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FOREIGN TECHNOLOGY DIVISION

GENERALIZATION OF A. N. KOLMOGOROV'S CRITERION FOR EVALUATING THE LAW OF DISTRIBUTION BY EMPIRICAL DATA

by

G. M. Maniya

EDITED TRANSLATION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α α *α*	Nu	Ν ν
Beta	Β β	Xi	Ξ ξ
Gamma	Γ γ	Omicron	Ο ο
Delta	Δ δ	Pi	Π π
Epsilon	Ε ε ϵ	Rho	Ρ ρ ϱ
Zeta	Ζ ζ	Sigma	Σ σ ς
Eta	Η η	Tau	Τ τ
Theta	Θ θ ϑ	Upsilon	Υ υ
Iota	Ι ι	Phi	Φ φ ϕ
Kappa	Κ ϰ κ	Chi	Χ χ
Lambda	Λ λ	Psi	Ψ ψ
Mu	Μ μ	Omega	Ω ω

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RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

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1672

GENERALIZATION OF A. N. KOLMOGOROV'S CRITERION FOR EVALUATING THE LAW OF DISTRIBUTION BY EMPIRICAL DATA

G. M. Maniya

(Presented by Academician A. N. Kolmogorov on 7 October 1949)

Let $x_1, x_2, \ldots, x_n$ be a set of independent values with the general continuous law of distribution F(x). Furthermore, let $x_1^*, x_2^*, \ldots, x_n^*$ be the same series, but in order of their magnitudes.

We will call the empirical distribution function step function $s_n(x)$:

$$S(x) = \begin{cases} 0 \text{ at } x < x_1^*, \\ k/n \text{ at } x_k^* \leqslant x < x_{k+1}^*, \\ 1 \text{ at } x \geqslant x_n^*. \end{cases}$$

We will designate

$$D_n = \sup_{-\infty < x < \infty} |S_n(x) - F(x)|,$$
$$D_n^+ = \sup_{-\infty < x < \infty} \{S_n(x) - F(x)\}.$$

According to the known theorem proven by A. N. Kolmogorov [1], for each $\lambda \geqslant 0$ and random continuous distribution function F(x)

$$P\left\{D_n < \frac{\lambda}{\sqrt{n}}\right\} \xrightarrow[n \to \infty]{} \Phi(\lambda) = 1 - 2\sum_{\nu=1}^{\infty} (-1)^{\nu-1} e^{-2\nu^2\lambda^2}. \quad (1)$$

In one of his results [2], N. V. Smirnov establishes the asymptotic formula for distribution D_n^+:

$$P\left\{D_n^+ < \frac{\lambda}{\sqrt{n}}\right\} \xrightarrow[n \to \infty]{} 1 - e^{-2\lambda^2}. \quad (2)$$

We will generalize A. N. Kolmogorov and N. V. Smirnov's correspondence criteria, considering the maximum deviation for a specific section $(0 < \theta_1 < \theta_2 < 1)$ of the growth of function F(x).

We will find two random values:

$$D_n^+(\theta_1, \theta_2) = \sup_{\theta_1 < F(x) < \theta_2} \{S_n(x) - F(x)\},$$

$$D_n(\theta_1, \theta_2) = \sup_{\theta_1 < F(x) < \theta_2} |S_n(x) - F(x)|.$$

The results we obtained can be stated in the form of the following two theorems:

Theorem 1. Let F(x) be the continuous function of the distribution of each of the independent values x_i $(i = 1, 2, \ldots, n)$,

$$\theta_1^{(n)} = \frac{m_1}{n} = \theta_1 + o\left(\frac{1}{\sqrt{n}}\right), \text{ и } \theta_2^{(n)} = \frac{m_2}{n} = \theta_2 + o\left(\frac{1}{\sqrt{n}}\right), \; 0 < \theta_1 < \theta_2 < 1.$$

Then

$$P\left\{D_n^+(\theta_1^{(n)}, \theta_2^{(n)}) \leqslant \frac{\lambda}{\sqrt{n}}\right\} \xrightarrow[n \to \infty]{} \Phi(\theta_1, \theta_2; \lambda),$$

where

$$\Phi^+(\theta_1, \theta_2; \lambda) = \frac{1}{2\pi\sqrt{1-R^2}} \int_{-\infty}^{a} \int_{-\infty}^{b} e^{-\frac{1}{2}\bar{\theta}(z_1, z_2)}\, dz_1\, dz_2 -$$

$$- \frac{e^{-2\lambda^2}}{2\pi\sqrt{1-R^2}} \int_{-\infty}^{a'} \int_{-\infty}^{b'} e^{-\frac{1}{2}\theta(z_1, z_2)}\, dz_1\, dz_2,$$

$$a = \frac{\lambda}{\sqrt{\theta_1(1-\theta_1)}}, \quad b = \frac{\lambda}{\sqrt{\theta_2(1-\theta_2)}}, \quad a' = \frac{\lambda - 2\lambda\theta_1}{\sqrt{\theta_1(1-\theta_1)}}, \quad b' = \frac{\lambda - 2\lambda(1-\theta_2)}{\sqrt{\theta_2(1-\theta_2)}},$$

$$R = \sqrt{\frac{\theta_1(1-\theta_2)}{\theta_2(1-\theta_1)}},$$

$$\theta(z_1, z_2) = \frac{1}{1-R^2}[z_1^2 + 2Rz_1z_2 + z_2^2], \quad \bar{\theta}(z_1, z_2) = \frac{1}{1-R^2}[z_1^2 - 2Rz_1z_2 + z_2^2].$$

Function $\Phi^+(\theta_1, \theta_2; \lambda)$ can also be represented as follows:

$$\Phi^{+}(\theta_1, \theta_2; \lambda) = \sum_{n=0}^{\infty} \frac{(+R)^n}{n!} \Phi^{(n)}(a) \Phi^{(n)}(b) - e^{-2\lambda^2} \sum_{n=0}^{\infty} \frac{(-R)^n}{n!} \Phi^{(n)}(a') \Phi^{(n)}(b').$$

Here $\Phi^{(n)}(x)$ is the n-th order derivative of the normal integral

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2} dz.$$

In the most interesting specific case, when $\theta_1 = 1 - \theta_2 = \theta$, we will have

$$\Phi^{+}(\theta; \lambda) = \frac{1}{2\pi\sqrt{1-R^2}} \int_{-\infty}^{c} \int_{-\infty}^{c} e^{-\frac{1}{2}\tilde{\theta}(z_1, z_2)} dz_1 dz_2 -$$

$$- \frac{e^{-2\lambda^2}}{2\pi\sqrt{1-R^2}} \int_{-\infty}^{c'} \int_{-\infty}^{c'} e^{-\frac{1}{2}\theta(z_1, z_2)} dz_1 dz_2,$$

where

$$c = \frac{\lambda}{\sqrt{\theta(1-\theta)}}, \quad c' = \frac{\lambda - 2\lambda\theta}{\sqrt{\theta(1-\theta)}}.$$

Whence we will obtain (2) at $\theta = 0$.

Theorem 2. Under the conditions in theorem 1

$$P\{D_n(\theta_1^{(n)}, \theta_2^{(n)}) \leqslant \lambda n^{-1/2}\} \xrightarrow[n\to\infty]{} \Phi(\theta_1, \theta_2; \lambda),$$

whereupon

$$\Phi(\theta_1, \theta_2; \lambda) = \frac{1}{2\pi\sqrt{1-R^2}} \int_{-a}^{a} \int_{-b}^{b} e^{-1/2\,Q(z_1, z_2)}\, dz_1\, dz_2 -$$

$$-\frac{2\sum_{k=1}^{\infty} (-1)^{k-1} e^{-2k^2\lambda^2}}{2\pi\sqrt{1-R^2}} \int_{-\alpha_k}^{\alpha_k} \int_{-\beta_k}^{\beta_k} e^{-1/2\,Q(z_1, z_2)}\, dz_1\, dz_2,$$

where

$$\alpha_k = \frac{\lambda - 2k\lambda\theta_1}{\sqrt{\theta_1(1-\theta_1)}}, \quad \beta_k = \frac{\lambda - 2k\lambda(1-\theta_2)}{\sqrt{\theta_2(1-\theta_2)}}.$$

We can represent function $\Phi(\theta_1, \theta_2; \lambda)$ in a different form as follows:

$$\Phi(\theta_1, \theta_2; \lambda) = \sum_{n=0}^{\infty} \frac{(-R)^n}{n!} \Phi^{(n)}(a)\, \Phi^{(n)}(b) -$$

$$-2\sum_{k=1}^{\infty} (-1)^{k-1} e^{-2k^2\lambda^2} \sum_{n=0}^{\infty} \frac{(-R)^n}{n!} \Phi^{(n)}(\alpha_k)\, \Phi^{(n)}(\beta_k).$$

In particular, when $\theta_1 = 0$, $\theta_2 = 1$ we obtain (1), i.e., the case which was first considered by A. N. Kolmogorov.

When $\theta_1 = 1 - \theta_2 = \theta$, we will have a more compact and symmetrical expression for the limiting function.

This method makes it possible to theoretically solve the problem of the applicability of the theoretical law at those boundaries where the material which is available to us is more reliable for comparison.

The proofs of theorems 1 and 2 are based on the theorems of continuity of random functions and the Laplace transform.

Moscow City Pedagogical Institute imeni V. P. Potemkin

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[1] А. Н. Колмогоров, Giorn. d. Att., 4, 83 (1933). [2] Н. В. Смирнов, Матем. сборн., 6 (48), 3 (1939). [3] W. Feller, Ann. of Math. Statistics, 19, 2, 177 (1948).

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