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QUASIPARTICLE DAMPING OF A POSITRON IN AN ELECTRON GAS. (U)  
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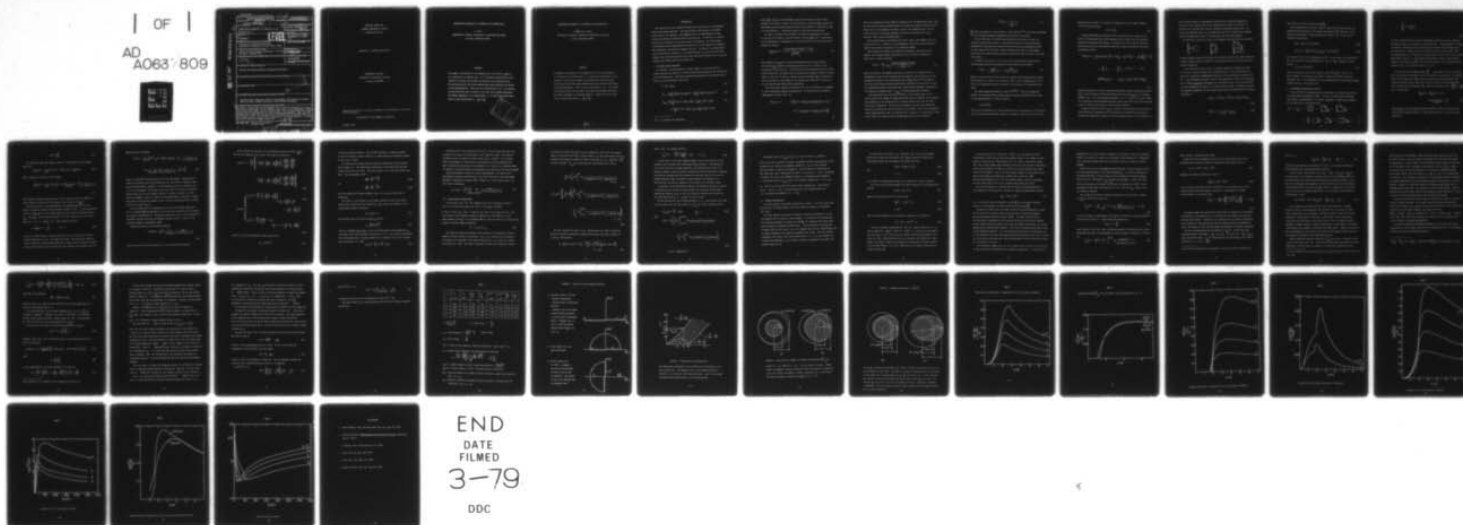
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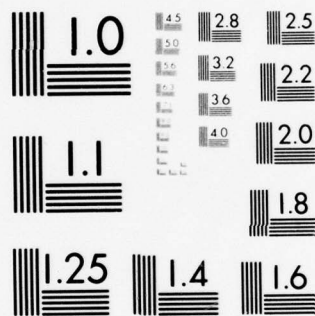
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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER TECHNICAL REPORT 148	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) QUASIPARTICLE DAMPING OF A POSITRON IN AN ELECTRON GAS.	5. TYPE OF REPORT & PERIOD COVERED TECHNICAL REPORT, 1/1/78-1/1/78	6. PERFORMING ORG. REPORT NUMBER NR17-653
7. AUTHOR(s) Joe Oliva	8. CONTRACT OR GRANT NUMBER(s) NONR N00014-76-C-0050	9. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of California San Diego Department of Physics, La Jolla, Calif. 92093	10. REPORT DATE November 1978	11. NUMBER OF PAGES 43
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research, Department of the Navy Arlington, Virginia 22217	12. SECURITY CLASS. (of this report) UNCLASSIFIED	13. DECLASSIFICATION/DOWNGRADING SCHEDULE
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 1244p.	15. DISTRIBUTION STATEMENT (of this Report) APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.	
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Positron Green's Function, Positron Self-Energy in Electron Gas, Positron Mean Free Path, Plasmon Excitation, Particle-Hole Excitation.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) We present a calculation of the imaginary part of the self energy $\Sigma_I$ of a positron in an electron gas. $\Sigma_I$ is inversely proportional to the inelastic collision rate between the positron and the electron gas. The calculation was done using standard many body techniques and within the RPA approximation. There are two contributions to $\Sigma_I$ : one arising from particle-hole pair excitation, the other from plasmon excitation. The former dominates at low energy where $\Sigma_I \sim E^2$ while the latter dominates at high energy where $\Sigma_I \sim \frac{1}{E^2} \ln \frac{E}{E_F}$ .		

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TECHNICAL REPORT #48  
CONTRACT NONR-N00014-76-C0050  
PROJECT NR 017-653

Supervisor: Professor Walter Kohn

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LA JOLLA, CALIFORNIA

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November 1978



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# QUASIPARTICLE DAMPING OF A POSITRON IN AN ELECTRON GAS

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We present a calculation of the imaginary part of the self energy  $\Sigma_I$  of a positron in an electron gas.  $\Sigma_I$  is inversely proportional to the inelastic collision rate between the positron and the electron gas. The calculation was done using standard many body techniques and within the RPA approximation. There are two contributions to  $\Sigma_I$ : one arising from particle-hole pair excitation, the other from plasmon excitation. The former dominates at low energy where  $\Sigma_I \sim E^2$  while the latter dominates at high energy where  $\Sigma_I \sim \frac{1}{E^2} \ln \frac{E}{E_F}$ .

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## INTRODUCTION

The collision rate of a positron moving in an electron gas is investigated using a self-energy approach. The quasi-particle self-energy is a complex-valued function of momentum. The imaginary part is a manifestation of real collisions between the positron and the electron gas. In Section I, a Green's function for a positron in an electron gas is introduced. The self energy  $\Sigma$  is identified and the quasiparticle collision rate is then given by an approximate expression involving its imaginary part  $\Sigma_I$ . In Section II an examination of  $\Sigma_I$  within RPA is carried out. Two contributions to  $\Sigma_I$  are found and investigated; one arising from particle-hole pair excitation (Section III), the other arising from plasmon excitation (Section IV).

### I. POSITRON GREEN'S FUNCTION

We consider a uniform system of a large number,  $N$ , of electrons and a single positron, all moving in a uniform neutralizing background and interacting via the Coulomb force. The second quantized Hamiltonian for the system may be written in the Schrodinger picture as\*

$$\hat{H} = \hat{H}_0 + \hat{H}_{INT} \quad (1a)$$

$$\hat{H}_0 \equiv -\frac{1}{2m} \sum_{\alpha} \int \psi_{\alpha}^{\dagger}(\underline{x}) \nabla_{\underline{x}}^2 \psi_{\alpha}(\underline{x}) d^3x - \frac{1}{2m} \sum_{\beta} \int \phi_{\beta}^{\dagger}(\underline{x}) \nabla_{\underline{x}}^2 \phi_{\beta}(\underline{x}) d^3x \quad (1b)$$

$$\begin{aligned} \hat{H}_{INT} = & \frac{e^2}{2} \sum_{\alpha\beta} \int d^3x d^3x' \psi_{\alpha}^{\dagger}(\underline{x}) \psi_{\beta}^{\dagger}(\underline{x}') \frac{1}{|\underline{x}-\underline{x}'|} \psi_{\beta}(\underline{x}') \psi_{\alpha}(\underline{x}) \\ & - e^2 \sum_{\alpha\beta} \int d^3x d^3x' \psi_{\alpha}^{\dagger}(\underline{x}) \psi_{\alpha}(\underline{x}) \frac{1}{|\underline{x}-\underline{x}'|} \phi_{\beta}^{\dagger}(\underline{x}') \phi_{\beta}(\underline{x}') \end{aligned} \quad (1c)$$

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\* $\hbar = 1$  throughout the discussion.

where  $\psi_{\alpha}^{+}(\underline{x})$  ( $\psi_{\alpha}(\underline{x})$ ) are Schrodinger electron field operators which create (destroy) an electron of spin  $\alpha$  at the position  $\underline{x}$ , and where  $\phi_{\alpha}^{+}(\underline{x})$  ( $\phi_{\alpha}(\underline{x})$ ) are Schrodinger positron field operators which create (destroy) a positron of spin  $\alpha$  at the position  $\underline{x}$ .  $m$  denotes the mass of the electron and positron.

We seek to calculate the lifetime of the positron quasiparticle states. To this end we introduce a positron Green's function  $G_{\alpha\beta}^P(\underline{x}t, \underline{x}'t')$  appropriate to the description of propagation of a single positron in an electron gas and defined as follows:

$$G_{\alpha\beta}^P(\underline{x}t, \underline{x}'t') = \frac{-i \langle \Psi_0 | T(\phi_{H\alpha}(\underline{x}t), \phi_{H\beta}^{+}(\underline{x}'t')) | \Psi_0 \rangle_H}{\langle \Psi_0 | \Psi_0 \rangle_H} \quad (2)$$

where  $\phi_{H\alpha}^{+}(\underline{x}t)$  and  $\phi_{H\beta}(\underline{x}'t')$  are the Heisenberg positron field operators and where  $|\Psi_0\rangle_H$  denotes the Heisenberg ground state of a uniform, fully interacting,  $N$ -electron gas.  $T$  denotes the Dyson chronological operator which orders operators in accordance with decreasing time arguments toward the right. Since, in our case,  $H$  does not involve the spin variables,  $G_{\alpha\beta}^P$  is diagonal in the spin indices; we only speak of the diagonal part  $G^P(\underline{x}t, \underline{x}'t')$  from here on out and suppress spin indices where appropriate.

That the Green's function as defined in Eq. (2) is pertinent, is suggested in the corresponding Lehmann representation. To obtain this we introduce a completeness relation in Eq. (2)

$$G^P(\underline{x}t, \underline{x}'t') = -i \sum_n \left\{ \theta(t-t') \langle \Psi_0 | \phi_H(\underline{x}t) | \Psi_n \rangle \langle \Psi_n | \phi_H^{+}(\underline{x}'t') | \Psi_0 \rangle - \theta(t'-t) \langle \Psi_0 | \phi_H^{+}(\underline{x}'t') | \Psi_n \rangle \langle \Psi_n | \phi_H(\underline{x}t) | \Psi_0 \rangle \right\} \quad (3)$$

where the intermediate states  $|\psi_n\rangle$  are eigenstates of the Hamiltonian  $H$  [Eq. (1)] for the fully interacting hybrid system of an arbitrary number of electrons and either zero or one positron; and where, as such, the summation in Eq. (3) includes states of all possible numbers of electrons and either zero or one positron. Note we have taken  $\langle\psi_0|\psi_0\rangle = 1$ .

Since  $|\psi_0\rangle$  contains no positrons, the second term of the summand of Eq. (3) vanishes for all  $n$ . Moreover, only intermediate states with  $N$  electrons and a single positron will contribute in the remaining summand.

Upon invoking the spatial and temporal translational invariance of the system, the following Lehmann representation for the Fourier transform of  $G^P(\underline{x}t, \underline{x}'t)$  is readily obtained:

$$G^P(\underline{k}, \omega) = \sum_n' \delta_{\underline{k}, \underline{p}_n} \frac{\langle\psi_0|\phi(o)|\psi_n\rangle\langle\psi_n|\phi^+(o)|\psi_0\rangle}{\omega - (E_n - E) + i\eta} \quad (4)$$

where the prime on the summation denotes the restriction to summation over intermediate states with  $N$  electrons and one positron, where  $\underline{p}_n$  and  $E_n$  are the momentum and energy of the intermediate state  $|\psi_n\rangle$ , where  $E$  is the energy of the fully interacting  $N$ -electron ground state and where  $\phi(o)$  and  $\phi^+(o)$  are now the Schrodinger positron destruction and creation field operators, evaluated at the origin of coordinate space.  $\delta_{\underline{k}, \underline{p}_n}$  is the Kronecker delta function.

Note importantly that the poles of  $G(\underline{k}, \omega)$  in the  $\omega$ -plane occur at the exact values of the change in energy associated with the addition to an electron gas originally in the ground state of a single positron such that the final system has a momentum  $\underline{k}$ . Note, by way of example, that when the interaction between the positron and the electron gas is set equal to zero, the intermediate states  $|\psi_n\rangle$  appearing in Eq. (4) separate into a product of a free positron state vector times an  $N$  electron (fully interacting) state vector, leading to



$$G^0_P(k, \omega) = \frac{1}{\omega - \frac{k^2}{2m} + i\eta} \quad (5)$$

$\frac{k^2}{2m}$  being the energy of a free positron. This form for  $G^0_P$  is obviously consistent with the previous statement regarding the positions of the poles.

Now we would like to describe these many body excitations in terms of a positron quasi-particle picture. Quasi-particle states are long lived quasi-eigenstates of the many body system which have properties approximating those of free single particle states, though differing from these in being damped with time, this reflecting the presence of interaction with the rest of the many body system.

In light of the form Eq. (5), we are led in the standard fashion to the following quasi-particle approximation for the fully interacting  $G^P$ :

$$G^P(k, \omega) = \frac{Z(k)}{\omega - \tilde{E}(k) + i\eta} = \frac{Z(k)}{\omega - E(k) - i\Gamma(k)} \quad (6)$$

where in the last step we have written the complex valued quasi-particle energy  $\tilde{E}(k)$  explicitly in terms of its real part,  $E(k)$ , and its imaginary part,  $\Gamma(k)$ .  $Z(k)$  is an unimportant function, related to the strength of the quasi-particle state.

The quasi-eigenstates behave in time as  $e^{-i\tilde{E}(k)t}$ . Thus the probability density of the quasi-particle having initial excitation energy  $\tilde{E}(k)$  will decay in time exponentially at a rate  $\gamma$  given by:

$$\gamma(k) = 2|\Gamma(k)| \quad (7)$$

This can be interpreted as a total collision rate of the positron in the electron gas, and the inverse therefore viewed as a lifetime. We point out that for the

quasi-particle concept to be useful the damping rate must be small compared to the excitation energy:

$$\Gamma(k) \ll \tilde{E}(k) \quad (8)$$

Having established the relevance of  $G^P$ , we may now bring to bear on the matter of determining the positron quasiparticle collision rate the standard methods of modern many body theory. We rewrite Eq. (2) in terms of the interaction picture via the Gell-Mann-Low theorem, thereby obtaining the following perturbation series:

$$\begin{aligned} iG^P(\underline{x}t, \underline{x}'t') &= \langle \psi_0 | T(\phi_H(\underline{x}t) \phi_H^+(\underline{x}'t')) | \psi_0 \rangle_H = \langle \phi_0 | S | \phi_0 \rangle_I^{-1} \cdot \langle \phi_0 | \sum_{v=0}^{\infty} (-i)^v \frac{1}{v!} \\ &\times \int_{-\infty}^{\infty} dt, \dots \int_{-\infty}^{\infty} dt_v e^{-\epsilon(|t_1| + \dots + |t_v|)} \\ &\langle \phi_0 | T(H_{INT_I}(t_1) \dots H_{INT_I}(t_v) \hat{\phi}_I(\underline{x}t) \hat{\phi}_I^+(\underline{x}'t')) | \phi_0 \rangle_I \end{aligned} \quad (9)$$

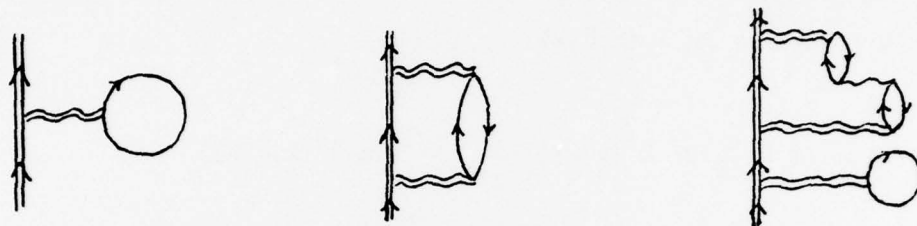
where the subscript I on state vectors and operators denotes interaction picture.  $|\phi_0\rangle_I$  is the N-electron non-interacting ground state and  $H_{INT_I}(t_1)$  is obtained from Eq. (1c). S is the scattering operator, i.e.,  $S = U_I(\infty, -\infty)$ ,  $U_I(t, t')$  being the conventional time development operator (which takes the form of the numerator of Eq. (9) with the two positron field operators absent).

Using the explicit form of  $H_{INT_I}$  in terms of field operators, this perturbation series may be expressed more conveniently by applying Wick's theorem to evaluate the expectation values of the time ordered products of field operators.



The resulting form has a diagrammatic interpretation completely analogous to that of the more familiar case of the single-particle Green's function of an electron in an electron gas, and with completely analogous Feynman rules. As in the just mentioned case, it is readily shown that the "disconnected" diagrams cancel the denominator in Eq. (9) leaving only the connected diagrams.

We illustrate a few of the lower order diagrams resulting from this procedure:



A double (single) simple line denotes a free positron (electron) Green's function; a double (single) wavy line denotes a bare positron-electron (electron-electron) Coulomb interaction.

It is clear that we may arrive at a Dyson's equation for  $G^P(\underline{x}t, \underline{x}'t')$  which will involve a positron proper self energy  $\Sigma^P(\underline{x}t, \underline{x}'t')$ , this latter being the sum of all irreducible subdiagrams arising out of the perturbation expansion for  $G^P$ . The subdiagrams referred to result from removing the "ends" of the diagrams for  $G^P$ . [The irreducible subdiagrams are those which cannot in turn be separated into two subdiagrams by removal of a single particle line.] Invoking the spatial and temporal homogeneity of the system, we may Fourier transform Dyson's equation, arriving at:

$$G^P(k, \omega) = G^O_P(k, \omega) + G^O_P(k, \omega) \Sigma^*(k, \omega) G^P(k, \omega). \quad (10a)$$

$$G^P(k, \omega) = \frac{1}{\omega - E^O(k) - \Sigma^*P(k, \omega)} \quad (10b)$$

where  $E^0(k)$  is the free particle energy  $\frac{k^2}{2m}$ .

Upon comparing Eq. (6) and Eq. (10b), we see that we can now relate the quasiparticle energy  $\tilde{E}(k)$  to the proper self energy  $\Sigma^*(k, \omega)$  [we here on out delete the superscript p (denoting positron) from  $G$  &  $\Sigma$ ]. In the case of long-lived excitations:

$$\tilde{E}(k) \approx E(k) + \text{Re } \Sigma^*(k, \tilde{E}(k)) \quad (11a)$$

$$\Gamma(k) \approx (1 - \frac{d}{d\omega} \text{Re } \Sigma^*(k, \omega)|_{\omega=\tilde{E}(k)})^{-1} \text{Im } \Sigma^*(k, \tilde{E}(k)) \quad (11b)$$

Our interest here is in the damping rate  $|2\Gamma|$ . We will make the approximation that the derivative term appearing in Eq. (11b) is small compared to unity, giving:

$$\Gamma(k) \approx \text{Im } \Sigma^*(k, \tilde{E}(k)) \quad (12)$$

This is certainly valid for very large  $k$ , since  $\Sigma^*(k, \tilde{E}(k)) \rightarrow 0$  for  $k \rightarrow \infty$ , and is likely to be valid for very small  $k$ , where  $\text{Re } \Sigma^*(k, \tilde{E}(k))$  should be relatively flat. For intermediate  $k$ , this approximation is questionable though; we shall return to this point later.

## II. APPROXIMATE EVALUATION OF $\text{Im } \Sigma^*$




We now evaluate the positron proper self energy in an electron gas within the random phase approximation (RPA). By this it is meant that the proper self energy is approximated by an infinite summation of bubble-type diagrams of the form shown in the following Eq.

$$\begin{aligned} \Sigma^* &\approx \Sigma_{RPA}^* = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \dots \\ &= \text{[diagram 4]} \times \left\{ \text{[diagram 5]} + \text{[diagram 6]} + \text{[diagram 7]} + \dots \right\} \end{aligned}$$

$$= \left\| \right\| \times \left\{ V_{\text{eff}}^p \right\}$$

(13)

We have introduced with an obvious diagrammatic identification the effective interaction,  $V_{\text{eff}}^p$ , of a positron in an electron gas. These diagrams for  $\Sigma^*(p, \omega)$  correspond to successive virtual excitations of electron particle-hole pairs by the positron. Though it is true, as with the case of electron RPA, that this approximation for  $\Sigma^*$  becomes very good (exact, in fact) only in the extreme high density limit  $r_s \ll 1$  [ $r_s$  being the Wigner-Seitz radius = mean distance between electrons], we probably can expect qualitative, even semi-quantitative validity for  $r_s$  in the metallic range  $\sim 2 < r_s < \sim 5$ .

We note that the exchange diagram, , appearing in the electron  $\Sigma_{\text{RPA}}^*$  has no analog here since there is only one positron in the system. Furthermore, since in all diagrams for  $V_{\text{eff}}^p$  there are exactly two electron-positron bare Coulomb interaction lines, each term is equal in value to the corresponding term in the effective potential,  $V_{\text{eff}}^e$ , seen by an electron (e.g.,  = , etc). We thus have

$$\begin{aligned} V_{\text{eff}}^p(k, \omega) &= V_{\text{eff}}^e(k, \omega) - v(k) \\ &= v(k) \left( \frac{1}{\epsilon_{\text{RPA}}(k, \omega)} - 1 \right) \end{aligned} \quad (14)$$

where  $\epsilon_{\text{RPA}}(k, \omega)$  is the propagating RPA (Lindhard) dielectric function of an N-electron gas [defined below, Eq. (28)] and where  $v(k)$  is the bare Coulomb interaction:

$$v(k) = \frac{4\pi e^2}{k^2} \quad (15)$$

The positron proper self energy is given, in accordance with the Feynman rules, by:

$$\Sigma_{RPA}^*(p, \omega) = \frac{i}{(2\pi)^4} \int d^3k \, d\omega' \, G^0(p-k, \omega-\omega') \, v_{eff}^p(k, \omega') \quad (16)$$

which, using Eq. (5) and Eq. (13), becomes

$$\Sigma_{RPA}^*(p, \omega) = \frac{i}{(2\pi)^4} \int d^3k \int_C d\omega' \, v(k) \left( \frac{1}{\epsilon_{RPA}(k, \omega')} - 1 \right) \frac{1}{\omega - \omega' - E^0(p-k) + i\eta} \quad (17)$$

The frequency integral appearing in Eq. (16) is along the contour C [shown in Figure (1a)], so chosen so as to properly avoid the poles of  $\epsilon_{RPA}(k, \omega')$ .

We have seen that in our approximation, the imaginary part of the self energy is directly related to the collision rate [Eq. (12)]. A rather convenient form for  $\text{Im } \Sigma_{RPA}^*(p, \omega)$  is now obtained [we from now on omit the subscript RPA on  $\Sigma^*$  and  $\epsilon_{RPA}$ ]. Note the free-positron G-F in the integrand of Eq. (17) has a pole at  $\omega' = \omega_1 \equiv \omega - E^0(p-k) + i\eta$ . Note further that since

$$\frac{1}{\epsilon(k, \omega')} - 1 \sim \frac{1}{\omega'^2}, \quad |\omega'| \rightarrow \infty \quad (18)$$

we may, by Jordan's lemma, close the contour C in Eq. (17) with the standard infinite semicircle in, e.g., the upper half plane. By rotating the resulting closed contour  $90^\circ$  counterclockwise [see Fig. (1c)], we have by Cauchy's theorem and upon noting that for both initial and final contours, the integral over the

semicircular part vanishes,

$$\Sigma^*(p, \omega) = \frac{i}{(2\pi)^4} \left\{ \int_{C'} d^3k \, d\omega' \, v(k) \left( \frac{1}{\epsilon(k, \omega')} - 1 \right) \cdot \frac{1}{\omega - \omega' - E^0(p-k) + i\eta} \right. \\ \left. - 2\pi i \int_{0 < E^0(p-k) < \omega} d^3k \left( \frac{1}{\epsilon(k, \omega - E^0(p-k) + i\eta)} - 1 \right) v(k) \right\} \quad (19)$$

where  $C'$  is the  $+90^\circ$  rotated version of  $C$  (with the dogleg). Realizing that there are no poles of the first integrand in Eq. (18) near the imaginary axis, we may now equivalently replace  $C'$  by the imaginary axis and in addition, set  $\eta = 0$  in the integrand. The second term in Eq. (18) is a residue contribution which arises since, as the original contour is rotated, the pole  $\omega_1$  of the positron propagator, when it is in the first quadrant [i.e., when  $k \rightarrow \omega > E(p-k)$ : note limits in the second integral of Eq. (1)] goes from being within to being outside the closed contour.

It can be shown that the imaginary part of the line integral appearing in Eq. (1) vanishes [Having replaced  $C$  by the real axis and setting  $\eta=0$ , introduce the change in variables  $u=i\omega$ . Noting then that since  $\epsilon(k, \omega')$  is the propagating dielectric function which has the symmetry property  $\epsilon(k, \omega') = \epsilon(k, -\omega')$ , the resulting integral is manifestly real].

Therefore we arrive at the useful expression

$$\text{Im } \Sigma^*(p, \omega) = \frac{-e^2}{2\pi^2} \text{Im} \int_{0 < E(p-k) < \omega} d^3k \frac{v(k)}{\epsilon(k, \omega - E^0(p-k) + i\eta)} \quad (20)$$

which will serve as the starting point for the subsequent development.



We now review the properties of the Lindhard dielectric function  $\epsilon(k, \omega)$ .  
RPA

Its real and imaginary parts  $\epsilon_r(k, \omega)$  and  $\epsilon_i(k, \omega)$  are given by

$$\epsilon_r(k, \omega) = 1 + \frac{2mp_o}{\pi k^2} \left\{ 1 - \frac{p_o}{2k} \left( 1 - \left( \frac{\omega m}{p_o k} - \frac{k}{2p_o} \right)^2 \right) \ln \left| \frac{1 + \left( \frac{\omega m}{p_o k} - \frac{p_o}{2k} \right)}{1 - \left( \frac{\omega m}{p_o k} - \frac{p_o}{2k} \right)} \right| \right. \\ \left. + \frac{p_o}{2k} \left( 1 - \frac{\omega m}{p_o k} + \frac{k}{2p_o} \right)^2 \ln \left| \frac{1 + \left( \frac{\omega m}{p_o k} + \frac{p_o}{2k} \right)}{1 - \left( \frac{\omega m}{p_o k} + \frac{p_o}{2k} \right)} \right| \right\} \quad (21a)$$

$$\epsilon_i(k, \omega) = \begin{cases} \epsilon_{i1}(k, \omega) = \frac{mp_o^2}{k^3} \left( 1 - \left( \frac{\omega m}{p_o k} - \frac{k}{2p_o} \right)^2 \right) , & \text{for } \frac{k}{p_o} + \frac{1}{2} \left( \frac{k}{p_o} \right)^2 \geq \frac{\omega m}{p_o^2} \\ \geq \left| \frac{k}{p_o} - \frac{1}{2} \left( \frac{k}{p_o} \right)^2 \right| \end{cases} \quad (21b)$$

$$\epsilon_i(k, \omega) = \begin{cases} \epsilon_{i2}(k, \omega) = \frac{mp_o^2}{k^3} \left( \frac{2\omega m}{p_o^2} \right) , & \text{for } \frac{k}{p_o} \leq 2 ; 0 \leq \frac{\omega m}{p_o^2} \leq \frac{k}{p_o} - \frac{1}{2} \left( \frac{k}{p_o} \right)^2 \end{cases} \quad (21c)$$

where  $p_o$  is the Fermi momentum of the electron gas,

$$p_o = (3\pi^2 n)^{1/3} \quad (22)$$

$n$  being the electron density. Note we have introduced a separate notation for the two distinct analytic forms of  $\epsilon_1$ , there applying in different regions of the  $\omega$  vrs.  $k$  plane.

We illustrate the single-particle excitation spectrum of a free electron gas in Figure (2). The shaded band corresponds to the region where excited particle-hole pairs can exist. Note that  $\epsilon_1(k, \omega)$  is non zero only within this band. The two bounding curves

$$\omega_{\max}(k) = \frac{k^2}{2m} + \frac{k p_0}{m} \quad (23a)$$

and

$$\omega_{\min}(k) = \frac{k^2}{2m} - \frac{k p_0}{m} \quad (23b)$$

give the maximum and minimum energies, resp., of a particle-hole pair of momentum  $k$ .

Also shown in this Figure is the plasmon excitation branch  $\omega_p(k)$  which appears when interactions are allowed for.  $\omega_p(k)$  solves the equation of the longitudinal resonance condition:

$$\epsilon(k, \omega_p(k)) = 0 \quad (24)$$

This branch starts at  $k=0$  with energy  $\omega_0$  given by

$$\omega_0 = \left( \frac{4\pi n e^2}{m} \right)^{1/2}, \quad (25)$$

risers to a maximum value  $\omega_p(k_c)$  at the so-called plasmon cutoff momentum,  $k_c$ , and ceases to be a well defined excitation for  $k > k_c$ . The cutoff is determined by the condition that it becomes possible for a plasmon to decay into a single pair excitation, i.e., when

$$\omega_{\max}(k) = \frac{k^2}{2m} + k \frac{p_0}{m} = \omega_p(k) \quad (26)$$



Returning now to the calculation for  $\text{Im } \Sigma^*$ , it will be seen that there are in general two types of contributions to  $\text{Im } \Sigma^*$  [see Eq. (20)]: one from the integration over the region of  $k$ -space where  $\epsilon_1(k, \omega - E(\underline{p}-\underline{k}) + i\eta)$  is finite [this corresponding to the positron's exciting particle-hole pairs] and the other arising from any poles in the integrand (i.e., zeros of  $\epsilon_1$ ) falling within the domain of integration [this corresponding to the positron's exciting plasmons].

Before proceeding, we make one further approximation: we take for the approximate collision rate  $2\Gamma$  (see Eq. (13)) twice the value of the self-energy  $\Sigma(\underline{k}, \tilde{E}(\underline{k}))$ , evaluated at frequency  $\tilde{E}(\underline{k}) = E^0(\underline{k})$ , i.e., the on shell value. We will thus be evaluating,

$$\text{Im } \Sigma^*(\underline{p}) = - \frac{e^2}{2\pi^2} \text{Im} \int_{0 < E^0(\underline{p}-\underline{k}) < E^0(\underline{p})} d^3k \frac{v(\underline{k})}{\epsilon(\underline{k}, E^0(\underline{p}) - E^0(\underline{p}-\underline{k}) + i\eta)} \quad (27)$$

### III. PARTICLE-HOLE CONTRIBUTION

As indicated in Eq. (21), the imaginary part of the dielectric function  $\epsilon(\underline{k}, \omega)$  is non-zero when either  $p_0 k + \frac{1}{2} k^2 \geq m\omega \geq |p_0 k - \frac{1}{2} k^2|$  or  $k < 2p_0$  &  $0 \leq m\omega \leq kp_0 - \frac{1}{2} k^2$ . Using the fact that in the integral of Eq. (27),  $\epsilon(\underline{k}, \omega)$  enters with frequency argument  $\omega = \frac{1}{2m} (2pk \cos \theta - k^2)$  [where  $\theta$  is the angle between  $\underline{p}$  and  $\underline{k}$ ], it is straightforward to show that this set of inequalities reduces to the one simple inequality:

$$|k - p \cos \theta| \leq p_0 \quad (28)$$

The region of  $k$ -space specified by this inequality is bounded by a surface known as a Pascal limaçon of revolution. For  $p > p_0$ , there are two branches of the surface (one contained in the other) whereas for  $p < p_0$ , there is a single branch [see Fig. (3)]. The region of  $k$ -space in which  $\text{Im } \epsilon(\underline{k}, \frac{1}{2m} (2pk \cos \theta - k^2))$

is nonzero and which contributes to the integral Eq. (27) is then the region which is both within the Pascal limaçon (when  $p > p_0$  this means between the two branches) and within the sphere of radius  $p$  centered at  $\underline{k} = \underline{p}$ . [see Fig. (3)]<sup>\*</sup>. We find, defining  $\Sigma_{I, p-h}^*(p)$  = particle-hole contribution to  $\text{Im } \Sigma^*(p)$ ,

$$\frac{-e^2}{2\pi^2} \text{Im} \int_0^1 dx \int_0^{2px} dk \frac{1}{\epsilon_r(k, \frac{1}{2m}(2pkx-k)) + \epsilon_{i_1}(k, \frac{1}{2m}(2pkx-k^2))} \quad (p < p_0)$$

(29a)

$$\Sigma_{I, p-h}^*(p) = \left\{ \frac{-e^2}{2\pi^2} \text{Im} \left( \int_0^{\frac{p_0}{p}} dx \int_0^{2px} dk \frac{1}{\epsilon_r(k, \frac{1}{2m}(2pkx-k^2)) + \epsilon_{i_1}(k, \frac{1}{2m}(2pkx-k^2))} \right. \right. \\ \left. \left. + \int_{\frac{p_0}{p}}^1 dx \int_{px-p_0}^{px+p_0} dk \frac{1}{\epsilon_r(k, \frac{1}{2m}(2pkx-k^2)) + \epsilon_{i_2}(k, \frac{1}{2m}(2pkx-k^2))} \right) \right\} \quad (p > p_0)$$

(29b)

We first consider the case  $p \ll p_0$ . Entering into Eq. (29a) is then the dielectric function evaluated for momentum and energy very small compared to  $p_0$  and  $E_0$ , respectively:

$$\epsilon(k, \frac{1}{2m}(2pk \cos\theta - k^2)) \approx \left\{ 1 + \frac{2mp_0}{\pi k^2} \right\} + i \left\{ \frac{2m^2 p_0^2}{k^3} \left( \frac{1}{2m}(2pk \cos\theta - k^2) \right) \right\} \quad (k \ll p_0) \quad (30)$$

see Eq. (21). We readily arrive at:

$$\Sigma_{I, p-h}^*(p) = -\frac{e^2 \pi a_0}{60} p_0^2 \left(\frac{p}{p_0}\right)^4 \sim r_s^2 p^4 \quad p \ll p_0 \quad (31)$$

We note that in this small momentum regime, the p-h contribution to the collision rate decreases with increasing electron density. Contributing to this is the fact that at higher densities, the characteristic size of the region of enhanced electron density screening the positron decreases, rendering the positron quasiparticle less effectively scattered, and the fact that at higher densities, where the kinetic energy dominates the dynamics, the electrons are less able to respond to the presence of an external charge.

In passing, we note the obvious physical correspondence between an electron of momentum  $k_e$  just outside the Fermi sphere and a positron of very small momentum  $k_p$  compared to  $p_0$ , in that the respective quasiparticle states are both very long lived ( $\tau_e^{-1} \sim (k_e - p_0)^2$  for an electron.)

We now consider the case of high momentum  $p \gg p_0$ . Starting with Eq. (29b) it can be shown that for this range of  $p$ ,  $\Sigma_{I, p-h}^*(p)$  varies inversely with  $p$ :

$$\Sigma_{I, p-h}^*(p) \approx \frac{p_0}{p} C(p_0) \quad \frac{p}{p_0} \rightarrow \infty \quad (32a)$$

with

$$C(p_0) \equiv \text{Im} \left\{ \int_1^\infty du \int_{p_0(u-1)}^{p_0(u+1)} dk \frac{1}{\epsilon_r(k, \Omega(k, u)) + \epsilon_{11}(k, \Omega(k, u))} \right. \\ \left. + \int_0^1 du \int_0^{2p_0 u} dk \frac{1}{\epsilon_r(k, \Omega(k, u)) + \epsilon_{12}(k, \Omega(k, u))} \right\},$$

(32b)

$$\Omega(k, u) \equiv \frac{1}{2m} (2p_0 k u - k^2)$$

- We present plots of  $|\Sigma_{I_{p-h}}^*(p)/E_0|$  vrs.  $p/p_0$  for various  $r_s$  (Graph 1).

Since it is quite difficult to obtain a manageable analytic approximation to the integrals in Eq. (28) for intermediate momenta ( $p \sim p_0$ ), we have resorted where necessary to numerical integration. From the numerical integration we have shown that the  $p^4$  dependence for small  $p$  is valid up to  $\sim .1p_0$ . Increasing  $p$  further, the curves rise somewhat rapidly to a relatively broad maximum at  $p \sim 2p_0$  (with values of  $\Sigma_{I_{p-h}}^*(p)/E_0$  at the maximum ranging from  $\sim .3$  for  $r_s=3$  to  $\sim .05$  for  $r_s=2$ ) and then fall rather slowly, dropping only  $\sim 10\%$  in going from  $p \sim 2p_0$  to  $p \sim 6p_0$ . We found numerically that the  $\frac{1}{p}$  variation for  $p \gg p_0$  becomes discernable for  $p > \sim 20p_0$ .

#### IV. PLASMON CONTRIBUTION

As noted earlier, the plasmon contribution to  $\Sigma_I^*(p) \equiv \text{Im } \Sigma^*(p)$  arises when the pole of  $\epsilon^{-1}(k, E^0(k) - E^0(k-p) + i\eta)$  falls within the domain of integration (Eq. (27)).

We first address the question of whether a positron of momentum  $p < p_0$  is in principle able to excite a (real) plasmon. Of course we are assuming that the system has plasmons of energy less than  $\frac{p^2}{2m}$ , for otherwise excitation would not be possible on energetic grounds alone. From the geometric construction of Figure (4a) for the case  $p < p_0$ , it is apparent that  $\text{Im } \epsilon(k, E^0(k) - E^0(p-k) + i\eta)$  is nonzero throughout the region of integration for  $\Sigma_I^*(p)$ . Thus the positron necessarily will excite only electron-hole pairs, i.e., it is not possible for a positron of momentum  $p < p_0$  to excite a plasmon, regardless of how small the plasmon energy may be.

We turn now to the case  $p > p_0$ . From Eqs. (24), (27) and the comment after (25), we see that the poles of the inverse dielectric function are encountered during the  $k$ -integration for  $\Sigma_I^*(p)$  when both

$$E^0(\underline{p}) - E^0(\underline{p}-\underline{k}) = \omega_p(k) \quad (33a)$$

and

$$k < k_c, \quad (33b)$$

i.e., when it is dynamically possible for a particle to excite a plasmon.

Let us for the time being neglect the small dispersion of the plasmon mode and set

$$\omega_p(k) = \omega_p(0) \equiv \omega_0 \quad (34)$$

whence the conditions (33) for a pole become:

$$\frac{1}{2m}(p^2 - |\underline{p}-\underline{k}|^2) = \omega_0 \quad (35a)$$

$$k < k_c^0 \quad (35b)$$

with the cutoff momentum in the absence of dispersion,  $k_c^0$  given by:

$$k_c^0 = (p_0^2 + 2m\omega_0)^{1/2} - p_0 \quad (36)$$

The set of points  $\underline{k}$  satisfying Eq. (35a) is a sphere centered at  $\underline{k} = \underline{p}$  and of radius  $(p^2 - 2m\omega_0)^{1/2}$ . When  $(p^2 - 2m\omega_0)^{1/2} < p_0$ , points  $\underline{k}$  on this sphere are such that  $k > k_c^0$ , and thus do not satisfy the second requirement for a pole, Eq. (35b). [Alternatively put, as is evident from Figure (4a), these points lie within a region of the domain of integration for  $\Sigma_I^*(p)$  in which  $\text{Im } \epsilon(k, E^0(\underline{p}) - E^0(\underline{p}-\underline{k}) + i\eta) \neq 0$ ]. There then is no plasmon contribution to  $\Sigma_I^*(p)$  in this case.



On the other hand, for radius  $(p^2 - 2m\omega_0)^{1/2} > p_0$ , the sphere of points satisfying Eq. (35a) does at least pass through regions of the domain of integration in which  $\text{Im } \epsilon(k, E^0(p) - E^0(p-k) + i\delta) = 0$ , namely, referring to Figure (4b), the crescent shaped region bounded by both the outer branch of the Pascal limaçon and the sphere of radius  $p$ , and the oval region bounded by the inner branch of the limaçon. Only in the oval region, however, do the points on the sphere also satisfy the additional condition for a pole, Eq. (35b) ( $\text{Re } \epsilon(k, E^0(p) - E^0(p-k) + i\eta) \neq 0$  in the crescent region) and it is therefore from only this region that we get a plasmon contribution for  $\Sigma_I^*(p)$ .

From this discussion, we conclude that the value of positron momentum at the threshold for excitation of a plasmon is, neglecting dispersion, given by

$$p_{th}^0 = (2m\omega_0 + p_0^2)^{1/2}, \quad (37)$$

i.e., the positron has at threshold an energy  $\frac{p^2}{2m}$  such that upon losing an energy  $\omega_0$  to the plasmon, it ends up with the Fermi energy  $\frac{p_0^2}{2m}$ .

Precisely the same value of plasmon excitation threshold momentum is found for the case of an electron probe interacting with an electron gas [again neglecting plasmon dispersion (Quinn, 1962)]. One might offhand say that the value  $p_{th}^0$  is reasonable for the electron case since the external electron cannot, after exciting a plasmon, end up with an energy less than  $E_0$ , the corresponding states already being occupied. This line of reasoning would seem to lead to the following "paradox." For a positron scattering from an electron gas, one is then tempted to say that the plasmon excitation threshold should occur at  $E^0(p) = \omega_0$ , with the scattered positron, being distinguishable from the electrons, ending up in the  $E^0(p) = 0$  state.

The discrepancy suggested by the last argument lies of course in its failing to take account of the implications of momentum conservation. If a positron of

energy  $E^0(p) = \omega_0$  were to excite a plasmon, it would have to give up a momentum  $(2m\omega_0)^{1/2}$ . But this is in excess of the plasmon cutoff momentum  $k_c^0 = (p_0^2 + 2m\omega_0)^{1/2} - p_0$ , therefore implying that such a positron is after all unable to excite a plasmon.

Note in fact that at the actual threshold  $p_{th}^0$  (Eq. (37)) the plasmon excited will have momentum equal to the cutoff momentum  $k_c^0$ . As positron momentum is increased above  $p_{th}^0$ , it will be able to excite plasmons with momenta ranging from a low of  $p - (p^2 - 2m\omega_0)^{1/2} \equiv k_{min}$  [forward scattering] to a high of  $k_c^0$  [the positron scattering at an angle  $\cos^{-1} \frac{p_0}{p}$  from its original direction]. These statements follow simply from Figures (3c & d). Note that  $k_{min}$  tends to zero as  $p$  tends to infinity.

For  $p > p_{th}^0$  we thus have a plasmon contribution to  $\Sigma_I^*(p)$  which we denote by  $\Sigma_{I_{pl}}^*(p)$ , and which is given by:

$$E_{I_{pl}}^*(p) = - \frac{e^2}{2\pi^2} \int_{\text{oval}} d^3k \operatorname{Im} \frac{v(k)}{\epsilon(k, E^0(p) - E^0(p-k) + i\eta)}, \quad p > p_{th}^0 \quad (38)$$

In the oval region we approximate the dielectric function by its high frequency, low wave vector form (i.e., again neglecting dispersion).

$$\epsilon(k, \omega) = 1 - \frac{\omega_0^2}{\omega^2} \quad (39)$$

Upon using Eq. (39) in Eq. (38), introducing spherical coordinates with  $p$  taken along the polar axis, and performing the trivial azimuthal integration, we arrive at:

$$\Sigma_{I_{pl}}^*(p) = - \frac{e^2}{4\pi} \operatorname{Im} \int_0^1 dx \int_0^{px-p_0} dk \frac{k^2 (2px-k)^2}{(\frac{1}{2} k(2px-k) + i\eta)^2 - \omega_0^2} \quad (40)$$



where  $x \equiv \cos\theta$ ,  $\theta$  being the polar angle.

Doing first the  $k$ -integration, only one of the four simple poles of the integrand falls within the region of integration, this being the one at

$$k = px - (p^2x^2 + 2m\omega_p - i\eta)^{1/2} \quad (41)$$

Applying the symbolic identity (for real  $x$ )

$$\frac{1}{x \pm i\epsilon} = P \frac{1}{x} \mp i\pi\delta(x) \quad (42)$$

it is trivially seen that the only contribution to the imaginary part of the integral arises from the residue at this pole. Proceeding in a straight forward manner, we eventually find:

$$\Sigma_{I_{pl}}^*(p) = -\frac{\omega_o}{2a_o p} \ln \left( \frac{p_o^2 + 2m\omega_o^{1/2} - p_o}{p - (p^2 - 2m\omega_o)^{1/2}} \right) \quad p > p_{th}^o \quad (43)$$

We should remark that precisely the same result is obtained for the plasmon contribution to the imaginary part of the electron self energy in an electron gas (see Quinn (1962)). Though this is consistent with our earlier observation of the equality of the positron and electron plasmon excitation threshold momenta, the equality of the two self-energy contributions is slightly surprising.

We note the following about the behavior of  $\Sigma_{I_{pl}}^*(p)$ . Recall that this expression applies only when  $p > p_{th}^o = p_o + k_c^o$ . Of course at  $p = p_{th}^o$ ,  $\Sigma_{I_{pl}}^*(p)$  must (and does\*) vanish.  $\Sigma_{I_{pl}}^*(p)$  drops abruptly just beyond  $p_{th}^o$ , with a finite derivative w.r.t.  $p$  at  $p = p_{th}^o +$ .

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\* Note that the numerator in the argument of the log in (43) is precisely  $k_c^o$ .

For  $p \gg p_0$ ,

$$\Sigma_{I_{pl}}^*(p) \approx -\frac{\omega_0}{a_0 p} \ln \frac{p}{p_0} \quad \left(\frac{p}{p_0} \rightarrow \infty\right) \quad (44)$$

As does the particle-hole contribution to  $\Sigma_I^*(p)$ , the plasmon contribution then decreases with increasing positron momentum.

In view of Eqs. (32) and (44), and using  $\Sigma_I^*(p) = \Sigma_{I_{p-h}}^*(p) + \Sigma_{I_{pl}}^*(p)$ , we thus arrive at the sensible conclusion that the total collision rate of a positron moving in an electron gas decreases with increasing momentum. [i.e., that for increasingly high momentum, the particle behaves as an increasingly nondisturbing probe of the electron gas]. Moreover, comparing Eq. (32) and Eq. (44) we see that the plasmon contribution to the collision rate dominates at high momentum. Thus,

$$\gamma(p) \approx 2|\Sigma_I^*(p)| \rightarrow 2|\Sigma_{I_{pl}}^*(p)| \sim \frac{2\omega_0}{a_0 p} \ln \frac{p}{p_0}, \quad \frac{p}{p_0} \rightarrow \infty \quad (45)$$

Let us remark that though this last result was derived for a charged particle, which is distinguishable from the electrons in the gas, the same result applies even to an energetic, "external" electron, since at very high momentum it "loses sight" of the Pauli restrictions. These restrictions would only come into play if the electron were to "try" to undergo a highly unlikely, strongly inelastic collision which would, in the absence of the Pauli restrictions, have brought it down below the Fermi sphere. Indeed, the particle-hole contributions to the electron and positron self-energies should thus (and do) approach one another in the high momentum limit.

Before presenting numerical results for  $\Sigma_{I_{pl}}^*(p)$  we make a worthwhile modification of the formula Eq. (43). Recall that we have until now neglected plasmon dispersion. This was done for reasons of calculational expediency and was

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\* It can be shown that the sign of the particle charge does not affect the result, Eq. (44).

justified since the dispersion, being relatively weak, should not enter into the self energy in a way that would qualitatively alter any conclusion based on neglecting it. However there is one quantitative aspect which we would like to be (and which is not) contained in the form Eq. (43) for  $\Sigma_{I_{pl}}^*(p)$ , namely, the correct value of particle momentum at the plasmon excitation threshold. We have seen earlier that this occurs at  $p = p_{th}^0 = p_0 + k_c^0$  if the dispersion is neglected. Now allowing for dispersion, at the plasmon excitation threshold, the plasmon produced will have the different plasmon cutoff value,  $k_c$ , for its momentum. However, at this high momentum the plasmon energy  $\omega_p(k_c)$  will be somewhat greater than  $\omega_p(0) = \omega_0$  (usually by  $\approx 20\%$  for most metals). Since at threshold the positron still ends up with momentum  $p_0$  after excitation\*, it must now have an initial momentum greater than the previous value  $p_{th}^0$ .

We see then that account must be taken of dispersion in order to obtain the correct threshold momentum. On the other hand, for particle momentum much higher than threshold, the plasmons which are excited (mostly in the near forward direction) will have small momenta, and energy close to  $\omega_0$

$[E^0(p) - E^0(p-k) = \omega_p(k)$  is satisfied by  $k \rightarrow 0$  as  $p \rightarrow \infty$  since  $\omega_p(k) \rightarrow \text{constant}$  as  $k \rightarrow 0]$ . Thus in the high  $p$  regime, the result Eq. (43) is quantitatively correct (within our overall approximation).

A device for shifting the threshold momentum to the correct value while preserving the correct high  $p$  behavior of the formula Eq. (43) is to everywhere replace  $\omega_0$  there by the expression  $\omega_p(k(p))$  :

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$$^*E_{th}^0(p) - E_{th}^0(p - k_c) = \omega_p(k_c) = (\text{from Eq. (26)}) \frac{1}{2m} (k_c^2 + k_c p_0) \Rightarrow p_{th} = p_0 + k_c.$$

$$\Sigma_{I_{pl}}^*(p) = -\frac{\omega_p(k(p))}{2a_0 p} \ln \left( \frac{(p_0^2 + 2m\omega_p(k(p)))^{\frac{1}{2}} - p_0}{p - (p^2 - 2m\omega_p(k(p)))^{\frac{1}{2}}} \right) ; \quad k(p) < k_c \quad (46)$$

Here  $k(p)$  ( $< k_c$ ) satisfies

$$E(p) - E(p-k) = \omega_p(k) \quad (47)$$

where we set  $k \parallel p$ . Note that the important  $k$ 's actually are parallel to  $p$  both for  $p$  near  $p_{th}$  and for  $p \rightarrow \infty$ .

It is seen that for  $p$  at the proper threshold  $p_{th} = k_c + p_0$ ,  $\Sigma_{I_{pl}}^*(p)$  vanishes, as desired\*. Moreover, since  $k(p) \rightarrow 0$  for high  $p$ , we obviously recover in this range the earlier correct result Eq. (43). The expression Eq. (46) then will serve as our approximation to  $\Sigma_{I_{pl}}^*(p)$ .

In practice we will make the quadratic approximation for  $\omega_p(k)$ :

$$\omega_p(k) = \omega_0 + \frac{3}{10} \frac{p_0^2}{m^2 \omega_0} k^2 \quad (48)$$

Using Eq. (48) in Eq. (47) we find  $k(p)$  which is then substituted back into  $\omega_p(k)$  to give  $\omega_p(p)$  :

$$\omega_p(k(p)) = \omega_0 + \beta \left( \frac{1}{2m\beta+1} \right)^2 \left\{ 2p^2 - (2m\beta+1)2\pi\omega_0 - 2p(p^2 - (2m\beta+1)2\pi\omega_0)^{\frac{1}{2}} \right\} \quad (49)$$

with,

$$\beta \equiv \frac{3}{10} \frac{p_0^2}{m^2 \omega_0} \quad (50)$$

In this approximation, the cutoff momentum  $k_c$  is given by:

$$k_c = \left\{ \frac{p_0}{m} - \left( \left( \frac{p_0}{m} \right)^2 + 4\omega_0 \left( \frac{1}{2m} - \beta \right) \right)^{\frac{1}{2}} \right\} / \left( 2 \left( \beta - \frac{1}{2m} \right) \right) \quad (51)$$

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\* The numerator of the argument of the logarithm is precisely  $k_c$ .

Values of the original and corrected threshold momentum for relevant values of  $r_s$  appear in Table (1). Differences of the order of  $\sim 10\%$  are seen. A comparison [for  $r_s = 2$ ] of  $\Sigma_{I_{pl}}^*(p)$  and  $\Sigma_{I_{pl}}^*(p)$  [using Eq. (49) in the latter] appears in Graph (2). It is apparent that both results are even quantitatively very similar, save for the discrepancy in thresholds. We point out that derivative w.r.t.  $p$  of  $\Sigma_{I_{pl}}^*(p)$  is again nonzero at  $p = p_{th}^+$ .

Finally, we present plots of  $|\Sigma_{I_{pl}}^*(p)|/E_0$  vrs  $p/p_0$  for relevant  $r_s$  Graph (3). The following qualitative features emerge: an abrupt rise in  $|\Sigma_{I_{pl}}^*(p)|/E_0$  just beyond  $p = p_{th}$ , a fairly broad maximum of magnitude  $\sim 1/10$  at  $p/p_0 \sim 2-4$ , followed by a rather gradual decrease to zero.

The total value for  $\Sigma_I^*(p)$  is given by the sum  $\Sigma_{I_{p-h}}^*(p) + \Sigma_{I_{pl}}^*(p)$  (Eqs. (29) and (46)). Before proceeding to a discussion of the character of  $\Sigma_I^*(p)$  it is necessary that we verify the notion implicit until this point that the probe interacting with the electron gas forms a reasonably well defined quasiparticle state. More precisely put, we must verify that the energy width of our quasi-eigenstates, namely  $\Sigma_I^*(p)$  is small compared to the excitation energy  $E^0(p)$ . Plots of  $\Sigma_I^*(p/p_f)/E^0(p/p_f)$  vrs  $p/p_f$  are presented in Graph (4) for representative  $r_s$ . It is seen that only rarely does this ratio approach even a tolerable  $\sim 20\%$ . For the most part, the fractional uncertainty is considerably smaller. We can be assured then that our results are reasonably sensible.

We are now able to present the following picture for the RPA total collision rate of a positron moving through an electron gas. [see Eqs. (29) and (46)]. Plots of  $|\Sigma_I^*(p)|/E_0 = \frac{1}{2} \gamma/E_0$  vrs.  $p/p_0$  appear in Graph (5) for relevant  $r_s$ .

For  $p < p_0$ , only particle-hole pairs are excited.  $|\Sigma_I^*(p)|$  starts off with a  $\sim p^4 r_s^2$  dependence and increases smoothly through the region  $p \approx p_0$ .



At a momentum  $p = p_{th} \sim 2p_0$  (for  $r_s$  of interest) the particle begins to excite plasmons [in addition to p-h pairs], this being manifested in a definite jump in  $|\Sigma_I^*(p)|$  there. There is then a rather broad maximum, of magnitude ranging from  $\sim .2 E_0$  for  $r_s = 1$  to  $\sim .7 E_0$  for  $r_s = 3$ , reached at  $p \sim 2-4 p_0$ . This is followed by a relatively slow drop off with an asymptotic  $\sim \frac{1}{p} \ln \frac{p}{p_0}$  dependence, originating from the dominating plasmon excitation channel.

For reference and convenience we present plots of  $\Sigma_I^*(E^0)$  vrs  $E^0$  in Graph (6).

In Graph (7) we present a comparison between our results for  $\Sigma_I^*(p)$  for a positron and Quinn's results for the same for an electron. The larger magnitude of the imaginary part of the positron self energy is readily understood: Since the positron does not see the Pauli restrictions operative for the electron, it has more available phase space to scatter into, resulting in its having a higher collision rate.

The mean free path  $\lambda(p)$  of a positron quasi-particle with excitation energy  $\tilde{E}(p)$  may be taken as

$$\lambda(p) = \frac{d \tilde{E}(p)}{dp} \cdot \frac{1}{\gamma(p)} \quad (52)$$

Now  $\tilde{E}(p)$  is given approximately by Eq. (11a). We will here ignore the k-dependence of  $\text{Re } \Sigma^*(p, \tilde{E}(p))$  and take simply:

$$\lambda(p) = \frac{p}{m} \cdot \frac{1}{\gamma(p)} \quad (53)$$

A plot of  $\lambda(E^0)$  vrs  $E^0$  appears in Graph (8). The correspondence between the structure in  $\Sigma_I^*$  described above and that in  $\lambda$  is apparent.

For  $E^0(p) \ll E_0$ .

$$\lambda(E^0) \sim \left( \frac{\sqrt{2} 15}{e^2 a_0 \pi} \cdot \frac{E_0}{m^{3/2}} \right) \frac{1}{E_0^{3/2}} \sim \frac{1}{E_0^{3/2}} \quad (54)$$

and for  $E^0(p) \gg E_0$

$$\lambda(E^0) \sim 4a_0 \frac{E^0}{\omega_0} \frac{1}{\ln(E^0/E_0)} \sim \frac{E^0}{\ln E^0} \quad (55)$$

A minimum of the order of a few Angstrom's occurs at  $E^0 \sim 2E_0$ .

The author would like to thank Professor Walter Kohn for numerous valuable discussions.



TABLE 1

$r_s$	$p_o$ (a.u.)	$E_o$ (eV)	$\frac{k_c^o}{p_o}$	$\frac{k_c}{p_o}$	$\omega_p(o)$ (eV)	$\omega_p(k_c)$ (eV)	$\frac{p_{th}^o}{p_o}$	$\frac{p_{th}}{p_o}$
2	.960	.12.5	.528	.640	16.7	21.4	1.53	1.64
3	.640	5.57	.621	.739	9.07	11.3	1.62	1.74
4	.480	3.13	.698	.820	5.89	7.23	1.70	1.82
5	.384	2.00	.764	.882	4.22	5.11	1.76	1.88

$$r_s = \left( \frac{3}{4\pi a_o^3 n} \right)^{1/3}$$

$$a_o = \text{Bohr radius} = \frac{\hbar^2}{me^2}$$

$$p_o = \text{Fermi momentum} = \left( \frac{9\pi}{4} \right)^{1/3} \frac{1}{r_s} \quad (\text{atomic units})$$

$$E_o = \text{Fermi energy} = \frac{p_o^2}{2m}$$

$$k_c^o = \text{plasmon cutoff momentum, neglecting dispersion} = (p_o^2 + 2\pi\omega_o)^{1/2} - p_o.$$

$$k_c = \text{plasmon cutoff momentum, including quadratic dispersion.}$$

$$k_c = \frac{\left( \frac{p_o}{m} - \left( \left( \frac{p_o}{m} \right)^2 + 4\omega_o \left( \frac{1}{2m} - \beta \right) \right)^{1/2} \right)}{2(\beta - \frac{1}{2m})}; \quad = \frac{3}{10} \frac{p_o^2}{m^2\omega_o}$$

$$\omega_p(o) = \text{plasmon energy at cutoff, neglecting dispersion} = \left( \frac{4\pi ne^2}{m} \right)^{1/2}.$$

$$\omega_p(k_c) = \text{plasmon energy at cutoff, including quadratic dispersion.}$$

$$p_{th}^o = \text{positron momentum at plasmon excitation threshold, neglecting dispersion:}$$

$$p_{th}^o = k_c^o + p_o.$$

$$p_{th} = \text{positron momentum at plasmon excitation threshold, including quadratic}$$

$$\text{dispersion: } p_{th} = k_c + p_o.$$

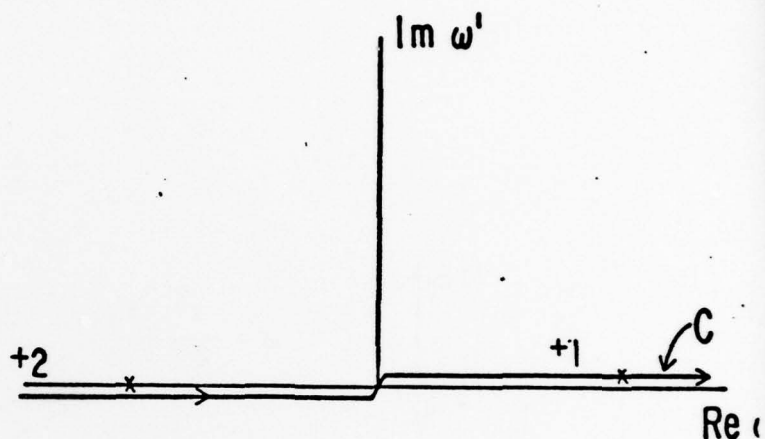
FIGURE 1: Contours for Self Energy Evaluation

a) Original contour  $C$  for the frequency integration.

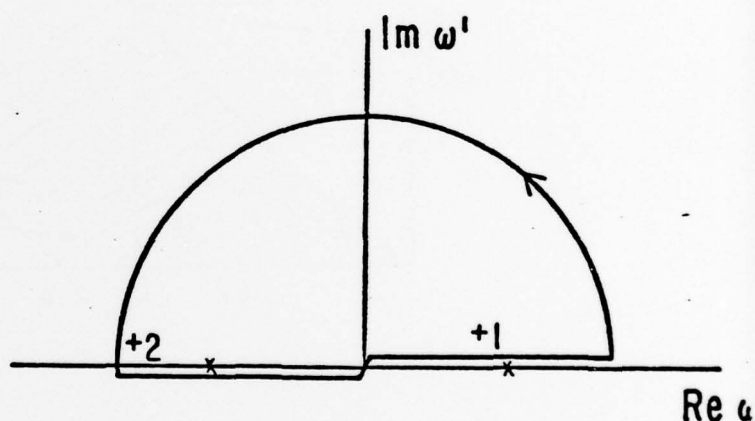
$x$  denotes poles of dielectric function.

$+$  denotes a pole of the free positron Green's function.

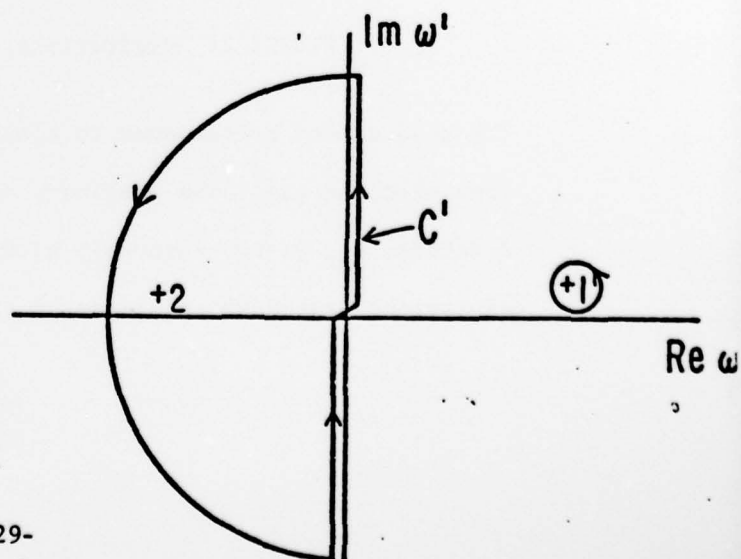
$+$  occurs in the 1st quadrant when  $\omega > E^0(\underline{p}-\underline{k})$ , e.g.,  $+_1$  and it occurs in the 2nd quadrant when  $E^0(\underline{p}-\underline{k}) > \omega$ , e.g.,  $+_2$ .



b) Close contour in, e.g., upper half plane.



c) Rotate contour in b by  $+90^\circ$ . A residue contribution arises when the pole  $+$  lies in the 1st quadrant. The contour  $C'$  may be now deformed into the imaginary axis.



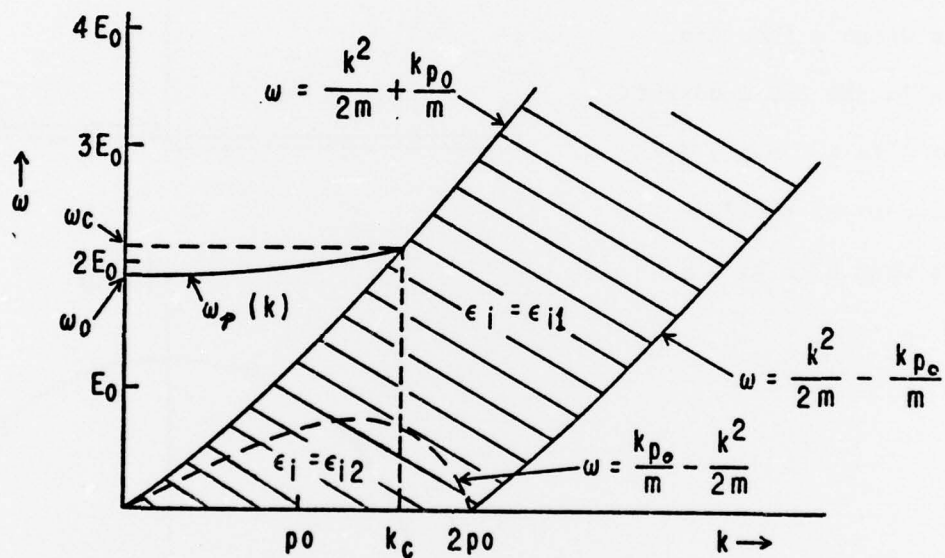


FIGURE 2: . Excitations in an Electron Gas

The shaded band corresponds to electron-hole pair excitations in the free electron gas. The imaginary part of the Lindhard dielectric function,  $\epsilon_i$ , is non-zero only within this band.  $\omega_p(k)$  is the plasmon excitation branch which arises in the interacting gas.

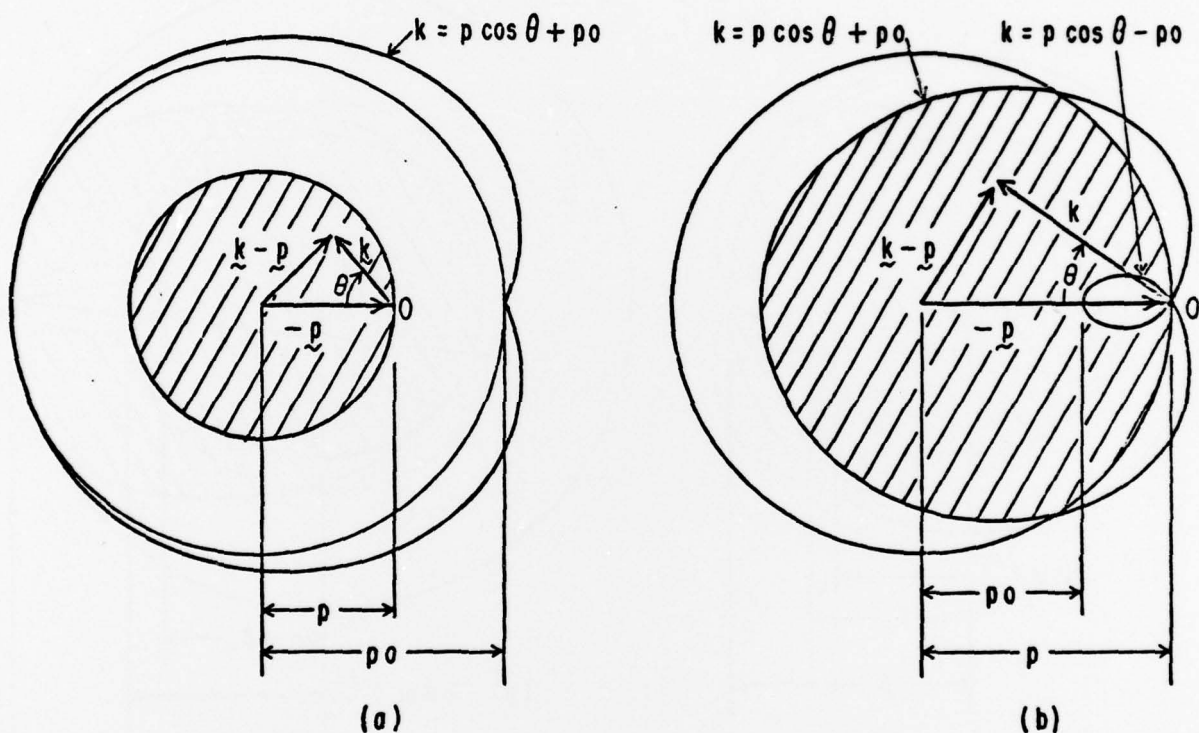
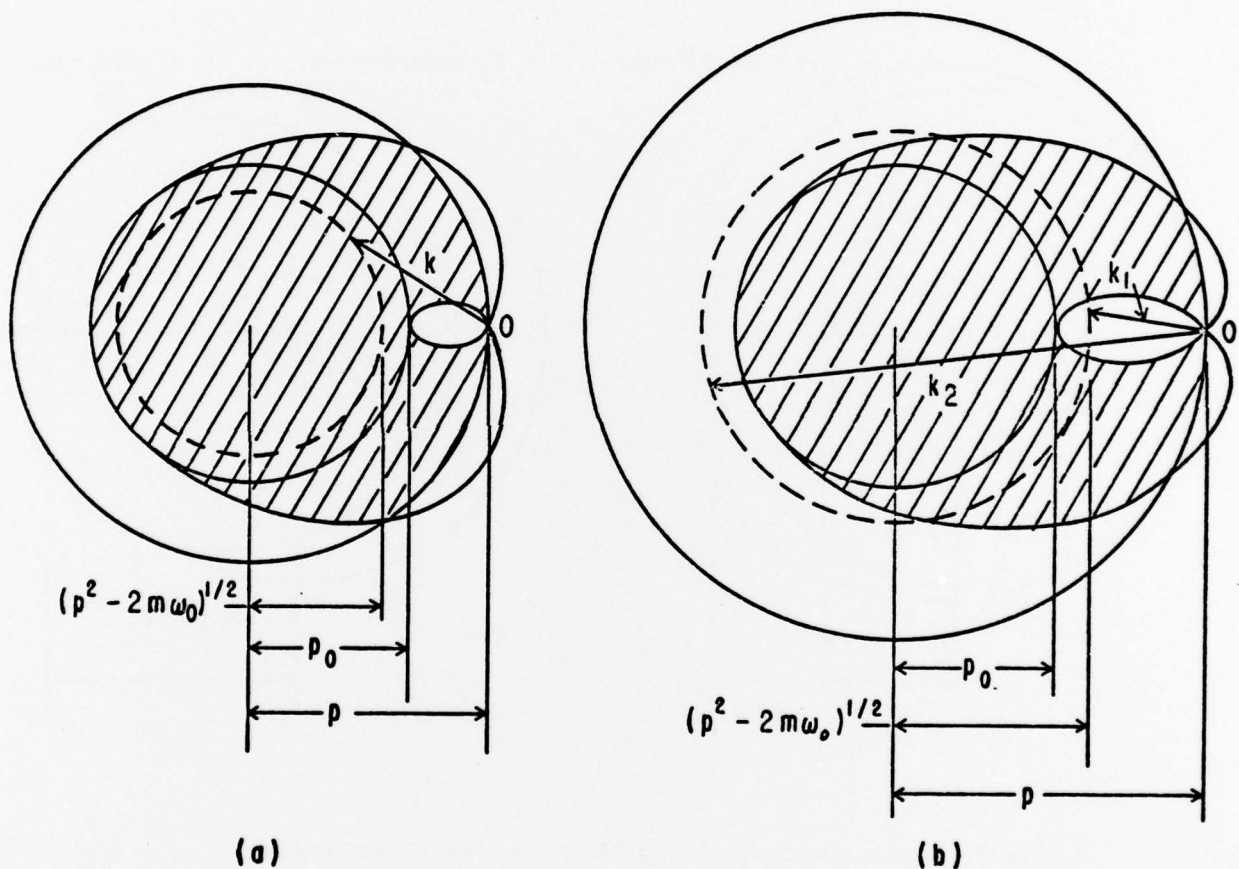


FIGURE 3: Cross Section of Region of  $\underline{k}$ -Space Contributing to  $\sum_{ph}^* (p)$ .

Figure a,  $p < p_0$ ; Figure b,  $p > p_0$ .  $O$  is the origin of  $\underline{k}$ -space. Shaded regions correspond to points  $\underline{k}$  lying both within the domain of integration in Eq. (21) (within the sphere of radius  $p$  centered at  $\underline{k} = \underline{p}$ ) and within the region of  $\underline{k}$ -space in which  $\text{Im } \epsilon(\underline{k}, \frac{1}{2m}(2pk \cos \theta - k^2))$

FIGURE 4: Plasmon-Contribution to  $\text{Im} \Sigma^*(p)$ .

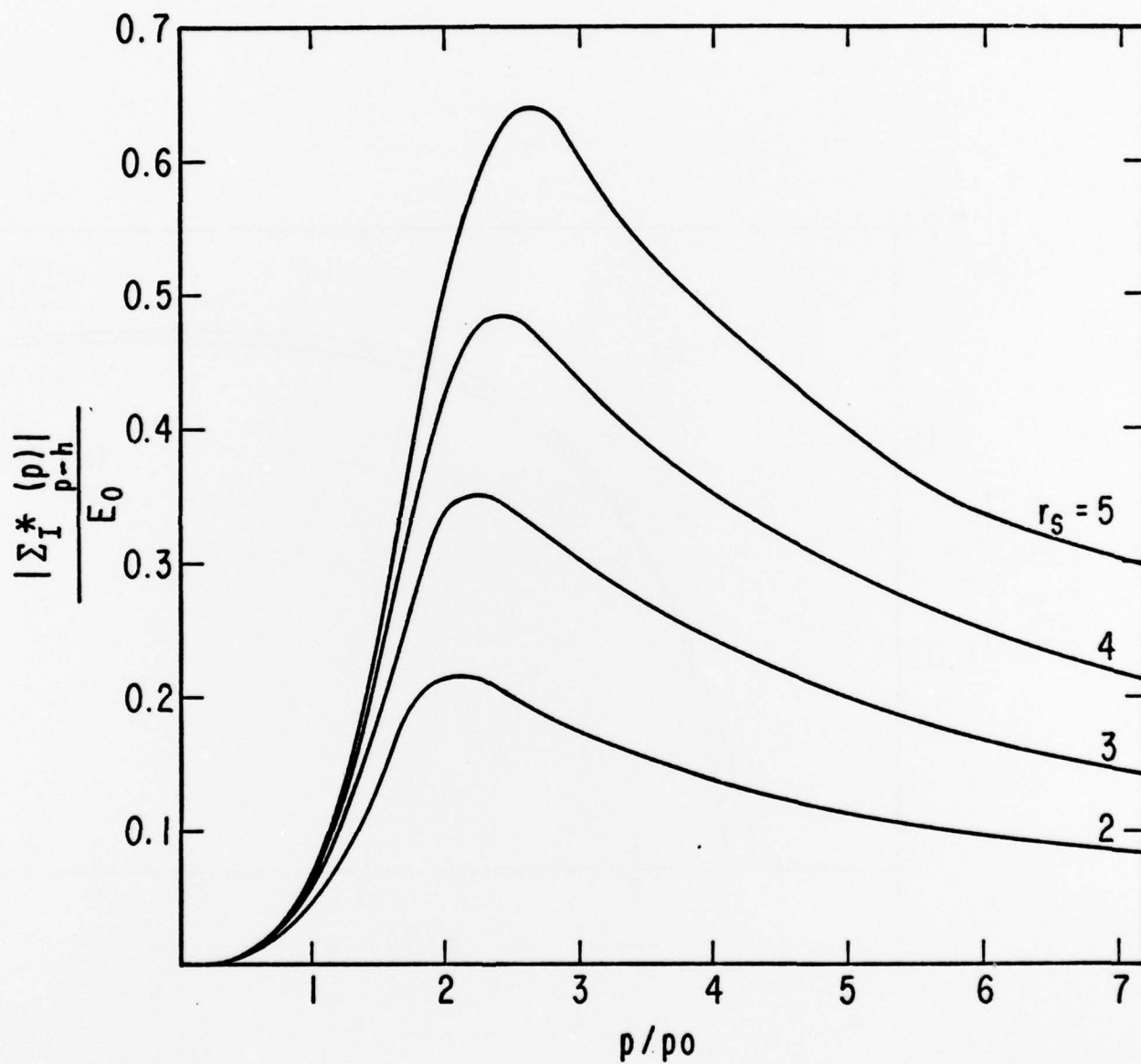


The energy conservation condition  $\omega_p(k) = E^0(p) - E^0(p-k)$  is satisfied for  $k$  on the dotted spheres in both cases. Only when its radius is greater than  $p_0$  do the points on this sphere lie in regions where  $\text{Im} \epsilon(k, \frac{1}{2m} (2kp \cos \theta - k^2)) = 0$  (b), and then, only points such as  $k_1$  in the small oval give a zero of  $\epsilon(k, \frac{1}{2m} (2kp \cos \theta - k^2))$ . ( $\text{Re} \epsilon(k, \frac{1}{2m} (2kp \cos \theta - k^2)) \neq 0$  for points such as  $k_2$ ). Obviously, threshold corresponds to the  $p$  for which the dotted sphere coincides with the sphere centered at  $k = p$  and of radius  $p_0$ .



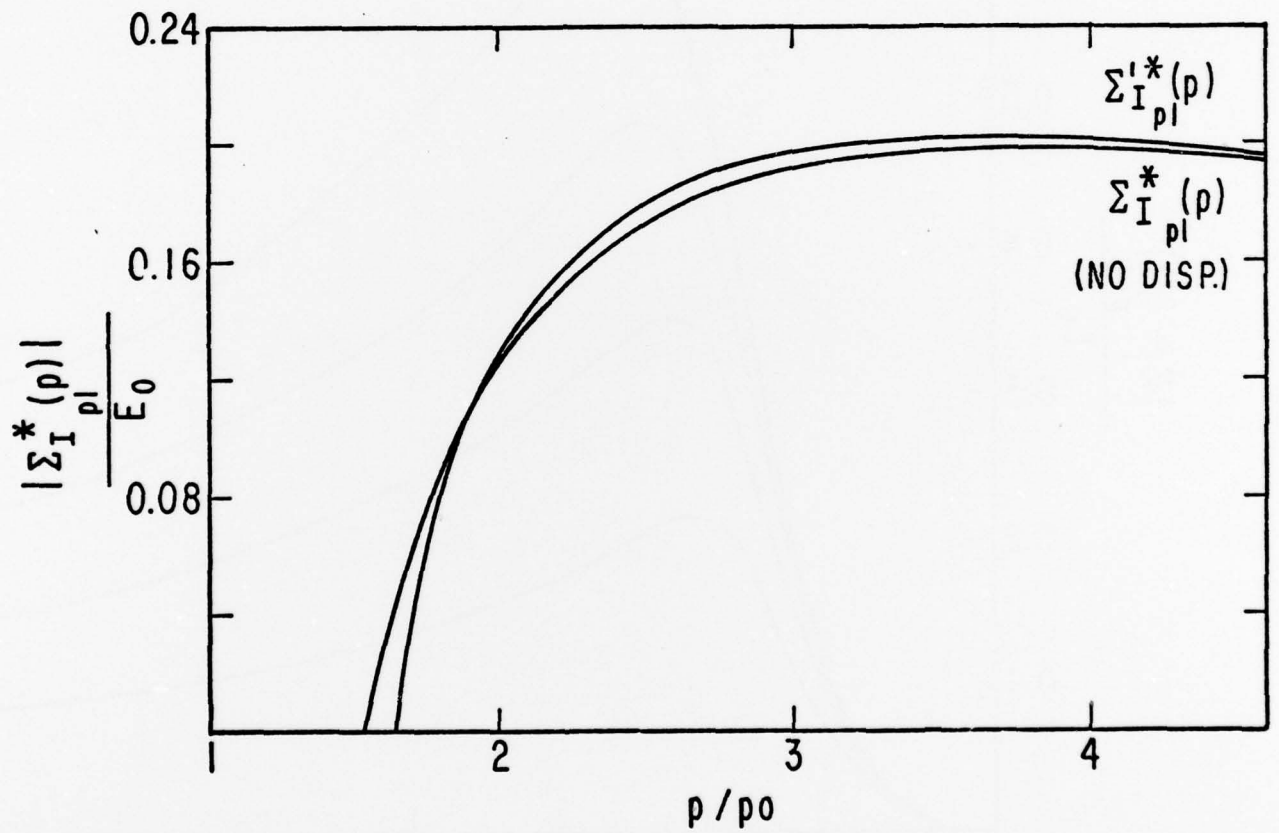
GRAPH 1

Particle-Hole Contribution to Imaginary Part of Self Energy vrs Momentum

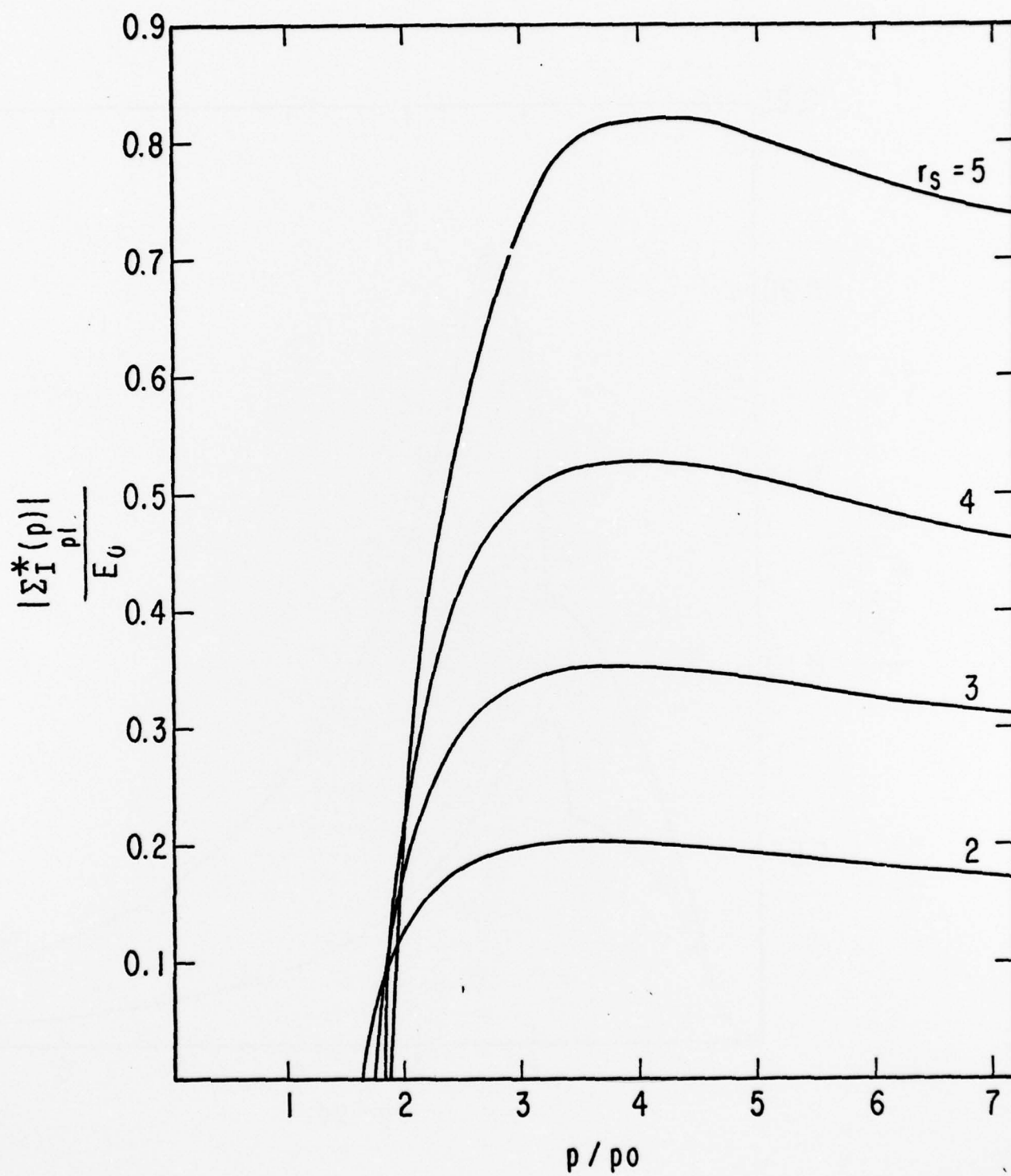


GRAPH 2

Comparison Between  $\Sigma_{I_{pl}}^*$  With and Without Including Dispersion ( $r_s = 2$ )

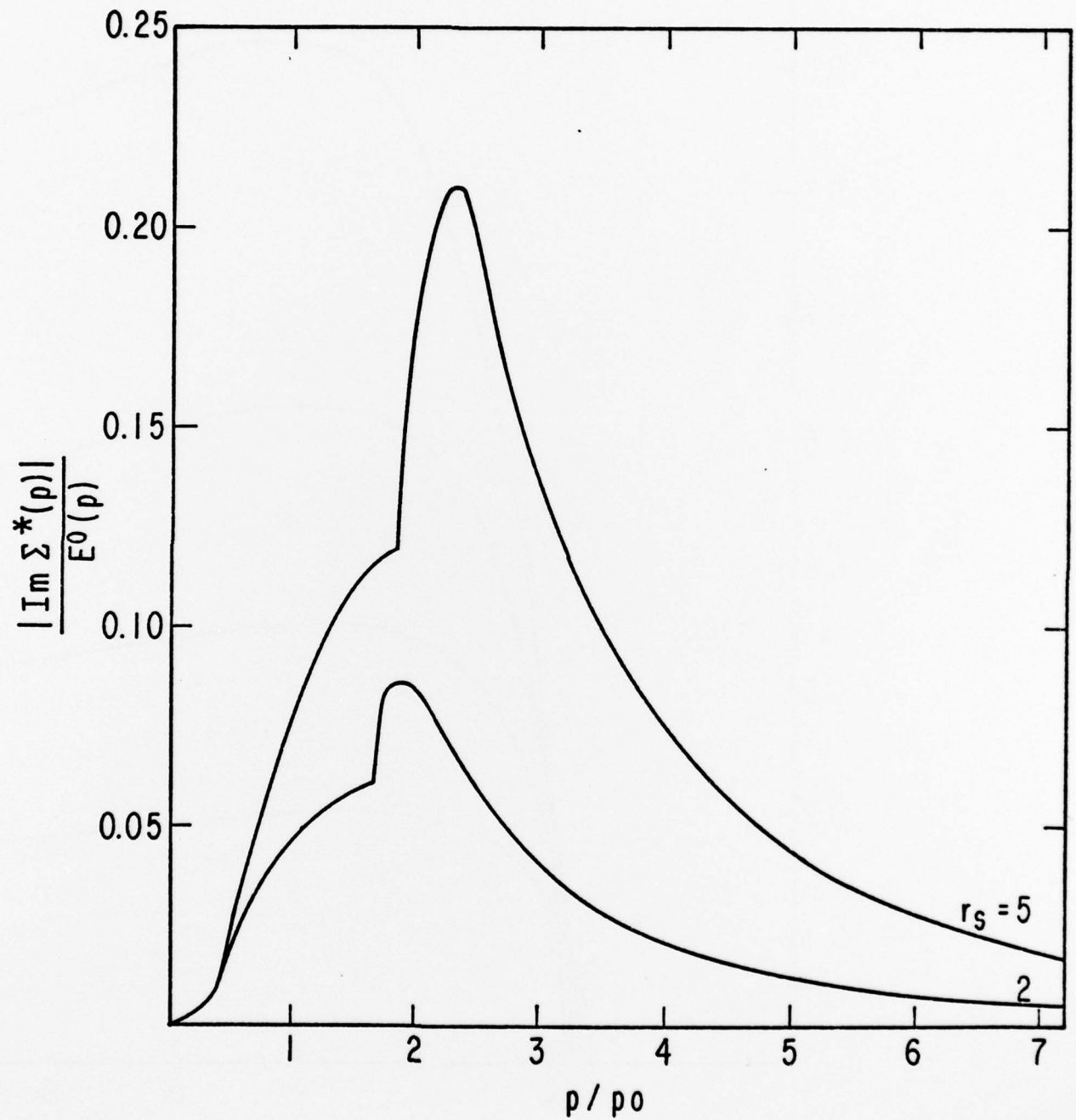


GRAPH 3



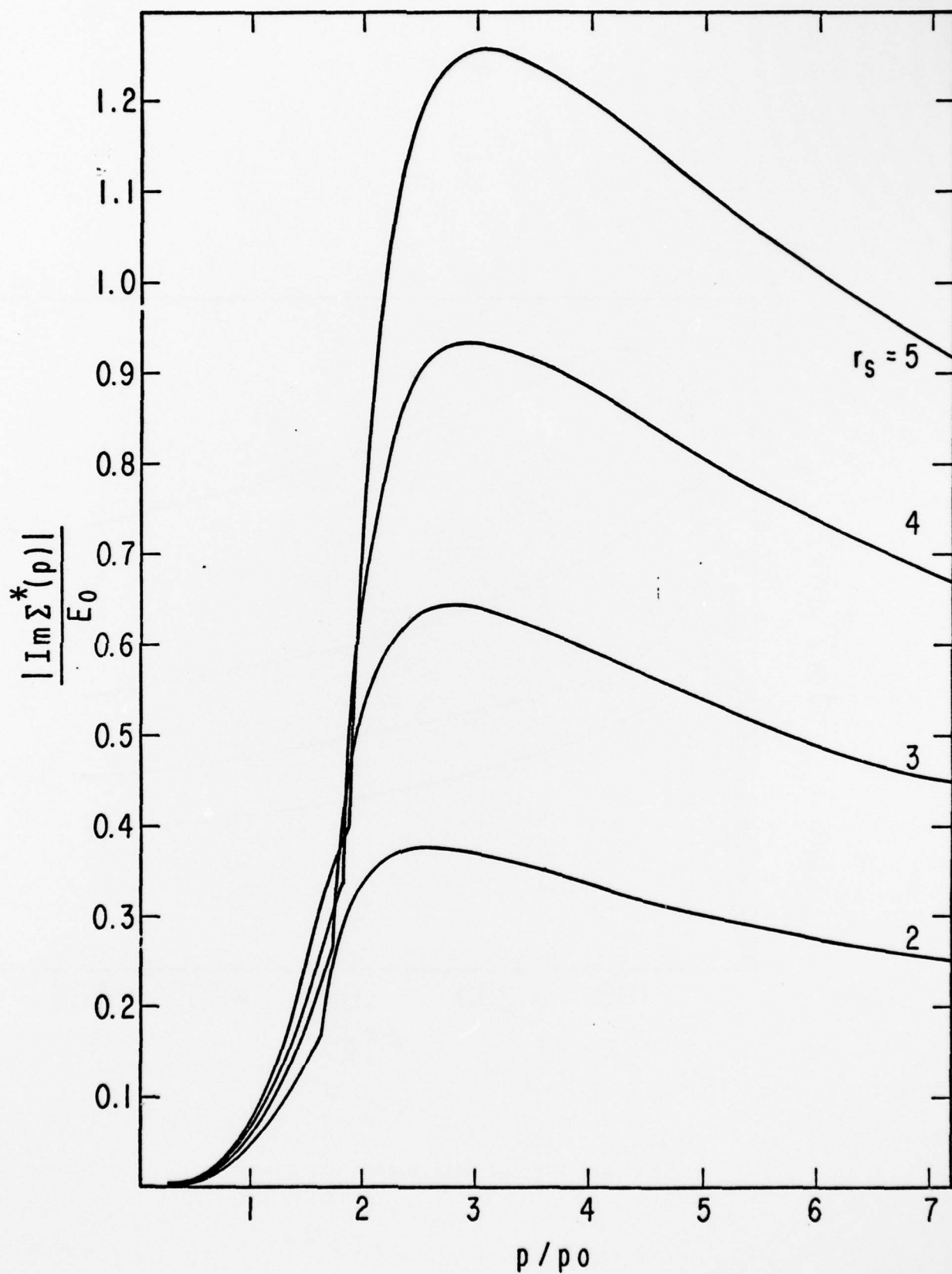
Plasmon Contribution to Imaginary Part of Self Energy vrs Momentum

GRAPH 4



Quasi-Particle Fractional Uncertainty vrs Momentum

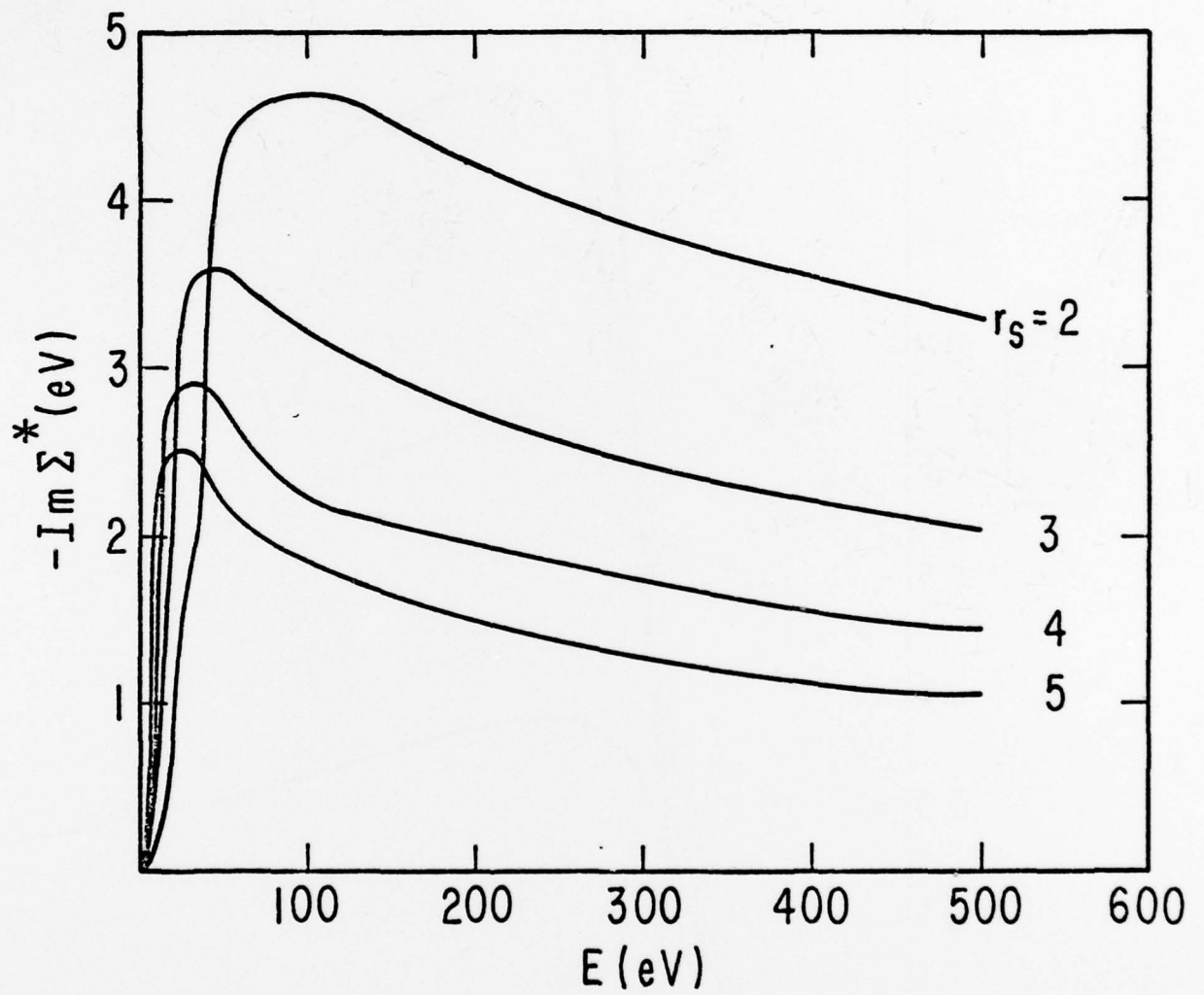
GRAPH 5



Imaginary Part of Self Energy vrs Momentum

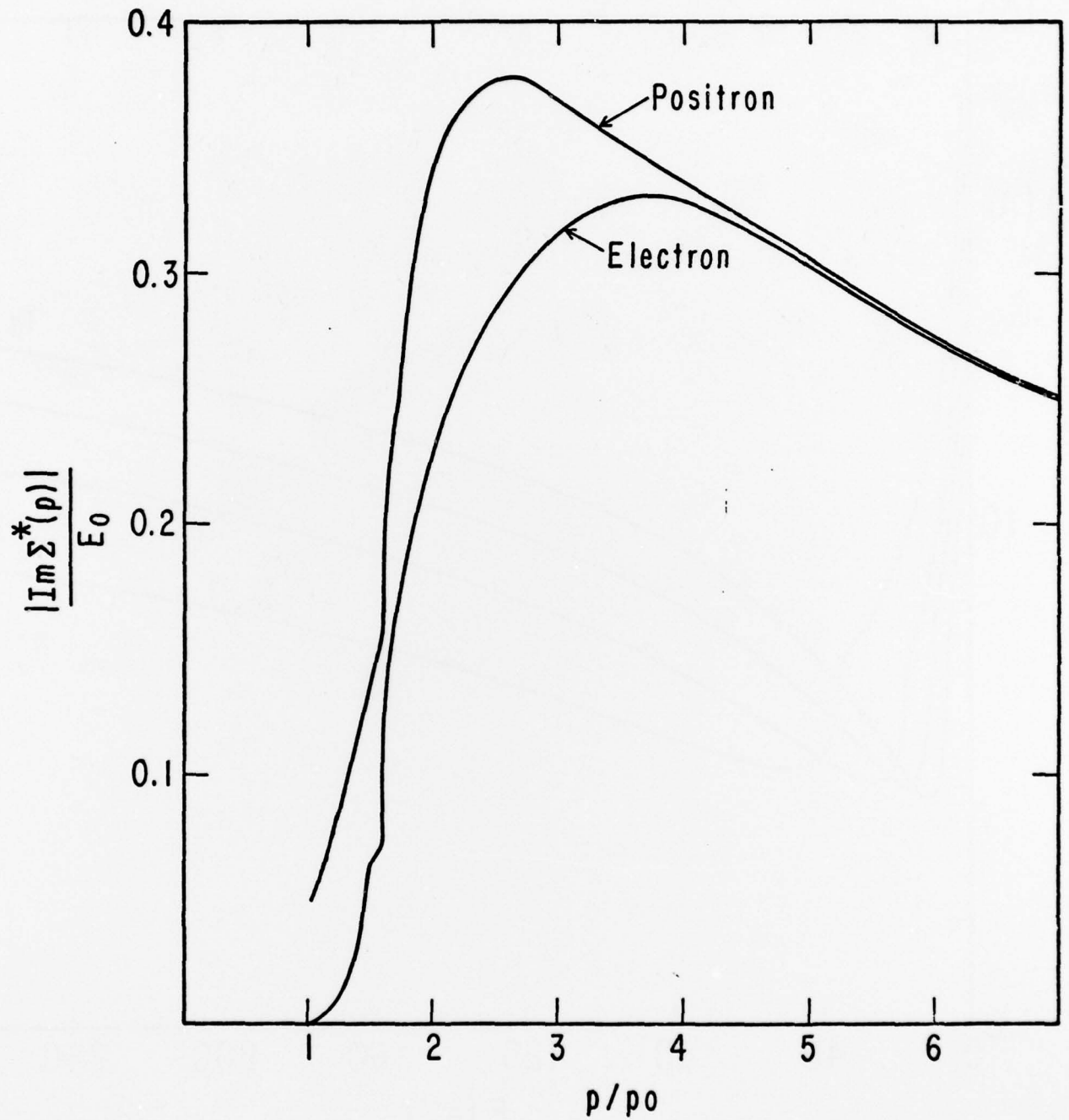


GRAPH 6



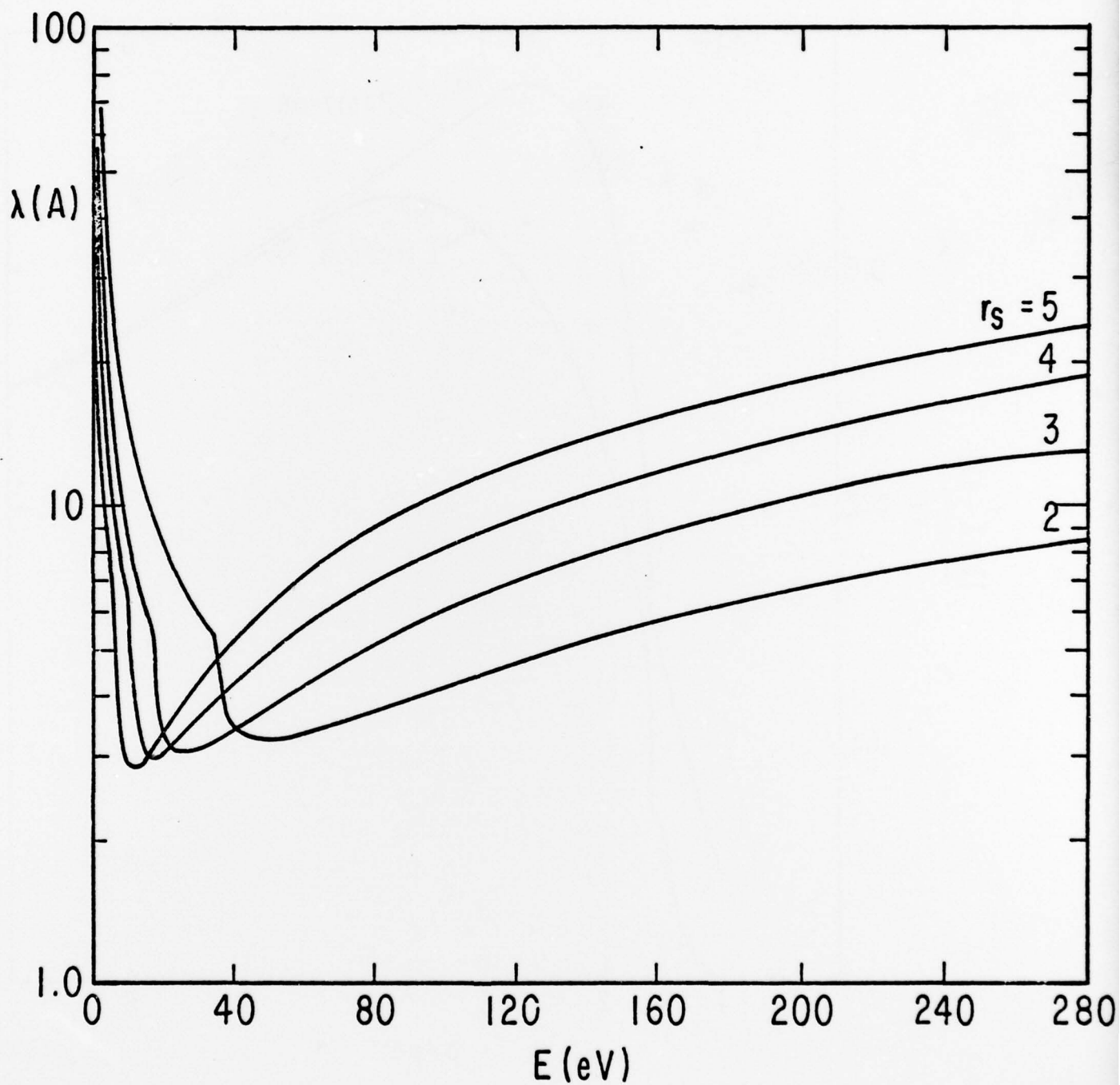
Imaginary Part of Self Energy vrs Energy

GRAPH 7



Comparison Between Imaginary Parts of Electron and Positron Self Energy

GRAPH 8



Mean Free Path vrs Energy

# BIBLIOGRAPHY

1. Bose, Bardasis, Glick, Hone and Longe, Phys. Rev. 155, 379 (1967).
2. Fetter and Walecka, Quantum Theory of Many Particle Systems, McGraw-Hill Book Co. (1971).
3. Lundqvist, Phys. Status Solidi 32, 273 (1969).
4. Penn, Phys. Rev. B13, 5248 (1976).
5. Quinn, Phys. Rev. 126, 1453 (1962).
6. Quinn and Ferrel, Phys. Rev. 112, 812 (1958).