



MOST Proj MOSI E Ject LEVEL 7 Code NAUSHIPS(000) **OOVI LIBRARY COPY** Copy No. USL Problem No. D01187 A-650-01-00 9 Tehnical memo. A A0 6703 U. S. NAVY UNDERWATER SOUND LABORATORY FORT TRUMBULL, NEW LONDON, CONNECTICUT AN ANALYSIS OF A SLACK TOWLINE . 2133 - 374-69 by 0 J. W./Schram MS S./Rupinski USL Technical Memorandum No. 2133-374-69 27 May \$69 1979 USL-7M-2133-374 INTRODUCTION ABSTRACT An important consideration in towed system dynamics is the determination of the conditions that cause the towline to become slack. The slack towline creates a problem in that the recapture of the freefalling body causes the system to experience large tensile forces. Sudden applications of high tensions often cause the towline to eventually fracture, with resulting loss of the towed body. This memorandum presents an analysis of slack line dynamics. ASSTRALI BACKGROUND In order to develop a criterion for the occurrence of a slack towline, some assumptions must be made concerning the form of the ship's motion and the manner in which the disturbance is transferred to the body. In reference (a) it was found that the transverse disturbances, see Figure 1, are diminished to a large extent as they travel down the towline. However, longitudinal disturbances are relatively undamped. Therefore, it will be assumed that the component of the ship motion 1811 This document is subject to special experies compole and each transmittar to foreign governments of DISTRIBUTION STATEMENT Aforeign nationals may be made only with prior and Approved for public release: OL Dr U. S Underwater Sound Distribution Unlimited

directed along the towline is transferred directly to the towed body, while the transverse motion is completely damped.

A slack towline is likely to occur in rough seas. Since a ship makes relatively little headway in high seas, resulting in  $\phi_S$  (towline inclination angle)  $\approx 90^\circ$ , most of the disturbances will be directed along the towline. Therefore, the assumption that the disturbance experienced by the body is only the longitudinal component of the ship motion is not unrealistic. If it is assumed that the longitudinal disturbances are transmitted instantaneously to the body as long as there is tension in the towline, then the body will have the same motion as the longitudinal component of the ship's motion.

#### NOMENCLATURE

Before proceeding further, the following notation is defined:

- H = amplitude of motion
- $A_{\rm p}$  = plan area of body

 $C_{\rm D}$  = coefficient of body drag and plan direction

- $F_{p}$  = hydrodynamic body forces in plan direction
- K = spring constant of towline
- M = mass
- T = towline tension
- W = weight in water
- x = displacement
- $\rho$  = density of fluid
- $\phi$  = towline angular inclination
- E = effective modulus of elasticity of towline



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- w = frequency of disturbances
- v = velocity
- a = acceleration

Subscripts:

- B = body
- d = change from steady state
- L = component along the towline

## SLACK TOWLINE CRITERION

Under conditions where a slack line can occur, if the towline is buoyant a slack towline will initially appear at the ship. However, in most practical applications the towline of itself is not buoyant and the longitudinal component of its weight is usually larger than the inertia and hydrodynamic effects caused by the ship's motion. Therefore, in general, the towline initially goes slack at the towed body. Hence the body forces govern the slack towline criterion. With this in mind, Newton's second law of motion for the body becomes:

$$-W_{R} + T + F_{R} = M_{R}a_{T}$$
(1)

where:  $F_B = C_{D_B} \frac{f}{2} A_B v_L^2$  = hydrodynamic body force

By definition, a slack towline occurs when the tension in the towline is zero. Therefore, rearranging the quantities in equation (1) and setting T = 0, the criterion that the towline does <u>not</u> become slack is:

$$W_{\rm B} > F_{\rm B} - M_{\rm B} a_{\rm L} \qquad (2)$$

The exact point in time when the towline initially goes slack depends on the relative magnitude of  $M_{\rm B}a_{\rm L}$ , the inertia of the body, and  $F_{\rm B}$ , the hydrodynamic force. Therefore, as the amplitude of the disturbance increases, the hydrodynamic forces become more important to the criterion for a slack towline.

From equation (2), it can be seen that if

$$F_{B} - M_{B}a_{L} max$$
(3)

is greater than one, the towline remains taut. However, if the above ratio is less than one, the towline goes slack.

If the displacement of the ship is considered sinusoidal in the form:

$$x_{T} = H_{T} \sin \omega t$$
,

then the velocity and acceleration are given by:

$$v_L = \dot{x}_L = H_L \omega \cos \omega t$$
  
 $a_L = \dot{x}_L = -H_L \omega^2 \sin \omega t$ 

If these expressions are substituted for  $v_{T}$  and  $a_{T}$  into

$$\begin{vmatrix} \mathbf{F}_{B} - \mathbf{M}_{B} \mathbf{a}_{L} \end{vmatrix},$$
$$\begin{vmatrix} \mathbf{F}_{B} - \mathbf{M}_{B} \mathbf{a}_{L} \end{vmatrix} = \begin{vmatrix} c_{D_{B}} \frac{\rho}{2} \mathbf{A}_{B} \mathbf{H}_{L}^{2} \omega^{2} \cos^{2} \omega t + \mathbf{M}_{B} \mathbf{H}_{L} \omega^{2} \sin \omega t \end{vmatrix}$$
(4)

the maximum of this quantity is found by setting:

$$\frac{d}{dt} (F_B - M_B a_L) = 0, \text{ or}$$
  
-C<sub>D<sub>B</sub></sub>  $\rho A_B H_L^2 \omega^3 \sin \omega t + M_B H_L \omega^3 \cos \omega t = 0.$ 

The two solutions are:

$$\cos \omega t = 0$$
(5)  
$$\sin \omega t = \frac{M_B}{C_{D_B} \rho A_B H_L}$$
(6)

Substituting (5) into (3):

$$\frac{W_{B}}{F_{B} - M_{B} a_{L} |_{max}} = \frac{W_{B}}{M_{B} H_{L} \omega^{2}} > 1 \quad (7)$$

It should be noted that  $M_B$  is the total enclosed plus hydrodynamic mass of the body, and  $W_B$  is the weight of the body in water. In particular,  $W_B \neq M_B$  g.

For (6) to be a valid solution, it must be that

$$\frac{M_{B}}{C_{D_{B}} \rho A_{B} H_{L}} < 1$$

If this is the case,

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$$|F_{B} - M_{B}a_{L}|_{max} = \left(C_{D_{B}} \frac{\rho}{2} A_{B}H_{L}^{2} \omega^{2}\right) \left(1 - \frac{M_{B}^{2}}{C_{D_{B}}\rho A_{B}H_{L}}\right) + M_{B}H_{L}\omega^{2} \frac{M_{B}}{C_{D_{B}}\rho A_{B}H_{L}} = \frac{H_{L}\omega^{2}}{2} \left(C_{D_{B}}\rho A_{B}H_{L} + \frac{M_{B}^{2}}{C_{D_{B}}\rho A_{B}H_{L}}\right),$$

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and the criterion (3) becomes:

$$\frac{2W_{B}}{H_{L}\omega^{2}} \left(C_{D_{B}}\rho A_{B}H_{L} + \frac{M_{B}^{2}}{C_{D_{B}}\rho A_{B}H_{L}}\right) = \frac{W_{B}}{H_{L}\omega^{2}} \left(\frac{2Z}{1+Z}\right) > 1 , \qquad (8)$$

where:

$$Z = \frac{M_B}{C_{D_B} \rho A_B H_L}$$

The quantity  $\frac{2Z}{1+Z^2}$  is always between zero and unity for 0 < Z, since if  $\frac{2Z}{1+Z^2} > 1$  and Z > 0, then:

> $Z^{2} + 1 \leq 2Z$   $Z^{2} - 2Z + 1 \leq 0$  $(Z - 1)^{2} \leq 0$

which is absurd, and hence (8) is a more restrictive criterion than (7); however, Z < 1 is a restriction on the use of (8).

Contrata (b) has made measurements on a slack towline at sea, the results of which are given in Appendix A.

For this particular case  $M_B = 266 \times 1.3$ , where 1.3 is the factor used to account for the hydrodynamic mass of the body.

$$Z = \frac{M_B}{C_{D_B} \rho A_B H_L} = \frac{(266)(1.3)}{(0.3)(2)(30.66)(8)} = 2.35 > 1$$

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Hence, criterion (7) must be used.

$$\frac{W_{B}}{M_{B}a_{L}max} = \frac{3260}{(266)(1.3)(11.3)} = .834 < 1.$$

The criterion predicts a slack towline for the experiment reported in (b).

### TOWLINE "RECAPTURE" TENSION

After the slack towline occurs, the towline tension is sharply increased when the body is recaptured. Since the mass of a destroyer is at least three orders of magnitude (1000 times) greater than the mass of the body, the motion of the ship is unaffected by the recapture of the body. Hence, the velocity of the body must change from its free fall velocity to the component of velocity along the towline when it is recaptured.

An energy balance can be written for the system before and after recapture. Referring to Figure 2, the potential energy datum is taken at the point of recapture, where the towed system regains its steady state configuration. The whole system is considered to be moving at the longitudinal velocity of the ship  $V_L$ . Therefore the energy balance becomes:

KEBody + PEBody + PETowline = Work of Hydrodynamic Forces

$$\frac{1}{2}M_{\rm B}(V_{\rm B}-V_{\rm L})^2 - W_{\rm B}x_{\rm d} + \frac{1}{2}Kx_{\rm d}^2 = \overline{F}_{\rm BL}x_{\rm d}$$
 (9)

The value of  $V_B$  is the absolute velocity of the body at the time of recapture, K is the equivalent spring constant of the towline, and  $\overline{F}_{BL}$  is the average hydrodynamic force on the body at the time of recapture. The towline can be simulated by two linear springs in series as shown in Figure 2: the elastic spring constant, K<sub>e</sub>; and the spring constant K<sub>x</sub> which is due to the change in the towline's shape when a force is applied to the towline. The equivalent spring constant K is given by:

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$$K = \frac{K_{x} K_{e}}{K_{x} + K_{e}}$$

where:  $K_{\epsilon} = \frac{AE}{L}$ 

and  $K_x$  will be discussed in a later section. Assuming that K is known, then:

$$\mathbf{T} = \mathbf{K}\mathbf{x}_{a} \tag{10}$$

Substituting (10) into (9),

$$\frac{1}{2}M_{B}(V_{B} - V_{L})^{2} - W_{B}\frac{T}{K} + \frac{1}{2}K\frac{T^{2}}{K^{2}} - \overline{F}_{BL}\frac{T}{K} = 0$$

$$T^{2} - 2(W_{B} - \overline{F}_{BL})T - KM_{B}(V_{B} - V_{L})^{2} = 0$$

$$T = W_{B} + \overline{F}_{BL} \pm \sqrt{(W_{B} + F_{BL})^{2} + KM_{B}(V_{B} - V_{L})^{2}}$$

The negative square root is discarded, since it represents negative tension, i.e., compression, which cannot be supported by a towline.

For 
$$V_B - V_L \rightarrow 0$$
,  
 $T = 2 (W_B + \overline{F}_{BL})$ ,

the well-known result for suddenly applied loads.

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If the towline is considered inextensible away from the body in any computer solution, the change in towline shape can be associated with the accompanying change in tension. There,  $K_x$  can be found by developing a functional relation between the tension and the longitudinal displacement of the towline. Such a relationship has been computed, using the dynamic computer program of reference (a), for the system towed by Contrata (b). The parameters of this system are given in Appendix A. Figure 3 is a plot of dynamic change in tension,  $T_d$ , versus longitudinal displacement of the towline,  $X_d$ .

 $X_{d} = 1.25 \times 10^{-5T} d$ 

 $0 < T_d < 1200$  pounds

Hence K<sub>x</sub> becomes:

$$K_x = \frac{1}{1.25 \times 10^{-5}} = .8 \times 10^5 \ \#/ft.$$

Using the quantities of Appendix A,

$$K_e = \frac{1.42 \times 10^7}{75} = 1.89 \times 10^5 \ \#/ft.$$

Although  $K_x$  could only be evaluated for tensions up to 1200 pounds, it is significant to note than  $K_x$  is the same order of magnitude as  $K_\epsilon$ . Also as more tension is added,  $K_x$  will increase. Therefore the equivalent spring constant may be slightly underestimated by using the linear approximation as the tension in the towline becomes large. Using the above values of  $K_x$  and  $K_\epsilon$  the equivalent spring constant becomes:

$$K = 5.6 \times 10^4 \ \#/ft.$$

Substituting the values of  $W_B$ ,  $M_B$ ,  $V_B$ , and  $V_L$  from Appendix A, and K from above into (11):

T = 
$$3260 + (0.3)(30.66) \left(\frac{12.5 + 8.15}{2}\right)^2$$

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$$+\sqrt{\left[3260+(0.3)(30.66)\left(\frac{12.5+8.15}{2}\right)^2\right]^2+(5.6\times10^4)(1.3)(266)(12.5-8.15)^2}$$

T = 23,842 pounds

This compares favorably to the experimental value of Contrata, which was:

T = 22,000 to 30,000 pounds.

### CONCLUSIONS

Certain general conclusions can be made about the slack towline study:

(1) A slack towline is the result of the component of ship motion directed along the towline. Since,  $\phi$ , towline angle inclination at the surface (and hence the motion along the towline) decreases as the towing speed and the towline length increase, the possibility of experiencing a slack towline at high towing speeds and with long towlines is decreased.

(2) The "recapture" tension increases with increasing equivalent spring constant K of the towline. However, the equivalent spring constant decreases with increasing towline length and towing speed. Therefore, the recapture tension decreases with increasing towline length and towing speed.

(3) The quantity  $H_{T} \omega^2$  in (7) and (8) represents the maximum acceleration amplitude experienced by the stern of the ship. In this report, a sinusoidal ship's motion was assumed, however, this is not in general the case. Kreitner (c) has done a statistical study of the accelerations experienced by the fantail of a destroyer in various seaways at a speed of 17 knots. If this study were extended to include other speeds, and some assumptions were made regarding velocity behavior, realistic criteria based on ship speed, sea state, and towline length could be developed for slack towline behavior. In general, it is clear that high stern acceleration decreases the

ratio  $\left(\frac{W_{\rm B}}{(a_{\rm L})_{\rm max}}\right)$ , and if this ratio becomes less than unity, then

certainly slack towline conditions appear.

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#### REFERENCES

- (a) Schram, Jeffrey W., "A Three Dimensional Analysis of a Towed System," Rutgers University, July 1967.
- (b) Contrata, F. J., "Results of Initial Experiments to Confirm Calculations of Slack Towline," USL Technical Memorandum No. 933-121-64, 1964.
- (c) Kreitner, C. W., "A Computer Aided Investigation of Destroyer Stern Acceleration," Webb Institute, 24 May 1965.

# APPENDIX A

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The following are the parameters of the system tested by Contrata:

A <sub>B</sub> V	=	30.66 ft <sup>2</sup>
A <sub>L</sub> max	=	ll.3 ft/sec <sup>2</sup>
°	<b>s</b>	•3
W <sub>B</sub>	=	3260 12.
HL	=	8 ft.
MB	=	1.3 x 266 slugs
P/2	=	1.0 slugs/ft <sup>3</sup>
A	=	1.42 in <sup>2</sup>
E	=	$10 \times 10^6$ lb/in <sup>2</sup>
L	\$	75 ft
v <sub>L</sub> *	=	8.15 ft/sec
V <sub>B</sub> *	=	12.5 ft/sec

\* These values were computed by Contrata (b).

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FIGURE 2

USL Tech Memo No. 2133-374-69 0 1200 1000 H 0 CHANGE IN TOWLINE TENSION, 800 FIGURE 3 600 400 0 200 00 ·015 .010 600. .020 0.0 **p**<sub>x</sub>q CONCITUDINAL DISPLACEMENT OF TOWLINE,

