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DIGITAL SIMULATION OF AN ANALOG FILTER.(U)
APR 66 W W ANDERSON, A S FIELDS, S S WOLFF
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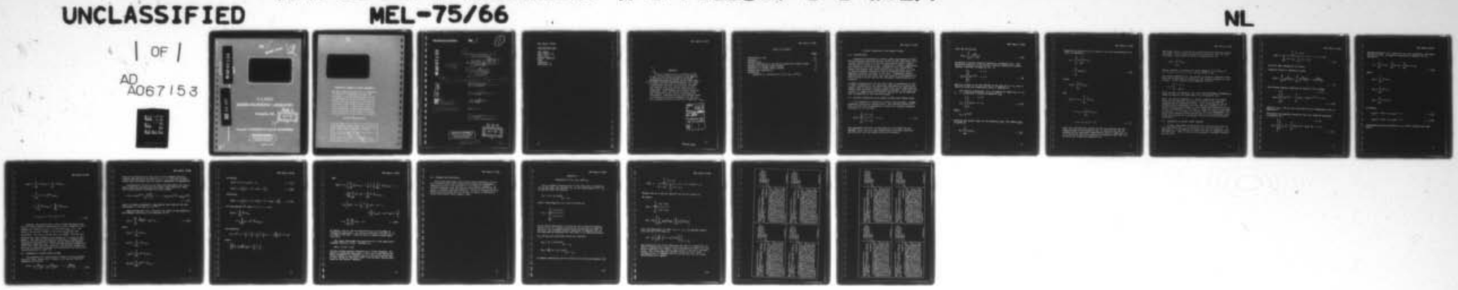

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ABSTRACT

Digital simulation of an analog filter is one of the signal processing requirements of "Shipboard ASW Magnetometer Systems", (BUSHIPS Sub-project S-F101 03 16, Task 1522). A simple digital simulation method utilizing a recurrence relation to eliminate the need for summing over the infinite past of the input data in performing the convolution is developed. Both simple and multiple poles in the transform of the filter's equation are considered. Formulations of the residues used in the partial fraction expansion of the filter transfer function are given in Appendix A.

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DIGITAL SIMULATION OF AN ANALOG FILTER

1.0 INTRODUCTION

Shipboard antisubmarine warfare (ASW) magnetometer systems, being developed under Sub-project S-F101 03 16, Task 1522, will utilize analog filters of various band passes wherein signal information is known to exist. Frequently, however, experiments are conducted with analog data processing over considerably larger band passes than those operationally desirable in order to acquire maximum information and to cooperate with other agencies having an interest in the data. Subsequent digital computer analysis must take this into account by processing the data through a filter having the band-pass characteristics of interest.

A common characteristic of the above type of processing is that it must correspond to the operations performed by a relatively simple class of analog filters, those which can be described as linear time-invariant, lumped-parameter systems. With this constraint, a simple method of digital simulation can be employed and is described on the following pages.

2.0 DIGITAL SIMULATION OF AN ANALOG FILTER WITH SIMPLE POLES

It is desired to simulate a linear time-invariant, lumped-parameter filter with simple poles. The transfer function of such a filter may be expressed in terms of a Laplace transform variable, s, in the form

$$H(s) = \frac{\prod_{i=1}^{r} (s + b_i)}{\prod_{j=1}^{p} (s + c_j)} \; ; \; r < p \qquad \text{.....(1)}$$

The reasonable criteria of realizability and stability are assumed; thus, the c_j's are constrained to be real, positive, and distinct (as only simple poles are allowed).

H(s) can be written

$$H(s) = \sum_{i=1}^{p} \frac{K_i}{(s + c_i)} \qquad .....(2)$$

by making a partial fraction expansion of Equation (1). The weighting function of the filter, found by taking the inverse Laplace transform of Equation (2), is

$$h(t) = \begin{cases} 0 & , t < 0 \\ \sum_{i=1}^{p} K_i e^{-c_i t} & , t \geq 0. \end{cases} \qquad .....(3)$$

Each K_i is seen to be the residue of the pole at $s = -c_i$ and is found by using the formulation given in Appendix A.

For digital simulation, h(t) is sampled at times $n\Delta t$, $n = 0, 1, ...,$ producing a weighting sequence $\{h_n\}$

$$h_n = \begin{cases} 0 & , n < 0 \\ \sum_{i=1}^{p} K_i r_i^n & , n \geq 0, \end{cases} \qquad .....(4)$$

where

$$r_i = e^{-c_i \Delta t}$$

Denoting the filter input by the sequence $\{x_n\}$, the output, $\{y_n\}$, is given by

$$y_t = \sum_{n=0}^{\infty} h_n \, x_{t-n} \qquad .....(5)$$

Substituting Equation (4) in Equation (5) and interchanging the order of summation:

$$y_t = \sum_{i=1}^{p} K_i \sum_{n=0}^{\infty} r_i^n x_{t-n}$$

$$= \sum_{i=1}^{p} K_i z_i(t), \qquad .....(6)$$

where

$$z_i(t) = \sum_{n=0}^{\infty} r_i^n x_{t-n},$$

and

$$z_i(t) = x_t + \sum_{n=1}^{\infty} r_i^n x_{t-n}$$

$$= x_t + r_i \sum_{n=0}^{\infty} r_i^n x_{t-n-1}$$

$$= x_t + r_i z_i (t - 1) \qquad .....(7)$$

Thus, the calculations implied by the convolution over the infinite past of the sequence $\{x_t\}$ in Equation (5) can be reduced to the recurrence relationship given in Equation (7) y_t is found by using the summation of Equation (6), the z_i's of which are found with Equation (7)

The filter must, of course, be initialized with starting values, $\{z_i(0)\}_1^p$. If the input sequence were available into the infinite past, these would be given via Equation (7) as

$$z_i(0) = \sum_{n=0}^{\infty} r_i^n \, x_{-n} \; .$$

Since, however, we assume the input sequence $\{x_t\}$ begins at $t = 0$ (i.e., $x_t = 0,\ t < 0$), the $\{z_i(0)\}_1^p$ is x_0.

The output sequence will now contain a starting transient whose form may be derived by considering the impulse response of the filter. A unit impulse input sequence would be

$$x_t = \begin{cases} 1 \; , & t = 0 \\ 0 \; , & \text{else,} \end{cases}$$

which yields, via Equation (6), the output sequence whereupon y_t takes on exactly the same form as h_n in Equation (4).

Thus, the output sequence, y_t, may be treated as the steady-state filter response as soon as the terms of the impulse response become negligibly small. Such a calculation can be automatically programmed into the computer together with an error criteria and a number of time increments derived for each filter corresponding to the time it takes for filter transients to die out. It is clear, of course, from this development that the filter transient difficulties are in no way different from those encountered with simulated filtering by straightforward convolution.

3.0 EXTENSION TO SECOND ORDER FILTERS

Project requirements dictated that attention be given to the case where the c_j's of Equation (1) are not distinct. In particular, it was desired to study the case of repeated roots of Order two. Such a filter would possess a transfer function of the form

$$H(s) = \frac{\prod_{i=1}^{r} (s + b_i)}{\prod_{j=1}^{p} (s + c_j) \cdot \prod_{\ell=1}^{q} (s + c_\ell)^2} \; ; \; r < p + 2q, \quad \text{.....(8)}$$

with the same assumption as before.

A partial fraction expansion yields

$$H(s) = \sum_{j=1}^{p} \frac{K_j}{(s + c_j)} + \sum_{\ell=1}^{q} \frac{M_{o\ell}}{(s + c_\ell)^2} + \frac{M_{1\ell}}{(s + c_\ell)} . \quad \text{..... (9)}$$

The inverse Laplace transform of Equation (10) yields

$$h(t) = \begin{cases} 0 & , t < 0 \\ \sum_{j=1}^{p} K_j e^{-c_j t} + \sum_{\ell=1}^{q} (M_{o\ell} t + M_{1\ell}) \, e^{-c_\ell t}, & t \geq 0, \end{cases} \quad \text{.....(10)}$$

where K_j, $M_{o\ell}$, and $M_{1\ell}$ are calculable by the formations given in Appendix A.

Once again for sampled systems we have the equation analogous to Equation (4)

$$h_n = \begin{cases} 0 & , n < 0 \\ \sum_{j=1}^{p} K_j \, r_j^n + \sum_{\ell=1}^{q} (M_{o\ell} \, n\Delta t + M_{1\ell}) \, r_\ell^n & , n \geq 0. \end{cases} \quad \text{.....(11)}$$

Applying Equation (5), reversing the order summation, and breaking up the sums, an equation analogous to Equation (6) is arrived at

$$y_t = \sum_{j=1}^{p} K_j \, z_{sj} + \sum_{\ell=1}^{q} M_{1\ell} \, z_{o\ell} + \sum_{\ell=1}^{q} M_{o\ell} \, \Delta t \, z_{1\ell}, \quad \text{.....(12)}$$

where

$$z_{sj} = \sum_{n=0}^{\infty} r_j^n \, x_{t-n}$$

$$z_{o\ell} = \sum_{n=0}^{\infty} r_\ell^n \, x_{t-n}$$

$$z_{1\ell} = \sum_{n=0}^{\infty} n \, r_\ell^n \, x_{t-n}$$

As before,

$$z_{sj}(t) = x(t) + r_j \, z_{sj}(t - 1) \quad \text{.....(13)}$$

$$z_{o\ell}(t) = x(t) + r_\ell \, z_{o\ell}(t - 1) \quad \text{.....(14)}$$

and applying similar analysis to $z_{1\ell}$ which contains the time term,

$$z_{1\ell}(t) = \sum_{n=0}^{\infty} n\, r_\ell^n\, x_{t-n} = \sum_{n=1} n\, r_\ell^n\, x_{t-n}$$

$$= \sum_{n=0}^{\infty} (n + 1)\, r_\ell^{n+1}\, x_{t-n-1}$$

$$= r_\ell \sum_{n=0}^{\infty} r_\ell^n\, x_{t-n-1} + \sum_{n=0}^{\infty} n\, r_\ell^n\, x_{t-n-1}$$

$$= r_\ell\, z_{o\ell}\, (t - 1) + z_{1\ell}\, (t - 1) \qquad .....(15)$$

Clearly, the calculation of the z's must be done in the order of their numerical subscripts as $z_{1\ell}(t)$ (Equation (15)) depends not only upon $z_{1\ell}(t - 1)$ but also $z_{o\ell}(t - 1)$. As before, the initial values of the sets, $\{z_{sj}(0)\}_1^p$ and $\{z_{o\ell}(0)\}_1^q$, will be x_o and the set, $\{z_{1\ell}(0)\}_1^q$, will be zero, assuming the input sequence x(t) begins at t = 0 and is zero before. The time for the starting transient to become negligible may be calculated as before, now by using the impulse response of Equation (10). Thus the formulations of Equations (12), (13), (14), and (15) along with the initial values are all that is required to perform the simulation of a filter containing poles up to order two.

4.0 EXTENSION TO HIGHER ORDER SYSTEMS

The presence of a single pole of order m in the transfer function, H(s), gives rise to terms in the partial fraction expansion as follows.

$$H'(s) = \frac{M_o}{(s + c_j)^m} + \frac{M_1}{(s + c_j)^{m-1}} + \ldots . \frac{M_{m-1}}{(s + c_j)} \qquad .....(16)$$

where H'(s) denotes only that part of the transfer function arising from the pole of order m at $s = -c_j$, and the M coefficients are calculable from the general expression in Appendix A.

Consideration of only H'(s) shows the form which the recurrence relations arising from the higher order pole will take. The inverse Laplace transform of H'(s) yields:

$$h'(t) = M_o t^{(m-1)} \frac{e^{-c_j t}}{(m-1)!} + \ldots . + M_{m-2}\, t e^{-c_j t} + M_{m-1} e^{-c_j t}, \quad \ldots . .(17)$$

which is readily extended to the sampled data case by the substitutions $t = n\Delta t$ and $r_j = e^{-c_j \Delta t}$.

Applying Equation (5), inverting the order of the summation, and summing according to the exponent of $n\Delta t$

$$y(t) = \sum_{i=0}^{m-1} \frac{M_{m-1-i}}{i!} (\Delta t)^i z_i , \quad \ldots . .(18)$$

where

$$z_o(t) = \sum_{n=0}^{\infty} r_j^n x_{t-n}$$

$$z_1(t) = \sum_{n=0}^{\infty} n\, r_j^n x_{t-n}$$

$$z_2(t) = \sum_{n=0}^{\infty} (n)^2 r_j^n x_{t-n}$$

$$\vdots$$

$$z_{m-1}(t) = \sum_{n=0}^{\infty} (n)^{m-1} r_j^n x_{t-n}$$

As before,

$$z_o(t) = x_t + r_j\, z_o(t - 1) \qquad .....(19)$$

$$z_1(t) = r_j \left[z_o(t - 1) + z_1(t - 1)\right]. \qquad .....(20)$$

Similarily,

$$z_2(t) = r_j \left[z_o(t - 1) + 2z_1(t - 1) + z_2(t - 1)\right]. \qquad .....(21)$$

For the general k^{th} term, $k = 0, 1, ...m-1$,

$$z_k(t) = \sum_{n=1}^{\infty} r_j^n\, x_{t-n}$$

$$= r_j \sum_{n=0}^{\infty} (n + 1)^k\, r_j^n\, x_{t-n-1}\ ,$$

and recalling

$$(n + 1)^k = 1 + \binom{k}{k - 1} n + \binom{k}{k - 2} n^2 + ...+ \binom{k}{1} n^{k-1} + n^k$$

where

$$\binom{k}{r} \triangleq \frac{k!}{r!\,(k - r)!} = \binom{k}{k - r}\ .$$

then

$$z_k(t) = r_j \left[\sum_{n=0}^{\infty} r_j^n x_{t-n-1} + \binom{k}{k-1} \sum_{n=0}^{\infty} n\, r_j^n x_{t-n-1} + \ldots \right.$$

$$\left. + \binom{k}{2} \sum_{n=0}^{\infty} n^{k-1} r_j^n + \sum_{n=0}^{\infty} n^k r_j^n x_{t-n-1} \right]$$

$$= r_j \left[z_o(t-1) + \binom{k}{k-1} z_1(t-1) + \ldots \right.$$

$$\left. + \binom{k}{2} z_{k-1}(t-1) + z_k(t-1) \right]$$

$$= r_j \sum_{i=0}^{k} \binom{k}{i} z_i(t-1) \; . \qquad .....(22)$$

As before, the z's can be calculated only in the order of increasing subscripts. As initial conditions, the set of z's is taken to be zero. Settling time is computed using Equation (17).

The result developed here applies only to the restricted transfer function, $H'(s)$. However, as

$$H(s) = H'(s) + F(s)$$

and the inverse Laplace transform is a linear operator, the result is generally applicable. Thus, any filter having the form of Equation (1), wherein the c_j's are not distinct and, in fact, poles of arbitrary order m are allowed, may be simulated on the digital computer.

5.0 SUMMARY AND CONCLUSION

The method described above permits easy simulation of signal filtering when input signals are sampled and subsequent computations are to be carried out in a digital computer. It is only necessary that the equation describing the filters be known in Laplace transform notation. Errors introduced by this method are calculable and can be kept as small as desired by using sufficiently small sample times.

Appendix A

Formulation of K_j, $M_{o\ell}$, and $M_{1\ell}$

For a transfer function H(s) of the form given in Equation (1) of the text, the residue, K_j, of any simple pole at $s = -c_j$ is derived from the relation

$$K_j = (s + c_j)\, H(s) \Big|_{s = -c_j}$$

which, using Equation (1), can be written as

$$K_j = \frac{\prod_{i=1}^{r} (-c_j + b_i)}{\prod_{\substack{i=1 \\ i \neq j}}^{p} (-c_j + c_i)}$$

As the above formulation is valid for H(s) containing nondistinct c_i's (i.e., higher order poles) as long as c_j itself is distinct, the transfer function expressed in Equation (8) of the text may be evaluated for this case in an identical manner.

$M_{o\ell}$ and $M_{1\ell}$ are calculated using the relations

$$M_{o\ell} = (s + c_\ell)^2\, H(s) \Big|_{s = -c_\ell}$$

$$M_{o\ell} = \frac{d}{ds} (s + c_\ell)^2\, H(s) \Big|_{s = -c_\ell}$$

A clearer formulation can be arrived at by rewriting Equation (8)

$$H(s) = \frac{\prod_{i=1}^{r} (s + b_i)}{(s + c_\ell)^2 \prod_{i=1}^{m} (s + c_i)} \; ; \; m = p + 2(q - 1),$$

Wherein the c_i's are not distinct but do not contain c_ℓ

We obtain

$$M_{o\ell} = \frac{\prod_{i=1}^{r} (-c_\ell + b_i)}{\prod_{i=1}^{m} (-c_\ell + c_i)}$$

$$M_{1\ell} = M_{o\ell} \left[\sum_{i=1}^{r} \frac{1}{(-c_\ell + b_i)} - \sum_{i=1}^{m} \frac{1}{(-c_\ell + c_i)} \right]$$

For a multiple pole of order n at $s = -c_i$, the general expression for the j^{th} M term is:

$$M_j = \frac{1}{j!} \left\{ \frac{d^j}{ds^j} \left[(s + c_i)^n H(s) \right] \right\} \Bigg|_{s = -c_i}$$

the formulation for H'(s) (Equation 16) and h'(t) (Equation 17) shown in section 4.0 consist of only one pole of order m for the entire function H'(s); hence, only the M_o th term has a non-zero value. For a more general transfer function, H(s), which includes H'(s), a complete set of M as derived from the above relationship will appear.

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2. Magnetometer systems (shipboard)

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