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ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CATEGORIES VI: PROPORTIONED SUM PROCEDURE IN THE CONDITIONAL P.D.F. APPROACH

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ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CATEGORIES VI: PROPORTIONED SUM PROCEDURE IN THE CONDITIONAL P.D.F. APPROACH

ABSTRACT

Following Simple Sum Procedure and Weighted Sum Procedure, another method, Proportioned Sum Procedure, is introduced in the context of the Conditional P.D.F. Approach. The new method is somewhat different from the previous two, however, in the sense that the set of conditional density functions is not exclusively recategorized into the item score groups, but they are proportioned into each item score category. The same hypothetical data, i.e., the maximum likelihood estimates of the five hundred hypothetical subjects and their responses to the ten binary items, each of which follows the normal ogive model, are used to try the method. The criterion item characteristic function for each binary item is obtained and compared with those obtained by the other two procedures. The Pearson System Method and the Two-Parameter Beta Method are used for both Degree 3 and 4 Cases, and the results are compared with the previous ones. The mean square errors are adopted in evaluating both the resultant estimated item characteristic functions and probability density functions of ability. The two item parameters in the normal ogive model are also estimated for each item, and the results are compared with the previous ones.

The research was conducted at the principal investigator's laboratory, 409 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked for her as assistants at various times include Paul S. Changas, Robert L. Trestman and Philip S. Livingston.

TABLE OF CONTENTS

I Introduction

Following previous studies (Samejima, 1977b, 1977d, 1978a, 1978b, 1978c, 1978d), the present paper proposes another method for estimating the operating characteristics of the graded items (Samejima, 1969, 1972), in which:

(1) no mathematical forms are assumed for the operating characteristics;

(2) a relatively small number of examinees are used in the whole process of estimation;

(3) the maximum likelihood estimates of the examinees are available because of a previous testing; and

(4) the test which was used for (3) has a constant test information function for the range of ability of our interest, and the amount of information is substantially large.

The process of the maximum likelihood estimation using the response pattern of a specified examinee has been described (Samejima, 1969). The asymptotic property of the maximum likelihood estimate (Samejima, 1975), i.e., the normality of its conditional distribution, given ability, with the given ability itself and the inverse of the test information function as its two parameters, is fully used. Since the constancy of the test information has many theoretical advantages (Samejima, 1977a), the estimation of the conditional moments of ability, given its maximum likelihood estimate, is relatively easy*, provided that the density function of the maximum likelihood estimate is given. By virtue of the concept of weakly parallel tests (Samejima, 1977c), the results are directly comparable with those of similar studies using

* Refer to Appendix II.

different sets of test items but following the same principles, (1) through (4).

The actual data used in the present study are simulated data of five hundred hypothetical examinees, and our target is to estimate the item characteristic function, i.e., the operating characteristic of the item score 1 when the item is binary, of each of the ten binary items. A brief description of these data is given as Appendix I. The ten binary items follow the normal ogive model, and the two parameters, discrimination and difficulty indices, are given in Tables 6-3, 6-4, etc., for each item.

Throughout the study, the method of moments (Elderton and Johnson, 1969, Johnson and Kotz, 1970) is frequently used. Among others, the density function of the maximum likelihood estimate is approximated by a polynomial of degree 3 or 4, applying the method of moments for the set of 500 maximum likelihood estimates.

II Conditional P.D.F. Approach

Conditional P.D.F. has been presented and developed (Samejima, 1978a, 1978b, 1978d). Let $f(\theta)$ be the density function of ability, or latent trait, θ , $\psi(\hat{\theta}|\theta)$ be the conditional density of $\hat{\theta}$, given θ , which is approximated by $n(\theta,\sigma^2)$ where σ^2 is the inverse of the test information function, $I(\theta) = 21.63$, and $g(\hat{\theta})$ is the density function of the maximum likelihood estimate, $\hat{\theta}$. We can write for the conditional density function of $\hat{\theta}$, given θ ,

$$(2.1) \qquad \phi(\theta|\hat{\theta}) = \psi(\hat{\theta}|\theta)\, f(\theta)\, [\int_{-\infty}^{\infty} \psi(\hat{\theta}|\theta)\, f(\theta)\, d\theta]^{-1}$$

$$= \psi(\hat{\theta}|\theta)\, f(\theta)\, [g(\hat{\theta})]^{-1} .$$

Let $P_{x_g}(\theta)$ be the operating characteristic of the graded item score category x_g . The estimate of the operating characteristic, $\hat{P}_{x_g}(\theta)$, in the Conditional P.D.F. Approach can be written in the general form

$$(2.2) \qquad \hat{P}_{x_g}(\theta) = \sum_{s\varepsilon G_{x_g}} w(\hat{\theta}_s)\, \hat{\phi}(\theta|\hat{\theta}_s)\, [\sum_{s=1}^{N} w(\hat{\theta}_s)\, \hat{\phi}(\theta|\hat{\theta}_s)]^{-1} ,$$

where G_{x_g} indicates the group of examinees having the item score x_g , N (=500) is the number of examinees, and $w(\hat{\theta}_s)$ is the weight assigned to the maximum likelihood estimate of the examinee s . When this weight is constant for all the examinees, (2.2) is called <u>Simple Sum Procedure</u>, to distinguish itself from the <u>Weighted Sum Procedure</u> (Samejima, 1978d).

To use (2.2), it is necessary to obtain the estimated conditional density of θ , given $\hat{\theta}$, and so far three methods, i.e., Pearson System Method, Normal Approach Method and Two-Parameter Beta Method, have been used. In the first method, $\phi(\theta|\hat{\theta}_s)$ is approximated by one of the Pearson System density functions, depending upon the value of Pearson's criterion κ , which is obtained from the first four conditional moments of θ , given $\hat{\theta}$. In the second method, $\phi(\theta|\hat{\theta}_s)$ is approximated by a normal density function, using the first two conditional moments of θ , given $\hat{\theta}$, as the parameters. In the third method, we use a Beta density function for $\hat{\phi}(\theta|\hat{\theta}_s)$, with a priori set two parameters, which are the two endpoints of the interval for which the Beta density function assumes positive values, and the other two parameters estimated from the first two conditional moments of θ , given $\hat{\theta}$. (For further detail, cf. Samejima, 1977d, 1978a, 1978b, 1978d).

In the present study, the concept of Conditional P.D.F. Method is expanded and generalized to mean that the estimated conditional density $\hat{\phi}(\theta|\hat{\theta})$ is fully utilized, without using any estimated joint density of θ and $\hat{\theta}$. In the following section, therefore, a new procedure is proposed which is <u>not</u> based on (2.2).

III Proportioned Sum Procedure

Let $p(s \varepsilon G_{x_g})$ be the probability with which the examinee s belongs to the group whose item score for item g is x_g, and $\hat{p}(s \varepsilon G_{x_g})$ be its estimate. In the present method, the estimated operating characteristic, $\hat{P}_{x_g}(\theta)$, is defined by

$$(3.1) \qquad \hat{P}_{x_g}(\theta) = \sum_{s=1}^{N} \hat{p}(s \varepsilon G_{x_g}) \hat{\phi}(\theta|\hat{\theta}_s) [\sum_{s=1}^{N} \hat{\phi}(\theta|\hat{\theta}_s)]^{-1} ,$$

where

$$(3.2) \qquad \sum_{x_g=0}^{m_g} p(s \varepsilon G_{x_g}) = \sum_{x_g=0}^{m_g} \hat{p}(s \varepsilon G_{x_g}) = 1 .$$

The main difference between (3.1) and (2.2) is that in the numerator of (3.1) the estimated conditional densities for all the $\hat{\theta}_s$'s participate, while that of (2.2) includes $\hat{\phi}(\theta|\hat{\theta}_s)$ for the examinees who belong to the group G_{x_g} only. The procedure represented by (3.1) will be called Proportioned Sum Procedure of the Conditional P.D.F. Approach.

Since our data used in the present study are for estimating the operating characteristics of binary items, it is sufficient to estimate the item characteristic function, i.e., the operating characteristic of $x_g = 1$, of each item. Let $P_g(\theta)$ be the item characteristic function of item g. We can rewrite (3.1) such that

$$(3.3) \qquad \hat{P}_g(\theta) = \sum_{s=1}^{N} \hat{p}(s \varepsilon G) \hat{\phi}(\theta|\hat{\theta}_s) [\sum_{s=1}^{N} \hat{\phi}(\theta|\hat{\theta}_s)]^{-1} ,$$

where G is the group of examinees whose item score for item g is 1.

The estimated probability, $\hat{p}(s\varepsilon G)$, or the proportioned weight, in the present study is given by the proportion of the examinees who belong to the group G within a specified interval whose midpoint is $\hat{\theta}_s$. This interval is more or less arbitrary, and we have taken $\hat{\theta}_s \pm \sigma^2$ and $\hat{\theta}_s \pm 2\sigma^2$, where σ^2 is the inverse of the constant test information (=21.63, cf. Appendix I) and also the second parameter of the conditional distribution of $\hat{\theta}$, given θ . Thus these intervals are $\hat{\theta}_s \pm 0.215$ and $\hat{\theta}_s \pm 0.430$, respectively.

If we replace $\hat{\phi}(\theta|\hat{\theta}_s)$ in (3.3) by $\phi(\theta|\hat{\theta}_s)$, then we will obtain the estimated item characteristic function which indicates the maximal possible attainment for the present procedure and data. This function is not obtainable from empirical data, and only the simulation study can provide us with it. As usual, we shall call it the <u>criterion item characteristic function</u>. Since we adopted two different sets of proportioned weights, as was mentioned in the preceding paragraph, we shall obtain two criterion item characteristic functions for each item.

For the same reason described in the preceding study (Samejima, 1978d), we use the two methods, i.e., <u>Pearson System Method</u> and <u>Two-Parameter Beta Method</u>, for obtaining $\hat{\phi}(\theta|\hat{\theta}_s)$, and exclude Normal Approach Method.

IV Results I: Proportioned Weights

Figure 4-1 presents the estimated probability, or proportioned weight, $\hat{p}(s \in G)$, using each of the two intervals, $\hat{\theta}_s \pm 0.215$ and $\hat{\theta}_s \pm 0.430$, for each of the 500 maximum likelihood estimates $\hat{\theta}_s$. We can see that these two sets of results, plotted by solid triangles and crosses respectively, are for the most part overlapping, except for some extreme values of $\hat{\theta}_s$. This finding suggests that we are likely to obtain similar estimated item characteristic functions as the results of using these two sets of proportioned weights.

It should be noted that these proportioned weights adopted in the Proportioned Sum Procedure in the present study themselves can be regarded as estimates of the item characteristic function of each binary item. Since we have the relationship (Samejima, 1977b)

(4.1) $$\mathrm{Var.}(\hat{\theta}) \doteq \mathrm{Var.}(\theta) + \sigma^2 ,$$

and we obtain

(4.2) $$E(\theta) = 0 ,$$

(4.3) $$\mathrm{Var.}(\theta) = \int_{-\infty}^{\infty} \theta^2 f(\theta)\, d\theta = 0.2 \left[\frac{1}{3}\theta^3 \Big|_{-2.5}^{2.5}\right] = \frac{25}{12} \doteq 2.083333 ,$$

(4.4) $$\mathrm{Var.}(\hat{\theta})^{1/2} \doteq [2.083333 + 0.046225]^{1/2} \doteq 1.459301$$

and

(4.5) $$\mathrm{Var.}(\theta)^{1/2} \doteq 1.443376 ,$$

it is expected that the abscissa be stretched by approximately 1.1 %

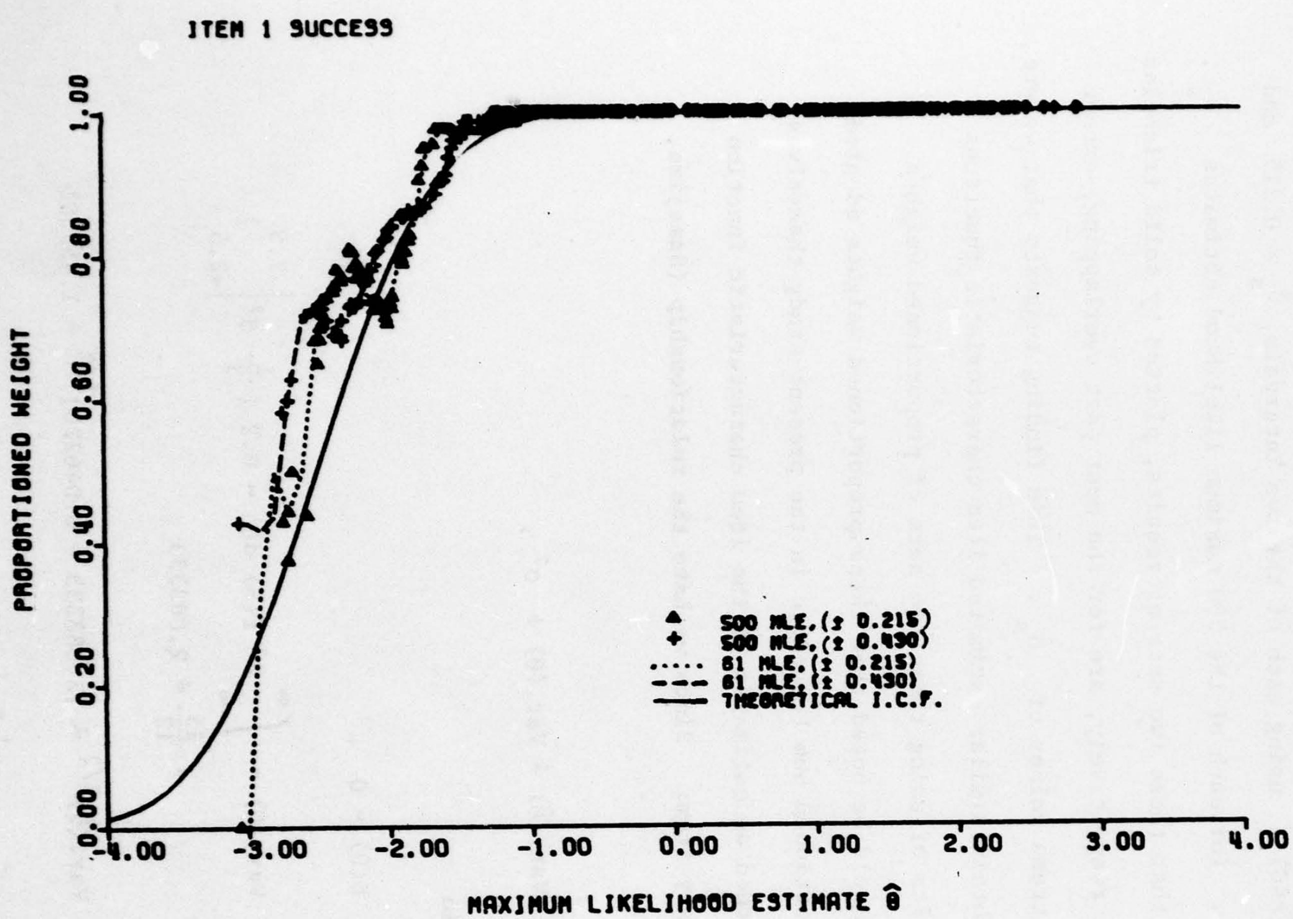
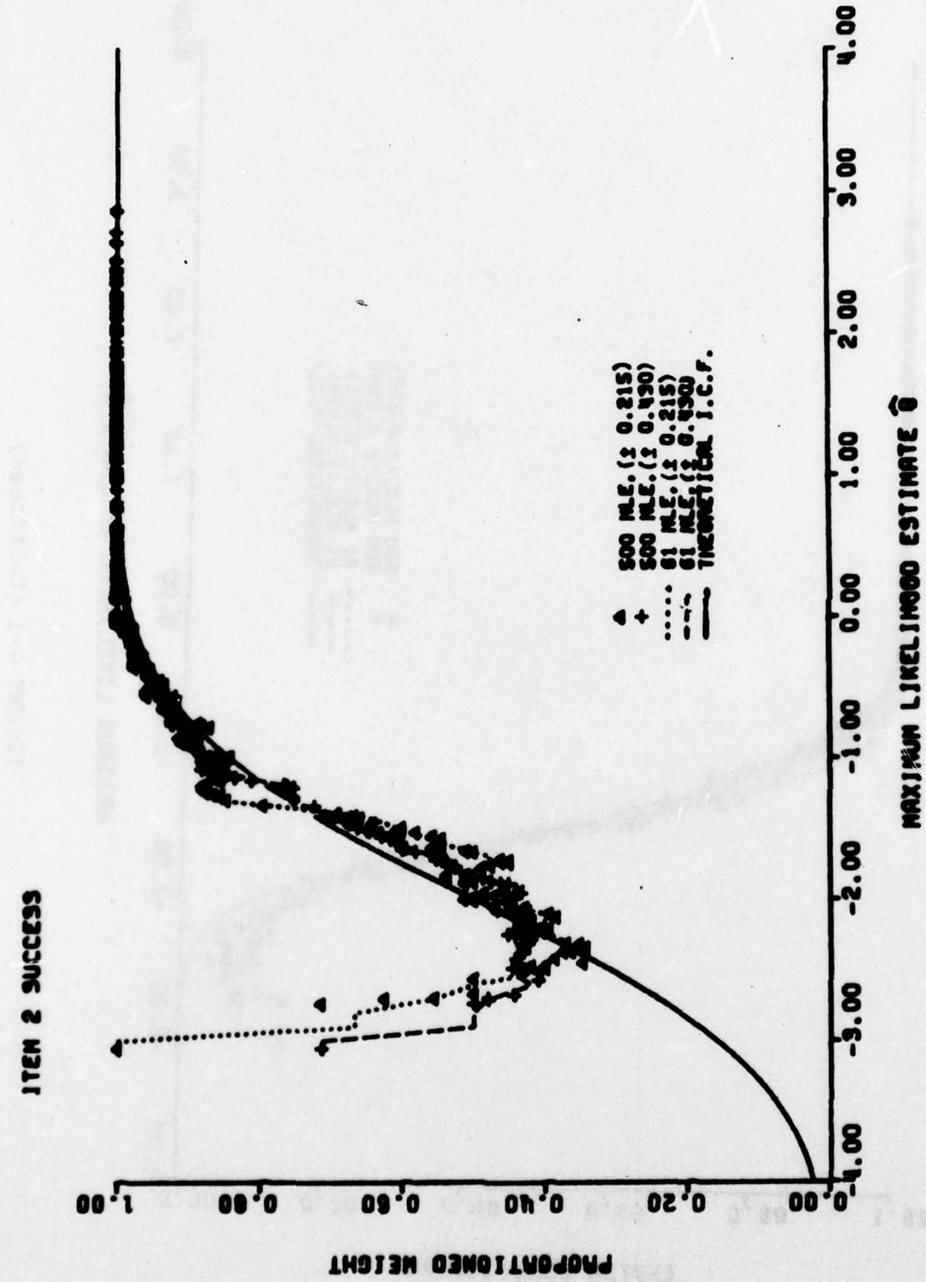


FIGURE 4-1

Four Sets of Proportioned Weights for the Success Group of the Item, Used in the Proportioned Sum Procedure, Presented with the Item Characteristic Function. Two of Them Are Based on the 500 Maximum Likelihood Estimates (MLE), with the Intervals ±0.215 and ±0.430, Respectively, and the Other Two Are Based on the 61 Equally Intervalled Points of θ with the Respective Intervals.



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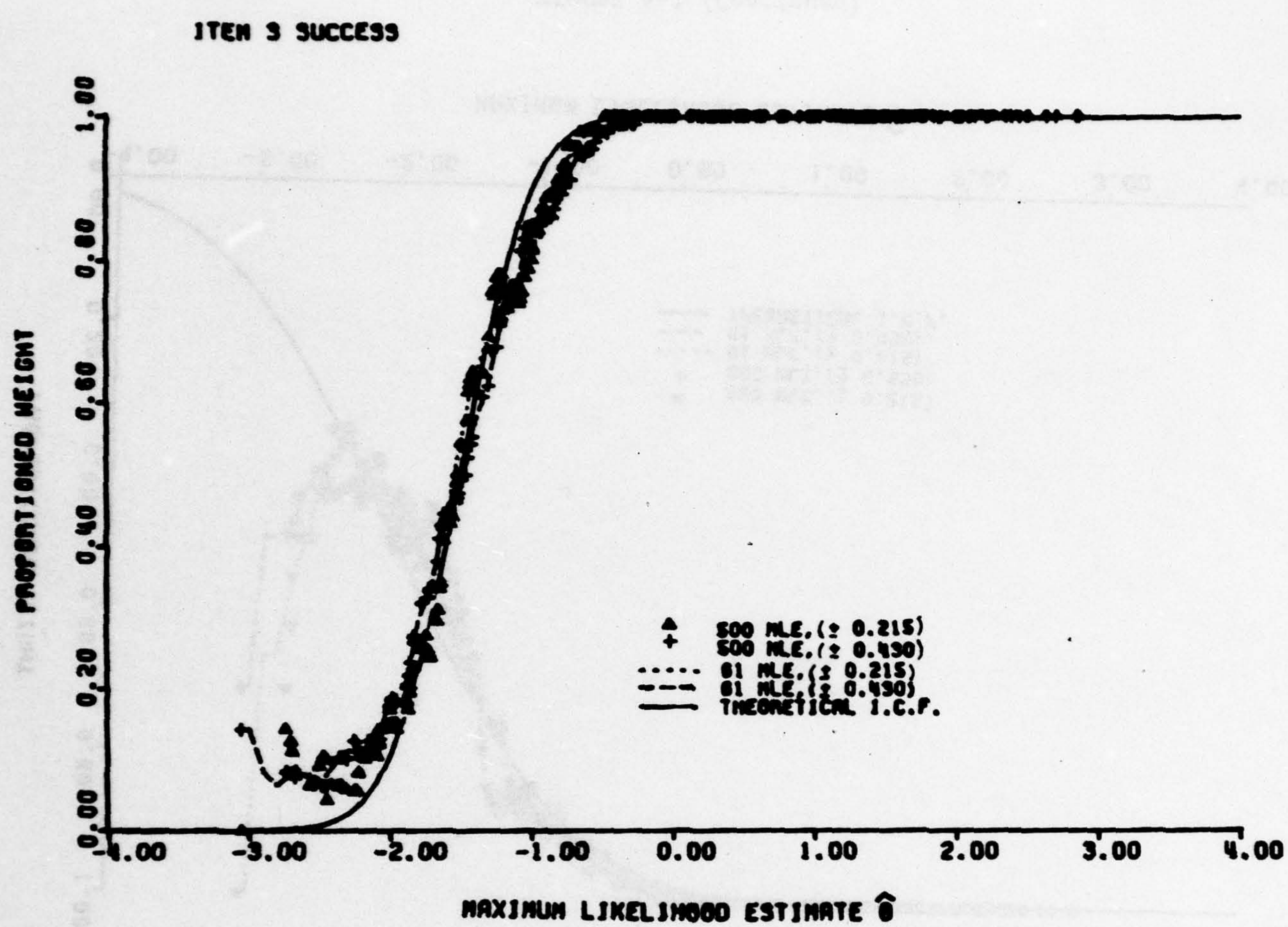

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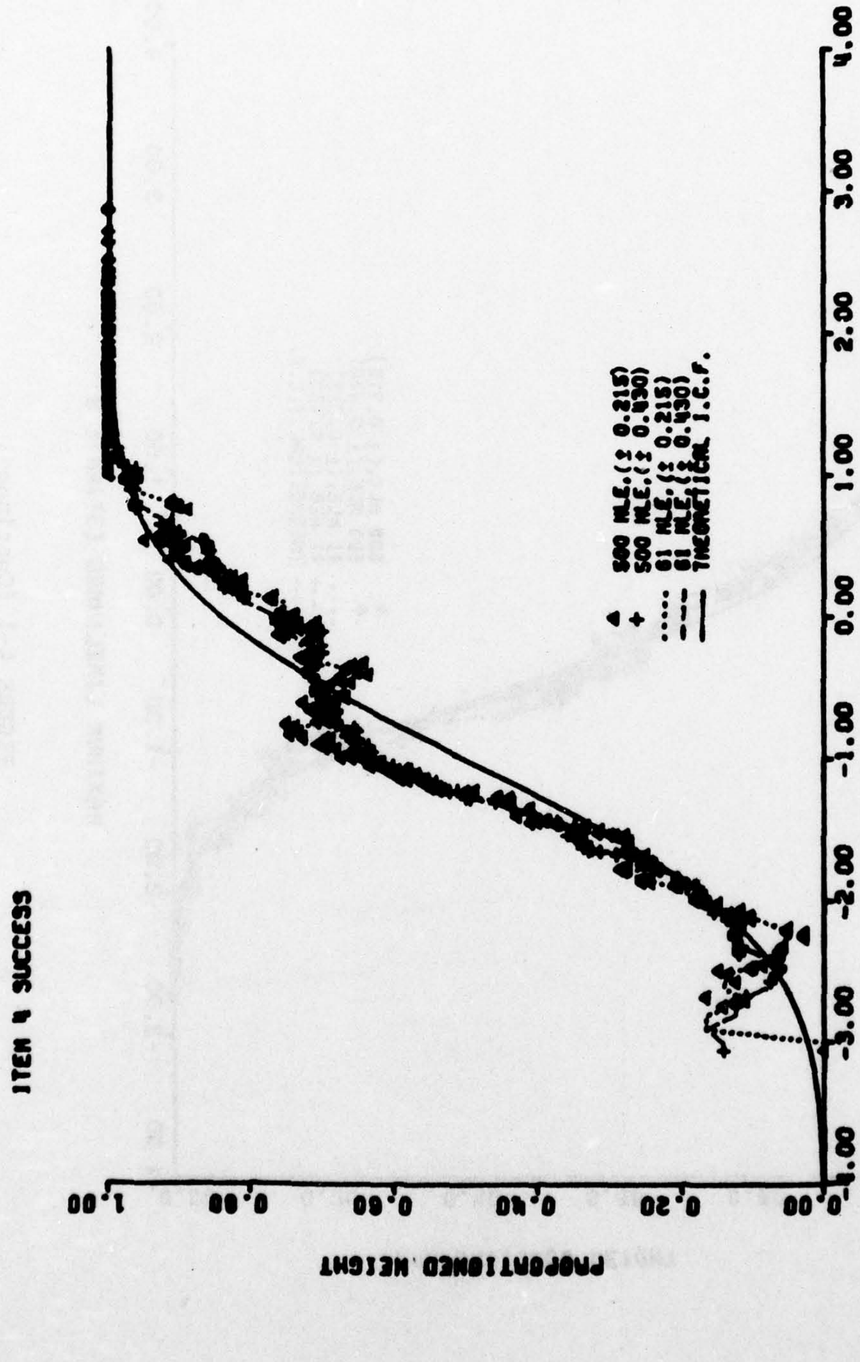

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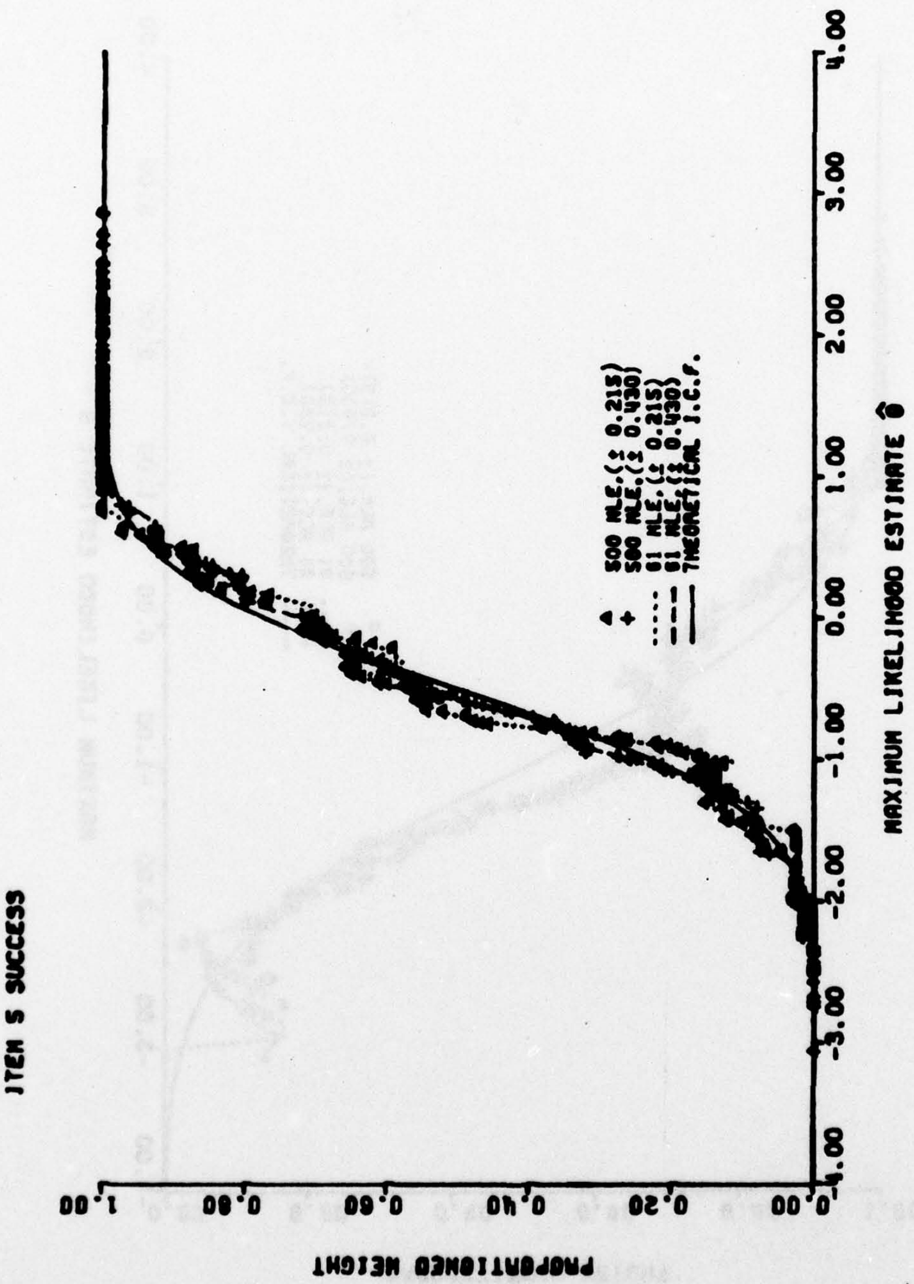

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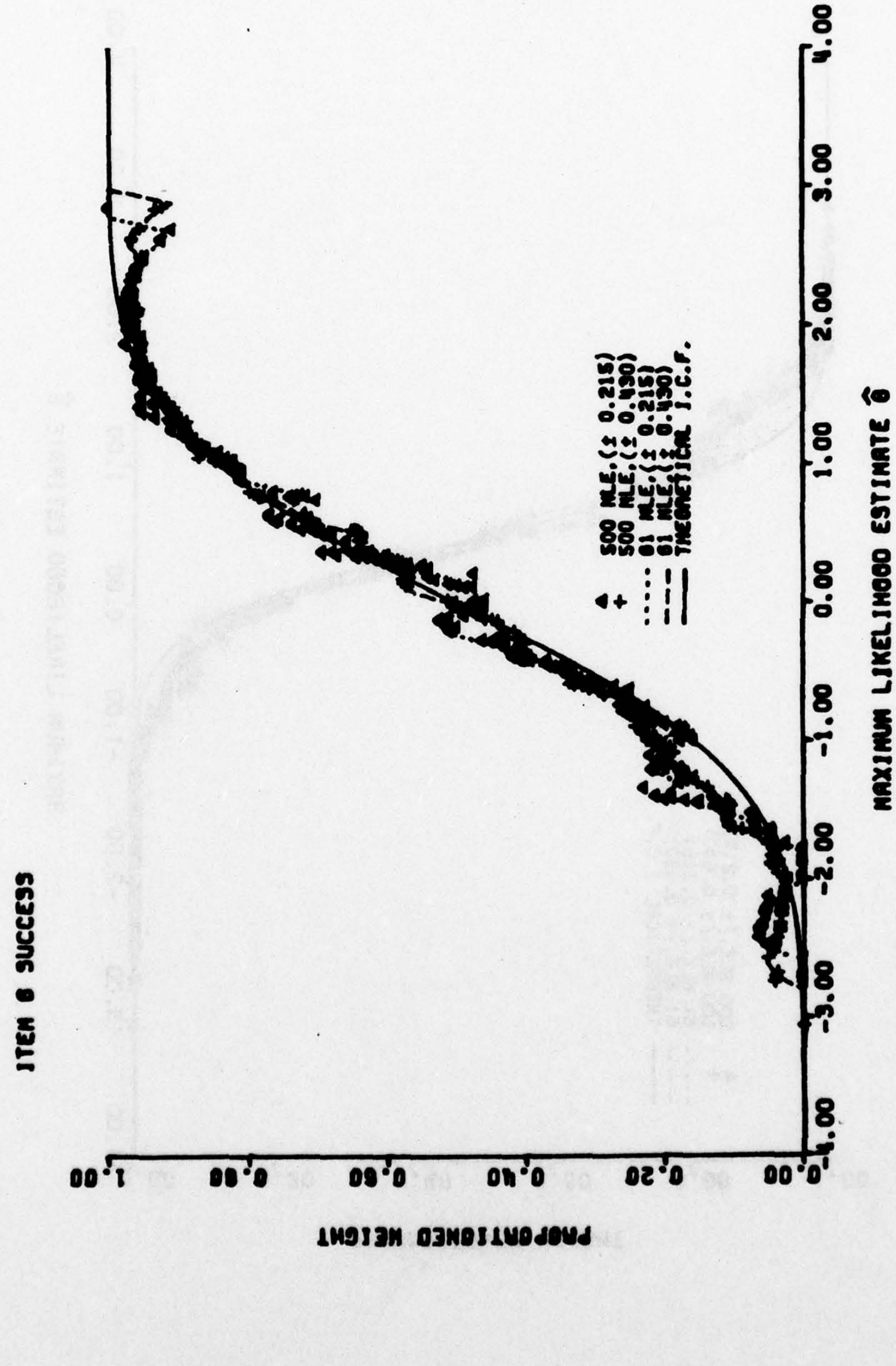

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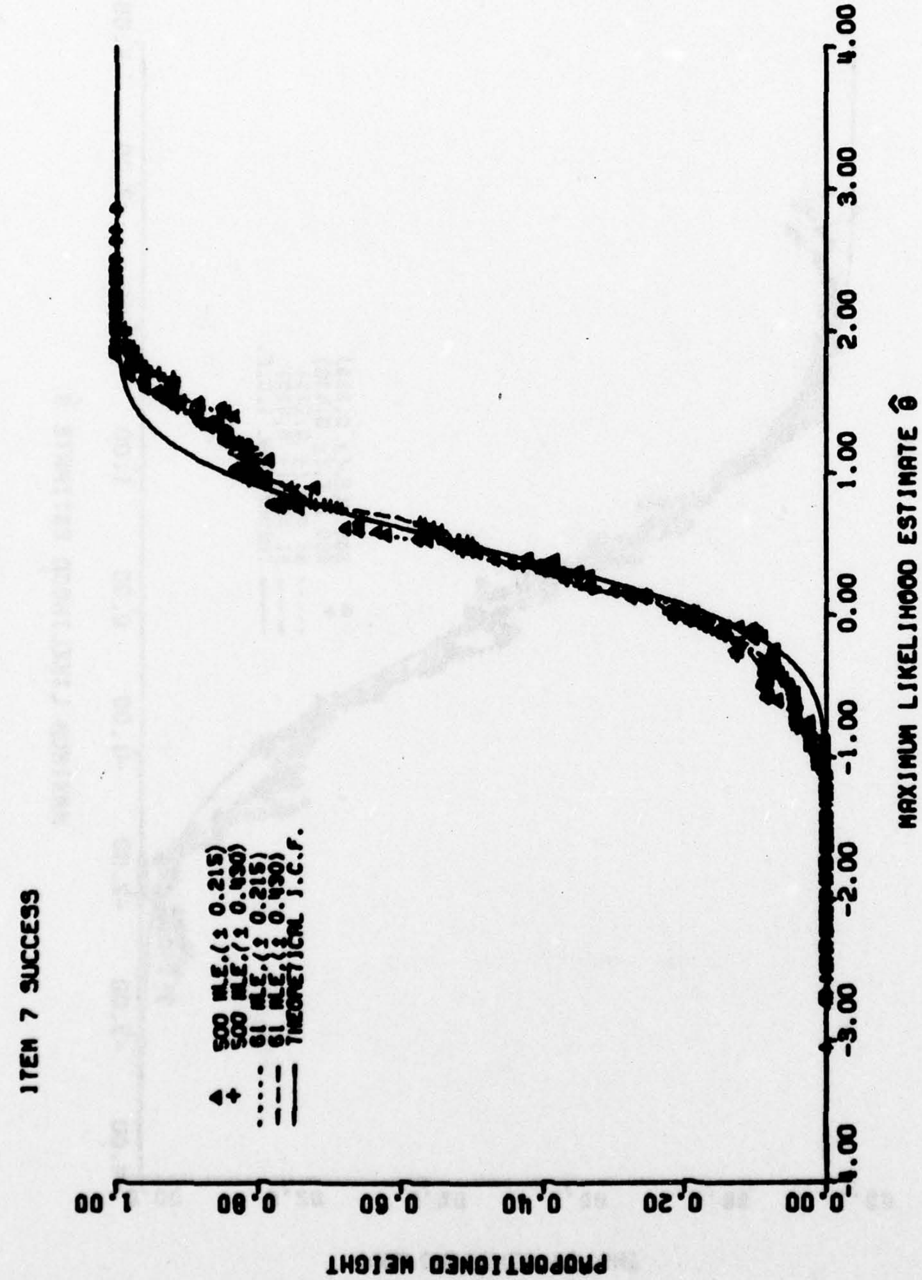

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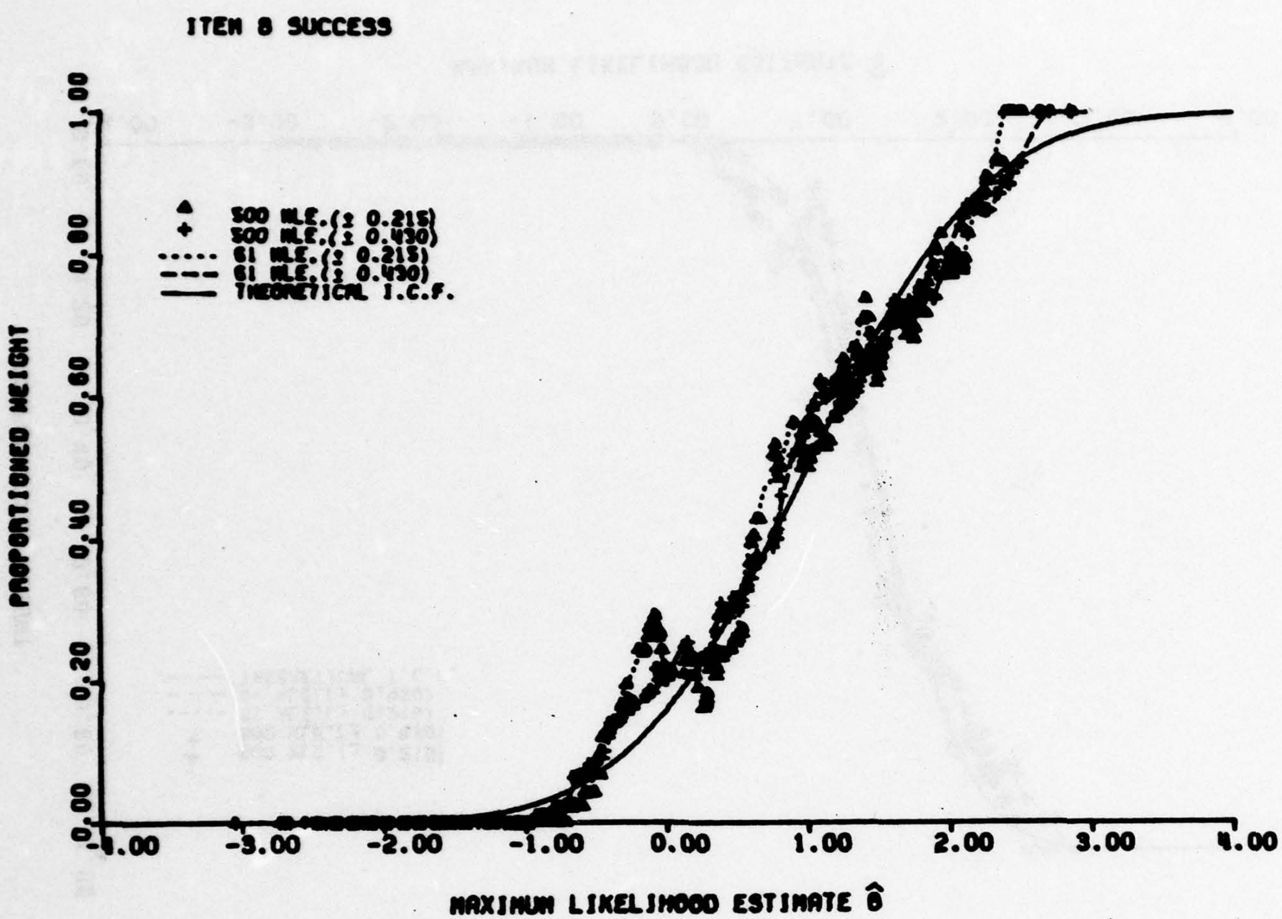

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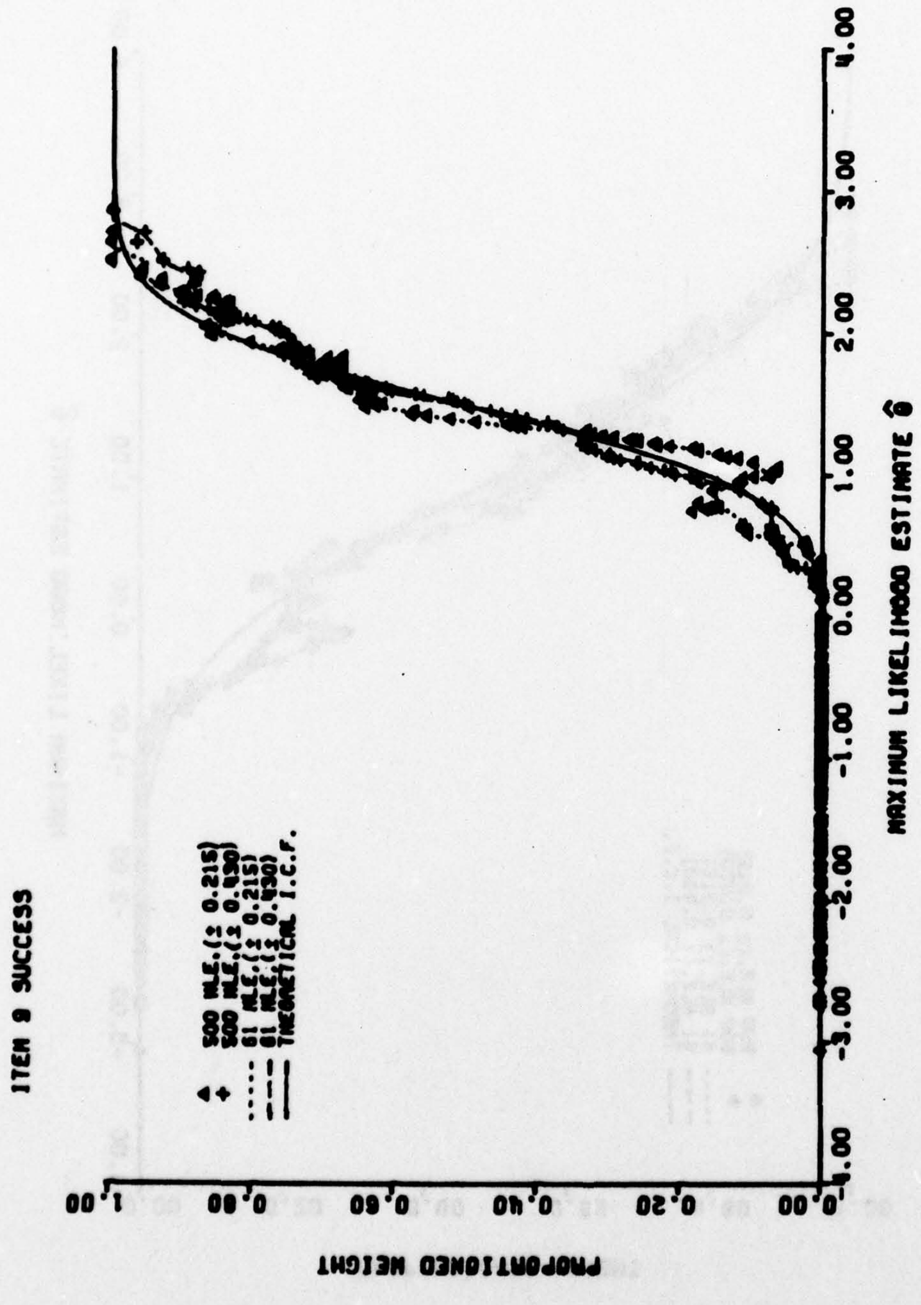

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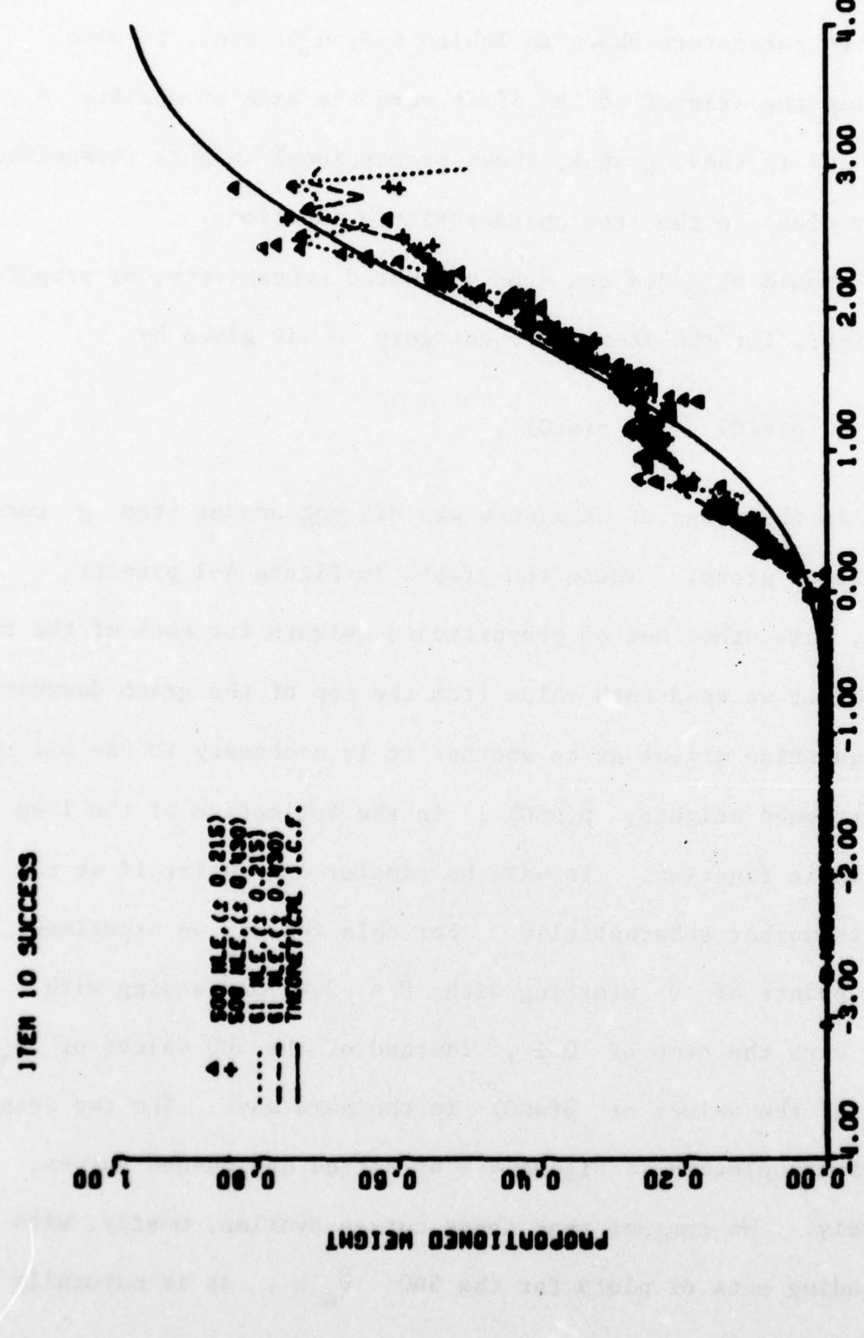


FIGURE 4-1 (Continued)

for these estimates. By virtue of the fact that this percentage is negligibly small, in each graph of Figure 4-1, the theoretical item characteristic function, which follows the normal ogive model with the two parameters shown in Tables 6-3, 6-4, etc., is also drawn, using the axis of $\hat{\theta}$ as if it were the axis of ability θ . As we can see in these graphs, these proportional weights themselves are fairly close to the item characteristic functions.

It should be added that the estimated probability, or proportioned weight, for the item score category 0 is given by

$$\hat{p}(s \varepsilon \overline{G}) = 1 - \hat{p}(s \varepsilon G) , \tag{4.6}$$

where $\overline{G}$ is the group of examinees who did <u>not</u> answer item g correctly, or the failure group. These ten graphs in Figure 4-1 present, therefore, this other set of proportioned weights for each of the two cases also, if we read each value from the top of the graph downwards.

A question arises as to whether it is necessary to use all the 500 proportioned weights, $\hat{p}(s \varepsilon G)$, in the estimation of the item characteristic function. It will be simpler and easier if we can reduce this number substantially. For this reason, we experiment using 61 points of $\hat{\theta}$ starting with $\hat{\theta} = -3.0$ and ending with $\hat{\theta} = 3.0$ with the step of 0.1 , instead of the 500 values of $\hat{\theta}_s$, and compute the values of $\hat{p}(s \varepsilon G)$ in the same way. The two sets of results are plotted in Figure 4-1 by dotted and dashed curves, respectively. We can see that these curves overlap, mostly, with correspoinding sets of plots for the 500 $\hat{\theta}_s$'s , as is naturally expected.

V Results II: Estimated Density Functions of Ability

Figure 5-1 presents the estimated density function, $\hat{f}(\theta)$, obtained by the Proportioned Sum Procedure, using the 500 $\hat{\theta}_s$'s , by dotted line, which is also the denominator of (3.3) with $\hat{\phi}(\theta|\hat{\theta}_s)$ replaced by its theoretical counterpart, $\phi(\theta|\hat{\theta}_s)$, and is divided by the total number of examinees, 500 . Note that this estimated density function is the same as the one obtained by the Simple Sum Procedure (Samejima, 1978d) of the Conditional P.D.F. Approach. For comparison, the estimated density function of θ obtained by the Pseudo Criterion Degree 4 Case of the Weighted Sum Procedure (Samejima, 1978d) is drawn by broken and dotted line, together with the theoretical density function, $f(\theta)$. We can see that the $\hat{f}(\theta)$ obtained by the Proportioned Sum Procedure is reasonably close to the theoretical density function, although not as close as the one by the Pseudo Criterion Degree 4 Case of the Weighted Sum Procedure.

As an additional information, the same denominator computed for the 61 points of $\hat{\theta}$ and divided by 61 is presented in the same figure by dashed line. This is by no means an estimate of the density function $f(\theta)$, and it is interesting to note that the curve is substantially different from the others.

The mean square error which is obtained from

$$(5.1) \qquad \frac{1}{m} \sum_{j=1}^{m} [\hat{f}(\theta_j) - f(\theta_j)]^2 \quad ,$$

where $m = 25$ and θ_j's are -2.4 through 2.4 with the step 0.2 is computed, and it turned out to be 0.00018 (0.01324), which is a little greater than 0.00009 (0.00972) for the $\hat{f}(\theta)$ by the Pseudo Criterion

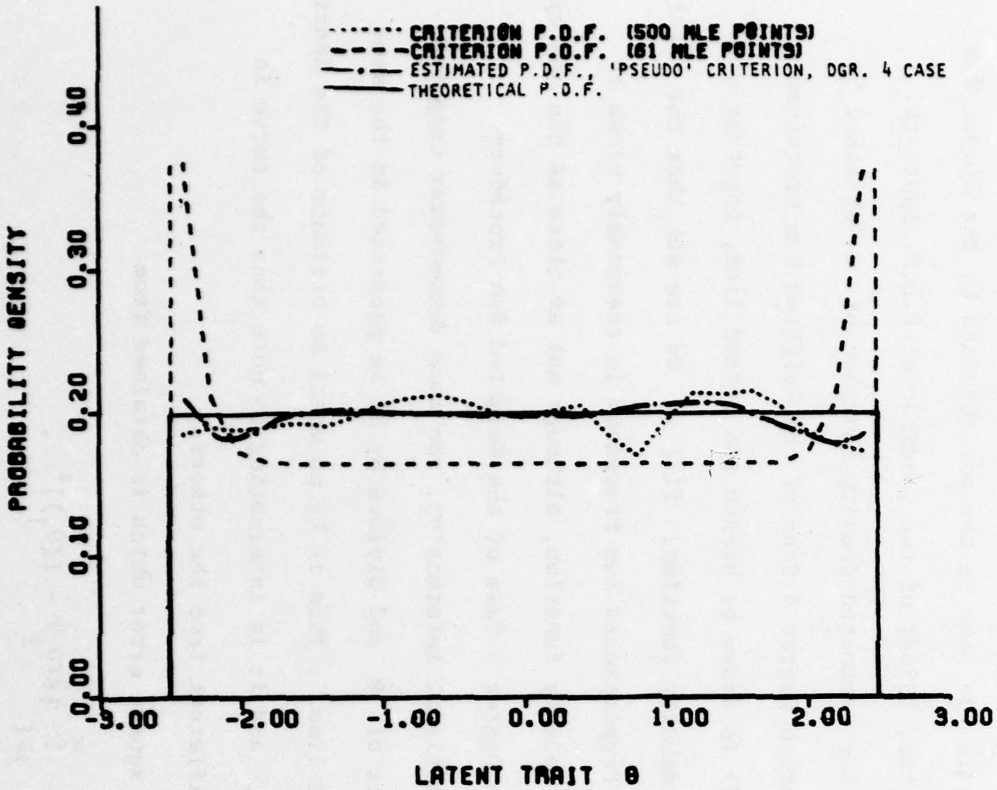


FIGURE 5-1

Estimated Density Function of Ability θ Obtained by the Proportioned Sum Procedure of the Conditional P.D.F. Approach, Which Is Based on the 500 True Conditional Density Functions, $\phi(\theta|\hat{\theta}_s)$, Together with the One Obtained by the Pseudo Criterion Degree 4 Case of the Weighted Sum Procedure and the Theoretical Density Function $f(\theta)$. Also Presented Is the One Based on the 61 Points of $\hat{\theta}$.

Degree 4 Case of the Weighted Sum Procedure. (The numbers in parentheses are the corresponding square roots of the mean square errors.) For an additional information, if we calculate the value given by (5.1) for the 61 point case, it will be 0.00350 (0.05918).

The estimated shared density function (Samejima, 1978d) for each of the failure and success groups of each item is presented as Figure 5-2, for the two criterion cases of the present study, together with the one obtained by the Simple Sum Procedure, the actual frequency ratios of θ and the theoretical shared density. (Again, for an additional information, the corresponding curves for the 61 point case are also presented in the same figure, which are by no means estimated shared densities.) We find that these two curves are fairly close to the one by the Simple Sum Procedure, which indicates that they are not very close to the three curves provided by the Pseudo Criterion Degree 3, 4 and 5 Cases of the Weighted Sum Procedure (Samejima, 1978d).

Figure 5-3 presents the estimated density functions, $\hat{f}(\theta)$, obtained by Degree 3 and 4 Cases* of the Pearson System Method. We can see that these results, especially the one obtained by Degree 3 Case, are very similar to the one obtained by the criterion case.

The means square errors of the $\hat{f}(\theta)$'s obtained by Degree 3 and 4 Cases of the Pearson System Method and their square roots were calculated through (5.1), and they are 0.00076 (0.02752) and 0.00105 (0.03242) respectively. As was expected, they are reasonably small, but not as small as those for the criterion case, which turned out

* Refer to Appendix II.

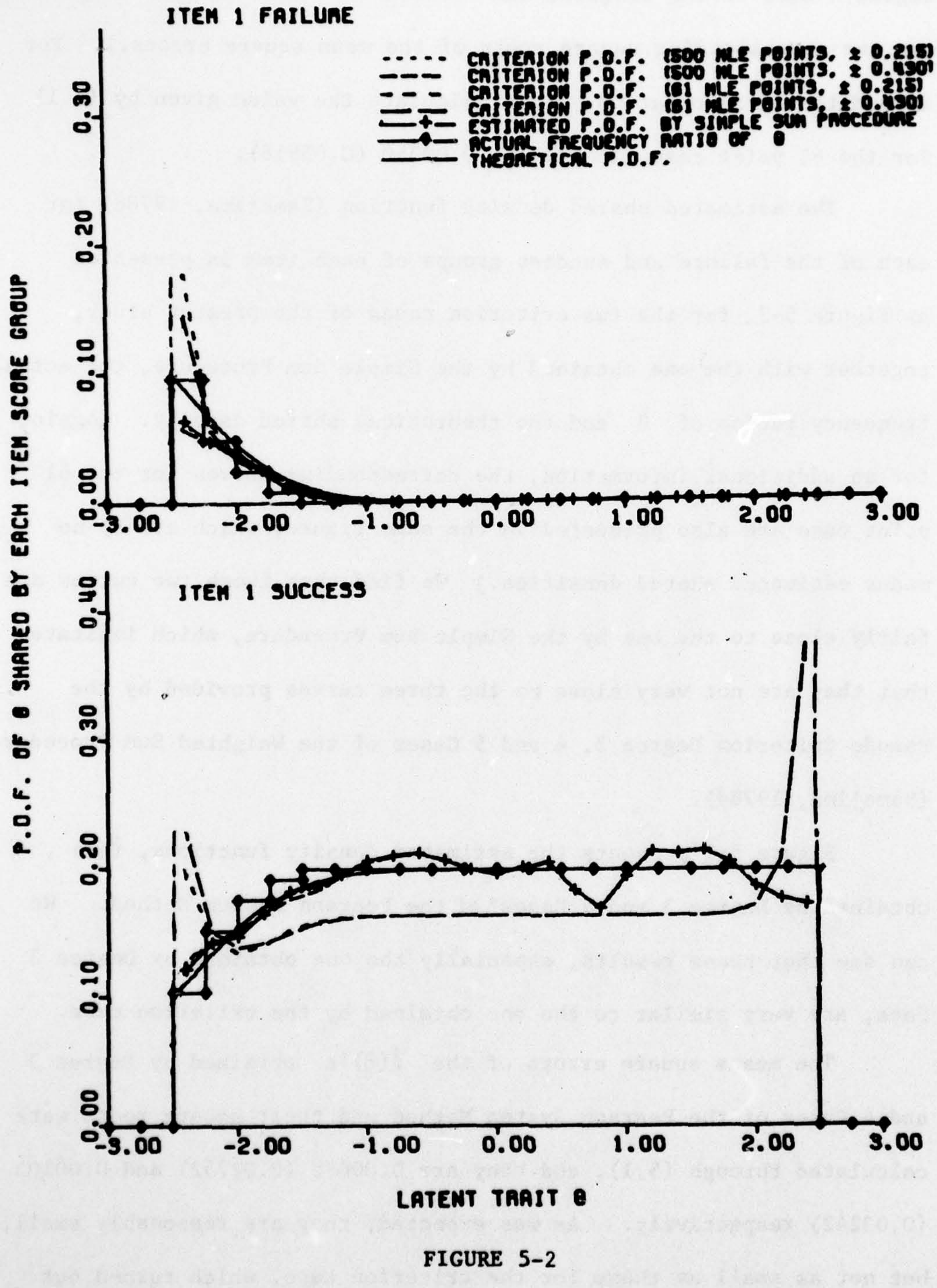


FIGURE 5-2

Comparison of the Four Estimated Shared Density Functions of Ability θ of Each of the Two Item Score Groups, Which are Obtained by the Proportioned Sum Procedure, Using the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, with the One Obtained by the Simple Sum Procedure. Theoretical Shared Density Function and Actual Frequency Ratios Are Also Presented.

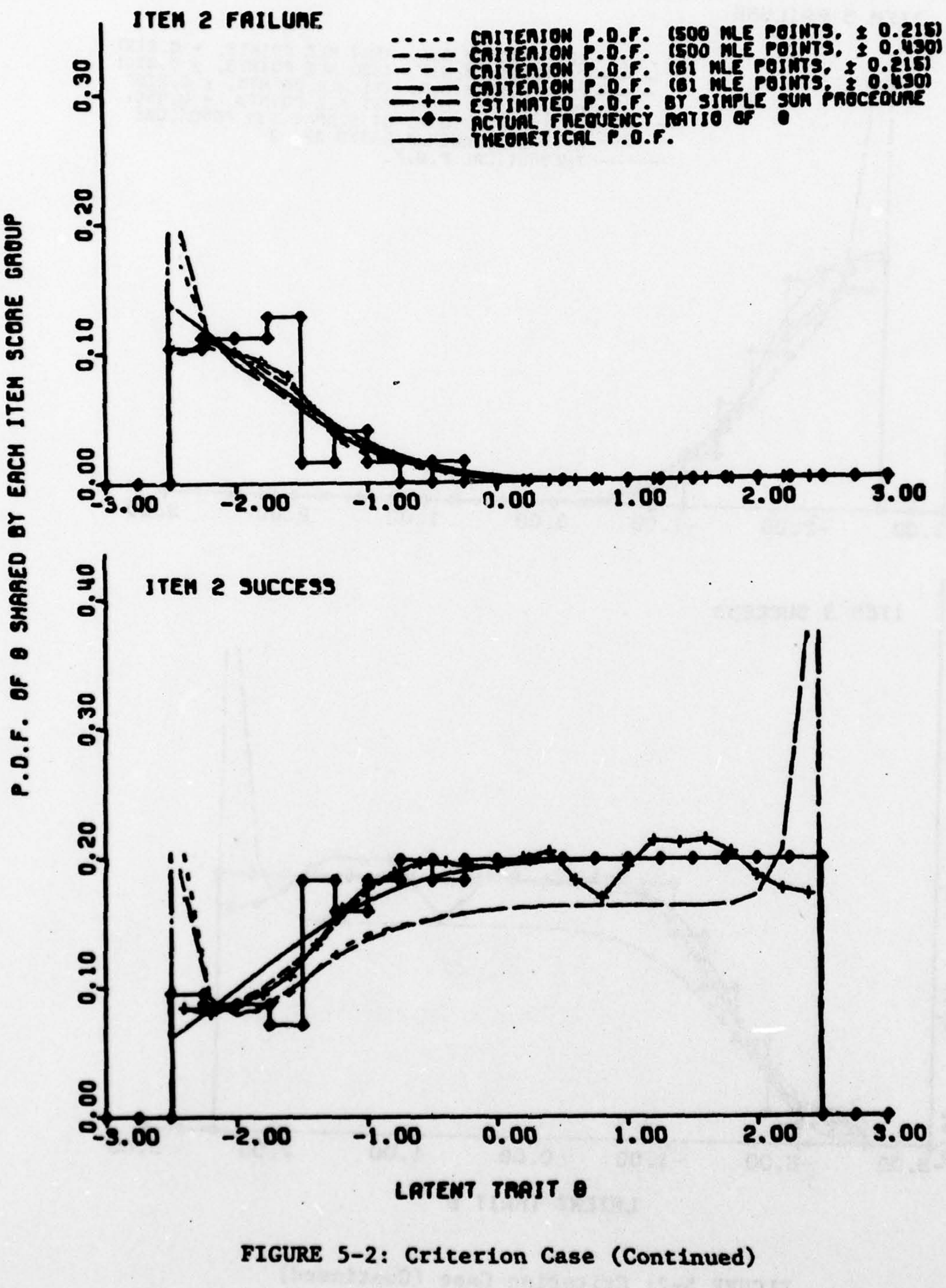
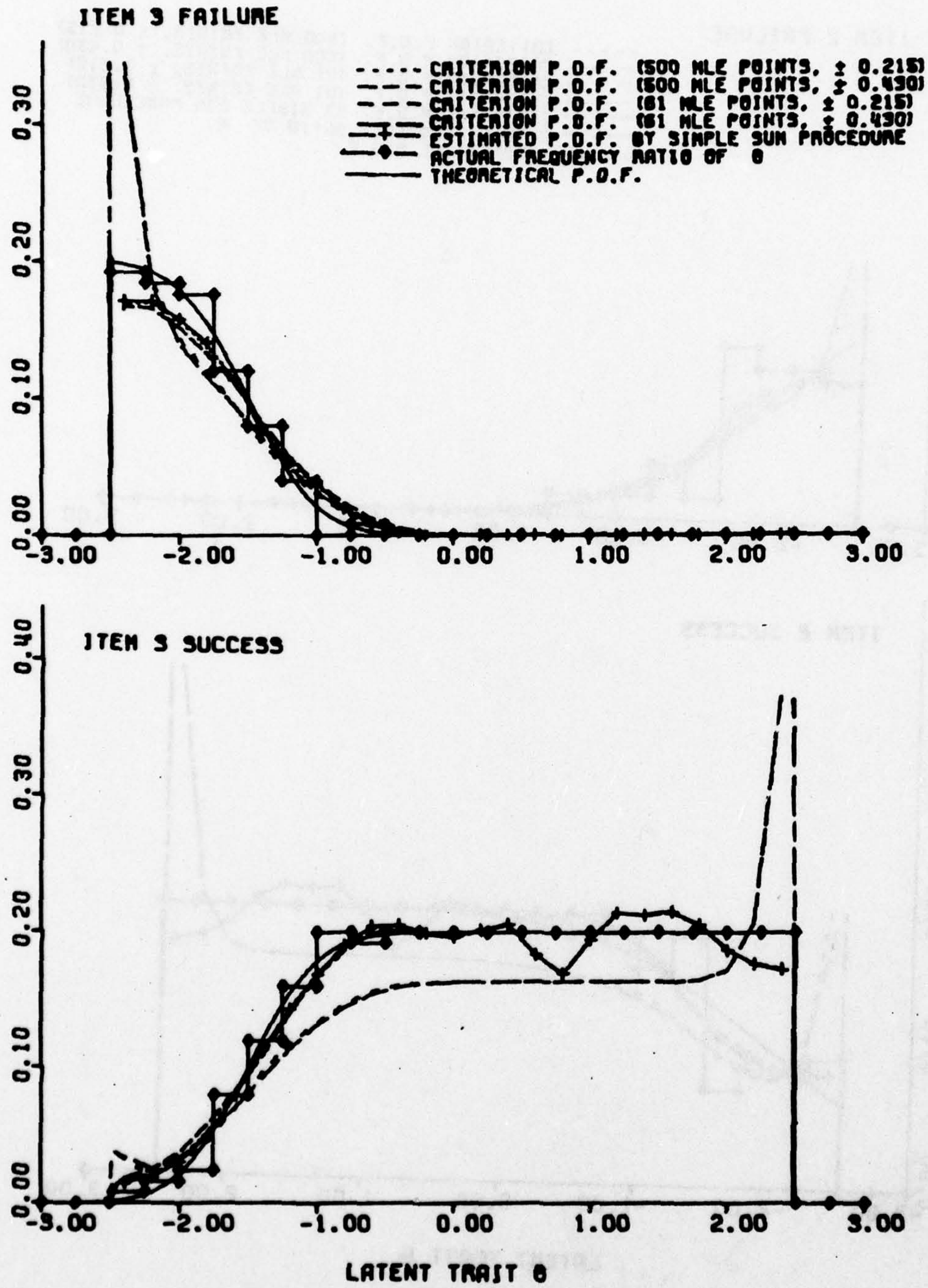


FIGURE 5-2: Criterion Case (Continued)



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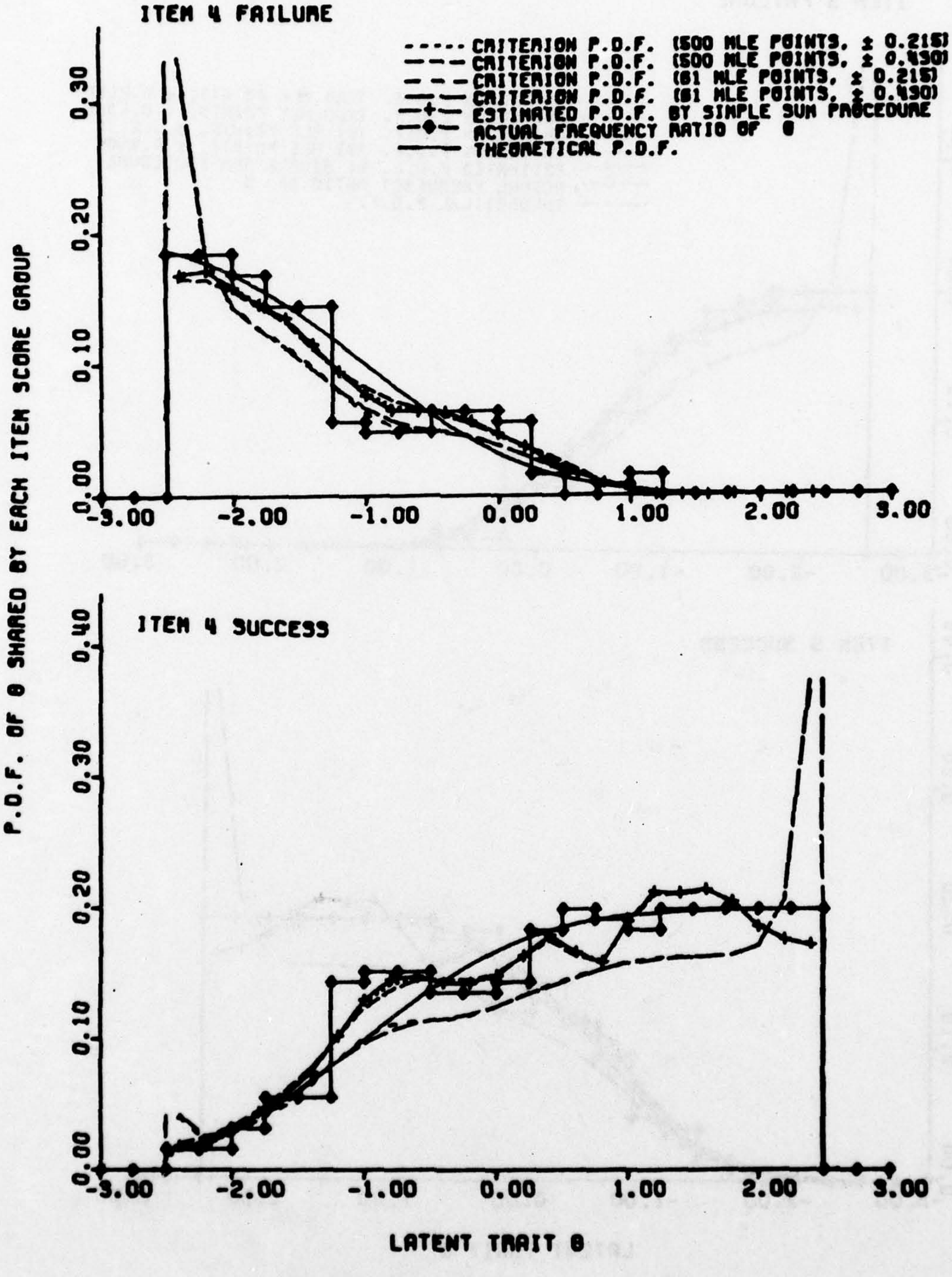

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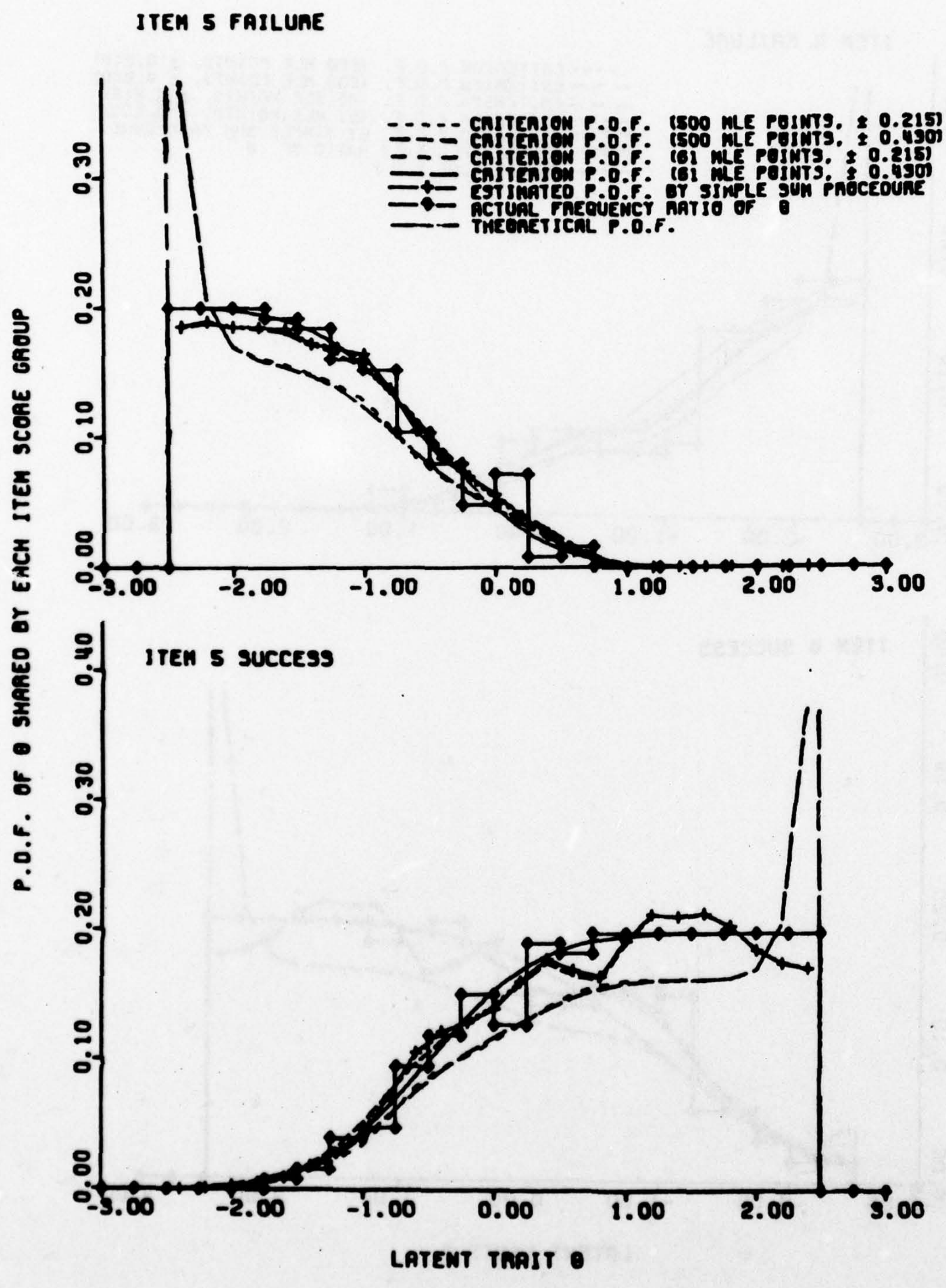

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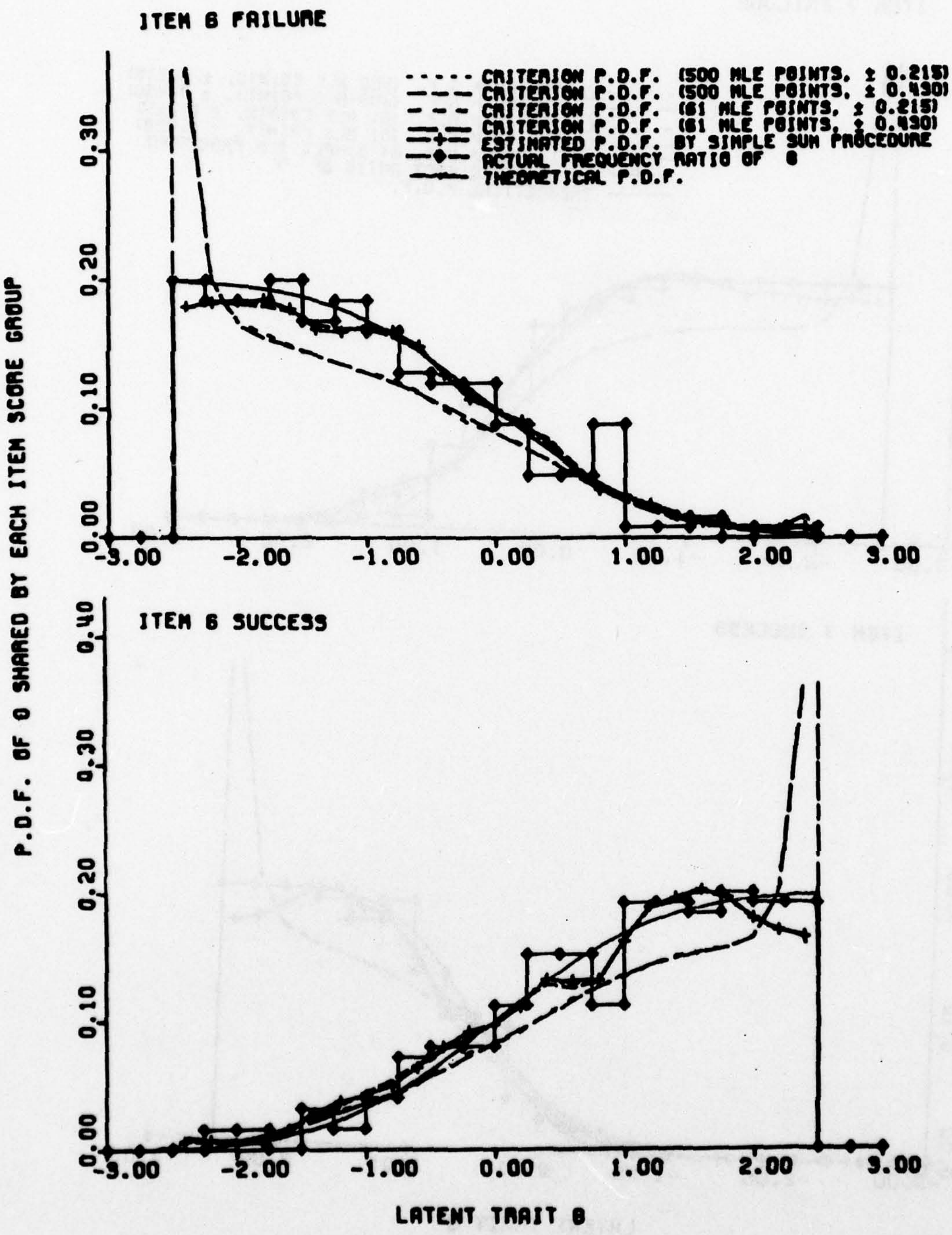

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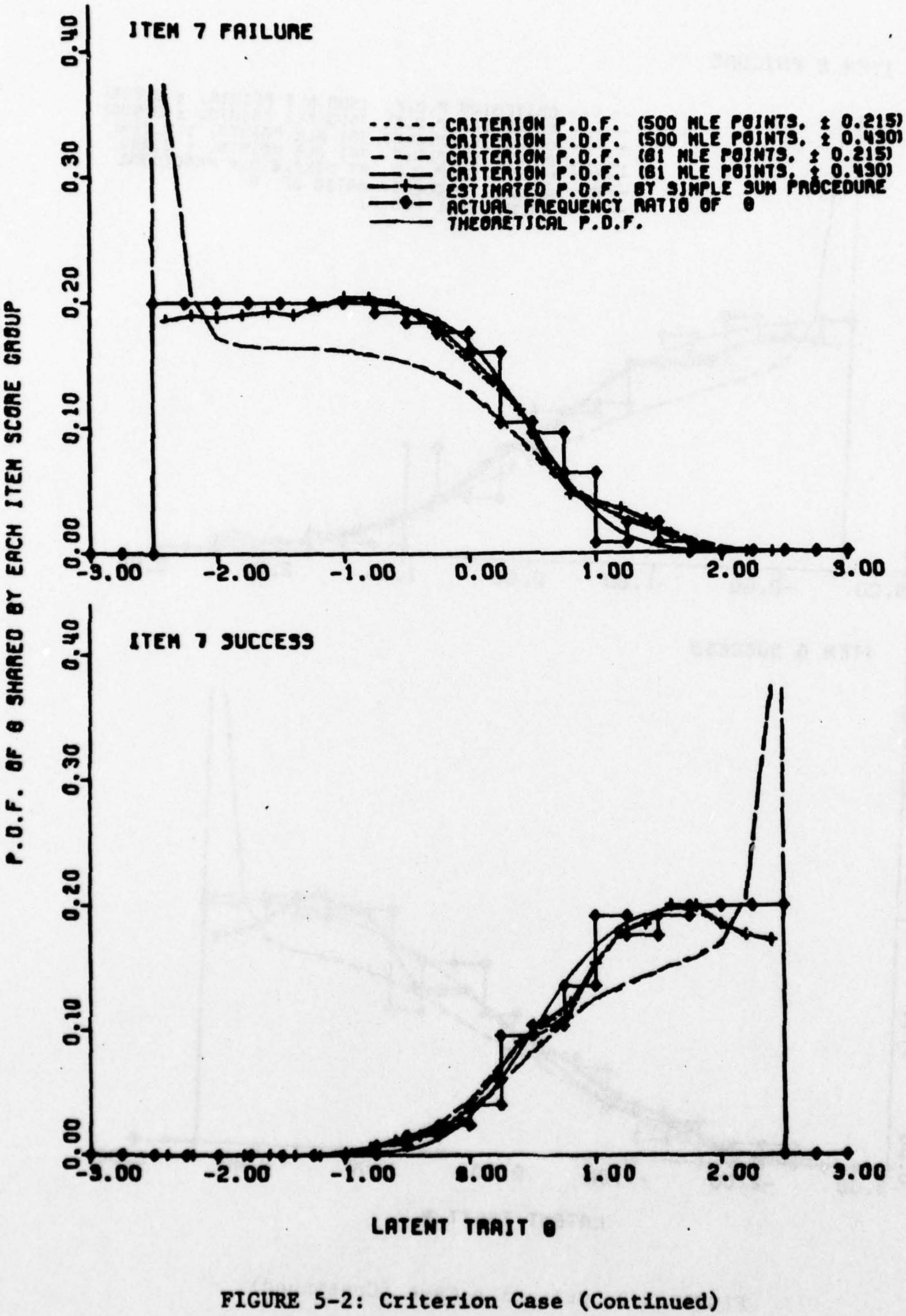

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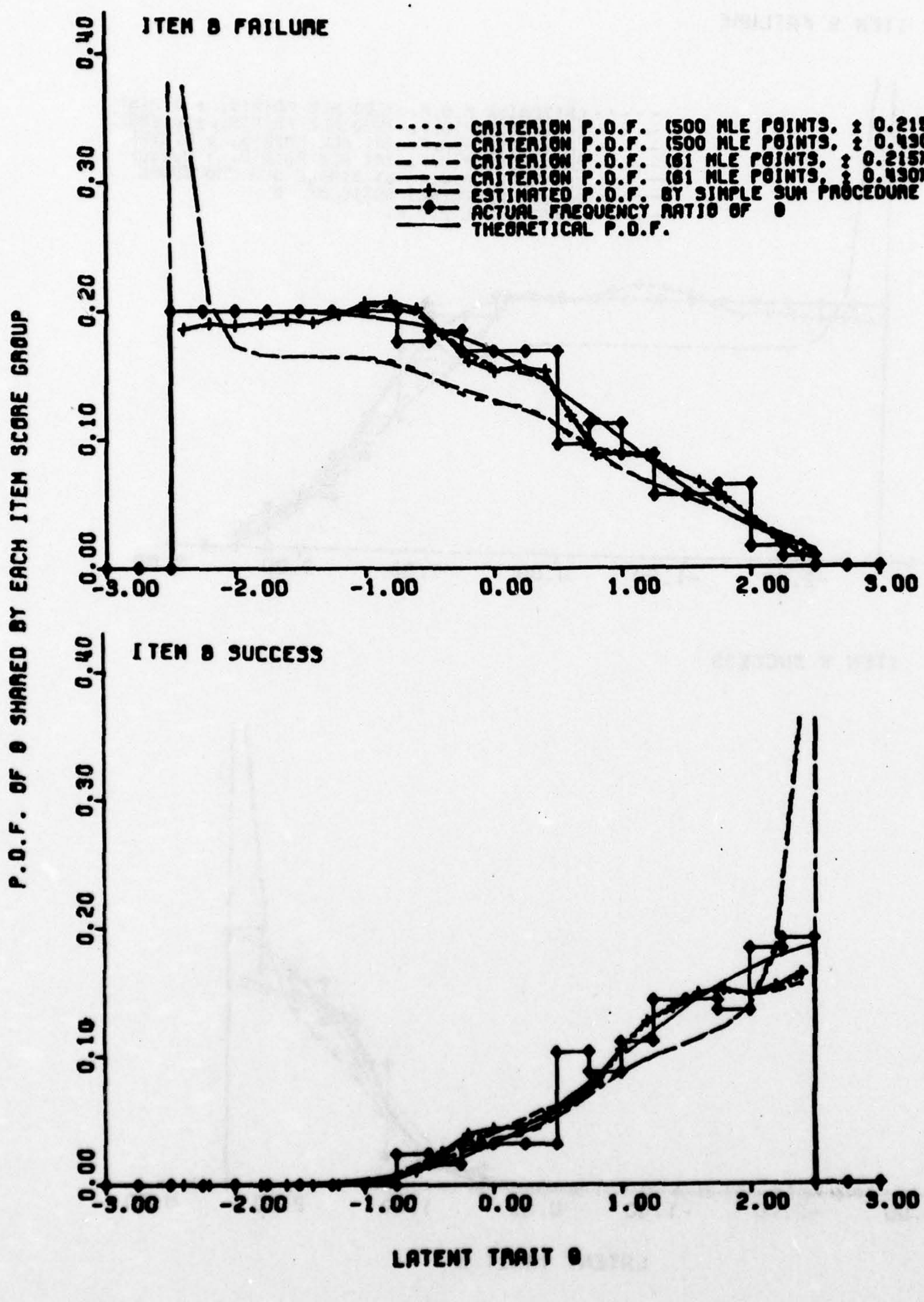

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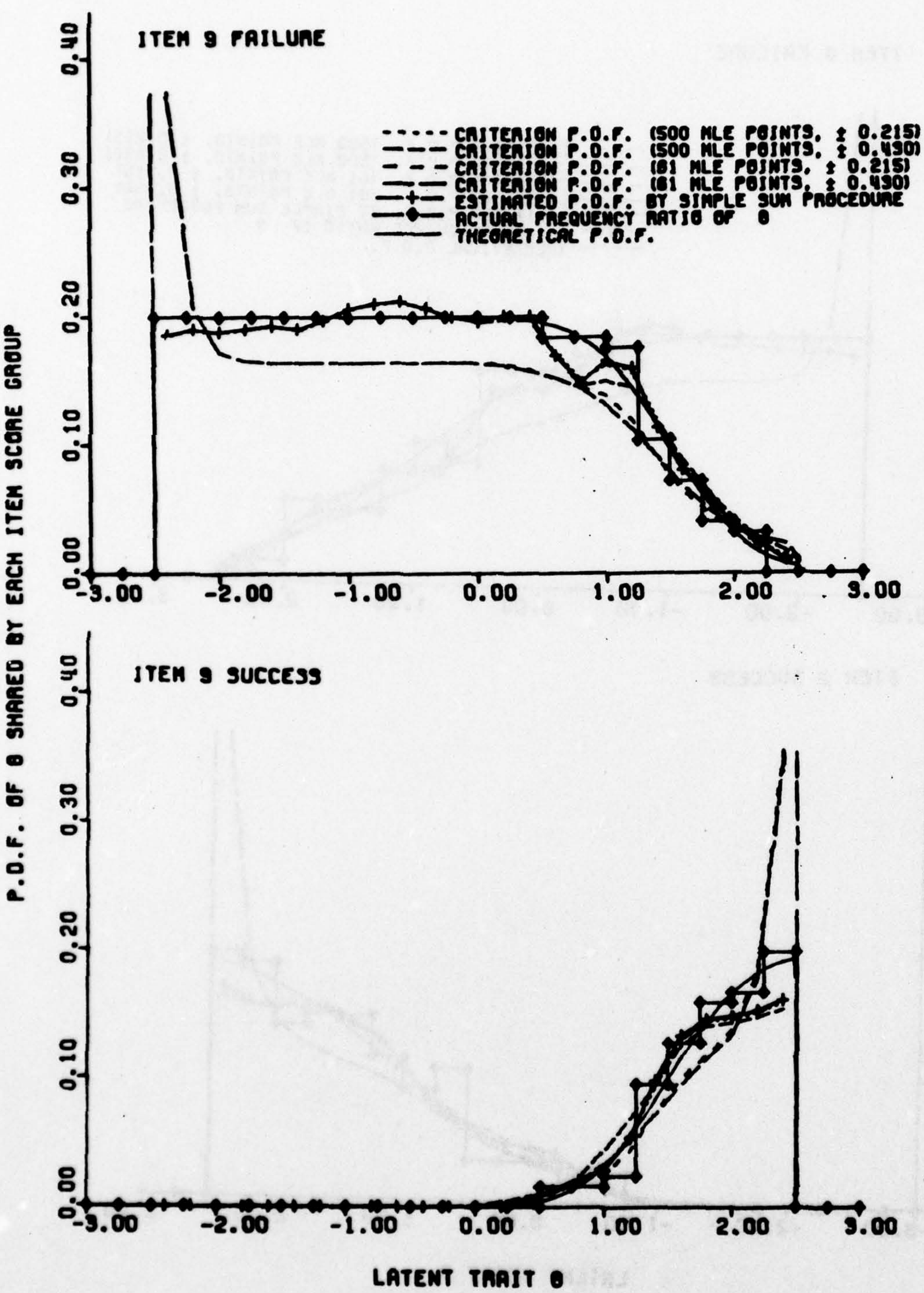

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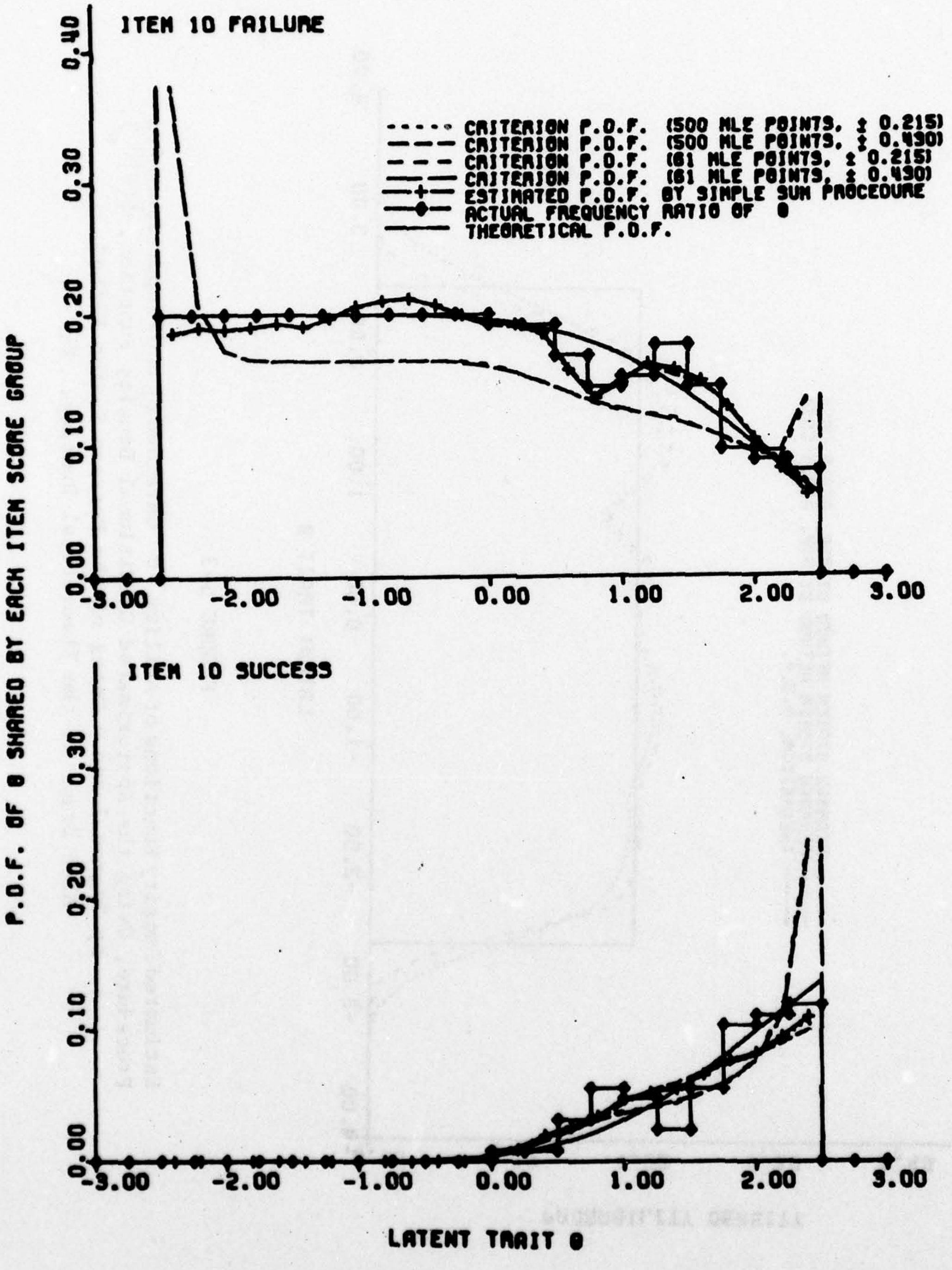

FIGURE 5-2: Criterion Case (Continued)

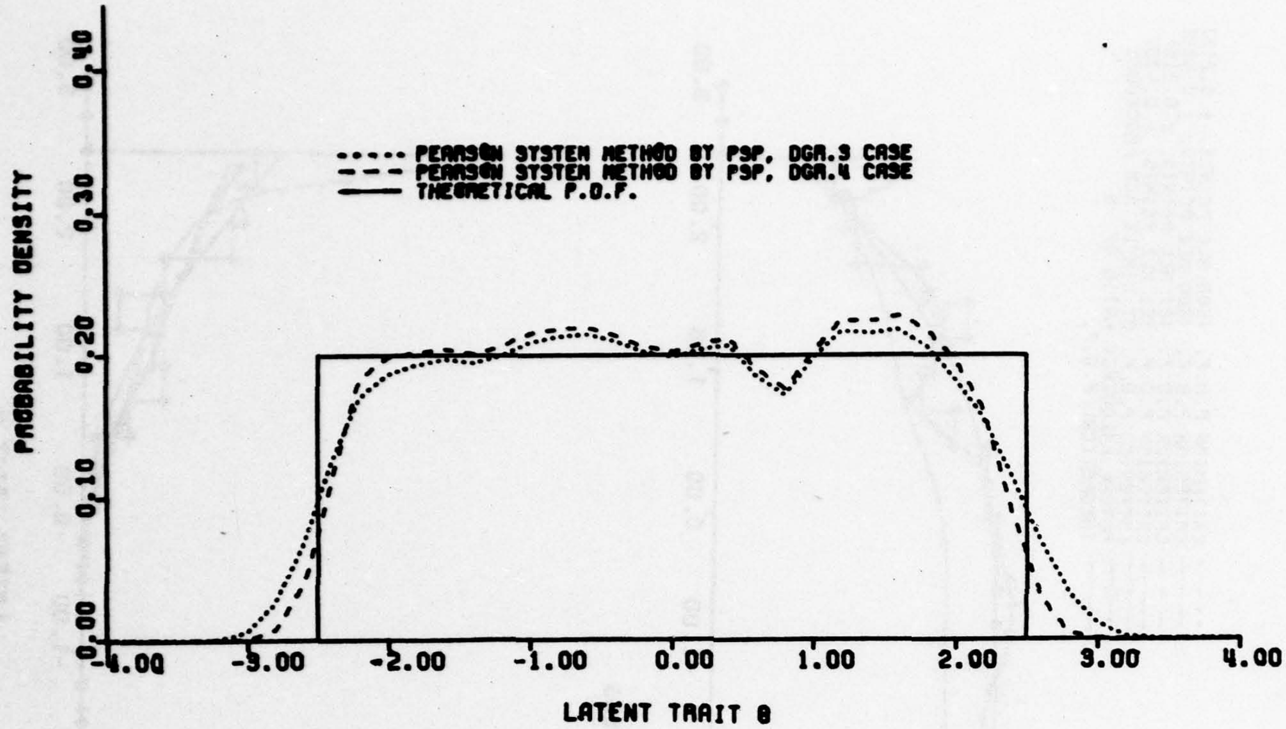

FIGURE 5-3

Estimated Density Functions of Ability θ Obtained by the Proportioned Sum Procedure, Using the Approximated Conditional Density Function, $\hat{\phi}(\theta|\hat{\theta}_s)$, by Degree 3 and 4 Cases of the Pearson System Method. Also Drawn is the Theoretical Density, $f(\theta)$.

to be 0.00018 (0.01324), as was mentioned earlier.

The estimated shared density functions by the failure and success groups obtained by Degree 3 and 4 Cases of the Pearson System Method for the two intervals of $\hat{\theta}_s$ each are presented in Figure 5-4, along with those obtained by the criterion case of the Simple Sum Procedure, the actual frequency ratios and the theoretical shared density functions. We find that the two curves for the different proportioned weights are very close to each other, in each of Degree 3 and 4 Cases.

Figure 5-5 presents the estimated density functions, $\hat{f}(\theta)$, obtained by Degree 3 and 4 Cases of the Two-Parameter Beta Method. Just like those obtained by the Pearson System Method, these two curves have "tails" outside the interval of θ, (-2.5, 2.5), for which the theoretical density, $f(\theta)$, assumes a constant positive value, 0.2. It is noticed that within the interval of θ these two curves are very close to each other, and, in fact, more so than those obtained by the Pearson System Method.

The mean square errors for these two estimated density functions from the theoretical density functions, which were computed through (5.1), turned out to be 0.00077 (0.02766) and 0.00064 (0.02529) for Degree 3 and 4 Cases, respectively. They are both comparable to those by Degree 3 Case of the Pearson System Method, i.e., 0.00076 (0.02752).

The estimated shared density functions of θ by the failure and success groups obtained by the Two-Parameter Beta Method are presented as Figure 5-6, by the four curves each of which represents Degree 3 Case with the interval $\hat{\theta}_s \pm 0.215$, Degree 3 Case with $\hat{\theta}_s \pm 0.430$, Degree 4 Case with $\hat{\theta}_s \pm 0.215$, or Degree 4 Case with

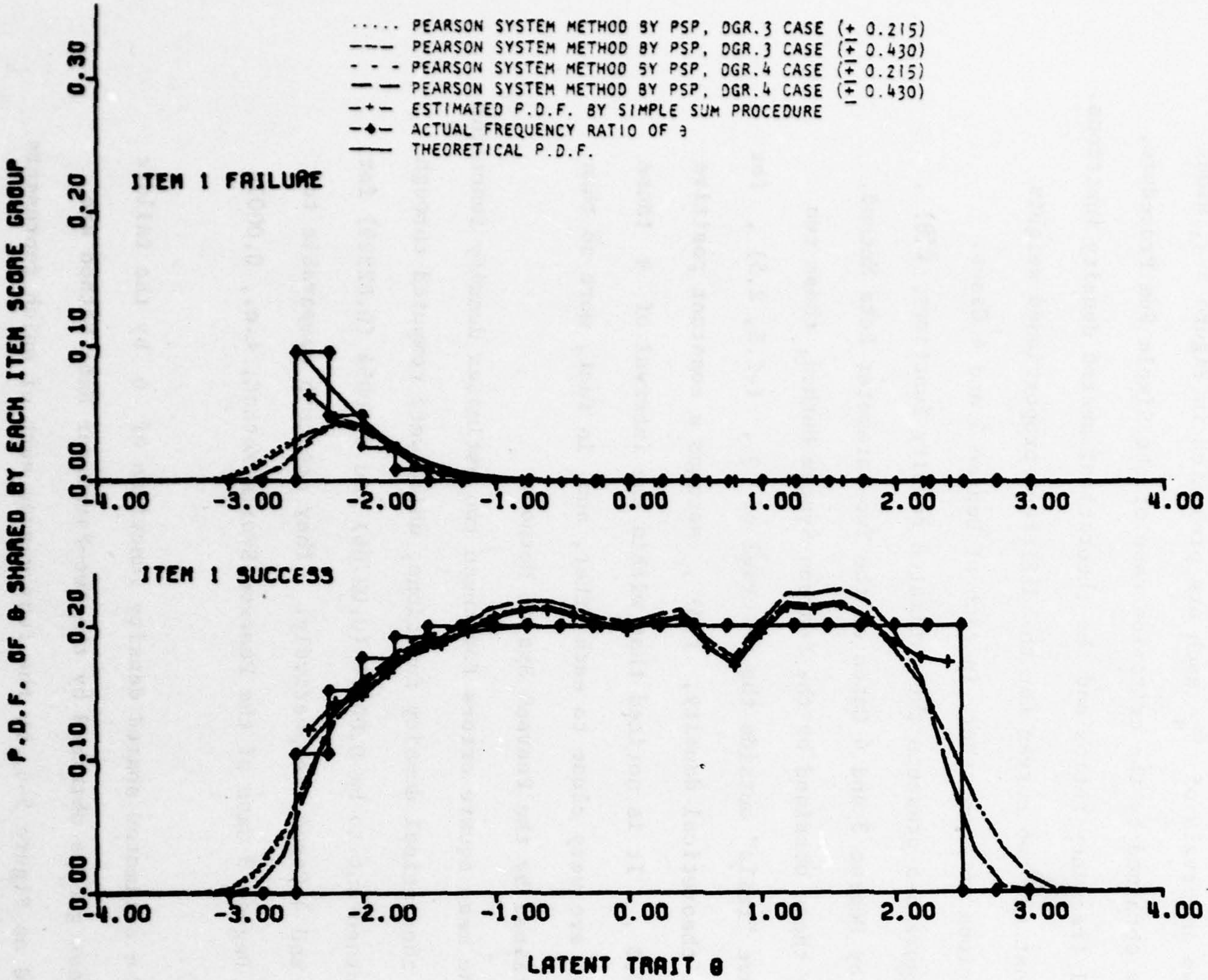

FIGURE 5-4

Four Estimated Shared Density Functions of Ability θ of Each of the Two Item Score Groups, Which Are Obtained by the Proportioned Sum Procedure Using the Approximated Conditional Density Function, $\hat{\phi}(\theta|\hat{\theta}_s)$, by Degree 3 and 4 Cases of the Pearson System Method. Also Presented Are the One Obtained by Using the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, by the Simple Sum Procedure, Actual Frequency Ratios of Ability θ, and Theoretical Shared Density Function.

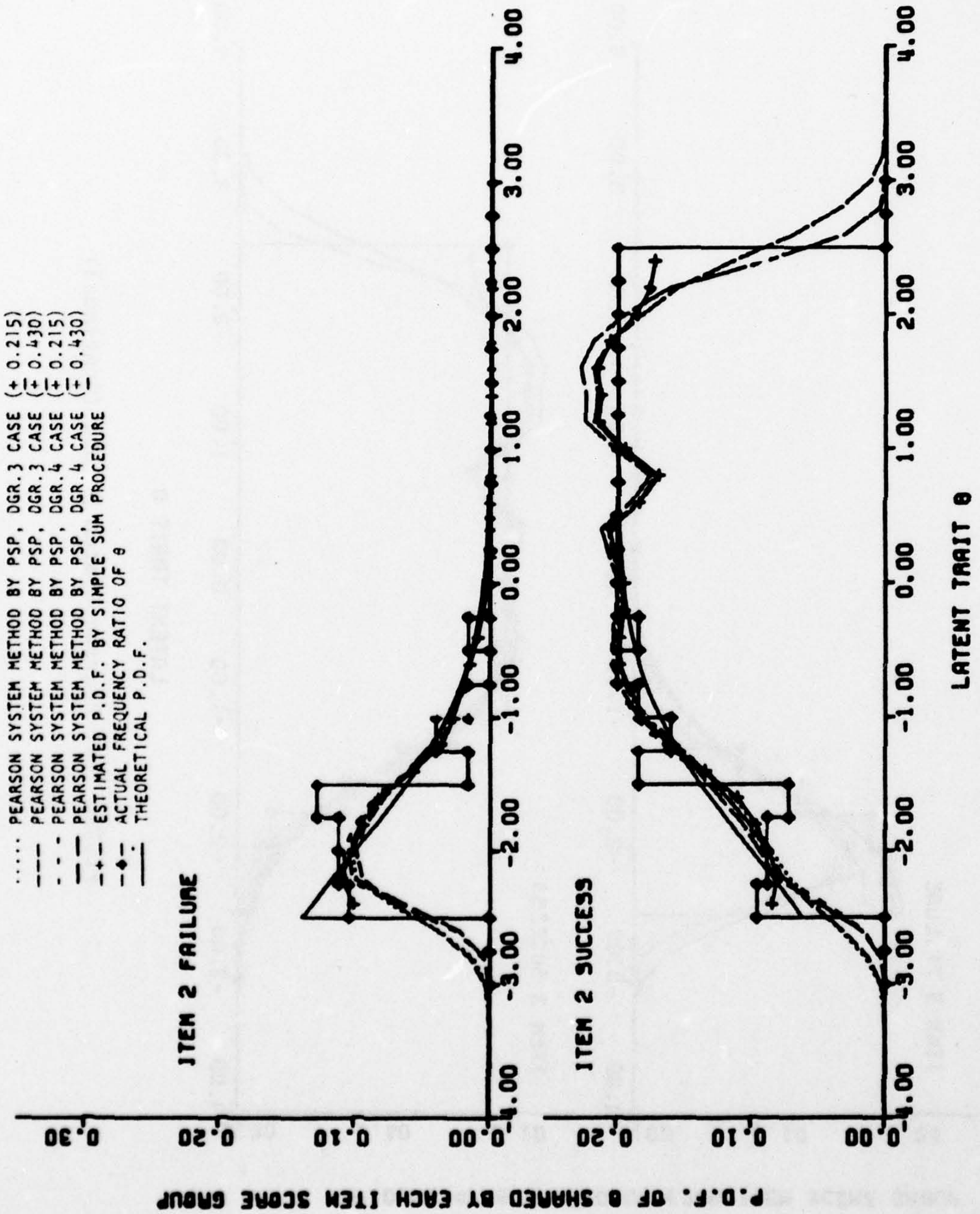

FIGURE 5-4: Pearson System Method (Continued)

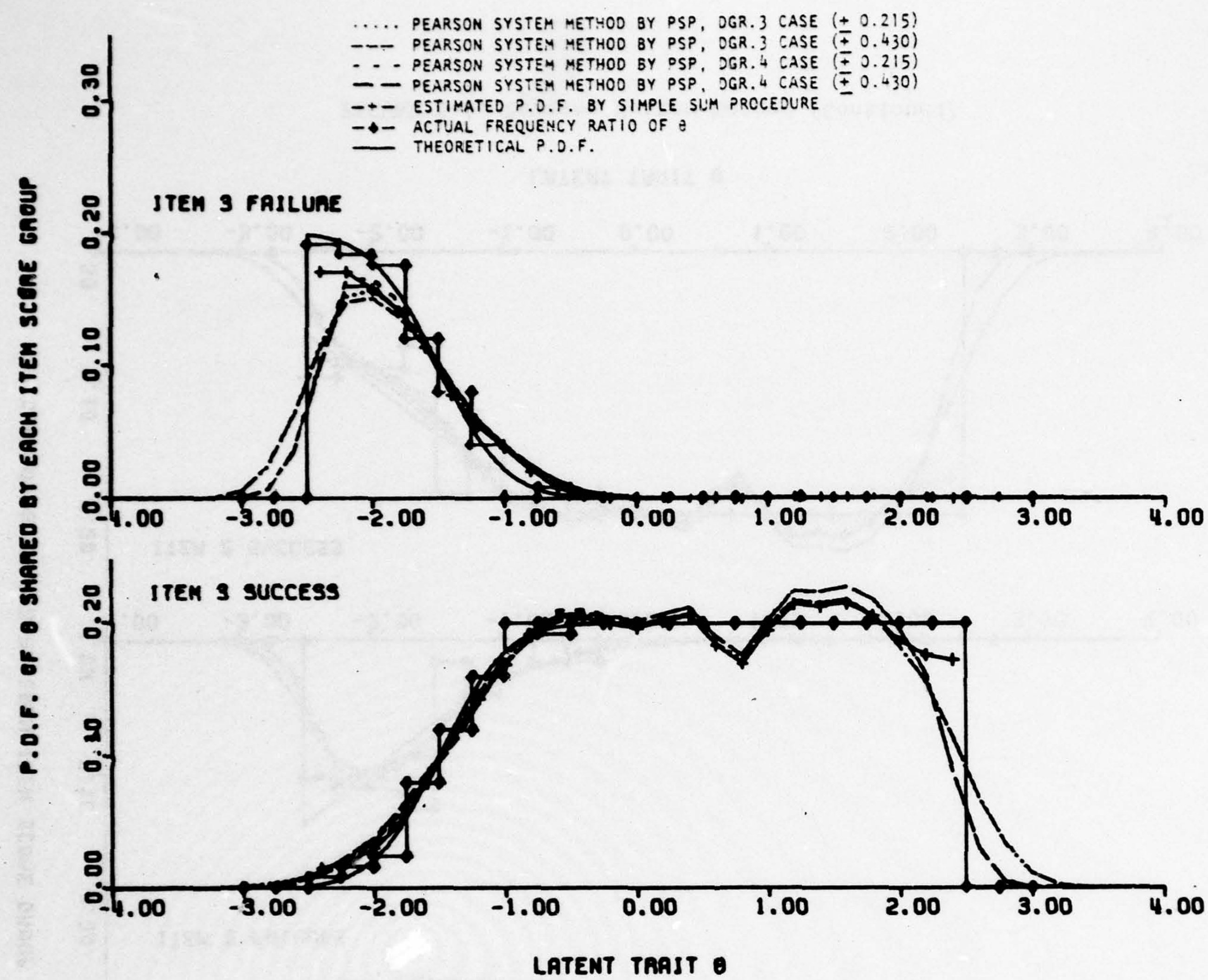

FIGURE 5-4: Pearson System Method (Continued)

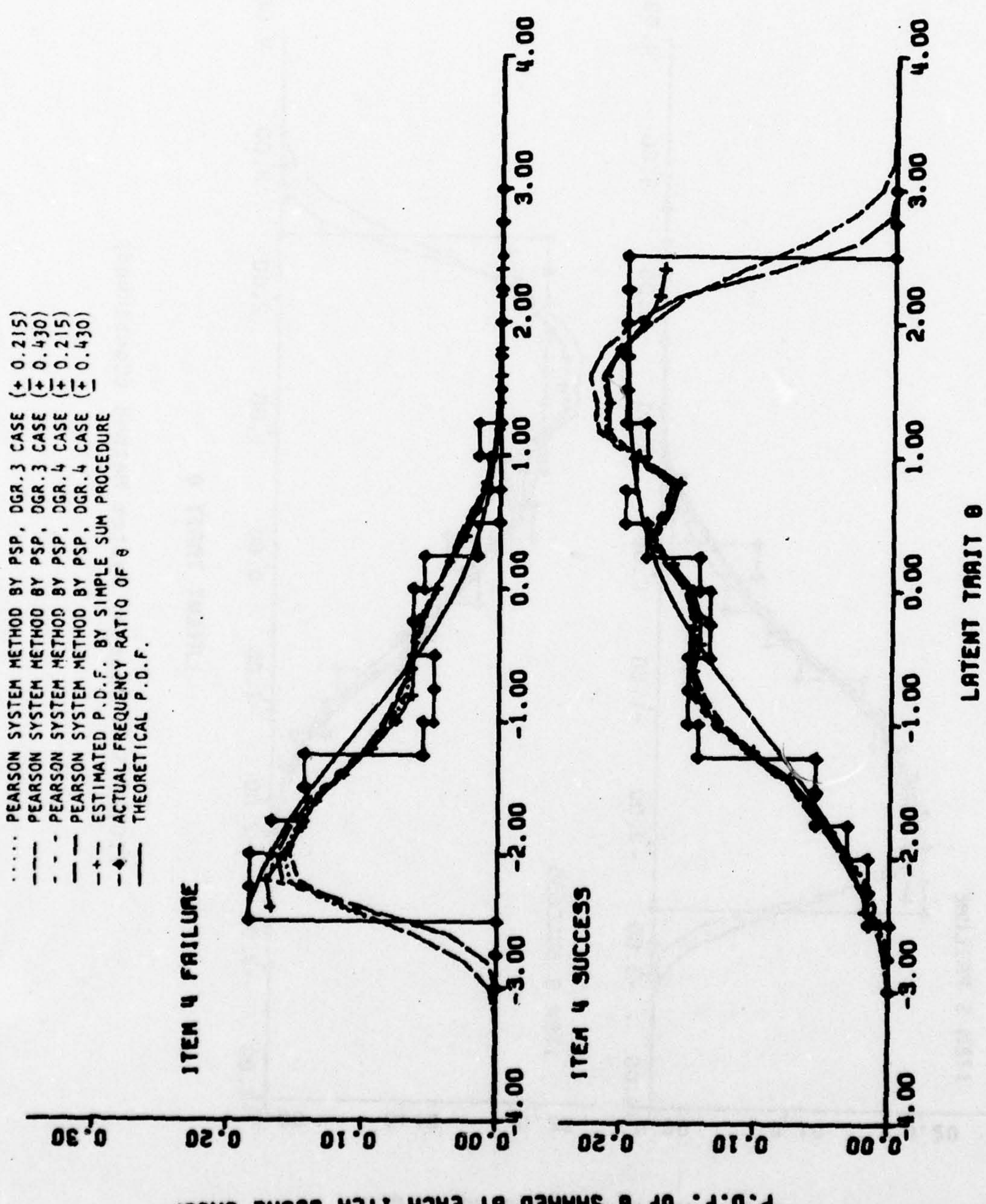

FIGURE 5-4: Pearson System Method (Continued)

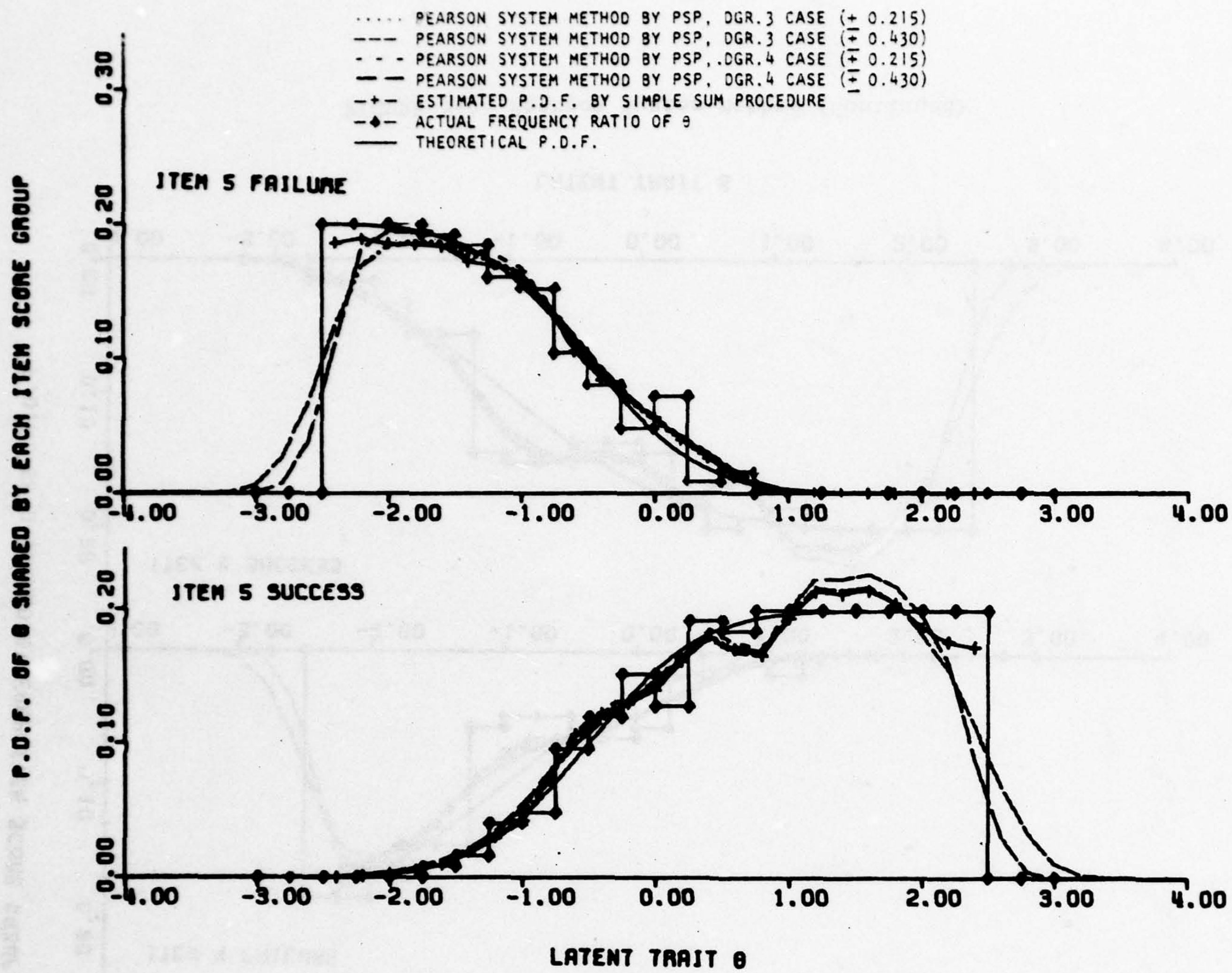

FIGURE 5-4: Pearson System Method (Continued)

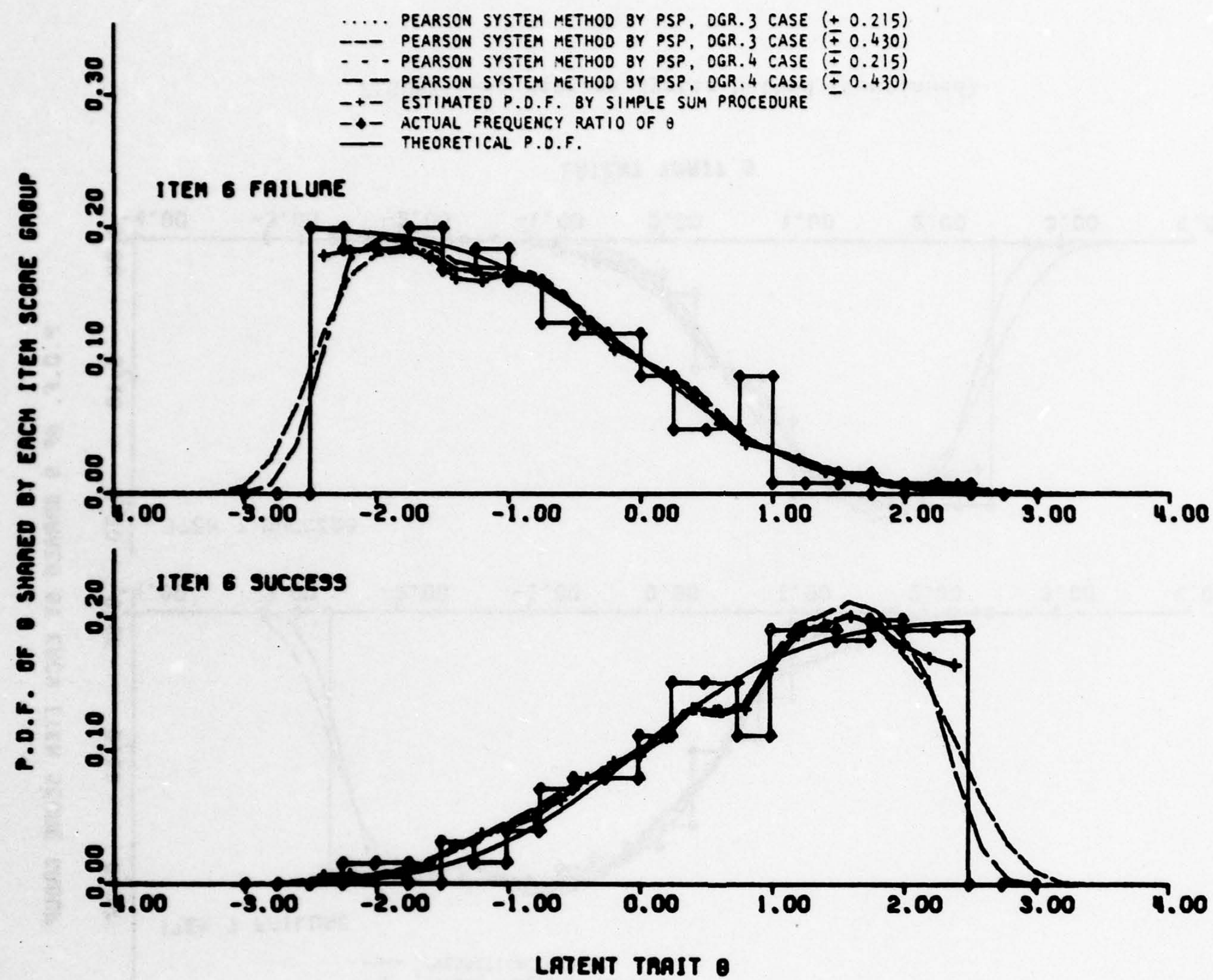

FIGURE 5-4: Pearson System Method (Continued)

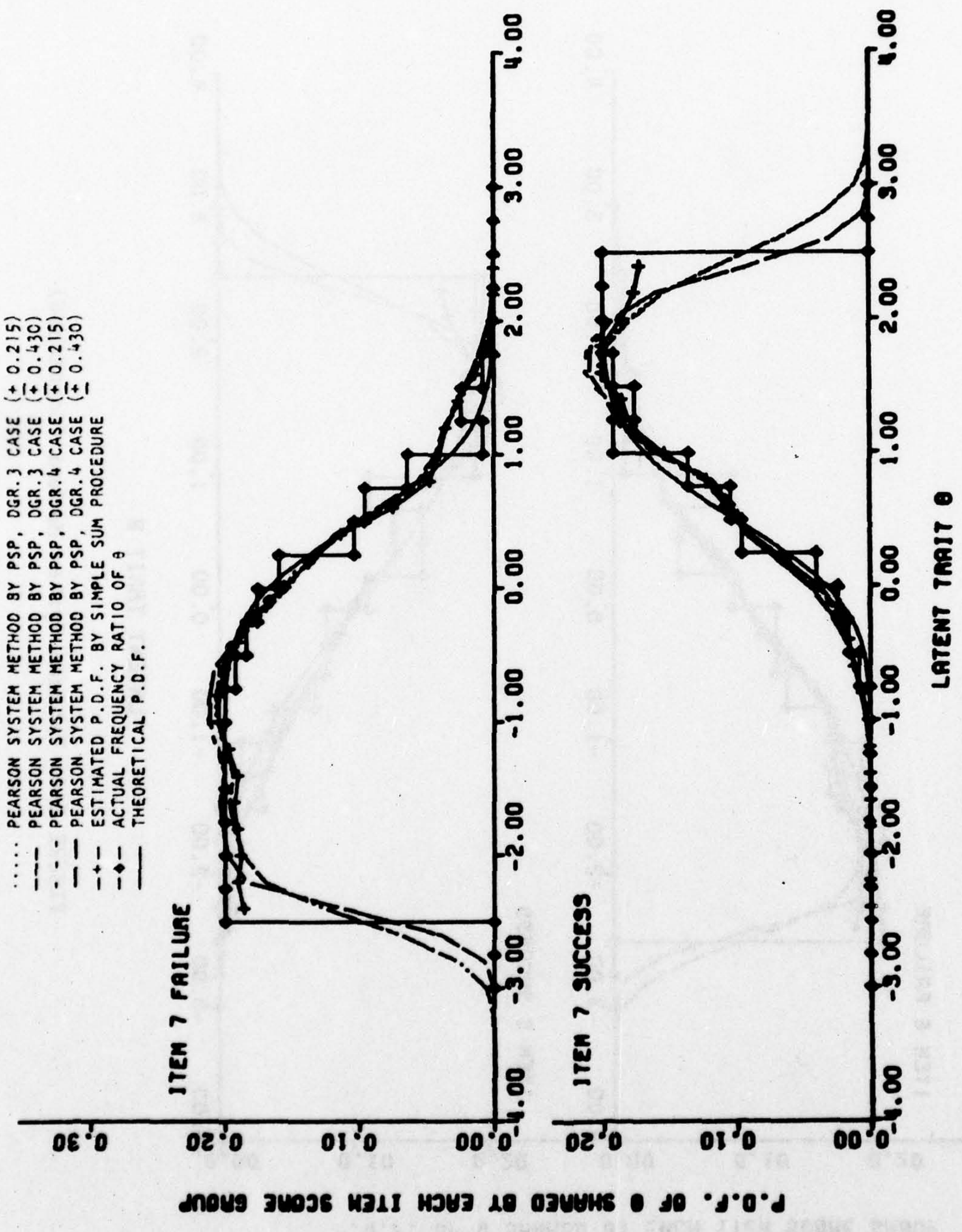

FIGURE 5-4: Pearson System Method (Continued)

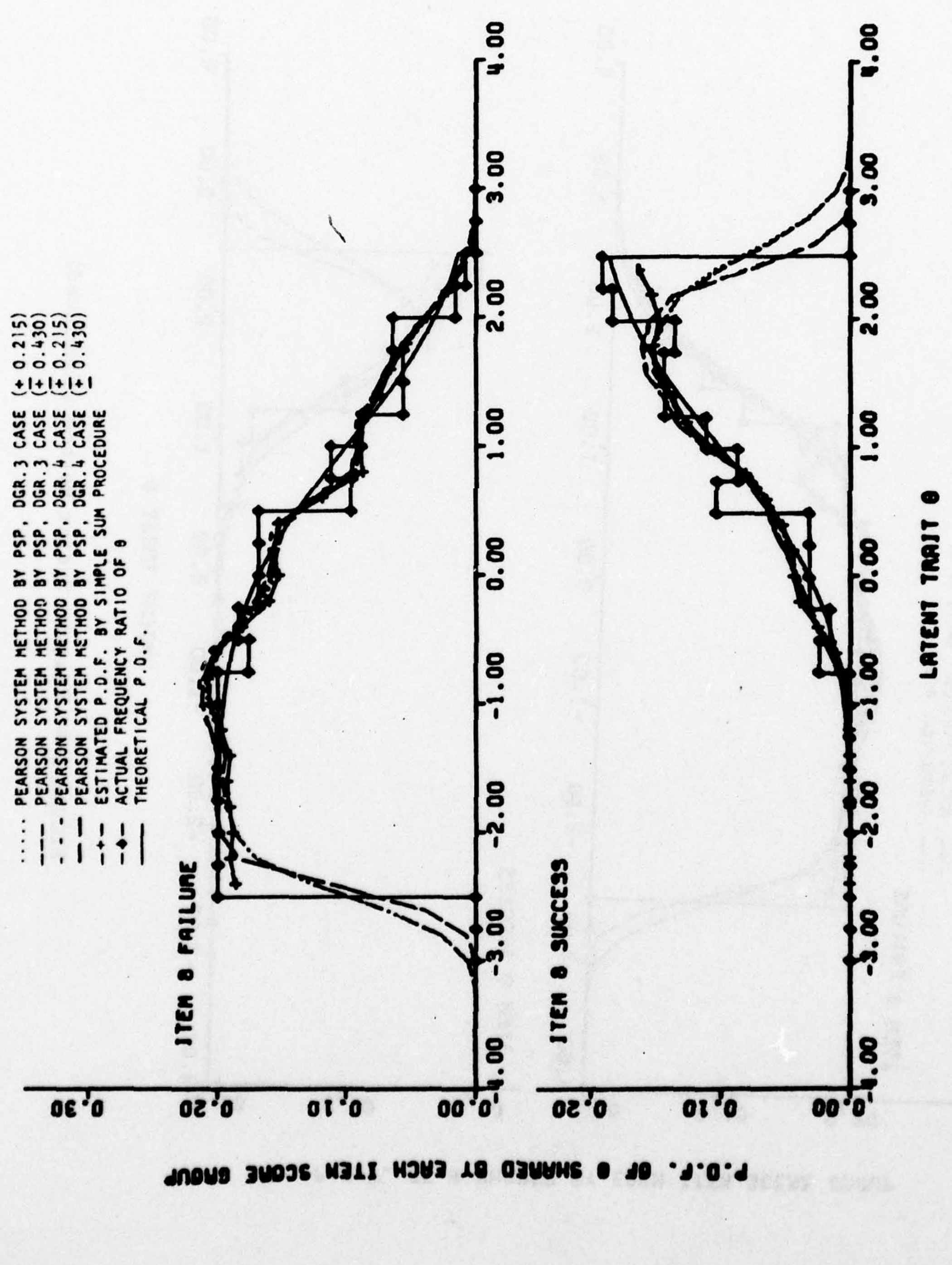

FIGURE 5-4: Pearson System Method (Continued)

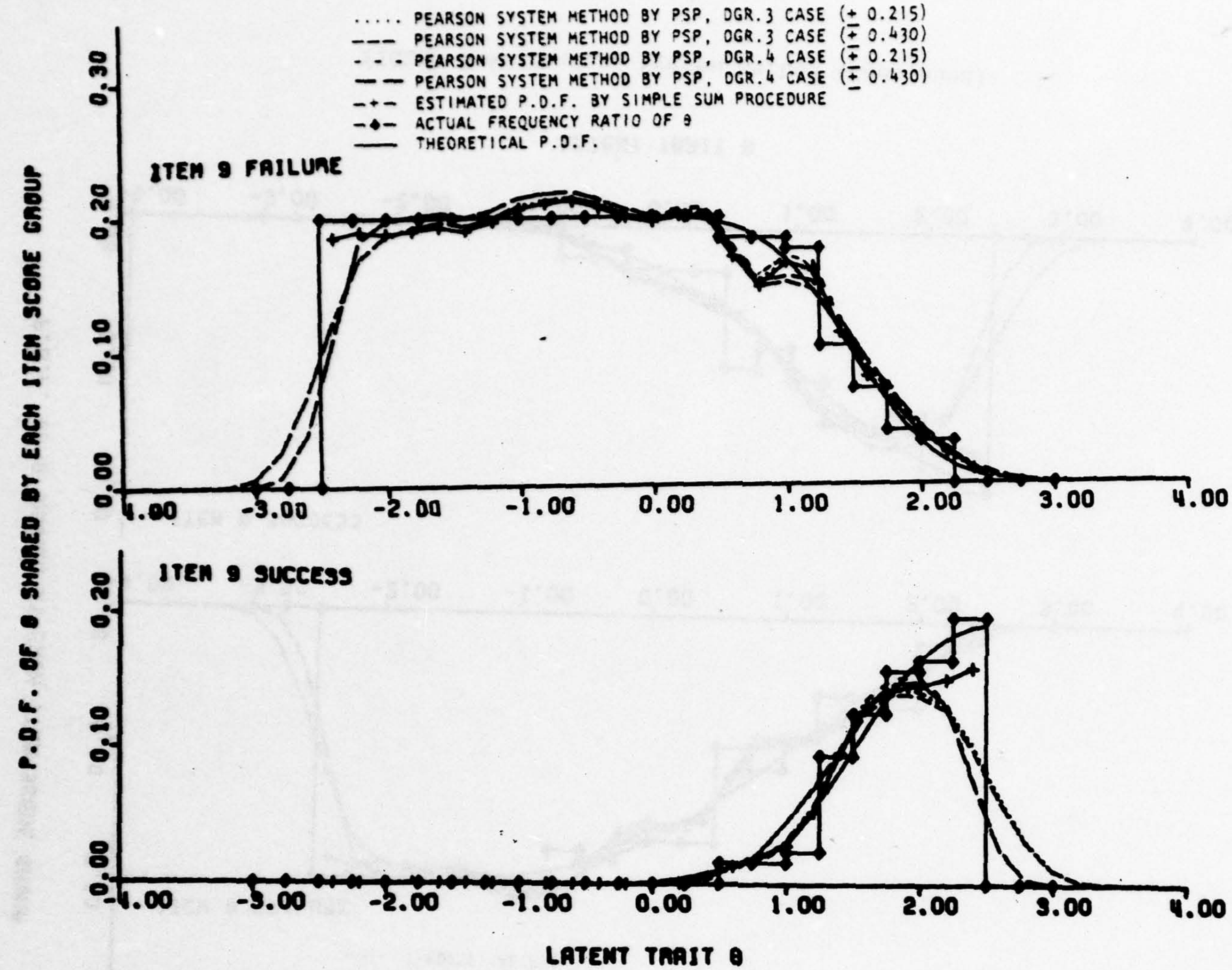

FIGURE 5-4: Pearson System Method (Continued)

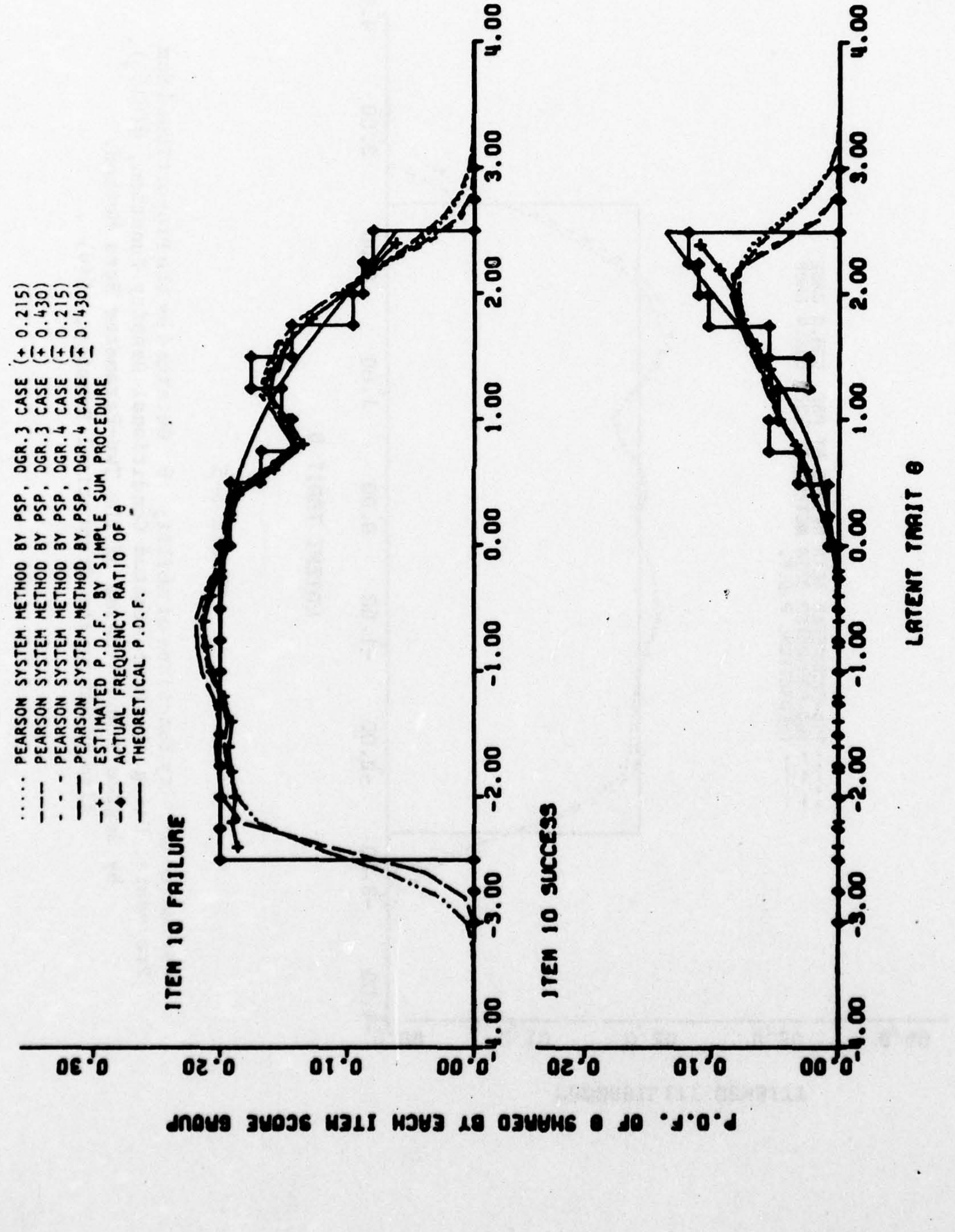

FIGURE 5-4: Pearson System Method (Continued)

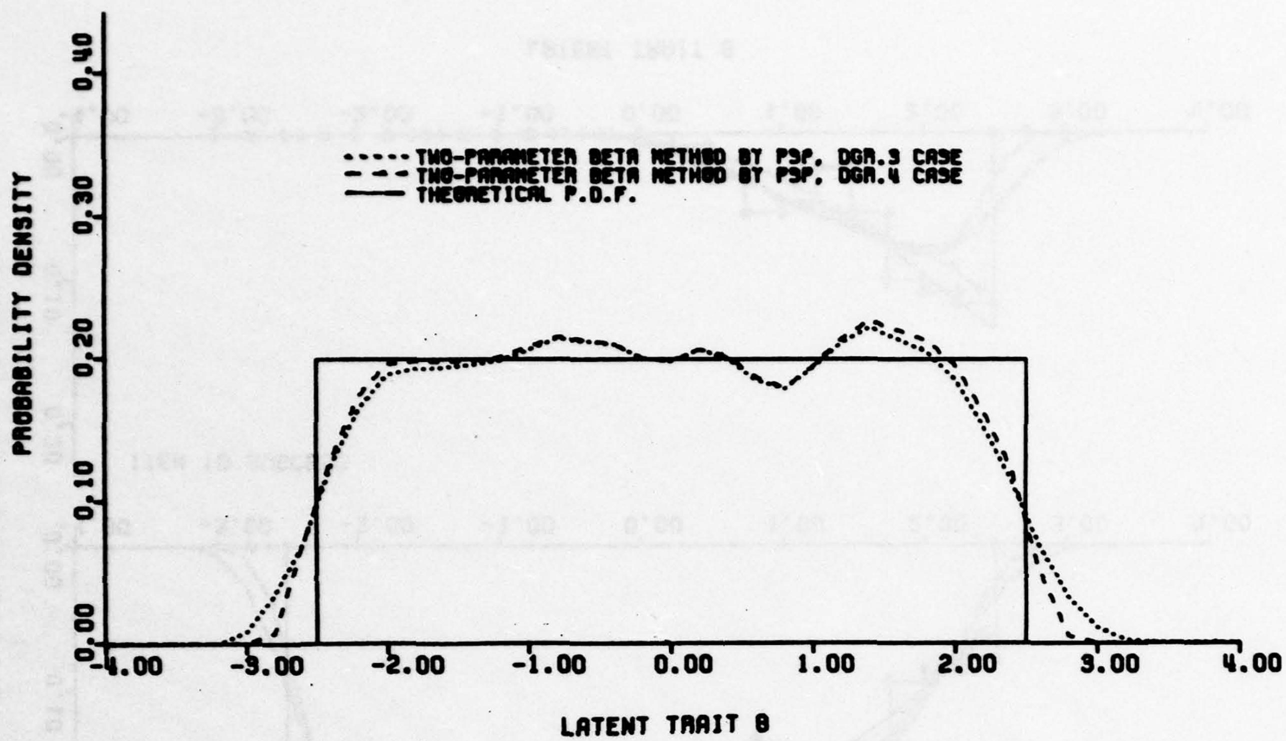

FIGURE 5-5

Estimated Density Functions of Ability θ Obtained by the Proportioned Sum Procedure, Using the Approximated Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, by Degree 3 and 4 Cases of the Two-Parameter Beta Method. Also Drawn is the Theoretical Density, $f(\theta)$.

$\hat{\theta}_s \pm 0.430$. Presented in the same figure are those obtained by the criterion case of the Simple Sum Procedure, the actual frequency ratios and the theoretical shared densities. Just as we have observed in the results obtained by the Pearson System Method, in each of Degree 3 and 4 Cases, the two curves for the estimated shared density functions using the two different intervals of $\hat{\theta}_s$ in obtaining the proportioned weights are very close to each other. It is also noticed that the set of four curves for each item is not too different from the set of those obtained by the Pearson System Method, which are presented in Figure 5-4.

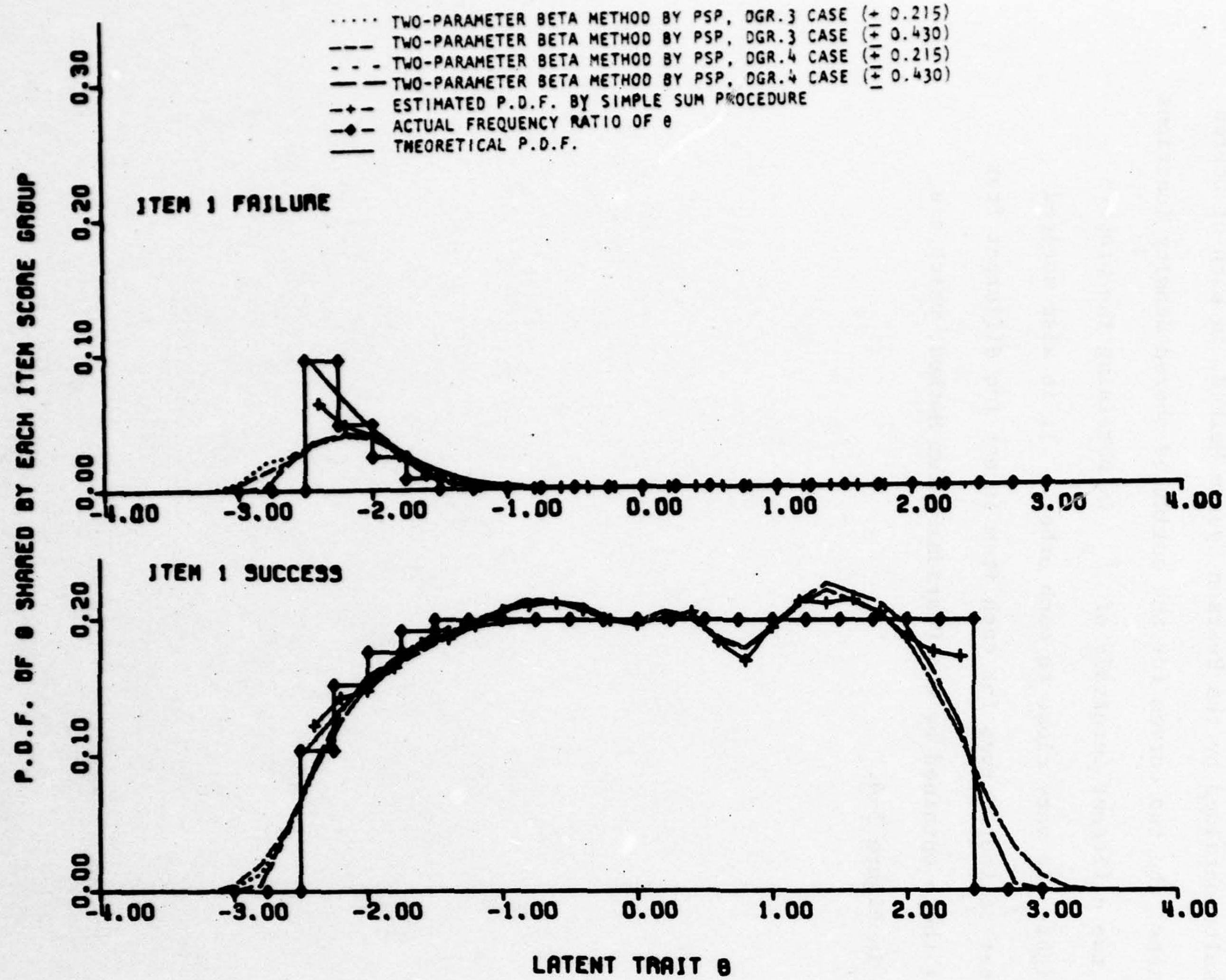

FIGURE 5-6

Four Estimated Shared Density Functions of Ability θ of Each of the Two Item Score Groups, Which Are Obtained by the Proportioned Sum Procedure Using the Approximated Conditional Density Function, $\hat{\phi}(\theta|\hat{\theta}_s)$, by Degree 3 and 4 Cases of the Two-Parameter Beta Method, Respectively. Also Presented Are the One Obtained by Using the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, by the Simple Sum Procedure, Actual Frequency Ratios of Ability θ, and Theoretical Shared Density Function.

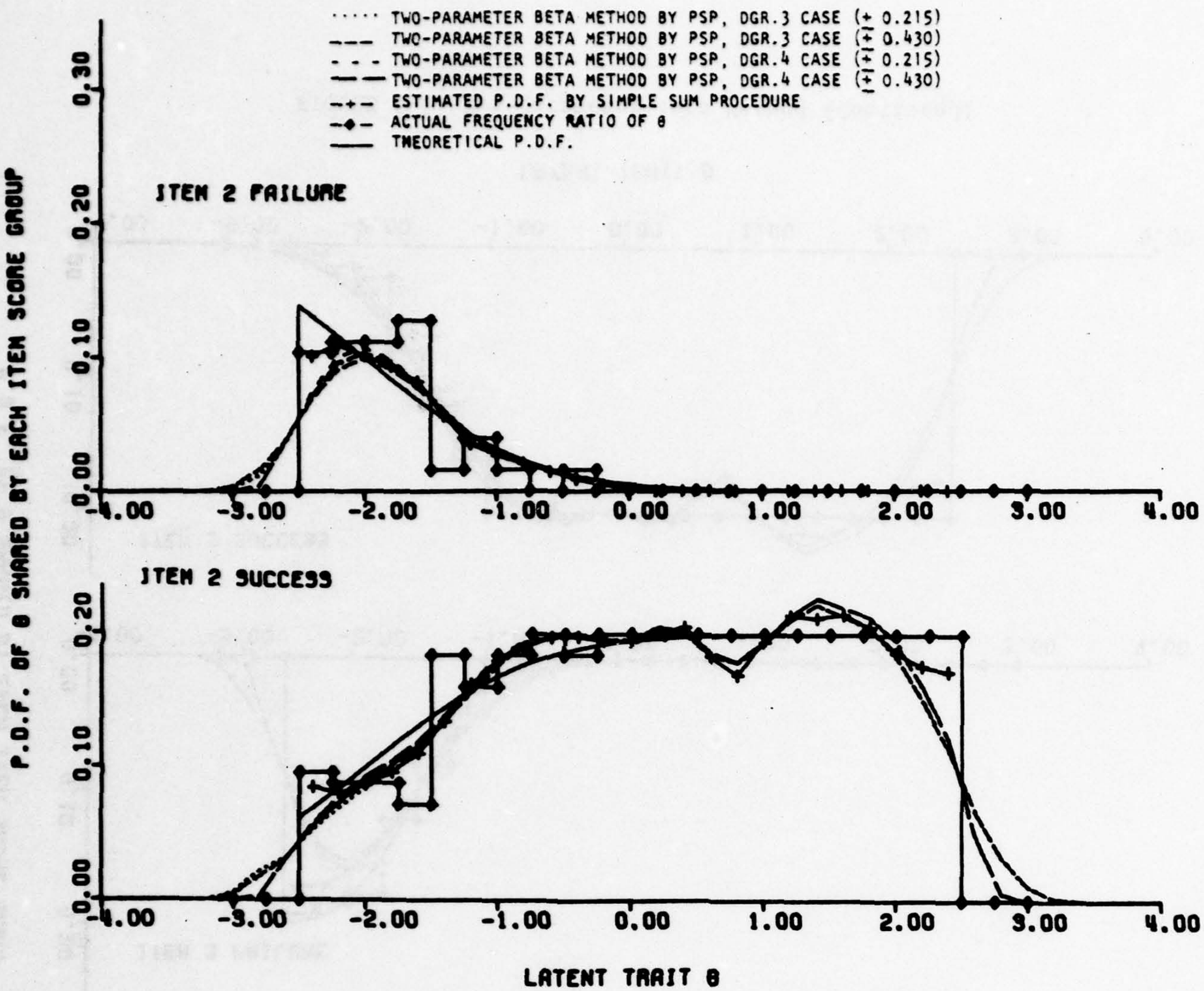

FIGURE 5-6: Two-Parameter Beta Method (Continued)

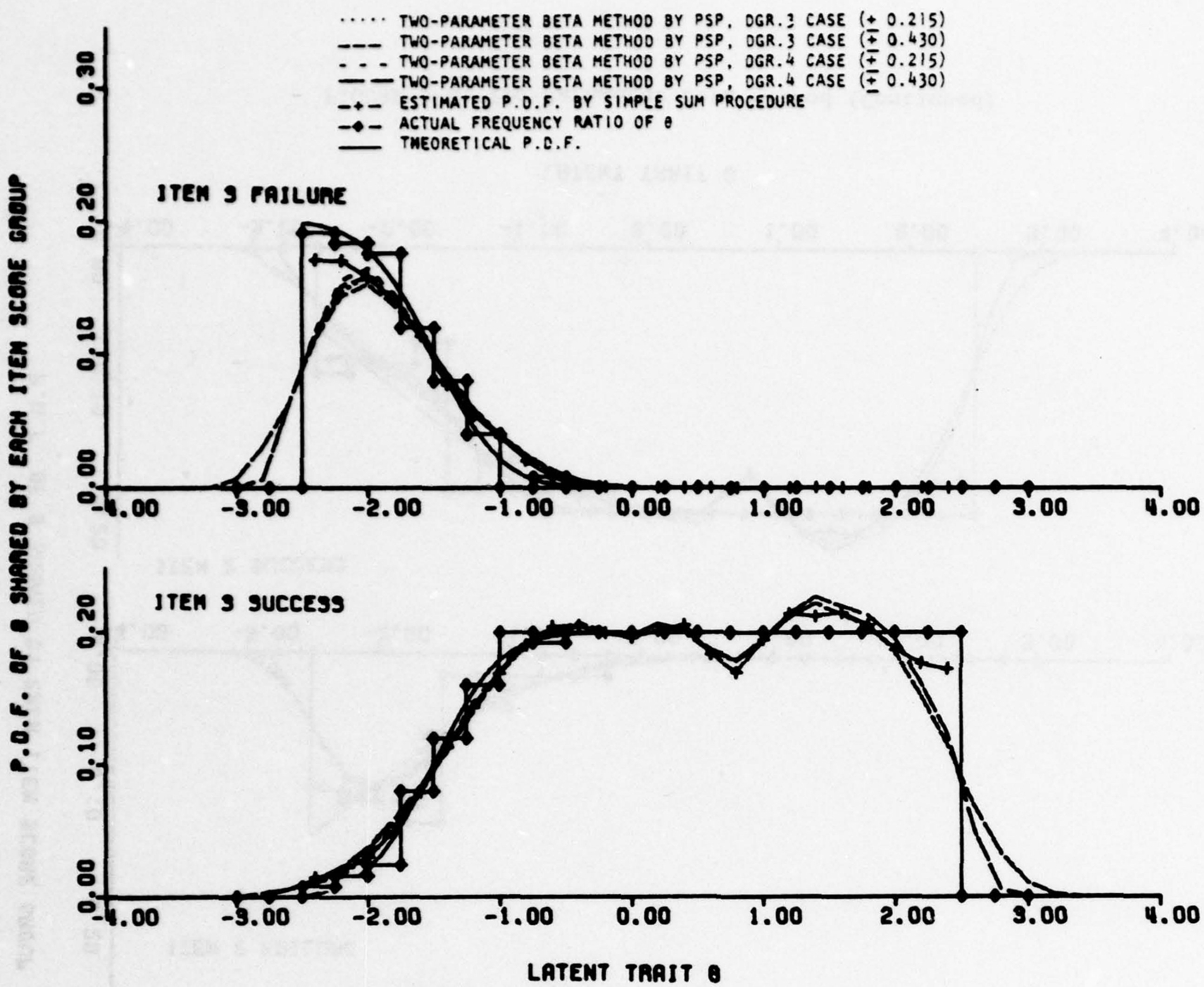

FIGURE 5-6: Two-Parameter Beta Method (Continued)

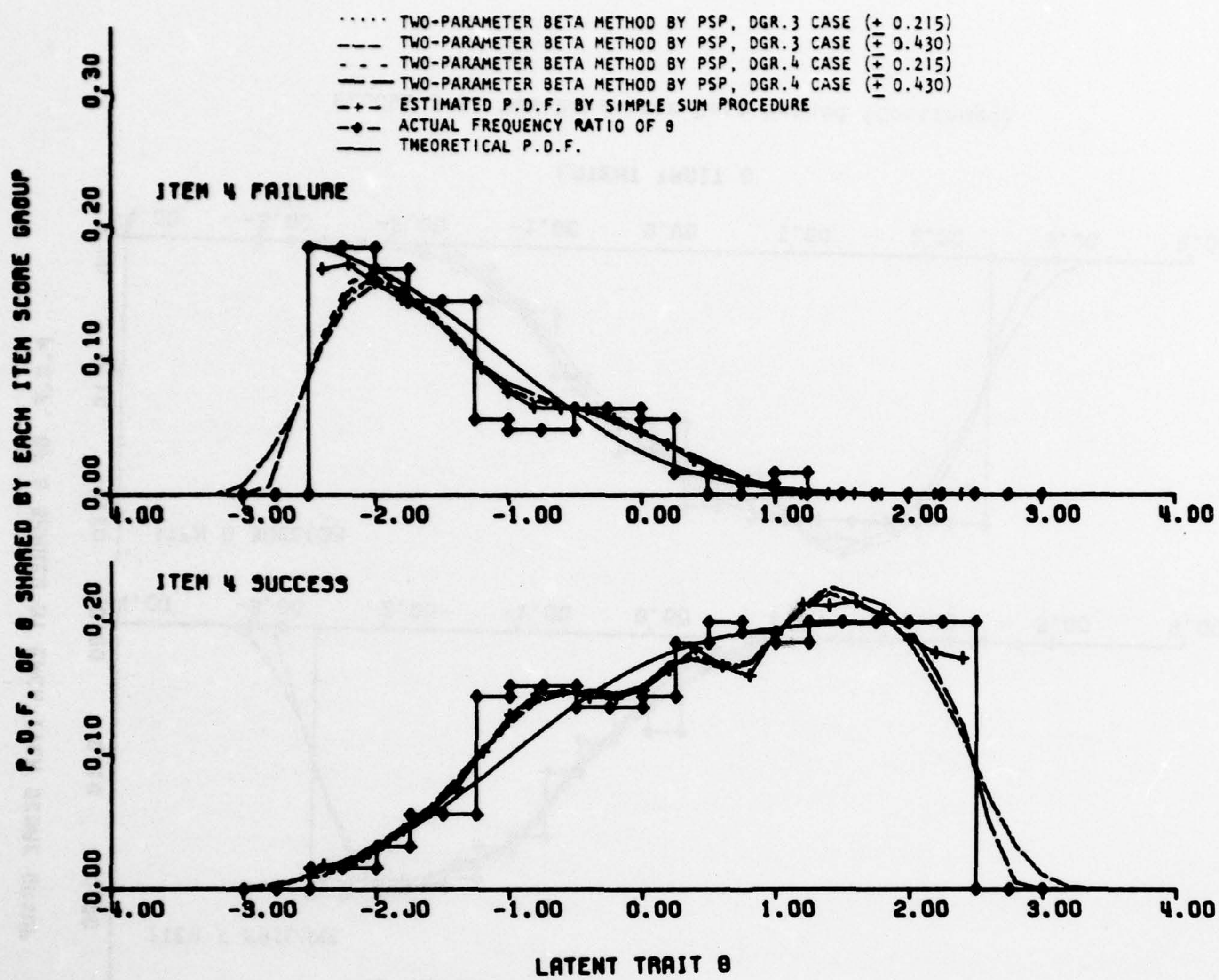

FIGURE 5-6: Two-Parameter Beta Method (Continued)

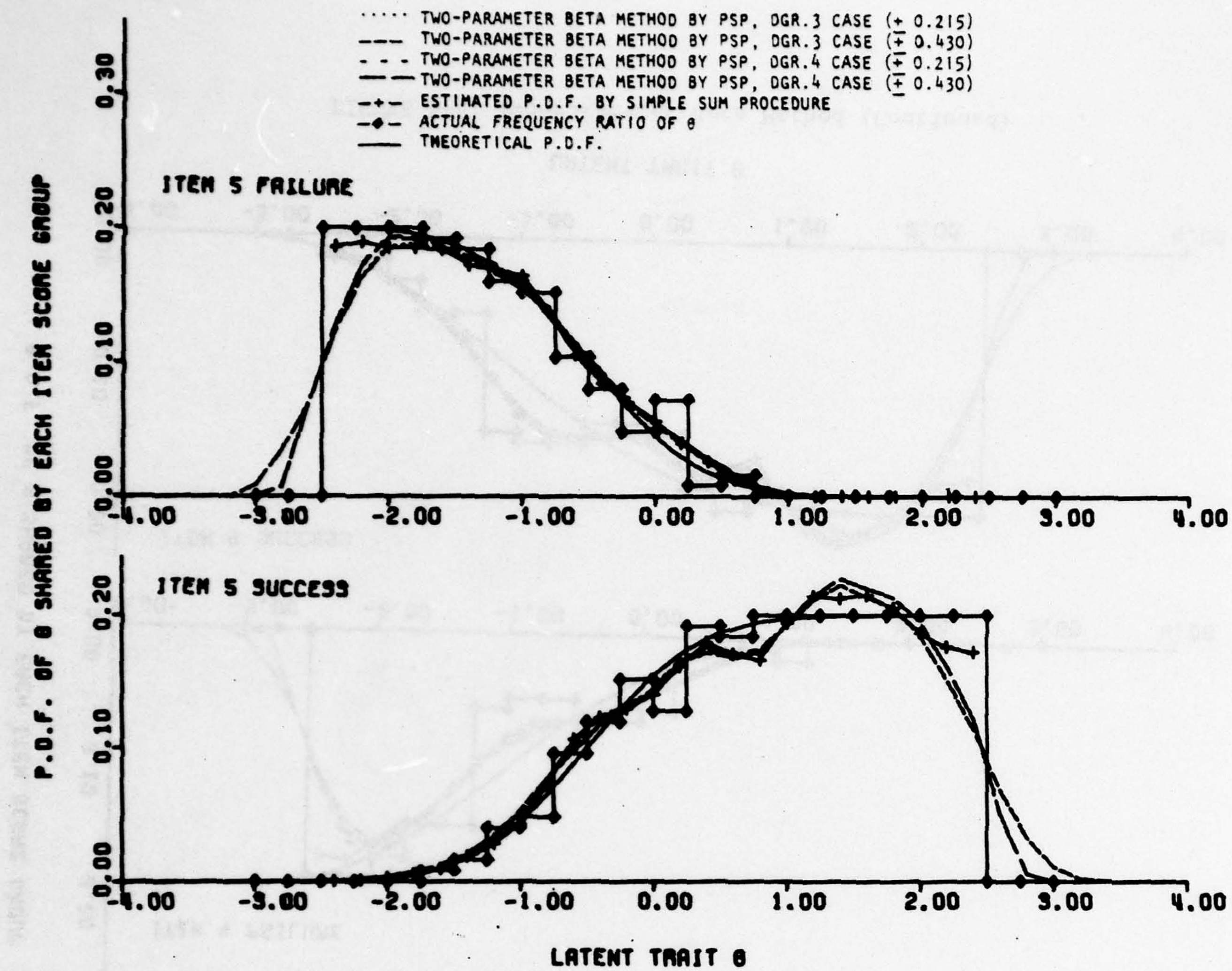

FIGURE 5-6: Two-Parameter Beta Method (Continued)

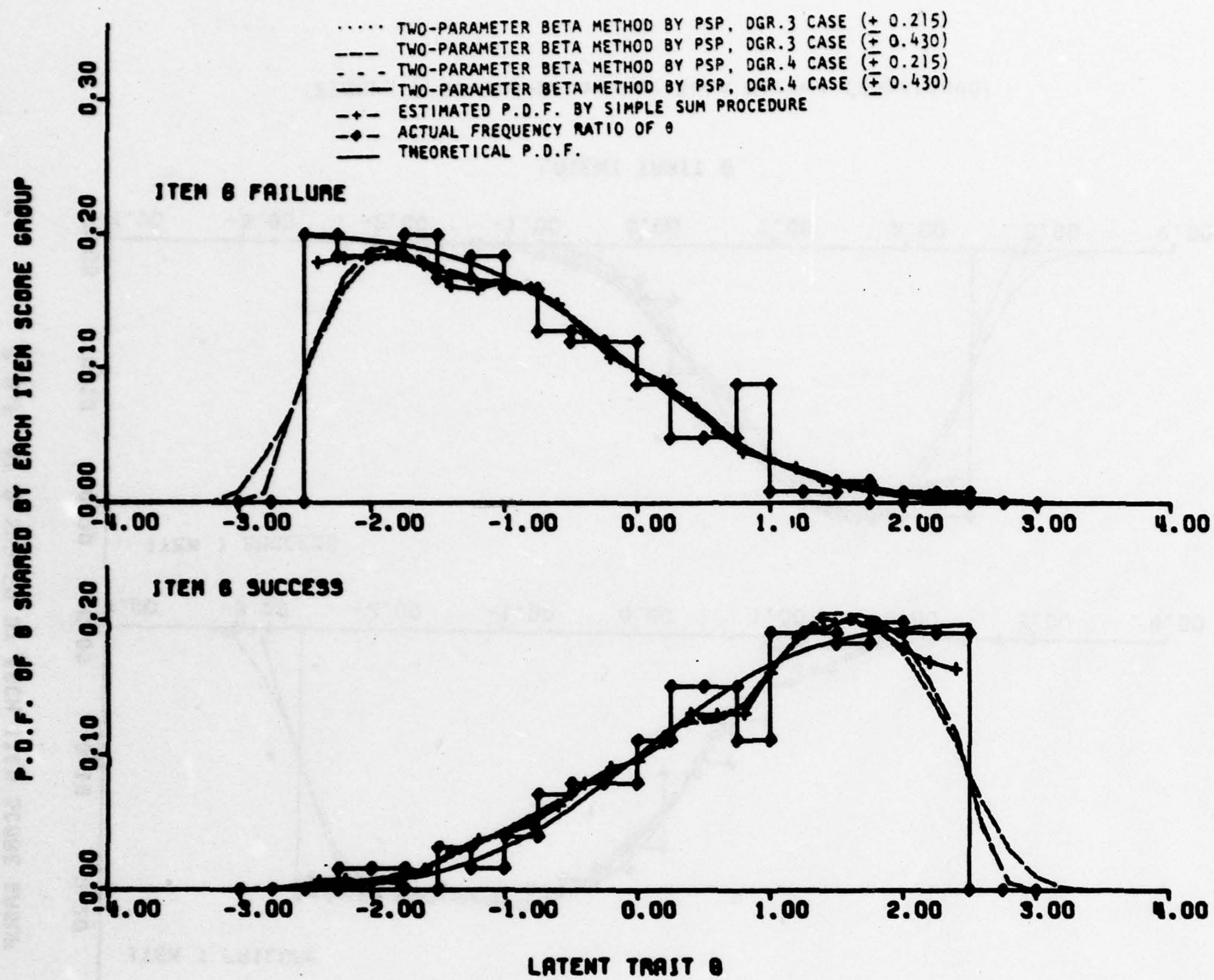

FIGURE 5-6: Two-Parameter Beta Method (Continued)

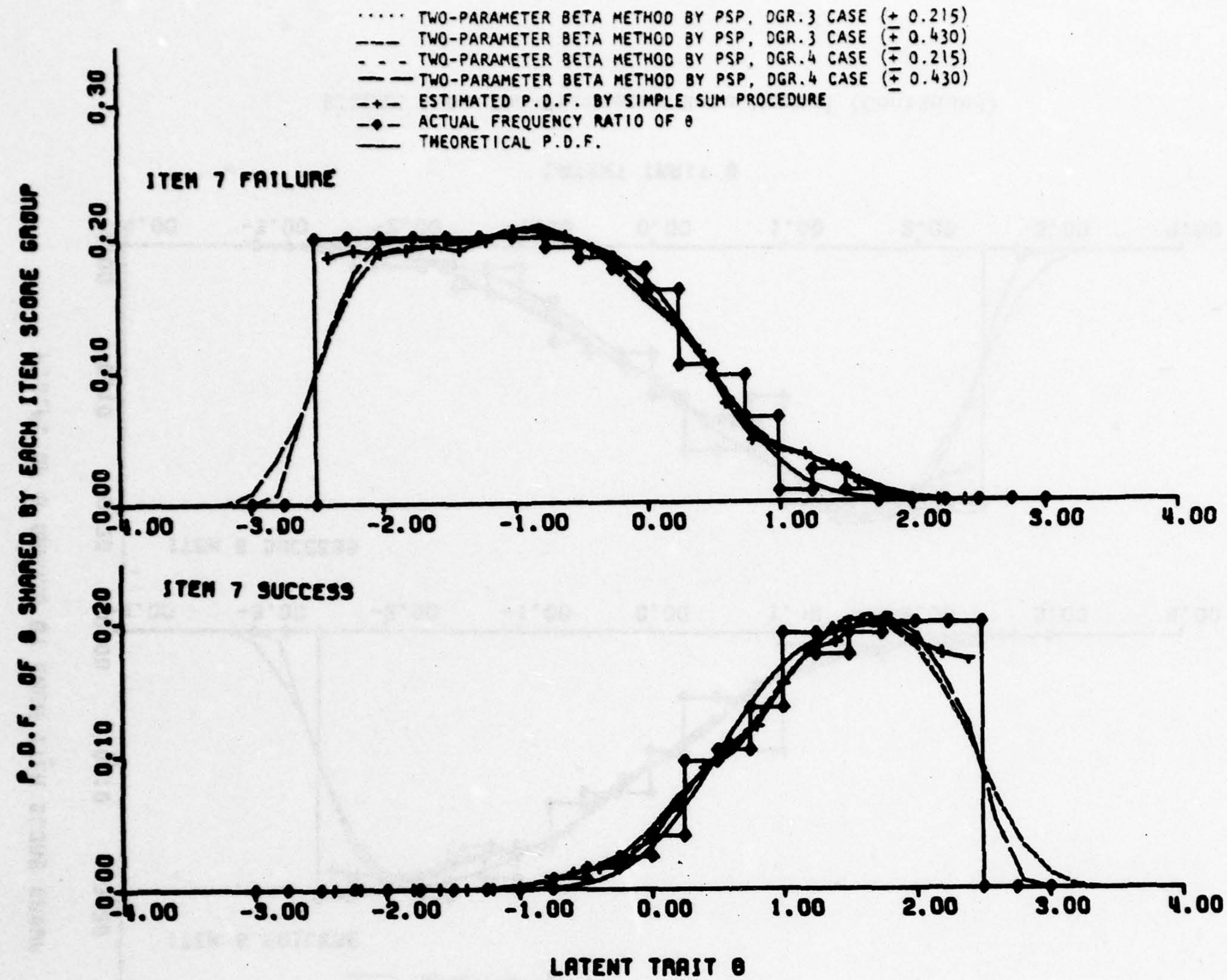

FIGURE 5-6: Two-Parameter Beta Method (Continued)

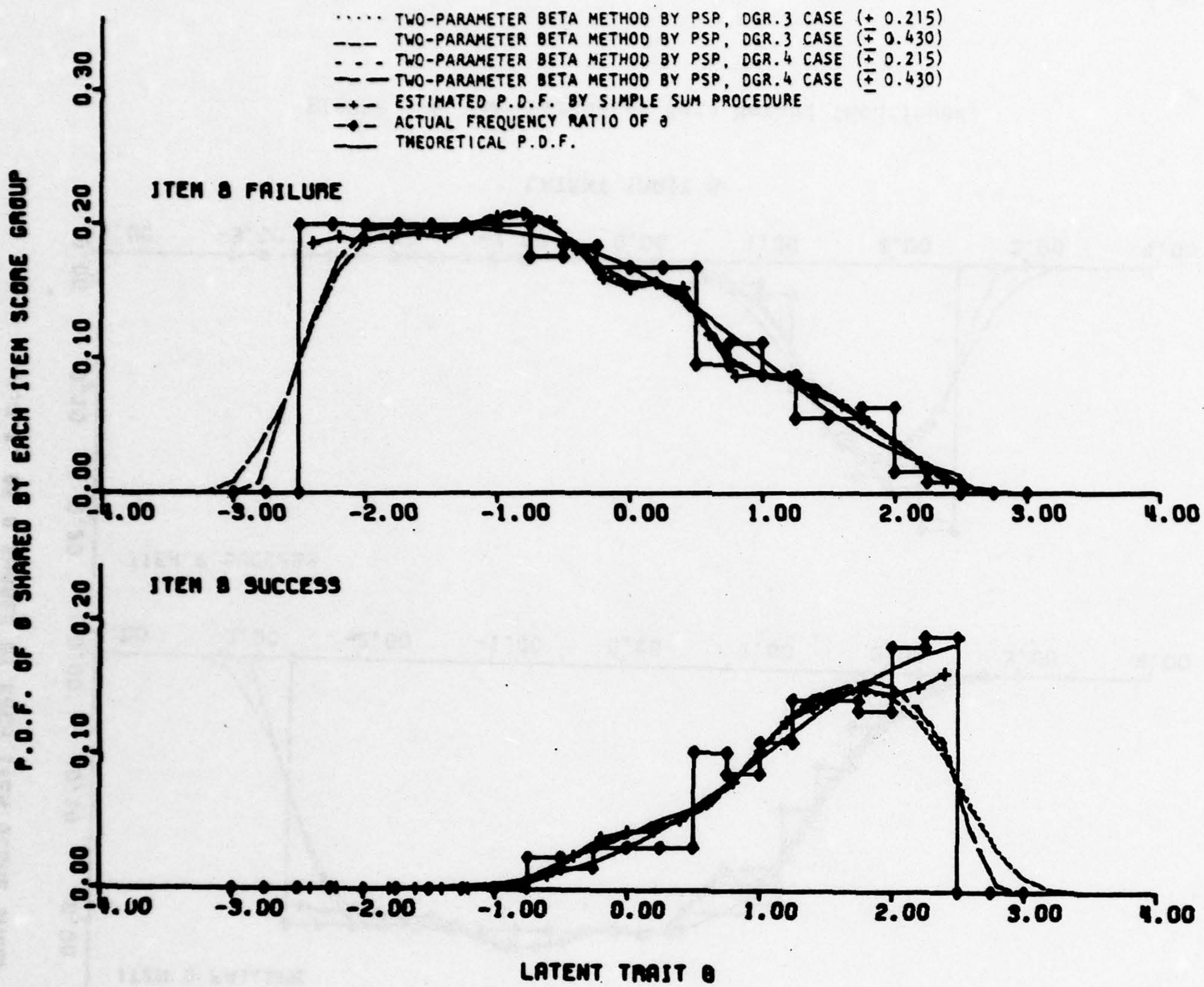

FIGURE 5-6: Two-Parameter Beta Method (Continued)

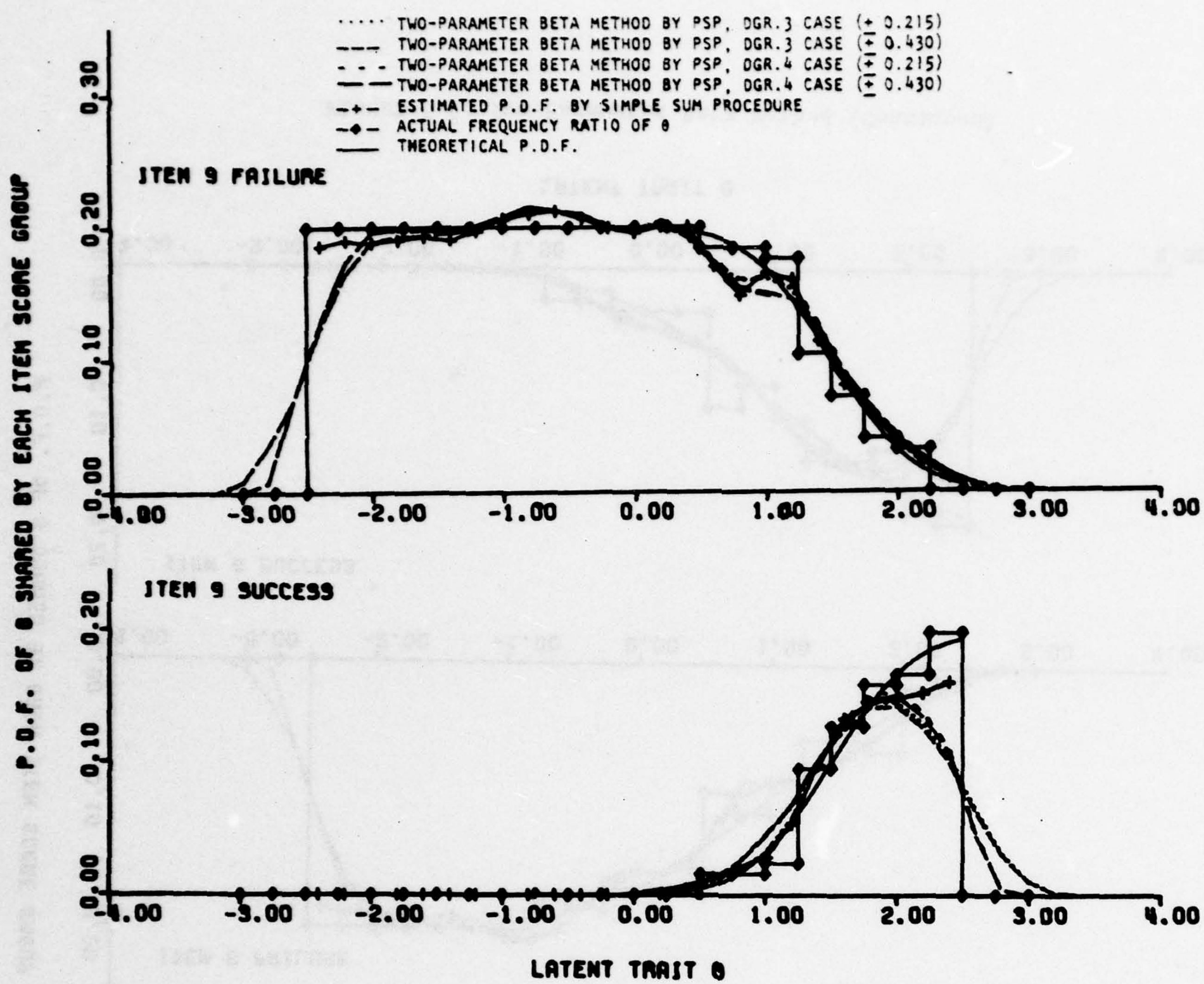

FIGURE 5-6: Two-Parameter Beta Method (Continued)

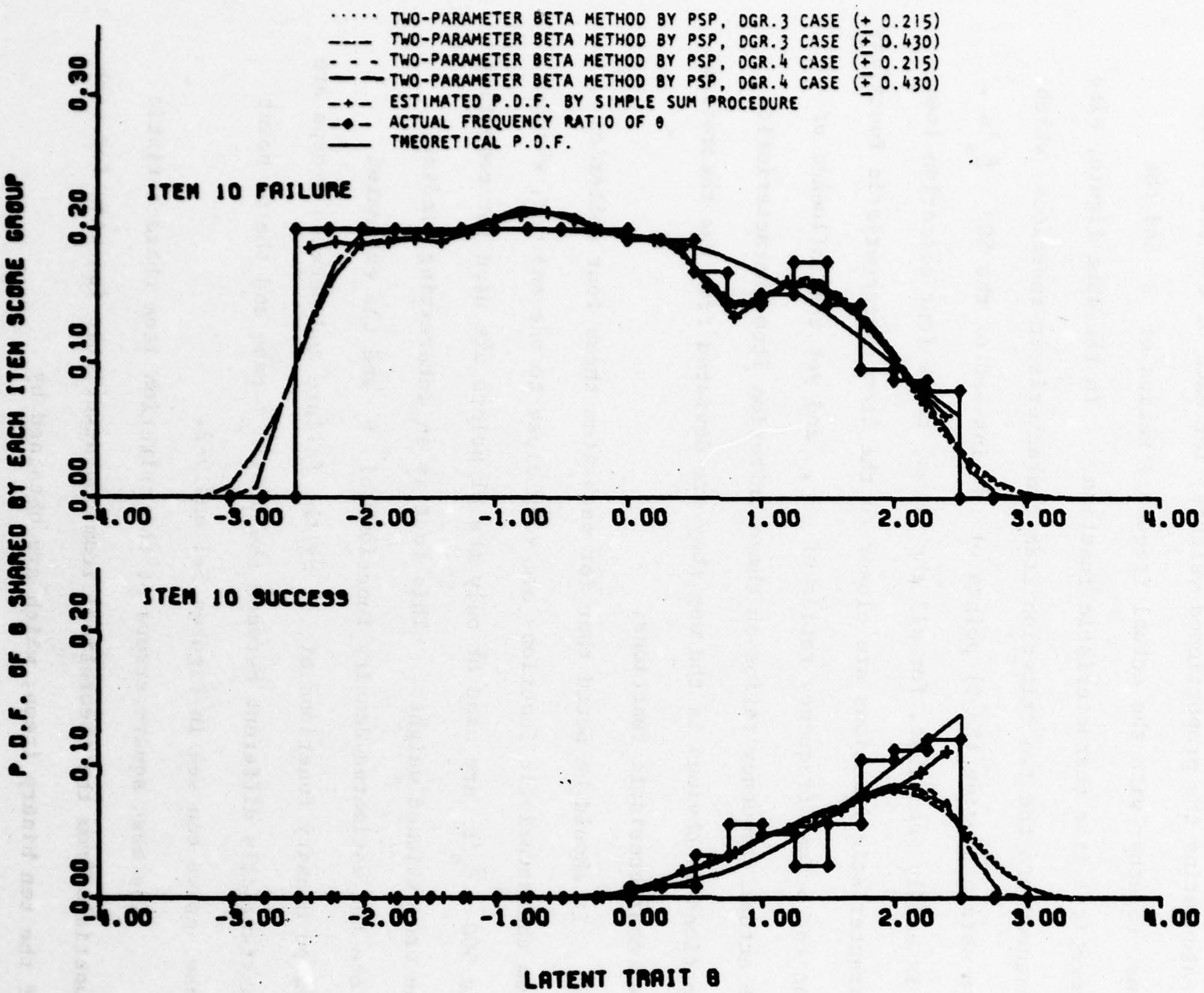

FIGURE 5-6: Two-Parameter Beta Method (Continued)

VI Results III: Estimated Item Characteristic Functions

Figure 6-1 presents the resultant criterion item characteristic functions using two different intervals, $\hat{\theta}_s \pm 0.215$ and $\hat{\theta}_s \pm 0.430$, in calculating the proportioned weights, for each of the ten binary items, together with the actual frequency ratios of θ and the theoretical item characteristic function. In the same figure, also presented are the two criterion item characteristic functions, which were obtained using the 61 points of $\hat{\theta}$ instead of the 500 $\hat{\theta}_s$'s. It is easily seen that, for all the items, these four criterion item characteristic functions are closer to the item characteristic functions than the actual frequency ratios of θ, and yet the influence of the actual frequency ratios on these criterion item characteristic functions is obvious in the way they are deviated from the theoretical item characteristic functions.

It should be noted that for each item these four criterion item characteristic functions are very close to one another, whether the 500 $\hat{\theta}_s$'s are used or only the 61 points are used for computing the proportioned weights. This fact is an interesting finding, since the estimated density functions of θ and the estimated shared density functions of θ by the failure and success groups are substantially different between the 500 $\hat{\theta}_s$ case and the 61 point case, as we can see in Figures 5-1 and 5-2.

The mean square errors of the criterion item characteristic functions from the theoretical item characteristic function for each of the ten binary items, which are obtained by

$$(6.1) \qquad \frac{1}{m} \sum_{j=1}^{m} [\hat{P}_g(\theta_j) - P_g(\theta_j)]^2 \quad ,$$

where $m = 25$ and $\hat{\theta}_j$'s are -2.4 through 2.4 with the step of 0.2, were computed for each criterion item characteristic function of each item, and these values and their square roots are presented in Tables 6-1 and 6-2.

It is noticed that among these four sets of mean square errors, or their square roots, there are no substantial differences, which indicates that these four sets of criterion item characteristic functions are just as good as one another. There is even a slight tendency that the two sets for the 61 point case have smaller values than those for the 500 $\hat{\theta}_s$'s case, which encourages the idea of reducing the number of points of the maximum likelihood estimate for which the conditional density, $\phi(\theta|\hat{\theta})$, is estimated and used for the estimation of the operating characteristics.

The discrimination parameter, a_g, and the difficulty parameter, b_g, in the normal ogive model* were estimated by the usual least square method for each of the ten binary items, and are presented as Tables 6-3 and 6-4. It is obvious that these estimated parameters are just as good as those obtained by the Simple Sum Procedure, which are also presented in these tables, if not substantially better. This indicates that they are also comparable with those obtained from the Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases, as we can see in one of the previous studies (Samejima, 1978d). As before, in the present results of the Proportioned Sum Procedure, in general, the estimates of the difficulty parameters b_g are closer to the true values than those of the discrimination parameters a_g.

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* Refer to Appendix I.

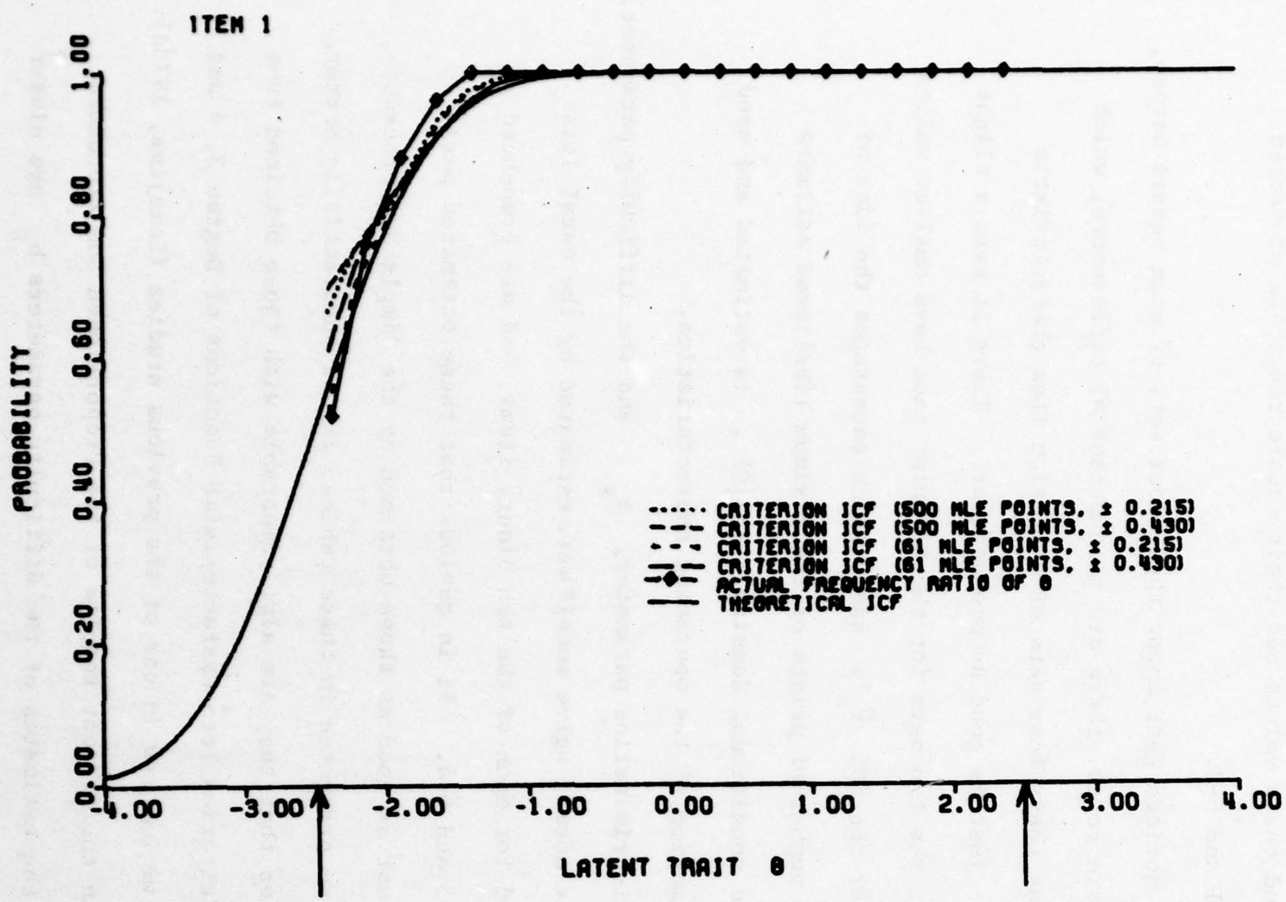

FIGURE 6-1

Comparison of the Four Criterion Item Characteristic Functions by the Proportioned Sum Procedure of Conditional P.D.F. Approach with the Actual Frequency Ratios and Theoretical Item Characteristic Function.

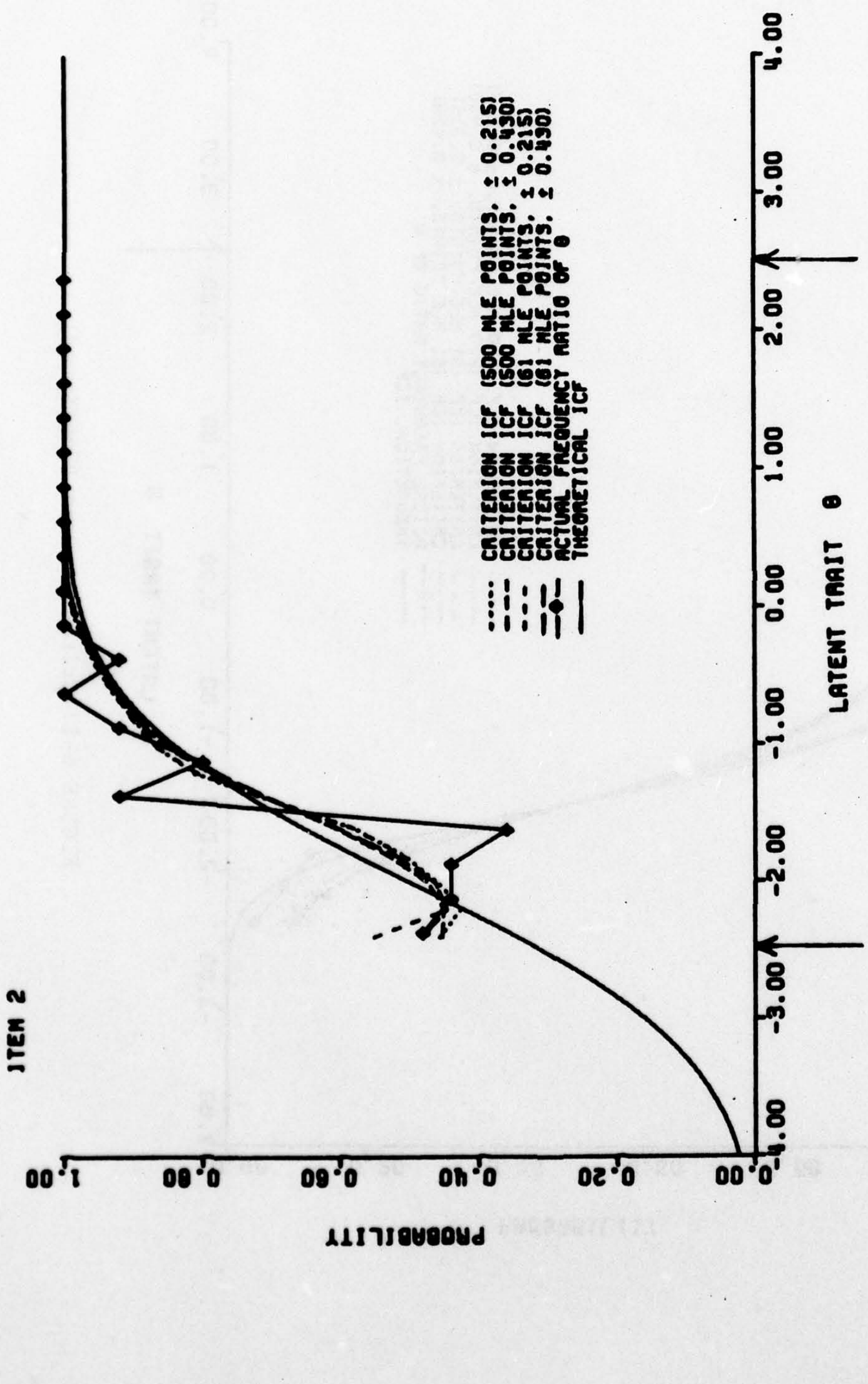

FIGURE 6-1: Criterion Case (Continued)

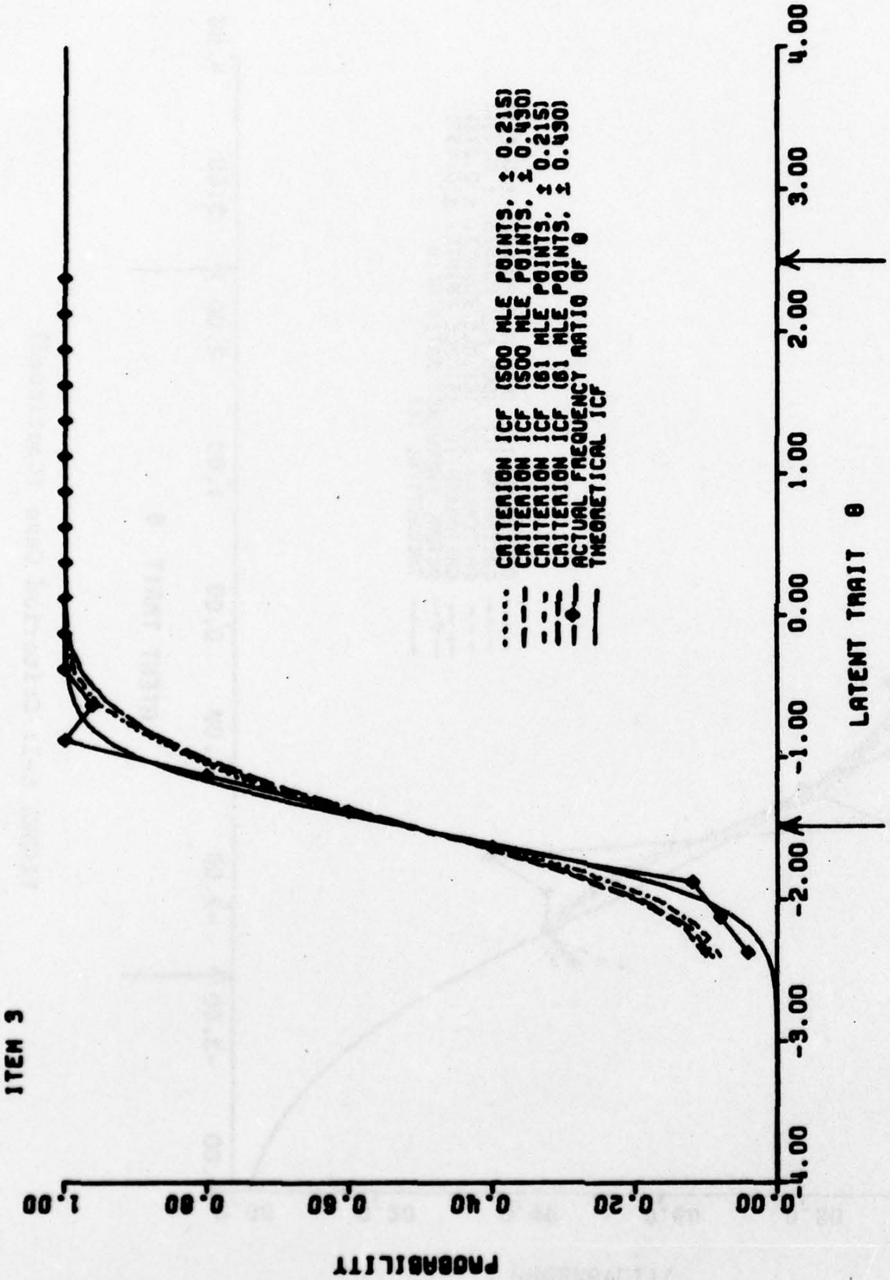

FIGURE 6-1: Criterion Case (Continued)

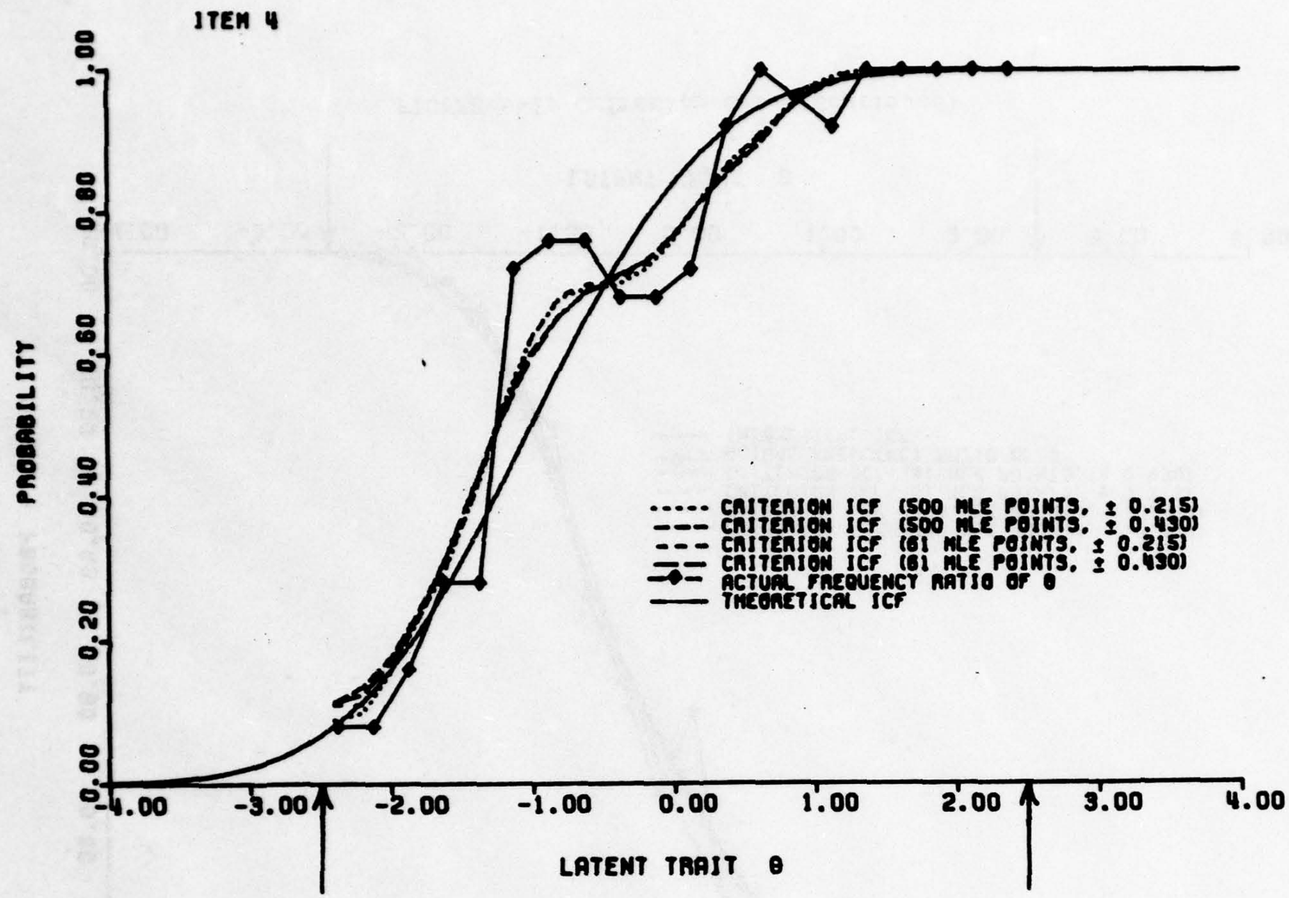

FIGURE 6-1: Criterion Case (Continued)

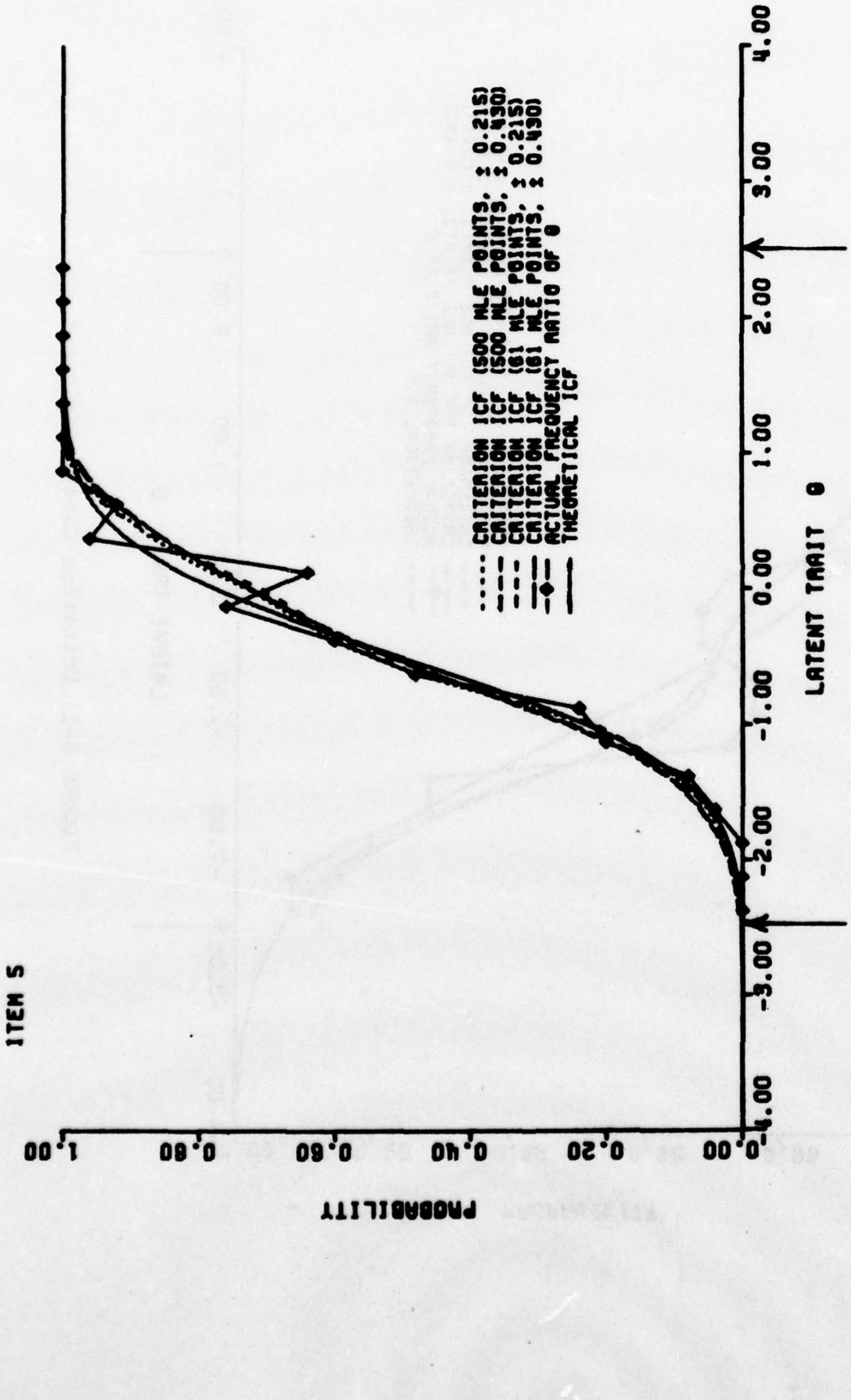

FIGURE 6-1: Criterion Case (Continued)

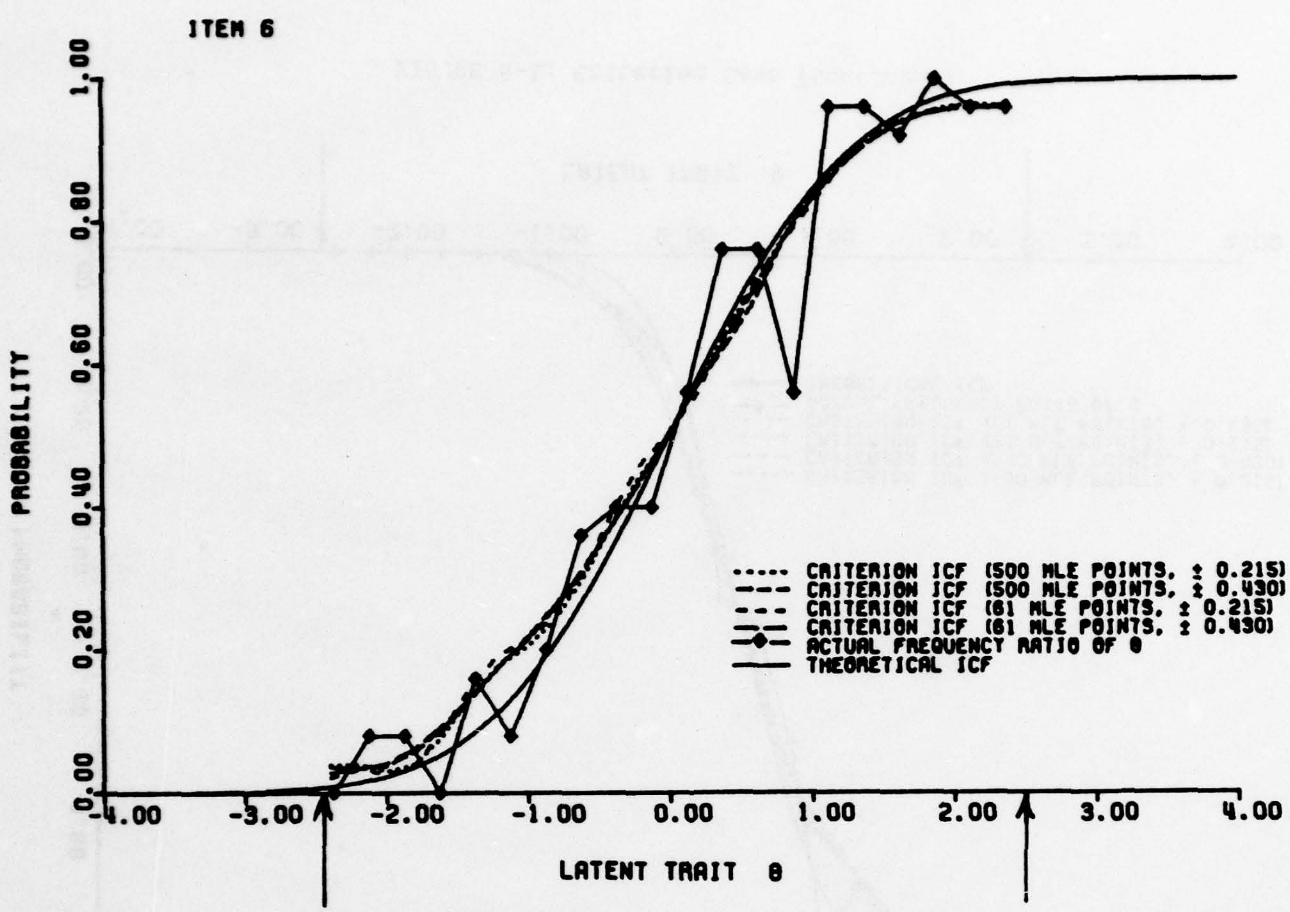

FIGURE 6-1: Criterion Case (Continued)

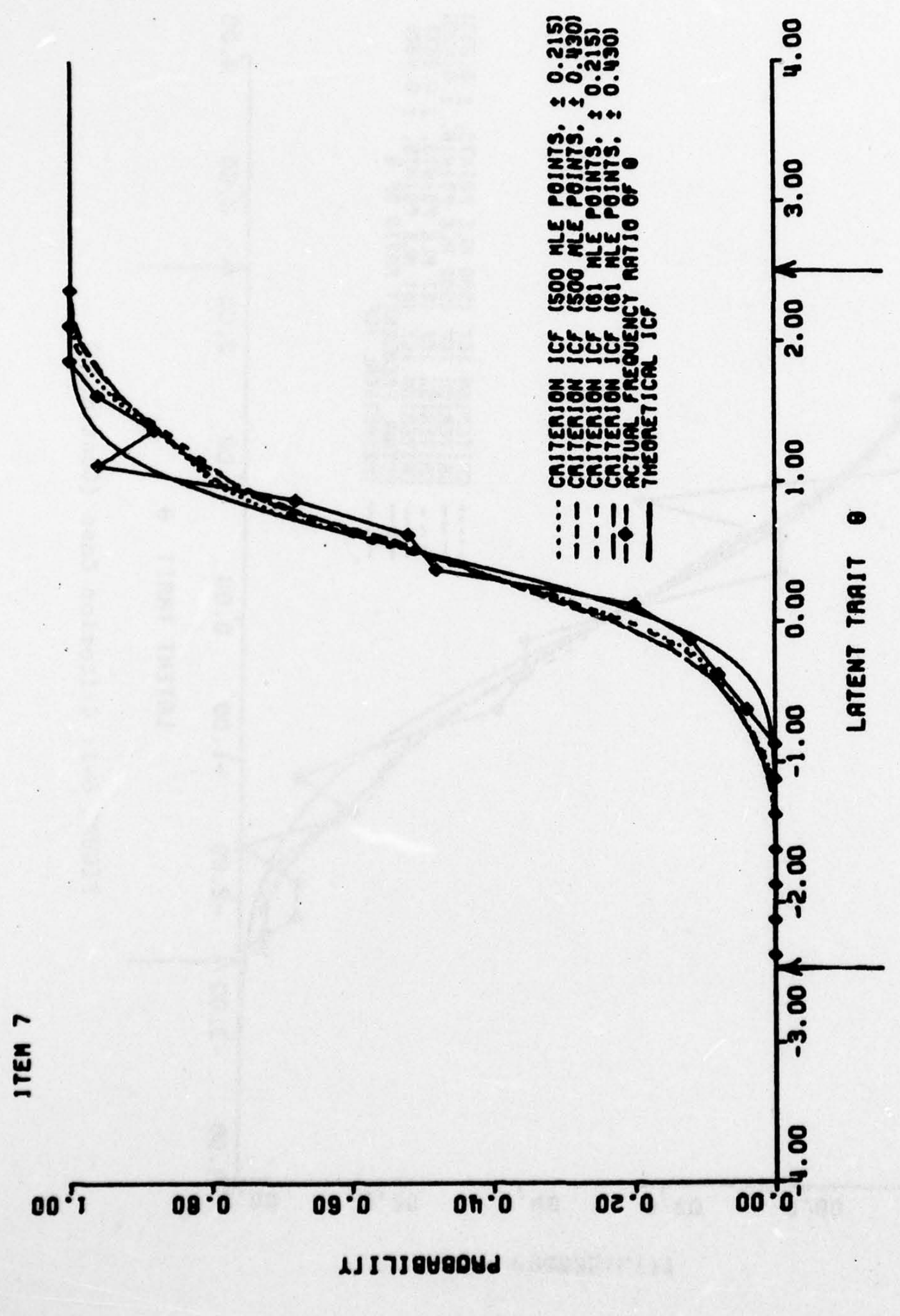

FIGURE 6-1: Criterion Case (Continued)

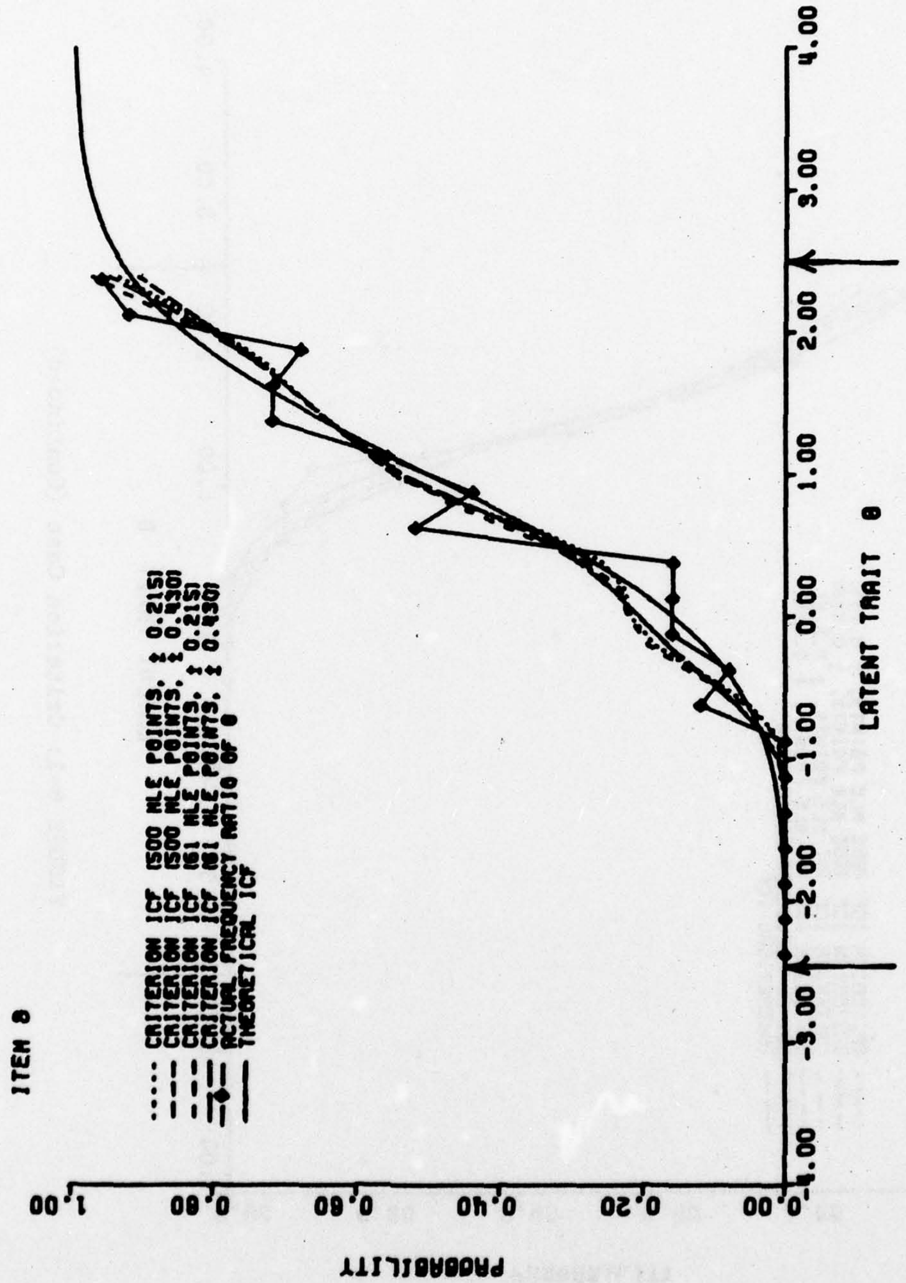

FIGURE 6-1: Criterion Case (Continued)

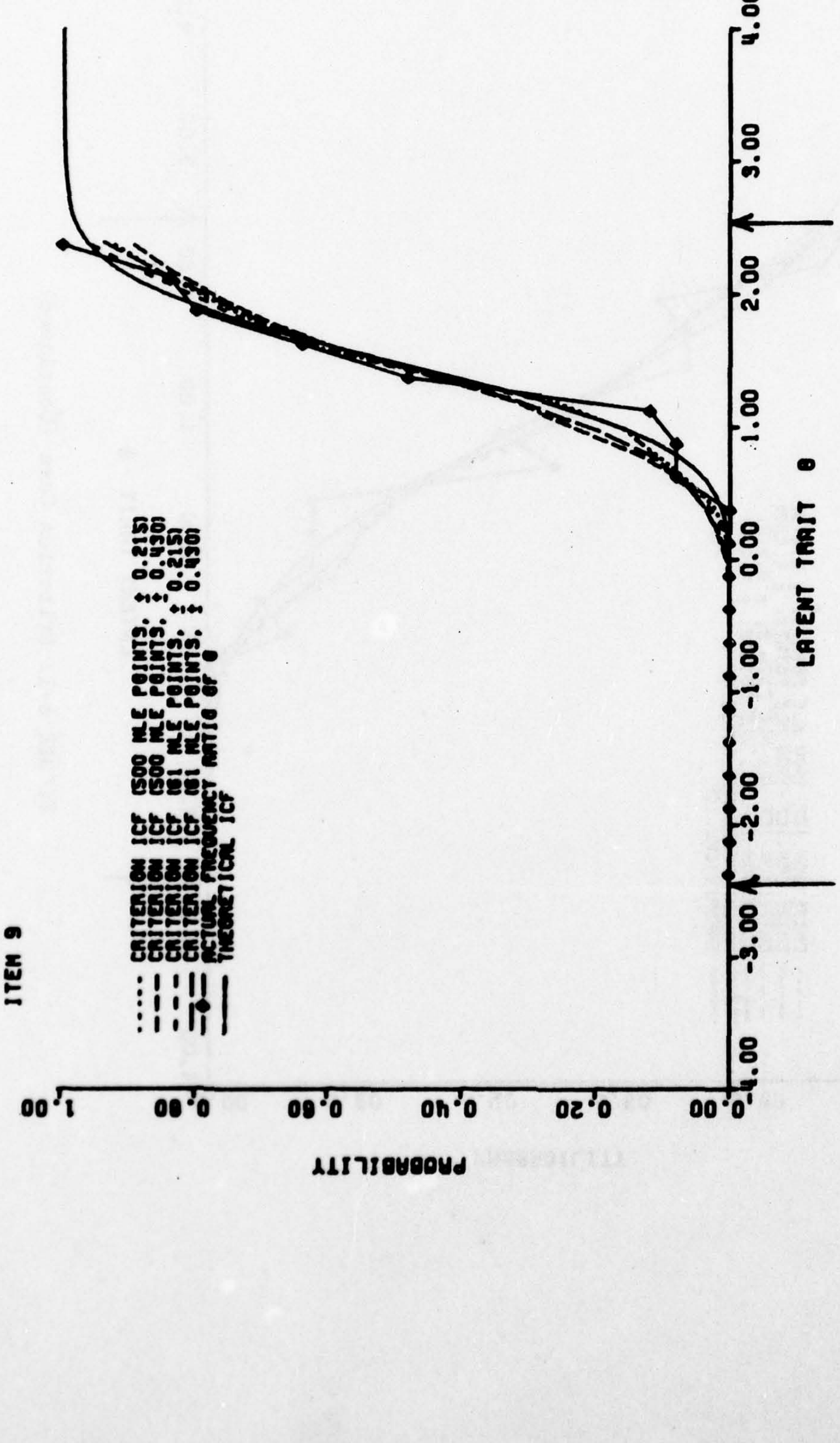

FIGURE 6-1: Criterion Case (Continued)

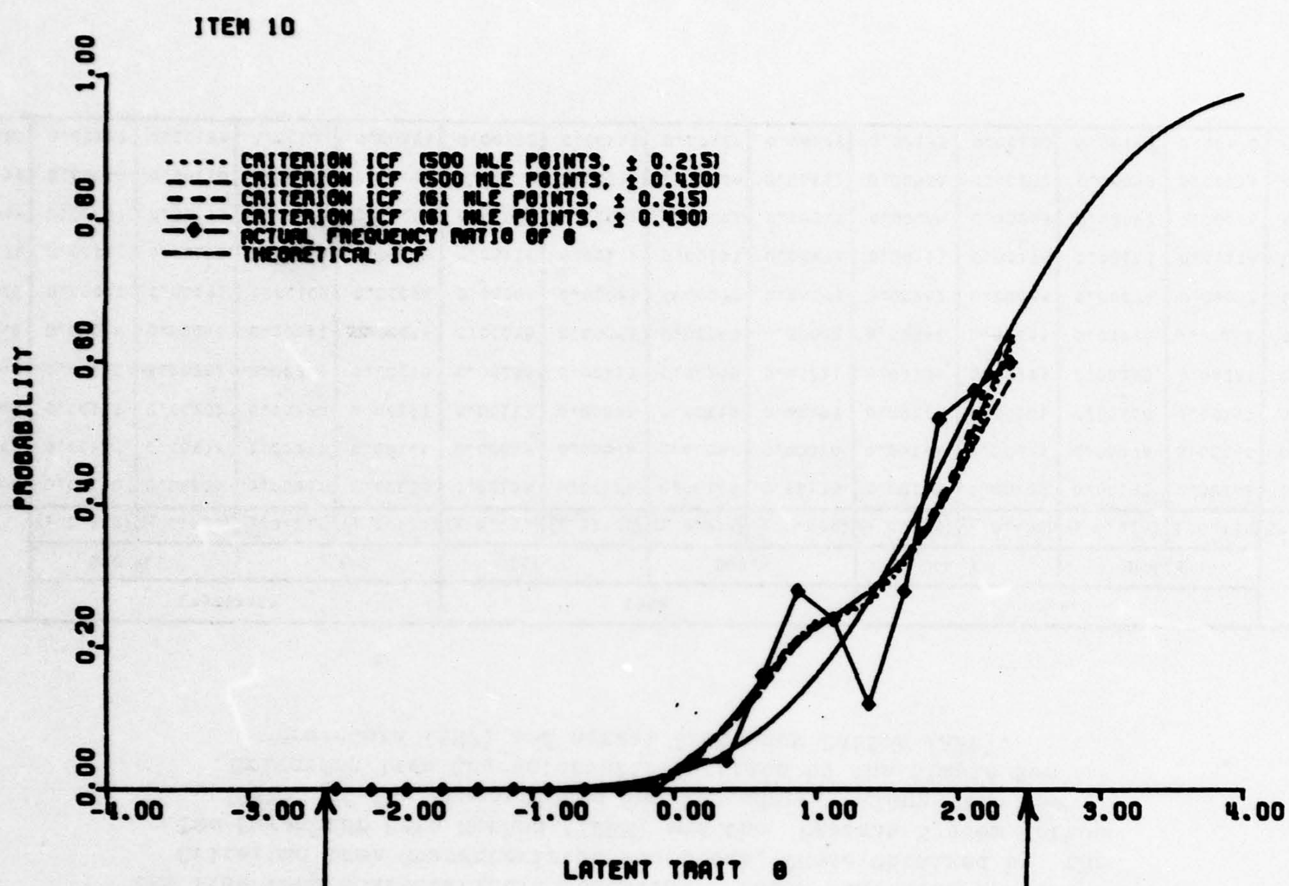

FIGURE 6-1: Criterion Case (Continued)

TABLE 6-1

Mean Square Errors of the Estimated Item Characteristic Functions Against the True Item Characteristic Function. These Functions Are the Four Criterion Item Characteristic Functions, Those Obtained by the Two-Parameter Beta Method (TPBM) and the Pearson System Method (PSM), of the Proportioned Sum Procedure, Along with the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP) and Actual Frequency Ratios (AFR).

		CRITERION				TPBM				PSM				
		500 MLE		61 MLE		DGR.3		DGR.4		DGR.3		DGR.4		SSP
ITEM	AFR	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	CRITERION
1	0.00068	0.00080	0.00109	0.00013	0.00030	0.00126	0.00129	0.00129	0.00135	0.00133	0.00129	0.00195	0.00148	0.00070
2	0.00782	0.00110	0.00076	0.00231	0.00111	0.00095	0.00076	0.00087	0.00070	0.00094	0.00075	0.00076	0.00069	0.00121
3	0.00049	0.00118	0.00202	0.00120	0.00197	0.00117	0.00201	0.00110	0.00195	0.00116	0.00201	0.00108	0.00197	0.00098
4	0.00940	0.00251	0.00232	0.00245	0.00210	0.00233	0.00215	0.00249	0.00231	0.00236	0.00215	0.00252	0.00231	0.00269
5	0.00242	0.00044	0.00060	0.00061	0.00054	0.00038	0.00055	0.00043	0.00062	0.00041	0.00055	0.00045	0.00061	0.00059
6	0.00601	0.00072	0.00053	0.00103	0.00056	0.00065	0.00085	0.00072	0.00093	0.00067	0.00085	0.00074	0.00052	0.00089
7	0.00179	0.00117	0.00179	0.00164	0.00183	0.00111	0.00171	0.00115	0.00178	0.00111	0.00170	0.00115	0.00178	0.00120
8	0.00368	0.00087	0.00079	0.00106	0.00069	0.00083	0.00082	0.00081	0.00077	0.00086	0.00083	0.00087	0.00086	0.00109
9	0.00135	0.00040	0.00116	0.00028	0.00060	0.00047	0.00134	0.00039	0.00121	0.00051	0.00132	0.00058	0.00134	0.00051
10	0.00325	0.00127	0.00135	0.00103	0.00105	0.00135	0.00149	0.00127	0.00138	0.00135	0.00150	0.00142	0.00170	0.00124

TABLE 6-2

Square Roots of the Mean Square Errors of the Estimated Item Characteristic Functions Against the True Item Characteristic Function. These Functions Are Criterion Item Characteristic Functions, Those Obtained by the Two-Parameter Beta Method (TPBM) and the Pearson System Method (PSM), of the Proportioned Sum Procedure, Along with the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP) and Actual Frequency Ratios (AFR).

		CRITERION				TPBM				PSM				
		500 MLE		61 MLE		DGR.3		DGR.4		DGR.3		DGR.4		SSP
ITEM	AFR	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	(± 0.215)	(± 0.430)	CRITERION
1	0.02610	0.02830	0.03301	0.01149	0.01728	0.03546	0.03595	0.03597	0.03681	0.03646	0.03585	0.04421	0.03842	0.02650
2	0.08843	0.03313	0.02762	0.04801	0.03325	0.03085	0.02750	0.02955	0.02648	0.03062	0.02746	0.02756	0.02621	0.03473
3	0.02223	0.03429	0.04496	0.03465	0.04436	0.03414	0.04480	0.03310	0.04413	0.03404	0.04479	0.03293	0.04442	0.03138
4	0.05656	0.05013	0.04820	0.04947	0.04587	0.04830	0.04635	0.04995	0.04802	0.04854	0.04640	0.05020	0.04805	0.05187
5	0.04923	0.02054	0.02452	0.02476	0.02331	0.01943	0.02349	0.02062	0.02481	0.02020	0.02352	0.02132	0.02479	0.02421
6	0.07754	0.02680	0.03051	0.03212	0.03057	0.02554	0.02918	0.02685	0.03050	0.02586	0.02916	0.02714	0.03037	0.02982
7	0.04236	0.03420	0.04229	0.04053	0.04282	0.03327	0.04130	0.03397	0.04224	0.03332	0.04127	0.03398	0.04219	0.03458
8	0.06070	0.02952	0.02807	0.03252	0.02630	0.02884	0.02871	0.02840	0.02783	0.02927	0.02874	0.02942	0.02538	0.03306
9	0.03671	0.02011	0.03405	0.01662	0.02453	0.02172	0.03658	0.01984	0.03480	0.02265	0.03635	0.02417	0.03666	0.02260
10	0.05701	0.03558	0.03673	0.03205	0.03236	0.03676	0.03865	0.03561	0.03717	0.03674	0.03877	0.03767	0.04118	0.03526

TABLE 6-3

Estimated Item Discrimination Parameters, $\hat{a}_g$, Obtained from the Four Criterion Item Characteristic Functions of the Proportioned Sum Procedure of the Conditional P.D.F. Approach, with the One from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95.

		CRITERION				
		500 MLE		61 MLE		SSP
ITEM	TRUE	± 0.215	± 0.430	± 0.215	± 0.430	CRITERION
1	1.5	1.330_5	1.064_5	1.683_5	1.327_5	1.400_5
2	1.0	1.023	0.970	0.945	0.939	1.024
3	2.5	1.740	1.642	1.755	1.660	1.788
4	1.0	0.873	0.821	0.854	0.836	0.868
5	1.5	1.377	1.286	1.372	1.303	1.368
6	1.0	0.894	0.882	0.872	0.872	0.895
7	2.0	1.475	1.415	1.420	1.405	1.473
8	1.0	0.910	0.873	0.885	0.916	0.886
9	2.0	1.708	1.522	1.741	1.653	1.716
10	1.0	0.757	0.740	0.797	0.769	0.725

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

TABLE 6-4

Estimated Item Difficulty Parameters, $\hat{b}_g$, Obtained from the Four Criterion Item Characteristic Functions of the Proportioned Sum Procedure of the Conditional P.D.F. Approach, with the One from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based On the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95.

ITEM	TRUE	CRITERION				
		500 MLE		61 MLE		SSP
		± 0.215	± 0.430	± 0.215	± 0.430	CRITERION
1	-2.5	-2.692_5	-2.857_5	-2.467_5	-2.638_5	-2.651_5
2	-2.0	-2.001	-2.031	-2.075	-2.036	-2.002
3	-1.5	-1.512	-1.526	-1.509	-1.513	-1.507
4	-1.0	-0.997	-1.030	-1.016	-1.022	-1.005
5	-0.5	-0.471	-0.468	-0.454	-0.467	-0.472
6	0.0	-0.063	-0.061	-0.083	-0.070	-0.075
7	0.5	0.522	0.522	0.533	0.527	0.527
8	1.0	0.976	0.977	0.951	0.940	0.981
9	1.5	1.518	1.511	1.490	1.496	1.502
10	2.0	2.116	2.157	2.048	2.080	2.118

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

As was the case of the Simple Sum and Weighted Sum Procedures, we notice that the estimates of the discrimination parameters a_g are further from the true values when a_g's are large, like for items 3, 7 and 9. It is interesting to note that, although the parameter estimation tends to be more inaccurate for very difficult and very easy items in most cases, this tendency is ameliorated when the 61 point case is used, in both the discrimination and difficulty parameter estimations.

Figure 6-2 presents the resulting estimated item characteristic functions obtained by Degree 3 and 4 Cases of the Pearson System Method, each of which has two results using the intervals, $\hat{\theta}_s \pm 0.215$ and $\hat{\theta}_s \pm 0.430$, in calculating the proportioned weights, respectively. In the same figure, for each item, also presented are the criterion item characteristic function obtained by the Simple Sum Procedure, the actual frequency ratios of θ and the theoretical item characteristic function. We notice that these four curves for each item are fairly close to one another, and also to the criterion item characteristic function obtained by the Simple Sum Procedure, for the interval of θ , (-2.5, 2.5). It is also noted that these estimated item characteristic functions are similar to the corresponding criterion item characteristic functions, if we compare Figure 6-2 with Figure 6-1.

The mean square errors and their square roots, which were calculated through (6.1), are shown in Tables 6-1 and 6-2, respectively. We can see that these values are very close to those for the criterion item characteristic functions, which are presented in the same tables. There are no tendencies that those obtained by Degree 4 Case are smaller than those obtained by Degree 3 Case of the Pearson System Method, or vice versa.

Tables 6-5 and 6-6 present the estimated discrimination parameter, $\hat{a}_g$, and difficulty parameter, $\hat{b}_g$, by the least square method, for each of the ten binary items, in comparison with those obtained from the criterion item characteristic function by the Simple Sum Procedure. These results are very similar to those obtained from the criterion item characteristic functions, and the same tendencies as were described earlier for the estimates from the criterion item characteristic functions are also observed among these estimates, although the latter tend to be slightly more inaccurate than the former. It is interesting to note that the estimates of the discrimination parameters in Degree 3 Case with the interval $\hat{\theta}_s \pm 0.215$ tend to be better than the others, although the differences are small.

Figure 6-3 presents the estimated item characteristic functions of each of the ten binary items obtained by Degree 3 and 4 Cases of the Two-Parameter Beta Method, each having two functions by using the two intervals, $\hat{\theta}_s \pm 0.215$ and $\hat{\theta}_s \pm 0.430$ for calculating the proportioned weights, along with the criterion item characteristic function by the Simple Sum Procedure of the Conditional P.D.F. Approach, the actual frequency ratios of θ and the theoretical item characteristic function. Again, these resultant functions are close to one another, to the criterion item characteristic function by the Simple Sum Procedure, and to the criterion item characteristic functions by the present procedure, which are presented in Figure 6-1, for the interval of θ , (-2.5, 2.5).

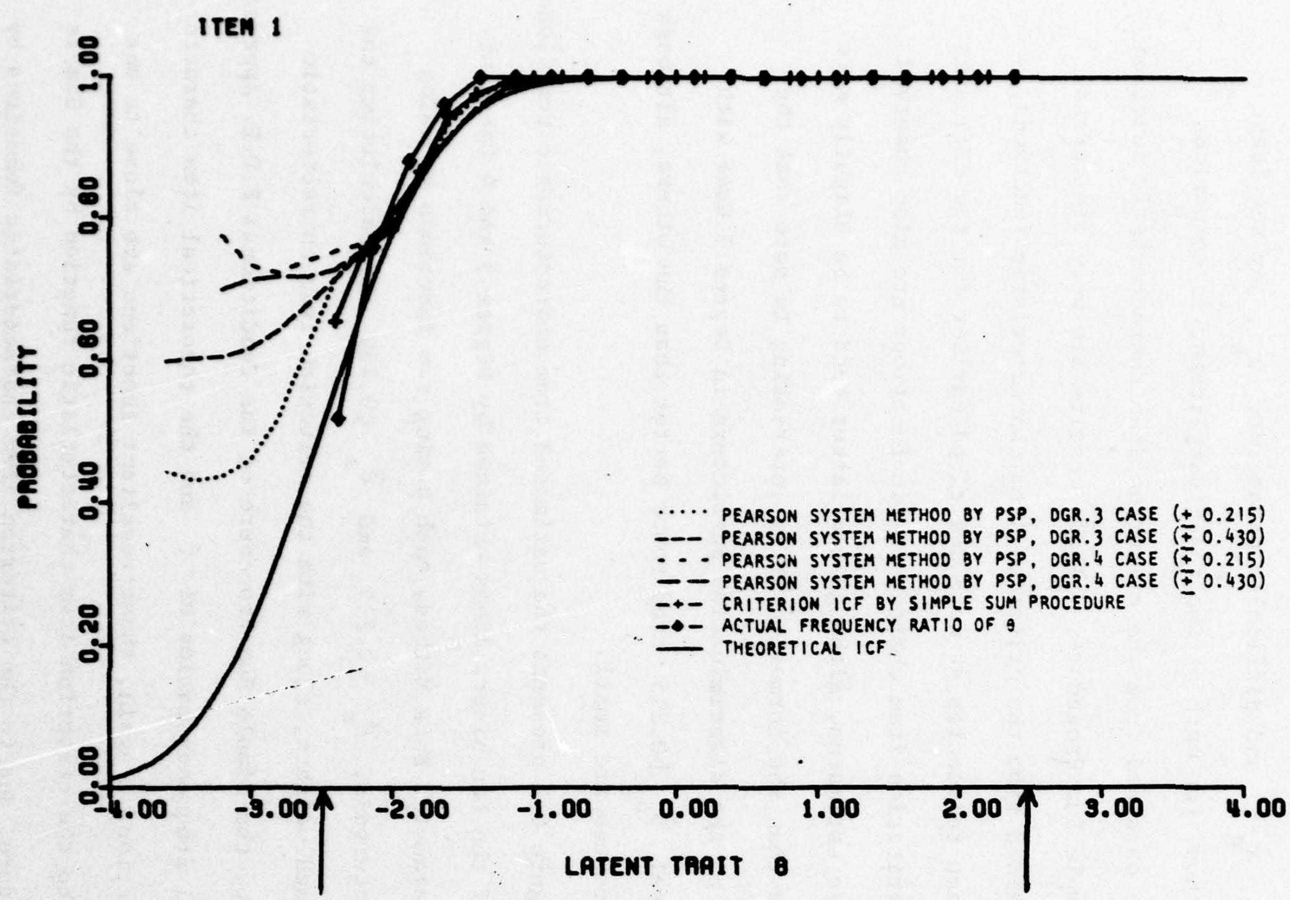

FIGURE 6-2

Comparison of the Four Estimated Item Characteristic Functions of Degree 3 and 4 Cases by the Pearson System Method of the Proportioned Sum Procedure with the Criterion Item Characteristic Function by the Simple Sum Procedure. Actual Frequency Ratios and Theoretical Item Characteristic Function Are Also Presented for Comparison.

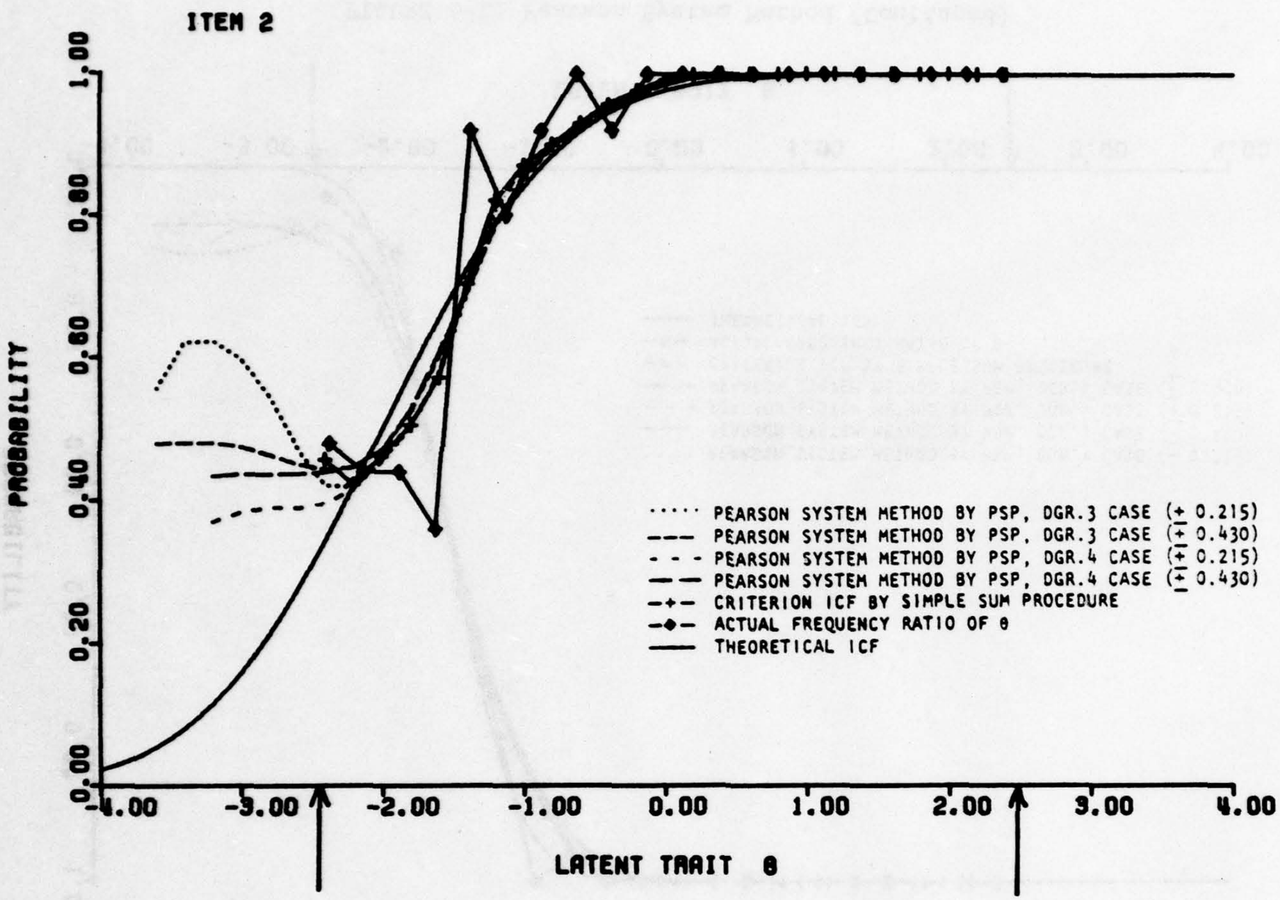

FIGURE 6-2: Pearson System Method (Continued)

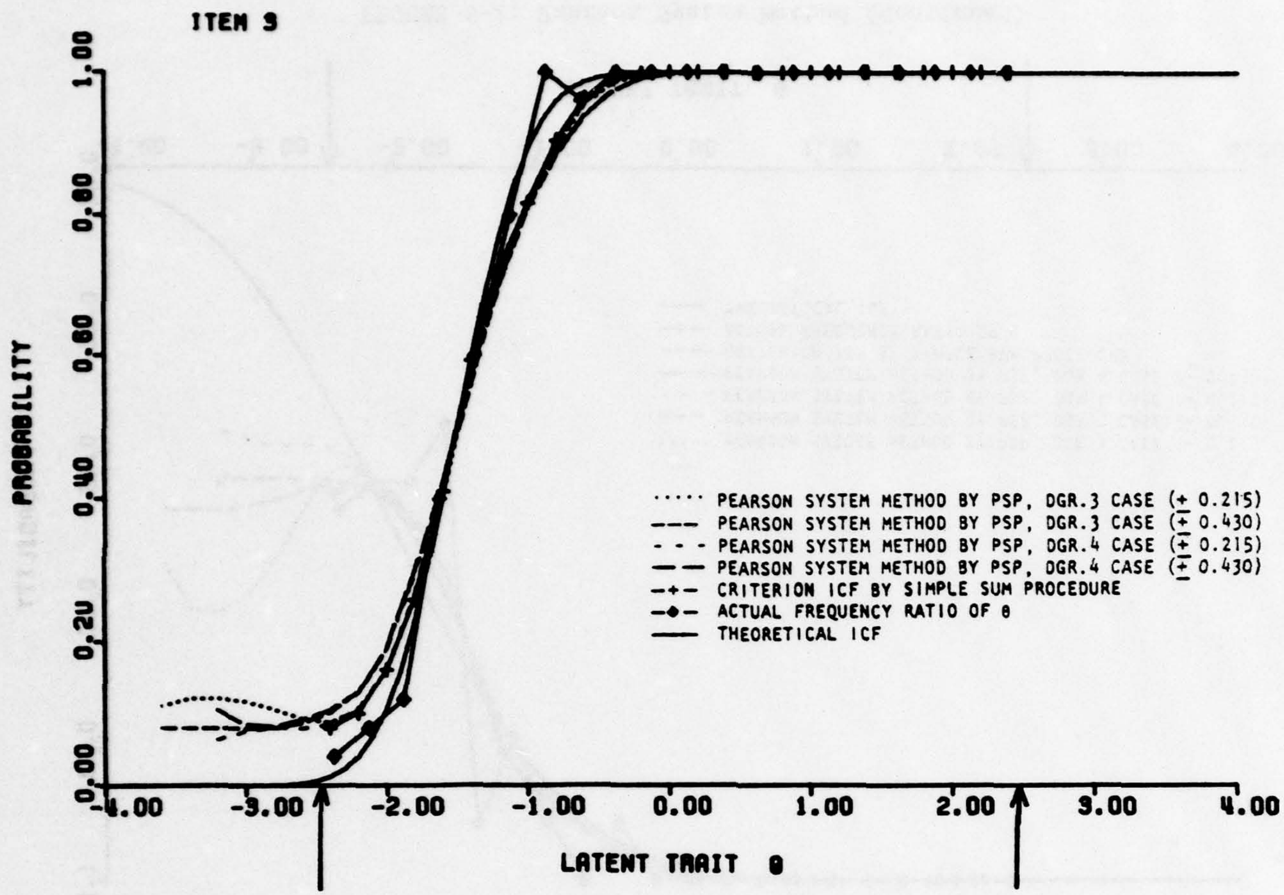

FIGURE 6-2: Pearson System Method (Continued)

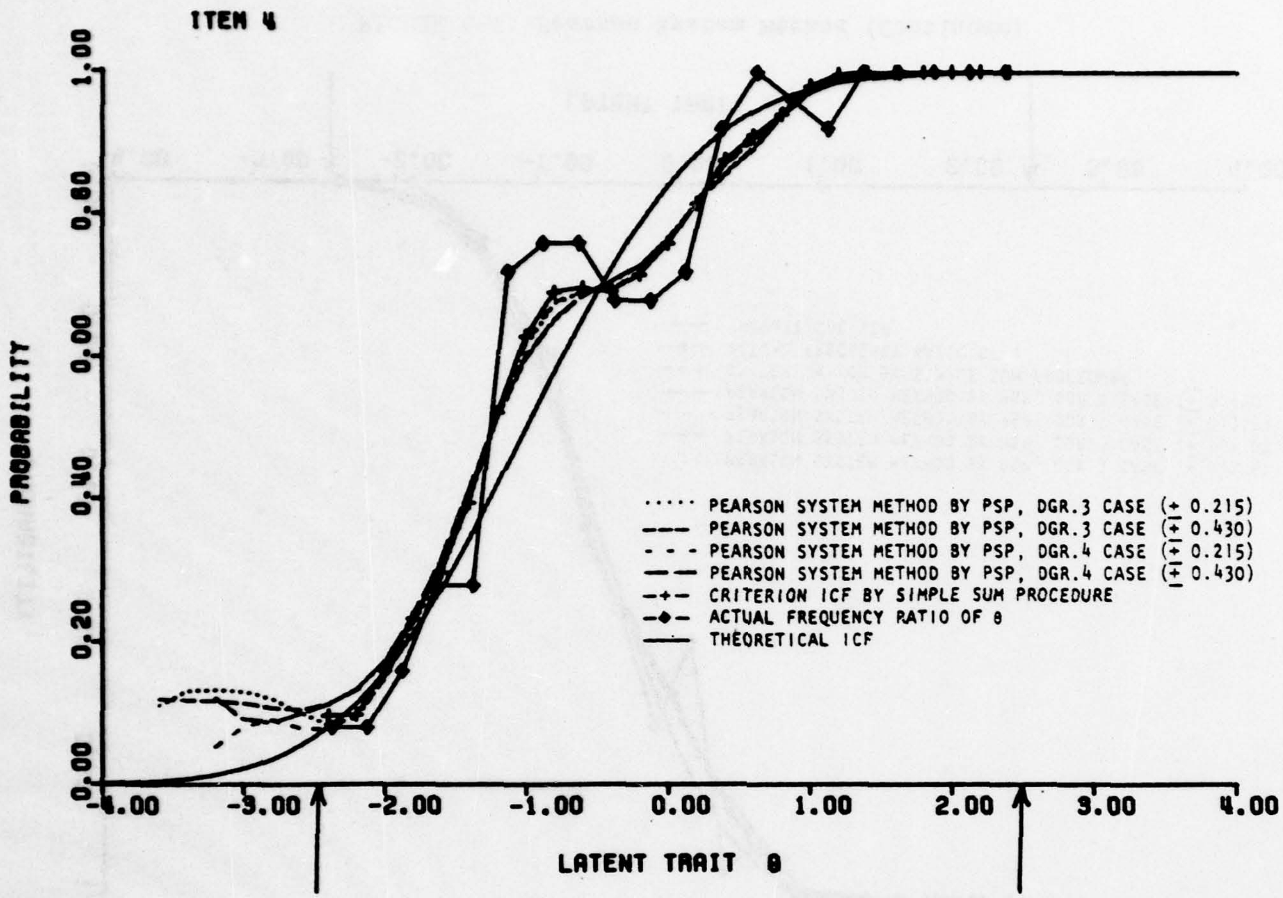

FIGURE 6-2: Pearson System Method (Continued)

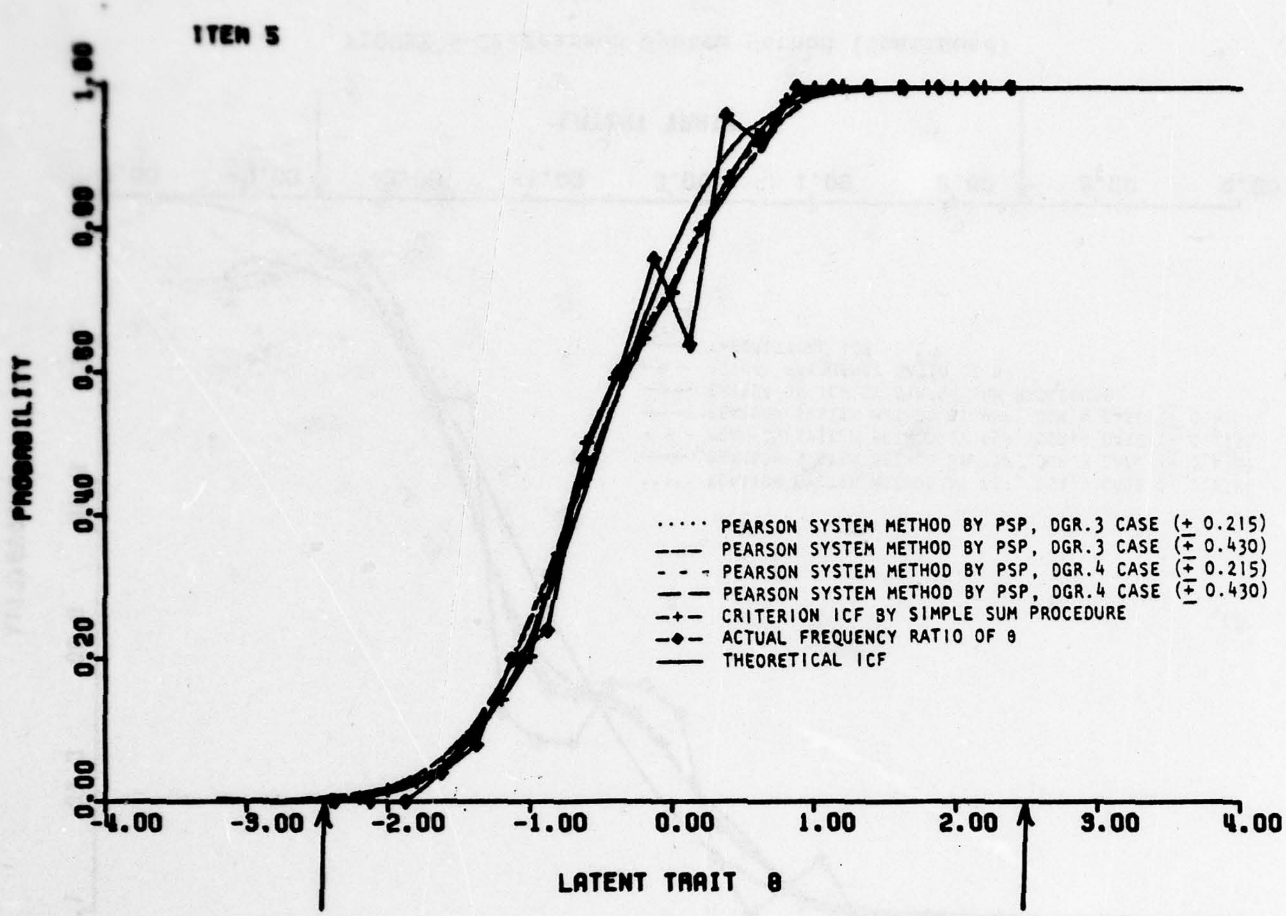

FIGURE 6-2: Pearson System Method (Continued)

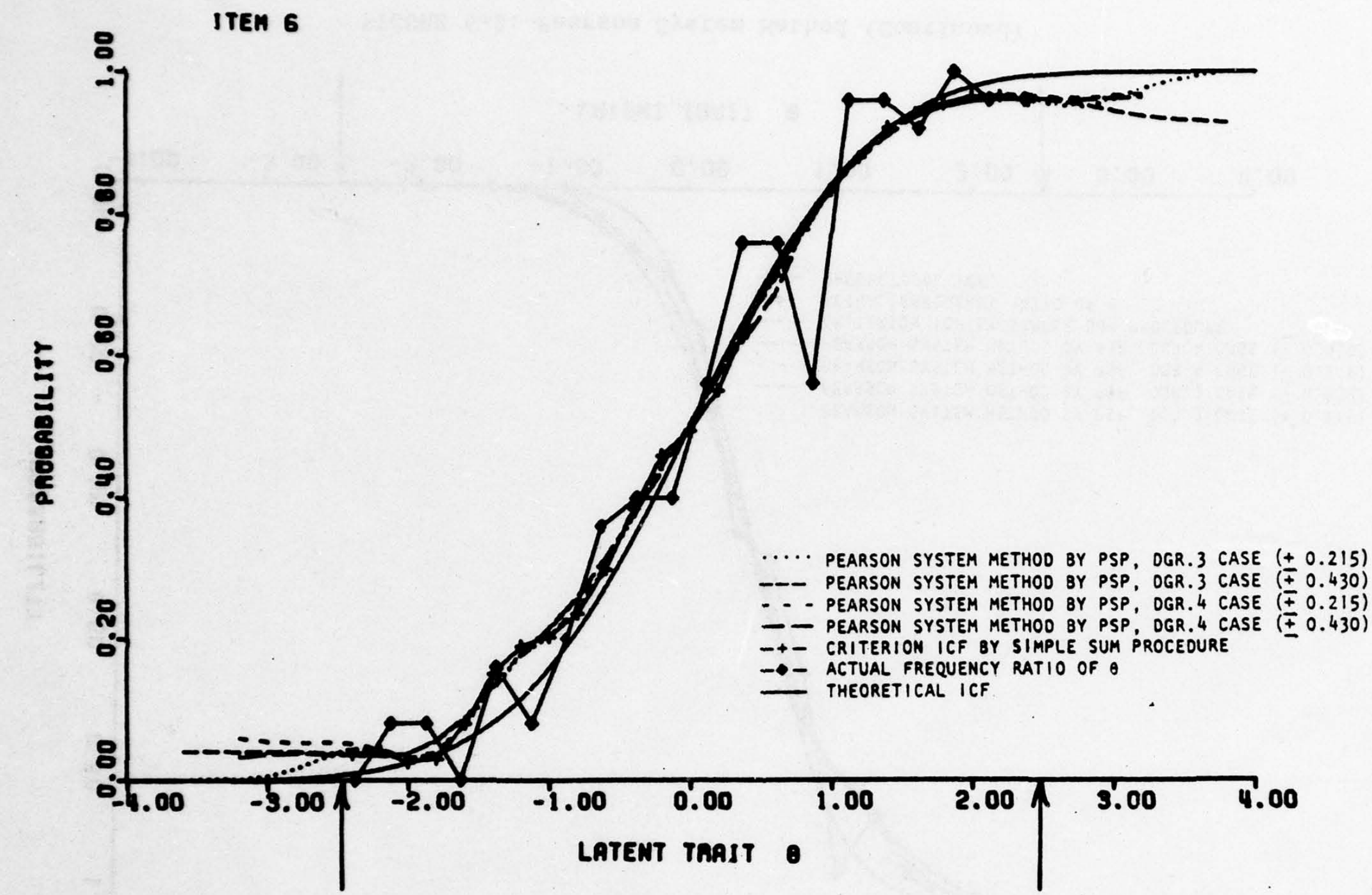

FIGURE 6-2: Pearson System Method (Continued)

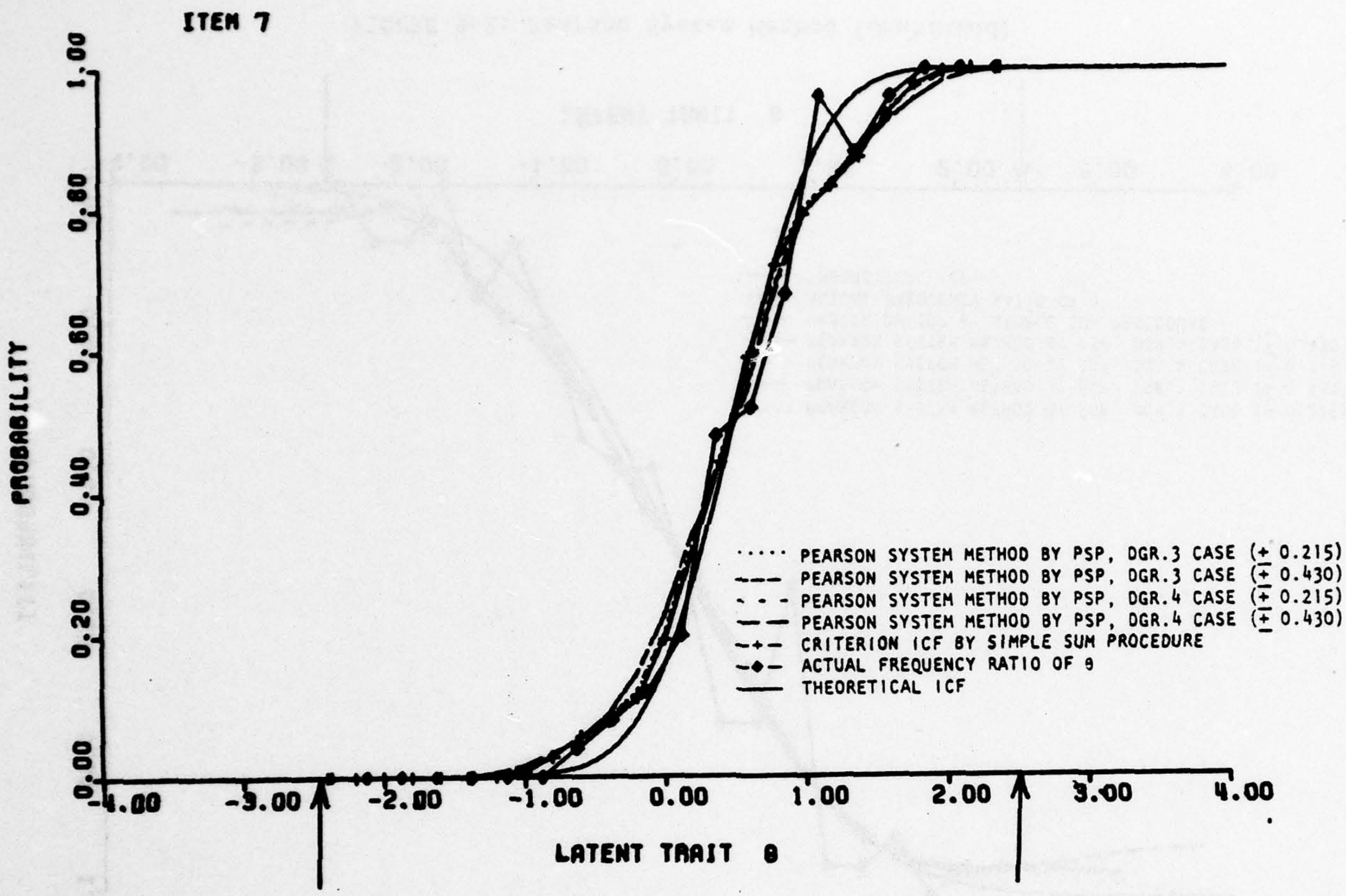

FIGURE 6-2: Pearson System Method (Continued)

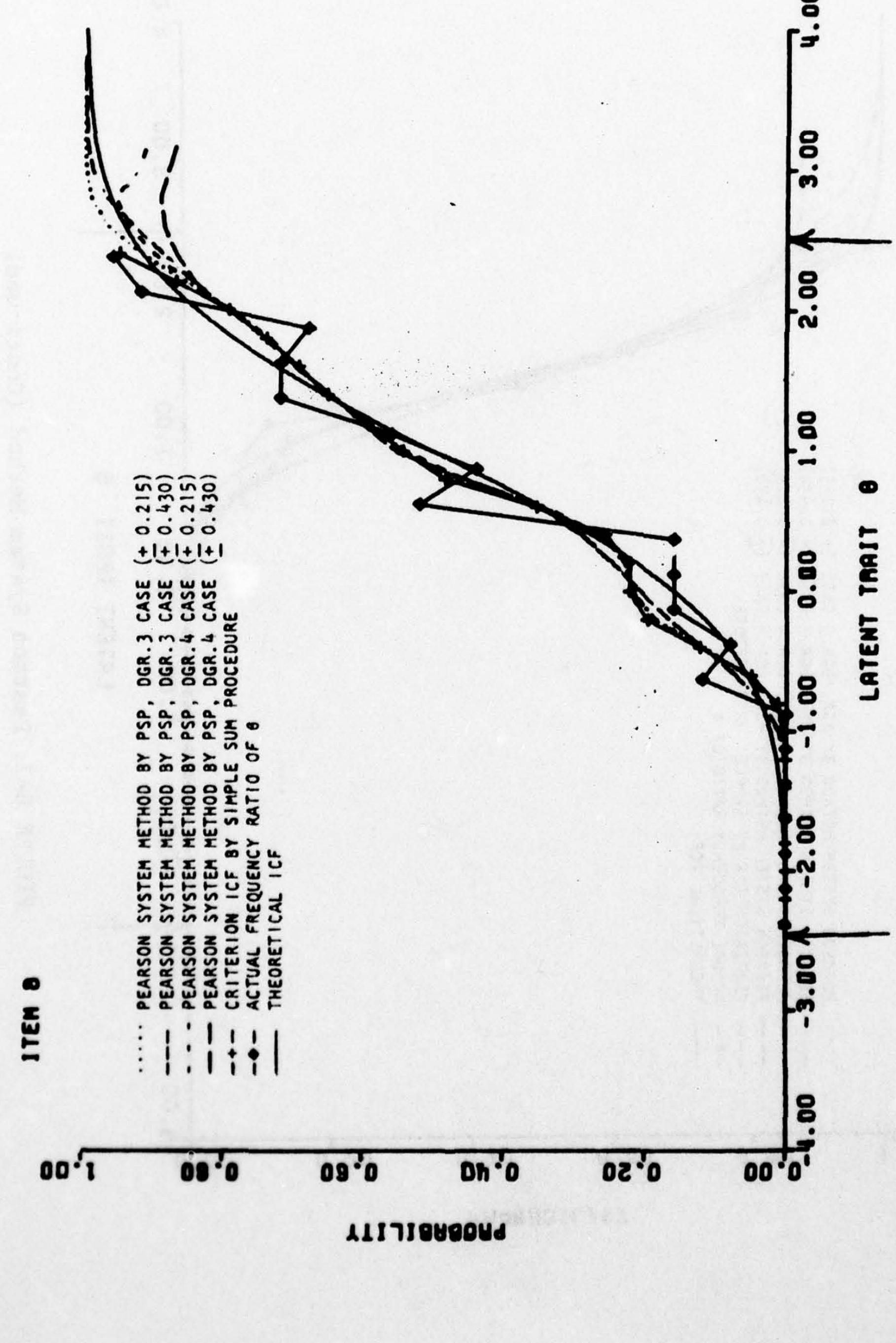

FIGURE 6-2: Pearson System Method (Continued)

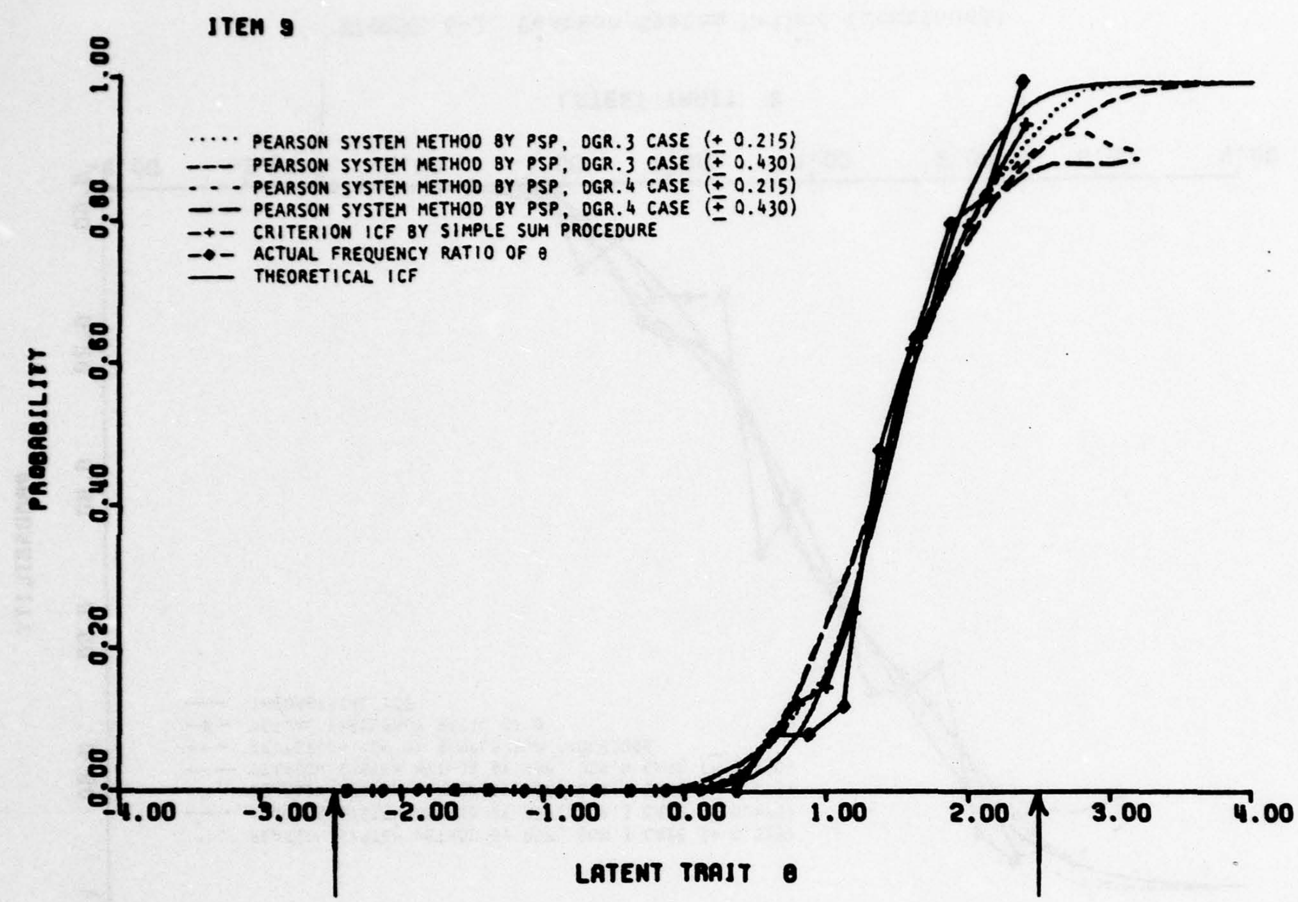

FIGURE 6-2: Pearson System Method (Continued)

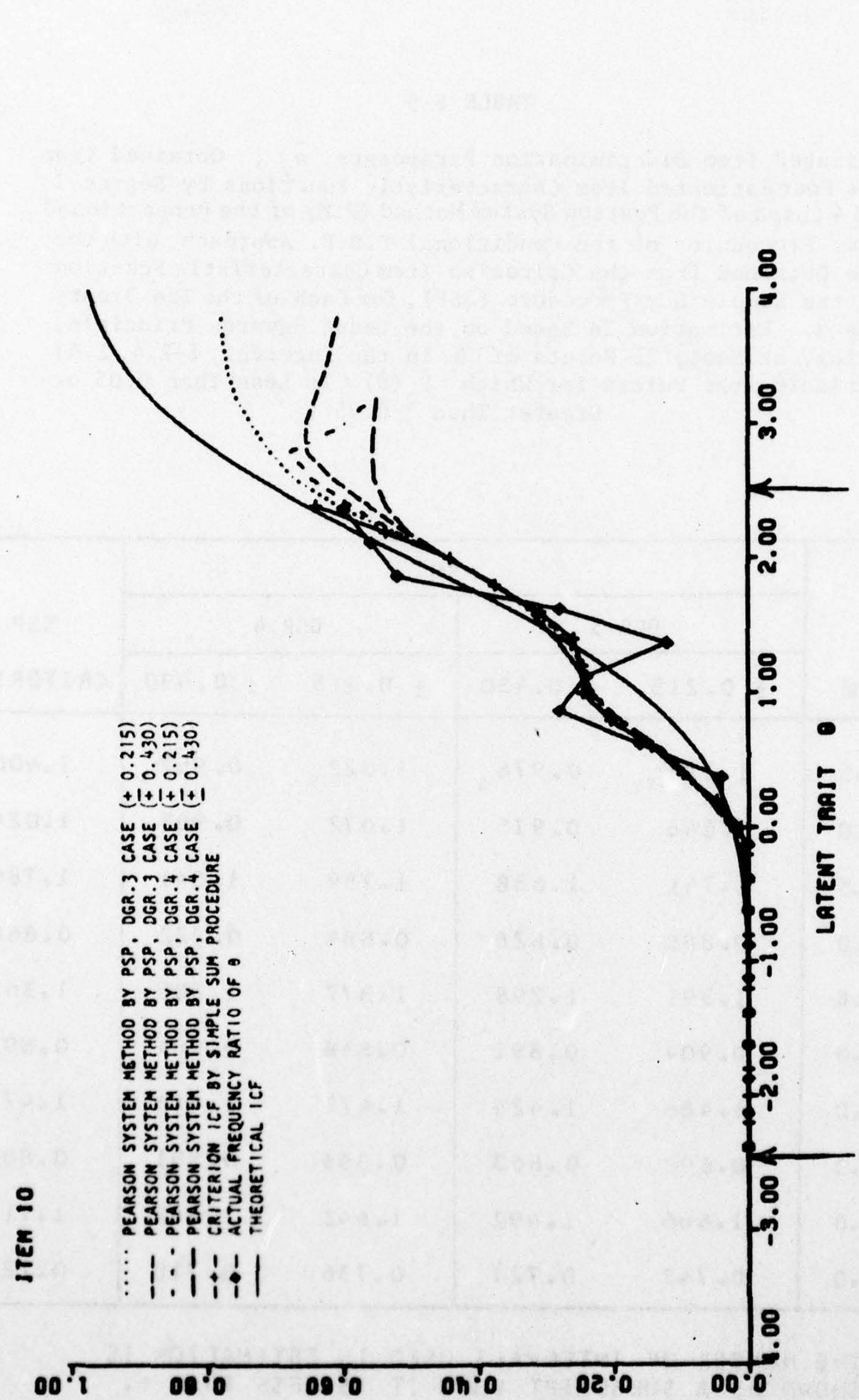

FIGURE 6-2: Pearson System Method (Continued)

TABLE 6-5

Estimated Item Discrimination Parameters, $\hat{a}_g$, Obtained from the Four Estimated Item Characteristic Functions by Degree 3 and 4 Cases of the Pearson System Method (PSM) of the Proportioned Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4] Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95 .

		PSM				SSP
		DGR.3		DGR.4		
ITEM	TRUE	± 0.215	± 0.430	± 0.215	± 0.430	CRITERION
1	1.5	1.123_5	0.976_5	1.022_5	0.957_5	1.400_5
2	1.0	1.048	0.975	1.072	0.983	1.024
3	2.5	1.741	1.638	1.759	1.641	1.788
4	1.0	0.886	0.828	0.884	0.822	0.868
5	1.5	1.391	1.298	1.377	1.285	1.368
6	1.0	0.904	0.891	0.848	0.884	0.895
7	2.0	1.486	1.424	1.477	1.416	1.473
8	1.0	0.898	0.863	0.883	0.851	0.886
9	2.0	1.666	1.492	1.642	1.482	1.716
10	1.0	0.742	0.727	0.736	0.718	0.725

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

TABLE 6-6

Estimated Item Difficulty Parameters, $\hat{b}_g$, Obtained from the Four Estimated Item Characteristic Functions by Degree 3 and 4 Cases of the Pearson System Method (PSM) of the Proportioned Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less than 0.05 or Greater Than 0.95 .

ITEM	TRUE	PSM DGR.3 ± 0.215	PSM DGR.3 ± 0.430	PSM DGR.4 ± 0.215	PSM DGR.4 ± 0.430	SSP CRITERION
1	-2.5	-2.833_5	-2.933_5	-2.952_5	-2.968_5	-2.651_5
2	-2.0	-1.970	-2.015	-1.960	-2.019	-2.002
3	-1.5	-1.500	-1.517	-1.507	-1.525	-1.507
4	-1.0	-0.984	-1.023	-0.984	-1.027	-1.005
5	-0.5	-0.467	-0.464	-0.471	-0.468	-0.472
6	0.0	-0.062	-0.061	-0.088	-0.060	-0.075
7	0.5	0.517	0.517	0.522	0.522	0.527
8	1.0	0.983	0.982	0.996	0.994	0.981
9	1.5	1.519	1.509	1.533	1.521	1.502
10	2.0	2.134	2.172	2.154	2.198	2.118

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

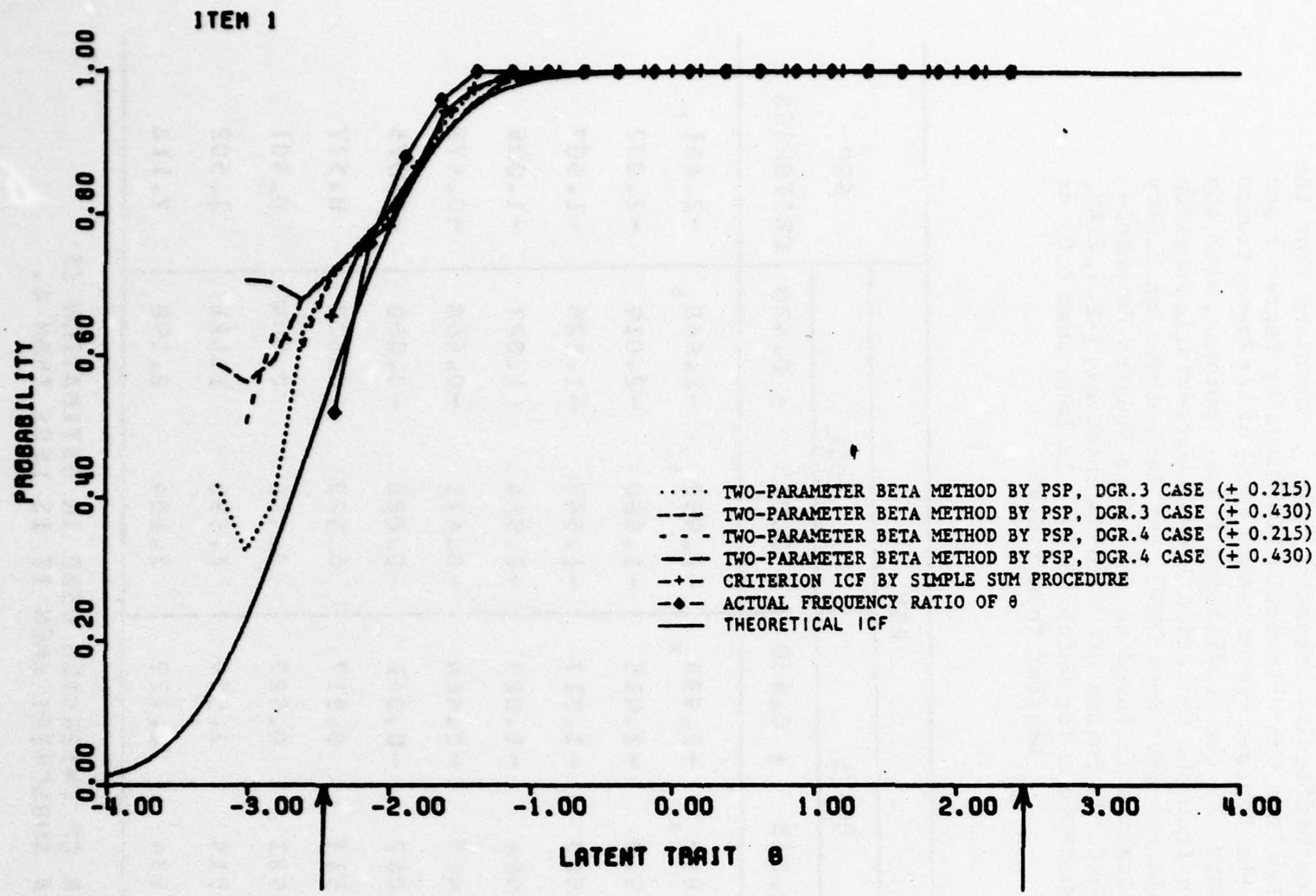

FIGURE 6-3

Comparison of the Four Estimated Item Characteristic Functions of Degree 3 and 4 Cases by the Two-Parameter Beta Method of the Proportioned Sum Procedure with the Criterion Item Characteristic Function by the Simple Sum Procedure. Actual Frequency Ratios and Theoretical Item Characteristic Function Are Also Presented for Comparison.

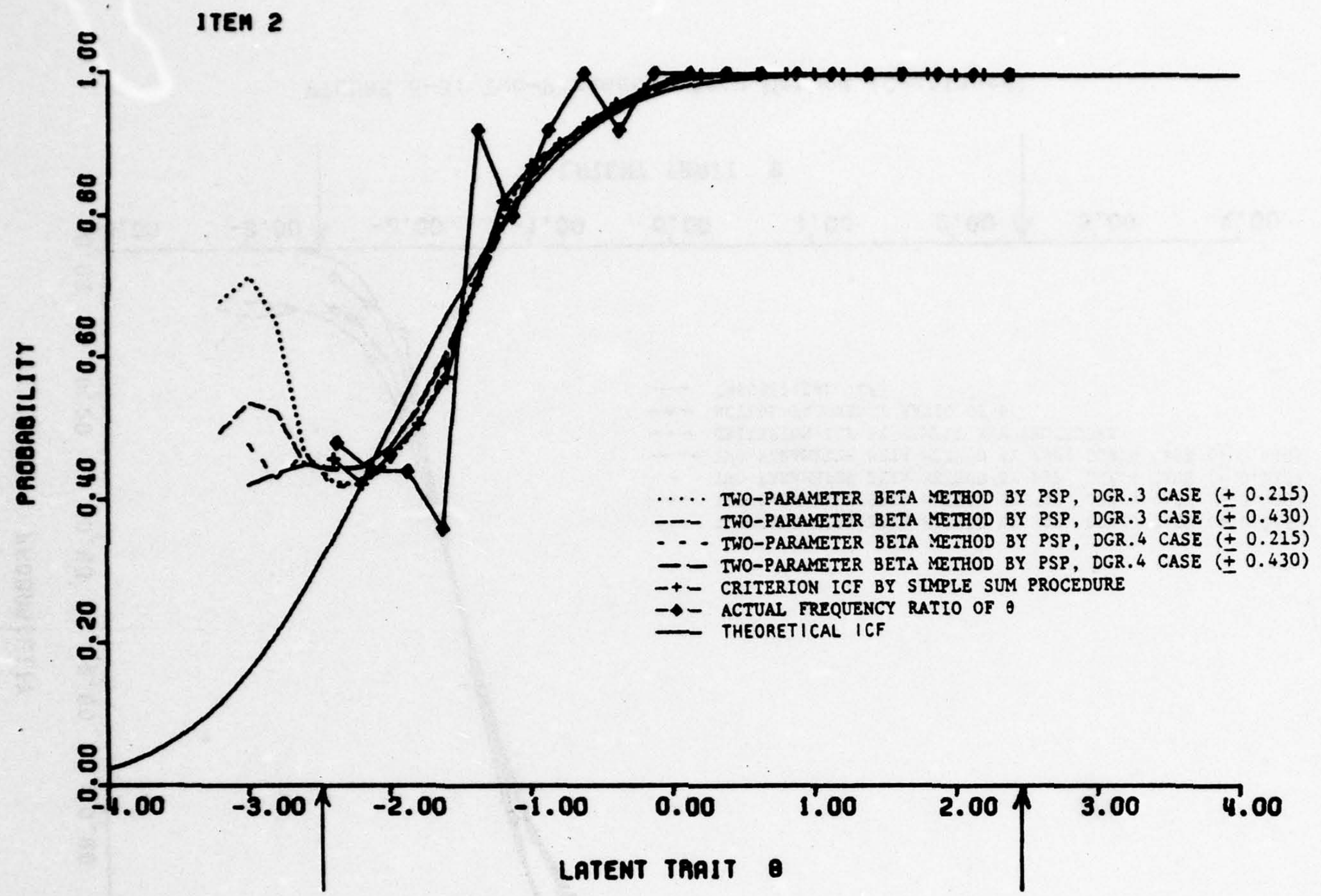

FIGURE 6-3: Two-Parameter Beta Method (Continued)

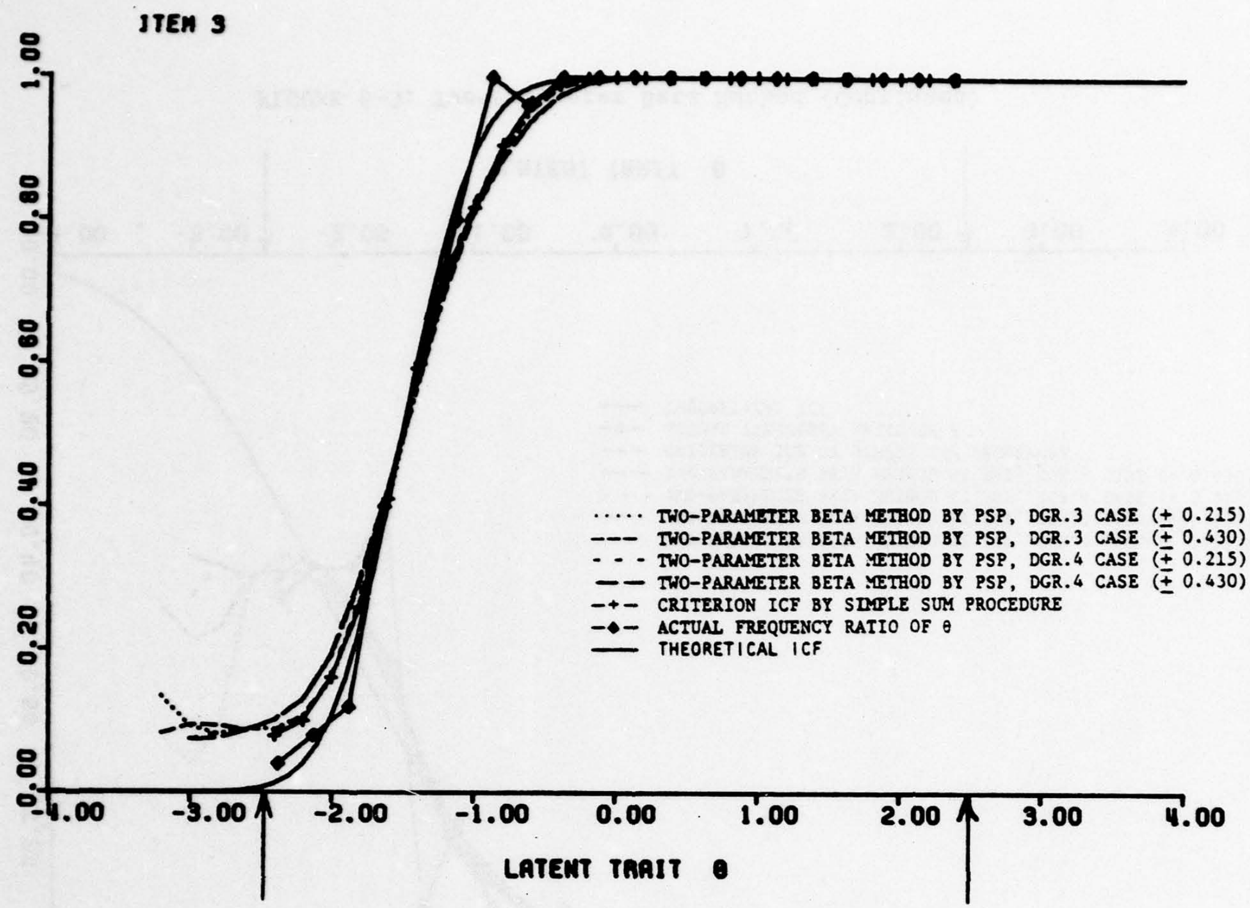

FIGURE 6-3: Two-Parameter Beta Method (Continued)

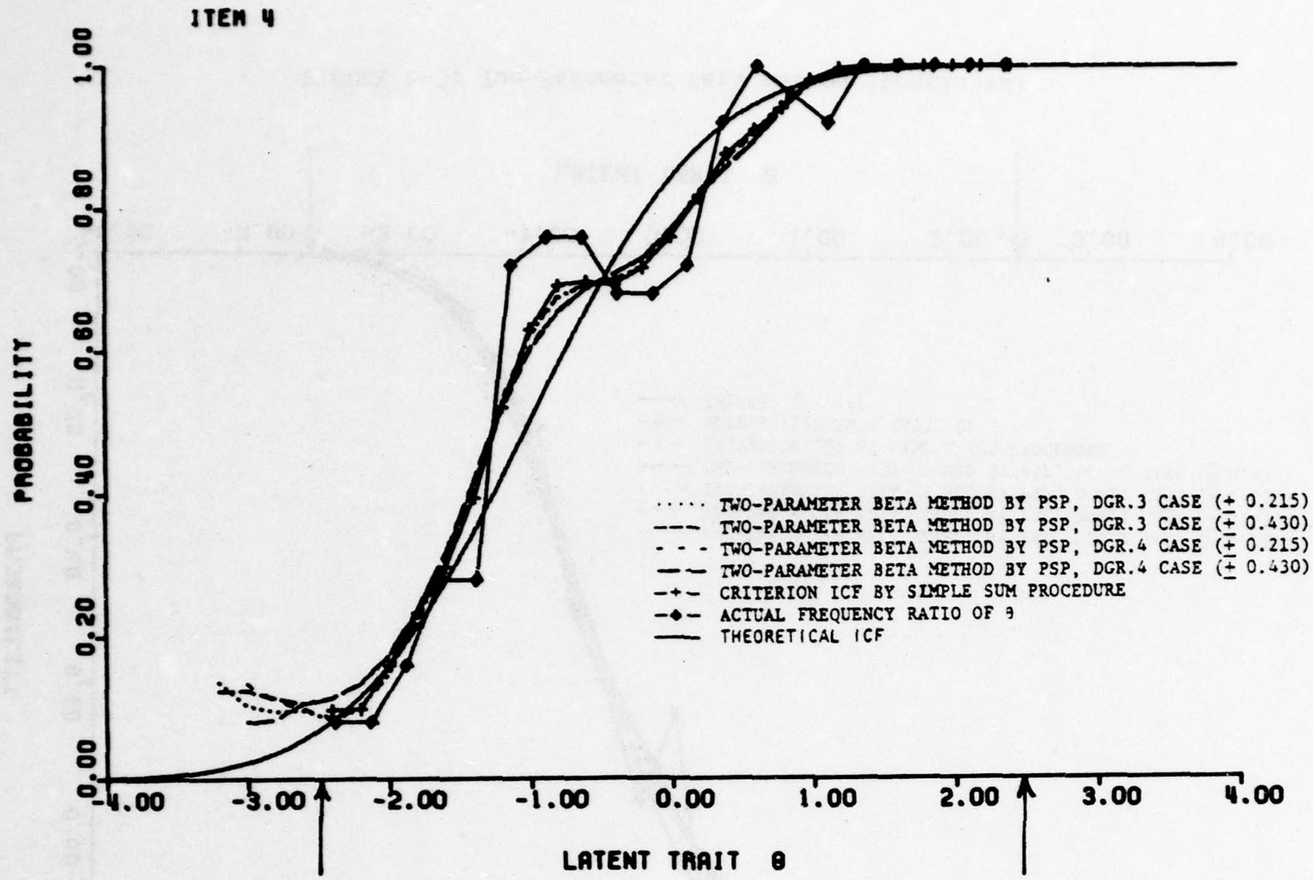

FIGURE 6-3: Two-Parameter Beta Method (Continued)

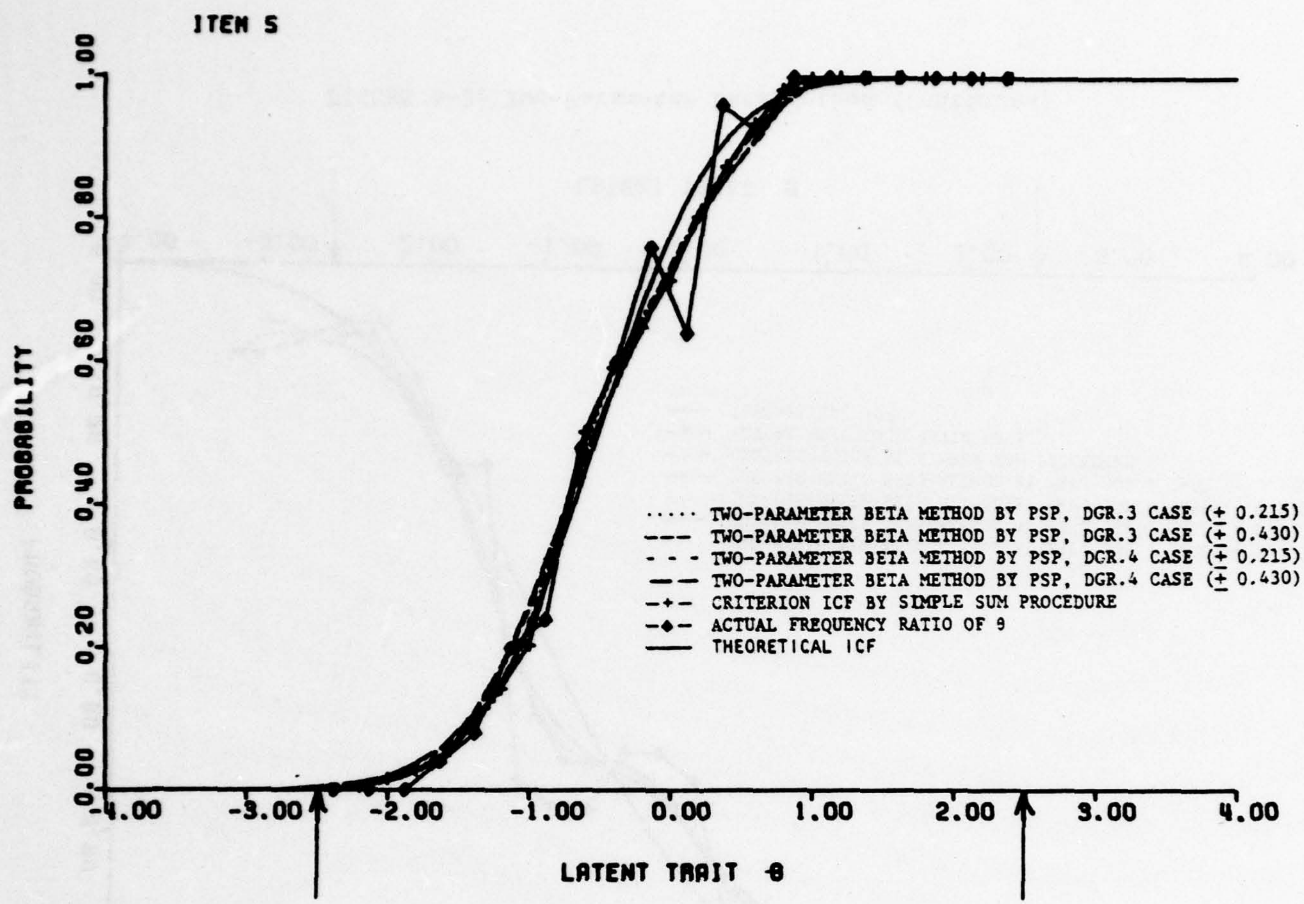

FIGURE 6-3: Two-Parameter Beta Method (Continued)

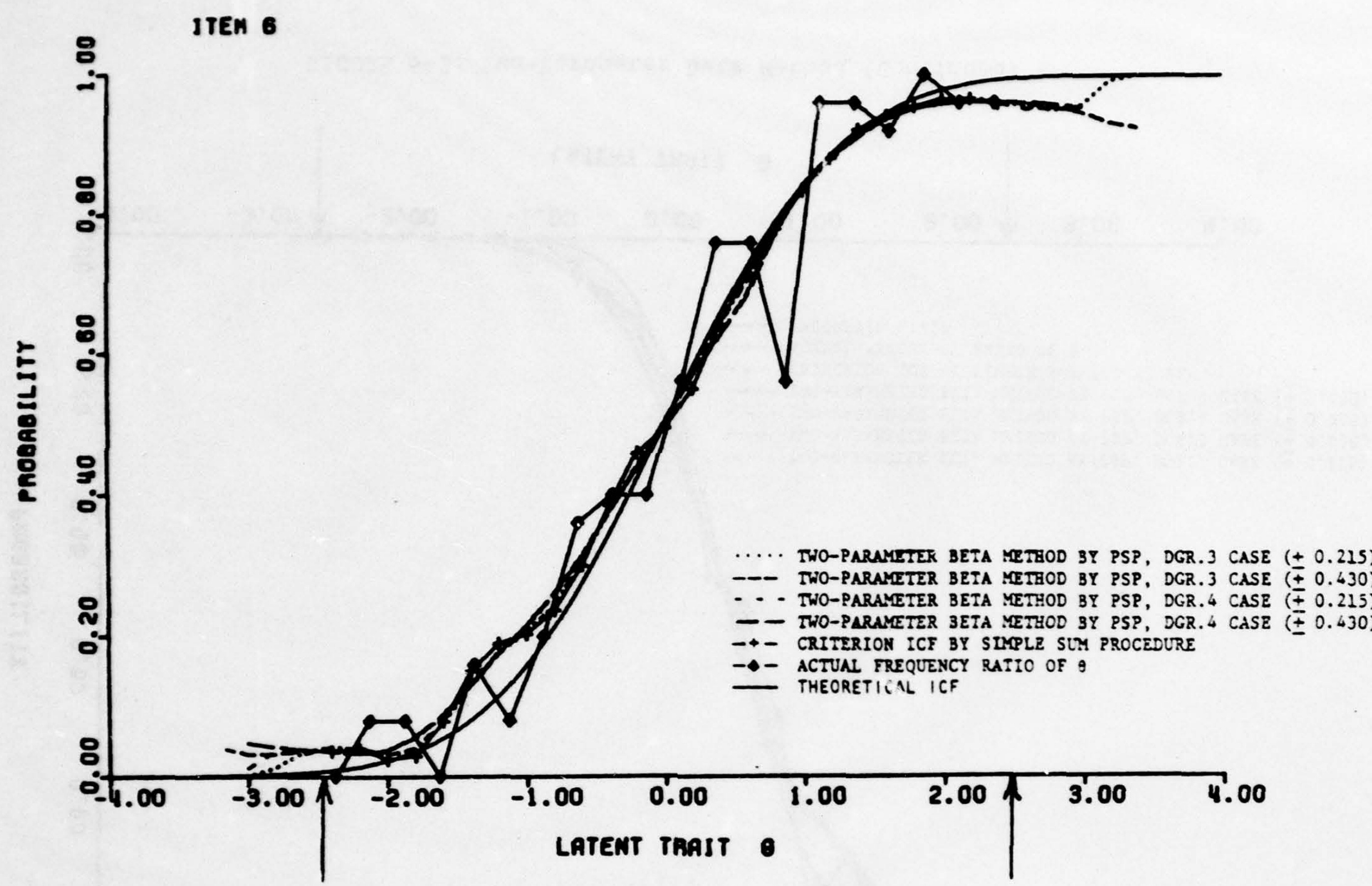

FIGURE 6-3: Two-Parameter Beta Method (Continued)

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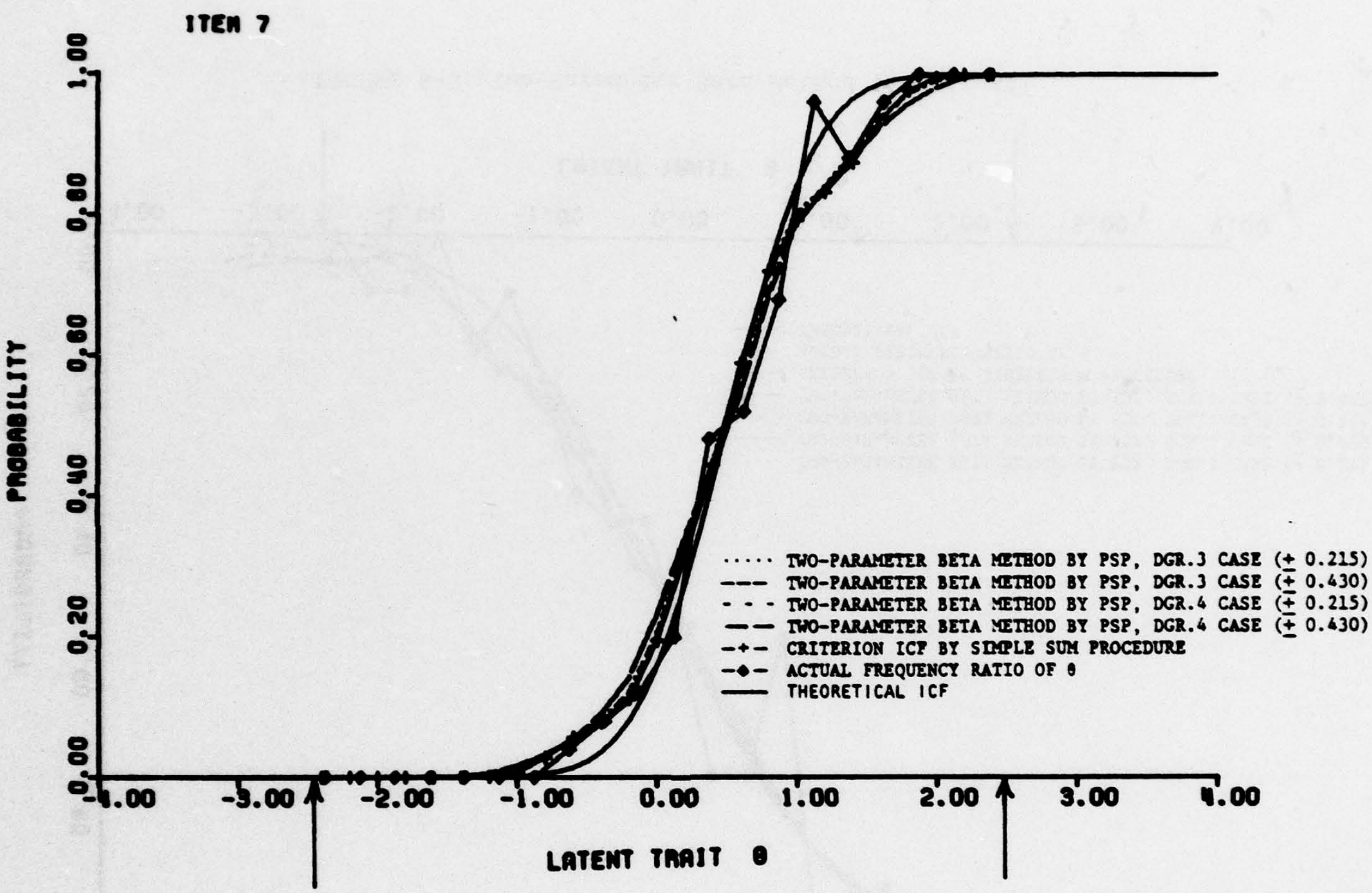

FIGURE 6-3: Two-Parameter Beta Method (Continued)

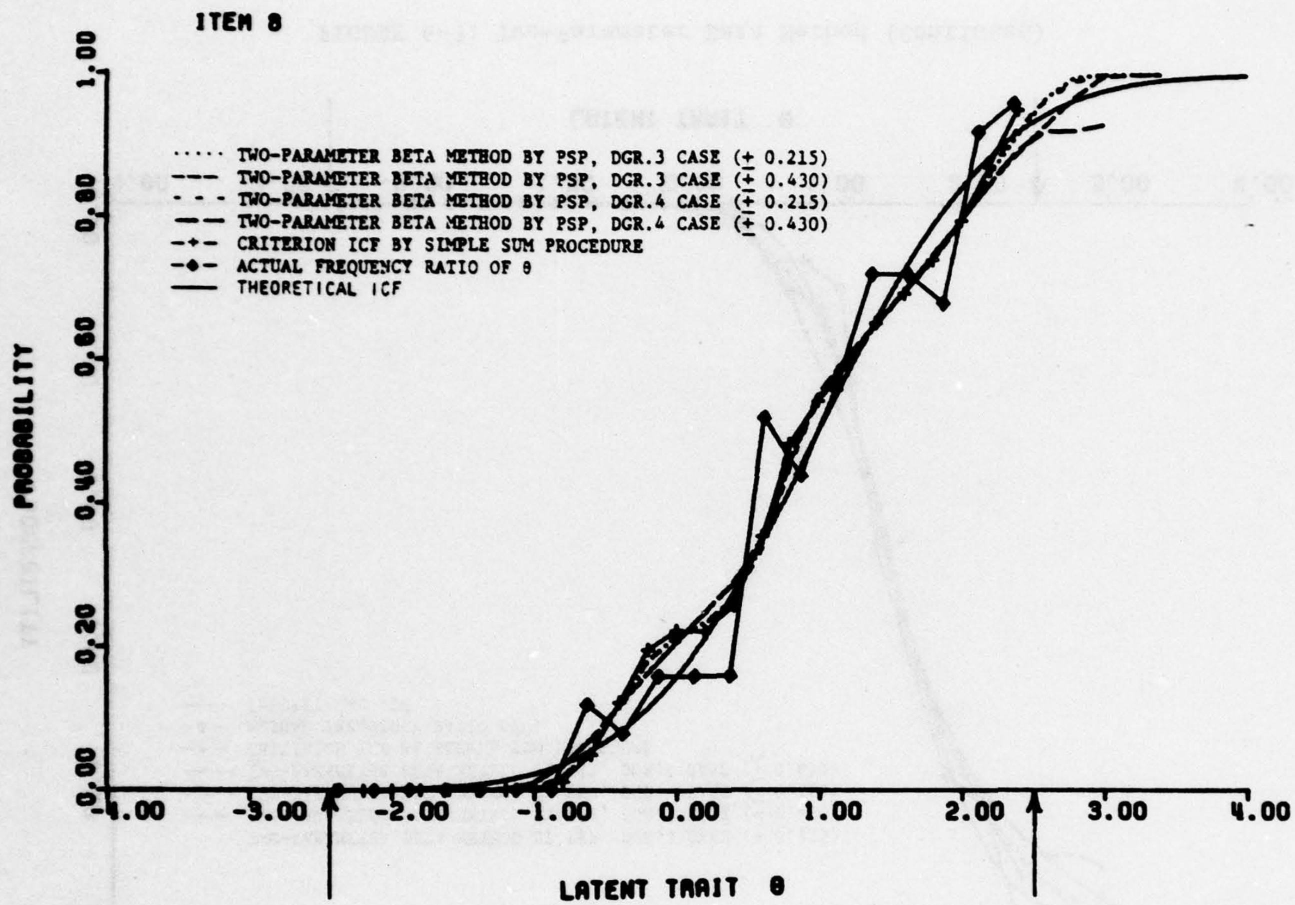

FIGURE 6-3: Two-Parameter Beta Method (Continued)

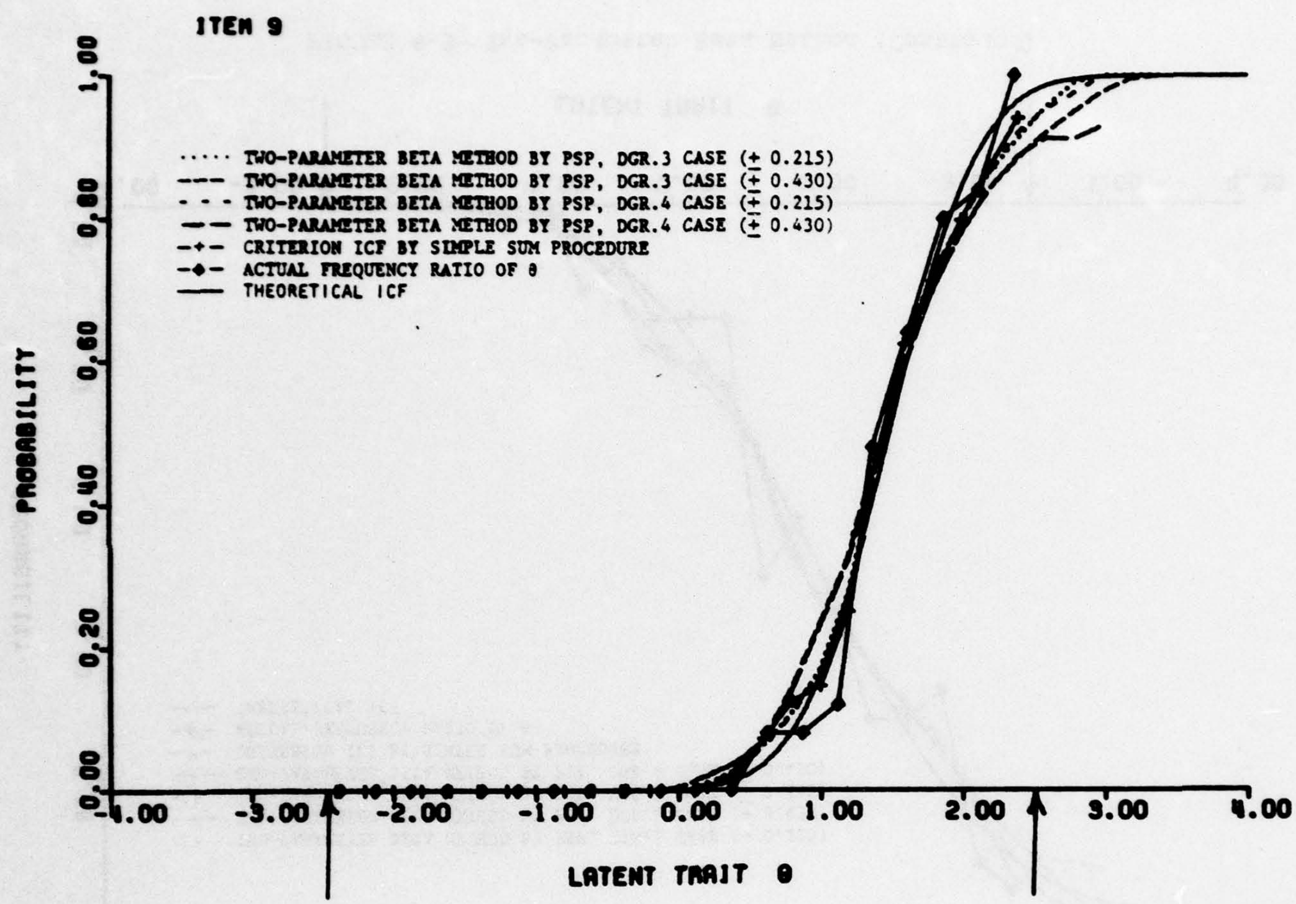

FIGURE 6-3: Two-Parameter Beta Method (Continued)

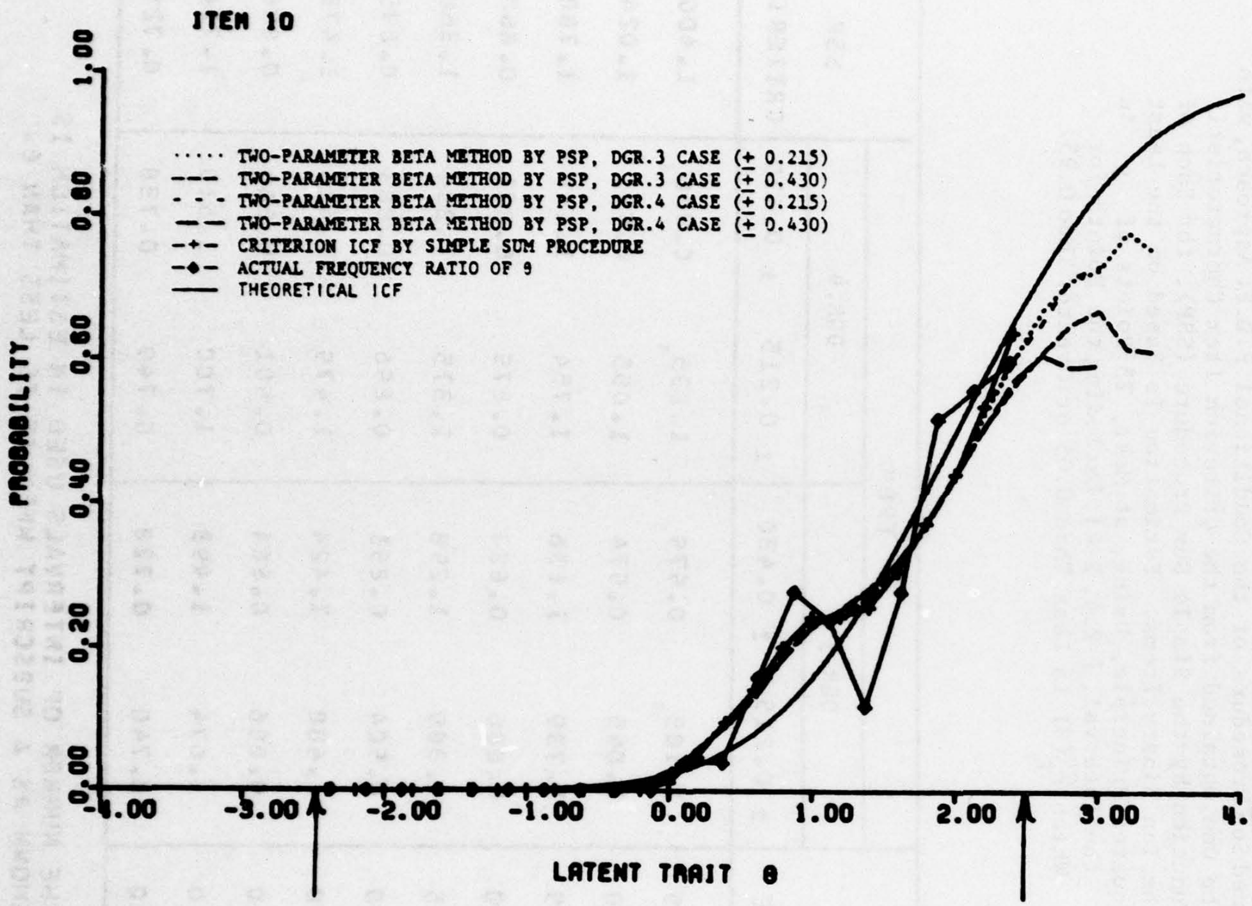

FIGURE 6-3: Two-Parameter Beta Method (Continued)

TABLE 6-7

Estimated Item Discrimination Parameters, $\hat{a}_g$, Obtained from the Four Estimated Item Characteristic Functions by Degree 3 and 4 Cases of the Two-Parameter Beta Method (TPBM) of the Proportioned Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95.

		TPBM				SSP
		DGR.3		DGR.4		
ITEM	TRUE	± 0.215	± 0.430	± 0.215	± 0.430	CRITERION
1	1.5	1.105_5	0.979_5	1.135_5	0.980_5	1.400_5
2	1.0	1.045	0.974	1.055	0.981	1.024
3	2.5	1.739	1.636	1.754	1.644	1.788
4	1.0	0.886	0.827	0.879	0.821	0.868
5	1.5	1.389	1.298	1.375	1.284	1.368
6	1.0	0.904	0.891	0.896	0.883	0.895
7	2.0	1.488	1.424	1.479	1.415	1.473
8	1.0	0.896	0.864	0.901	0.866	0.886
9	2.0	1.674	1.493	1.700	1.510	1.716
10	1.0	0.740	0.728	0.749	0.738	0.725

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

TABLE 6-8

Estimated Item Difficulty Parameters, b_g, Obtained from the Four Estimated Item Characteristic Functions by Degree 3 and 4 Cases of the Two-Parameter Beta Method (TPBM) of the Proportioned Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95.

		TPBM				SSP
		DGR.3		DGR.4		
ITEM	TRUE	± 0.215	± 0.430	± 0.215	± 0.430	CRITERION
1	-2.5	-2.839_5	-2.931_5	-2.827_5	-2.938_5	-2.651_5
2	-2.0	-1.972	-2.015	-1.973	-2.019	-2.002
3	-1.5	-1.500	-1.516	-1.508	-1.524	-1.507
4	-1.0	-0.986	-1.023	-0.991	-1.026	-1.005
5	-0.5	-0.467	-0.463	-0.471	-0.466	-0.472
6	0.0	-0.061	-0.060	-0.062	-0.060	-0.075
7	0.5	0.518	0.518	0.523	0.522	0.527
8	1.0	0.985	0.982	0.981	0.982	0.981
9	1.5	1.520	1.509	1.523	1.514	1.502
10	2.0	2.138	2.171	2.130	2.162	2.118

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

The mean square errors, and their square roots, of these estimated item characteristic functions from the true item characteristic functions, which are obtained through (6.1), are presented in Tables 6-1 and 6-2. Again these results are similar to those obtained by the Pearson System Method, and those from the criterion item characteristic functions by the Simple Sum Procedure and from those by the Proportioned Sum Procedure.

The estimated discrimination and difficulty parameters, $\hat{a}_g$ and $\hat{b}_g$, are presented in Tables 6-7 and 6-8. They are comparable to those obtained by the Pearson System Method, and, again, there is a slight tendency that the estimates of the discrimination parameters are better for Degree 3 Case with the interval, $\hat{\theta}_s \pm 0.215$.

VII Discussion and Conclusion

Conditional P.D.F. Approach was expanded, and a new procedure, i.e., Proportioned Sum Procedure, was introduced, and tried on the simulated data, which have been used to test many different methods and approaches. As the result, we have obtained sufficiently good estimates of the item characteristic functions of the ten binary items. They are similar to the results obtained in previous studies, using Simple Sum Procedure and Weighted Sum Procedure, however, and we cannot say that the present method is superior to the others. Thus we ended up with expanding our repertory of methods of estimating the operating characteristics of the graded item categories.

One interesting finding is that, using as small a number of points of $\hat{\theta}$ as 61, we have succeeded in producing criterion item characteristic functions which are practically the same as those obtained on the 500 $\hat{\theta}_s$'s . This result is all the more interesting if we call our attention to the fact that neither of the numerator and the denominator of (3.3), which are divided by 61, is close to the density function of θ and the shared density function of θ by the success group, as we can see in Figures 5-1 and 5-2. Although it is too early to jump to the conclusion, if this is further tested with different types of data and proves to be successful, this will contribute to saving our time and trouble in the estimation process. In such a case, the set of a certain larger number of maximum likelihood estimates is used, mainly, for obtaining the estimates of the moments, and to specify a polynomial of a certain degree using the method of moments to approximate the density function of the maximum likelihood, $g(\hat{\theta})$, for all the conditional moments of ability θ, given $\hat{\theta}$, are estimated from $g(\hat{\theta})$ and σ^2 (cf. Appendix II).

As was the case in most of the previous studies, we have found no substantial differences between the set of results obtained by Degree 3 Case and that by Degree 4 Case, in spite of the fact that these two polynomials of degree 3 and 4 are substantially different (cf. Appendix II). It seems that the accuracy of estimation of the density function $g(\hat{\theta})$ is not too important, although we should try not to generalize this tentative conclusion too far. Again, we find no significant differences between the results obtained by the Pearson System Method and the Two-Parameter Beta Method, and both of them proved to be very close to the criterion item characteristic functions.

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APPENDIX I

A-I <u>Simulated Data</u>

The simulated data used in the present study are characterized as follows.

(1) There are 500 hypothetical examinees.

(2) Their ability, or latent trait, distributes uniformly for the interval of θ , (-2.5, 2.5). Actually, we use 100 discrete points of θ , such as -2.475, -2.425, -2.375, -2.325, ..., 2.375, 2.425 and 2.475, i.e., the midpoints of the 100 subintervals with the width of 0.05, and at each point five examinees are located.

(3) There is a hypothetical test of 35 graded items, each of which has four item score categories, and which provides us with an approximately constant test information function, 21.63, for the interval of θ , [-3.0, 3.0], following the normal ogive model of the graded response level (Samejima, 1969, 1972). The test is called the Old Test, to distinguish from the New Test, which will be described later.

(4) Each of the 500 examinees is assumed to have taken the Old Test, and his response pattern on the 35 graded items has been calibrated by the Monte Carlo method. The score categories of each item are 0, 1, 2 and 3, and a typical response pattern looks like: (3,3,3,2,3,3,2,2,2,2,2,2,1,2,2,2,1,2,1,1,1,0,1,1, 1,0,1,0,1,1,0,0,0,0,0).

(5) From each response pattern, the maximum likelihood estimate of the examinee's ability has been obtained, using a computer program written for this purpose. In this process, out of 140 basic functions (Samejima, 1969, 1972), an appropriate set of 35 basic

functions are chosen depending upon the item scores in the response pattern, and, using the Newton-Raphson procedure, the point of θ at which the sum total of these 35 basic functions equals zero is searched.

(6) There is another hypothetical test of 10 binary items, each of which follows the normal ogive model of the dichotomous response level. This is called the New Test.

(7) Each of the 500 examinees is assumed to have taken the New Test also, and his response pattern on the New Test has been calibrated by the Monte Carlo method. A typical response pattern looks like: (1,1,1,0,1,0,0,1,0,0).

(8) The item characteristic functions of the test items of the New Test are assumed to be unknown, and they are the target of estimation. Each method of estimation is evaluated by the the "closeness" of the resultant estimated item characteristic functions to the true item characteristic functions, i.e.,

$$P_g(\theta) = (2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta-b_g)} e^{-\frac{t^2}{2}} dt \quad .$$

APPENDIX II

A-II Conditional Moments and Degree 3, 4 and 5 Cases

Let $\phi(\theta|\hat{\theta})$ be the conditional density function of ability θ, given its maximum likelihood estimate, $\hat{\theta}$. We can write

$$\text{(A.2.1)} \qquad \phi(\theta|\hat{\theta}) = \psi(\hat{\theta}|\theta)\ f(\theta)\ [\int_{-\infty}^{\infty}\psi(\hat{\theta}|\theta)\ f(\theta)\ d\theta]^{-1}$$
$$= \psi(\hat{\theta}|\theta)\ f(\theta)\ [g(\hat{\theta})]^{-1},$$

where $\psi(\hat{\theta}|\theta)$ is the conditional density function of $\hat{\theta}$, given θ, $f(\theta)$ is the probability density function of ability θ, and $g(\hat{\theta})$ is the probability density function of the maximum likelihood estimate $\hat{\theta}$.

By virtue of the asymptotic property of the maximum likelihood estimate and the constancy of the test information function of the Old Test, the conditional density $\psi(\hat{\theta}|\theta)$ is approximated by the normal density $n(\theta,\sigma^2)$, where $\sigma^2 = (21.63)^{-1} \doteq 0.046225$. Because of this fact, although the density function $\phi(\theta|\hat{\theta})$ given by (A.2.1) is not observable in the empirical situation, it is possible to estimate the conditional moments of ability θ, given the maximum likelihood estimate $\hat{\theta}$, provided that the density function $g(\hat{\theta})$ is estimated. That is to say, we can derive the following equations.

$$\text{(A.2.2)} \qquad E(\theta|\hat{\theta}) \doteq \hat{\theta} + \sigma^2\cdot\frac{d}{d\theta}\log g(\hat{\theta}) = \hat{\theta} + \sigma^2[\frac{d}{d\theta}g(\hat{\theta})][g(\hat{\theta})]^{-1}.$$

$$\text{(A.2.3)} \qquad \mathrm{Var.}(\theta|\hat{\theta}) \doteq \sigma^2[1 + \sigma^2\frac{d^2}{d\theta^2}\log g(\hat{\theta})]$$
$$= \sigma^2[1 + \sigma^2\{\frac{d^2}{d\theta^2}g(\hat{\theta})\cdot g(\hat{\theta}) - [\frac{d}{d\theta}g(\hat{\theta})]^2\}\{g(\hat{\theta})\}^{-2}].$$

$$\text{(A.2.4)} \qquad E[\{\theta - E(\theta|\hat{\theta})\}^3|\hat{\theta}] \doteq -\sigma^6[\frac{d^3}{d\theta^3}\log g(\hat{\theta})]$$
$$= -\sigma^6[\{g(\hat{\theta})\}^2\cdot\frac{d^3}{d\theta^3}g(\theta) - 3\ g(\theta)\cdot\frac{d}{d\theta}g(\theta)\cdot\frac{d^2}{d\theta^2}g(\theta)$$
$$+ 2\{\frac{d}{d\theta}g(\theta)\}^3]\{g(\theta)\}^{-3}].$$

(A.2.5) $$E[\{\theta - E(\theta|\hat{\theta})\}^4|\hat{\theta}] \doteq \sigma^4[3 + 6\sigma^2\{\frac{d^2}{d\theta^2}\log g(\hat{\theta})\} + 3\sigma^4\{\frac{d^2}{d\theta^2}\log g(\hat{\theta})\}^2 + \sigma^4\{\frac{d^4}{d\theta^4}\log g(\hat{\theta})\}]$$

$$= \sigma^4[3 + 6\sigma^2\{[g(\hat{\theta})\cdot\frac{d^2}{d\hat{\theta}^2}g(\hat{\theta}) - \{\frac{d}{d\hat{\theta}}g(\hat{\theta})\}^2][g(\hat{\theta})]^{-2}\}$$

$$+ 3\sigma^4[g(\hat{\theta})\cdot\frac{d^2}{d\theta^2}g(\hat{\theta}) - \{\frac{d}{d\theta}g(\hat{\theta})\}^2]^2[g(\hat{\theta})]^{-1}$$

$$+ \sigma^4[\{g(\hat{\theta})\}^3\cdot\frac{d^4}{d\theta^4}g(\hat{\theta}) - 4\{g(\hat{\theta})\}^2\cdot\frac{d}{d\theta}g(\hat{\theta})\cdot\frac{d^3}{d\theta^3}g(\hat{\theta})$$

$$- 3\{g(\hat{\theta})\}^2\{\frac{d^2}{d\theta^2}g(\hat{\theta})\}^2 + 12\, g(\hat{\theta})\{\frac{d}{d\theta}g(\hat{\theta})\}^2\frac{d^2}{d\theta^2}g(\hat{\theta})$$

$$- 6\{\frac{d}{d\theta}g(\hat{\theta})\}^4][g(\hat{\theta})]^{-4}].$$

Thus it is obvious that we need an estimate of $g(\hat{\theta})$ in order to obtain the above four conditional moments. This is done by approximating $g(\hat{\theta})$ by a polynomial of degree 3, 4 or 5 using the method of moments (Elderton and Johnson, 1969, Johnson and Kotz, 1970). These three polynomials obtained for the five hundred maximum likelihood estimates, which have been used as our simulated data, are as follows.

(A.2.6) $$g(\hat{\theta}) = 0.22416 - 0.00351\hat{\theta} - 0.01873\hat{\theta}^2 + 0.00095\hat{\theta}^3$$

(A.2.7) $$g(\hat{\theta}) = 0.19620 + 0.00238\hat{\theta} + 0.01319\hat{\theta}^2 - 0.00062\hat{\theta}^3 - 0.00427\hat{\theta}^4$$

(A.2.8) $$g(\hat{\theta}) = 0.19539 - 0.00638\hat{\theta} + 0.01449\hat{\theta}^2 + 0.00405\hat{\theta}^3 - 0.00449\hat{\theta}^4 - 0.00048\hat{\theta}^5$$

We distinguish three different cases, depending upon the polynomial used for $g(\hat{\theta})$, and call them Degree 3, 4 and 5 Cases. Since the polynomial of degree 5, shown as (A.2.8), is close to that of degree 4, in many cases we deal only with Degree 3 and 4 Cases.

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