 \delta_o - G_o x^{(1)} x \, \delta_1 \tag{VI-5.E}$$

$$\delta_i = \alpha_i - a_i, \; \alpha_i = \text{parameter estimate} \tag{VI-6.E}$$

It is suggested to equalize the average eigenvalue of (VI-5.E) and hence pick G, so that

$$\overline{G_1 (x^{(1)})^2} = \overline{G_o x^2} = C = \text{constant} \tag{VI-7.E}$$

If excessive noise is present, reduce C somewhat. For the nonlinear terms with partitioned variable $y^{(k)}$

$$G_{ij_p} \simeq \frac{C}{\overline{(y^{(k)})^2}_p} \simeq \frac{C}{\overline{(y^{(k)})^2}\ (2p-1)} \qquad \text{(VI-8.E)}$$

where

$$\overline{y_r^{(k)^2}} = \frac{1}{\tau} \int_0^{\tau} y_r^{(k)^2}\, dt \qquad \text{(VI-9.E)}$$

is the interval average and p is the interval number. For linear parameter terms, $1 \leq C \leq 50$ has proven useful. For nonlinear parameters, C needs to be much bigger, typically 10 to 100 times that for linear terms [162].

For sinusoidal inputs, $C \simeq (\omega_{low})$ rad/sec. is a design guideline where ω_{low} is the lowest expected input frequency.

6. A number of identifier gain selections are given in [184].

7. A procedure for minimizing noise and accuracy in the selection of the design parameter is given in this chapter, section A, subsection 2.

F. Computational Considerations

In order to determine the computational burden requirements, a simplified analysis of a number of the algorithms was performed as regards hardware/software needs. Continuous-time systems were evaluated as to the number of integrators and multipliers, and discrete systems in terms of multiply and add operations. The calculations for these methods are given in Appendix I and [109]. The results are approximate due to some simplifying assumptions and to the limited degree of computational optimization employed.

In Tables VI-5 through VI-8 are the results for some of the methods, Table VI-5 restricted to continuous linear time-invariant, Table VI-6 to discrete time-invariant, and Table VI-7 to nonlinear time-invariant methods. Table VI-5 assumes a plant of the form

$$G(s) = \frac{\sum_{i=0}^{m} b_i s^i}{s^n + \sum_{i=1}^{n} a_i s^{i-1}} \qquad \text{(VI-1.F)}$$

is to be identified, Table VI-6 a plant

$$G(z) = \frac{\sum_{i=0}^{m-1} b_i z^{-i}}{1 + \sum_{i=1}^{n} a_i z^{-i}} \qquad \text{(VI-2.F)}$$

and Table VI-7 a plant

$$y(t) = \sum_{h=1}^{\ell} \frac{F_h(s)}{A(s)} n_h(u(t)) \qquad \text{(VI-3.F)}$$

where

$$A(s) = s^n + \sum_{i=0}^{n-1} a_i s^i \qquad \text{(VI-4.F)}$$

$$F_h(s) = f_{hm} s^m + f_{h(m-1)} s^{m-1} + \cdots + f_{h_1} s + f_{h_o} \qquad \text{(VI-5.F)}$$

$n_h(u(t))$ a nonlinear static function of the scalar input $u(t)$. In each case, the "type" of identifier, either response error (RE) representing "parallel" MRAS, or generalized equation error (GEE) representing "series-parallel" MRAS, is indicated.

Shown in Table VI-8 are the computation requirements for the λ and W memory shaping methods for time varying system identification using the

Landau MRAS identifier [116]. The calculation details are given in Appendix I. With the W method an averaging requirement occurs because every N samples a special set of update equations must be employed, so the multiplication entry is an average cost/cycle, the actual burden being non-symmetrically distributed with time.

From a study of the tables, based on the limited number of methods reviewed, a number of findings can be stated. As regards Table VI-5:

Finding 1.

> The hardware requirements of the RE and basic GEE methods are about equal.

Finding 2.

> Lion's extended-GEE method, although providing fast convergence rate [102], requires prohibitive hardware.

From Table VI-5, it can be seen that:

Finding 3.

> The Narendra and Landau methods, although exact and insuring stability, are far more complicated than the simplified method of Johnson. This points up the tremendous hardware savings possible by simplifying the identifier algorithms.

Finding 4.

> The GEE method has lower hardware requirements than the RE method. However, the GEE does not guarantee stability, the RE does.

Finding 5.

> The additional computational burden for the time-varying identifiers is nominal, so decisions as to their use depends on time-invariant burden. The λ method, however, requires less additional computations than the W methods.

That comparison of computational requirements is not trivial, see [224]. More recently, [131,225] develop and calculate computational demands of various identifier structures. Such work is of key importance in

TABLE VI-5. HARDWARE REQUIREMENTS OF CONTINUOUS LINEAR IDENTIFIER METHODS

METHOD	TYPE	I	M
Hang [109]	RE	5n-2	4n
Tomizuka [142] (no nonlinearities)	RE	5n	4n
Young [223]	GEE	4n	4n
Young [5]	GEE-IV	6n	6n
Lion [102]	GEE - Extended	8n-2	$8n^2$

Assumes:

m = n

Addition, Subtraction can be additionally performed with integers

I = integrator

M = multiplier

TABLE VI-6. HARDWARE REQUIREMENTS OF DISCRETE LINEAR IDENTIFIER METHODS

Method	Type	A	M
Narendra [113]	GEE	$(9n^2 + 3n)$	$(7n^2 + 2n+1)$
Landau [116]	RE	$(4n^2 + 23n + 8)$	$3(2n+1)^2 + 15n+11$
Johnson [128]	RE	$6n + 5$	$10n + 7$

Assumes:

Add (A) = Subtract (S)

Multiply (M) = Divide (D)

$m = n$

TABLE VI-7. HARDWARE REQUIREMENTS FOR CONTINUOUS NONLINEAR IDENTIFIERS

Method	Type	I	M
Sehitoglu [144]	GEE	$4n + \ell$	$4n + 2\ell$
Tomizuka [142]	RE	$n(3 + 2\ell)$	$n(2 + 2\ell)$

Assumes:

Simple nonlinearities only considered

ℓ single-valued nonlinearities

$m = n$

Add, Subtract can be additionally performed with integrators

TABLE VI-8. TIME-VARYING IDENTIFIER CALCULATIONS

Method	Type	A	M
λ Method	RE	$(4n^2 + 23n + 8)$	$3(2n+1)^2 + 15n + 12$
W Method*	RE	$(4n^2 + 23n + 8)$	$(3+\frac{1}{N})(2n+1)^2 + 15n + 11 + \frac{1}{N}$ *

*Average Calculations/cycle

N = # samples in update cycle period Δt;

$\Delta t = NT$.

digital implementation on a microprocessor, where the real-time demands are most severe. Not developed, but of great importance also, are the storage requirements of the various identifier algorithms.

G. Model Order Determination

Although many different identification algorithms have been developed, of which parallel and series-parallel MRAS methods for linear, nonlinear, and time-varying plant have been developed, compared and discussed in this report. All, however, assume the structure and order of the dynamics are known *a priori*. In practical application, this is never true, and hence some sort of "systematic" guesswork is required. To aid in the structure and order determination, a number of techniques will be given.

In order to develop an identification package which is implementable in real time without need of restrictive *a priori* human operator knowledge, some form of real-time parameter-space model-order search is needed. To the knowledge of the writer, no previous work along these lines has been done previously. Traditionally, off-line determination is employed by selecting a model structure and obtaining a "best-fit" parameterization. These off-line methods include Woodside [226], Unbehauen and Göhring [227], Van Den Boom and Van Den Enden [228], Söderström [229], Wellstead [230], and Wellstead [231].

The problem can be stated as follows:

> Given a plant with unknown coefficients, unknown number of poles, zeros, and (any) transport lag, using only input/output measurements determine the "best" model structure and complexity, given that the smaller the parameterization the better.

It is desirable that some straightforward decision criterion be available for ranking the methods as to their structural accuracy.

1. Off-Line Model Order Determination

First, typically off-line approaches will be surveyed, with comment as to their applicability to the present MRAS study. The disadvantages of these approaches are that they generally require statistical information concerning the quality of parameter estimates, a parameter not available from many MRAS approaches because the MRAS method is designed to be statistically transparent to noise asymptotically. Hence the MRAS algorithms process noisy data to eliminate noise effects without explicitly indicating such information.

1a) Determinant Ratio Test [226]

This method is only used to limit the possible model orders. Suppose the unknown plant is discrete-time of order n_o with input u(k) and noisy output y(k). Based on any a priori information, a test order n is selected and used to define

$$\underline{h}_r^T(n) = [u(k-1),\ y(k-1),\ u(k-2),\ y(k-2),\ \cdots \\ u(k-n),\ y(k-n)] \qquad \text{(VI-1.G)}$$

With N an integer >> n and representing an analyst's choice of a data smoothing region,

$$H_r(n) = \frac{1}{N} \sum_{k=n+1}^{n+N} \underline{h}_r(n)\ \underline{h}_r^T(n) \qquad \text{(VI-2.G)}$$

Using the decision function

$$J = \left| H_r(n) \right| \qquad \text{(VI-3.G)}$$

where $|L|$ represents determinant of the matrix L, it can be decided whether $n \simeq n_o$ from

$$\begin{aligned} |H_r(n)| &> 0 \quad \text{for} \quad n \leq n_o \\ |H_r(n)| &= 0 \quad \text{for} \quad n > n_o \end{aligned} \tag{VI-4.G}$$

In practice, the equality is hard to meet precisely, and hence the selection of an optimum n is not so simple. The determinant ratio

$$V(n) = \frac{|H_r(n)|}{|H_r(n+1)|} \tag{VI-5.G}$$

can be employed to determine if increasing n improves the "fit". If V(n) markedly increases as compared to V(n-1), then $n \simeq n_o$, where $n = 1,2,\cdots$, n_{max} with n_{max} some *a priori* selected maximum value of model order to be considered.

1.b) Condition Number Test [232]

Using a direct least-squares formulation [233] for a model

$$\frac{y_p}{u}(z) \simeq \frac{\sum_{i=1}^{n} b_i z^{-i}}{1 + \sum_{i=1}^{n} a_i z^{-i}} \tag{VI-6.G}$$

with measured output $y = y_p + \varepsilon$, ε a noise , this can be parameterized as

$$\underline{y}(N) = M(N)\ \underline{p}(N) + \underline{\varepsilon}(N) \tag{VI-7.G}$$

$$\underline{y}^T(N) = [y(n+1) \quad y(n+2) \quad \cdots \quad y(n+N)] \tag{VI-8.G}$$

$$\underline{p}^T(N) = [a_1 \quad a_2 \cdots a_n \quad b_1 \quad b_2 \cdots b_n] \tag{VI-9.G}$$

$$\underline{\varepsilon}^T(N) = [\varepsilon(n+1) \quad \varepsilon(n+2) \quad \cdots \quad \varepsilon(n+N)] \tag{VI-10.G}$$

$$M(N) = \begin{bmatrix} -y(n) & -y(n-1) & \cdots & -y(1) & u(n) & u(n-1) & \cdots & u(1) \\ -y(n+1) & -y(n) & \cdots & -y(2) & u(n+1) & u(n) & \cdots & u(2) \\ \vdots & & & & & & & \vdots \\ -y(n+N-1) & -y(n+N) & \cdots & -y(N-1) & u(n+N-1) & \cdots & & \end{bmatrix} \quad \text{(VI-11.G)}$$

Testing is based on the cost criterion

$$J = ||A|| \cdot ||A^{-1}|| \quad \text{(VI-12.G)}$$

where

$$||A|| = \max_i \left\{ \sum_{j=1}^{n} a_{ij} \right\} \quad \text{(VI-13.G)}$$

$$A = M^T(N)\, M(N) \quad \text{(VI-14.G)}$$

If $n > n_o$, then A^{-1} can become singular (or very large), which can be used as a selection guideline. A suggested threshold for $n > n_o$ decision is [234]

$$J \geq 10^7 \ . \quad \text{(VI-15.G)}$$

1.c) Polynomial Test

Given a noisy plant SISO formulation

$$A(z)\, y(z) - B(z)\, u(z) = C(z)\, \varepsilon(z)$$

with A, B, C filter polynomials of degree n, testing can be performed to determine if any of A, B, or C have common factors. Söderström [229] presents a search technique for accomplishing this. In general, ill-conditioning [235,236] makes this approach difficult.

1.d) Test For Independence

This test determines whether the model errors ε are uncorrelated. Define

$$R_{\varepsilon\varepsilon}(\tau_N) = \frac{1}{N} \sum_{k=n+1}^{n+N} \varepsilon(k)\, \varepsilon(k+\tau_n) \quad \text{(VI-17.G)}$$

where $\tau_N = 0,1,2,\cdots,10$. If $R_{\varepsilon\varepsilon}(\tau_N \neq 0) \simeq 0$, then the $\varepsilon(k)$ are uncorrelated and an acceptable model fit (from which ε were obtained) results. This is also referred to as a *test for correlation* [236,237].

1.e) Test for Normality

This check is used to determine if the statistical distribution of ε is normally distributed, given a normal random exciting input was applied. This off-line method requires human interpretation data analysis by [227] techniques.

1.f) Statistical F-Test

Aström [238] provides a test for statistical independency of I_1 and I_2, where

$$I_1 = \frac{1}{N} \underline{\varepsilon}^T(N)\, \underline{\varepsilon}(N) \qquad \text{(VI-18.G)}$$

with $\underline{\varepsilon}(N)$ from (VI-10.G) and I_1 is determined from a model with order n_1, and similarly for I_2. If $n_2 > n_1 \geq n_o$, then

$$t = \frac{I_1 - I_2}{I_2} \cdot \frac{N - (2n_2+1)}{2(n_2 - n_1)} \qquad \text{(VI-19-G)}$$

has degrees of freedom

$$f_1 = N - (2n_2 + 1) \qquad \text{(VI-20.G)}$$

$$f_2 = 2(n_2 - n_1)$$

and F distribution $F(f_2,f_1)$. For a risk level of 5% and $N > 100$, the significance test is $t < 3$. This means that if $t < 3$, then $n_o \simeq n_1$ is the correct estimated model order [228,239].

A variety of other approaches have been developed but are not applicable here. These include whiteness of residuals, determinant behavior, pole zero cancellation, and likelihood ratio [226]. An excellent comparison of some of these and other approaches is [228].

2. Pseudo On-Line Model Order Determination

The key to any useful order determination technique which has the possibility of operating "on-line" (or practically so) is that it a) must utilize directly identified parameters, and b) work in parallel with the identification algorithm. One such candidate for this is an auxiliary system approach by Wellstead [230] for SISO discrete-time systems. The principle of operation is the fact that an auxiliary system, made up of unknown plant parameter estimates plus other parameter estimates, must equal an auxiliary model if the guessed order of the plant is correct. Similarly, the auxiliary system parameter estimates are biased for incorrect model orders. The beauty of this approach is that any simple auxiliary "model" is created and an identifier, which uses simultaneous parameter estimates, used to check it. The problem of model order determination is thus simultaneously imbedded in the form of an identification problem.

The general algorithm is applied to a plant of the form

$$y(z) = -A_p(z)\, y(z) + B_p(z)\, u(z) + \mathcal{S}(z) \qquad \text{(VI-22.G)}$$

where z is the traditional z-transform, u the input and y the output, $\mathcal{S}$ an unobservable white noise sequence of random variables (it can also be applied to colored noise), and

$$A_p(z) = \sum_{i=1}^{n_a} a_i\, z^{-i} \qquad \text{(VI-23.G)}$$

$$B_p(z) = \sum_{j=k}^{n_b} b_j\, z^{-j} \qquad \text{(VI-24.G)}$$

where kT is the transport lag, n_a the number of poles, and n_b the number of zeros. A series-parallel model of (VI-22.G) is

$$y(z) = -A_m(z)\, y(z) + B_m(z)\, u(z) + e(z) \qquad \text{(VI-25.G)}$$

where $e(z)$ is a model fitting error, and

$$A_m(z) = \sum_{i=1}^{n_\alpha} \alpha_i\, z^{-i} \qquad \text{(VI-26.G)}$$

$$B_m(z) = \sum_{j=\kappa}^{n_\beta} \beta_j\, z^{-j} \qquad \text{(VI-27.G)}$$

The plant structure is then characterized by the triple (n_a, n_b, k) and the model by $(n_\alpha, n_\beta, \kappa)$.

An augmented system plant is defined as

$$\tilde{y} = y(z) + r(z) \qquad \text{(VI-28-G)}$$

$$r(z) = T_p(z)\, m(z) \qquad \text{(VI-29-G)}$$

$$T_p(z) = \sum_{i=0}^{n_t} t_i\, z^{-i} \qquad \text{(VI-30.G)}$$

$$m(z) = u(z) + \delta(z)$$

where $\delta(z)$ is a white noise sequence with zero mean and known variance, and t_i are known designer selected constants. Similarly an augmented model is defined by

$$y = -A_m(z)\, y(z) + B_m(z)\, u(z) + T_m(z)\, m(z) + e(z) \qquad \text{(VI-32.G)}$$

$$T_m(z) = \sum_{i=0}^{n_\tau} \tau_i\, z^{-i} \qquad \text{(VI-33.G)}$$

This configuration is shown in Figure VI-3. Since the augmented plant is known, $T(z)$ can always be correctly parameterized by selecting $n_\tau \geq n_\tau$. The order testing algorithm is

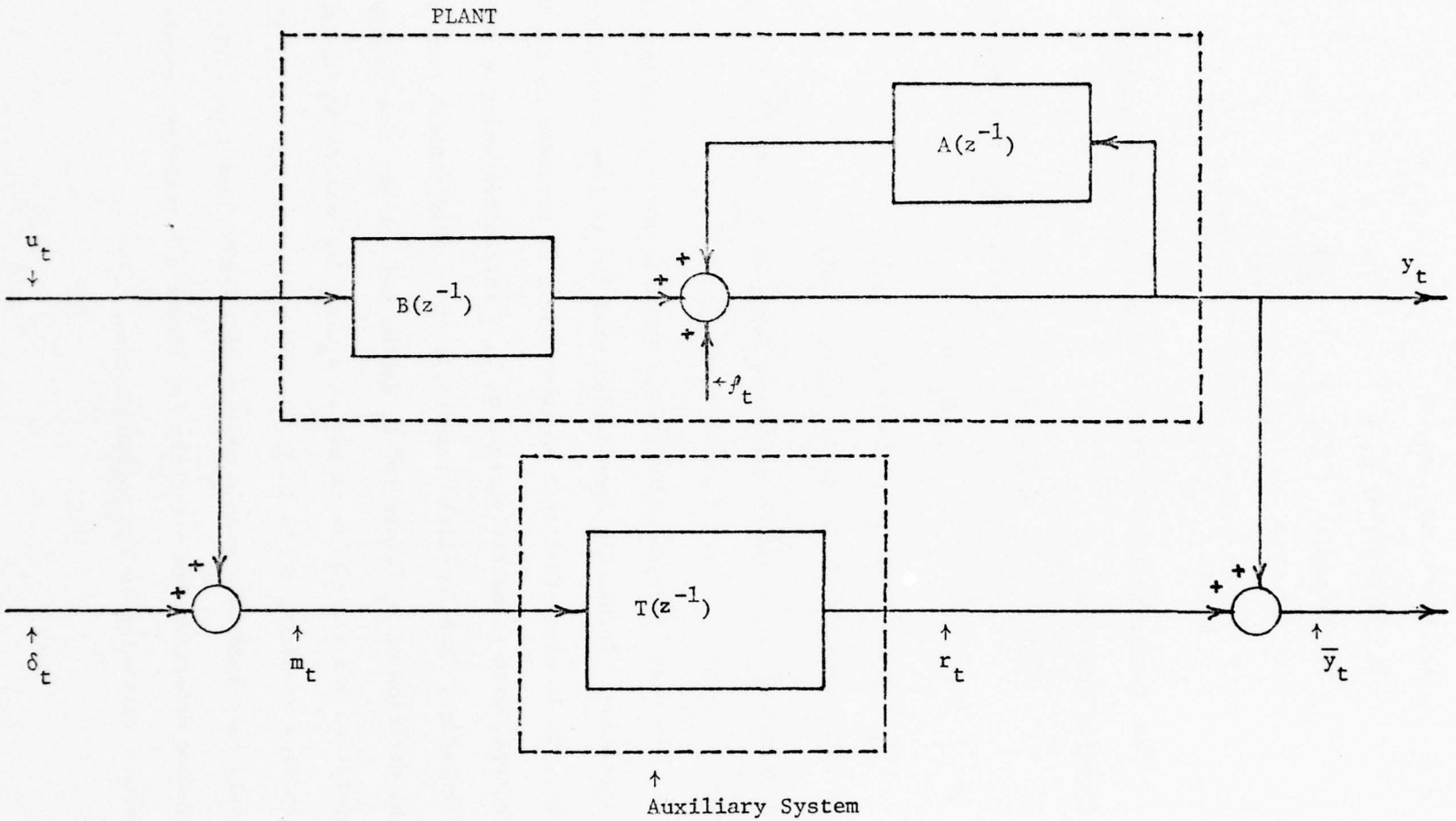


Figure VI-3. General form of augmented system, comprising system under study and auxiliary system.

1) select $(n_\alpha, n_\beta, \kappa)$

2) identify α, β, τ

3) determine if $\{\tau_0 \quad \tau_1 \cdots \tau_{n_\tau}\}$ are equal to $\{t_0 \quad t_1 \cdots t_{n_t}\}$

4) adjust augmented model order (select new $(n_\alpha, n_\beta, \kappa)$

For determination of "fit", item 3 above can be analyzed by a decision ordering

$$\text{If } \{d_\ell < \frac{\tau_i}{t_i} < d_u\}, \quad D_i = 1 \qquad i=1,2,\cdots w_j = \min\{n_t, n_\tau\}$$

$$\text{If NOT}, \quad D_i = 0 \qquad \text{(VI-34.G)}$$

d_u = upper bound on ratio (i.e. 1.1)

d_ℓ = lower bound on ratio (i.e. .91)

In theory d_u, d_ℓ could be different for each τ_i. If $\bigcap_{i=1}^{w} D_i = 1$, then the fit is good, if not then it is not.

The order determination system shown is for a series-parallel configuration. It has not been implemented due to time limitations. However, it is clear that this approach could be extended to the MRAS parallel concept, with parameter vectors $\underline{\alpha}$, $\underline{\beta}$, $\underline{\tau}$ determined using an MRAS configuration. The "on-line" possibilities of this concept come about because the decision of τ_i "close to" t_i can be made at any time during system operation and a decision to adjust n_α, n_β, n_τ made <u>during</u> a run by simply removing or adding terms in $\underline{\alpha}$, $\underline{\beta}$. The further refinement of this concept could then lead to a pseudo on-line model order and transportation lag measure determination algorithm for linear discrete-time models, with obvious extension to the nonlinear case.

3. Other Approaches

In addition to the model order determination techniques discussed before, a few other promising concepts will be mentioned for completeness. Structural distance is a method for determining model parameter matching to an unknown plant, but requires non-measurable information (the plant parameters), to decide whether a model is of the proper structure. A performance index (IP) of this type is

$$IP = ||\underline{p} - \hat{\underline{p}}|| \qquad \text{(VI-35.G)}$$

where $\underline{p}$ is the plant parameter vector and $\hat{\underline{p}}$ the estimate [70].

Another, but this time realizable, IP is

$$IP = \sum_{i=1}^{m} (x_m(k) - y_p(k)^2 \qquad \text{(VI-36.G)}$$

The problem here is if y_p is noise corrupted (i.e. $y_p = x_p + \eta$, η a noise), then m must be large to "average out" the noise. Such an approach could, however, be used in principle to decide if $IP < IP_{max}$ so that the model represented by x_m could be a "reasonable fit".

Mehra [4, 236] uses a whiteness test on the innovations sequence of Kalman Filtering identification approach to provide a goodness-of-fit test for the estimated parameters. Such an approach allows use of the standard Kalman Filter structure to obtain an optimal, efficient identifier as well as the correct model on-line and simultaneously.

CHAPTER 7. SIMULATION RESULTS

In order to indicate the overall operation of the identifiers for realistic systems, a series of simulation tests were run with the various forms of MRAS identifiers. Following the main report sequence, linear time-invariant, nonlinear time-invariant, and linear time-varying identifiers will be addressed in that order. These results are not meant to be exhaustive but to demonstrate the operation of the various MRAS identifier algorithms.

A. Linear Time-Invariant Case

Consider the first-order process

$$x_p(k+1) = a_1 x_p(k) + b_o\, u(k+1) + b_1\, u(k) \qquad \text{(VII-1.A)}$$

or

$$x_p(k+1) = \underline{\phi}^T\, \underline{s}(k+1) \qquad \text{(VII-2.A)}$$

$$\underline{\phi}^T = [a_1 \quad b_o \quad b_1] \qquad \text{(constant)} \qquad \text{(VII-3.A)}$$

$$\underline{s}(k+1) = [x_p(k) \quad u(k+1) \quad u(k)] \qquad \text{(VII-4.A)}$$

The output is corrupted by an additive noise, η,

$$y_f(k) = x_p(k) + \eta(k) \qquad \text{(VII-5.A)}$$

where η is a nonstationary zero-mean Guassian noise sequence whose standard deviation $\sigma(k)$ is variable according to

$$\sigma(k) = p\, x_p(k) \qquad \text{(VII-6.A)}$$

p varies between $0 \leq p \leq 1$, and the "percent noise" is $p \times 100\%$. This nonstationary noise which depends on the output is a first step towards the human operator problem where a remnant depends on an input signal magnitude. Initial parameter estimates were found using Eqns. (III-218.B)-(III-221.B) and noisy measurements y_f,

$$A(0) = \begin{bmatrix} \underline{v}^T(-2) \\ \underline{v}^T(-1) \\ \underline{v}^T(0) \end{bmatrix} \qquad \text{(VII-7.A)}$$

$$\underline{v}^T(k) = [y_p(k-1) \quad u(k) \quad u(k-1)] \qquad \text{(VII-8.A)}$$

$$\underline{q}^T(0) = [y_p(-2) \quad y_p(-1) \quad y_p(0)] \qquad \text{(VII-9.A)}$$

yielding

$$\hat{\underline{\phi}}(0) \simeq A(0)^{-1} \, \underline{q}(0) \qquad \text{(VII-10.A)}$$

The MRAS algorithm used is given in Eqns. (III-218.B)-(III-221.B), with

$$F(0) = (A(0)^{-1})(A(0)^{-1}))^T. \qquad \text{(VII-11.A)}$$

Since three parameters are present, the frequency richness criterion was satisfied by using

$$x(k) = \sum_{i=1}^{4} \sin(\frac{ik}{4}) \qquad \text{(VII-12.A)}$$

Using (VII-12) as input to (VII-1) with available measurements u and y_p, simulations were run to demonstrate system operation, with $a_1 = .8016$, $b_o = .0274$, $b_1 = .0286$.

Shown in Figure VII-1 is the effect of different c_i values (see Eqns. (III-109.B) and (III-112.B) on tracking accuracy, with $p = .4$. Case 1 is for $c_1 = \hat{a}_i(k)$ which allows for regular updating of c_1 for the SPR condition, but this case does <u>not</u> insure asymptotic stability. The other four cases are for constant c_1, Case 2 being for c_1 equal to the exact plant pole (unknown), Cases 3 and 5 for c_1 equal to $\pm\{$the (biased) least-squares estimate of $\hat{a}_1(0)\}$, and Case 4 just a number insuring that (III-112.B) is satisfied. Results clearly demonstrate the improved response for c_1 = constant as evidenced by the decreasing structural distance $\frac{||\underline{\phi} - \hat{\underline{\phi}}||}{||\underline{\phi}||}$.

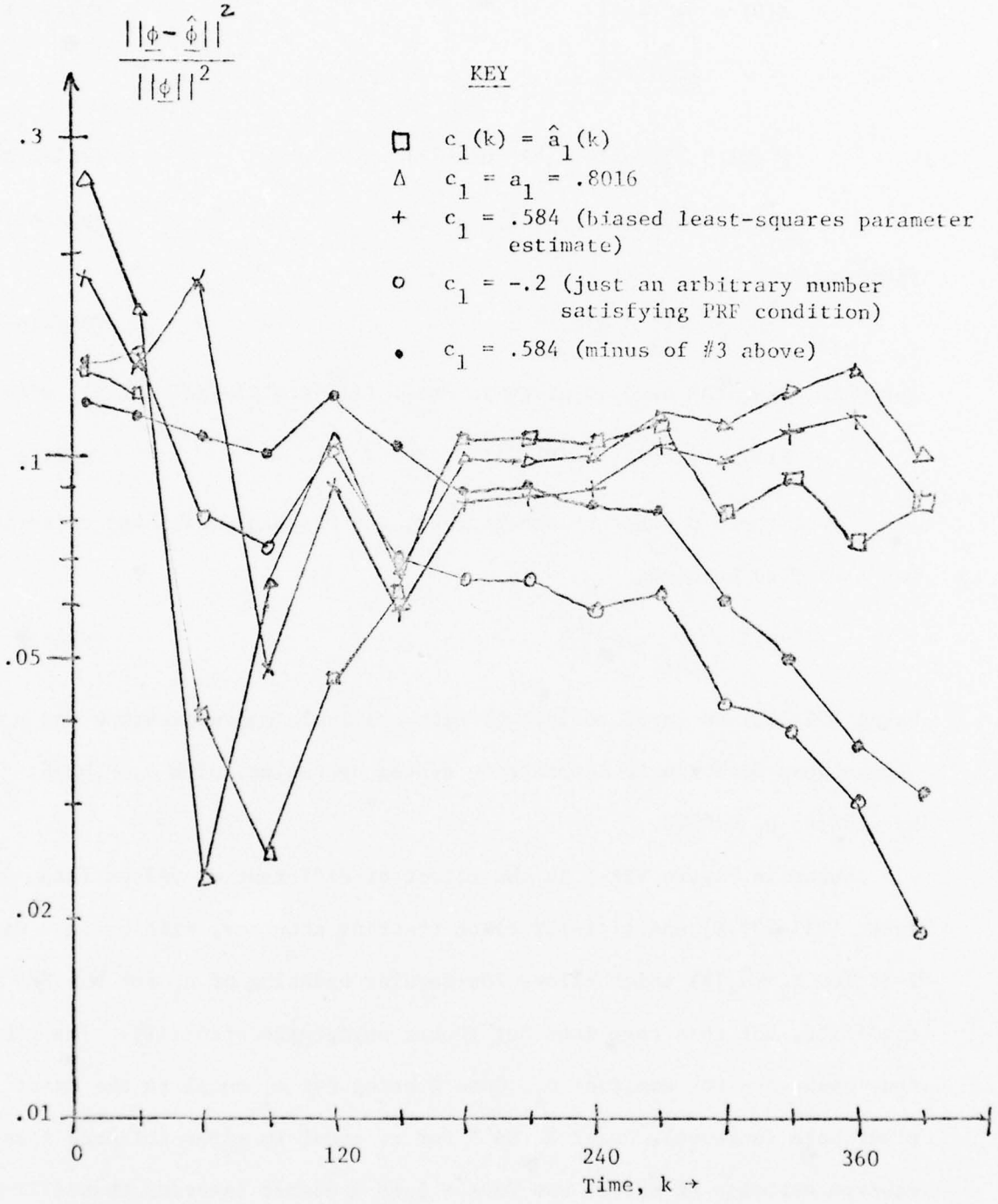


Figure VII-1. Landau MRAS Parameter Convergence Characteristic As A Function of the Designer-Controlled Parameter c_1.

Figure VII-2 demonstrates the tracking accuracy versus c_i = constant at various time instants for the previous simulation case. Note the general parabolic nature of each fixed-time-curve suggesting that at different portions of the response cycle, different fixed c_1 values are best.

As a second example demonstrating the guaranteed asymptotic converge of the MRAS identifier in the presence of output noise, consider the second-order case

$$x_p(k+1) = a_1 x_p(k) + a_2 x_p(k-1) + b_1 u(k) + b_2 u(k-1) \qquad \text{(VII-13.A)}$$

$$y_p = x_p(k) + \alpha\, \eta(k) \qquad \text{(VII-14.A)}$$

where $\eta(k)$ is a colored noise

$$\eta(k+1) = \alpha_1\, \eta(k) + \alpha_2\, \omega(k) \qquad \text{(VII-15.A)}$$

and $\omega(k)$ a Gaussian white noise. For the particular case $\underline{\phi}^T$ = [.975, .223, .079, .047], $E\{u(k)\} = E\{\omega(k)\} = 0$, $E\{u^2(k)\} = E\{\omega^2(k)\} = 1$, $E\{\omega(k)\, u(k)\}$, $\alpha = .08$, $\gamma_1 = .8$, $\gamma_2 = .7$, and the identifier constants $c_1 = c_2 = 0$ (which satisfies the SPR condition of (III-112.B)). The model is assumed the same order as the plant, and for this case the asymptotic convergence characteristic (as guaranteed from Hyperstability) of the normalized structural distance squared $\left(\frac{||\underline{\phi} - \underline{\hat{\phi}}||^2}{||\underline{\phi}||^2}\right)$ is shown in Figure VII-3. Note that although the time-invariant system convergence rate is slow, this can be improved by adjustment of the c_i and F(0) values by the designer.

B. Nonlinear Identification

A number of computer simulations were run to verify the equation error nonlinear identifier of Sehitoglu, et al [144], to indicate the ability of such an algorithmic approach to handle nonlinear dynamic systems with and without memory. Using (IV-41.C)-(IV-51.C), investigations

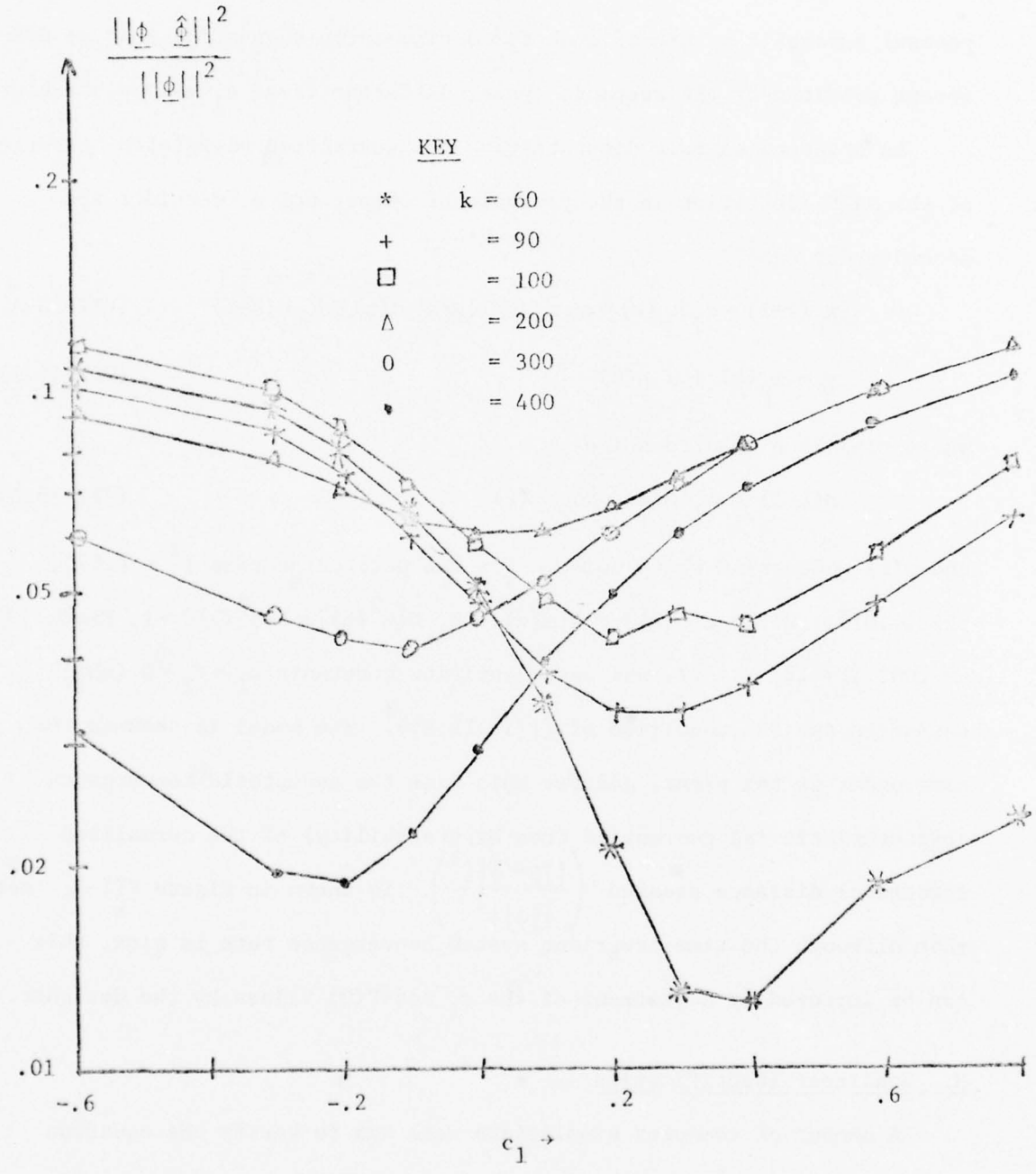


Figure VII-2. Structural Distance vs. c_1 at various Frozen Time Instants.

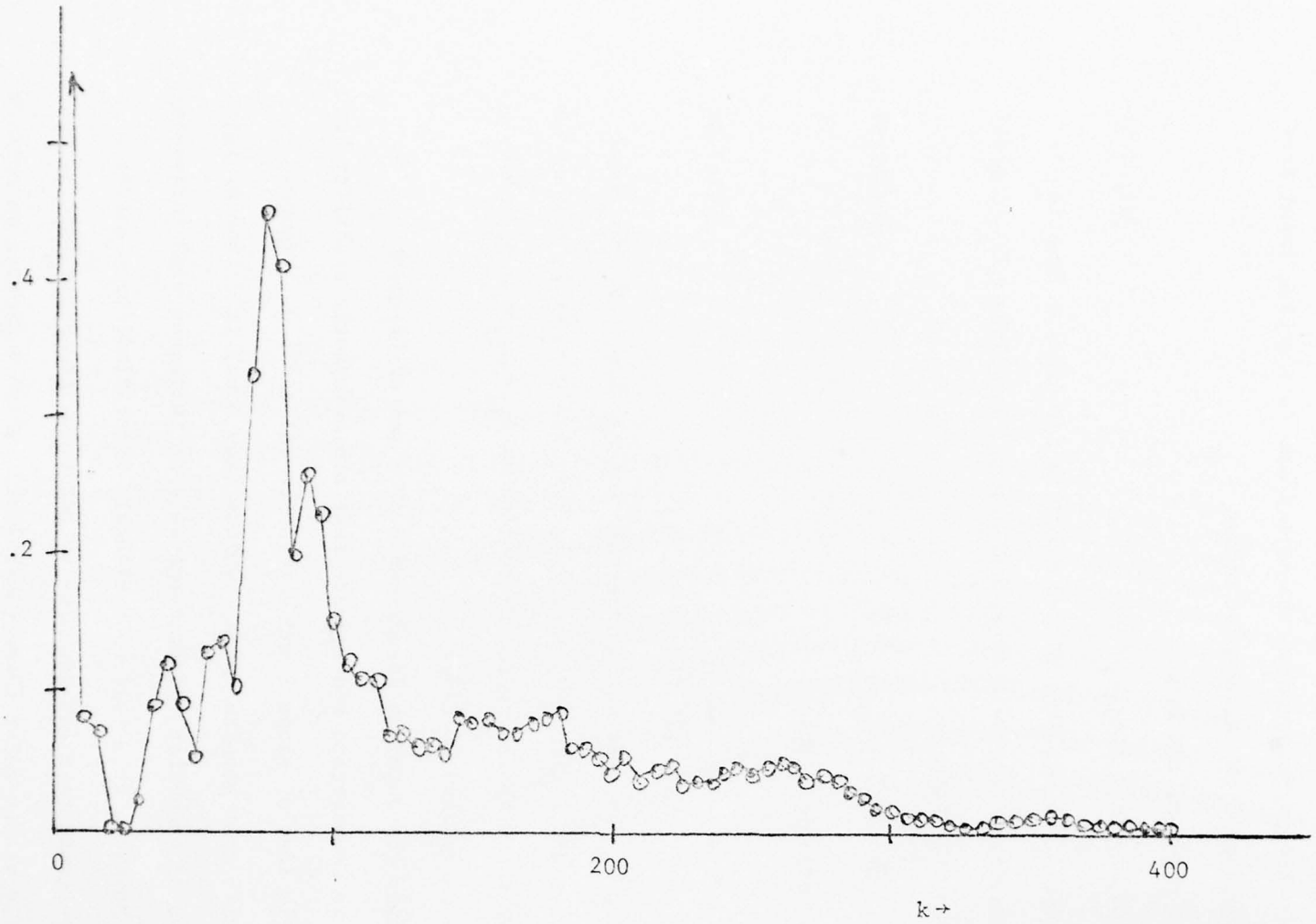


Figure VII-3. Landau MRAS Parameter Convergence Characteristic In The Presence of Colored Output Measurement Noise.

were made into the performance of the identifier to changing input frequency richness and with measurement noise.

First, a plant of the form

$$A_2 \ddot{x}_p + f(\dot{x}_p) + A_0 x_p = u \qquad \text{(VII-1.B)}$$

was used, where $A_2 = A_0 = 1$ (known) and $f(\dot{x}_p)$ is the unknown function (a hysterisis nonlinearity of slope = 1, and saturation value = ± 1). A model was selected of the form

$$A_2 \hat{\ddot{x}}_p + \hat{f}(\hat{\dot{x}}_p) + A_0 \hat{x}_p = \hat{u} \qquad \text{(VII-2.B)}$$

with a state variable filter

$$\frac{\hat{x}_p}{x_p}(s) = \frac{D_0}{D_o + D_1 s + s^2} \qquad \text{(VII-3.B)}$$

with $D_0 = 600.$, $D_1 = 50$. Using 22 intervals for $\hat{x}_p$, i.e. $-2.2 \leq \hat{x}_p \leq 2.2$ with increments of .2 with midpoints 2.1, 1.9, 1.7, $\cdots$, -2.1, the system (VII-1.B) was run with the operating conditions $A_{11} = A_{12} = 0$, $G_{11} = 8.0$, $G_{12} = 30$, and $u = 3.0 \sin \omega t$ [144].

Identifier response was determined over a set of frequencies $.4 \leq \omega \leq .9$ to demonstrate convergence rate and asymptotic stability for the hysterisis loop of slope 1 and saturation value $\pm$ 1.0. Figures VII-4 and VII-5 show results for A_{11} values (key term) in different intervals. It can be seen that the convergence rate increases with increasing frequency. However, at $\omega = .1$, no noticeable identification occurred (either convergence rate was too slow to notice or the identifier is not asymptotically stable under those conditions), which indicates an optimum frequency range is present to accomplish identification in the fastest possible time. The total convergence of the loop at $\omega = .5$ is shown in

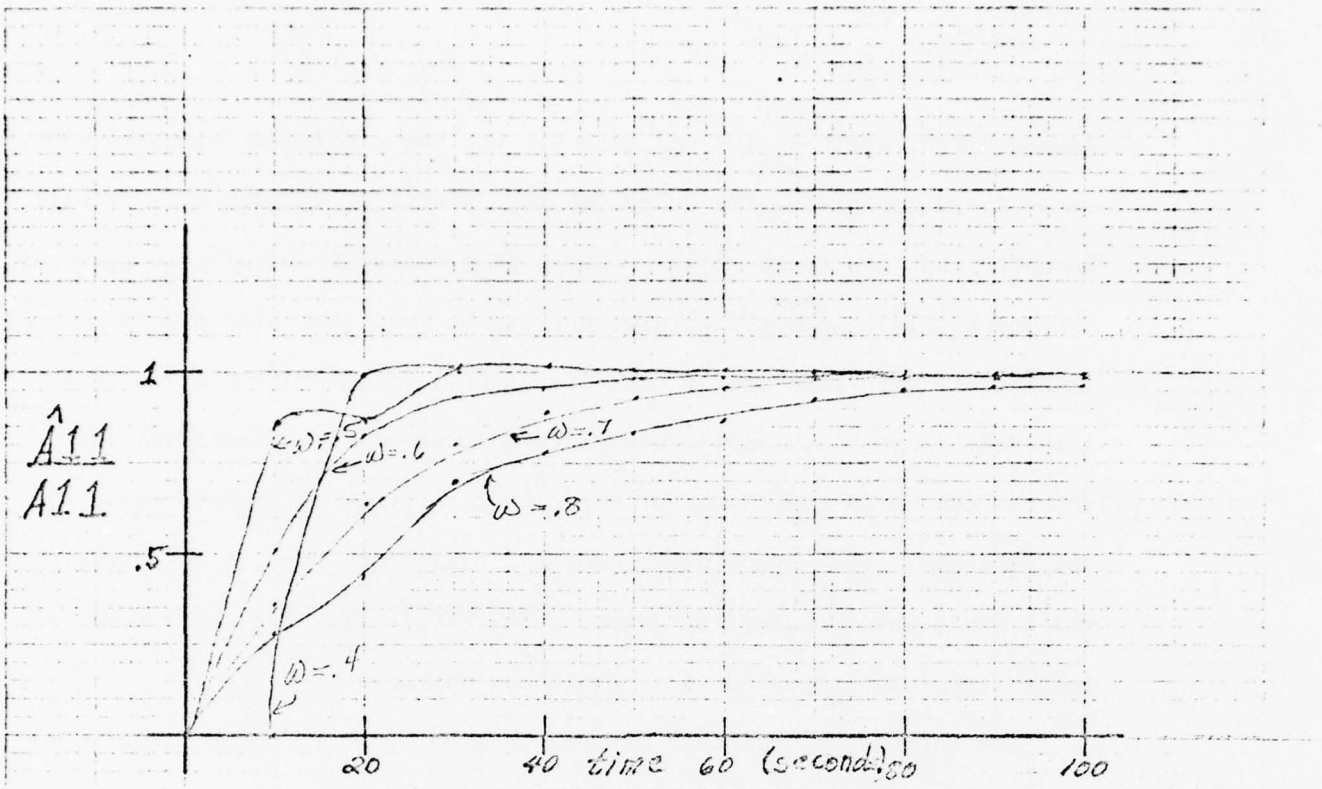


Figure VII-4. Convergence Rate of $\hat{A}_{11}$ in the Interval $-.8 < \dot{x} < -.6$, $\ddot{x} > 0$.

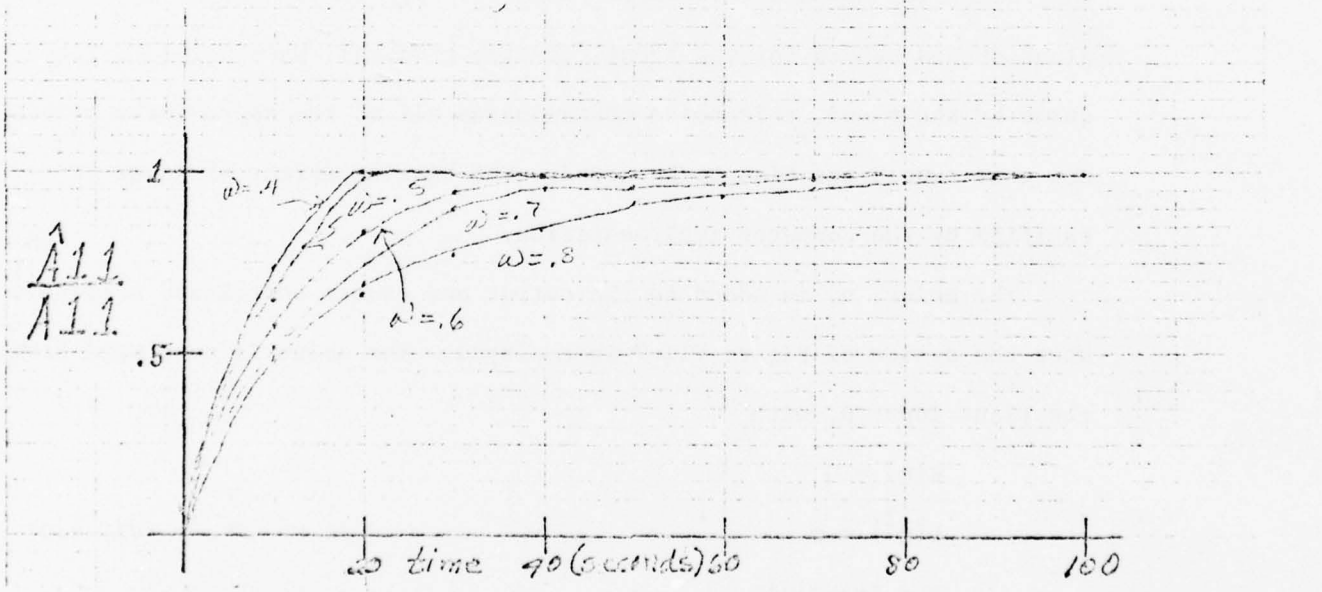


Figure VII-5. Convergence Rate of $\hat{A}_{11}$ in the Interval $.6 < \dot{x} < .8$, $\ddot{x} < 0$.

Figure VII-6, where $A_{11}(0) = A_{12}(0) = 0$. For the particular example, $\omega = .4$ seems to be an optimum frequency for the identifier to work at. It will be noted that the curve shows a smooth progression from point to point on the graph. This is not accurate in that the identification process occurs in discrete intervals and adjusts itself while the variable is in that interval. When the variable leaves that interval, the identification process ceases in that interval until that interval is "swept" again by the variable. This procedure produces a series of jumps in the convergence which would make reading the graphs quite difficult. For this reason the information was presented in a continuous manner.

The theoretical response of the equation error to noise either correlated or uncorrelated would be to add an asymptotic bias to the results obtained by the identifier. This is because the plant output is used by the identifier and state variable filter to approximate all of the missing plant states needed for the identification process. When noise is introduced by the plant, this noise contaminates all of the approximate states and this causes the bias to be added. The biasing effect of noise is verified by the computer implementation.

The noise, η, is added to the output and causes correlated residuals when the system of Figure VII-7 is employed. The noise is developed from the plant input u, with

$$E\{\eta\} = 0$$
$$E\{\eta^2\} = \sigma \qquad \text{(VII-4.B)}$$
$$\sigma = \text{Frac}*|u|$$

where $< \text{Frac} \leq 1$ is a constant representing "percent" noise. This case is particularly severe because noise bias occurs because η is correlated with u.

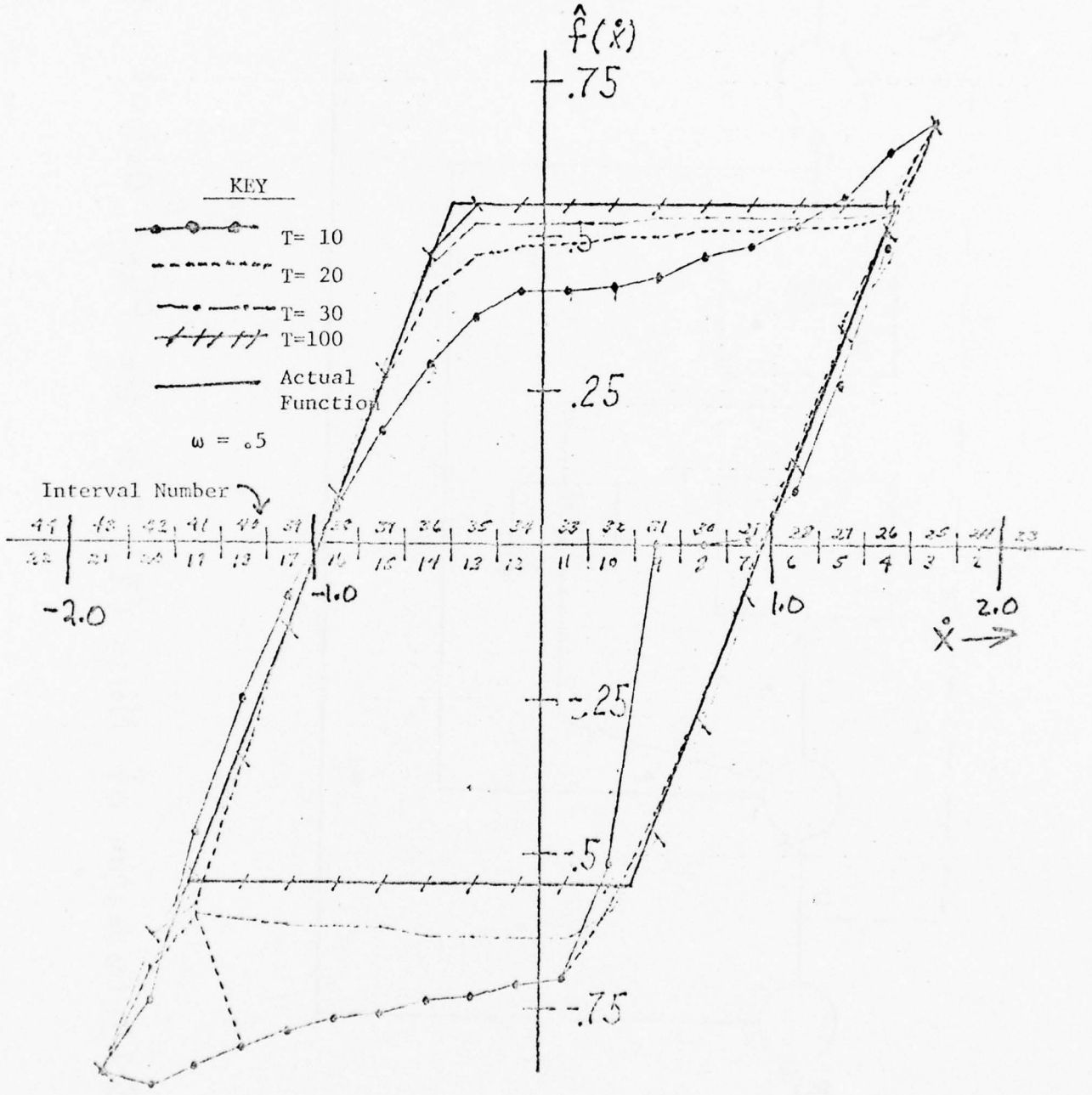


Figure VII-6. Convergence Rate of the Hysterisis Loop For Various Frozen Times from Start of Identification.

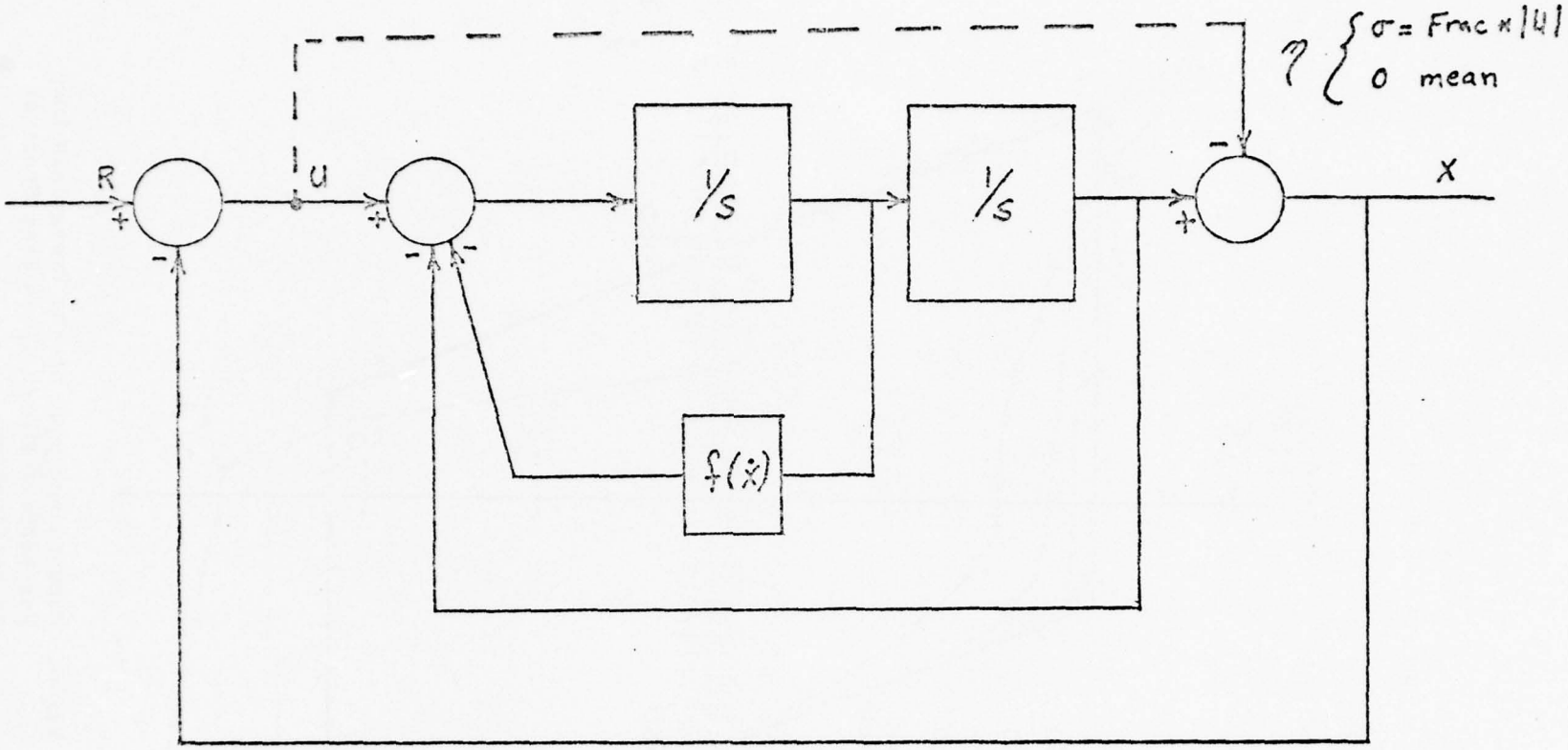


Figure VII-7. Introduction of Noise (η) Into the Plant Output

Two functions were tested on the identifier with this type of noise. In both, no information or assumptions about $f(\dot{x})$ were made except that it could have been linear, non-linear, contain memory, or be memoryless. The first function tested was a hysterisis loop with slope 1.0 and saturating at +1.0 and -1.0. The standard deviation of the noise was allowed to vary between 10% and 100% in this instance. The results of this test are shown in Figure VII-8. From this figure, it can be seen that, in general, the higher the standard deviation of the noise, the less accurate the results of the identification. However, the identifier did not breakdown completely and even at 100% standard deviation noise it still attempted identification, although, extremely inaccurate.

The other function which was used to test the effect of noise on the identifier was $f(\dot{x}) = 2\dot{x}^3$. The results of this test are shown in Figure VII-9. From this figure, it can be seen that the identifier produced a generally accurate representation of the actual curve for noise levels of 10% to 50% standard deviation. The convergence of three of the intervals with respect to time is shown in Figure VII-10. Examination reveals that these intervals do, indeed, converge but the convergence is not exact.

C. Time-Varying Identification

From Chapter 5, some of the time-varying approaches were investigated as regards their tracking ability. Considerations of bias, convergence rate and stability are addressed.

First, consider the plant

$$x_p(k+1) = a(k)\, x_p(k) + b_o u(k+1) + b_1 u(k) \qquad \text{(VII-1.C)}$$

$$y_p = x_p + \eta \quad \text{(output)} \qquad \text{(VII-2.C)}$$

$$u \text{ Gaussian, } E\{u\} = 0,\ E\{u^2\} = 1 \qquad \text{(VII-3.C)}$$

$$E\{\eta\} = 0 \qquad E\{\eta^2\} = p \cdot |x_p|, \quad 0 < p \leq 1.$$

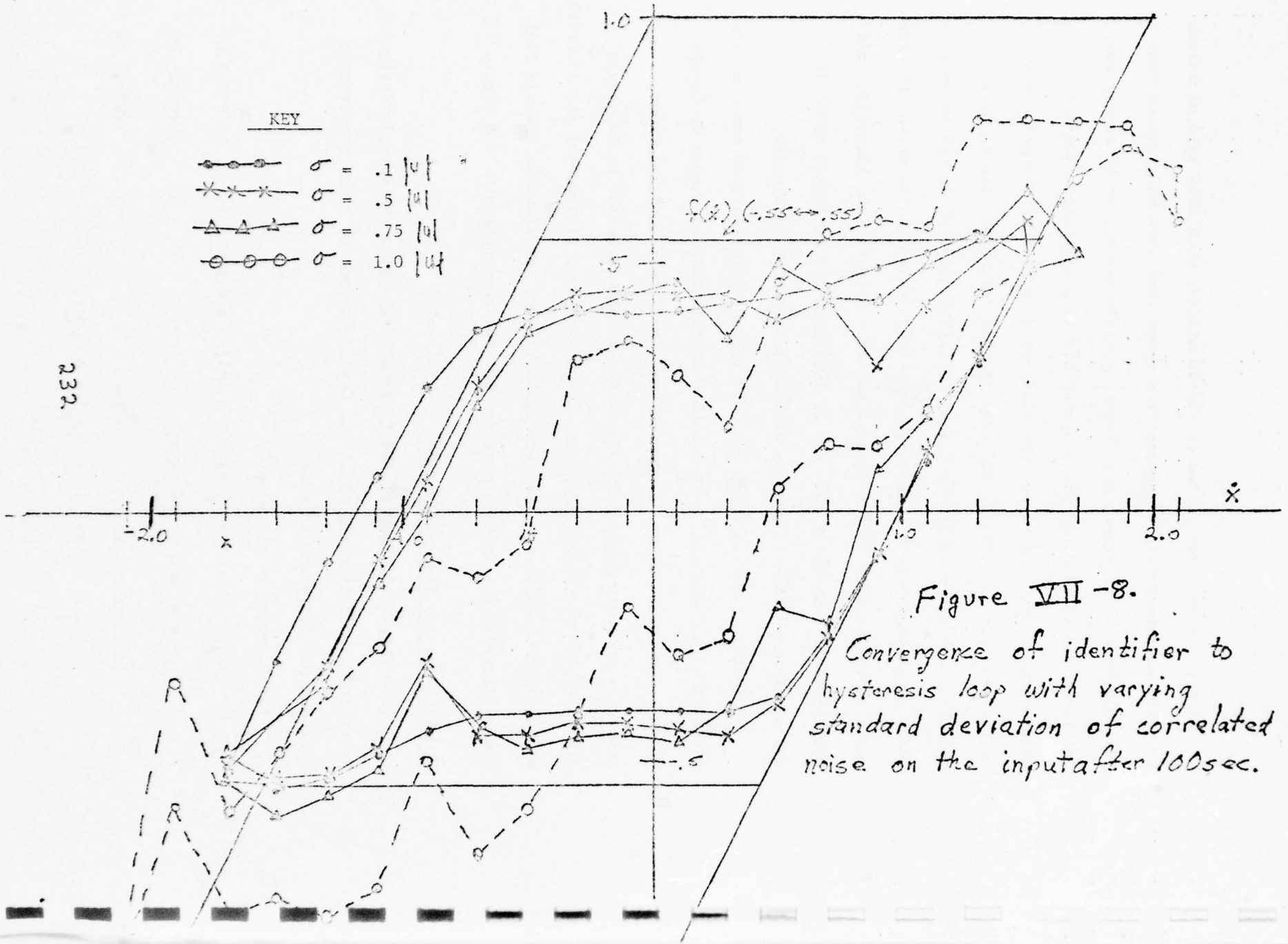


Figure VII-8.

Convergence of identifier to hysteresis loop with varying standard deviation of correlated noise on the input after 100 sec.

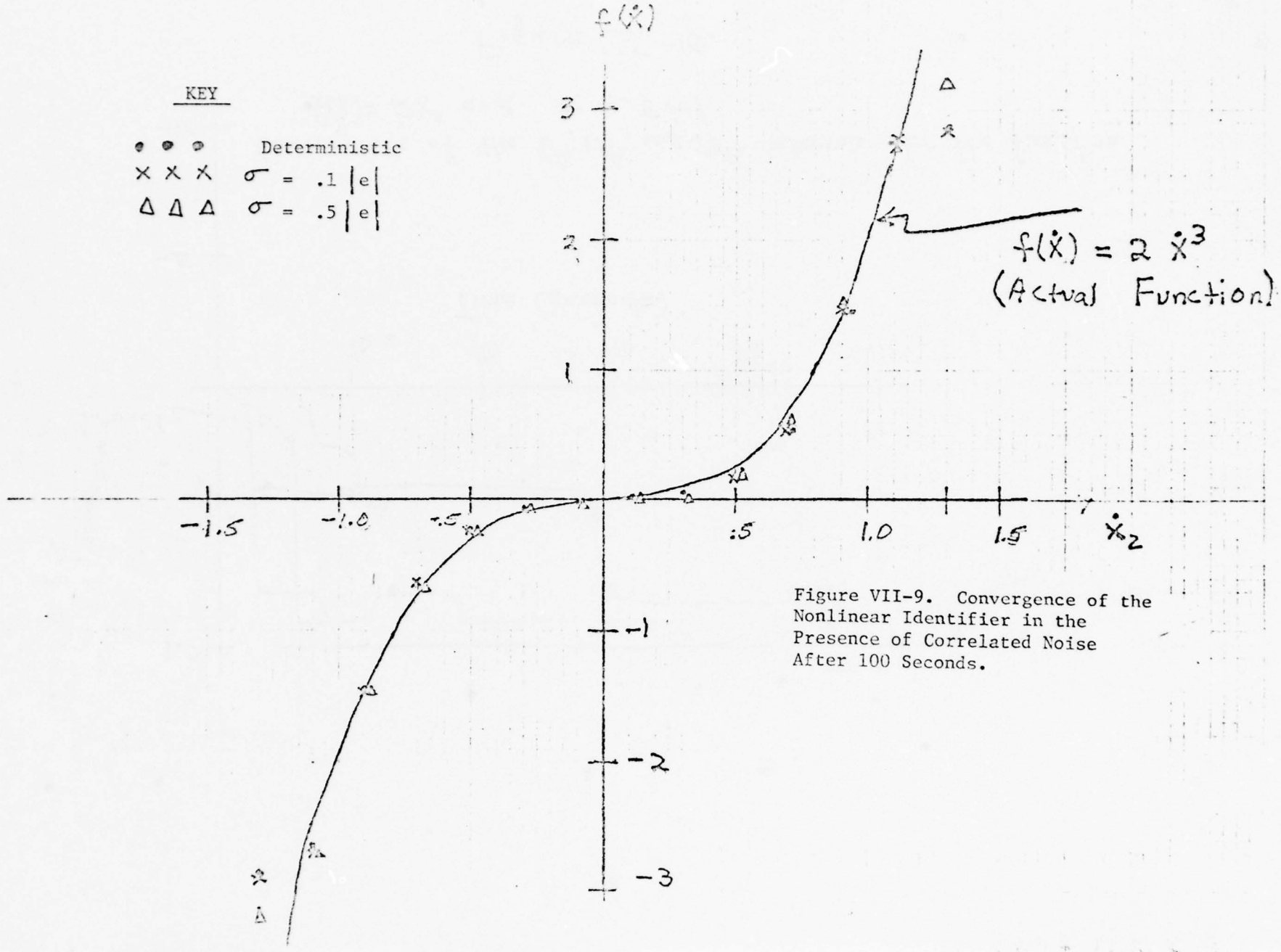
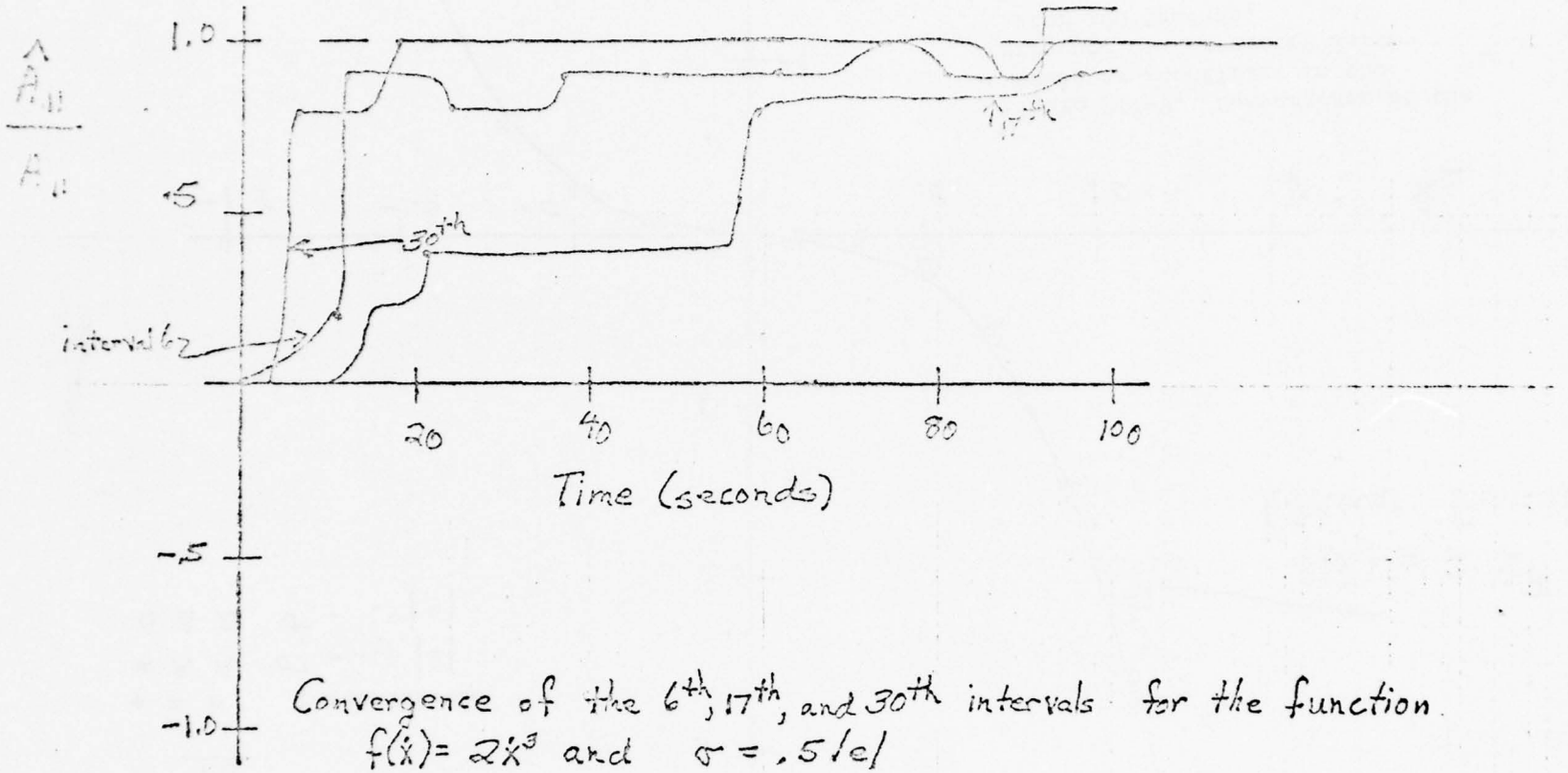


Figure VII-9. Convergence of the Nonlinear Identifier in the Presence of Correlated Noise After 100 Seconds.



Convergence of the 6th, 17th, and 30th intervals for the function $f(x) = 2x^3$ and $\sigma = .5|e|$

Figure VII-10.

$$b_o = .0274,\ b_1 = .0286 \qquad \text{(VII-4.C)}$$

$\underline{\hat{\phi}}(0)$: from (III-218.B)-(III-221.B)

A cost function on the structural distance is defined as

$$J = \sum_{k=20}^{400} ||\underline{\phi}(k) - \underline{\hat{\phi}}(k)|| \qquad \text{(VII-5.C)}$$

Shown in Figure VII-11 is the effect of both the time-invariant Landau MRAS identifier with that of the W method with 0 and 10% noise (K_2=10); the equations employed are given in Chapter 3 and 5. Note the relatively good tracking of the W identifier in tracking a(k) whereas the time-invariant Landau case provides poorer estimates as $k \to \infty$. The "reweighting" or updating employed by the W and λ time-varying identifiers can be seen in Figures VII-12(a),(b) for the example given. Note the reset or cyclical increase in $||F(k)||$ every N (= 10) terms for W, but for the λ method $||F(k)||$ is relatively continuous if $.8 < \lambda < 1$.

Using J in (VII-5.C), the same plant as before with a different a(k) was employed to determine the effect of varying λ and W. Shown in Figures VII-13 and VII-14 are identifier results for the λ and W methods for different output noise percentages. Results suggest a range of λ, $.85 < \lambda < .97$ because of noise considerations, with K_2 optimum about $1 < K_2 < 100$. Similar results hold for other plants of higher order with more (and different) time-varying coefficients.

As another case, consider a third order plant, with $n = m = n' = m' = 3$,

$$\underline{\phi}^T(0) = [.4 \quad .49 \quad -1.96 \quad 1.0 \quad .5 \quad .06]$$

$$\underline{\hat{\phi}}^T(0) = [.3199 \quad .4806 \quad -.144 \quad .983 \quad .557 \quad .119]$$

Time varying parameters were generated from

$$\phi_i(k+1) = \alpha\, \phi_i(k) + (1-\alpha)\, r_i(k) \qquad i = 1, 2, \cdots, 7 \qquad \text{(VII-6.C)}$$

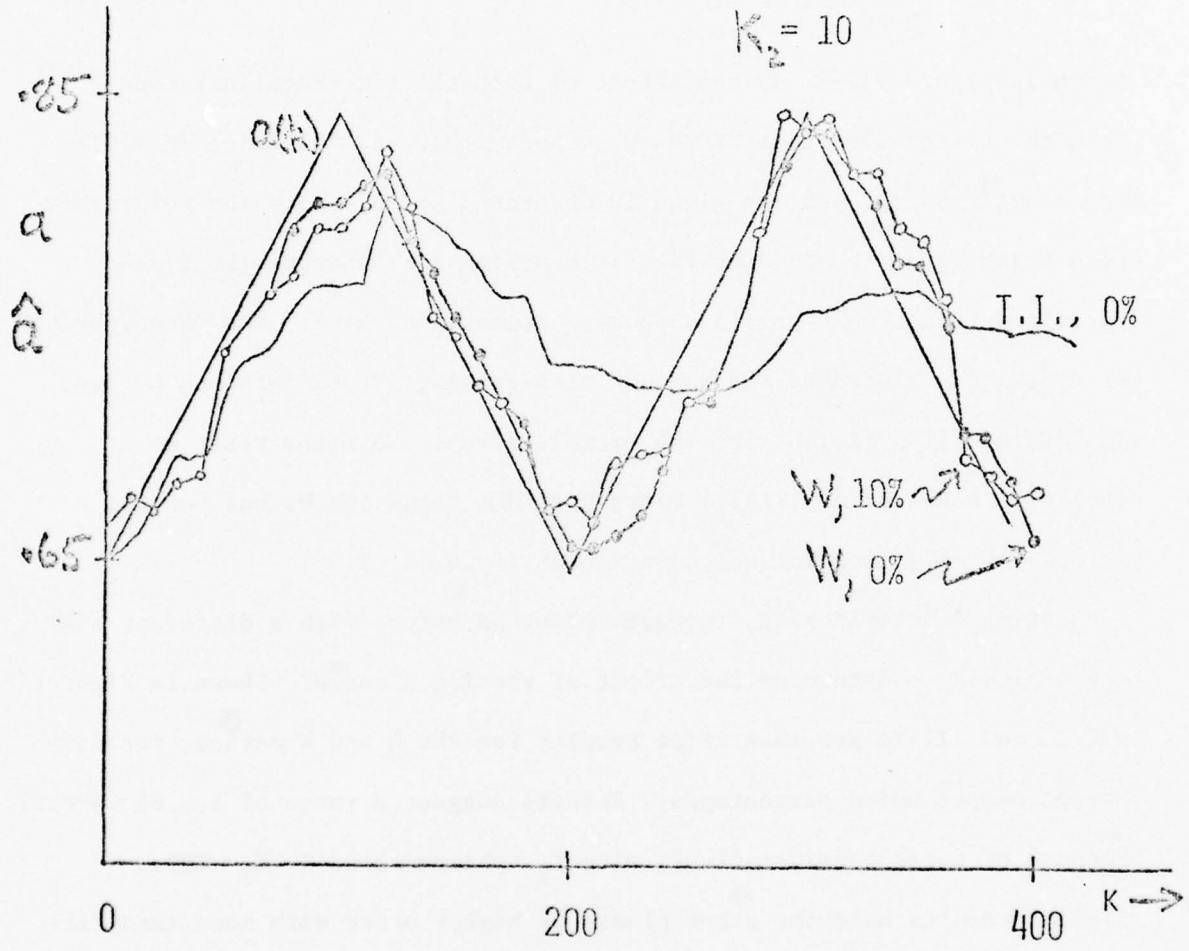


Figure VII-11. Parameter Tracking of the W and Time-Invariant Identifiers for a First Order System.

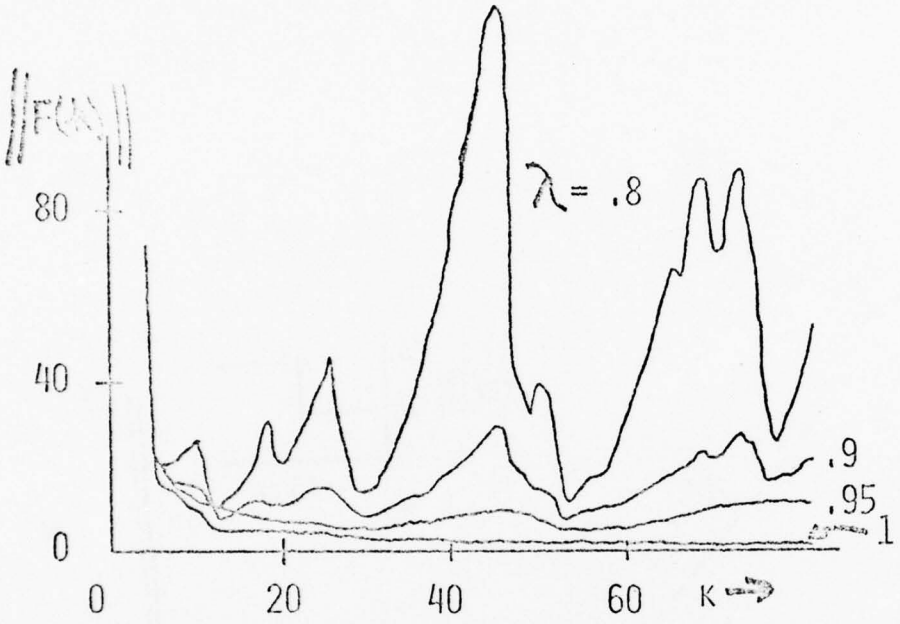


Figure VII-12(a). Normed F Matrix Response for the λ Method.

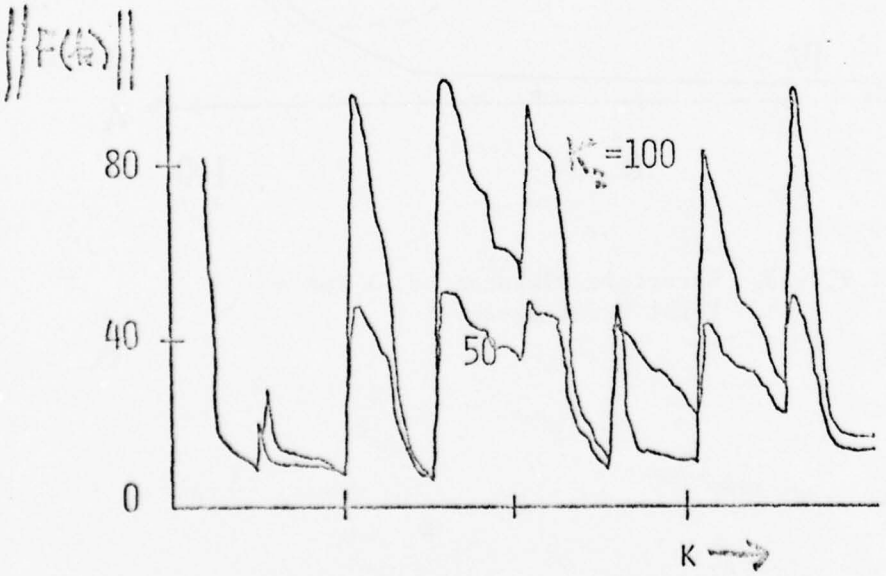


Figure VII-12(b). Normed F Matrix Response for the W Method.

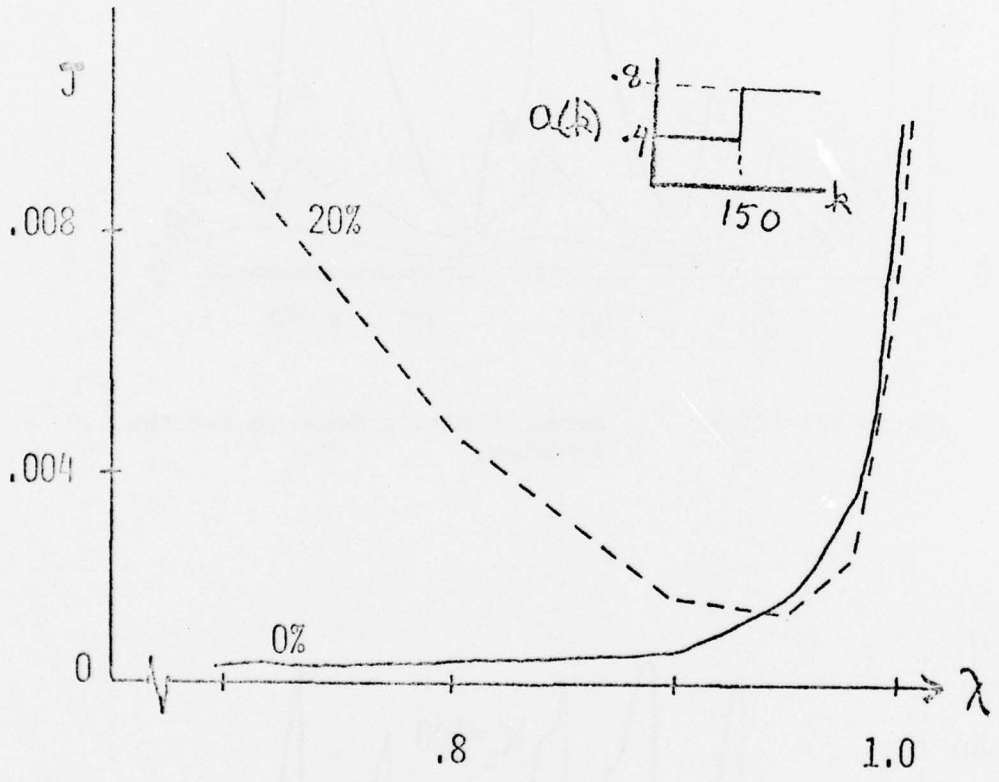


Figure VII-13. Structure Distance vs. λ For a First Order System.

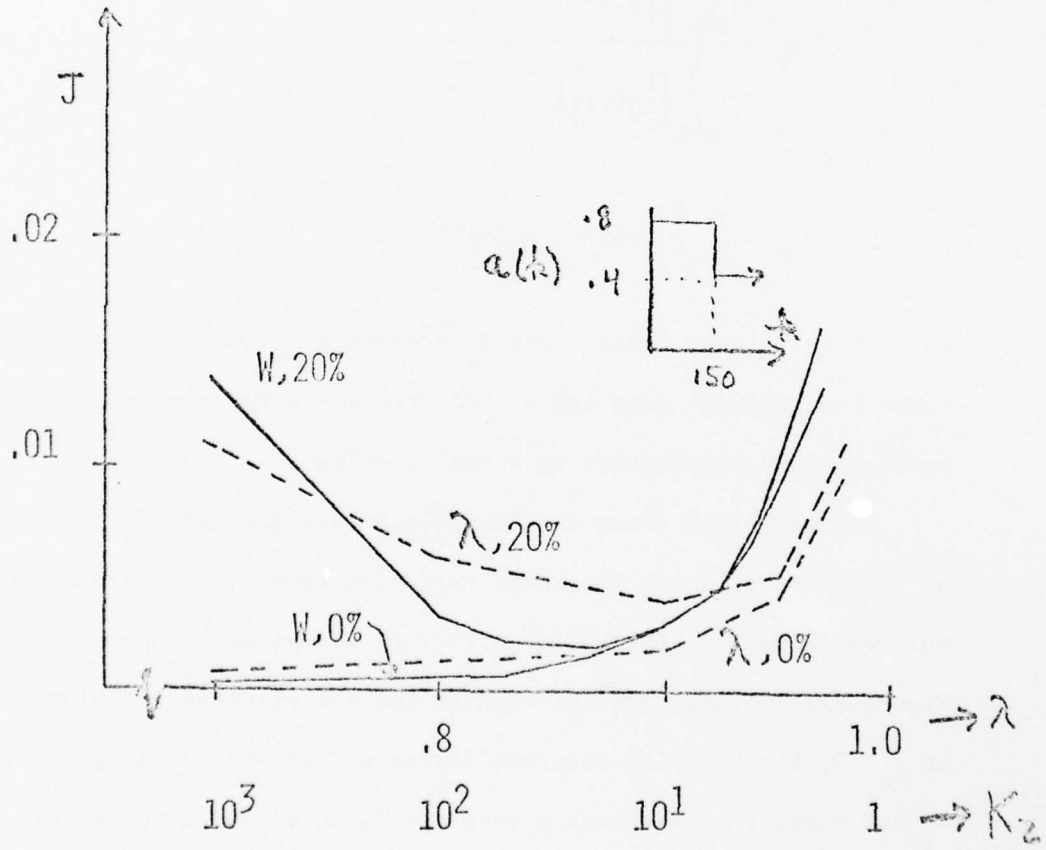


Figure VII-14. J vs. λ and W Methods For Comparison Purposes for a First Order System.

where $0 << \alpha < 1$ and $r_i(k)$ are independent zero mean Gaussian white noises with unit variances. The output noise, η, ($y_p = x_p + \eta$) is given by

$$\eta(k+1) = .8\ \eta(k) + .7\ r_7(k) \qquad \text{(VII-7.C)}$$

The colored noise η is employed because it violates the ideal assumption stated in Method 13 of Chapter 3. Using the λ and W methods with the parallel MRAS formulation with $[C_1\ C_2\ C_3] = [-.4\ \ -.49\ \ .196]$, simulations were run for varying α. Penalty costs were defined as

$$J_\phi = \frac{\sum_{i=1}^{N} \left|\left| \underline{\phi}(i) - \hat{\underline{\phi}}(i) \right|\right|^2}{\sum_{i=1}^{N} \left|\left| \underline{\phi}(i) \right|\right|^2} \qquad \text{(VII-8.C)}$$

$$J_\varepsilon = \frac{1}{N} \sum_{i=1}^{N} [y(i) - x_m(i)]^2 \qquad \text{(VII-9.C)}$$

where N is a large number, and J_ϕ represents a parameter structural distance misalignment cost and J_ε the tracking error penalty. Only J_ε is available for computation in a real problem.

Figure VII-15 shows the output tracking accuracy J_ε as λ and K_2 (for W) vary for different parameter variation rates (α changing). Clearly noticeable is the "parabolic" effect of an optimal range of λ and K_2. Figure VII-16 shows similar results for the structural distance J_ϕ; clearly as $\dot{a}_i \to 0$, the tracking accuracy improves. Figure VII-17 compares J_ε and J_ϕ and shows that although a range of λ, K_2 minimize the errors, the mins occur at different λ values, showing that "optimum" depends on the tracking accuracy definition (i.e. $\lambda_{opt} \simeq .9$ for J_ε). Due to lack of space and time, oscillating memory results are not given, although trends are similar. Figure VII-18 shows the good parameter tracking accuracy possible.

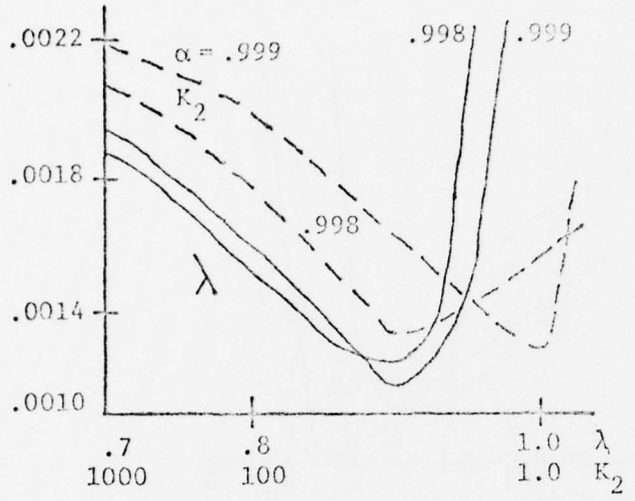


Figure VII-15. Output Error Cost vs.λ, K_2.

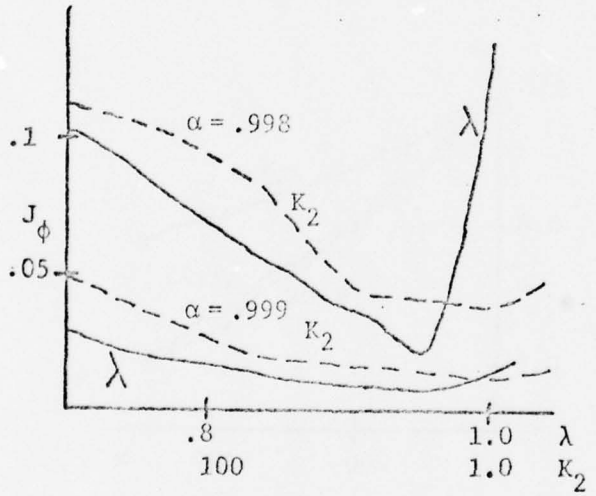


Figure VII-16. Effect of $\dot{\phi}$ on J_ϕ.

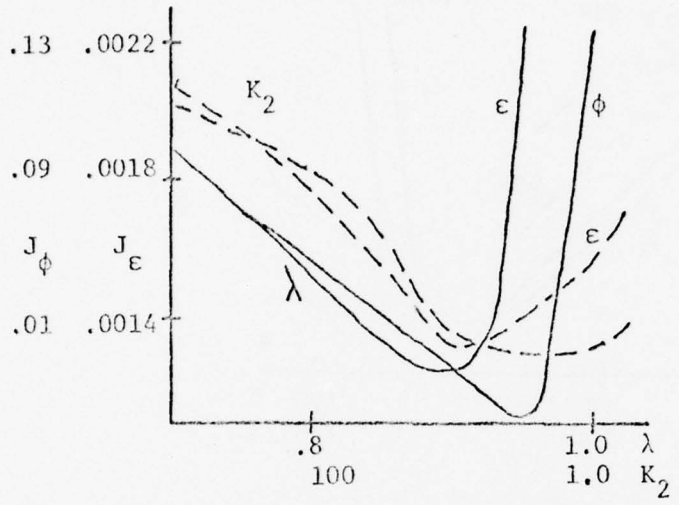


Figure VII-17. Parameter and Ouput Cost vs. λ, K_2.

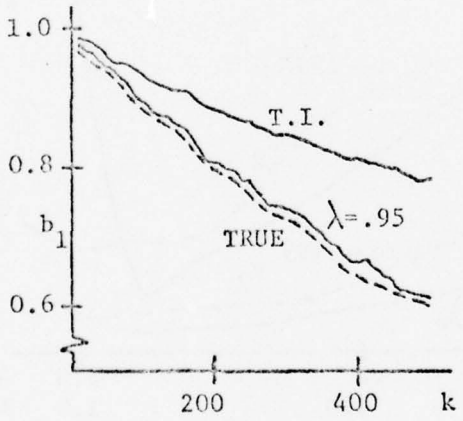


Figure VII-18. Parameter Tracking Ability of the λ Method.

To compare computational requirements, Table VII-I was generated assuming $n' = m' = 3$ for various methods using an 8080 μP. Clearly, no significant added burden occurs for the W or λ methods over time-invariant approaches, but the oscillating memory requires about double the W method and the SHARF much less than all others. Unfortunately, SHARF does not insure global stability, but from the times shown it is the only method easily amenable to on-line use.

D. Analysis and Summary of Time-Varying MRAS Identifiers

From the limited results presented, it is not clear that the parallel MRAS approach for moving window time-varying identifiers is "best", but it has been shown it functions well with appropriate tuning. Experimental results shows no one method is better, but some design guidelines are:

W Method

$N_L < N < N_u$

N_L = lower bound (> 0), N_u = upper bound

$N \to 1 \Rightarrow$ λ Method

$N \to 0 \Rightarrow$ Ignore Everything

$N \to \infty \Rightarrow$ Time-Invariant Identifier

$K = f(\dot{a}_i$, # parameters to be identified)

As $\dot{a}_i \uparrow$, so should K_2

$K_2 \to 0$ Reject Present Data

$K_2 \to \infty$ Ignore Everything

λ Method

$\lambda \to 1 \Rightarrow$ Infinite Memory

$\lambda < 1 \Rightarrow$ Shed Memory

Table VII-I. Computational Burden of Identifiers

	METHOD	COST	8080 μp* UPDATE TIME (sec), $n' = 3$
A.	TIME-INVARIANT	$J_A = 4n'^2 + 21n' + 4] + s[2n' + 4] + M[3(2n+1)^2 + 15n + 9] + D[2]$	.42
B.	λ METHOD	$J_B = J_A + 1M$	.42
C.	W METHOD (N = 10)	$J_C = J_A + \frac{1}{N} \cdot \{(D + (2n' + 1)^2 M)\}$	.43
D.	OSCILLATING MEMORY ($N_1 = \frac{1}{2} N_2$)	$J_D \simeq 2 J_A$	.84
E.	SHARF - λ METHOD	$J_E = A[5n' + 3] + s[2n' + 2] + M(10n' + 7[+ D[1]$	.08

*8080 μp SPECIFICATIONS [242]

2 MHz

SOFTWARE-IMPLEMENTED +, -, *, 1

A: 410 μs

S: 440 μs

M: 1800 μs

D: 3600 μs

Slow Parameter Variation	$.95 < \lambda < 1$
Fast	$.85 < \lambda < .95$

Oscillating Memory

N_2 - Maximum Memory Length

N_1 - Minimum Memory Length

Fix N_1; as $N_2 \uparrow$	Poor Tracking
Fix N_2; as $N_1 \to N_2$	Moving Rectangular Window Results

One selection criterion for N_1, N_2 is that

$N_1 = h_1(\dot{a}, \#$ of parameters)

$N_2 = h_2$(output measurement noise)

where $h_i \Longrightarrow$ "is a function of"

It is difficult to indicate graphically the relative data weighting of the time-varying parameter identifiers because data is added and subtracted in different ways at different times. To give a rough idea, however, Figure VII-19 is presented, where *relative* data weighting is plotted vs. present time k. This shows that as $k \to \infty$, the individual weighting given any piece of data by the time invariant identifier approaches zero. The λ method levels off to provide a finite weighting for data and the W and oscillating memory (O.M.) methods have weights which vary between upper and lower bounds (although the upper and lower bounds are not the same for the two methods).

It *appears* the λ method is less sensitive to both plant-model order differences and parameter time-variations, although in isolated cases the W or oscillating memory method could be better. In all cases time-variation control occurs through controlling F(k) (or equivalent for SHARF). The weighting philosophies of the various approaches are compared to the time-invariant case in Figure VII-19.

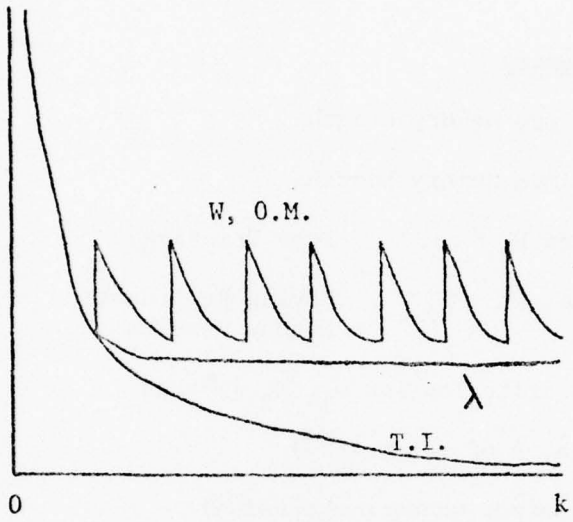


Figure VII-19. Relative Weighting Applied to Each Successive Piece of Data.

CHAPTER 8. FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

Based on this study and results presented in this report, a series of findings and conclusions regarding MRAS-type identifiers and their suitability for system identification of a type involving a human operator model are given. This chapter will dileneate those key findings, formulate general conclusions, and then based on these statements offer recommendations for new work regarding new work in research, development, and hardware implementation.

From a study of the material developed in Chapters 2-7, the following general findings can be given:

(1) A great variety of MRAS identifiers exist which can be used to identify linear systems

(2) The designer-selected parameters for the adaptive identifiers are somewhat arbitrarily chosen, with the result being an ill-defined convergence rate. Although a variety of convergence analysis tools are available, few if any, provide a definitive estimate of the parameter estimator convergence rate.

(3) MRAS identifier methods are available which can handle time-varying and nonlinear plants with output noise-corrupted measurements.

(4) Although the parallel MRAS identifiers can yield asymptotically unbiased estimates (under appropriate noise conditions), implementation is difficult in practice because of the (theoretical) need for *a priori* knowledge of plant parameters, as in (III-112.B) for (III-109.B)

(5) Series-Parallel MRAS can lead to biased parameter estimates in the presence of noise [109]. Recent new work provides a means of eliminating such bias in practice, however [243], under certain conditions.

(6) Much work has already been done on determining optimum transport lag filters for nonlinear identification, as developed in Chapter 4 and Section D of Chapter 6. All are easily implemented in practice; listings of common methods are given in Table form in Section D of Chapter 6,

along with conditions on use of the state-variable filters.

(7) As shown in Chapter 7, the time-varying MRAS identifiers developed in this report can track time-varying parameters much better than traditional time-invariant MRAS identifiers.

(8) From Table VII-1, computational burden can be large insofar as microprocessor implementation of some of the MRAS identifiers is concerned for real time operation. Simplifications must be made to allow for on-line, real-time identification.

(9) From Chapters 4 and 7, the MRAS nonlinear identifiers investigated appear to not be amenable to on-line identification due to the slow convergence rate (caused by the relatively short time the independent variable stays in a given identification increment in phase-space).

(10) Straightforward design guidelines can be formulated as to the effect of measurement and input noise on parameter tracking accuracy (see Chapter 6, Section B).

(11) Although a variety of parameterized human operator models have been developed by other researchers, there appears to be no one best *structure* or parameterization that is best (Chapter 2).

From a study of the results of the analytical developments in Chapters 2-6 and simulation results in Chapter 7, the following general conclusions can be reached concerning the use of MRAS identifiers:

(1) Simple analytical prediction tools for determining the relationship between design parameter selection and parameter convergence rate for MRAS linear identifiers is sparce.

(2) Parameter convergence rate is strongly a function of the identifier *structure* [134,244].

(3) Accurate *off-line* nonlinearity identification is possible for moderate noise obscured output measurements (Chapter 7).

(4) For "fast" time-varying parameters, traditional MRAS time-invariant identifiers cannot accurately track parameters well for large times, as shown both analytically in Chapter 5 and via example in Chapter 7.

(5) No single MRAS appears best, whether based on convergence rate, noise sensitivity, design simplicity, identifier modeling structure, etc.

(6) MRAS identifiers can properly handle identification problems of a type similar to the human operator in a compensatory tracking loop.

(7) Further analytical work as to convergence design and analysis techniques for one or two of the parallel MRAS identifiers is needed.

(8) Analytical determination of accuracy of fit vs. number of linearization terms, frequency richness requirements, parameter convergence rate, number of nonlinear parameters, etc.

(9) Determine under what exact set of conditions (if any) certain MRAS identifier approaches can provide better results than the Optimal Control Model (OCM) approach.

With regard to time-varying identifiers, areas of work being investigated include

1) prediction of parameter rates of change, then use "shooting methods" to improve tracking speed.

2) adaptive adjustment of N (for W method), N_1 and N_2 (for oscillating memory method), and c_i by using decision logic on the output error.

3) use of series-parallel methods since they are easier to realize physically.

4) determination of the band limiting effect on $\hat{\underline{\phi}}(k)$ due to the identifier makeup (the time-invariant case has a passband that asymptotically approaches zero).

5) determine the parameter estimate phase lag as a function of identified plant dynamics.

6) determine analytically the effects of high order systems (with many time-varying terms) on parameter convergence rate, tuning, etc.

Based on the work performed and reported on this report, the following recommendations are given:

(1) Perform an analytical study to determine whether any of the MRAS identifiers are superior to maximum likelihood, etc. To date, no definitive work has been done on comparing efficiencies of MRAS identifiers to others.

(2) No definitive work has been done with on-line model order determination and it would be fruitful to further investigate. One conceptual approach with promise has been developed in Chapter 6 as a guideline.

(3) Investigate the nonlinear identifier developed in Chapter 4 as regards convergence rate, noise biasing etc.

(4) Determine the uniqueness capability of a multivariable identifier as a function of the nature of the input. Since in many practical cases one cannot control the input but only monitor it, one would like to know the size of the parameter estimate error bounds if the frequency-richness condition is not present (and hence the parameter error does not approach zero asymptotically.

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APPENDIX

To complement the computational requirements of Chapter 6, the computation calculations are detailed here for completeness. It should be noted the results are approximate because 1) computational optimization was not performed, although obvious computational repetitions were avoided, 2) only key identifier integrators, summers, adders, multipliers, etc. were counted, with no direct accounting of the operating system overhead, and 3) nonlinear state-variable "region" calculator requirements as for [144] were not considered directly. Only a few of the identifier methods in this report were studied due to time constraints.

A. Tomizuka [142]

ℓ = number of nonlinear terms

n' = model order

m' = number of numerator terms of model function

y to y_o: requires 1 SVF (L(s)) made up of n' Integrators

Model: n' Integrators (for $\int \dot{\hat{x}}_i \, dt$)

n' Multipliers (for $\hat{a}_i \, \hat{x}_i$)

$\underbrace{\ell m' \text{ Multipliers}}$

to account for $f_{n_j}^{*} \quad n_{h_o}$

$\dot{\hat{a}}_i$: 1 Integrator (for $\dot{\hat{a}}_i$)

1 Multiplier (for $v \cdot \frac{d^i}{dt^i} (\hat{y}_o)$)

$\dot{\hat{f}}_{h_j}$: 1 Integrator (for $\dot{\hat{f}}$)

1 Multiplier

n' Integrator (for SVF L(s) for each $\hat{f}_{h_j}$)

i Multipliers (represents nonlinearity n_ℓ requirements for implementation)

$$\text{Cost} = I[3n' + \ell m' + \ell n'] + M[2n' + \ell m' + \ell i + \ell m']$$

If

$$i = 0$$

$$m' = n'$$

then

$$\text{Cost} = I[n'(3 + 2\ell)] + M[n'(2 + 2\ell)]$$

B. Schitoglu [144]

n' = model order

m' = degree of numerator polynomial

ℓ = number of simple single-valued nonlinearities

$j = 0$ or $1 \Rightarrow$ number of multi-valued nonlinearities

$n' + m' - (\ell + j)$ = number of linear parameters

Multi-valued Nonlinearity:

2 Add

1 Multiply

Single-valued Nonlinearity:

2 Add

2 M

SVF for x_1:

n' I

SVF for u:

n' I

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Gains

Linear:	1I	1M		
Nonlinear Single-Valued:	2I	3M	1S	1M
Nonlinear Multi-Valued:	2I	1M		1M

Error $\hat{E}$:

$n' + m' + 1$ Add

$n' + m' - (\ell + j)$ linear terms to multiply

$(\ell + j)$ nonlinear factors to multiply

Cost = $M[2n' + 2m' + 2\ell] + I[3n' + m' + \ell + j]$

If $m' = n'$, $j = 0$ (standard single-valued nonlinearities only), then

Cost = $M[4n' + 2\ell] + I[4n' + \ell]$

C. Johnson [128]

$e(k)$:

1 S

Model:

$n' + m' + 1$	A
$n' + m' +$	M

Gains:

$n' + m' + 1$	S
$2(n' + m' + 1)$	M

$\varepsilon(k)$:

1	A
1	M

$(F+G)\underline{s}$: $n'+m'+1$ M
$n'+m'$ A

$\underline{s}^T(F+G)\underline{s}$: $n'+m'+1$ M
$n'+m'$ A

$1+\{\ \}$: 1 A

$\frac{1}{1+\{\ \}}$: 1 D

$\frac{1}{1+\{\ \}} \cdot \varepsilon(k)$: 1 M

$\hat{\phi}^P$:

$\underline{Cs}$: $n'+m'+1$ M
$n'+m'$ A

$\frac{\underline{Cs}}{1+\{\ \}}\ \varepsilon$: 1 M

$\hat{\phi} = \hat{\phi}^I + \hat{\phi}^P$:

$n'+m'+1$ A

$\varepsilon(k)$:

$x_p - (\ )$: 1 S

$\phi^T\underline{s}$: $n'+m'+1$ M
$n'+m'$ A

+: 1 A

Σ: n' M
$(n'-1)$ A

F(k): Subtract List

$F - (\overset{\downarrow}{\ })$: $n'+m'+1$ S

$(\underline{Fs})(\underline{Fs})^T$: $(n'+m'+1)^2$ M
$(n'+m')^2$ A

set G = 0: $(n'+m'+1)^2$ M

$\frac{(\underline{Fs})(\underline{Fs})^T}{1+\{\ \}}$: 1 D

α:

$2(n' + m' + 1)$ M

1 D

$n' + m' + 1$ A

Cost = $[5n' + 5m' + 6]M + 1D + A[2n' + 2m' + 3] + S[n' + m' + 2]$

If $M = D$, $A = S$, and $m' = n'$, then

Cost = $M[10n' + 7] + A[6n' + 5]$

D. Landau [116]

Model:

$n' + m' + 1$ M

$n' + m'$

$e^{\circ}(k)$:

1 S

$e(k)$:

1 S

x°_m:

$n' + m' + 1$ M

$n' + m'$ A

$\hat{\phi}^I$:

$\phi + \Delta\phi$: $n' + m' + 1$ A

Fs: $n' + m' + 1$ M

$n' + m'$ A

F + G: $n' + m' + 1$ A

G(k):

$(n'+m'+1)^2$ M

$$\text{Cost} = A[(n'+m')^2 + 11n' + 10m' + 4] + S[n'+m'+4] + M[3(n'+m'+1)^2 + 8n' + 7m' + 9] + D[2]$$

If $n' = m'$, A = D, and M = D, then

$$\text{Cost} = M[3(2n'+1)^2 + 15n' + 11] + A[4n'^2 + 23n' + 8]$$

E. Narendra [113]

Model:

n'^2 M
n'^2 A $\quad \hat{A}\,\underline{x}_p$

$n'm'$ M
$n'm'$ A $\quad \hat{B}\,\underline{u}$

e(k):

n' S

Gains:

$\frac{1}{\lambda_m \underline{y}^T \underline{y}}$: 1 D

$\underline{y}^T\underline{y}$: n' M
n' A

$-\frac{1}{(\ )}$: $n'(n'+m')$ S

αP: n'^2 M
n'^2 A

$(\alpha P)\ \underline{e}$: n' M; n' A

$[(\alpha P)\underline{e}]\underline{y}^T$: $n'(n'+m')$ M; $n'(n'+m')$ A

$\frac{1}{(\ )} \cdot (\ )$: $n'(n'+m')$ M; $n'(n'+m')$ A

If A = S, M = D, then

$$\text{Cost} = M[4n'^2 + 3n'm' + 2n' + 1] + A[5n'^2 + 4n'm' + 3n']$$

If $n' = m'$,

$$\text{Cost} = A[9n'^2 + 3n'] + M[7n'^2 + 2n' + 1]$$

F. Hang [109]

From [109], the calculated cost is

$$\text{Cost} = I[5n' - 2] + M[4n']$$

G. Young [240]

From [109], the calculated cost is

$$\text{Cost} = I[4n'] + M[4n']$$

H. Young [5]

From [109], the calculated cost is

$$\text{Cost} = I[6n'] + M[6n']$$

I. Lion [102]

From [109], the calculated cost is

$$\text{Cost} = I[8n'-2] + M[8n'^2]$$

J. Landau + λ Method for Time-Varying Plants

$\underline{\phi}$ update the same

F update the same except for one additional multiplication of

$\frac{1}{\lambda} * (\cdot) \Rightarrow$ 1 extra multiplication (M)

K. Landau + W Method for Time-Varying Plants

$\underline{\phi}$, F updates the same for N samples,

N = # periods in an update cycle.

@ N+1, perform $\frac{K_2}{f^*_{(N+1)}} * F(N+1)$

1 extra division (D)

$(n' + m' + 1)^2$ extra multiplications (M)

Cost average/cycle = "Old Cost" + $\frac{1}{N}$ (1D + $(n' + m' + 1)^2$M)

With $n' = m'$, then

Cost average/cycle = "Old Cost" + $\frac{1}{N}$ (1D + $(2n' + 1)^2$M)

where "Old Cost" = Time Invariant Landau MRAS Identifier Cost

L. Oscillating Memory (O.M.) for Time-Varying Plants

Because the calculations are so complicated, the details will not be dilineated here. In general, the computation burden is

$\text{Cost}_{\text{O.M.}} \approx 2 \cdot \{$"Old Cost"$\}_f$, as defined in Case K above.