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THE DELAY-DOPPLER SPECTRAL ANALYSIS OF NONSTATIONARY TIME VARIA--ETC(U)
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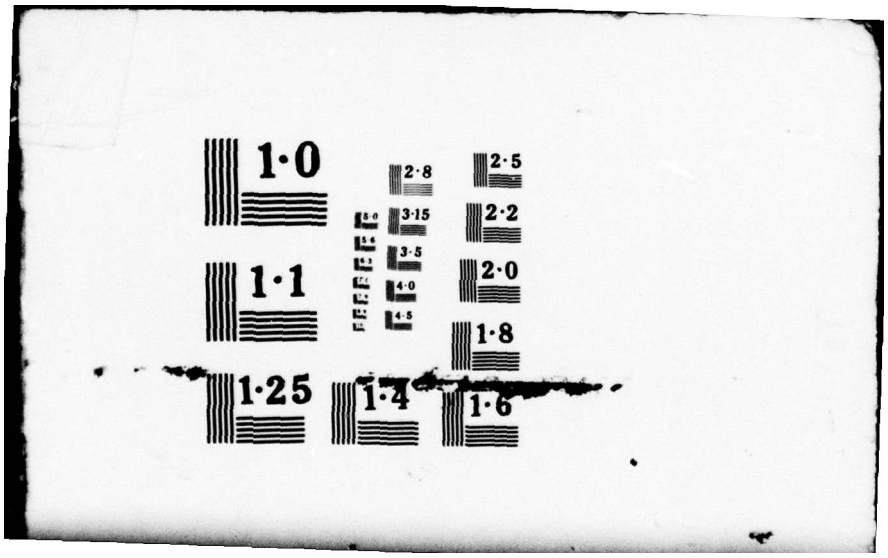
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THE DELAY-DOPPLER SPECTRAL ANALYSIS OF NONSTATIONARY TIME VARIANT LINEAR CHANNELS

by

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I. INTRODUCTION

The channel characterization and measurement problem for physical time variant channels is complicated by nonstationary statistical behavior. This complication is, in part, analogous to that perceived in the analysis of speech signals, particularly with a view to the characterization of such signals in identifiable classes. The earliest and still popular approach to speech analysis involves a short-term running spectral analysis in order to follow the nonstationary changes and to provide identifiable characteristics (e.g., formants). A similar approach is necessary in the measurement of nonstationary channels and is developed in some detail in the discussion below. In brief, a short-term running two-dimensional delay-Doppler spectral analysis is studied as a means of characterizing the channel on a short-term basis and allowing observance of nonstationary behavior. The analysis draws upon prior theory developed in [1] and [2] on the characterizations and measurement of randomly time variant linear channels.

II. THE SPECTRAL REPRESENTATION OF TIME VARIANT CHANNELS

A variety of system functions exist for characterizing the input-output relations of time variant linear channels. We use the categorization and definition of system functions developed in [1], where system functions are arranged in time-frequency dual pairs. To simplify the discussion we shall deal primarily with two system functions $T(f,t)$ and $U(\xi,\nu)$ called, respectively, the time variant transfer function and the delay-Doppler spread function.* If $z(t)$ is the complex envelope of the channel input and $w(t)$ is the complex envelope of the channel output, then $T(f,t)$ and $U(\xi,\nu)$ provide the input-output relations,

$$w(t) = \int Z(f)T(f,t)e^{j2\pi ft}df \tag{1}$$

$$w(t) = \iint z(t-\xi)e^{j2\pi\nu t}\, U(\xi,\nu)d\xi d\nu \tag{2}$$

where $Z(f)$ is the spectrum of $z(t)$.

The system functions $T(f,t)$ and $U(\xi,\nu)$ are double Fourier transform pairs

$$U(\xi,\nu) = \iint T(f,t)e^{j2\pi f\xi - j2\pi\nu t}\, dfdt \tag{3}$$

$$T(f,t) = \iint U(\xi,\nu)e^{-j2\pi\xi f}\, e^{j2\pi\nu t}\, d\xi d\nu \tag{4}$$

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*The entire paper can be phrased in terms of the dual system functions $M(t,f)$ and $V(\nu,\xi)$ defined in [1].

An examination of Eq. (2) reveals that the delay-Doppler spread function provides a phenomenological model of the channel as a continuum of differential "scatterers" subjecting the transmitted signal to a complex gain $U(\xi,\nu)d\xi d\nu$ for the scatterers providing delays (ξ) and Doppler shifts (ν) in the region $(\xi,\xi+d\xi) \times (\nu,\nu+d\nu)$. The function $U(\xi,\nu)$ provides the formal spectral representations of the time variant channel that we shall use in this paper. It will be used in the same way that the Fourier transform of a time function is used to provide a formal spectral representation of a time series. It should be noted that we do not propose to present a rigorous mathematical exposition of the Fourier transform of random processes but rather an engineering presentation following the approach developed by the author in [3], Section III. Thus we shall sidestep questions concerning the existence of integrals and appeal to physical interpretation for the justification of results.

The spectral occupancy of a time function is defined as the set of frequencies over which the spectrum of the function is "significantly" different from zero. The bandwidth of a time function is generally defined as the highest frequency of its spectral occupancy. However, in the case of narrow band processes, it is more meaningful to define a "center" frequency for the spectral occupancy and then define bandwidth as the "width" of the spectral occupancy, i.e., the difference between the highest and lowest frequencies in the spectral occupancy.

For a stationary random process $z(t)$ with power spectrum $P_z(f)$ and autocorrelation function $R_z(\tau)$, the spectrum of $z(t)$, defined as

$$Z(f) = \int z(t)e^{-j2\pi ft}\,dt \tag{5}$$

has the autocorrelation function (see [3], Section III),

$$\overline{Z^*(f)Z(\ell)} = P_z(f)\delta(f-\ell) \tag{6}$$

where $\delta(\cdot)$ is the unit impulse and

$$P_z(f) = \int R_z(\tau) e^{-j2\pi f\tau} d\tau \tag{7}$$

$$\overline{z^*(t)z(t+\tau)} = R_z(\tau) \tag{8}$$

The spectral occupancy of z(t) will clearly not include any frequencies for which $P_z(f)$ is identically zero. Where $P_z(f)$ is not identically zero some criterion of "significantly different from zero" must be established. When z(t) is a non-stationary random process the power spectrum does not have any meaning since the latter is defined only for a stationary random process. However the concepts of spectral occupancy, bandwidth, and center frequency can still have meaning for some nonstationary processes.

Turning now to time-varying channels we define by analogy the delay-Doppler occupancy pattern or just delay-Doppler occupancy of the channel as the set of delays and Doppler shifts over which the two-dimensional spectrum $U(\xi,\nu)$ is "significantly" different from zero. Examples of possible occupancy patterns are shown in Fig. 1 (repeated from Fig. 1 of [2]). The parameters B_{max} and L_{max} denoted on these patterns are analogous to bandwidth for a narrow band process. B_{max}, the Doppler spread, is the difference between the maximum and minimum Doppler shifts and L_{max}, the delay spread, is the difference between the maximum and minimum delays in the occupancy pattern.

The analog of the stationary process z(t) is the WSSUS (wide sense stationary uncorrelated) channel defined in [1] for which

$$\overline{T^*(f,t)T(f+\Omega, t+\tau)} = R(\Omega,\tau) \tag{9}$$

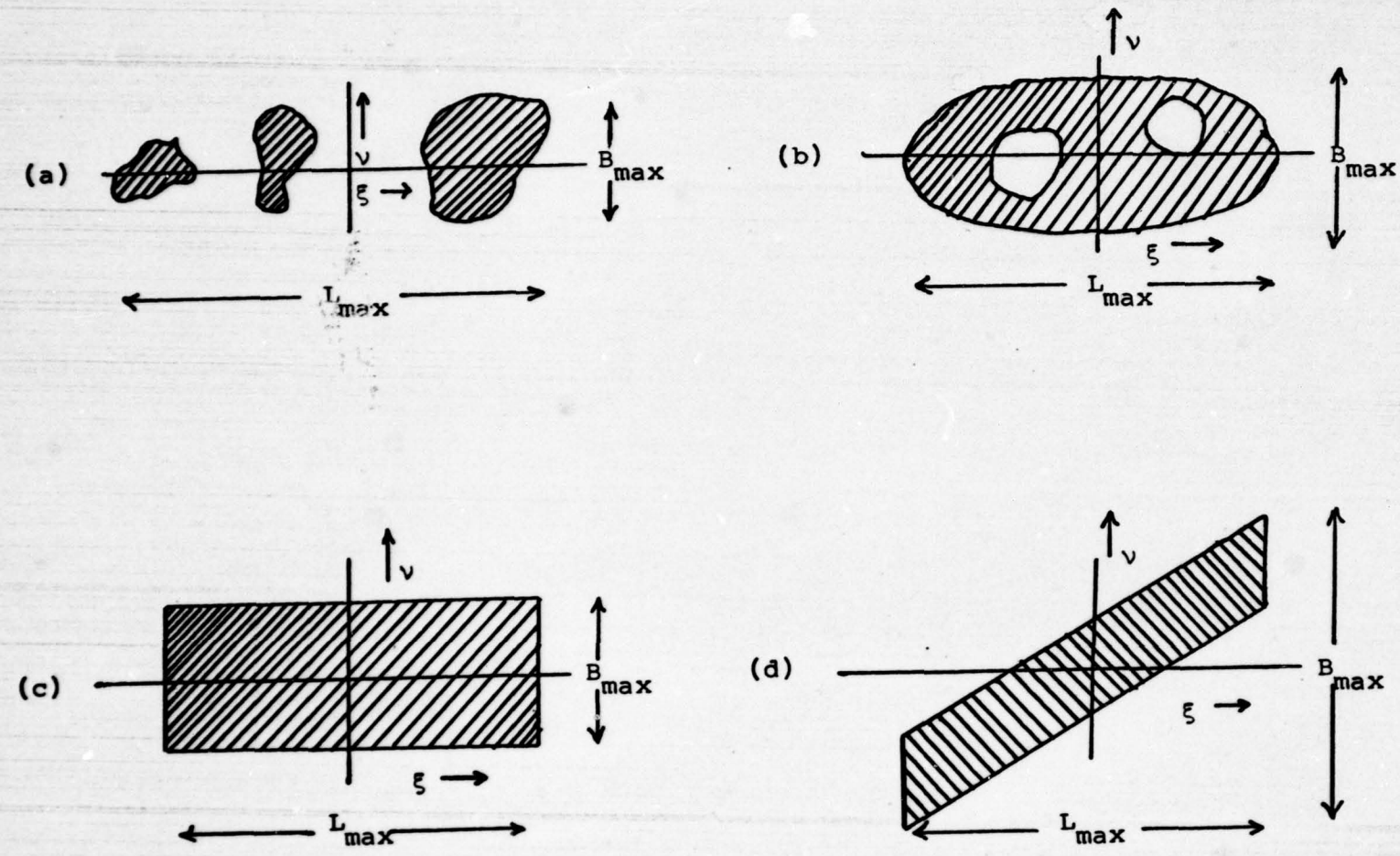

Figure 1 EXAMPLES OF POSSIBLE DELAY DOPPLER OCCUPANCY PATTERNS

Note that the two dimensional correlation function of the time variant transfer function depends only on time and frequency differences. It is readily shown (see [1] that for the WSSUS channel

$$\overline{U^*(\xi,\nu)U(\xi+\eta,\ \nu+\mu)} = S(\xi,\nu)\,\delta(\eta-\xi)\,\delta(\mu-\nu) \qquad (10)$$

where the two dimensional power density spectrum $S(\xi,\nu)$ is called the scattering function [1]. The delay-Doppler occupancy of the channel will clearly not include any (ξ,ν) values for which $S(\xi,\nu)$ vanishes. Where $S(\xi,\nu)$ is not identically zero some criterion of "significantly different from zero" must be established. When the channel is nonstationary the scattering function has no meaning. However the concepts of delay-Doppler occupancy, Doppler spread, and delay spread can still have significance for some nonstationary channels.

While the spectral representation of a channel is a clear generalization to two dimensions of the spectral representation of a time function, this is not true for the spectral measurement of a time series. We note that in the channel measurement case one must transmit probing waveforms through the channel to generate signals related to the time variant transfer function. In addition, under certain conditions the channel system functions are nonmeasurable even in the absence of additive noise. Thus the author has shown [2] that a time variant channel is measurable if and only if the area of the delay-Doppler occupancy pattern is less than unity. Finally, the measurement problem in the presence of noise [2] is considerably more difficult to deal with in the channel measurement case even apart from the increase in dimensionality of the problem.

III. SHORT-TERM SPECTRAL ANALYSIS OF TIME VARIANT CHANNELS

The integrals in (1) and (3) involve integration over all time and frequency and are thus convenient mathematical fictions. Although only finite records of a process in time and frequency can ever be available and although statistical stationarity is the exception rather than the rule, this has not prevented spectral analysis from being an extremely useful tool to the communication engineer in the characterization and processing of communication signals and interference. The type of spectral analysis that is employed in practice is based upon finite intervals of time and sometimes short-term running intervals of time. A case in point is the analysis of speech signals with a Sonogram [3] which presents a short-term intensity spectrum as a function of time. The advantage of such a running analysis is that "quasi-stationary" spectral characteristics can be observed. The purpose of this section is to define such a short term delay-Doppler spectral analysis for time variant channels and discuss some of its properties.

3.1 The Segmented Channel

Just as the basis for the short-term spectral analysis of an analog waveform is the Fourier analysis of a finite segment of the waveform, the basis for the short-term spectral analysis of a time variant channel is the Fourier analysis of a "finite segment" of the channel. Here "finite segment" is interpreted as a hypothetical channel whose time variant transfer function is identical to the transfer function $T(f,t)$ of the actual channel in a rectangular region of t-f space and is zero elsewhere. Thus we define a truncated time variant transfer function

$$\hat{T}(f,t) = \text{Rect}(f/W)T(f,t)\text{Rect}(t/T) \tag{11}$$

where

$$\text{Rect}(x) = \begin{cases} 1 & ; \quad |x| < \frac{1}{2} \\ 0 & ; \quad |x| \geq \frac{1}{2} \end{cases} \tag{12}$$

One may obtain a physical representation of a channel with time variant transfer function $\hat{T}(f,t)$ in terms of the channel with time variant transfer function $T(f,t)$ by preceding the latter with a filter having transfer function Rect(f/W) (i.e., an ideal band pass filter of bandwidth W Hz) and following it with a time gate Rect(t/T) of duration T seconds. Using the Fourier transform property (3), (4) it is readily shown that the spectral representation of the segmented channel is related to that of the original channel by

$$\hat{U}(\xi,\nu) = \iint TW \operatorname{sinc}\,[W(\eta-\xi)]\, T \operatorname{sinc}\,[T(\mu-\nu)]U(\eta,\mu)\, d\eta d\mu \tag{13}$$

where

$$\operatorname{sinc}\, x = \frac{\sin \pi x}{\pi x} \tag{14}$$

The spectrum of the segmented channel is obtained by a two dimensional convolution of the spectrum of the original channel with the spectrum $TW\,\operatorname{sinc}(W\xi)\operatorname{sinc}(T\nu)$. It is important to note that this latter spectrum is non-vanishing in any finite non-zero interval of ξ or ν and decreases very slowly from its maximum value of unity. Thus although the delay-Doppler occupancy pattern of the original channel may be identically zero outside a given region of the ξ,ν plane. The occupancy pattern of the segmented channel cannot be identically zero over any nondegenerate region. Of course, the occupancy pattern of the segmented channel will become small sufficiently far from the occupancy pattern of the original channel and will eventually become sufficiently small to be regarded as negligible. However the occupancy pattern will be a spread-out version of the original, the amount of spreading decreasing as T and W increase.

A discrete spectral representation of the segmented channel is provided by the use of sampling theory applied to $\hat{U}(\xi,\eta)$.

Thus since the transform of $\hat{U}(\xi,\eta)$, $\hat{T}(f,t)$, is "band-limited" in both the variables t and f we may reconstruct $\hat{U}(\xi,\eta)$ from the samples

$$U_{mn} \equiv \frac{1}{TW}\,\hat{U}\left(\frac{m}{W}\,,\,\frac{n}{T}\right) \tag{15}$$

The impulse sampled version of $\hat{U}(\xi,\eta)$ is given by

$$\tilde{U}(\xi,\eta) = \sum_m \sum_n U_{mn}\,\delta\left(\xi - \frac{m}{W}\right)\delta\left(\nu - \frac{n}{T}\right) \tag{16}$$

where, from (15) and (13),

$$U_{mn} = \iint \operatorname{sinc}\left[W\left(\xi - \frac{m}{W}\right)\right] \operatorname{sinc}\left[T\left(\nu - \frac{n}{T}\right)\right] U(\xi,\nu)\,d\xi\,d\eta \tag{17}$$

The quantity U_{mn} is readily shown to be equal also to the generic coefficient in the Fourier expansion of T(f,t) (and $\hat{T}(f,t)$)

$$U_{mn} = \frac{1}{TW}\int_{-T/2}^{T/2}\int_{-W/2}^{W/2} T(f,t)\,e^{j2\pi f\frac{m}{W} - j2\pi\frac{n}{T}t}\,df\,dt \tag{18}$$

The segmented channel is "recovered" from the channel which has the impulse lattice (16) for its delay-Doppler spread function by the use of band limiting before and time gating after the latter channel as shown in Fig. 2. This is the discrete delay-Doppler representation discussed in [1] and [2]. The input-output relation of the segmented channel is uniquely determined by the coefficients $\{U_{mn}\}$. Thus if the segmented channel input z(t) is limited to the band of frequencies $-W/2 < f < W/2$, the output w(t) is given by

$$w(t) = \operatorname{Rect}(t/T) \sum_{mn}\sum U_{mn}\,z(t - \tfrac{m}{W})\,e^{j2\pi\frac{n}{T}t} \tag{19}$$

If the input is not bandlimited, then $z(\cdot)$ in (19) should be replaced by only those frequency components of $z(\cdot)$ which lie within $-W/2 < f < W/2$.

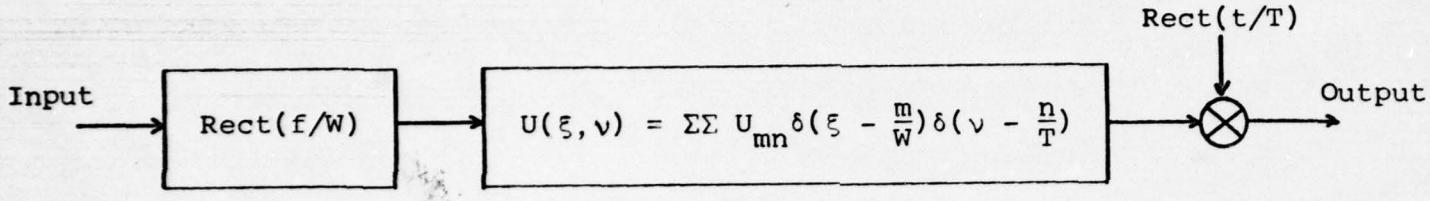

Figure 2 DISCRETE SPECTRAL REALIZATION OF SEGMENTED CHANNEL

The number of U_{mn} coefficients which are significantly different from zero depends upon the occupancy pattern of $\hat{U}(\xi,\nu)$ and the values of T and W as illustrated in Fig. 3. Also shown in Fig. 3 is the occupancy pattern of the original unsegmented channel in addition to that of the segmented channel, showing the spreading of the latter relative to the former. The spreading of the occupancy pattern after segmentation is unfortunate because it increases the number of spectral coefficients that need to be measured in order to characterize the segmented channel. In Section 3.3 we consider the use of window functions as a means of reducing the spreading of the occupancy pattern of segmented channels.

3.2 Representation of Channel in Terms of Segmented Channels

The above discussion was confined to a segmented version of the actual channel which accepted frequencies in the band $-W/2 < f < W/2$ and had output only in the time interval $-T/2 < t < T/2$. To provide spectral estimates over wider frequency and time intervals one may either increase T and W or else keep T and W fixed and use more segmented channels to characterize the additional time and frequency intervals. In order to observe nonstationary characteristics in time and frequency one should keep T and W as small as possible and build up the actual channel with segmented channels.

Thus we consider a channel with time-variant transfer function given by the series

$$\hat{T}(f,t) = \sum_{p=-P}^{P} \sum_{q=-Q}^{Q} \hat{T}_{pq}(f,t) \tag{20}$$

where $\hat{T}_{pq}(f,t)$ is the time variant transfer function of a component segmented channel

$$\hat{T}_{pq}(f,t) = \text{Rect}\left(\frac{f}{W} - p\right) T(f,t) \text{Rect}\left(\frac{t}{T} - q\right) \tag{21}$$

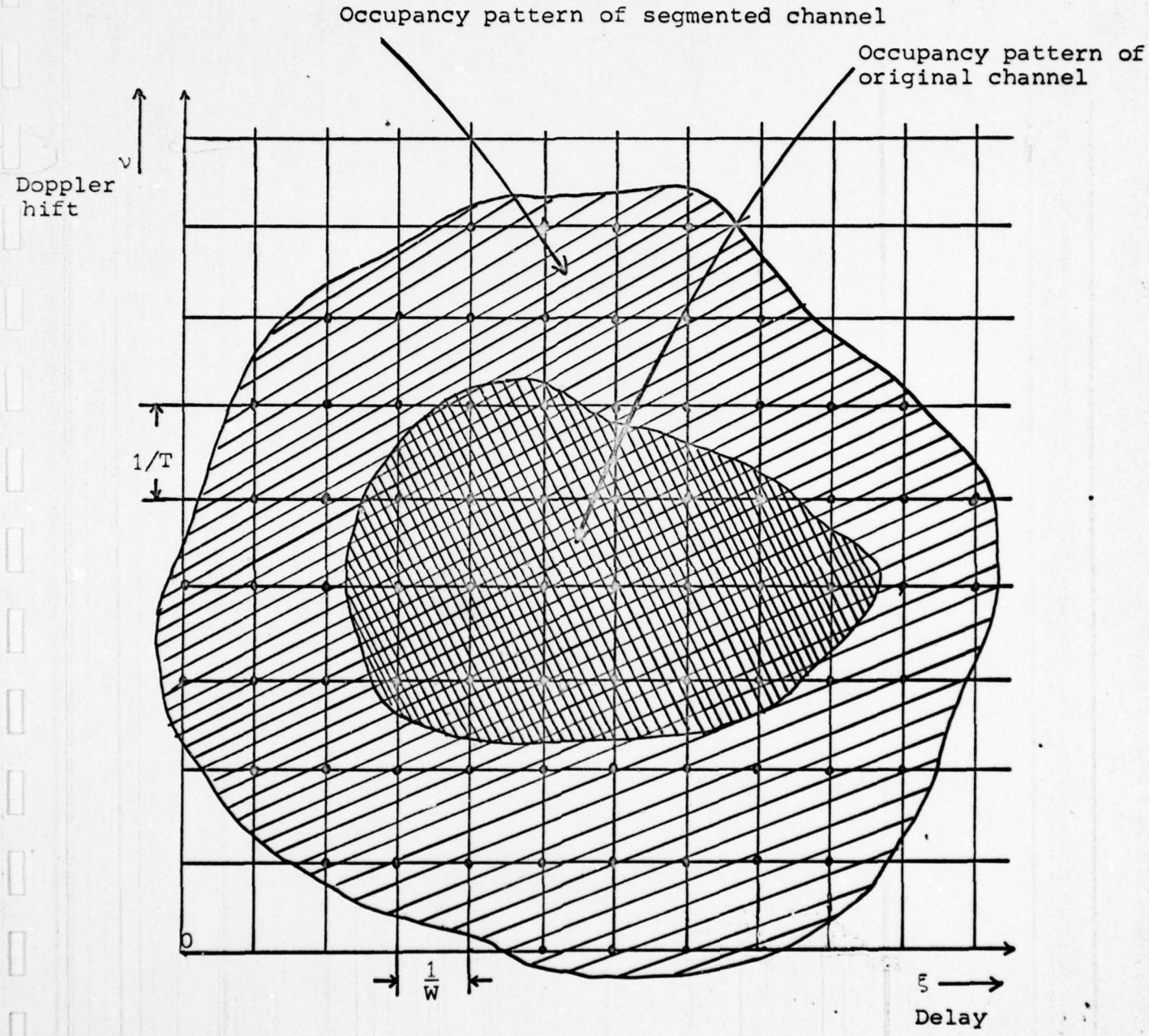

Figure 3 DISCRETE SPECTRAL REPRESENTATION OF SEGMENTED CHANNEL

in which the input frequency band is confined to $pW - W/2 < f < pW + W/2$ and the output time to $qT - T/2 < t < qT + T/2$.

Note that since

$$\sum_{p=-P}^{P} \text{Rect}\left(\frac{f}{W} - p\right) = \text{Rect}\left[\frac{f}{(2P + 1)W}\right]$$

$$\sum_{q=-Q}^{Q} \text{Rect}\left(\frac{t}{T} - q\right) = \text{Rect}\left[\frac{t}{(2Q + 1)T}\right] \tag{22}$$

the summation (19) is identical to

$$\hat{T}(f,t) = \text{Rect}\left[\frac{f}{(2P + 1)W}\right] T(f,t) \text{Rect}\left[\frac{t}{(2Q + 1)T}\right] \tag{23}$$

which is a representation of the channel for input frequencies $-(P + 1/2)W < f < (P + 1/2)W$ and output time instants $-(Q + 1/2)T < t < (Q + 1/2)T$.

By analogy with Eq. (13), the delay-Doppler spread function of the component segmented channel (20) is readily found to be

$$\hat{U}_{pq}(\xi,\nu) = \iint e^{-j2\pi pW(\eta-\xi)} e^{j2\pi qT(\mu-\nu)} \tag{24}$$
$$TW \text{ sinc } [W(\eta-\xi)] \text{ sinc } [T(\mu-\nu)] U(\eta,\mu) d\eta d\mu$$

The impulse sampled version of $\hat{U}_{pq}(\xi,\nu)$ analogous to (16) is

$$\hat{U}_{pq}(\xi,\nu) = \sum_m \sum_n U_{mn}^{pq} \delta\left(\xi - \frac{m}{W}\right) \delta\left(\xi - \frac{n}{T}\right) \tag{25}$$

where the spectral samples are

$$U_{mn}^{pq} = \frac{1}{TW}\,\hat{U}_{pq}\left(\frac{m}{W}\,,\,\frac{n}{T}\right)$$

$$= \iint e^{-j2\pi pW\xi} e^{j2\pi qT\nu}\ \mathrm{sinc}\left[W\left(\xi - \frac{m}{W}\right)\right]\mathrm{sinc}\left[T\left(\nu - \frac{n}{T}\right)\right]$$

$$U(\xi,\nu)\,d\xi\,d\nu \qquad (26)$$

As in (18) the spectral samples are expressible as Fourier coefficients

$$U_{mn}^{pq} = \frac{1}{TW}\int_{qT - T/2}^{qT + T/2}\ \int_{pW - W/2}^{pW + W/2} T(f,t)\,e^{j2\pi f\frac{m}{W}}\,e^{-j2\pi\frac{n}{T}t}\,df\,dt \qquad (27)$$

The set of spectral samples $\{U_{mn}^{pq}\}$ for fixed values of p and q provide a "snapshot" spectral analysis of the channel. As p and q are changed one obtains a discrete "running" spectral analysis in the (output) time and (input) frequency variables.

3.3 Window Functions

In practice only a finite number of spectral samples can be used to characterize a component segmented channel. As discussed at the end of Section 3.1 and illustrated in Fig. 3, the number of spectral samples needed is determined by the area of the occupancy pattern of the component segmented channel. This area in turn is dependent upon some arbitrary criterion as to what values of $\hat{U}_{pq}(\xi,\nu)$ are "significantly" different from zero. In the discussion that follows we assume that a threshold criterion has been established which allows us to regard $\hat{U}_{pq}(\xi,\nu)$ as negligible when it is less than some constant c. Thus if $\hat{S}_{pq}$ denotes the set of points in the occupancy pattern of the segmented channel,

$$(\xi,\nu) \in \hat{S}_{pq} \Longrightarrow |\hat{U}_{pq}(\xi,\nu)| > c \, \text{Max}|\hat{U}_{pq}(\xi,\nu)| \qquad (28)$$

The occupancy pattern of the segmented channel is a somewhat spread-out version of the actual channel occupancy pattern. Thus in the case of a channel consisting of a single delay and Doppler shift, i.e.,

$$U(\xi,\nu) = U_1 \delta(\xi - \xi_1)\delta(\nu - \nu_1) \qquad (29)$$

we note from Eq. (23) that the magnitude of the spectrum of the segmented channel is given by

$$|\hat{U}_{pq}(\xi,\nu)| = U_1 TW \, \text{sinc}\,[W(\xi_1 - \xi)] \, \text{sinc}\,[T(\nu_1 - \nu)] \qquad (30)$$

Since the sinc function drops to zero very slowly and in an oscillatory fashion, the zero-area occupancy pattern of the original channel expands into a pattern of occupied regions. For example, if c in (27) is defined as 10%, the occupancy pattern of the segmented channel takes the form shown in solid lines in Fig. 4. While the total area occupied by this pattern is approximately $4.5/TW$, note that it extends almost to $\pm 3/W$ in ξ and $\pm 3/T$ in ν. When c is decreased to 3% the pattern broadens drastically as shown in Fig. 5. The area has increased to approximately $27.8/TW$ and the pattern extends to $\pm 8/W$ in ξ and $\pm 8/T$ in ν.

In order to observe the (time and frequency) non-stationary characteristics of the channel it is desirable to make T and W as small as possible. However we see that the penalty is an increase in the area of the segmented channel's occupancy pattern and a consequent increase in the number of discrete spectral samples needed to characterize each segmented channel. Note that if TW becomes small enough the segmented channel could become non-measurable!

In order to alleviate this spreading problem one may use an expansion of the time variant transfer function over the frequency band $-(P + 1/2)W < f < (P + 1/2)W$ and time interval $-(Q + 1/2)T < t < (Q + 1/2)T$ which is different from that shown in Eq. (19) in that rectangular time and frequency gates are not used.

Consider the representation

$$\hat{T}(f,t) = \sum_{p=-(P+\alpha)}^{P+\alpha} \sum_{q=-(Q+\beta)}^{Q+\beta} H(f - pW)G(t - qT)T(f,t) \tag{31}$$

where

$$\sum_{-\infty}^{\infty} H(f - pW) = 1 \tag{32}$$

$$\sum_{-\infty}^{\infty} G(t - qT) = 1 \tag{33}$$

$$H(f) = 0 \quad ; \quad |f| > \alpha W/2 \tag{34}$$

$$G(t) = 0 \quad ; \quad |t| > \beta T/2 \tag{35}$$

We call any function which satisfies (32) and (34) or (33) and (35) a window function of bandwidth αW or time duration βT, respectively. Nyquist [4] has shown that large classes of functions can be generated to satisfy what we call the window property by starting with a rectangle and adding odd symmetric functions at the edges, keeping the resulting function symmetrical. Typical normalized $(W,T = 1)$ window functions are $c(\cdot)$ the "raised-cosine" function and $Tr(\cdot)$ the triangle function, defined by

$$C(x) = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos(\pi x) & ; \quad |x| < 1 \\ 0 & ; \quad |x| \geq 1 \end{cases} \tag{36}$$

and

$$Tr(x) = \begin{cases} 1 - |x| & ; \quad |x| < 1 \\ 0 & ; \quad |x| > 1 \end{cases} \tag{37}$$

For purposes of discussion let us use

$$\begin{aligned} H(f) &= C(f/W) \\ G(t) &= C(t/T) \end{aligned} \tag{38}$$

Note that

$$\alpha = \beta = 2 \tag{39}$$

and that

$$\sum_{p=-(P+1)}^{P+1} \sum_{q=-(Q+1)}^{Q+1} C\left(\frac{f}{W} - p\right) C\left(\frac{t}{T} = q\right) = 1 \; ;$$

$$\left\{ \begin{array}{l} -(P+1/2)W < f < (P+1/2)W \\ -(Q+1/2)T < t < (Q+1/2)T \end{array} \right\} \tag{40}$$

Thus

$$\hat{T}(f,t) = \sum_{p=-(P+1)}^{P+1} \sum_{q=-(Q+1)}^{Q+1} C\left(\frac{f}{W} - p\right) C\left(\frac{t}{T} - q\right) T(f,t) \tag{41}$$

is identical to $T(f,t)$ over the input frequency range $-(P+1/2)W < f < (P+1/2)W$ and the output time interval $-(Q+1/2)T < t < (Q+1/2)T$. If we define

$$\hat{T}_{pq}(f,t) = C\left(\frac{f}{W} - p\right) C\left(\frac{t}{T} - q\right) T(f,t) \tag{42}$$

then $\hat{T}(f,t)$ is represented as a summation of component segmented channels with time and frequency weightings applied by the window functions. Equation (41) may be compared to Eq. (20) which can now be viewed as the case of rectangular window functions. It is important to note that the duration of the windows in (41) are 2W and 2T as opposed to W and T in (20). Since adjacent frequency windows are separated by W Hz and adjacent time windows by T sec, the windows overlap in (40) and not in (19). Also note that the summation in (40) extends one unit farther in each direction compared to (19) so that somewhat more component segmented channels are required.

These additional complexities are offset by a reduced spreading of the occupancy pattern of the segmented channel. Thus upon Fourier transforming (41), we note that the spectrum of the segmented channel with raised cosine windows is given by

$$\hat{U}_{pq}(\xi,\nu) = \iint e^{-j2\pi pW(\eta-\xi)} e^{j2\pi qT(\mu-\nu)} TW \frac{\text{sinc}\,[2W(\eta-\xi)]\,\text{sinc}\,[2T(\mu-\nu)]}{(1-4W^2(\eta-\xi)^2)(1-4T^2(\mu-\nu)^2)} U(\eta,\mu)\,d\eta d\mu \tag{43}$$

where

$$c(y) = \frac{\text{sinc}\,2y}{1-4y^2} \tag{44}$$

is the Fourier transform of the raised cosine window function C(x).

To see the reduced spreading of the segmented channel's occupancy pattern associated with raised cosine windows we consider the case of a channel consisting of a single delay and Doppler shift, Eq. (28). Using (28) in (42) we find that for raised cosine weighting

$$\frac{|\hat{U}_{pq}(\xi,\nu)|}{\text{Max } |\hat{U}_{pq}(\xi,\nu)|} = \frac{\text{sinc } [2W(\xi_1 - \xi)] \text{ sinc } [2T(\nu_1 - \nu)]}{[1 - 4W^2(\xi_1 - \xi)^2][1 - 4T^2(\nu_1 - \nu)^2]} \tag{45}$$

For ϵ = 10% the occupancy pattern takes the form shown by dashed lines in Fig. 4. The area of this pattern is 2.3/TW and extends to around $\pm 1/W$ in ξ and $\pm 1/T$ in ν. The improvement of raised cosine weighting over rectangular weighting becomes very noticeable for ϵ = 3% as shown in Fig. 5. Here the area has increased to only 3.6/TW and the pattern still extends to only around $\pm 1/W$ in ξ and $\pm 1/T$ in ν.

For general window functions it is clear that the spectrum of the segmented channel can be expressed as

$$\hat{U}_{pq}(\xi,\nu) = \iint e^{-j2\pi pW(\eta-\xi)} e^{j2\pi qT(\mu-\nu)} h(\eta-\xi) g(\mu-\nu) U_{pq}(\eta,\mu) d\eta d\mu \tag{46}$$

where $h(\xi)$, $g(\nu)$ are the Fourier transforms of the window functions $H(f)$ and $G(t)$, respectively.

Consider now the discrete representation of each of the segmented channels. We note first that the application of general window functions as described above causes an increase in the input bandwidth and output time duration over which the segmented channel is characterized to αW and βT, respectively. The sampling grid then becomes finer with samples occuring at $(m/\alpha W, n/\beta T)$ where m,n are integers. For the case of the raised cosine windows the samples are at $(m/2W, n/2T)$. Thus, in the latter case, the discrete spectral samples needed to represent the channel are given by

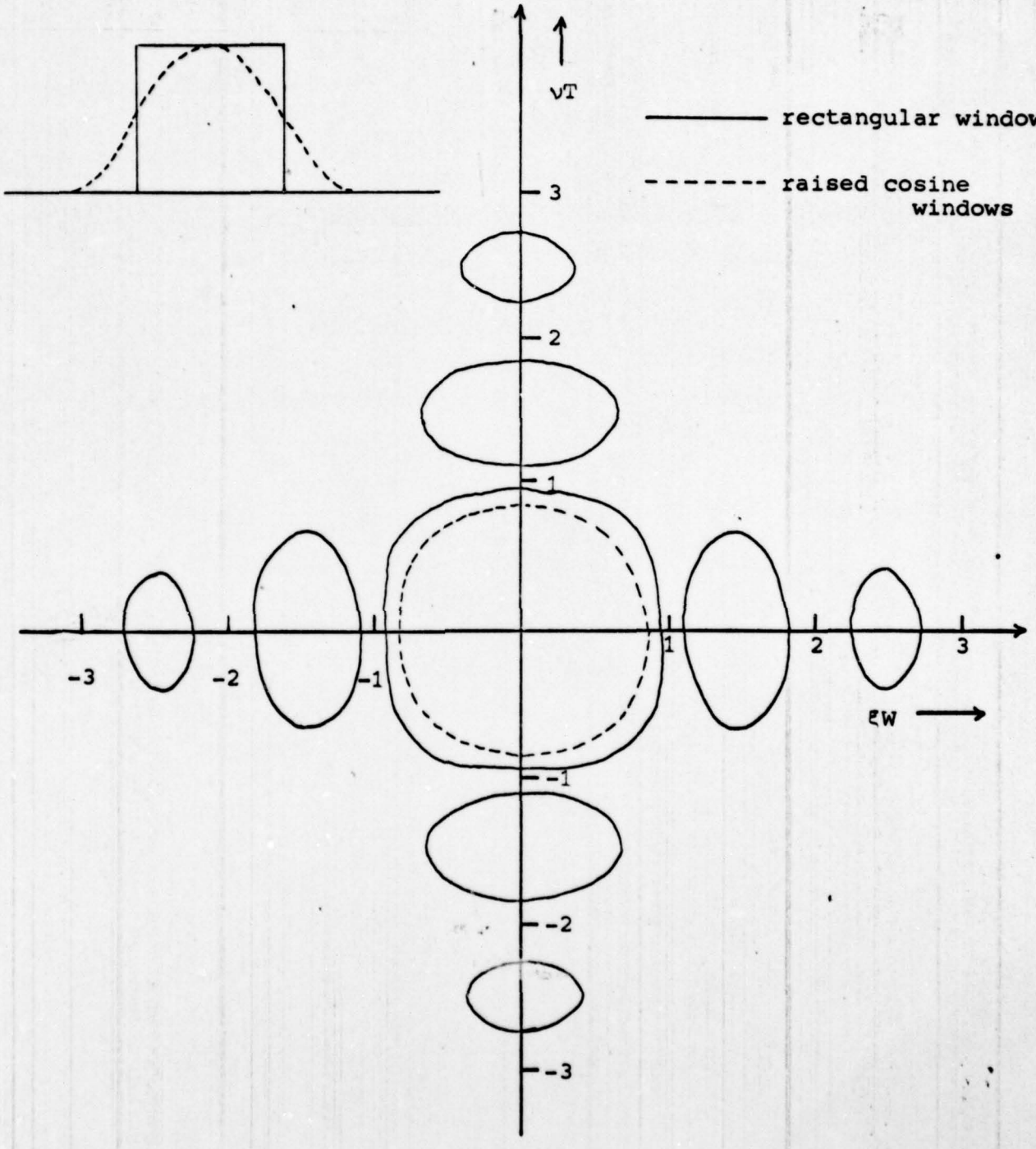

Fig. 4 Occupancy Patterns of Segmented Channels With Raised Cosine and Rectangular Gates for a 10% Threshold Level.

$$U_{mn}^{pq} = \frac{1}{4TW}\,\hat{U}_{pq}\left(\frac{m}{2W}\,,\ \frac{n}{2T}\right)$$

$$= \frac{1}{4}\iint e^{-j\pi pm}e^{j\pi qn}e^{-j2\pi pW\xi}e^{j2\pi qT\nu} \qquad (47)$$

$$\frac{\text{sinc } 2W(\xi - m/2W)\ \text{sinc } 2T(\nu - n/2T)}{[1 - 4W^2(\xi - m/2W)^2][1 - 2T^2(\nu - n/2T)^2]}\,U(\xi,\nu)\,d\xi d\nu$$

It should be clear that the number of discrete spectral coefficients required to characterize the segmented channel with raised cosine windows of durations 2T, 2W is around four times that required for rectangular windows of durations T, W, at least ignoring the differences of spreading of the occupancy patterns for the two cases.

One may use raised cosine windows of smaller width than 2T, 2W, but then more segmented channels would be needed to characterize the channel over large intervals of input bandwidth and output time. For example, suppose the raised cosine windows were of duration T, W. Roughly four times as many segmented channels would be required but each segmented channel would require roughly the same number of spectral samples as rectangular gates of duration T, W if the occupancy patterns of the segmented channels had the same area in both cases. However, as Fig. 5 shows, the spreading caused by the rectangular gates can be rather severe as compared to raised cosine gates.

Assuming that the window widths of 2T and 2W are used and thus the spectral samples are given by (46), the input-output relation for the segmented channel becomes

$$w_{pq}(t) = \text{Rect}\left(\frac{t}{2T} - \frac{q}{2}\right)\sum_m\sum_n U_{mn}^{pq}\,z_p\left(t - \frac{m}{2W}\right)e^{j\pi\frac{n}{T}t} \qquad (48)$$

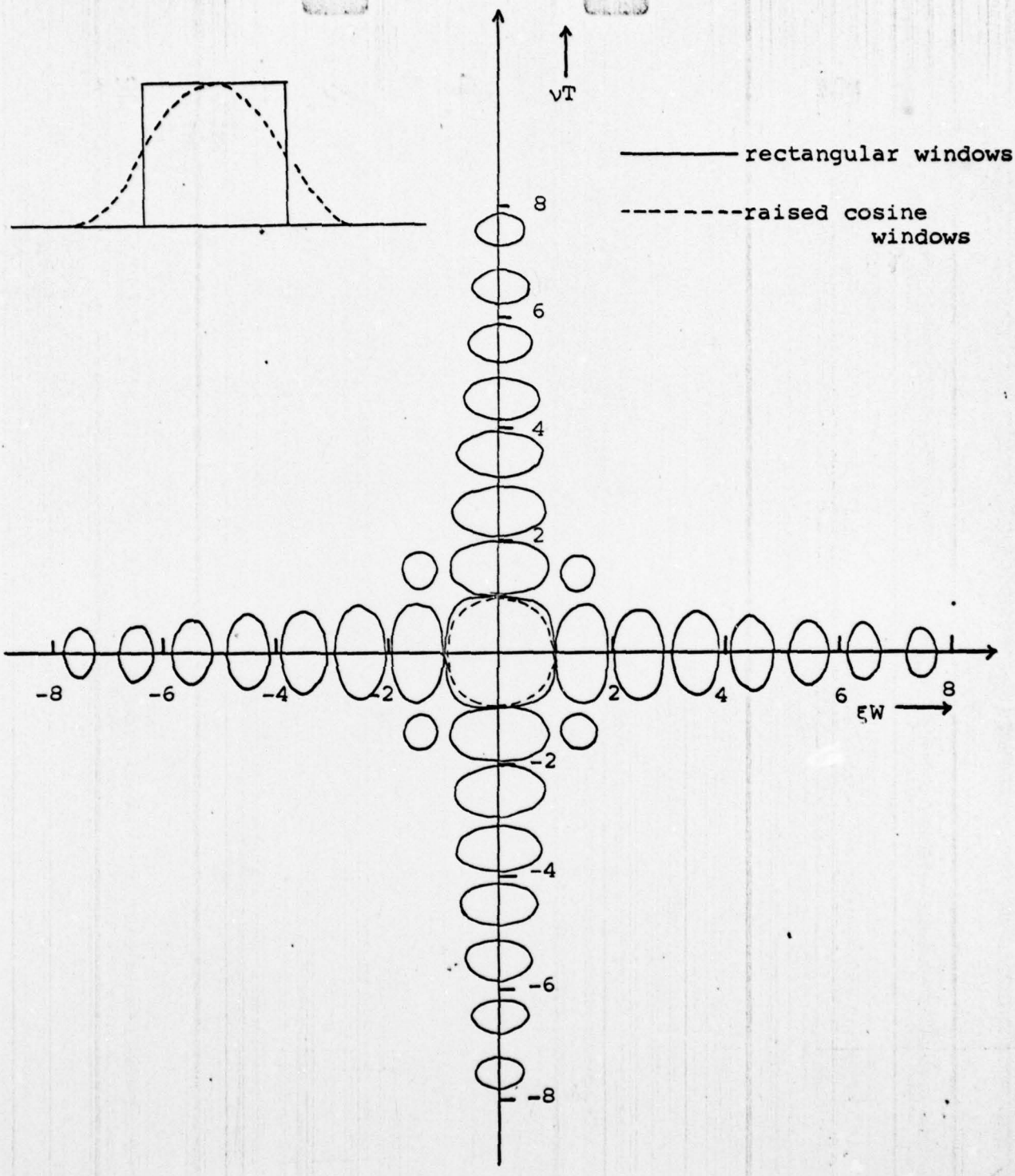

Fig. 5 Occupancy Patterns of Segmented Channels with Raised Cosine and Rectangular Gates for a 3% Threshold Level.

where $z_p(t)$ is that portion of the input signal $z(t)$ contained within the spectral band $(p - 1)W < f < (p + 1)W$.

IV. SPECTRAL MEASUREMENT

In a previous paper [2] the channel measurement problem was formulated for a single segmented channel with rectangular windows on the input time and output frequency. Here we wish to consider the more practical running discrete spectral analysis with shaped windows outlined in the previous section. For concreteness we will assume overlapping raised cosine windows of duration 2T and 2W for input frequency and output time. We first confine our attention to the case of a single input frequency window but a large number of overlapping output time windows. As shown in Fig. 6, the frequency window is assumed to cover the band $-W < f < W$ Hz. Also it is assumed that the input signal $z(t)$ contains no frequency components outside of this band and has a non-zero spectrum at all frequencies within this band, except possibly for $f = \pm W$. From (48) we find that the output of a particular time window located at $(q - 1)T < t < (q + 1)T$ may be expressed as

$$w_q(t) = \text{Rect}\left(\frac{t}{2T} - \frac{q}{2}\right) \sum_m \sum_n U_{mn}^q \, z\left(t - \frac{m}{2W}\right) e^{j\pi \frac{n}{T} t}$$

$$+ C\left(\frac{t}{T} - q\right) n(t) \qquad (49)$$

where $n(t)$ is an additive noise, assumed white in the present discussion. The "short-term" spectral samples $\{U_{mn}^q\}$ are given by

$$U_{mn}^q = \frac{e^{j\pi qn}}{4} \iint e^{j2\pi qT\nu} \, c(W\xi - m)c(T\nu - n)U(\xi,\nu)d\xi d\nu \qquad (50)$$

where $c(\cdot)$ is the transform of the raised cosine spectrum (see (4-31)).

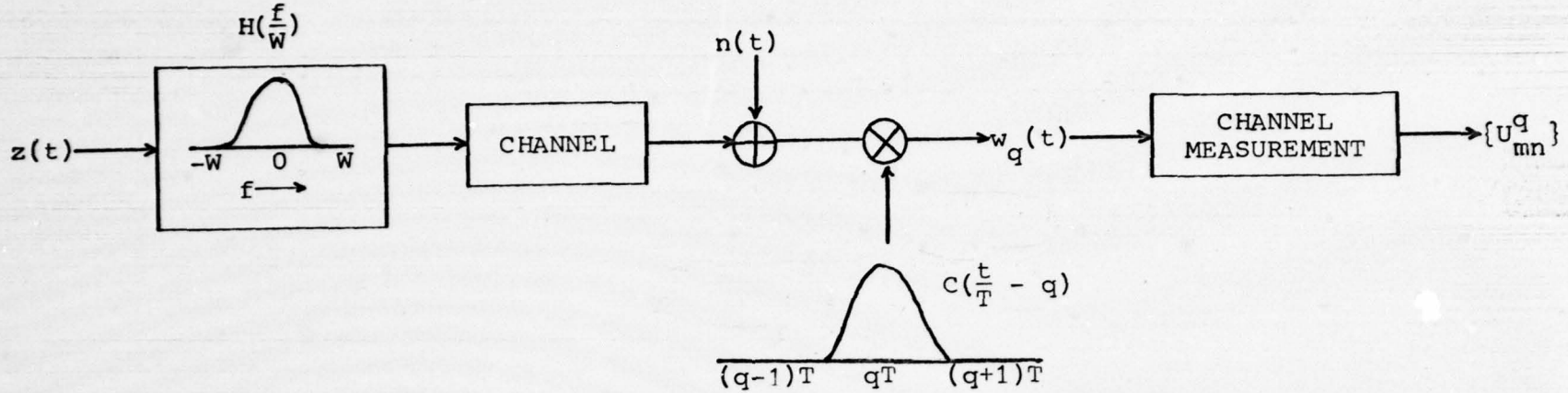

Figure 6 Short Time Spectral Measurement of a Segmented Channel

The least-squares measurement technique to extract the spectral samples $\{U^q_{mn}\}$ from $w_q(t)$ can be approached exactly as in [2]. However, here we follow a slightly different procedure using a continuous rather than discrete representation of $w_q(t)$ (see, for example, the formulation by Levin [5]). To simplify the notation we assume the set of possible discrete delay-Doppler-shift pairs has been ordered in some convenient fashion. The k^{th} delay-Doppler shift pair (m_k, n_k) is denoted by the vector $\underset{\sim}{d}_k$. The received waveform can then be represented by the vector product

$$w_q(t) = \text{Rect}\left(\frac{t}{2T} - \frac{q}{2}\right) \underset{\sim}{U}^T_q \underset{\sim}{Z}(t) + C\left(\frac{t}{T} - q\right)n(t) \qquad (51)$$

where the vectors $\underset{\sim}{U}^T_q$ and $\underset{\sim}{Z}^T(t)$ are given by

$$\underset{\sim}{U}^T_q = [U^q_{\underset{\sim}{d}_1}, U^q_{\underset{\sim}{d}_2}, \ldots U^q_{\underset{\sim}{d}_s}] \qquad (52)$$

$$\underset{\sim}{Z}^T(t) = [z_{\underset{\sim}{d}_1}(t), z_{\underset{\sim}{d}_2}(t), \ldots z_{\underset{\sim}{d}_s}(t)] \qquad (53)$$

in which

$$\underset{\sim}{d}_k = [m_k, n_k] \qquad (54)$$

$$U^q_{\underset{\sim}{d}_k} = U^q_{m_k, n_k} \qquad (55)$$

$$z_{\underset{\sim}{d}_k}(t) = z(t - \frac{m_k}{2W})e^{j\pi \frac{n_k}{T} t} \qquad (56)$$

The discrete spectral representation of the q^{th} channel is given by the vector $\underset{\sim}{U}_q$. The least squares estimate of $\underset{\sim}{U}_q$ is given by (see [5], pg. 40)

$$\hat{\underset{\sim}{U}}_q = \underset{\sim}{E}^{-1}_q \underset{\sim}{M}_q \qquad (57)$$

where the typical element of $\underset{\sim}{E}_q$ and $\underset{\sim}{M}_q$ are given by

$$E^q_{k\ell} = \int_{(q-1)T}^{(q+1)T} z_{\underset{\sim}{d}_k}(t) z^*_{\underset{\sim}{d}_\ell}(t)dt \tag{58}$$

$$M^q_\ell = \int_{(q-1)T}^{(q+1)T} z_{\underset{\sim}{d}_\ell}(t) w_q(t)dt \tag{59}$$

The matrix $\underset{\sim}{M}_q$ is essentially the set of matched filter outputs. One may write (59) in the alternate form

$$\hat{\underset{\sim}{U}}_q = \int_{(q-1)T}^{(q+1)T} \underset{\sim}{E}_q^{-1} \underset{\sim}{Z}^*(t) w_q(t)dt \tag{60}$$

If we define the column matrix

$$\underset{\sim}{e}_q(t) = \underset{\sim}{E}_q^{-1} \underset{\sim}{Z}^*(t) \tag{61}$$

then

$$\hat{\underset{\sim}{U}}_q = \int_{(q-1)T}^{(q+1)T} \underset{\sim}{e}_q(t) w_q(t)dt \tag{62}$$

The matrix $\underset{\sim}{e}_q(t)$ is a column vector of s time functions which may be computed once the occupancy pattern of the channel is known. Equation (62) represents the estimate of each spectral coefficient as an inner product of the received signal and one of the component time functions of $\underset{\sim}{e}_q(t)$. Alternatively, Eq. (57) shows that this estimate may also be represented as a weighted sum of "matched filter" outputs.

The least square estimate (57) is the minimum variance linear estimator only when the noise is white and WSS (wide sense stationary). For more general noise the minimum variance linear estimate is the Markov estimate [see [5]]. In the channel under study we consider only the case of white and WSS noise at the receiver input. However, it should be noted that the imposition of the segmenting time gate makes the noise observed after the time gate nonstationary. Thus it might appear as if the more general Markov estimate should be used. For our special case we may obtain the Markov estimate by the simple artifice of dividing both sides of (49) or (51) by the time gate $C(\frac{t}{T} - q)$, which removes the raised cosine gate from both the noise and observed data, replacing it by the rectangular gate $\text{Rect}(t/2T - q/2)$. Equation (51) becomes

$$w(t)\text{Rect}(t/2T - q/2) = \frac{\text{Rect}(t/2T - q/2)}{C(t/T - q)} \underset{\sim q}{U^T} \underset{\sim}{Z}(t) + n(t) \tag{63}$$

where $w(t)$ is the receiver input waveform. Since the noise in (63) is white and WSS now one may employ least squares to obtain the minimum variance linear estimate. Thus if this estimate is denoted by $\hat{\underset{\sim q}{U}}$ we find

$$\hat{\underset{\sim q}{U}} = \underset{\sim q}{F^{-1}} \underset{\sim q}{N} \tag{64}$$

where the typical elements of $\underset{\sim q}{F^{-1}}$ and $\underset{\sim q}{N}$ are

$$F^q_{k\ell} = \int_{(q-1)T}^{(q+1)T} \frac{z_{\underset{\sim k}{d}}(t) z^*_{\underset{\sim \ell}{d}}(t)}{|C(t/T - q)|^2} \, dt \tag{65}$$

$$N^q_\ell = \int_{(q-1)T}^{(q+1)T} \frac{z^*_{\underset{\sim \ell}{d}}(t) w(t)}{C^*(t/T - q)} \, dt \tag{66}$$

An examination of (65) and (66) reveals that certain difficulties may occur in applying (64) since the gating function can drop to zero and the coefficients $F^q_{k\ell}$ and N^q_ℓ might become infinite. This singular case has not been examined yet. The covariance matrix of the measurement error for the least squares and Markov estimates are [see [5] and [2]].

$$C_{ov}(\hat{\underset{\sim}{U}}_q) = \langle \hat{\underset{\sim}{U}}_q \hat{\underset{\sim}{U}}^{*T}_q \rangle - \langle \hat{\underset{\sim}{U}}_q \rangle \langle \hat{\underset{\sim}{U}}^{*T}_q \rangle$$

$$= (\underset{\sim}{E}_q^{-1} \underset{\sim}{D}_q \underset{\sim}{E}_q^{-1}) \langle |n(t)|^2 \rangle \tag{67}$$

$$C_{ov}(\hat{\underset{\sim}{U}}_q) = \underset{\sim}{F}_q^{-1} \langle |n(t)|^2 \rangle \tag{68}$$

where the typical element of the matrix $\underset{\sim}{D}_q$ is given by

$$D^q_{k\ell} = \int_{(q-1)T}^{(q+1)T} |C(t/T - q)|^2 z_{\underset{\sim}{d}_k}(t) z^*_{\underset{\sim}{d}_\ell}(t)\, dt \tag{69}$$

We consider now the more general case in which there are 2P+1 input raised cosine frequency windows each of width 2W cps, with the "center" window located at -W<f<W Hz. From Eq. (48) we see that the output of a particular time window located at (q-1)T <f<(q+1)T may be expressed as

$$w_q(t) = \sum_{p=-P}^{P} w_{pq}(t)$$

$$= \text{Rect}(t/2T - q/2) \sum_p \sum_m \sum_n U^{pq}_{mn} z_p(t - \tfrac{m}{2W}) e^{j\pi \frac{n}{T} t}$$

$$+ C(t/T - q) n(t) \tag{70}$$

where $z_p(t)$ is that portion of the input signal $z(t)$ located within the spectral band $(p-1)W<f<(p+1)W$. Equation (51) may still be used to represent $w_q(t)$ provided we used the definitions

$$\underset{\sim}{U}_q^T = [\underset{\sim}{U}_{-P,q}^T | \underset{\sim}{U}_{1-P,q}^T | \ldots | \underset{\sim}{U}_{-1,q}^T | \underset{\sim}{U}_{0,q}^T | \underset{\sim}{U}_{1,q}^T | \ldots | \underset{\sim}{U}_{P,q}^T] \quad (71)$$

$$\underset{\sim}{Z}^T(t) = [\underset{\sim}{Z}_{-P}^T(t) | \underset{\sim}{Z}_{1-P}^T(t) | \ldots | \underset{\sim}{Z}_{-1}^T(t) | \underset{\sim}{Z}_0^T(t) | \underset{\sim}{Z}_1^T(t) | \ldots | \underset{\sim}{Z}_P(t)] \quad (72)$$

in which

$$\underset{\sim}{U}_{p,q}^T = [U_{\underset{\sim}{d}_1}^{pq}, U_{\underset{\sim}{d}_2}^{pq}, \ldots, U_{\underset{\sim}{d}_s}^{pq}] \quad (73)$$

$$\underset{\sim}{Z}_p^T(t) = [z_{pd_{\underset{\sim}{1}}}(t), z_{pd_{\underset{\sim}{2}}}(t), \ldots, z_{pd_{\underset{\sim}{s}}}(t)] \quad (74)$$

where

$$\underset{\sim}{d}_k = [m_k, n_k] \quad (75)$$

$$U_{\underset{\sim}{d}_k}^{pq} = U_{m_k,n_k}^{pq} \quad (76)$$

$$z_{pd_k}(t) = z_p(t - \frac{m_k}{2W}) e^{j\pi \frac{n_k}{T} t} \quad (77)$$

In the definitions above it has been assumed, for simplicity, that the occupancy pattern is the same for each segmented channel. If desired, one may extend the notation to a different occupancy pattern for each segmented channel. The discrete spectral representation of the segmented channel with the p^{th} frequency gate

and q^{th} time gate is the vector $\underset{\sim}{U}_{pq}$. The set of spectral representations for all the frequency gates and the q^{th} time gate is given by $\underset{\sim}{U}_{q}$. The least squares estimate of $\underset{\sim}{U}_{q}$, $\hat{\underset{\sim}{U}}_{q}$ can be expressed in the general form (57) where $\underset{\sim}{U}_{q}$, $\underset{\sim}{E}_{q}$ and $\underset{\sim}{M}_{q}$ must be redefined as the following partitioned matrices

$$\underset{\sim}{E}_{q} = \left[\begin{array}{c|c|c} \underset{\sim}{E}_{q'-P,-P} & \underset{\sim}{E}_{q'-P,1-P} & \cdots \\ \hline \underset{\sim}{E}_{q'1-P,-P} & \underset{\sim}{E}_{q'1-P,1-P} & \cdots \\ \hline \vdots & \vdots & \end{array}\right. \qquad \left.\begin{array}{c|c} \vdots & \vdots \\ \hline \cdots & \underset{\sim}{E}_{q'P-1,P-1} \quad \underset{\sim}{E}_{q'P-1,P} \\ \hline \cdots & \underset{\sim}{E}_{q'P,P-1} \quad \underset{\sim}{E}_{q'P,P} \end{array}\right] \tag{78}$$

$$\underset{\sim}{M}_{q}^{T} = [\underset{\sim}{M}_{-P,q}^{T} | \underset{\sim}{M}_{1-P,q}^{T} | \cdots | \underset{\sim}{M}_{0,q}^{T} | \cdots | \underset{\sim}{M}_{P,q}^{T}] \tag{79}$$

$$\underset{\sim}{U}_{q}^{T} = [\underset{\sim}{U}_{-P,q}^{T} | \underset{\sim}{U}_{1-P,q}^{T} | \cdots | \underset{\sim}{U}_{0,q}^{T} | \cdots | \underset{\sim}{U}_{P,q}^{T}] \tag{80}$$

The typical elements of $\underset{\sim}{E}_{qp_1p_2}$ and $\underset{\sim}{M}_{pq}$ are

$$E_{k\ell}^{qp_1p_2} = \int_{(q-1)T}^{(q+1)T} z_{p_1}(T - \frac{m_k}{2W}) z_{p_2}^{*}(t - \frac{m_\ell}{2W}) e^{j\frac{\pi}{T}(n_k - n_\ell)t} dt \tag{81}$$

$$M_{\ell}^{qp} = \int_{(q-1)T}^{(q+1)T} z_p^*(t - \frac{m_{\ell}}{2W}) e^{-j\frac{\pi}{T}n_{\ell}t} w_q(t)\, dt \qquad (82)$$

Since the time functions $z_{p_1}(t)$ and $z_{p_2}(t)$ occupy portions of the same frequency band only when p_1 and p_2 differ by unity, as long as $TW >> 1$ and the Doppler spread of the channel is much less than W,

$$E_{k\ell}^{qp_1p_2} \approx 0 \text{ for } |p_1 - p_2| > 1 \qquad (83)$$

and the matrix $\underset{\sim}{E}_q$ will be sparse.

Markov estimates and error variances for the spectral coefficients may be formed as in the single-frequency gate case.

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