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FORM OF INSTABILITY OF STEADY CONVECTIVE MOVEMENT CAUSED BY INT--ETC(U)
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AIR FORCE SYSTEMS COMMAND

FORM OF INSTABILITY OF STEADY CONVECTIVE
MOVEMENT CAUSED BY INTERNAL HEAT SOURCES

by

A. A. Yakimov

FOREIGN TECHNOLOGY DIVISION

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EDITED TRANSLATION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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FORM OF INSTABILITY OF STEADY CONVECTIVE MOVEMENT CAUSED BY INTERNAL HEAT SOURCES

A. A. Yakimov

The stability of steady convective movement caused by internal heat sources was studied earlier in [1, 2]. The spectra of the decrements of the disturbances and the neutral curves were obtained for different values of the Prandtl number. This report considers the form of the disturbances of convective movement.

1. Heat sources with volumetric density Q are uniformly distributed in a plane vertical layer of viscous fluid with a width of 2h. The layer is assumed to be closed from above and below, and the vertical walls which bound it are held at the same temperatures. Stationary plane-parallel convective movement with velocity and

temperature profiles which are even relative to the axis of the channel, found from the ordinary equations of convection with consideration of internal heat sources [1], originate in this channel due to internal heating. If we take h, h^2/ν, $g\beta q h^4/2\nu$, $qh^2/2$, $\rho g\beta h^2/2$ ($q = Q/\rho c_p\chi$) as the units of distance, time, velocity, temperature, and pressure, respectively, the dimensionless velocity and temperature profiles will be

$$v_0 = 1/60(1-6x^2+5x^4), \quad (1)$$

$$T_0 = 1 - x^2. \quad (2)$$

Small plane normal disturbances

$$\psi(x, z, t) = \varphi(x)\exp(-\lambda t + ikz), \quad (3)$$

$$T(x, z, t) = \theta(x)\exp(-\lambda t + ikz), \quad (4)$$

are imposed on the main flow, where φ and θ are the amplitudes of the oscillations, λ is the decrement, and k is the wave number.

Substituting (3) and (4) in the convection equation and considering the smallness of the disturbances, we will obtain a system of linear homogeneous differential equations for determining intensities $\varphi(x)$ and $\theta(x)$ [1]:

$$\Delta^2\varphi - ikGH\varphi + \theta' = -\lambda\Delta\varphi, \quad (5)$$

$$P^{-1}\Delta\theta + ikG(T_0'\varphi - v_0\theta) = -\lambda\theta \quad (6)$$

$$(\Delta\varphi \equiv \varphi'' - k^2\varphi,\ H\varphi \equiv v_0\Delta\varphi - v_0''\varphi,\ G = g\beta q h^5/2\nu^2,\ P = \nu/\chi)$$

with the homogeneous boundary conditions

$$\varphi=\varphi'=0,\ \theta=0 \qquad \text{at } x=\pm 1. \tag{7}$$

2. We will use the Galerkin method to solve boundary problem (5)-(7). We will find φ and θ in the form of the superimposition of the basic functions

$$\varphi=\sum_{i=1}^{N} a_i\varphi_i,\ \theta=\sum_{k=1}^{M} b_k\theta_k. \tag{8}$$

We will take the intensity of the disturbances in a quiescent fluid, determined from the boundary problem

$$\Delta^2\varphi_i=-\mu_i\Delta\varphi_i,\ \varphi_i=\varphi_i'=0, \text{ at } x=\pm 1, \quad (i=1,2,\ldots,N), \tag{9}$$

as the basic functions φ_i, and the intensity of the temperature perturbations, determined by the problem

$$P^{-1}\Delta\theta_k+\nu_k\theta_k=0,\quad \theta_k=0 \text{ at } x=\pm 1 \quad (k=1,2,\ldots,M) \tag{10}$$

as the basic functions θ_k (the explicit form of the basic functions is given in [3], for example).

The requirement of the orthogonality of the discrepancies in the basic functions leads to a system of homogeneous linear algebraic equations for coefficients a_i and b_k.

The condition of the existence of a nonzero solution for this system determines the spectrum of the characteristic decrements of disturbances λ depending on the Grashof number N, the Prandtl number P and the wave number k. The characteristic decrements λ are defined as the intrinsic values of the system matrix.

The expansion coefficients a_l and b_k are the components of the latent vector which corresponds to characteristic number λ. This vector was found as follows. The value of λ was substituted in the characteristic equation

$$|A-\lambda E|=0, \tag{11}$$

where A is the system matrix and E is the unit matrix. One equation was deleted from the system obtained. For the best conditionality of the matrix obtained, the equation with the minimum modulus of the coefficient in the diagonal term was selected as this equation. One of the unknowns (a_k or b_k) was assigned a random value (e.g., "-1"), and the system thus obtained (with complex elements) was solved by the method of primary elements.

3. The normal disturbances ψ(x, z, t) and T(x, z, t) with a certain amplitude a, which remains arbitrary when staying within the bounds of the linear theory of stability, are added to the main flow and distort it. We will plot the current lines and isotherms of the disturbed total movement.

At an arbitrary fixed point in time t_0, the equation of the current line of disturbed movement is

$$\psi_0(x)+a\psi(x, z, t_0)=C_1,$$

where $\psi_0(x)$ is the current function of the primary flow, and C_1 is a certain constant.

If disturbances $\psi(x, z, t)$ are determined from formula (3), the current line equation assumes the following form:

$$\psi_0(x)+a_1\varphi(x)e^{ikz}=C_1. \qquad (12)$$

Factor $e^{\lambda t_0}$ only affects the intensity of the disturbances; therefore, we can set $t_0 = 0$ without affecting continuity.

Considering the complex form of $\varphi(x)$ and C_1, equation (12) can be rewritten as:

$$\psi_0(x)+a_1[\varphi_r(x)\cos kz-\varphi_i(x)\sin kz]=A_1, \qquad (13)$$

where

$$\varphi_r(x)=Re\,\varphi(x),\ \varphi_i(x)=\operatorname{Im}\varphi(x),$$

and a_1 and A_1 are real constants.

The isolines are plotted as follows. The corresponding values of x are found from equation (13) for a given value of A_1 and a fixed value of x. Breaking down the change interval x into a sufficiently large number of parts, we will obtain the current line for the selected value of A_1. Varying A_1 with a certain spacing, we will find the family of equidistant current lines. The equation for the isotherms

$$T_0(x) + a_1[\theta_r(x)\cos kz - \theta_i(x)\sin kz] = B_1, \qquad (14)$$

is found analogously, where

$$\theta_r(x) = Re\,\theta(x),\ \theta_i(x) = \mathrm{Im}\,\theta(x),$$

and the family of isotherms of the disturbed movement is plotted.

4. The results of the numerical calculations are given below. The intrinsic values of the system matrix were found using the QR algorithm realized on Aragats and M-220M computers of the Computer Center of the Perm' State University [4]. Six to fifteen functions of each type were taken in expansions (8). The latent vector, isolines and isotherms were found on an M-220M computer.

Figure 1 shows the neutral curve for the Prandtl number P = 10 plotted from the materials in [2]. As this study establishes, at sufficiently large values of the Prandtl number, the neutral curve consists of two branches. The short-wave branch corresponds to hydrodynamic disturbances drifting slowly along the channel. The long-wave branch corresponds to heat wave disturbances, the phase velocity of which is close to the maximum flow velocity[1].

Footnote: [1]For comparison, the broken line in Fig. 1 shows the neutral curve for P = 0 taken from [1], which characterizes the development of hydrodynamic disturbances alone. End footnote

It is interesting to trace the form of the disturbances corresponding to both branches.

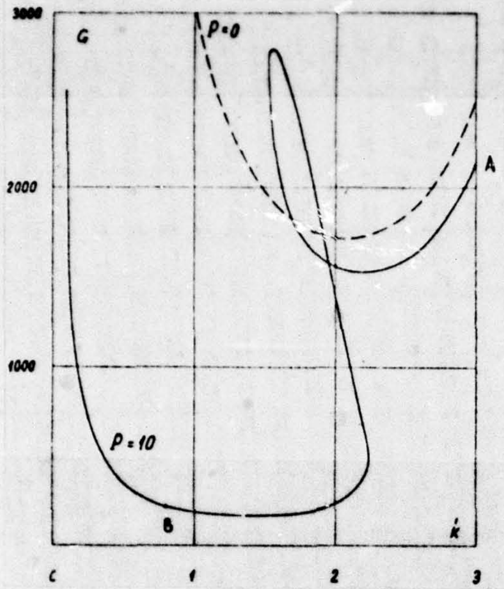


Fig. 1.

Figure 2 shows the current lines and isotherms of the total convective movement plotted for point A (see Fig. 1), which lies on the hydrodynamic branch of the neutral curve ($k = 3$; $G = 2180$). Figure 3 corresponds to point B, which is located on the thermal branch ($k = 0.8$; $G = 230$). For convenience of illustration, the vertical scale in this figure is one-fourth of the full scale.

Fig. 2. KEY: (1) Current lines. (2) Isotherms.

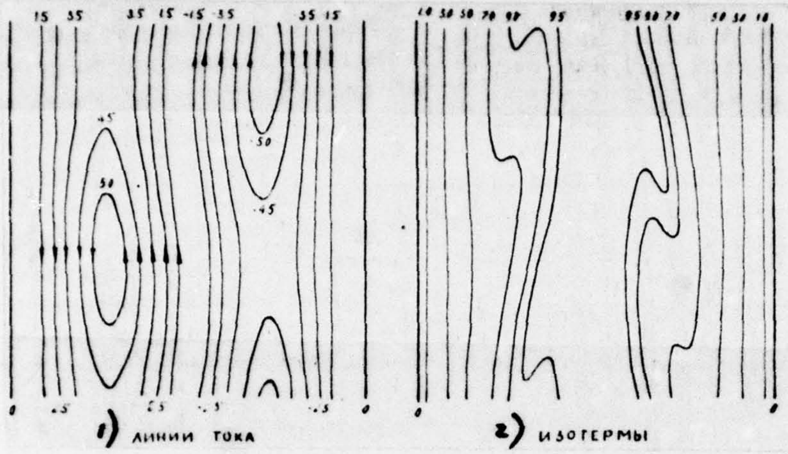
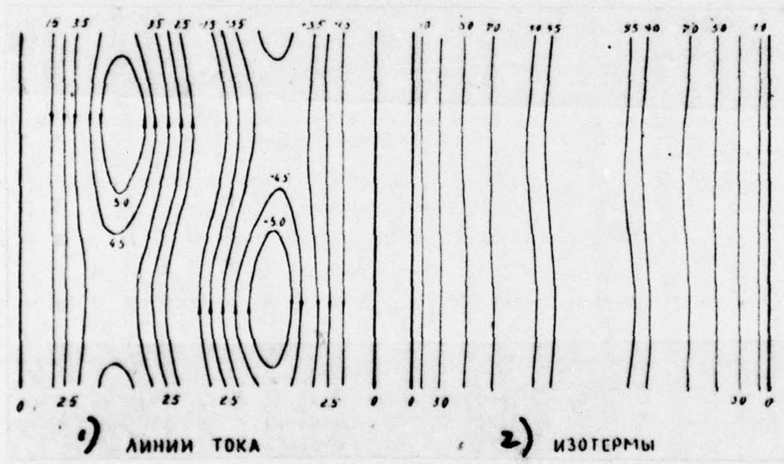


Fig. 3. KEY: (1) Current lines. (2) Isotherms.



The values of the current function indicated in the figures are increased 10^3 times, and the temperature values - ten times. When comparing the figures one should remember that constant a_1 used in formulae (13) and (14) was 2.5 times larger for point A than for point B when plotting the isolines.

These figures show that in both cases, instability develops in the form of two vortex chains which alternate on the interfaces of the convective flows. Thus, although hydrodynamic disturbances and rising heat flux disturbances are related to two different modes of the instability spectrum, there is no essential difference in their form. However, the difference is that heat waves have a relatively high phase velocity compared to hydrodynamic disturbances.

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