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SINGULAR PLASTIC ELEMENT: NASTRAN IMPLEMENTATION AND APPLICATION

M. A. Hussain
S. L. Pu
W. E. Lorensen

December 1978

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The elastic and plastic singularities near a crack tip are obtained from higher order isoparametric elements. This is simply accomplished by collapsing the quadrilateral element into a triangular element and by judicious choice of adjacent mid-side nodes.

Specifically for the cubic element, the elastic singularity is obtained by

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placing the mid-side nodes adjacent to the crack tip at 1/9th and 4/9th locations. The plastic singularity is constructed using the sliding node concept. These elements have been implemented in NASTRAN as user dummy elements.

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ILLUSTRATIONS

SYMBOLS

$(x,y),(r,\theta)$	cartesian and cylindrical coordinates
(ξ,η)	curvilinear coordinates
$x_i,y_i,\ \xi_i,\eta_i$	grid point coordinates
N_i	shape function at grid point i
u,v	cartesian displacements
ε_{ij}	strain tensor
σ_{ij}	stress tensor
$S_{ij},\ e_{ij}$	deviatoric stress and strain tensors
W	strain energy density
J	path independent integral
$[J]$	Jacobian matrix
n	strain hardening exponent

INTRODUCTION

In recent years there has been a wide acceptance of linear fracture mechanics resulting in the development of new structural alloys having high fracture toughness and maintaining yield strength close to previous levels.

However, plasticity plays a major role in the application of these materials either in thin cross sections or under mixed mode conditions. Also in some cases, to meet the ASTM requirement for plane strain fracture toughness testing, the specimens required are too large for economical testing. To alleviate some of these problems a number of methods have been proposed, e.g., Irwin's equivalent 'Elastic Crack Length'[1], Well's Crack Opening Displacement[2], Rice's Path Independent J-Integral[3] and Non-linear Energy Methods proposed by Liebowitz and his coworkers[4], the last two being quite promising. Hence it is necessary to model the plastic condition near the crack tip as accurately as possible.

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[1] Irwin, G. R., Fracture Testing of High-Strength Sheet Materials Under Conditions Appropriate for Stress Analysis, Naval Research Laboratory, Rpt. 5486, July 1960.

[2] Wells, A. A., Unstable Crack Propagation in Metals, Proc. Conf. Crack Propagation, Cranfield, England, 1960, p. 120.

[3] Rice, J. R., A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks, Trans. Am. Soc. Mech. Engrs., Journal of Applied Mechanics, 1968, p. 379.

[4] Eftis, J., Jones, D. L., and Liebowitz, H., On Fracture Toughness in the Nonlinear Range, Engr. Fract. Mech., Vol. 7, 1975, p. 491.

In this paper, we implement higher order isoparametric elements (quadratic and cubic) in NASTRAN's piecewise linear (plasticity) module. By judicious choice of intermediate grid points, and using proper constraints, we develop elastic and elastic-plastic singular elements.

Specifically, the elastic singular cubic element embodying the square root ($1/\sqrt{r}$) singularity is constructed by placing the midside nodes, adjacent to the crack tip, at 1/9th and 4/9th locations. The plastic singular element is constructed for the Ramberg-Osgood type of material with zero hardening exponent (ideally plastic material) using the 'Sliding Node Concept' of Barsoum.[5]

'Sliding Nodes' are simply achieved by collapsing one side of an element and surrounding the crack tip with these elements, so that the crack tip has multiple independent nodes at one physical location which slide with respect to each other during deformation, due to loading. The proper order for plastic singularity (i.e., $1/r$) is achieved by locating the adjacent midside nodes at 1/9th and 4/9th of the length of the side of the element, as done for the elastic element.

After a brief review of the theory proving the existence of crack tip singularities, we discuss the implementation of these elements in NASTRAN as user dummy elements. The results of the analysis are compared to a Prandtl slip-line field solution.

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[5]Barsoum, R. S., Triangular Quarter-Point Elements as Elastic and Perfectly-Plastic Crack Tip Element, Int. J. Num. Meth. Engrg., Vol. 11, 1977, p. 85.

Many general purpose finite element codes as well as advanced versions of NASTRAN may have these elements. Hence, the method may be quite accessible to many users.

CRACK TIP SINGULARITIES

Consider the path integral J developed by Rice[3,6],

$$J = \int_{\Gamma} (Wdy - \bar{\tau} \cdot \frac{\partial \bar{u}}{\partial x} ds) \tag{1}$$

where W is the strain energy density, $\bar{\tau}$ and $\bar{u}$ traction and displacement vectors on the path Γ. Using a circular path of radius r surrounding a crack tip (1) reduces to,

$$J = r \int_{-\pi}^{\pi} \{W \cos\Theta - \bar{\tau} \cdot \frac{\partial \bar{u}}{\partial x}\} d\Theta \tag{2}$$

The terms in {...} in above are of the form:

(stress)(strain),

hence for the nonvanishing contribution to J (which is identical to energy release rate for the elastic case), we have

$$\sigma_{ij} \, \varepsilon_{ij} = 0(\frac{1}{r}) \text{as } r \to 0 . \tag{3}$$

Equation (3) is quite familiar for the elastic case for which stress and strain each have a singularity of the order of one half at the crack tip.

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[3]Rice, J. R., A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks, Trans. Am. Soc. Mech. Engrs., Journal of Applied Mechanics, 1968, p. 379.

[6]Rice, J. R. and Rosengren, G. F., Plane Strain Deformation Near a Crack Tip in a Power-Law Hardening Material, J. Mech. Phys. Solids, Vol. 16, p. 1.

Now consider the Ramberg-Osgood type of material given by

$$\tau = G\gamma = \frac{\tau_o}{\gamma_o}\gamma \ , \quad \text{for } \gamma \leq \gamma_o \tag{4}$$

$$\tau = \tau_o\left(\frac{\gamma}{\gamma_o}\right)^n \ , \quad \text{for } \gamma \geq \gamma_o \tag{5}$$

where $\tau = \sqrt{1/2 S_{ij} S_{ij}}$, $\gamma = \sqrt{2 e_{ij} e_{ij}}$, and τ_o, γ_o are yield stress and strain in shear and n is the hardening exponent. From (4), (5) and (3), outside the elastic range, we have

$$\begin{aligned} \sigma_{ij} &= 0\left(r^{\frac{-n}{n+1}}\right) \\ \varepsilon_{ij} &= 0\left(r^{-\frac{1}{1+n}}\right) \end{aligned} \tag{6}$$

From (6) we have the familiar elastic case for n = 1. However when n = 0, which is the case of ideally plastic material, we have from (6)

$$\begin{aligned} \sigma_{ij} &= 0(r^0) \\ \varepsilon_{ij} &= 0(r^{-1}) \end{aligned} \tag{7}$$

indicating a singularity of order one for the strains.

The existence of such singularities for quadratic elements have been given in ref. 5,7 . In the next section we briefly outline the case of the cubic elements.

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[5]Barsoum, R. S., Triangular Quarter-Point Elements as Elastic and Perfectly-Plastic Crack Tip Element, Int. J. Num. Meth. Engrg., Vol. 11, 1977, p. 85.

[7]Hussain, M. A., Lorensen, W. E., and Pflegl, G., The Quarter-Point Quadratic Isoparametric Element as a Singular Element for Crack Problems, NASTRAN Users' Experiences, NASA TM-X-3428, Oct. 1976, p. 419.

SINGULARITIES OF CUBIC ELEMENTS

Following the notation of ref. 8, the geometry of a 12-point cubic element is mapped into a normalized square in (ξ,η) plane $(-1 \le \xi \le 1, -1 \le \eta \le 1)$ through the transformation

$$x = \sum_{i=1}^{12} N_i(\xi,\eta)x_i \ ,$$
$$y = \sum_{i=1}^{12} N_i(\xi,\eta)y_i \ , \quad (8)$$

where the shape function is given by

$$\begin{aligned} N_i = \frac{1}{256}(1+\xi\xi_i)(1+\eta\eta_i)[-10+9(\xi^2+\eta^2)][-10+9(\xi_i^2+\eta_i^2)] \\ + \frac{81}{256}(1+\xi\xi_i)(1+9\eta\eta_i)(1-\eta^2)(1-\eta_i^2) \\ + \frac{81}{256}(1+\eta\eta_i)(1+9\xi\xi_i)(1-\xi^2)(1-\xi_i^2) \ , \end{aligned} \quad (9)$$

(x_i, y_i and ξ_i, η_i are the grid points.)

Collapsing the quadrilateral element as shown in Figure 1 and placing the midside nodes at 1/9th and 4/9th location, we have

$$x_1 = x_{10} = x_{11} = x_{12} = 0, \quad x_2 = x_9 = h/9, \quad x_3 = x_8 = 4h/9 \ ,$$

$$x_4 = x_5 = x_6 = x_7 = h \ ,$$

$$y_1 = y_{10} = y_{11} = y_{12} = 0, \quad y_2 = -y_9 = -\ell/9, \quad y_3 = -y_8 = -4\ell/9$$

$$y_4 = -y_7 = -\ell, \quad y_5 = -y_6 = -\ell/3 \quad (10)$$

[8] Zienkiewicz, C.O., The Finite Element Method in Engineering Science, McGraw Hill, London, 1971.

Substituting (10) into equation (8) we have

$$x = \frac{h}{4}(1 + \xi)^2$$

$$y = \frac{\ell\eta}{4}(1 + \xi)^2 \qquad (11)$$

Any point at a radial distance, $r = (x^2 + y^2)^{1/2}$, from the crack tip is given by

$$r = \frac{\ell}{4}(1 + \xi)^2 [(\frac{h}{\ell})^2 + \eta^2]^{1/2}$$

or

$$(1 + \xi) = \sqrt{r}\, \frac{1}{\{\frac{\ell}{4}[(\frac{h}{\ell})^2 + \eta^2]^{1/2}\}^{1/2}} \qquad (12)$$

The Jacobian [J] is given by

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{h}{2}(1 + \xi) & \frac{\ell}{2}\eta(1 + \xi) \\ 0 & \frac{\ell}{4}(1 + \xi)^2 \end{bmatrix} \qquad (13)$$

and the determinant is

$$\det |J| = \frac{h\ell}{8}(1 + \xi)^3 \qquad (14)$$

For the inverse functions, we have

$$[J]^{-1} = \begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\ \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{2}{h(1 + \xi)} & \frac{-4\eta}{h(1 + \xi)^2} \\ 0 & \frac{4}{\ell(1 + \xi)^2} \end{bmatrix} \qquad (15)$$

The displacement components of the point (ξ,η) for an isoparametric transformation are,

$$u = \sum_{i=1}^{12} N_i(\xi,\eta)u_i$$

$$v = \sum_{i=1}^{12} N_i(\xi,\eta)v_i \qquad (16)$$

The derivatives of u, v with respect to ξ, η are

$$\frac{\partial u}{\partial \xi} = \sum_{i=1}^{12} \frac{\partial N_i}{\partial \xi} u_i \quad , \quad \frac{\partial u}{\partial \eta} = \sum_{i=1}^{12} \frac{\partial N_i}{\partial \eta} u_i$$

$$\frac{\partial v}{\partial \xi} = \sum_{i=1}^{12} \frac{\partial N_i}{\partial \xi} v_i \quad , \quad \frac{\partial v}{\partial \eta} = \sum_{i=1}^{12} \frac{\partial N_i}{\partial \eta} v_i \qquad (17)$$

where

$$\frac{\partial N_i}{\partial \xi} = \frac{1}{256}(1 + \eta\eta_i)[-10 + 9(\xi_i^2 + \eta_i^2)](-10\xi_i + 9\xi_i\eta^2 + 18\xi + 27\xi_i\xi^2)$$
$$+ \frac{81}{256}(1 - \eta_i^2)\xi_i(1 + 9\eta\eta_i)(1 - \eta^2)$$
$$+ \frac{81}{256}(1 + \eta\eta_i)(1 - \xi_i^2)(9\xi_i - 2\xi - 27\xi_i\xi^2) \qquad (18)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{1}{256}(1 + \xi\xi_i)[-10 + 9(\xi_i^2 + \eta_i^2)](-10\eta_i + 9\eta_i\xi^2 + 18\eta + 27\eta_i\eta^2)$$
$$+ \frac{81}{256}(1 + \xi\xi_i)(1 - \eta_i^2)(9\eta_i - 2\eta - 27\eta_i\eta^2)$$
$$+ \frac{81}{256}(1 - \xi_i^2)\eta_i(1 + 9\xi\xi_i)(1 - \xi^2)$$

Substituting for nodal values and collecting terms, using MACSYMA,[9] equations (17) become

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[9]MACSYMA: Math Lab Group, MIT Laboratory for Computer Science (Symbolic Manipulation System), November 1975.

$$\frac{\partial u}{\partial \xi} = a_0 + a_1(1 + \xi) + a_2(1 + \xi)^2$$

$$\frac{\partial u}{\partial \eta} = b_0 + b_1(1 + \xi) + b_2(1 + \xi)^2 + b_3(1 + \xi)^3 \qquad (19)$$

where

$$a_0 = \frac{1}{32} [9(-3u_{12} + 3u_{11} - u_{10} + u_7 - 3u_6 + 3u_5 - u_4 + u_1)\eta^3$$
$$+ 9(u_{12} + u_{11} - u_{10} + u_7 - u_6 - u_5 + u_4 - u_1)\eta^2$$
$$+ (27u_{12} - 27u_{11} - 35u_{10} + 9u_9 - 9u_8 - u_7$$
$$+ 27u_6 - 27u_5 + u_4 + 9u_3 - 9u_2 + 35u_1)\eta$$
$$+ (-9u_{12} - 9u_{11} - 35u_{10} + 9u_9 - 9u_8 - u_7 + 9u_6$$
$$+ 9u_5 - u_4 - 9u_3 + 9u_2 - 35u_1)] \qquad (20)$$

$$a_1 = \frac{9}{8} [(2u_{10} - 5u_9 + 4u_8 - u_7)(1 + \eta) - (u_4 - 4u_3 + 5u_2 - 2u_1)(1 - \eta)]$$

$$a_2 = -\frac{27}{32} [(u_{10} - 3u_9 + 3u_8 - u_7)(1 + \eta) - (u_4 - 3u_3 + 3u_2 - u_1)(1 - \eta)]$$

$$b_0 = \frac{1}{16} [27(3u_{12} - 3u_{11} + u_{10} - u_1)\eta^2 - 18(u_{12} + u_{11} - u_{10} - u_1)\eta$$
$$+ u_1 - 27u_{12} + 27u_{11} - u_{10}] \qquad (21)$$

$$b_1 = -\frac{1}{32} [27(3u_{12} - 3u_{11} + u_{10} - u_7 + 3u_6 - 3u_5 + u_4 - u_1)\eta^3$$
$$- 18(u_{12} + u_{11} - u_{10} + u_7 - u_6 - u_5 + u_4 - u_1)\eta^2$$
$$+ (-27u_{12} + 27u_{11} + 35u_{10} - 72u_9 + 36u_8 + u_7$$
$$- 27u_6 + 27u_5 - u_4 - 36u_3 + 72u_2 - 35u_1)] \qquad (22)$$

$$b_2 = \frac{9}{16}(2u_{10} - 5u_9 + 4u_8 - u_7 + u_4 - 4u_3 + 5u_2 - 2u_1)$$

$$b_3 = -\frac{9}{32}(u_{10} - 3u_9 + 3u_8 - u_7 + u_4 - 3u_3 + 3u_2 - u_1)$$

The derivatives $\partial v/\partial\xi$, $\partial v/\partial\eta$ are the same except for replacing u_i by v_i. The derivatives of u with respect to x,y are obtained from

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x}$$

$$= -\frac{4\eta b_0}{h(1+\xi)^2} + \frac{2a_0 - 4\eta b_1}{h(1+\xi)} + \frac{1}{h}(2a_1 - 4\eta b_2) + \frac{1}{h}(1+\xi)(2a_2 - 4\eta b_3) \tag{23}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}$$

$$= \frac{4b_0}{\ell(1+\xi)^2} + \frac{4b_1}{\ell(1+\xi)} + \frac{4b_2}{\ell} + \frac{4b_3}{\ell}(1+\xi) \tag{24}$$

Similar expressions are obtained for $\partial v/\partial x$ and $\partial v/\partial y$ with u_i replaced by v_i in a's and b's.

The stresses and strains are singular when the Jacobian determinant vanishes at $\xi = -1$. From (23), (24) and (12), the singularity is $O(1/r)$ if $b_0 \neq 0$ and is $O(1/\sqrt{r})$ if $b_0 = 0$. A careful study of (21) indicates that b_0 depends on the displacements of nodal points at the crack tip. If the nodal points at the crack tip are tied together, i.e.,

$$u_1 = u_{10} = u_{11} = u_{12} \quad \text{and} \quad v_1 = v_{10} = v_{11} = v_{12} \tag{25}$$

then $b_0 = 0$ and the strain field has the inverse square root of r singularity, the correct singularity of linear fracture mechanics. On the other hand if the nodal points at the crack tip are allowed to move independently to one another, the strain field has the $(1/r)$ singularity, a characteristic of perfect plasticity.

NASTRAN IMPLEMENTATION

The NASTRAN implementation for the quadratic element follows the steps outlined in section 6.8.3.12 of reference 12. The following routines require modification: PLA1, which creates the ECPT's and EST's for the linear and non-linear elements; PLA31 and PLA32, which recover stresses for the non-linear elements; and PLAYBD, PLA41 and PLA42 which control generation of the updated stiffness matrix. The following new routines are required: PSDUM1, a driver for stress data recovery in PLA3; PSDM11 and PSDM12, phase I and II stress recovery routines; PKDUM1, a driver for stiffness generation for the non-linear elements; PKDM11 and PKDM12, stress recovery routines which generate stresses for the computation of the non-linear material matrix; and PKDM1S, the stiffness matrix generation routine for non-linear elements. The two driver routines, PSDUM1 and PKDUM1 can be modelled after the corresponding routines for the QUAD1 element. The remaining routines are modifications of the stiffness and stress recovery routines[7] required for rigid format 1, statics. The major modifications to switch from statics to piecewise linear include changing the labelled common areas, building the non-linear material matrix[10] and calculating incremental stress rather than total stresses.

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[7] Hussain, M. A., Lorensen, W. E., and Pflegl, G., The Quarter-Point Quadratic Isoparametric Element as a Singular Element for Crack Problems, NASTRAN Users' Experiences, NASA TM-X-3428, Oct. 1976, p. 419.

[10] The NASTRAN Theoretical Manual, Editor, MacNeal, R. H., NASA SP-221, Sep. 1970, p. 104.

[12] The NASTRAN Programmer's Manual, NASA SP-223(01), Sep. 1972.

NUMERICAL EXAMPLE

Consider the problem of small scale yielding. The problem is governed by the elastic field at points far away from the crack tip and asymptotically has the elastic singular field. Near the crack tip we have the plastic zone. This is schematically represented in Figure 2. The plane strain slip line field is also shown.

The problem is modelled in a fashion similar to Barsoum's[5]. The geometry is shown in Figure 3. The crack tip elements, 1-12, are the singular elements which can either be quadratic or cubic elements. For the symmetric case the corner nodes of the elements are placed on concentric semi-circles, $0 \leq \Theta \leq \pi$, at $\pi/12$ intervals, of radii, r = 0, .5, 1.0, 1.625, 1.5^2, 2^2, 2.5^2, 3^2, 4^2, 5.5^2.

The method of solution, for the plastic problem, is based on Swedlow's piece-wise linear analysis and is well documented in the NASTRAN theoretical manual.[10]

The procedure for the present problem is accomplished via two rigid formats. The static rigid format is first used to obtain the stress distribution and the equivalent stresses at the integration point ($\xi = \eta = 0$) for the elastic increment. This solution is performed with all the collapsed nodes at the crack tip having the same displacement

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[5]Barsoum, R. S., Triangular Quarter-Point Elements as Elastic and Perfectly-Plastic Crack Tip Element, Int. J. Num. Meth. Engrg., Vol. 11, 1977, p. 85.

[10]The NASTRAN Theoretical Manual, Editor, MacNeal, R. H., NASA SP-221, Sept. 1970, p. 104.

vector (see equation 25). This is accomplished with multipoint constraints. The outermost nodes are subjected to the displacements governed by Westergaard's solution, with K = 1,

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{K}{2G} \left(\frac{r}{2\pi}\right)^{1/2} \left(\frac{3-\nu}{1+\nu} - \cos\Theta\right) \begin{bmatrix} \cos\Theta/2 \\ \sin\Theta/2 \end{bmatrix} \quad (26)$$

where $E = 30 \times 10^6$ psi and $\nu = .3$. The value of $2K_0$ is established from the elastic solution based on the yield stress (σ_0) of 20×10^3 psi for the highest stressed element. For the plastic analysis the stress-strain curve is provided with the above constants and yield strain at .2% and hardening exponent $n \approx .3$ (this should be close to zero for perfect plasticity). The nodes at the crack tip are then released for sliding in order to obtain 1/r singularity at the crack tip. The load is incremented by $K_0/4$ till the plastic zone has reached the first layer of elements.

Preliminary results of the problem are indicated in Figures 4a-c and compared with those of ref. 11 . From the static solution it was found that the inception of yielding occurs at $\Theta = 68°$ compared to the theoretical value of $\Theta = 70°$.

In Figures 4a-c we have also plotted the slip line (plane strain) solution for comparison. The plastic zone also corresponds well with ref. 11.

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[11] Hutchinson, J. W., Singular Behavior at the End of a Tensile Crack in a Hardening Material, J. Mech. Phys. Solids, 1968, Vol. 16, p. 13.

CONCLUSION

Higher order isoparametric elements can be effectively used for modelling singular elastic as well as plastic problems that arise in the field of fracture mechanics. The procedure in obtaining these do not require any special crack tip elements but are simply constructed by adjusting the adjacent nodes at proper locations and proper constraints. The locations of these nodes should be adhered to as closely as possible for stable answers. Since many general purpose finite elements may have these elements in their library the method, for crack problems, may be accessible to many users.

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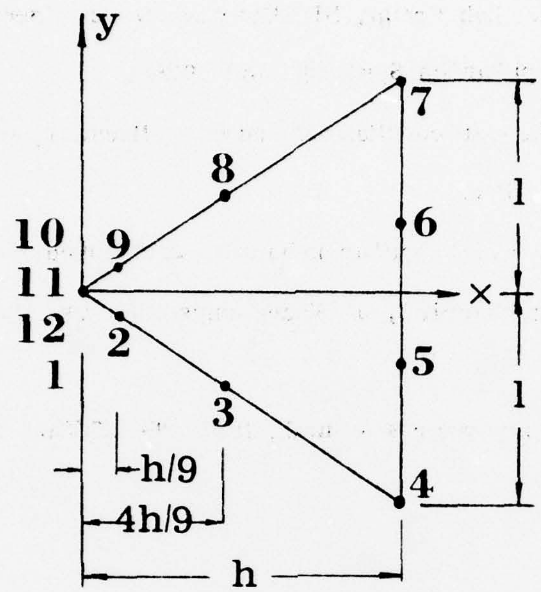
12. The NASTRAN Programmer's Manual, NASA SP-223(01), Sept. 1972.

(a)



(b)

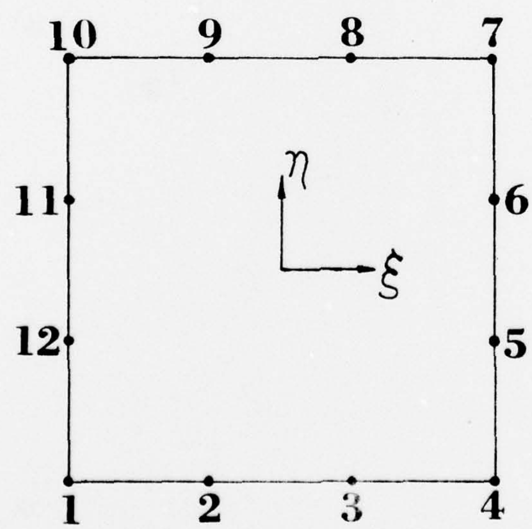


FIGURE 1. (a) 12-NODE CUBIC ELEMENT COLLAPSED TO FORM A SINGULAR ELEMENT; (b) THE PARENT ELEMENT.

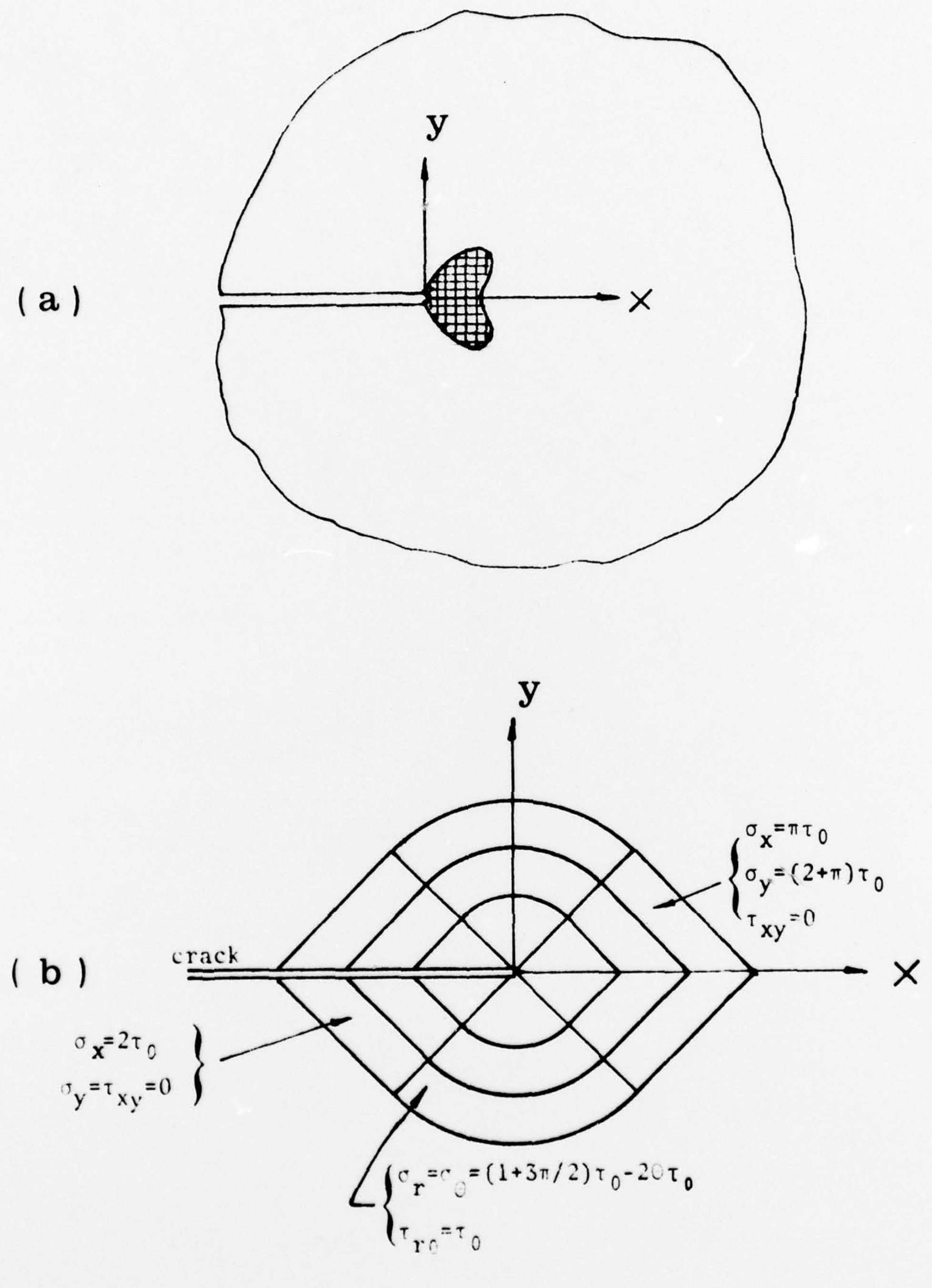


FIGURE 2. (a) SMALL-SCALE YIELDING NEAR A SEMI-INFINITE CRACK;
(b) PERFECTLY PLASTIC, PLANE STRAIN, SLIP-LINE FIELD AT THE CRACK TIP.

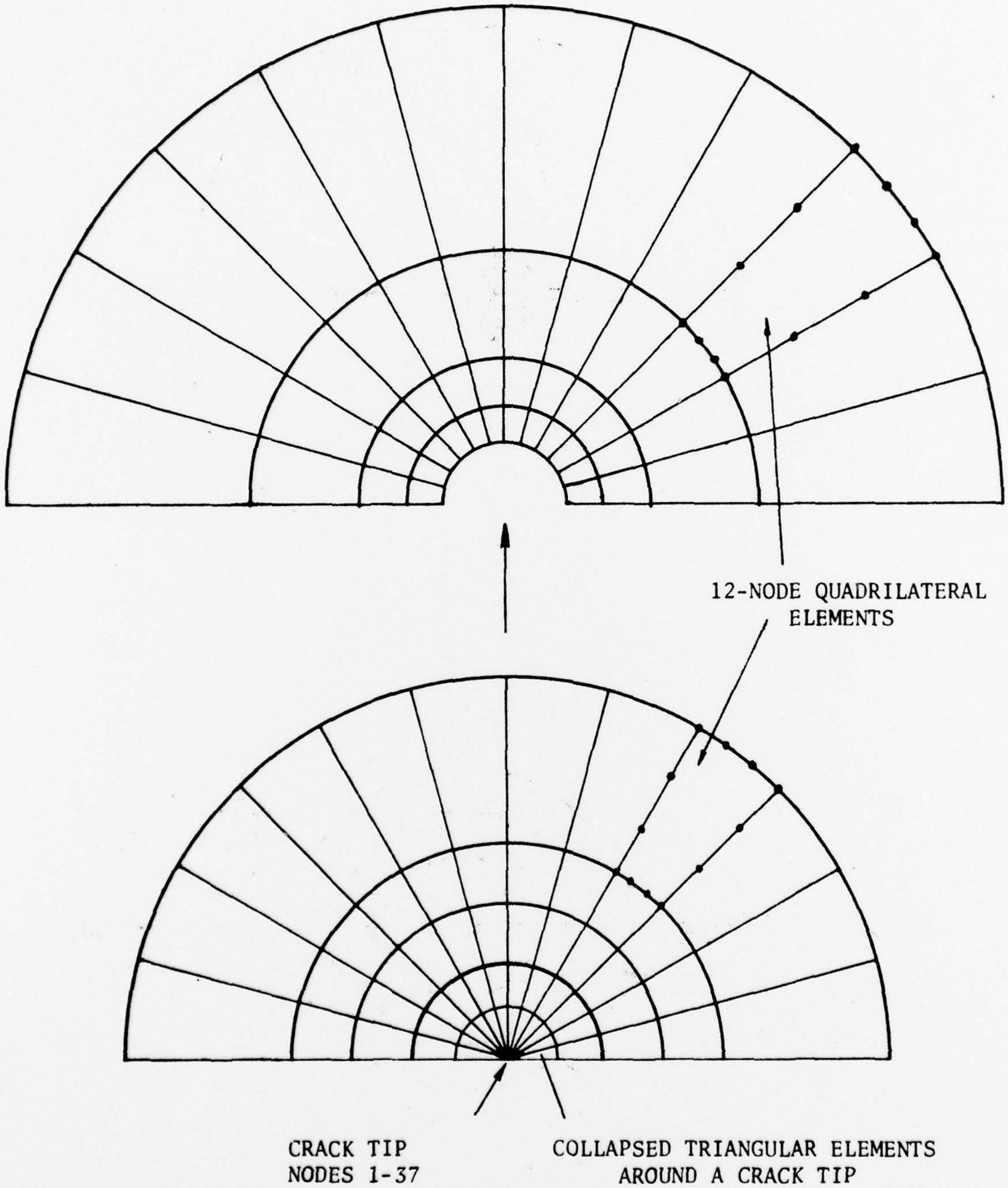


FIGURE 3. FINITE ELEMENT IDEALIZATION OF THE CRACK TIP NEAR FIELD.

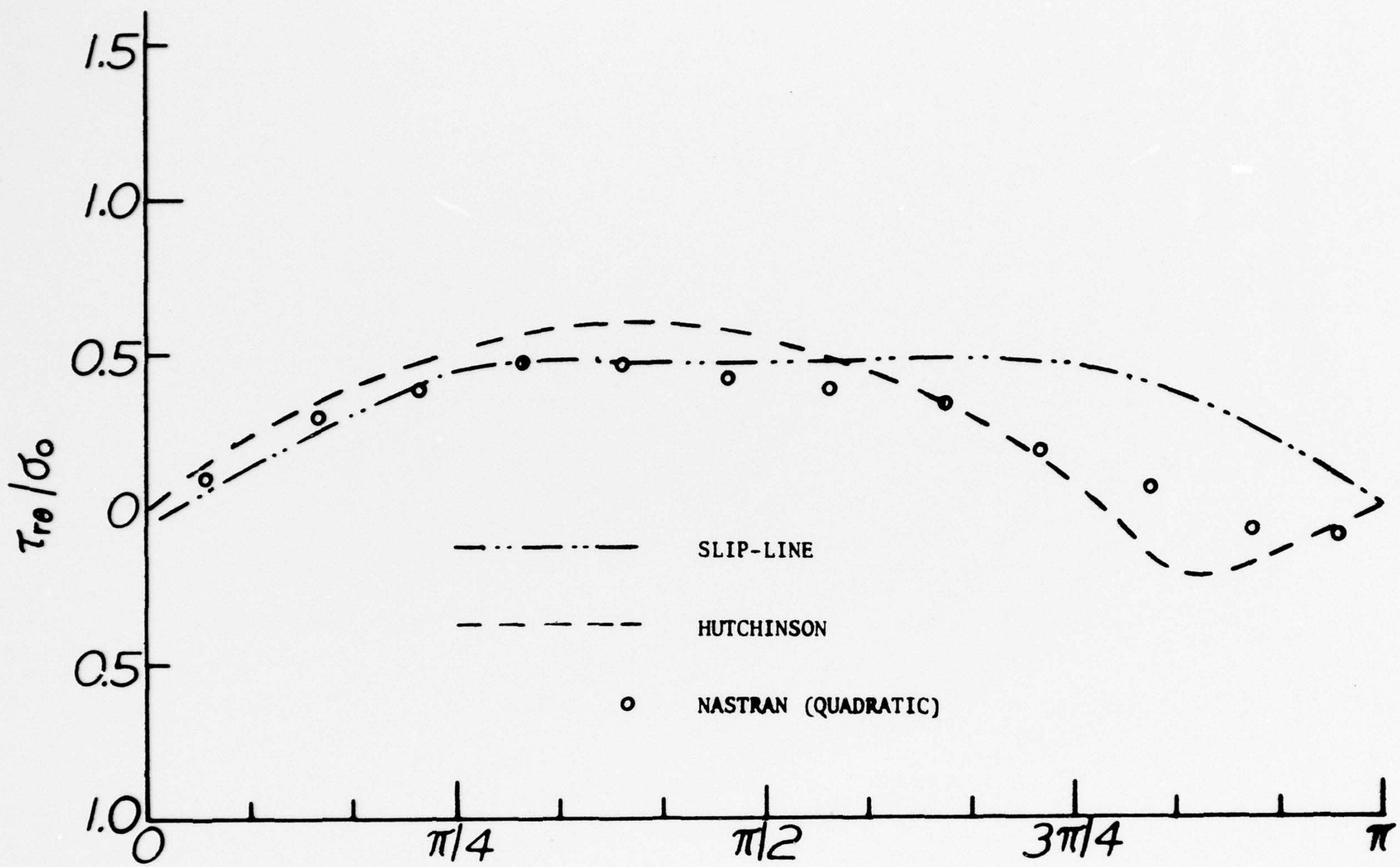


FIGURE 4. (a) SHEARING STRESS DISTRIBUTION FOR THE SINGULAR ELEMENTS COMPARED TO REF. (11) AND A SLIP-LINE SOLUTION (PRELIMINARY RESULTS).

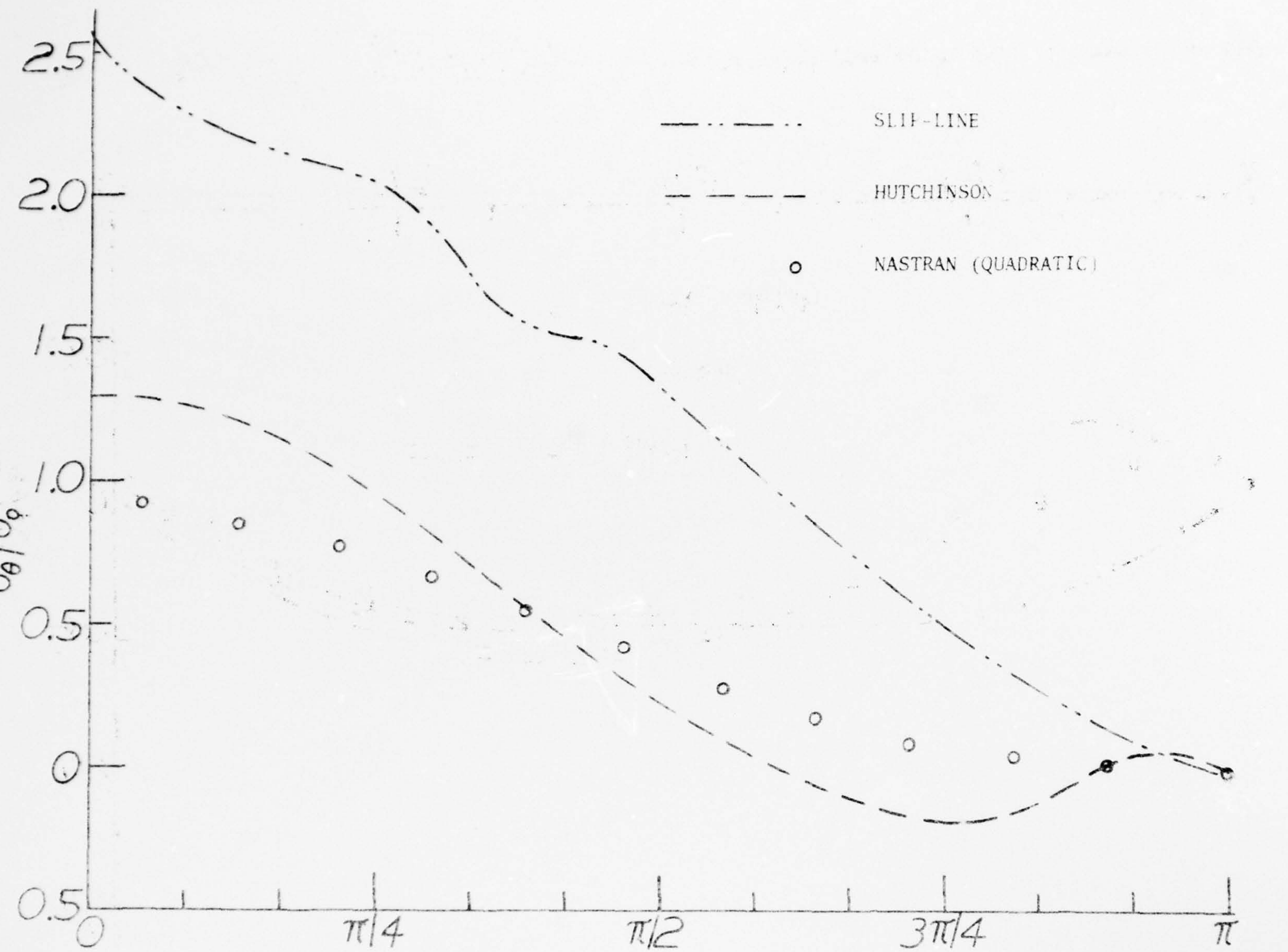


FIGURE 4. (b) TANGENTIAL STRESS DISTRIBUTION FOR THE SINGULAR ELEMENTS COMPARED TO REF. (11) AND A SLIP-LINE SOLUTION (PRELIMINARY RESULTS).

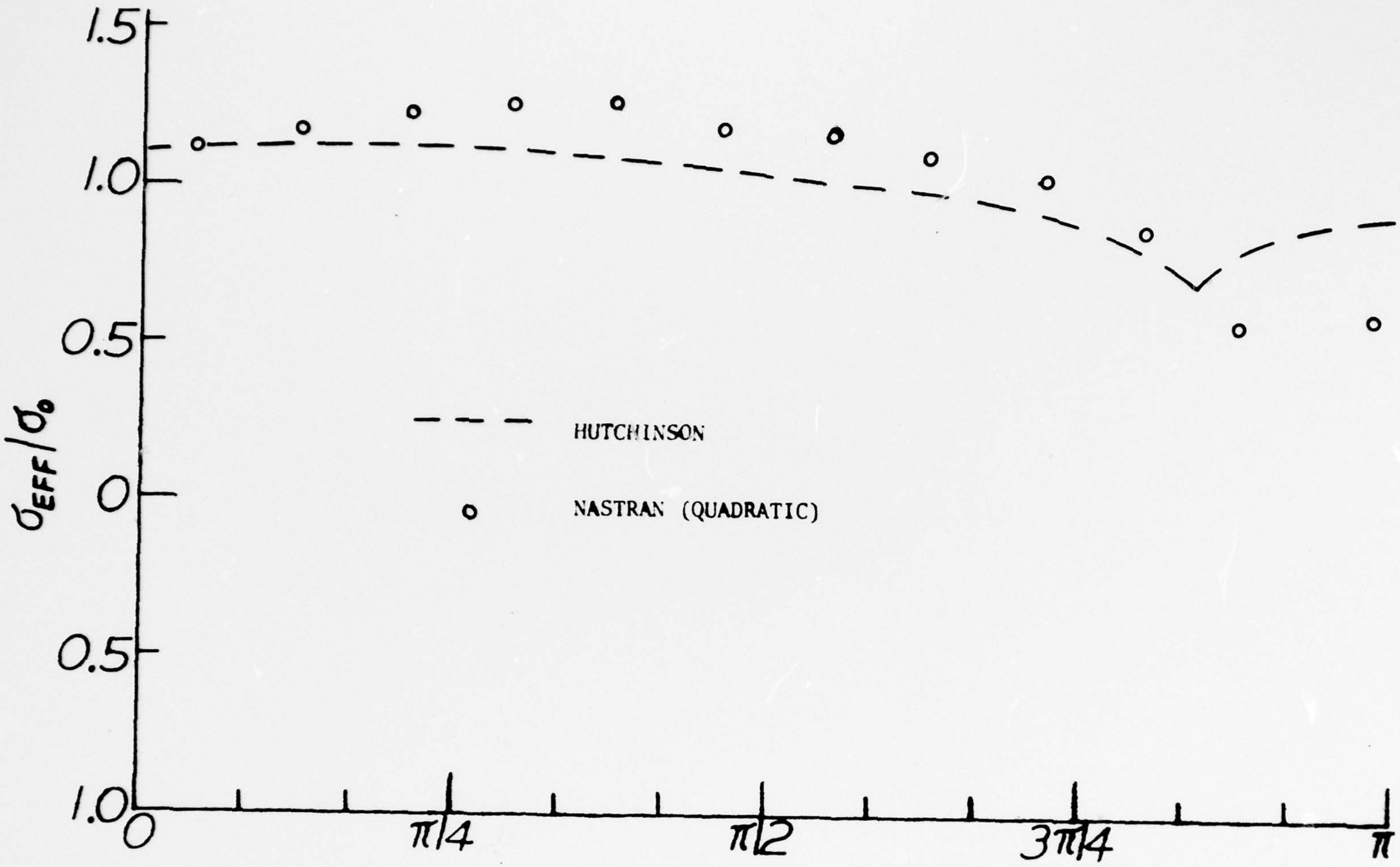


FIGURE 4. (c) EFFECTIVE STRESS DISTRIBUTION FOR THE SINGULAR ELEMENTS COMPARED TO REF. (11) AND A SLIP-LINE SOLUTION. (PRELIMINARY RESULTS)

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