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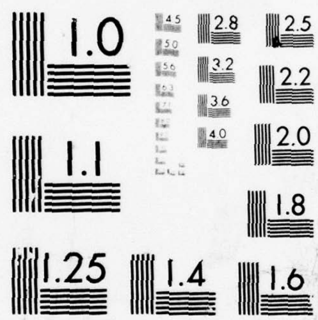

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REPORT SAMSO-TR-79-28

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The Short-Term Quality of NAVSTAR Tracking Data

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September 1978

Final Report

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RICHARD JOHNS, Lt Col, USAF
Director of Operational Applications

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

For several months, two Navstar satellites have been intensively tracked by three monitor stations located in California, Hawaii, and Alaska. Analysis of this tracking data shows that its short-term quality is high. This report discusses the tracking data, a technique for analyzing the data, and the results of this analysis.

The application of this technique to evaluate the performance of atomic frequency standards in the satellite is presented. The effect of two-frequency ionospheric correction on the short-term quality of the tracking data is also discussed.

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I. INTRODUCTION

The Navstar/Global Positioning System is a satellite-based navigation system that provides extremely accurate three-dimensional position and velocity information to properly equipped users anywhere on or near the earth. It is a Joint Service Program, managed by the Air Force with deputies from the Navy, Army, Marines, Defense Mapping Agency, and Coast Guard and with technical support provided by The Aerospace Corporation. The baseline program is divided into three phases:

I - Concept Validation Phase (1974-1979)
II - System Validation Phase (1979-1983)
III - Production Phase (1983-1987)

The major elements comprising the navigation payload on the satellites are the pseudo random noise signal assembly (PRNSA), atomic frequency standard, processor, and L-band antenna. The PRNSA includes the baseband generator, which produces the basic P (precise) and C/A (coarse/acquisition) ranging codes and encodes navigation data from the processor onto the pseudo random noise ranging signal; the amplifier/modulator units that supply the L_1 (1575.42 MHz) and L_2 (1227.6 MHz) carrier frequencies modulated by the PRN ranging signals; and the high-power amplifiers that amplify the carrier signals for transmission.

A user measures pseudo range and pseudo range rate using the navigation signal from each of four satellites. (Pseudo range is the true distance from the satellite to the user plus an offset due to the user's clock bias. Similarly, pseudo range rate is the true slant range rate plus an offset due to the frequency of the user's clock.) Each signal carries ephemeris data and system timing information for that satellite, which allows the user receiver/processor to convert the pseudo range and pseudo range rate to user three-dimensional position and velocity.

The control segment consists of a Master Control Station (MCS), a navigation message upload station, and widely separated monitor stations. The monitor stations passively track all satellites in view and accumulate ranging data, which is processed at the MCS to calculate the satellite ephemerides and clock offsets. At least once a day this information is transmitted by the upload station to the satellites for subsequent downlink transmission of the navigation data encoded on the carrier signals.

Under contract to the Space and Missile Systems Organization (SAMSO) of the U.S. Air Force, the Navstar satellites were developed and produced by Rockwell International Corp., Seal Beach, Calif. Magnavox Government and Industrial Electronics Co., Torrance, Calif., has developed a variety of user equipments, including the receivers now being used at the monitor stations. General Dynamics, Electronics Division, San Diego, Calif., which is responsible for the control segment, provided magnetic data tapes that were used in the preparation of this report.

II. SUMMARY

The results of the analyses and computations described in this paper are summarized as follows:

a. The rms short-term error in the pseudo range measurements is typically about 0.3 m. These measurements are made every 6 sec at the monitor stations, and this random error appears as uncorrelated white noise. A major part of the error is due to the quantization in the code tracking process.

b. When the two-frequency ionospheric correction is added to the pseudo range measurements, the error doubles to about 0.6 m. This increase in random error is a trivial price to pay for the overall improvement in accuracy associated with correcting for the ionospheric delay.

c. The noise in the delta range measurements over the 6-sec interval is about 1 cm (rms).

d. With a multiplicity of satellites and monitor stations, the random error in the delta range measurements can be separated, and the performance of the atomic frequency standards, both in the satellite and at the monitor stations, can be evaluated. At the 6-sec sampling interval, the fractional frequency variation of these frequency standards is about $3 - 4 \times 10^{-12}$, which exceeds the specification (1×10^{-11}).

III. OVERVIEW OF NAVSTAR PHASE I ORBIT CONFIGURATION

The baseline orbit configuration for Navstar Phase I is shown in Table 3-1. Navstar satellites 1, 2, and 3 are currently on-orbit. Navstar 1 occupies position 1, Navstar 2 position 5, and Navstar 3 position 6. For convenience, this data is referenced to the beginning of the day, midnight (0 hr) Greenwich Mean Time (GMT) on 1 January 1979. On this particular date the reference ground trace of satellite 1 will first cross the equator (north to south) at a longitude of 47.0 deg at 4 hr and 44.5 min GMT. Approximately 11 hr and 58 min later it will again cross the equator (south to north), this time at a longitude of 227.0 deg. The time of the ascending node crossings will differ for other days during the year.

The ground trace of the satellite orbits is fixed from day to day and therefore repeats itself every 23 hr 55 min and 56.6 sec. Most of the deviation from a 24-hr period is due to the difference between a solar and a siderial day, but a small part is due to the rotation of the orbit planes due to the earth's oblateness and to sun and moon effects. This longitudinal motion, or precession effect, is slightly different for each orbit but averages about 11.5 deg per year. The satellites thus appear 4 min and 3.4 sec earlier each day.

The information in Table 3-1 can be used to compute the times of visibility of the satellites for any location on earth as of 1 January 1979. For other days during 1979, the times are advanced 4 min and 3.4 sec for each day after 1 January.

Table 3-1. Navstar Phase I Orbits at First Ascending Node on 1 January 1979

Satellite Position No.	Nodal Period, sec	Inclination, deg	Nominal Longitude of First Ascending Node, deg	Right Ascension of the Ascending Node[a], deg	Nominal Time of First Ascending Node (GMT)	Eccentricity	Argument of Perigee, deg
1	43, 078.3	63.25[b]	47.0	217.98	$4^h\ 44^m\ 30^s$	0.0038	343
2	43, 078.3	63	68.5	219.49	$3^h\ 24^m\ 45^s$	0	0
3	43, 078.3	63	89.5	219.49	$2^h\ 00^m\ 59^s$	0	0
4	43, 078.3	63	310.5	99.48	$3^h\ 16^m\ 16^s$	0	0
5	43, 078.3	63.18[b]	332.0	100.27	$1^h\ 54^m\ 7^s$	0.0048	97
6	43, 078.3	63[b]	353.0	99.49	$0^h\ 27^m\ 15^s$	0	0

[a]Referenced to astronomical of 1950.0.

[b]Actual values of orbit. Values for other satellite orbits are nominal.

IV. SIGNAL STRUCTURE

Each satellite transmits a navigation signal on two L-band frequencies, one at 1575.42 MHz (L_1) and the other at 1227.6 MHz (L_2). These two carrier frequencies are biphase modulated by pseudo random sequences providing a spread spectrum modulation. The L_1 carrier is actually modulated by two such sequences in phase quadrature so that, strictly speaking, this carrier is actually quadraphase modulated. One pseudo random sequence is a precision (P) signal at a random pulse repetition rate of 10.23 MHz and is an extremely long code so that for all practical purposes it is a truly random sequence. The second pseudo random sequence is a coarse acquisition (C/A) signal, which is a short sequence used either for initial acquisition of the P signal or as a less accurate navigation signal for low-cost users. The L_2 carrier frequency is biphase modulated only by the P signal or, as a ground-controlled option, only by the C/A signal.

For the purpose of this discussion we will confine our attention only to the P signal on the L_1 carrier. This is the primary navigation signal used for tracking the Navstar satellites at the monitor stations. Note that the carrier frequency (1575.42 MHz) is an exact multiple (154) of the pseudo random sequence pulse rate (10.23 MHz). The wavelength of the carrier is only 19 cm, whereas the "chipping" rate of the pseudo random sequence is about 30 m.

The pseudo random sequence is generated by a feedback shift register, the output of which modulates the carrier as illustrated in Fig. 4-1. The P signal pseudo random sequence generator is functionally illustrated in Fig. 4-2. By combining four 12-stage feedback shift registers, the equivalent of a 48-stage shift register is obtained. Using a pseudo random sequence to biphase modulate the carrier results in the transmitted spectrum being spread as illustrated in Fig. 4-3. This spread permits an interference signal from being rejected in the receiver in the following manner: The receiver's pseudo random sequence generator modulates the incoming signal in the same manner as the generator on the satellite (the two generators are the same). The original carrier frequency is thus reconstructed and is collapsed to a very narrow band. The interference signal, however, is spread out over a wide spectrum, and this signal can therefore be filtered out so that only a small residue remains near the now-reconstructed carrier frequency.

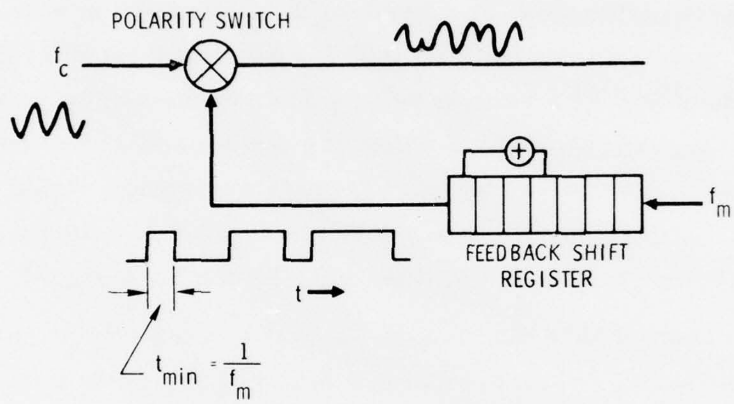
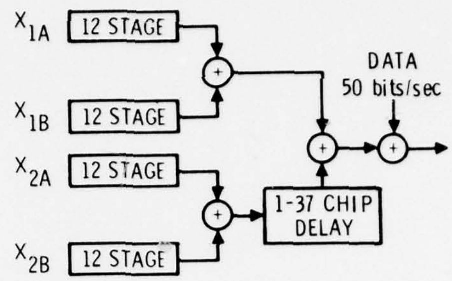


Fig. 4-1. Pseudo Random (Noise) Code



CHARACTERISTICS	CODE PROPERTIES
RATE	10.23×10^6 bits/sec
PERIOD	$\approx 2^{48} - 1$ 267 days
TYPE CODE	LINEAR, NON-MAX LENGTH
CORRELATION PROPERTIES	~ 36 dB

Fig. 4-2. Code Generator

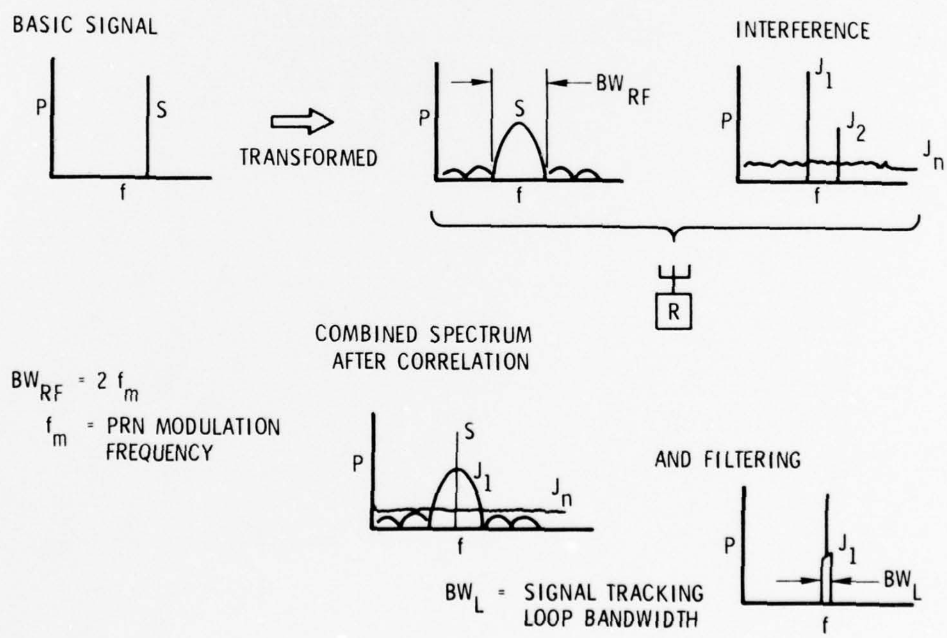


Fig. 4-3. Pseudo Random Noise-Spread Spectrum

V. GENERIC RECEIVER

Functionally, Navstar receivers used at the monitor stations incorporate two tracking loops that must operate simultaneously to properly track the Navstar navigational signal. The first is the code tracking loop, which tracks the pseudo random sequence by matching the locally generated sequence with the sequence on the received signal. Simultaneously, a phase lock loop is tracking the carrier frequency. The actual process is much more complicated because of the necessary intermediate frequency (IF) downconversion steps. For simplicity, however, these downconversion steps are omitted in the functional diagram shown in Fig. 5-1. Figure 5-2 expands on the function of such a receiver by illustrating the role of a feedback shift register and envelope detectors in the code lock loop that tracks the incoming pseudo random sequence. There are three outputs of the feedback shift register: an-on time sequence P_O, an early sequence P_E, and a late sequence P_L. The early and late codes modulate the carrier frequency C, which is synthesized in the phase lock loop. These signals are then mixed with the incoming signal, thereby generating voltages proportional to the extent that the sequences match the incoming sequence from the satellite. The difference between these two signals generates an error voltage that drives a voltage control oscillator (VCO) with which the feedback shift register is synchronized, thereby tracking the incoming random sequence (Fig. 5-3).

The on-time pseudo random sequence is mixed with the incoming signal to reconstruct the carrier signal. The phase lock loop (which is actually a bistable Costas loop) tracks this satellite-transmitted carrier. Binary data is added modulo-2 to the P signal pseudo random sequence at a rate of 50 bps. Since only the pseudo random sequence is removed from the incoming signal, the 50-bps data sequence still remains on the signal, but the single-sided bandwidth of this signal is now only about 50 Hz. In addition to maintaining phase lock on the carrier signal, the Costas loop also strips off the data remaining on the signal.

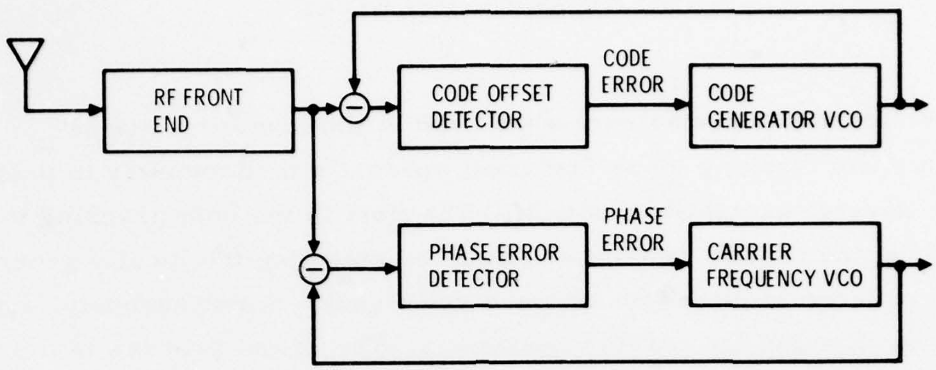


Fig. 5-1. Simplified Diagram of Generic Receiver

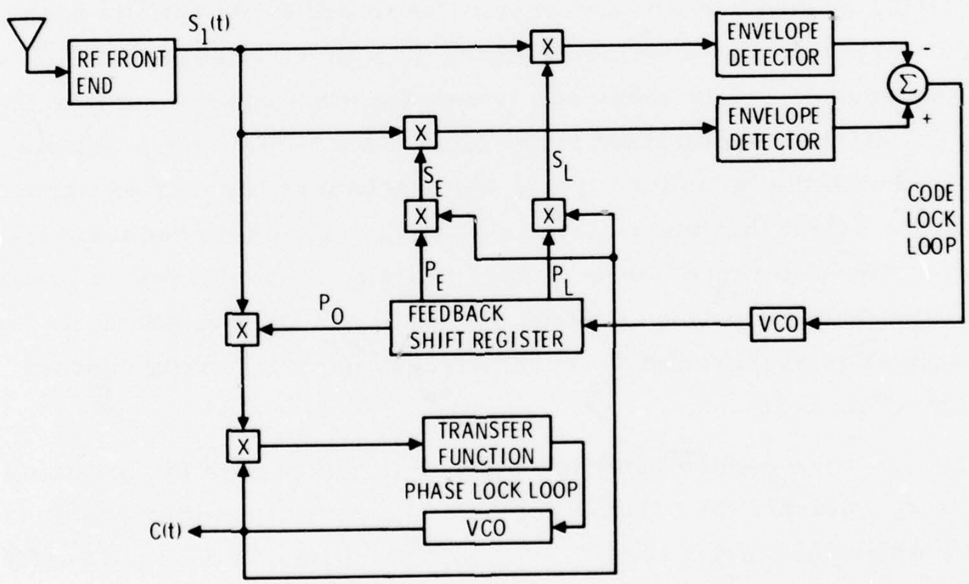


Fig. 5-2. Generic Pseudo Random Noise Receiver Functional Block Diagram

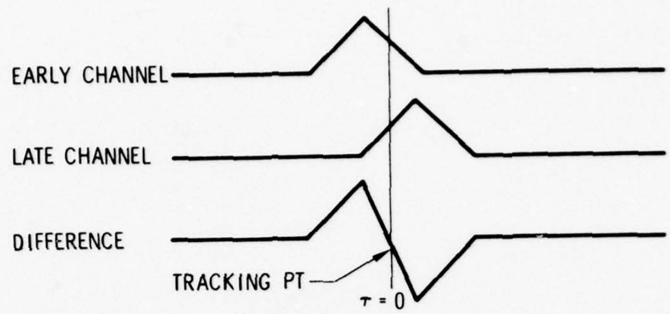


Fig. 5-3. Correlation Principle

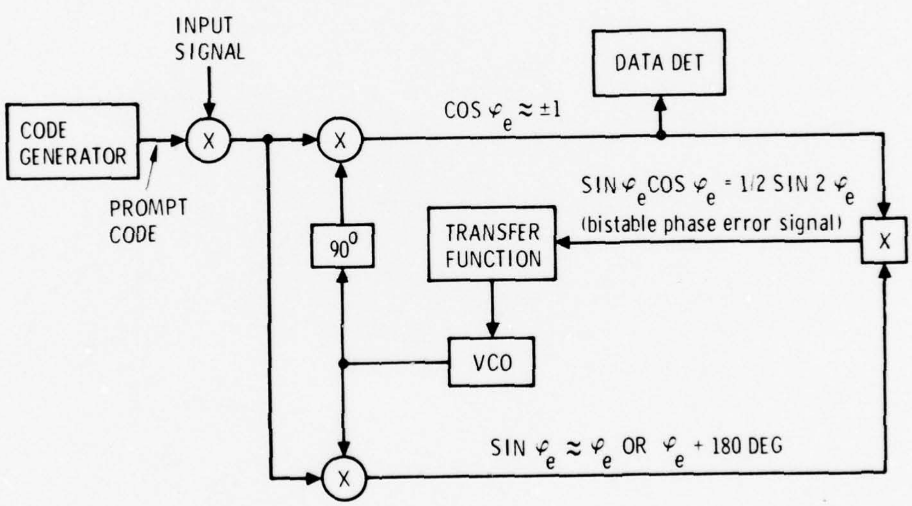


Fig. 5-4. Costas (Phase Lock) Loop

VI. NAVSTAR MONITOR STATIONS

Each monitor station has a receiver that tracks both the pseudo random sequence and the carrier signal. Tracking the incoming pseudo random sequence allows the receiver to make a range measurement to the satellite. If the atomic frequency standard at the monitor station were synchronized exactly with the standard in the satellite, this measurement would be a true measure of the distance from the monitor station to the satellite. In a practical situation, however, there is always some bias between the monitor stations and the satellite atomic frequency standards, and because of this relative clock difference the range measurement is referred to as pseudo range. This bias is determined as part of the ephemeris and clock parameter estimation process in the MCS computer, and since it is known to some degree of accuracy, the pseudo range measurements can be considered a measure of the absolute distance from the monitor station to the satellite.

Tracking the carrier signal, however, does not provide absolute ranging measurements; only changes in the range can be measured from carrier tracking. At the Navstar monitor stations, the changes in range are measured in the phase lock loop over successive 6-sec intervals. These range changes are referred to as delta range measurements.

An important device used in both the code and carrier tracking loops is an incremental phase modulator (IPM), a digital frequency synthesizer that is in effect the actual VCO. The IPM performs relative to the frequency standard in the monitor station. The quantization of each step is 1/64 of the wavelength of the quantity being tracked. Since the chipping rate of the pseudo random sequence corresponds to about 30 m, the quantization of the code tracking loop is about 0.46 m. Similarly, since the wavelength of the carrier frequency is about 19 cm, the quantization in the phase lock loop is about 0.3 cm. Because of this two-order-of-magnitude difference in quantization between the code and carrier tracking loops, the tracking accuracy of the phase lock loop is much greater than that of the code loop. The short-term quality of the Navstar tracking data is obtained by analysis of the 6-sec delta range data obtained from carrier tracking at the monitor stations.

VII. DELTA RANGE TRACKING DATA

In the laboratory, atomic frequency standards are tested by comparing the output of the frequency standard under test with another that (ideally) is much more stable. The delta range tracking data makes possible the same type of test. The high-performance cesium beam atomic frequency standard at the monitor station serves as a reference for evaluating the atomic frequency standard in the satellite even though they are about 20, 000 km apart.

A quantization of 0.3 cm over a 6-sec interval corresponds with an average range rate of 0.5×10^{-3} m/sec. Normalized by the velocity of light (3×10^{8} m/sec), this corresponds with a fractional frequency offset of the atomic frequency standard of 1.7×10^{-12}. Over 6-sec intervals, typical atomic frequency standards (rubidium tube or cesium beam) can be expected to exhibit frequency stability on the order of 10^{-11}; consequently, quantization of the Navstar tracking data is consistent with its use for verifying proper performance of the satellite atomic frequency standard.

VIII. POLYNOMIAL FITTING PROCESS

Successive pseudo range and delta range measurements change in a gradual systematic fashion, mainly because of satellite motion and earth rotation. There is also some variation in these measurements due to the long-term effects of biases in the satellite and monitor station frequency standard parameters. To evaluate the short-term fluctuations in the frequency standards, it is necessary to remove these longer term systematic effects. A straightforward technique for accomplishing this is to fit, in a least squares sense, a polynomical function of time to the tracking data. The order of the polynomial can vary from about five to fifteen, depending on the length of the span of data being used in the analysis.

A complete, uninterrupted span of data can be as long as six hours; typically, however, good uninterrupted spans are about one to three hours. The residuals of the original tracking data to the polynomial fit are computed from the polynomial expressions. These residuals represent the noise in the data and can be regarded as a measure of the short-term stability of the frequency standards. This noise is generally similar to uncorrelated white noise and is not sensitive to the degree of the polynomial used in the fitting process.

Since there are two satellites and three monitor stations, by a covariance analysis of the residuals from various simultaneous sets of tracking data, it is possible to separate the noise contributed by the satellite, that contributed by the monitor station clocks, and other sources of noise (usually the receiver tracking process, including the 0.3-cm quantization).

IX. AN EXAMPLE OF TWO-SATELLITE TRACKING DATA

On 20 July 1978, there was a 70-min interval during which data was obtained simultaneously from both Navstars 1 and 2 at both the Vandenberg and Hawaii monitor stations. A complete covariance analysis of the residuals from a ninth-order polynomial fit to the delta range data was performed. From this analysis, a breakdown of the residuals has been obtained. The total noise from each set of measurements was divided between the noise from the satellite, noise from the monitor stations clocks, and noise from other sources.

For each of the four sets of data, which covered exactly the same 70-min interval, 700 delta range measurements were analyzed. The rms of the residuals for each set was about 1 cm. The average of the squares of the residuals was computed, and the average of all six combinations of the products of the residuals was obtained. The computation can be conveniently formulated in matrix form. The four sets of 700 measurement residuals can be regarded as a 4×700 matrix A, an array of 4 columns and 700 rows. This covariance matrix is a 4×4 array C, which represents the expected values of the squares and products of the measurement errors, where the errors are taken to be the residuals computed from the ninth-order polynomial function of time. The relationship between the measurement residuals and the covariance matrix is given by

$$C = AA^T \div 700$$

The values of the individual terms of the covariance matrix are computed from

$$c_{ij} = \left(\sum_{k=1}^{700} a_{ik} a_{jk} \times \frac{1}{700} \right)$$

For the off-diagonal terms (the products of different sets of measurements), the normalized correlation coefficient is given by

$$c_{nij} = \frac{100\% \times c_{ij}}{\sqrt{c_{ii} \times c_{jj}}}$$

For the purpose of presenting results, the normalized correlation coefficients are sometimes shown in the upper right-hand triangle of the covariance matrix. It should be noted that the percentage correlation can be either positive or negative since the average of the product of two sets of measurement errors can have either sign. The quantities of interest are frequently the one-sigma of the measurement errors, and for this reason the square roots of the variances along the diagonal are frequently presented. Similarly, the square roots of the absolutes of the off-diagonal terms may be of interest and are thus presented in the lower left-hand triangle of the modified covariance matrix.

The results of this straightforward covariance analysis are given in Fig. 9-1. In this 4 × 4 modified covariance matrix, the normalized correlation coefficients in percent are given in the upper right. Along the diagonal of the matrix and at the lower left, the square roots of the absolute values of the elements of the covariance matrix are shown. The quantities on the diagonal are the computed rms of the residuals of the four sets of measurements. The quantities at the lower left are the parts of the residuals that can be attributed to isolated sources. For example, the 0.553 cm in the first column of the second row can be attributed to noise in the cesium beam atomic frequency standard at Vandenberg. The residuals for the cesium beam standard at Hawaii and the rubidium frequency standards in the two satellites were also isolated. The cross correlation between the various sets of data varies from 34.7 to 44.1 percent. Note that there appears to be a 2.25 percent cross correlation between Navstar 1/Vandenberg and Navstar 2/Hawaii as well as -0.68 percent between Navstar 2/Vandenberg and Navstar 1/Hawaii. The

CROSS CORRELATION IN PERCENT

RMS OF RESIDUALS IN CENTIMETERS

	NAVSTAR 1/ VANDENBERG	NAVSTAR 2/ VANDENBERG	NAVSTAR 1/ HAWAII	NAVSTAR 2/ HAWAII
NAVSTAR 1/VANDENBERG	0.916	34.685	36.685	2.251
NAVSTAR 2/VANDENBERG	0.553	0.964	-0.683	40.244
NAVSTAR 1/HAWAII	0.591	0.083	1.038	44.089
NAVSTAR 2/HAWAII	0.152	0.658	0.715	1.117

Fig. 9-1. Modified Covariance Matrix

expected cross correlation for these cases is, of course, zero, and the fact that finite values were computed simply reflects the statistical variability of a limited set of observation data.

The results shown in Fig. 9-1 are presented in a different form in Table 9-1. Noise from various atomic frequency standards and other sources are separable when there is a multiplicity of satellites and monitor stations. These results show that at a 6-sec sampling interval, all four frequency standards have comparable stability, about 3 - 4 $\times 10^{-12}$ fractional frequency variation. The category "other" was obtained by subtracting, in an rms sense, the satellite and monitor station clock residuals from the total residual.

The observed fractional frequency stability over a 6-sec sample time exceeds the specification level of 10^{-11} and is typical of the performance measured by Rockwell International in the laboratory. In Fig. 9-2, based

Table 9-1. Breakdown of Residuals (rms, cm)

Source	Navstar 1/ Vandenberg	Navstar 2/ Vandenberg	Navstar 1/ Hawaii	Navstar 2/ Hawaii
Satellite	0.591	0.658	0.591	0.658
Monitor Station Clocks	0.553	0.553	0.715	0.715
Other	0.429	0.436	0.466	0.551
Total	0.916	0.964	1.038	1.117

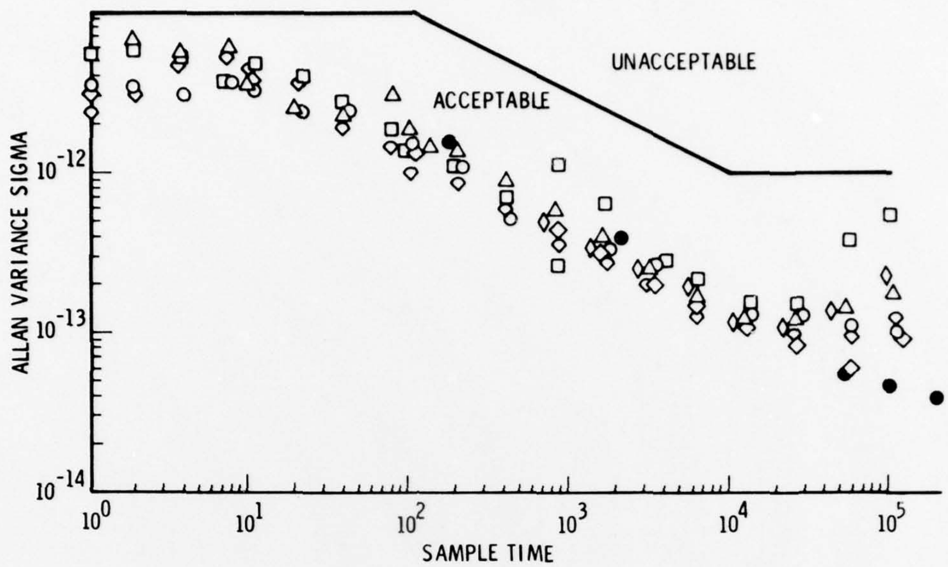


Fig. 9-2. Frequency Standard Stability Performance

on Rockwell data, the specification for the satellite rubidium frequency standard is shown as a heavy solid line. The many dots represent the results of laboratory test data taken from seven frequency standards. Note that at or near a 6-sec sample time, the data points cluster in the region from $3 - 6 \times 10^{-12}$. In conclusion, the analysis of the satellite tracking data indicates substantially the same level of frequency standard performance as generally observed in the laboratory.

Figure 9-3 shows a sample of the residuals for a 10-min interval over which 100 measurements were taken. This plot represents the first 10 min of the data taken at the Vandenberg monitor station tracking Navstar 1. This plot is typical of all the data obtained in tracking both satellites from both monitor stations.

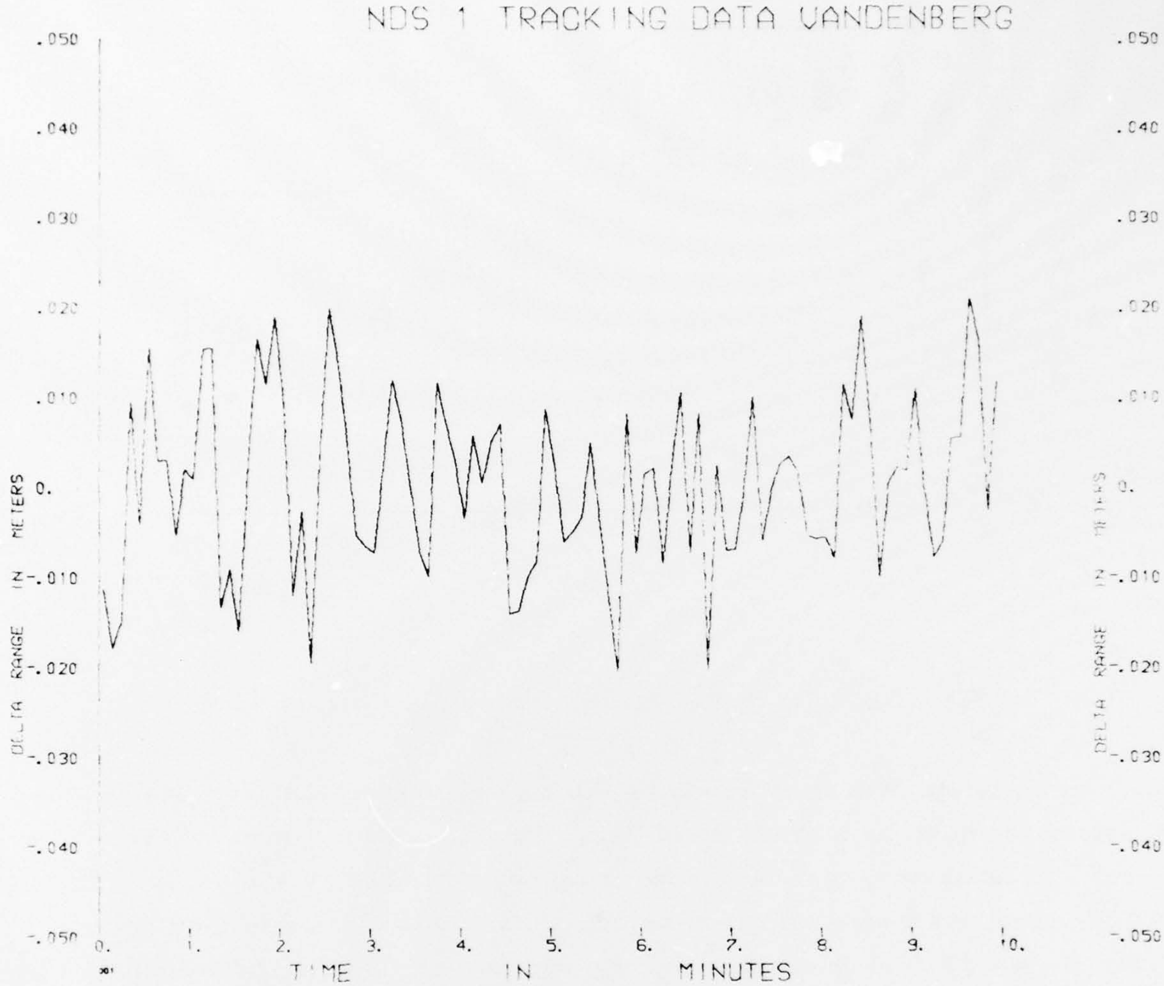


Fig. 9-3. Delta Range Residuals

X. PSEUDO-RANGE TRACKING DATA

While the monitor station is tracking both pseudo range and delta range on the L_1 frequency (1575.42 MHz), it is also measuring the difference between the pseudo range as measured on the L_1 and L_2 (1227.6 MHz) frequencies. This difference is measured to automatically correct for the ionospheric delay in the transmission of the navigation signal. The primary reason that two frequencies are employed on Navstar is to make possible this compensation, which is based on the fact that the ionospheric delay is inversely proportional to the square of the carrier frequency. The difference in the delay as measured on the two L-band frequencies is, in effect, a measurement of the ionospheric delay at each of these two frequencies. The expression for the necessary compensation of the pseudo range measurement at the L_1 frequency is given by

$$R_1' = R_1 + \left(\frac{f_2^2}{f_1^2 - f_2^2}\right)(R_1 - R_2)$$

or

$$R_1' = R_1 + 1.54573\,(R_1 - R_2)$$

where

R' = corrected pseudo range measurement at the L_1 frequency

R_1, R_2 = receiver outputs, i.e., uncorrected pseudo range measurements at the L_1, L_2 frequencies

The short-term quality of the pseudo range measurements can be obtained by the same technique previously described for the delta range measurements. The quality can be evaluated for both the uncompensated pseudo range measurement R_1 and the pseudo range measurement corrected for the ionospere R_1'. From this expression, the noise in the compensated pseudo range measurements would be expected to be larger than the measurements before the ionospheric compensation because there are two noise sources, the noise in tracking the pseudo random code on the L_1 frequency and the noise in the receiver tracking of the difference in pseudo range on the two L-band carrier frequencies.

For the 70-min interval on 20 July 1978, a ninth-order polynomial function of time was fitted to the pseudo range data taken at the Vandenberg monitor station while tracking Navstar 1. For the uncompensated pseudo range measurements the rms of the residuals was computed to be 0.29 m, and for the ionospheric-compensated pseudo range measurements the rms was computed to be 0.57 m. These results represent both the code tracking of the pseudo random sequences and the noise increase associated with adding the ionospheric correction. A representative 10-min sample of these residuals, which covers 100 measurements, is shown in Figs. 10-1 and 10-2 for the uncompensated and compensated cases, respectively.

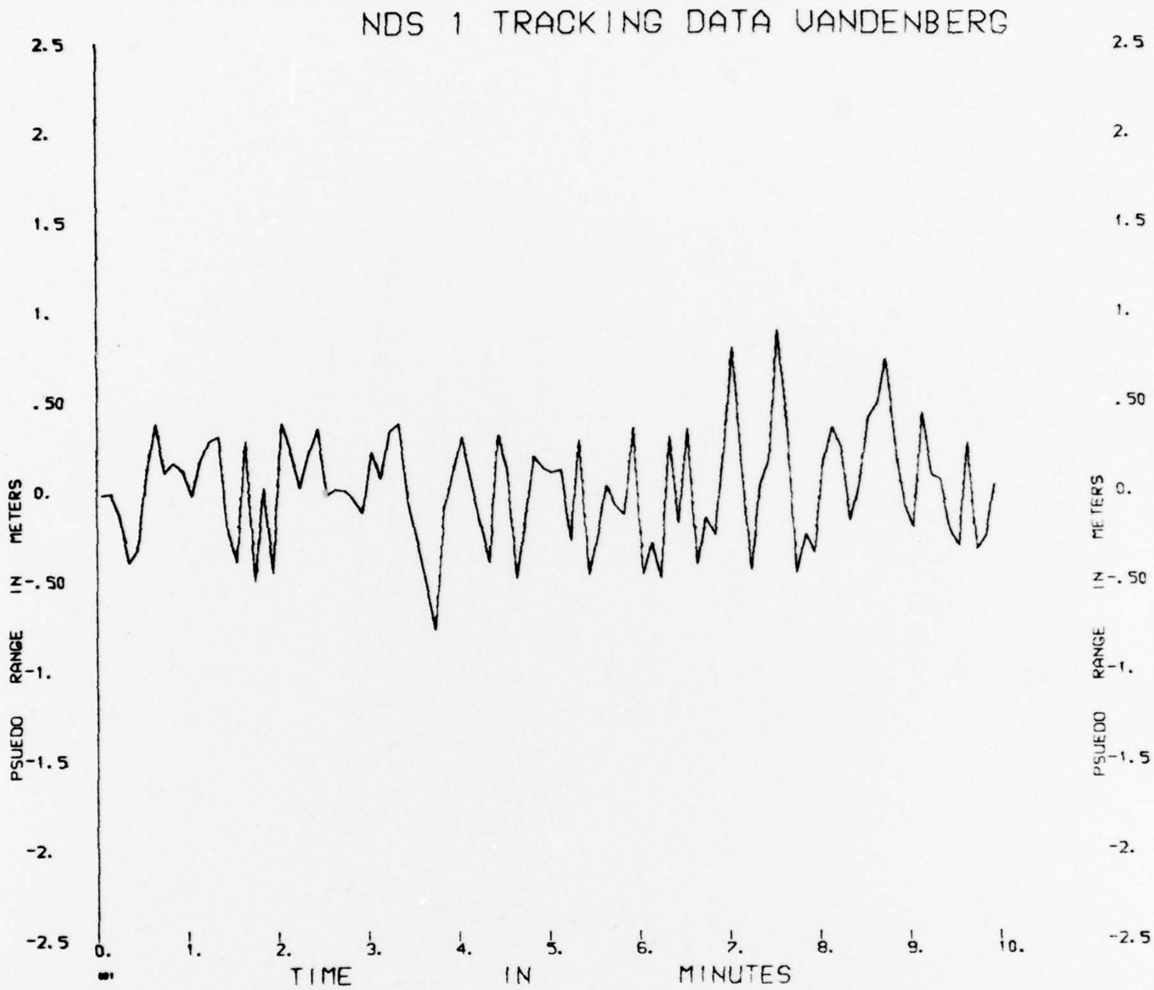


Fig. 10-1. Uncompensated Pseudo Range Residuals

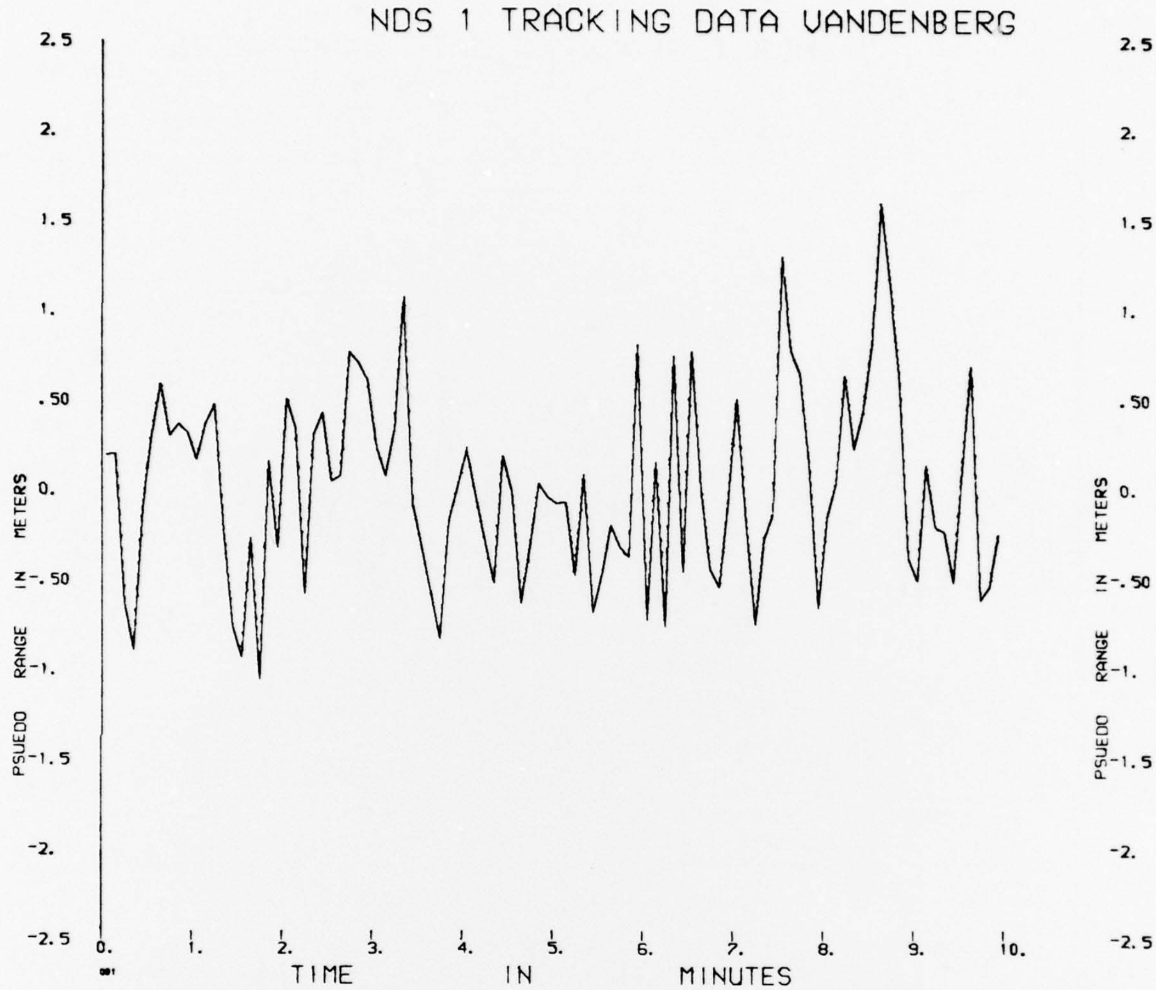


Fig. 10-2. Compensated Pseudo Range Residuals

XI. ANOTHER EXAMPLE OF TWO SATELLITE TRACKING DATA

On 22 August 1978, there was a two-hour span during which data was obtained simultaneously from both Navstars 1 and 2 at all three monitor stations, with high-quality data for all six combinations. The residuals from a ninth-order polynomial fit of the delta range data (1σ) are given in Table 11-1.

Table 11-1. Residuals from a Ninth-Order Fit

Combination	Residuals (1σ), cm
Navstar 1/Vandenberg	0.976
Navstar 2/Vandenberg	0.960
Navstar 1/Hawaii	1.039
Navstar 2/Hawaii	1.097
Navstar 1/Alaska	0.950
Navstar 2/Alaska	1.055

A covariance analysis of these residuals was performed, and the results are given in Fig. 11-1. As in the previous example, the 6×6 matrix shown on this figure is a modified covariance matrix in which the normalized correlation coefficients in percent are given in the upper right-hand triangular part of the matrix. Along the diagonal of the matrix and the lower left-hand triangle, the square roots of the elements of the covariance matrix are shown. The percentage cross correlations, the one-sigma measurement residuals, and the manner in which the noise error can be assigned to satellites, monitor station clocks, and other sources is similar to the results obtained from the first example.

CROSS CORRELATION IN PERCENT

RMS OF RESIDUALS IN CENTIMETERS

	NAVSTAR 1/ VANDENBERG	NAVSTAR 2/ VANDENBERG	NAVSTAR 1/ HAWAII	NAVSTAR 2/ HAWAII	NAVSTAR 1/ ALASKA	NAVSTAR 2/ ALASKA
NAVSTAR 1/VANDENBERG	0.976	40.540	31.441	-4.184	34.707	-1.263
NAVSTAR 2/VANDENBERG	0.616	0.960	-3.654	31.890	-1.915	39.015
NAVSTAR 1/HAWAII	0.564	0.191	1.039	45.608	35.930	0.813
NAVSTAR 2/HAWAII	0.211	0.579	0.721	1.097	-0.217	34.175
NAVSTAR 1/ALASKA	0.567	0.132	0.596	0.048	0.950	44.804
NAVSTAR 2/ALASKA	0.114	0.629	0.094	0.629	0.670	1.055

Fig. 11-1. Modified Covariance Matrix