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TRACOR-SD-67-013-C

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TECHNICAL NOTE

C/P ARRAY BEAM PATTERN WITH AMPLITUDE SHADING CONSISTING OF COSINE-SQUARED ON A PEDESTAL (U)

Submitted to
U. S. Navy Electronics Laboratory
San Diego, California 92152

Attention: Code 2110

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June 12, 1967

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1 TECHNICAL NOTE

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C/P ARRAY BEAM PATTERN WITH AMPLITUDE SHADING CONSISTING OF COSINE-SQUARED ON A PEDESTAL

In calculating C/P Array beam patterns, certain time-consuming summations can be avoided if they can be expressed in closed analytical form. Such is known to be the case, for example, for uniform amplitude shading plus the phasing required to steer the beam in some particular direction. Even for some types of non-uniform shading it appears that the summations involved in the beam pattern analysis can be carried out to closed form. One such type is that of shading consisting of cosine-squared on a pedestal. The purpose of this note is to derive the beam pattern equations in closed form for cosine-squared on a pedestal amplitude shading (plus phasing for steering) and to present some patterns computed from the results obtained.

Our starting point is the total far-field pressure expressed as

$$P(\varphi,\theta)=\rho(\varphi,\theta)\left[\sum_{x=-\frac{M-1}{2}}^{\frac{M-1}{2}}\left(p_{11}+p_{12}\cos^2\frac{\pi x}{M-1}\right)e^{-jAx}\right]\left[\sum_{z=-\frac{N-1}{2}}^{\frac{N-1}{2}}\left(p_{21}+p_{22}\cos^2\frac{\pi z}{N-1}\right)e^{-jBz}\right] \quad (1)$$

in the direction (φ,θ), where φ is the usual depression look angle and θ is the azimuthal look angle (measured from dead ahead). The function $\rho(\varphi,\theta)$ is the cardioid response function given by

$$\rho = \tfrac{1}{2}(1 + \cos w \cos \varphi \sin \theta + \sin w \sin \varphi) \quad (2)$$

w being the array tilt angle. Other pertinent relations and

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definitions are:

$$A = ka(\cos\varphi\cos\theta - \cos\varphi_o \cos\theta_o) \tag{3}$$

$$B = kb\left[\cos w(\sin\varphi - \sin\varphi_o) - \sin w(\cos\varphi \sin\theta - \cos\varphi_o \sin\theta_o)\right] \tag{4}$$

(φ_o, θ_o) = steered direction

$k = 2\pi f/c$ = wave number

f = frequency

c = speed of sound

a = column spacing

b = row spacing

M = number of columns

N = number of rows

p_{11} = pedestal value for horizontal direction

p_{12} = cosine-squared amplitude (horizontal direction)

p_{21} = pedestal value for vertical direction

p_{22} = cosine-squared amplitude (vertical direction)

The functions A and B above incorporate the phasing considerations required for steering such that the individual element pressures are all in phase in the steered direction (φ_o, θ_o). It should be

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noted that in (1) M and N may be odd or even.

Since the summations contained in (1) are both of the same form, it will suffice to work with only one of them. Let us consider

$$S_1 = \sum_x (p_{11} + p_{12} \cos^2 \frac{\alpha x}{2})\ e^{-jAx} \tag{5}$$

$$= \sum_x (p_{11} + p_{12} \cosh^2 \frac{j\alpha x}{2})\ e^{-jAx}$$

where the summation limits are as indicated in (1) and $\alpha/2 = \pi/(M-1)$. Now

$$\cosh^2 \frac{j\alpha x}{2} = \tfrac{1}{4}\ (2 + e^{j\alpha x} + e^{-j\alpha x})$$

so that S_1 becomes

$$S_1 = (p_{11} + \tfrac{1}{2}\ p_{12}) \sum_x e^{-jAx} + \tfrac{1}{4}\ p_{12} \left[\sum_x e^{-j(A+\alpha)x} + \sum_x e^{-j(A-\alpha)x} \right]$$

$$= (p_{11} + \tfrac{1}{2}\ p_{12})\ \frac{\sin MA/2}{\sin A/2} + \tfrac{1}{4}\ p_{12} \left\{ \frac{\sin[M(A+\alpha)/2]}{\sin[(A+\alpha)/2\ ]} + \frac{\sin[M(A-\alpha)/2]}{\sin[(A-\alpha)/2\ ]} \right\} \tag{6}$$

each of the three sums above being directly summable as a geometric series. Since

$$M\alpha/2 = \pi M/(M-1) = \pi + \pi/(M-1)$$

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we may rewrite (6) as

$$S_1 = (p_{11} + \tfrac{1}{2} p_{12}) \frac{\sin MA/2}{\sin A/2} - \tfrac{1}{4} p_{12} \left[\frac{\sin(\frac{MA}{2} + \frac{\pi}{M-1})}{\sin(\frac{A}{2} + \frac{\pi}{M-1})} + \frac{\sin(\frac{MA}{2} - \frac{\pi}{M-1})}{\sin(\frac{A}{2} - \frac{\pi}{M-1})} \right] \tag{7}$$

An analogous expression holds for S_2, the other sum contained in (1).

To find the beam pattern in conventional form, we normalize with respect to the pressure in the steered direction (φ_o, θ_o). In this direction A=B=0, and S_1 becomes

$$S_{1o} = (p_{11} + \tfrac{1}{2} p_{12})M - \tfrac{1}{2} p_{12}$$

with an analogous expression for S_{2o}. Defining $R_1 = S_1/S_{1o}$ and $R_2 = S_2/S_{2o}$, we have:

$$R_1 = \frac{\dfrac{\sin MA/2}{\sin A/2} - K_1 \left[\dfrac{\sin(\frac{MA}{2} + \frac{\pi}{M-1})}{\sin(\frac{A}{2} + \frac{\pi}{M-1})} + \dfrac{\sin(\frac{MA}{2} - \frac{\pi}{M-1})}{\sin(\frac{A}{2} - \frac{\pi}{M-1})} \right]}{M - 2K_1} \tag{8}$$

$$R_2 = \frac{\dfrac{\sin NB/2}{\sin B/2} - K_2 \left[\dfrac{\sin(\frac{NB}{2} + \frac{\pi}{N-1})}{\sin(\frac{B}{2} + \frac{\pi}{N-1})} + \dfrac{\sin(\frac{NB}{2} - \frac{\pi}{N-1})}{\sin(\frac{B}{2} - \frac{\pi}{N-1})} \right]}{N - 2K_2} \tag{9}$$

where $K_1 = p_{12}/(4\, p_{11} + 2\, p_{12})$ and $K_2 = p_{22}/(4\, p_{21} + 2\, p_{22})$. Additionally,

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$$\frac{\rho}{\rho_o} = \frac{1 + \cos w \cos \varphi \sin \theta + \sin w \sin \varphi}{1 + \cos w \cos \varphi_o \sin \theta_o + \sin w \sin \varphi_o} \qquad (10)$$

The beam pattern equation in dB relative to the intensity in the steered direction is

$$\beta(\varphi,\theta) = 20 \log (R_1 R_2 \rho/\rho_o) \qquad (11)$$

where the constituent factors are given by (8) through (10) and other pertinent definitions and relations are as stated earlier in this paper. Note that with appropriate values assigned to the shading constants the results obtained here cover the cases of natural beams and cosine-squared amplitude shading with or without pedestals.

The foregoing beam pattern equations were programmed for the computer by TRACOR and several runs made to produce the patterns plotted in Figs. 1 through 5. Inputs common to all these patterns are:

$c = 4900.36$ ft/sec.
$M = 224$
$N = 12$
$a = b = 0.656$ ft.
$w = 25^o$

Uniquely identifying information is shown in each figure. In general, the angle scales in the figures are different in order to present the detail of sidelobe structure.

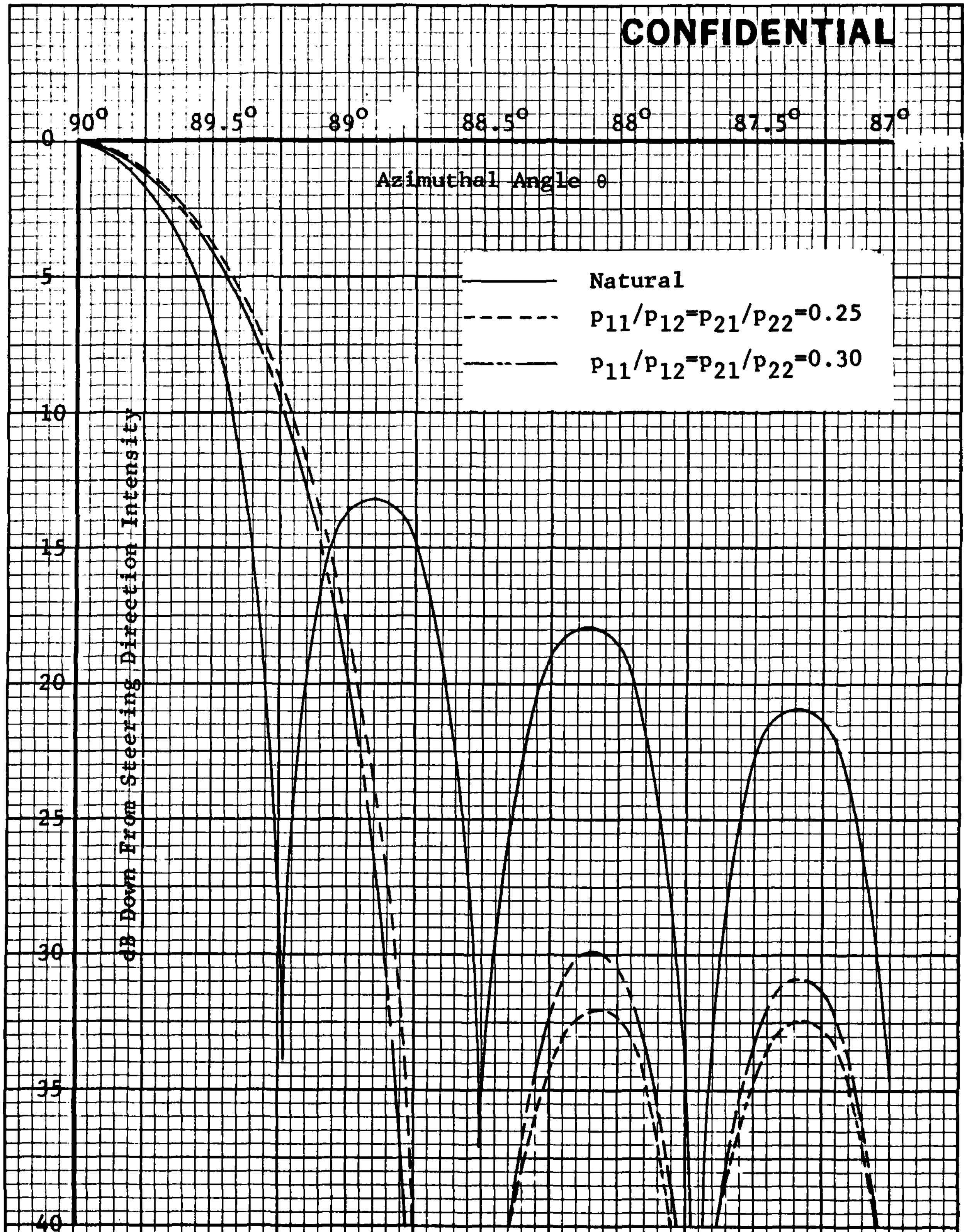

FIG. 1: AZIMUTHAL BEAM PATTERNS
2500 cps, $\varphi_o=0^{\circ}$, $\theta_o=90^{\circ}$

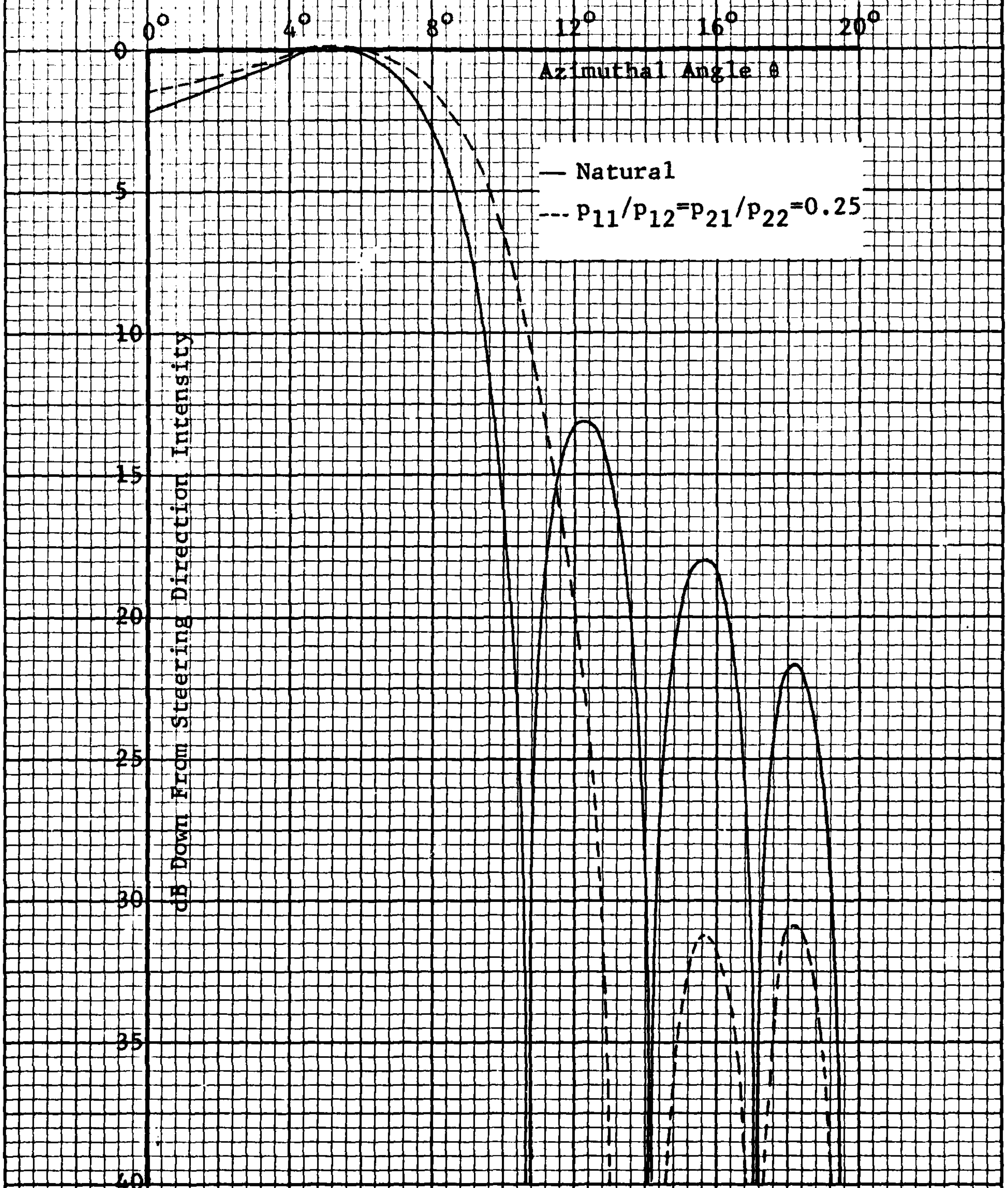

FIG. 2: AZIMUTHAL BEAM PATTERNS
2500 cps, $\varphi_o=0^o$, $\theta_o=5^o$

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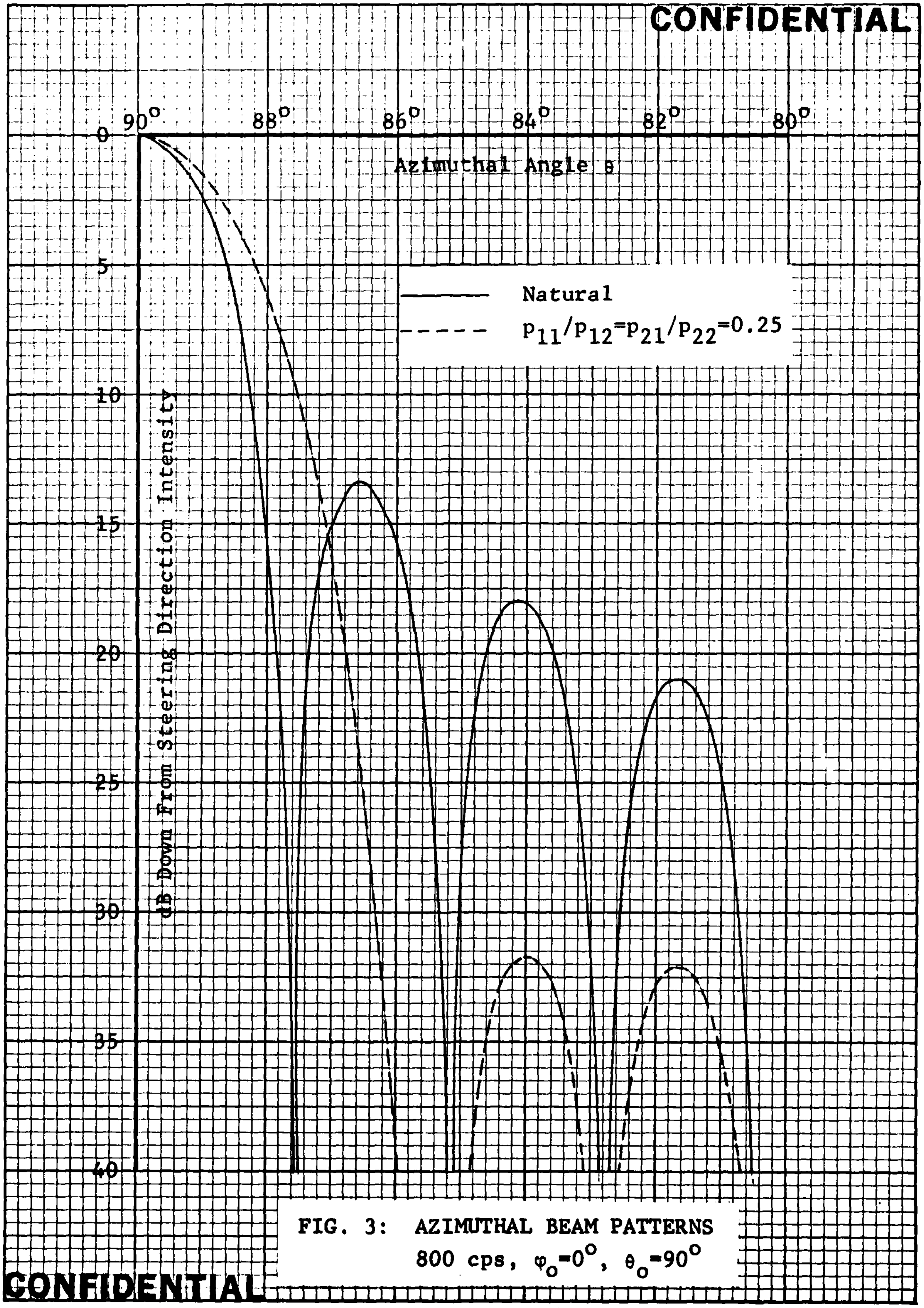

FIG. 3: AZIMUTHAL BEAM PATTERNS
800 cps, $\varphi_o=0^o$, $\theta_o=90^o$

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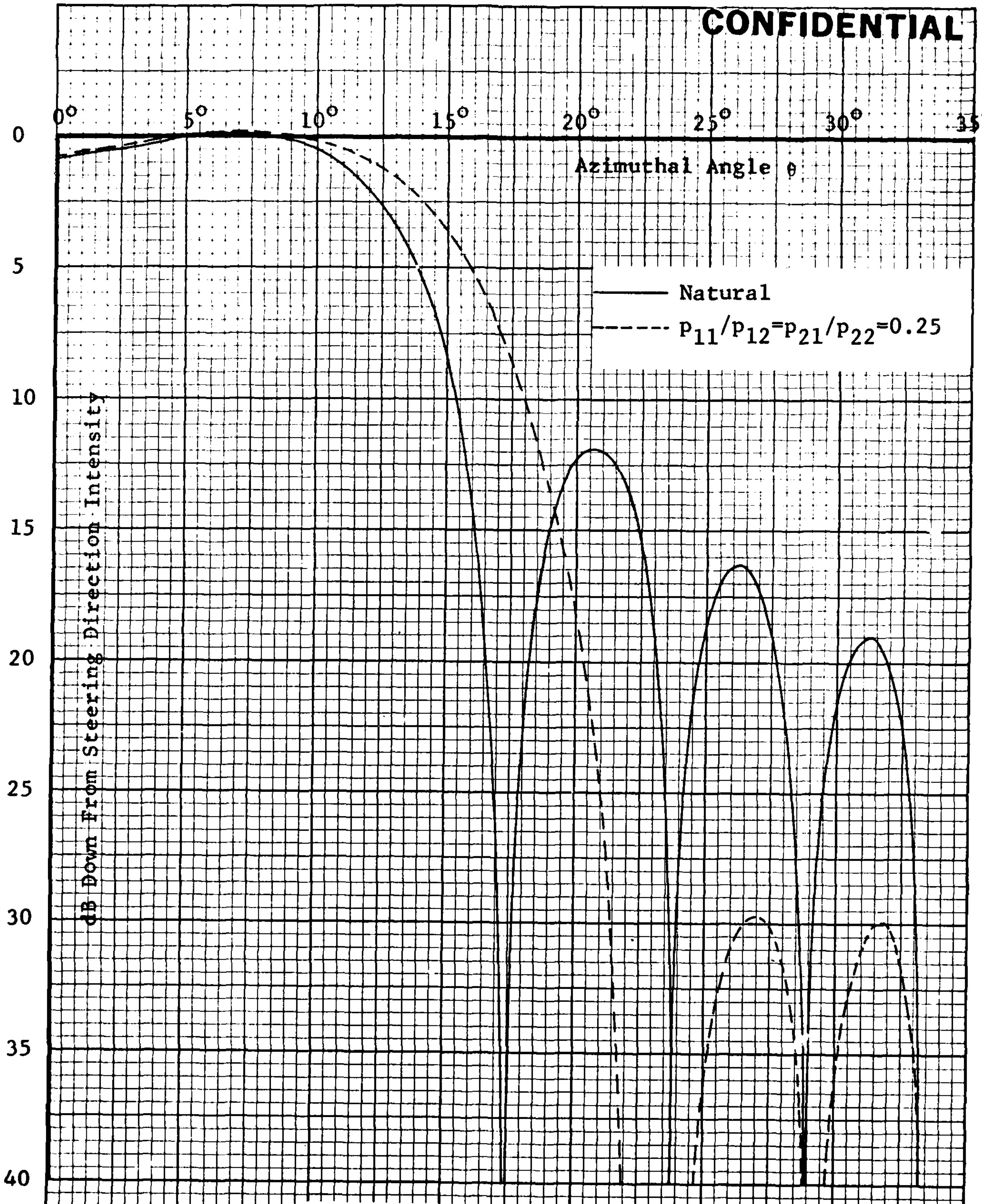

FIG. 4: AZIMUTHAL BEAM PATTERNS
800 cps, $\varphi_o=0^o$, $\theta_o=5^o$

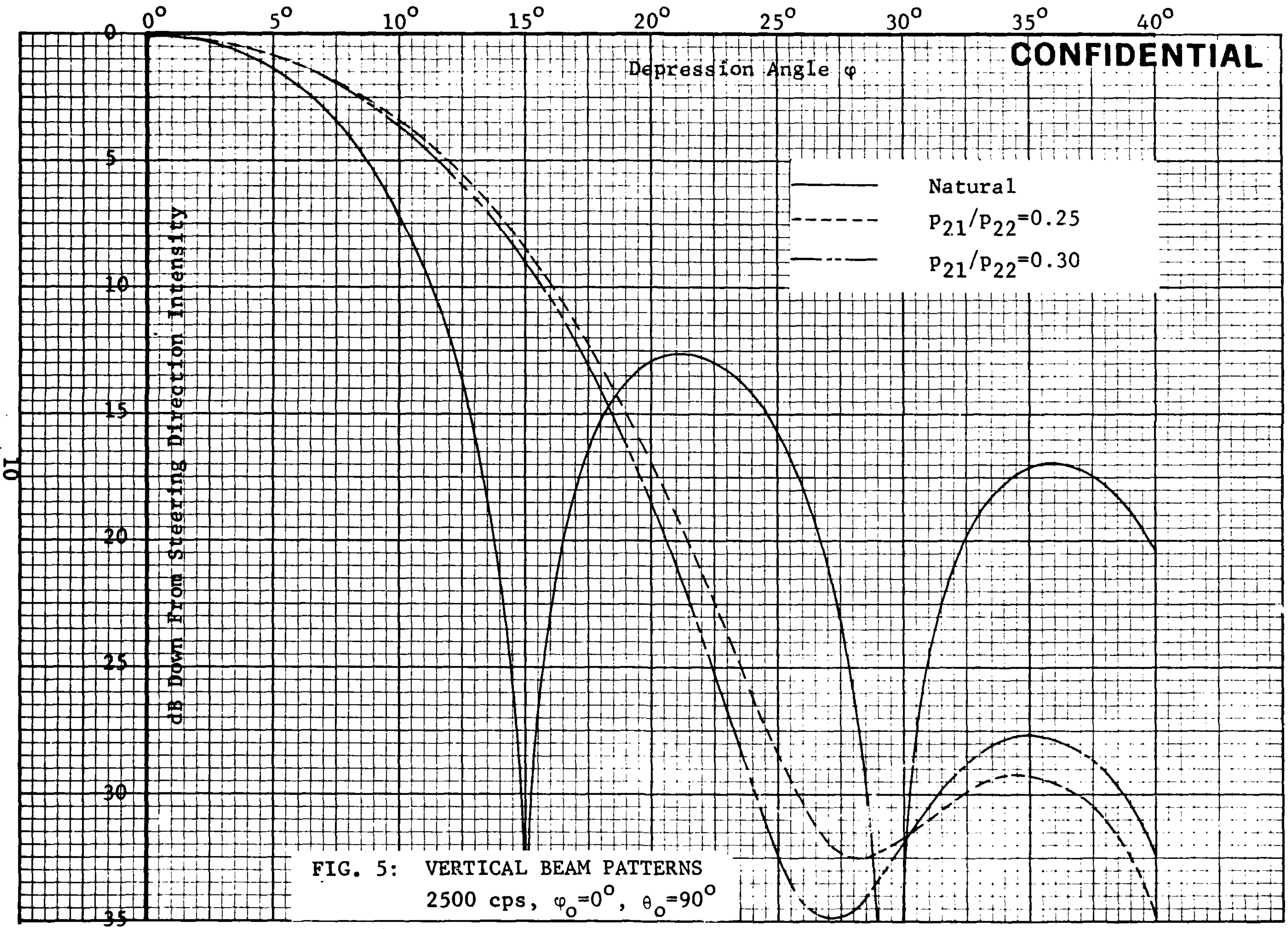

FIG. 5: VERTICAL BEAM PATTERNS
2500 cps, $\varphi_o=0^o$, $\theta_o=90^o$