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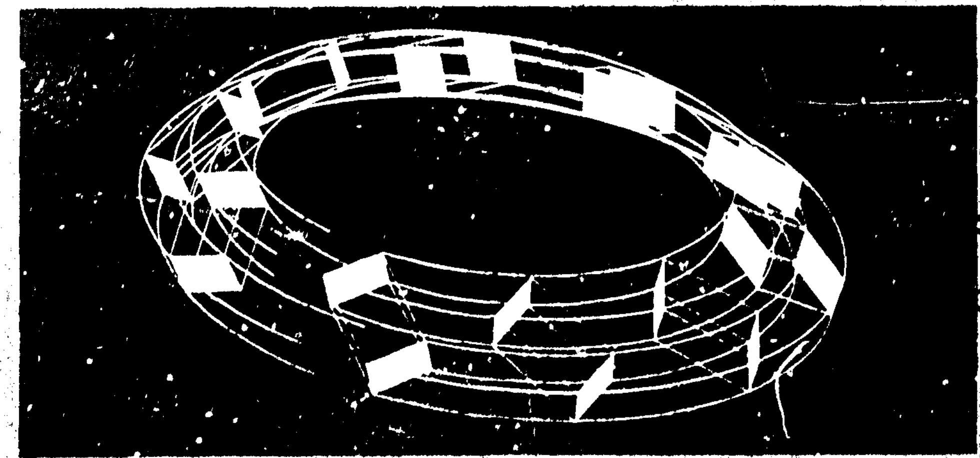

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MULTIBODY DYNAMICS INCLUDING TRANSLATION BETWEEN THE BODIES - WITH APPLICATION TO HEAD-NECK SYSTEMS

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ABSTRACT

This report presents new and recently developed concepts which are useful for obtaining and solving equations of motion of multibody mechanical systems with translation between the respective bodies of the system. These concepts are then applied in the study of human head/neck systems in high acceleration configurations.

The developed concepts include the use of Euler parameters, Lagrange's form of d'Alembert's principle, quasi-coordinates, relative coordinates, and body connection arrays. This leads to the development of efficient computer algorithms for the coefficients of the equations of motion. The developed procedures are applicable to "chain-link" systems such as finite-segment cable models, mechanisms, manipulators, robots, and human body models.

The application with human head/neck models consists of a 54 degree of freedom, three-dimensional system representing the head, the vertebrae, and the connecting discs, muscles, and ligaments. The computer results for the system in a high acceleration configuration agree very closely with available experimental data.

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INTRODUCTION

Recently there has been considerable interest in the development of equations of motion for multi-body mechanical systems--that is, systems containing many rigid bodies. There are two reasons for this interest: First, many complicated mechanical systems and devices such as manipulators, robots, and biosystems, can be effectively modelled by systems of rigid bodies; and second, it has just recently been possible, with the aid of high-speed digital computers, to obtain efficient numerical solutions of the governing dynamical equations. The emphasis of researchers working with multi-body systems has therefore been the formulation of equations of motion which can easily be developed into numerical algorithms for computer codes.

Most of this recent research interest has been with multibody systems consisting of linked rigid bodies - that is, systems of connected rigid bodies such that adjacent bodies share at least one common point and such that no closed loops or circuits are formed. Such systems are sometimes called "general-chain", "open-chain", or "chain-link" systems. Figure 1. depicts such a system. General chain systems are useful for modelling chains, cables, manipulators, teleoperators, antennas, and beams.

There are some systems, however, where the restriction to linked rigid bodies precludes a satisfactory modelling. For example, with a human body model it is frequently advantageous to simulate neck stretch during periods of high acceleration such as in crash environments. Such a simulation is not possible with a fully linked model. Therefore, it is of interest to generalize the multibody models to include translation between the bodies. Figure 2. depicts such a generalization of the system of Figure 1.

This report presents the results of recent research efforts to develop efficient, computer-oriented algorithms for obtaining and solving the governing dynamical equations of motion for these generalized multibody systems. The report also contains a summary of results of the application of these procedures with human head-neck systems in high acceleration configurations.

The balance of the report is divided into six parts with the first part providing a summary of earlier efforts to model multibody systems. This is followed by two parts which contain the general geometrical and kinematical background necessary for the development of the governing equations. The governing equations themselves are developed in the next part, and an application of the developed procedures in studying head-neck dynamics is presented in the subsequent part. The final part contains a summary discussion and suggestions for other applications of the developed procedures.

PREVIOUS MULTIBODY SIMULATION EFFORTS

References [1-36]* provide a summary of approaches taken to obtain efficient, computer-oriented formulation of the equations of motion of multibody systems such as in Figure 1. In one of these approaches [19,29,33], it is shown that it is possible to obtain expressions for the governing equations in a form where the coefficients are easily evaluated through computer algorithms. This approach uses Lagrange's form of d'Alembert's principle, as exposited by Kane and others [37,38,39], together with relative orientation coordinates [40,41,42], to obtain the governing equations. Although this principle is not as widely used as, for example, Newton's laws or Lagrange's equations, it has the advantage of automatic elimination of non-working internal constraint forces without the introduction of tedious differentiation or other calculations.

Recently, it has been suggested by Huston, et.al., [42,43], that further efficiencies in the development and solution of the governing equations could be obtained through the use of Euler parameters as described by Wittaker [44] and Kane and Likins [45], together with the quasi-coordinates suggested by Kane and Wang [46]. Specifically, it is claimed [42,43] that using Euler parameters together with relative angular velocity components as generalized coordinate derivatives allows for the avoidance of geometrical singularities encountered with using Euler angles or dextral orientation angles to define the relative orientation of bodies. (Recall that Euler angles may be defined by aligning mutually perpendicular axes fixed in the bodies and then successively rotating one body relative to the other about the third, first,

*Numbers in brackets refer to references at the end of the report.

and third axes, whereas dextral orientation angles may be defined by successive rotations about the first, second, and third axes.)

PRELIMINARY GEOMETRICAL CONSIDERATIONS

Body Connection Array

Consider a mechanical system such as depicted in Figure 1. To develop an accounting routine for the system's geometry, arbitrarily select one of the bodies as a reference body and call it B_1. Next, number or label the other bodies of the system in ascending progression away from B_1 as shown in Figure 1. Now, although this numbering procedure does not lead to a unique labeling of the bodies, it can nevertheless be used to describe the chain structure or topology through the "body connection array" as follows: Let L(k), k=1,...,N be an array of the adjoining lower numbered body of body B_k. For example, for the system shown in Figure 1., L(k) is:

$$L(k) = (0,1,1,3,1,5,6,7,6) \tag{1}$$

where

$$(k) = (1,2,3,4,5,6,7,8,9) \tag{2}$$

and where 0 refers to an inertial reference frame R. It is not difficult to see that, given L(k), one could readily describe the topology of the system. That is, Figure 1. could be drawn by simply knowing L(k). It is shown in the sequel that L(k) is useful in the development of expressions of kinematical quantities needed for analysis of the system's dynamics.

Transformation Matrices

Next, consider a typical pair of adjoining bodies such as B_j and B_m as shown in Figure 3. The general orientation of B_k relative to B_j may be defined in terms of the relative orientation of the dextral orthogonal unit vector sets $\underset{\sim}{n}_{ji}$ and $\underset{\sim}{n}_{ki}$ (i=1,2,3) fixed in B_j and B_k as shown in Figure 2. Specifically $\underset{\sim}{n}_{ji}$ and $\underset{\sim}{n}_{ki}$ are related to each other as

$$\underset{\sim}{n}_{ji} = SJK_{im}\underset{\sim}{n}_{km} \qquad (3)$$

where SJK is a 3 x 3 orthogonal transformation matrix defined as [47]:

$$SJK_{im} = \underset{\sim}{n}_{ji} \cdot \underset{\sim}{n}_{km} \qquad (4)$$

(Regarding notation, the J and K in SJK and the first subscripts on the unit vectors refer to bodies B_j and B_k, and repeated indices, such as the m, in Equation (3) signify a sum over the range (eg. 1,...,3) of that index. Thus, with a computer SJK_{im} would be the array SJK(I,M).)

From Equation (3), it is easily seen that with three bodies B_j, B_k, B_ℓ, the transformation matrix obeys the following chain and identity rules:

$$SJL = SJK\ SKL \qquad (5)$$

and

$$SJJ = I = SJK\ SKJ = SJK\ SJK^{-1} \tag{6}$$

where I is the identity matrix.

These expressions allow for the transformation of components of vectors referred to one body of the system into components referred to any other body of the system and, in particular, to the inertial reference frame, R. For example, if a typical vector, $\underset{\sim}{V}$, is expressed as

$$\underset{\sim}{V} = V_i^{(k)}\ \underset{\sim}{n}_{ki} = V_i^{(0)}\ \underset{\sim}{n}_{0i} \tag{7}$$

then

$$V_i^{(0)} = SOK_{ij}\ V_j^{(k)} \tag{8}$$

where 0 refers to the inertial frame, R.

Since these transformation matrices play a central role throughout the analysis, it is helpful to also have an algorithm for their derivative, especially the derivative of SOK. Using Equation (3), and noting that $\underset{\sim}{n}_{0i}$ are fixed in R, the following is obtained:

$$d(SOK_{ij})/dt = \underset{\sim}{n}_{0i} \cdot {}^{R}d\ \underset{\sim}{n}_{kj}/dt \tag{9}$$

where the R in ${}^{R}d\,\underset{\sim}{n}_{kj}/dt$ indicates that the derivative is computed in R. However, since the $\underset{\sim}{n}_{kj}$ are fixed in B_k, their derivatives may be written as $\underset{\sim}{\omega}_k \times \underset{\sim}{n}_{kj}$ where $\underset{\sim}{\omega}_k$ is the angular velocity of B_k in R. Equation (9) may then be written as:

$$d(SOK_{ij})/dt = -e_{imn}\omega_{kn}\,\underset{\sim}{n}_{0m} \cdot \underset{\sim}{n}_{kj} \qquad (10)$$

or as

$$d(SOK)/dt = WOK\ SOK \qquad (11)$$

where WOK is a matrix defined as

$$WOK_{im} = -e_{imn}\omega_{kn} \qquad (12)$$

and where ω_{kn} are the components of $\underset{\sim}{\omega}_k$ referred to $\underset{\sim}{n}_{on}$ and e_{imn} is the standard permutation symbol [47,48]. (WOK is simply the matrix whose dual vector [48] is $\underset{\sim}{\omega}_k$.) Equation (11) thus shows that the transformation matrix derivative may be computed by a simple matrix multiplication.

Euler Parameters

Finally, consider describing the relative orientation of B_j and B_k by using the so-called Euler parameters as discussed by Whittaker [44] and Kane and Likins [45]. It is well known [44] that B_k may be brought into any general orientation relative to B_j by means of a single rotation about an appropriate axis. If λ_k is a unit vector along this axis and if θ_k is the rotation angle, the four Euler parameters describing the orientation of B_k relative to B_j may be defined as:

$$
\begin{aligned}
\varepsilon_{k1} &= \lambda_{k1} \quad \sin(\theta_k/2) \\
\varepsilon_{k2} &= \lambda_{k2} \quad \sin(\theta_k/2) \\
\varepsilon_{k3} &= \lambda_{k3} \quad \sin(\theta_k/2) \\
\varepsilon_{k4} &= \cos\,(\theta_k/2)
\end{aligned}
\tag{13}
$$

where the λ_{ki} (i=1,2,3) are the components of λ_k referred to n_{ji}, the unit vector fixed in B_j. Clearly, the ε_{ki} (i=1,2,3,4) are not independent since:

$$\varepsilon_{k1}^2 + \varepsilon_{k2}^2 + \varepsilon_{k3}^2 + \varepsilon_{k4}^2 = 1 \tag{14}$$

These parameters may be related to angular velocity components by using the transformation matricies as follows: It is shown in [44,45] that SJK may be expressed in terms of these parameters as:

$$SJK = \begin{bmatrix} \varepsilon_{k1}^2-\varepsilon_{k2}^2-\varepsilon_{k3}^2+\varepsilon_{k4}^2 & 2(\varepsilon_{k1}\varepsilon_{k2}-\varepsilon_{k3}\varepsilon_{k4}) & 2(\varepsilon_{k1}\varepsilon_{k3}+\varepsilon_{k2}\varepsilon_{k4}) \\ 2(\varepsilon_{k1}\varepsilon_{k2}+\varepsilon_{k3}\varepsilon_{k4}) & -\varepsilon_{k1}^2+\varepsilon_{k2}^2-\varepsilon_{k3}^2+\varepsilon_{k4}^2 & 2(\varepsilon_{k2}\varepsilon_{k3}-\varepsilon_{k1}\varepsilon_{k4}) \\ 2(\varepsilon_{k1}\varepsilon_{k3}-\varepsilon_{k2}\varepsilon_{k4}) & 2(\varepsilon_{k2}\varepsilon_{k3}+\varepsilon_{k1}\varepsilon_{k4}) & -\varepsilon_{k1}^2-\varepsilon_{k2}^2+\varepsilon_{k3}^2+\varepsilon_{k4}^2 \end{bmatrix} \quad (15)$$

Now, by solving Equations (11) and (12) for the angular velocity components, one obtains:

$$\begin{aligned} \omega_{k1} &= SOK_{21}\, S\dot{O}K_{31} + SOK_{22}\, S\dot{O}K_{32} + SOK_{23}\, S\dot{O}K_{33} \\ \omega_{k2} &= SOK_{31}\, S\dot{O}K_{11} + SOK_{32}\, S\dot{O}K_{12} + SOK_{33}\, S\dot{O}K_{13} \\ \omega_{k3} &= SOK_{11}\, S\dot{O}K_{21} + SOK_{12}\, S\dot{O}K_{22} + SOK_{13}\, S\dot{O}K_{23} \end{aligned} \quad (16)$$

where the dot designates time differentiation. By using Equation (15), these expressions may be used to express the $\underset{\sim}{n}_{ji}$ components of the angular velocity of B_k relative to B_j in terms of the Euler parameters as:

$$\begin{aligned} \hat{\omega}_{k1} &= 2(\varepsilon_{k4}\, \dot{\varepsilon}_{k1} - \varepsilon_{k3}\, \dot{\varepsilon}_{k2} + \varepsilon_{k2}\, \dot{\varepsilon}_{k3} - \varepsilon_{k1}\, \dot{\varepsilon}_{k4}) \\ \hat{\omega}_{k2} &= 2(\varepsilon_{k3}\, \dot{\varepsilon}_{k1} + \varepsilon_{k4}\, \dot{\varepsilon}_{k2} - \varepsilon_{k1}\, \dot{\varepsilon}_{k3} - \varepsilon_{k2}\, \dot{\varepsilon}_{k4}) \\ \hat{\omega}_{k3} &= 2(-\varepsilon_{k2}\, \dot{\varepsilon}_{k1} + \varepsilon_{k1}\, \dot{\varepsilon}_{k2} + \varepsilon_{k4}\, \dot{\varepsilon}_{k3} - \varepsilon_{k3}\, \dot{\varepsilon}_{k4}) \end{aligned} \quad (17)$$

(Regarding notation, in the sequel "hats" refer to relative angular velocity vectors or their components. That is, the $\underset{\sim}{\omega}_k$ represent the angular velocity of B_k in R and $\hat{\underset{\sim}{\omega}}_k$ represent the angular velocity of B_k relative to B_j, its adjoining lower numbered body.) Equation (17) may now be solved for the $\hat{\varepsilon}_{ki}$ (i=1,...,4) in terms of the $\hat{\omega}_{ki}$, leading to the expressions:

$$\begin{aligned}
\dot{\varepsilon}_{k1} &= \tfrac{1}{2}(\varepsilon_{k4}\,\hat{\omega}_{k1} + \varepsilon_{k3}\,\hat{\omega}_{k2} - \varepsilon_{k2}\,\hat{\omega}_{k3}) \\
\dot{\varepsilon}_{k2} &= \tfrac{1}{2}(-\varepsilon_{k3}\,\hat{\omega}_{k1} + \varepsilon_{k4}\,\hat{\omega}_{k2} + \varepsilon_{k1}\,\hat{\omega}_{k3}) \\
\dot{\varepsilon}_{k3} &= \tfrac{1}{2}(\varepsilon_{k2}\,\hat{\omega}_{k1} - \varepsilon_{k1}\,\hat{\omega}_{k2} + \varepsilon_{k4}\,\hat{\omega}_{k3}) \\
\dot{\varepsilon}_{k4} &= \tfrac{1}{2}(-\varepsilon_{k1}\,\hat{\omega}_{k1} - \varepsilon_{k2}\,\hat{\omega}_{k2} - \varepsilon_{k3}\,\hat{\omega}_{k3})
\end{aligned} \tag{18}$$

This solution is quickly obtained by observing that if Equation (14) is differentiated and placed with Equation (17), the resulting set of equations could be written in the matrix form:

$$\begin{bmatrix} \hat{\omega}_{k1} \\ \hat{\omega}_{k2} \\ \hat{\omega}_{k3} \\ \hat{\omega}_{k4} \end{bmatrix} = 2 \begin{bmatrix} \varepsilon_{k4} & -\varepsilon_{k3} & \varepsilon_{k2} & -\varepsilon_{k1} \\ \varepsilon_{k3} & \varepsilon_{k4} & -\varepsilon_{k1} & -\varepsilon_{k2} \\ -\varepsilon_{k2} & \varepsilon_{k1} & \varepsilon_{k4} & -\varepsilon_{k3} \\ \varepsilon_{k1} & \varepsilon_{k2} & \varepsilon_{k3} & \varepsilon_{k4} \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}_{k1} \\ \dot{\varepsilon}_{k2} \\ \dot{\varepsilon}_{k3} \\ \dot{\varepsilon}_{k4} \end{bmatrix} \tag{19}$$

where ω_{k4} is equal to the derivative of Equation (14) and has the value zero. The square matrix in Equation (19) is seen to be orthogonal (ie. the inverse is the transpose) and hence, Equations (18) follow immediately from (19) upon letting ω_{k4} be zero.

KINEMATICS

Coordinates

A multibody system of N bodies, with translation permitted between the bodies will, in general, have 6N degrees of freedom. Let these be described by 6N generalized coordinates x_ℓ (ℓ=1,...,6N) and let the first 3N of these be divided into N triplets describing the relative orientation of the successive bodies of the system. Let the remaining 3N x_ℓ also be divided into N triplets representing the relative displacement of the successive bodies of the system. As before, let B_k be a typical body of the system and let B_j be its adjacent lower numbered body, as in Figure 3. The angular velocity of B_k relative to B_j (that is, the relative rate of change of orientation) may then be written as:

$$\hat{\underset{\sim}{\omega}}_k = \hat{\omega}_{k1} \underset{\sim}{n}_{j1} + \hat{\omega}_{k2} \underset{\sim}{n}_{j2} + \hat{\omega}_{k3} \underset{\sim}{n}_{j3} \tag{20}$$

where $\underset{\sim}{n}_{ji}$ (j=1,...,N_j i=1,2,3) are mutually perpendicular dextral unit vectors fixed in B_j. Next, let these bodies be displaced relative to each other with the displacement measured by the vector $\underset{\sim}{\xi}_k$ as shown in Figure 4., where O_j and O_k are arbitrarily selected reference points of B_j and B_k. O_k, which is fixed in B_j, is the connection point or "origin" of B_k. Then $\underset{\sim}{\xi}_k$ may be written in the form:

In general, Equations (23) are non-integrable. That is, they cannot be integrated to obtain generalized orientation coordinates x_{3k-2}, x_{3k-1}, x_{3k}. Thus, explicit parameters x_{3k-2}, x_{3k-1}, and x_{3k} do not in general exist—hence, the name "quasi-coordinates". However, since parameters are needed to relate the relative orientation of the bodies to the respective relative angular velocities, let the Euler parameters introduced in the foregoing section be used for this purpose. Hence, if the orientation of a typical body B_k relative to B_j is described by the four parameters ε_{ki} $(i=1,\ldots,4)$, the geometry and kinematics of the entire system may be expressed in terms of the 4N Euler parameters ε_{ki} $(k=1,\ldots,N;\ i=1,\ldots,4)$, the 3N relative angular velocity components $\hat{\omega}_{ki}$ $(k=1,\ldots,N;\ i=1,2,3)$, and the 3N displacement components ξ_{ki} $(k=1,\ldots,N;\ i=1,2,3)$.

Angular Velocity

The angular velocity of a typical body B_k in the inertial frame R is readily obtained by the addition formula as [38]:

$$\underset{\sim}{\omega}_k = \hat{\underset{\sim}{\omega}}_1 + \ldots + \hat{\underset{\sim}{\omega}}_k \tag{25}$$

where the relative angular velocities on the right side of this expression are each with respect to the respective adjacent lower numbered bodies and where the sum is taken over the bodies of the chain from B_1 outward through the branch containing B_k. The L(k) array introduced in the foregoing section can be useful in computing this sum: Consider for example, the system shown in Figure 1. The angular velocity of B_9 is:

$$\underset{\sim}{\omega}_9 = \hat{\underset{\sim}{\omega}}_1 + \hat{\underset{\sim}{\omega}}_5 + \hat{\underset{\sim}{\omega}}_6 + \hat{\underset{\sim}{\omega}}_9 \tag{26}$$

The subscript indices (ie. 9,6,5,1) may be obtained from L(k) as follows: Consider L(k) as a function mapping the (k) array (See Equation (2)) into the L(k) array. Then, using the notation that $L^0(k) = (k)$, $L^1(k) = L(L^0(k)$, $L^2(k) = L(L^1(k)$, ..., $L^j(k) = L(L^{j-1}(k))$, it is seen (see Equation (1)) that:

$$L^0(9) = 9,\ L^1(9) = 6,\ L^2(9) = 5,\ L^3(9) = 1 \tag{27}$$

Therefore, $\underset{\sim}{\omega}_9$ may be written as:

$$\underset{\sim}{\omega}_9 = \sum_{p=0}^{3} \hat{\underset{\sim}{\omega}}_q \quad , \quad q = L^p(9) \tag{28}$$

Hence, in general, the angular velocity of B_k may be written as:

$$\underset{\sim}{\omega}_k = \sum_{p=0}^{r} \hat{\underset{\sim}{\omega}}_q, \quad q = L^p(k) \tag{29}$$

where r is the index such that $L^r(k) = 1$ and it is obtained by comparing $L^p(k)$ to 1. The index r represents the number of bodies from B_1 to B_k in that branch of the chain system B_k. For example, for the system of Figure 1., if k=9, r=3. Equation (29) is thus an algorithm for determining $\underset{\sim}{\omega}_k$ once $\hat{\underset{\sim}{\omega}}_k$ and L(k) are known.

$$\underset{\sim}{\xi}_k = \xi_{k1}\, \underset{\sim}{n}_{j1} + \xi_{k2}\, \underset{\sim}{n}_{j2} + \xi_{k3}\, \underset{\sim}{n}_{j3} \tag{21}$$

Following Kane and Wang [46], introduce 6N parameters y_ℓ (ℓ=1,...,6N) defined as:

$$y_\ell = \dot{x}_\ell \quad \ell = 1,\ldots, 6N \tag{22}$$

where the first 3N of these are

$$\begin{aligned} y_{3k-2} &= \hat{\omega}_{k1} \\ y_{3k-1} &= \hat{\omega}_{k2} \\ y_{3k} &= \hat{\omega}_{k3} \end{aligned} \tag{23}$$

and the remaining 3N are:

$$\begin{aligned} y_{3(N+k)-2} &= \dot{\xi}_{k1} \\ y_{3(N+k)-1} &= \dot{\xi}_{k2} \\ y_{3(N+k)} &= \dot{\xi}_{k3} \end{aligned} \tag{24}$$

By examining Equations (20, (23), and (25) it is seen that $\underset{\sim}{\omega}_k$ may be written in the form

$$\underset{\sim}{\omega}_k = \omega_{k\ell m}\, y_\ell\, \underset{\sim}{n}_{om} \tag{30}$$

where there is a sum over the repeated indices and where $\omega_{k\ell m}$ ($k=1,\ldots,N$; $\ell=1,\ldots,3N$; $m=1,2,3$) form a block array of coefficients needed to express $\underset{\sim}{\omega}_k$ in terms of $\underset{\sim}{n}_{om}$. In view of Equations (3), (16), (20), and (23), it is seen that the elements of the $\omega_{k\ell m}$ array may be obtained from the SOK transformation matrices. Moreover, it can be shown that the matching between the elements of the $\omega_{k\ell m}$ and SOK arrays is solely dependent upon the body connection array L(k).

To see this, consider for example the angular velocity of B_4 of the system of Figure 1: From Equation (25), $\underset{\sim}{\omega}_4$ is

$$\underset{\sim}{\omega}_4 = \hat{\underset{\sim}{\omega}}_1 + \hat{\underset{\sim}{\omega}}_3 + \hat{\underset{\sim}{\omega}}_4 \tag{31}$$

where from Equations (3), (20), and (23) $\hat{\underset{\sim}{\omega}}_1$, $\hat{\underset{\sim}{\omega}}_3$, and $\hat{\underset{\sim}{\omega}}_4$ may be written as:

$$\hat{\underset{\sim}{\omega}}_1 = y_1 \underset{\sim}{n}_{01} + y_2\, \underset{\sim}{n}_{02} + y_3\, \underset{\sim}{n}_{03} = y_j\, \delta_{mj}\, \underset{\sim}{n}_{0m} \tag{32}$$

$$\hat{\underset{\sim}{\omega}}_3 = y_7\, \underset{\sim}{n}_{11} + y_8\, \underset{\sim}{n}_{12} + y_9\, \underset{\sim}{n}_{13} = y_{6+j}\, SO1_{mh}\, \underset{\sim}{n}_{0m} \tag{33}$$

$$\hat{\underset{\sim}{\omega}}_4 = y_{10}\, \underset{\sim}{n}_{31} + y_{11}\, \underset{\sim}{n}_{32} + y_{12}\, \underset{\sim}{n}_{33} = y_{9+j}\, SO3_{mj}\, \underset{\sim}{n}_{0m} \tag{34}$$

Hence, the $\omega_{4\ell m}$ are:

$$\omega_{4\ell m} = \begin{cases} \delta_{m\ell} & \ell = 1,2,3 \\ 0 & \ell = 4,5,6 \\ S01_{m\ell-9} & \ell = 7,8,9 \\ S03_{m\ell-9} & \ell = 10,11,12 \\ 0 & \ell > 12 \end{cases} \qquad m = 1,2,3 \tag{35}$$

where δ_{ij} are the identity matrix components [47,48].
Next, consider that the results such as Equation (35) may be obtained for the entire system of Figure 1. or Figure 2. from a table such as Table 1., where the "m" entries of the $\omega_{k\ell m}$ array are the column of the transformation matrices. Finally, note that the non-zero entries in a typical row, say the k^{th} row of Table 1. are obtained as follows: Let $P = L(k)$. Then SOP is placed in the k^{th} column of triplets of $\dot{x}_\ell$. Next, let $Q=L(P)$. The SOQ is placed in the P^{th} column to triplets of $\dot{x}_\ell$, etc. That is, SOM is placed in column $L^{j-1}(k)$ where $M = L^j(k)$, $j=1,\ldots,r+1$ with r determined from $L^r(k) = 1$.

Finally, it is interesting to note that the elements of the $\omega_{k\ell m}$ array (and hence, the transformation matrix columns of Table 1.) are components of the "partial rate of change of orientation vectors" as originally defined by Kane [37].

Angular Acceleration

The angular acceleration of B_k in R may be obtained by differentiating Equation (30). Noting that the $\underset{\sim}{n}_{om}$ are constant, this leads to:

$$\underset{\sim}{\alpha}_k = (\dot{\omega}_{k\ell m}\, y + \omega_{k\ell m}\, \dot{y}_\ell)\, \underset{\sim}{n}_{om} \tag{36}$$

A table containing the $\dot{\omega}_{k\ell m}$ can be constructed directly form the corresponding table for the $\omega_{k\ell m}$. For example, for the system of Figure 1., such a table is shown in Table 2.

Mass Center Velocities

The velocity and acceleration of the mass center G_k of a typical body B_k (k=1,...,N) may be obtained as follows: Let $\underset{\sim}{r}_k$ locate G_k relative 0_k as shown in Figure 4. Since 0_k is located relative to Q_k by $\underset{\sim}{\xi}_k$ and if Q_k is lccated relative to 0_j by the vector $\underset{\sim}{q}_k$ (See Figure 4.), then by continuing this procedure, G_k may ultimately be located relative to a fixed point 0 in R, the inertial reference frame. For example, for Body B_8 of Figure 2., the position vector $\underset{\sim}{P}_8$ of G_8 relative to 0 is:

$$\underset{\sim}{P}_8 = \underset{\sim}{\xi}_1 + \underset{\sim}{q}_5 + \underset{\sim}{\xi}_5 + \underset{\sim}{q}_6 + \underset{\sim}{\xi}_6 + \underset{\sim}{q}_7 + \underset{\sim}{\xi}_7 + \underset{\sim}{q}_8 + \underset{\sim}{\xi}_8 + \underset{\sim}{r}_8 \tag{37}$$

In general, for Body B_k, the position vector $\underset{\sim}{P}_k$ of B_k relative to 0 is:

$$\underset{\sim}{P}_k = [SOK_{ih}\, r_{kh} + \sum_{q=0}^{u} SOS_{ih}\,(q_{sh} + \xi_{sh})]\underset{\sim}{n}_{oi} \tag{38}$$

where $s = L^q(k)$, $S = L^{q+1}(k)$, and u is the index such that $L^u(k) = 1$, and where $\underset{\sim}{q}_1$ is 0. By differentiating, the velocity of G_k in R is obtained as:

$$\underset{\sim}{v}_k = \{\dot{SOK}_{ih}\, r_{kh} + \sum_{q=0}^{u} [\dot{SOS}_{ih}\,(q_{sh} + \xi_{sh})$$

$$+ SOS_{ih}\,\dot{\xi}_{sh}]\,\underset{\sim}{n}_{oi}\} \tag{39}$$

By using Equations (11), (12), and (30), $\underset{\sim}{v}_k$ may be written in the form:

$$\underset{\sim}{v}_k = v_{k\ell m}\, y_\ell\, \underset{\sim}{n}_{om} \tag{40}$$

where $v_{k\ell m}$ ($k=1,\ldots,N$; $\ell=1,\ldots,6N$; $m=1,2,3$) form a block array of coefficients needed to express $\underset{\sim}{v}_k$ in terms of $\underset{\sim}{n}_{om}$. In view of Equation (39), the non-zero $v_{k\ell m}$ are:

$$v_{k\ell m} = WK_{mh\ell}\, r_{kh} + \sum_{q=0}^{u} WS_{mh\ell}\,(\xi_{sh} + q_{sh})$$

$$(k=1,\ldots,N;\ \ell=1,\ldots,3N;\ m=1,2,3) \tag{41}$$

where WK_{mh} is defined as:

$$WK_{mh\ell} = \frac{\partial WOK_{mp}}{\partial 2y_\ell} SOK_{ph} = -e_{mpi}\ \omega_{k\ell i}\ SOK_{ph} \qquad (42)$$

and

$$v_{k(3N+\ell)m} = \omega_{k\ell m} \quad (k=1,\ldots,N;\ \ell=1,\ldots,3N;\ m=1,2,3)\ . \qquad (43)$$

Mass Center Accelerations

Similarly, by differentiation of Equations (40), the acceleration of G_k in R is

$$\underset{\sim}{a}_k = (\dot{v}_{k\ell m}\ y_\ell + v_{k\ell m}\ \dot{y}_\ell)\underset{\sim}{n}_{om} \qquad (44)$$

where the non-zero $\dot{v}_{k\ell m}$ are, by Equations (41) to (43),

$$\dot{v}_{k\ell m} = \dot{WK}_{mh\ell}\ r_{kh} + \sum_{q=0}^{u-1} [\dot{WS}_{mh\ell}(\xi_{sh} \quad (k=1,\ldots,N;\ \ell=1,\ldots,3N,\ m=1,2,3)$$

$$+ q_{sh} \quad + WS_{mh\ell}\ \dot{\xi}_{sh}] \qquad (45)$$

where $\dot{WK}_{mh\ell}$ is:

$$\dot{WK}_{mh\ell} = -e_{mpi}\ (\dot{\omega}_{k\ell i}\ SOK_{ph} + \omega_{k\ell i}\ \dot{SOK}_{ph}) \qquad (46)$$

and

$$\dot{v}_{k}(3_{N}+\ell)_{m} = \dot{\omega}_{k\ell m} \quad (k=1,\ldots,N;\ \ell=1,\ldots,3N,\ m=1,2,3) \tag{47}$$

EQUATIONS OF MOTION

Consider again a general chain system such as shown in Figure 2., and imagine the system to be subjected to an externally applied force field. Let the force field on a typical body B_k, be replaced by an equivalent force field consisting of a single force $\underset{\sim}{F}_k$, passing through G_k together with a couple with torque $\underset{\sim}{M}_k$. Then Lagrange's form of d'Alembert's principle leads to governing dynamical equations of motion of the form [38]:

$$F_\ell + F_\ell^* = 0 \quad \ell = 1,\ldots,6N \tag{48}$$

F_ℓ (ℓ=1,...,6N) is called the generalized active force and is given by:

$$F_\ell = v_{k\ell m} F_{km} + \omega_{k\ell m} M_{km} \tag{49}$$

where there is a sum from 1 to N on k and from 1 to 3 on m, and where F_{mk} and $\underset{\sim}{M}_{km}$ are the components of $\underset{\sim}{F}_k$ and $\underset{\sim}{M}_k$ with respect to $\underset{\sim}{n}_{om}$. F_ℓ^* (ℓ=1,...,6N) is called the generalized inertia force and is given by:

$$F_\ell^* = V_{k\ell m} F_{km}^* + \omega_{k\ell m} M_{km}^* \tag{50}$$

where the indices follow the same rules as in Equation (48), and where F_{km}^* and M_{km}^* are $\underset{\sim}{n}_{om}$ components of inertia forces, $\underset{\sim}{F}_k^*$, and inertia torques, $\underset{\sim}{M}_k^*$, given by [38]:

$$\underset{\sim}{F}_k^* = -m_k \underset{\sim}{a}_k \quad \text{(no sum)} \tag{51}$$

and

$$\underset{\sim}{M}_k^* = -\underset{\sim}{I}_k \cdot \underset{\sim}{\alpha}_k - \underset{\sim}{\omega}_k \times (\underset{\sim}{I}_k \cdot \underset{\sim}{\omega}_k) \quad \text{(no sum)} \tag{52}$$

where m_k is the mass of B_k and $\underset{\sim}{I}_k$ is the inertia dyadic of B_k relative to G_k $(k=1,\ldots,N)$. ($\underset{\sim}{F}_k^*$, with line of action passing through G_k together with $\underset{\sim}{M}_k^*$ are equivalent to the inertia forces on B_k [38].) Through use of the shifter transformation matrices, $\underset{\sim}{I}_k$ may be written in the form:

$$\underset{\sim}{I}_k = I_{kmn} \underset{\sim}{n}_{om} \underset{\sim}{n}_{on} \tag{53}$$

By substituting Equations (36) and (44) into Equations (51) and (52) and ultimately into Equation (47), the equations of motion may be written in the form:

$$a_{\ell p} \dot{y}_p = f_\ell \quad (\ell=1,\ldots,6N) \tag{54}$$

where there is a sum from 1 to 6N on p and where $a_{\ell p}$ and f_ℓ are given by:

$$a_{\ell p} = m_k v_{kpm} v_{k\ell m} + I_{kmn} \omega_{kpm} \omega_{k\ell n} \tag{55}$$

and

$$f_\ell = -(F_\ell + m_k v_{k\ell m} \dot{v}_{kqm} y_q + I_{kmn} \omega_{k\ell m} \dot{\omega}_{kqn} y_q$$

$$+ e_{nmh} I_{kmr} \omega_{kqn} \omega_{ksr} \omega_{k\ell n} y_q y_s) \tag{56}$$

where there is a sum from 1 to N on k, from 1 to 6N on q and s, and from 1 to 3 on the other repeated indices.

Recall that the first 3N y_p are relative angular velocity components. These may be related to the Euler parameters by N sets of first order equations of the form of Equations (18).

Equations (54), (20), and the 4N equations of the form of Equations (18) form a set of 13N simultaneous first-order differential equations for the 6N y_p, the 3N ξ_{ki}, and the 4N Euler parameters ε_{ki} (h=1,...,N; i=1,...,4). Since the coefficients $a_{\ell p}$ and f_ℓ in Equations (54) are algebraic functions of the physical parameters and the four block arrays $\omega_{k\ell m}$, $\dot{\omega}_{k\ell m}$, $v_{k\ell m}$ and $\dot{v}_{k\ell m}$, computer algorithms can be written for the numerical development of these governing equations. Moreover, once these arrays are developed, the system of equations consisting of Equations (54), (20), and 4N equations of the form of Equations (18), may also be solved numerically by using one of the standard numerical integration routines and a linear equation solver.

The development of these computer algorithms and the numerical development of Equations (54) might proceed as follows: First, let the body connection array L(k) (See Equation (1)) together with the geometrical and physical parameters $\underset{\sim}{r}_k$, $\underset{\sim}{\xi}_k$, $\underset{\sim}{I}_k$, and m_k (See Equations (38), (51), and (52).) and the applied forces and moments $\underset{\sim}{F}_k$ and $\underset{\sim}{M}_k$ (See Equation (48).) be read into the computer. (Let $\underset{\sim}{r}_k$, $\underset{\sim}{\xi}_k$, $\underset{\sim}{I}_k$ and, if desired, $\underset{\sim}{F}_k$ and $\underset{\sim}{M}_k$ be expressed in terms of $\underset{\sim}{n}_{ki}$.) Next, from assumed initial values of ε_{ki} form the transformation matrix arrays SOK using Equations (15) and (5). Use these arrays to express $\underset{\sim}{r}_k$, $\underset{\sim}{\xi}_k$, $\underset{\sim}{I}_k$ and possibly $\underset{\sim}{F}_k$ and $\underset{\sim}{M}_k$ in terms of $\underset{\sim}{n}_{ok}$. Next, using L(k) and SOK write an algorithm, with Tables 1. and 2. as a guide, to form $\omega_{k\ell m}$ and $\dot{\omega}_{k\ell m}$. For example, to obtain the non-zero $\omega_{k\ell m}$, observe that if L(k) = p, then $\omega_{k\ell m} = SOP_{m\ell}$ (m=1,2,3; ℓ=3p+1, 3p+2, 3p+3). Then, if L(p) = q, $L^2(k)$ = q and $\omega_{k\ell m} = SOQ_m$ (m=1,2,3; ℓ=3q+1, 3q+2, 3q+3). This assignment procedure is continued until unity is reached or r times where r is given by $L^r(k) = 1$ (See the remark following Equation (29).). $v_{k\ell m}$ and $\dot{v}_{k\ell m}$ may then be obtained using Equations (40) to (47). Finally, numerical values of the coefficients $a_{\ell p}$ and f_ℓ of the governing differential equations (54) may then be obtained from Equations (55) and (56). These equations may then be integrated numerically to obtain incremental values to the initial values of the parameters y_p, ε_{ki}, and x_q (p=1,...,3N+3; k=1,...,N; i=1,2,3,4; and q=1,2,3), at the end of a time interval, say t_1. New values of the transformation matrix arrays SOK may then be obtained and the entire process repeated until a history of the configuration and motion of the system is determined.

Specific computer algorithms following this general procedure have been written and validated. A listing together with a tape copy (or card deck) are available at reproduction cost from the authors.

APPLICATION WITH HEAD-NECK SYSTEMS

Previous Simulation Efforts

Recently, there has been considerable interest in using the foregoing procedures in the modelling of biodynamic systems. Specifically, there has been interest in modelling the human body - and particularly, head-neck systems - during periods of high acceleration, as experienced in vehicle accidents. This interest stems from the fact that accident injuries, including both direct and indirect (for example, "whiplash") impact, are basically mechanical phenomena. The emphasis on modelling the head-neck system is stimulated by the belief that as many as 60 - 70% of vehicle related accident fatalities are a direct result of injuries to the head-neck system.

There are a number of head-neck simulation models discussed in the technical literature. Specifically, in 1971, Orne and Liu [60] developed a discrete-parameter spine model which simultaneously accounts for axial, shear, and bending deformation of the discs, for the variable size and mass of the vertebrae and discs, and for the natural curvature of the spine. They also present an extensive literature review of spine models prior to 1970. Later in 1971, McKenzie and Williams [61] used the Orne-Liu model to develop a two-dimensional discrete-parameter head-neck-torso model for "whiplash" investigation. A two-dimensional mechanical linkage model simulating head-neck response to frontal impact has been presented by Becker [62]. This model allows for elongation of the neck. It concentrates the mass at the head mass center. Springs and dampers are used to control the elongation of the model. A three-dimensional neck-torso linkage vehicle-occupant model has been developed by Bowman and Robbins [63]. The model has two ball-and-socket joints and the neck can elongate with the motion limited by joint stopping moments.

In addition to these computer models, there have also been developed a number of anthropometric dummy models. (These are currently used extensively by the automotive industry.) In 1972, Melvin, et.al. [64] presented a mechanical neck for authropometric dummies. The neck consists of three steel universal joints pinned into aluminum discs with shaped rubber discs around the joints. The joints allow the neck to move in flexion, extension, and lateral flexion but do not allow for either rotation or elongation. A mechanical neck has also been presented by Culver, et.al. [65]. It consists of four ball-joint segments and one pin-connected "nodding" segment. Viscoelastic resistive elements inserted between the segments provide for bending resistance and energy dissipation with the primary objective being to model flexion and extension responses.

In this part of the report, there is presented, as an application of the foregoing procedures, a comprehensive, three-dimensional, head-neck computer model which has 54 degrees of freedom and includes the effects of discs, muscles, and ligaments. The model is developed by considering the skull and vertebrae as a chain system of rigid bodies which may translate relative to one another. The soft tissue effects of the discs, muscles, and ligaments are modelled by nonlinear springs and dampers between the bodies. The model is based primarily on the research of J. Huston and Advani [55,56,57].

The balance of this part of the report contains a description of the modelling itself and the development of the governing dynamical equations of motion. This is followed by a comparison of results from numerical integration of these equations, with available experimental data.

Head-Neck Modelling

A comprehensive presentation of the head-neck anatomy may be found in references [66-73]. The anatomy is conveniently divided into two categories: bones and soft tissue.

Bones

The largest and heaviest is the skull which consists of a large cranial cavity (enclosing the brain) and smaller bones of the face and jaw. The skull is actually composed of 21 closely fitted bones. The other bones of the head-neck system are seven cervical vertebrae (C1-C7) which support and provide mobility to the head. The first of these C1, called the "atlas", supports the skull. The second C2, called the "axis", is distinctive because of its adontoid process (or axis) which rises perpendicularly to the vertebrae. The five remaining cervical vertebrae are roughly annular in shape and are similar to each other with a slight increase in size going down from C3 to C7.

Soft Tissue

The soft tissue is composed primarily of the discs, the muscles, the ligaments, and the brain. The discs provide the cushioning or separation for the vertebrae. They are annular in shape. The ligaments connect the cervical vertebrae to each other and thus allow for the gross and fine movement of the head and neck. The muscles control the movement of the head and neck which may be classified grossly as: flexion, extension, and rotation. The muscles originate on the various cervical vertebrae, the skull, the spine, and the shoulder bones. The brain tissue is basically four mass volumes composed of two cerebral hemispheres in the upper half of the skull, the triangular shaped cerebellum in the lower posterior and the brain stem in the center of the skull.

Modelling

The head-neck system is modelled by a system of 9 rigid bodies representing the skull, vertebrae, and torso as shown in Figure 5. and springs and dampers representing the discs, ligaments, and muscles. The masses, inertia matrices, and overall geometry of the rigid bodies are adjusted to match the actual human values [70]. Each body has 6 degrees of freedom and hence, the entire system has a total of 54 degrees of freedom.

Following Orne and Liu [60] the discs are modelled in the axial direction as two-parameter viscoelastic solids with the uniaxial force-displacement relationship being:

$$F = (A/h)(d_1\delta + d_2\dot{\delta}) \tag{57}$$

In bending and shear the discs are modelled as linear elastic solids. Using the principles of strength of materials theory [70], the following force and moment equations are developed:

$$F_x = \left(\frac{6EI_1}{h^2}\right)\left(\frac{2\delta_x}{h} - \theta_y\right)/P_1 \quad (58)$$

$$F_y = \left(\frac{6EI_2}{h^2}\right)\left(\frac{2\delta_y}{h} + \theta_x\right)/P_2 \quad (59)$$

$$F_z = \left(\frac{A}{h}\right)(d_1\ \delta_z + d_2\delta_z) \quad (60)$$

$$M_x = \left(\frac{EI_2}{hP_2}\right)\left[\frac{6\delta_y}{h} + (P_2+3)\ \theta_x\right] \quad (61)$$

$$M_y = \left(\frac{EI_1}{hP_1}\right)\left[\frac{-6\delta_x}{h} + (P_1+3)\ \theta_y\right] \quad (62)$$

$$M_z = JG\theta_z/h \quad (63)$$

where P_1 and P_2 are:

$$P_1 = 1 + \frac{12EI_1k_x}{GAh^2} \quad (64)$$

and

$$P_2 = 1 + \frac{12EI_1k_y}{GAh^2} \quad (65)$$

where as shown in Figure 5., Z is in the axial (up direction, X is forward and Y is to the left.

The ligaments are modelled as non-linear elastic bands capable of exerting force only in tension. The force-displacement relation is taken as:

$$F = \ell_1 \delta + \ell_2 \delta^2 \tag{66}$$

The muscles are modelled as two-parameter, visco-elastic solids, which, like the ligaments, only exert force when in tension. The force-displacement relation is taken as:

$$F = m_1 \delta + m_2 \delta \tag{67}$$

The joint constraints (limiting the relative motion of the bodies) are modelled as one-way dampers. The force-displacement and moment-rotation relation are taken as:

$$F = \begin{cases} -C\dot{\delta} & \text{for } \dot{\delta} > 0 \\ 0 & \text{for } \dot{\delta} < 0 \end{cases}$$

and (68)

$$M = \begin{cases} -C\dot{\theta} & \text{for } \dot{\theta} > 0 \\ 0 & \text{for } \dot{\theta} < 0 \end{cases}$$

where the damping constant is

$$C = \begin{cases} C_o + C_1(X - X\text{max}) & \text{for } X > X\text{max} \\ C_o & \text{for } X\text{min} < X < X\text{max} \\ C_o + C_1(X - X\text{min}) & \text{for } X < X\text{min} \end{cases} \tag{69}$$

where X,Xmax, and Xmin are the values of the displacement or rotations variable and its corresponding maximum and minimum values.

The values of these various constants for the discs, ligaments, muscles, and joints for the various directions and motion are difficult to specify precisely due to a lack of experimental data. However, the values for the discs may be obtained from Markold and Steidal [74], Orne and Liu [60], and McKenzie and Williams [6]]. The ligament and muscle attach points may be obtained from Francis [75], Lanier [76], and Todd and Lindala [77], with the spring and viscoelastic constants obtained from Nunley [78] and Close [79].

Governing Equations

The procedures developed in the foregoing parts of the report are directly applicable to the model of Figure 5. including the simulated disc, muscle, and ligament forces. Specifically, as noted earlier, the model has 54 degrees of freedom (27 translation and 27 rotation). This leads to a system of 117 simultaneous first-order differential equations of the form of Equations (18), (20), and (54). The disc, muscle, and ligament forces are included in the generalized active forces F_ℓ of Equation (56).

Comparison with Experimental Data

It is difficult to obtain experimental data which is suitable for checking the model. This is due to the expense and impracticality of using dummies, cadavers, or animal surrogates and due to the limited experimental range with human volunteers. However, several experiments have been conducted which may be used to obtain a validation of the model. In one of these, a seated cadaver was subjected to head impacts by a rigid pendulum. Accelerometers were used to measure the resultant frontal and occipital head impact forces and accelerations. Using the impact force data as input, the acceleration was calculated using the computer model. A comparison of the results for two of the frontal impact experiments, 6-2 and 6-5 is shown in Figures 6.-9.

In the same set of experiments, high-speed cameras were used to measure the acceleration, velocity, and displacement of the mass center. A comparison of the results with those predicted by the computer model for experiments 6-1 and 6-2 are shown in Figure 10.

Finally, the model was checked against live human data generated by Ewing and Thomas [33] using elaborate testing facilities. A comparison of the results for the head angular acceleration, angular velocity, and angular displacement is shown in Figures 11., 12., and 13.

DISCUSSION AND CONCLUSIONS

The results of using the modelling procedures outlined herein and numerically integrating the resulting governing differential equations (54) for a number of other physical systems (in addition to head-neck systems) are reported and discussed in References [40,41,49,50,51,52,53].

The application of Equations (54) with these systems, however, is based on the use of relative orientation angles between the respective bodies of the system as the generalized coordinates (x_ℓ) as opposed to the use of Euler parameters and quasi-coordinates as outlined in the foregoing sections. A problem which arises in the numerical solution of Equations (54) where orientation angles are used is that there always exists values of the angles and hence, configurations of the system, for which the determinant of $a_{k\ell}$ is zero. A numerical solution will, of course, fail to converge at these singular configurations of the system, and convergence is very slow for configurations in the vicinity of a singularity. This problem is avoided by using Euler parameters to relate the orientation geometry to the angular velocity.

The advantages of using Lagrange's form of d'Alembert's principle to obtain the governing equations of motion for multi-body mechanical systems has been exposited in detail in References [29] and [39]. Basically, this principle has the advantages of Lagrange's equations or of virtual work in that non-working internal constraint forces, between the bodies of the system, are automatically eliminated from the analysis, and may therefore be ignored in the formulation of the governing equations. The principle, however, has the additional advantage of avoiding the differentiation of scalar energy functions. Indeed, the differentiation required to obtain velocities and accelerations are performed by vector cross products and multiplication algorithms — procedures which are ideally suited for numerical computation. As with Lagrange's equations, Lagrange's form of d'Alembert's principle requires the use of generalized coordinates to define the system geometry. The use of Euler parameters to avoid problems with singularities, as discussed above, leads naturally to the use of relative angular velocity components as the generalized coordinate derivatives. This in turn leads to additional computational advantages as observed by Kane and Wang [46] and Likins [54]. Specifically, by using relative angular velocity components as the principle parameters of the analysis, the coefficient matrices in the governing equations can be obtained directly from the body connection array L(k) (See Tables 1. and 2.).

The use of "relative" coordinates, that is, angular velocity components of the bodies with respect to their adjoining bodies, as opposed to "absolute" coordinates, that is, angular velocity components in inertial space, also contributes to the computational advantage. In applications with specific geometrical configurations [40,41,49-53], it is seen that the geometry is more easily described in terms of relative coordinates.

Finally, the generalization to allow translation between the bodies of the system makes the analysis applicable to a much broader class of problems than was possible with those previous analyses which are restricted to <u>linked</u> multibody systems. For example, with the head-neck system, the use of translation variables between the vertebrae is necessary to obtain a satisfactory model of the system. Moreover, this generalization to include translation is a natural extension of the analyses of [33,42,49,50,51].

Regarding the application to the head-neck system, Figures 7. - 13. show there is agreement between the experimental results and those predicted by the computer model. This is indeed encouraging and it suggests that this head-neck model represents one of the most sophisticated models available. However, more testing and refining needs to be done. Specifically, the three-dimensional features of the model need to be further checked with experimental data. Also, better experimental values for the soft tissue mechanical properties need to be obtained. Finally, the effect of muscle time delay needs to be incorporated into the model.

Beyond this, as injury criteria becomes better established, the model can serve as an effective and economical tool for predicting injury in a variety of high-acceleration/high-accident configuration environments. It could then be used for the development and design of safety and restraining devices.

Finally, the entire analysis and the procedures outlined in this report are developed with the intent of obtaining efficiencies in a computer or numerically oriented development and solution of the governing dynamical equations of large multibody systems. As such, its most productive application is likely to be with systems such as finite-segment biodynamic models, chains, cables, robots, manipulators, teleoperators, etc.

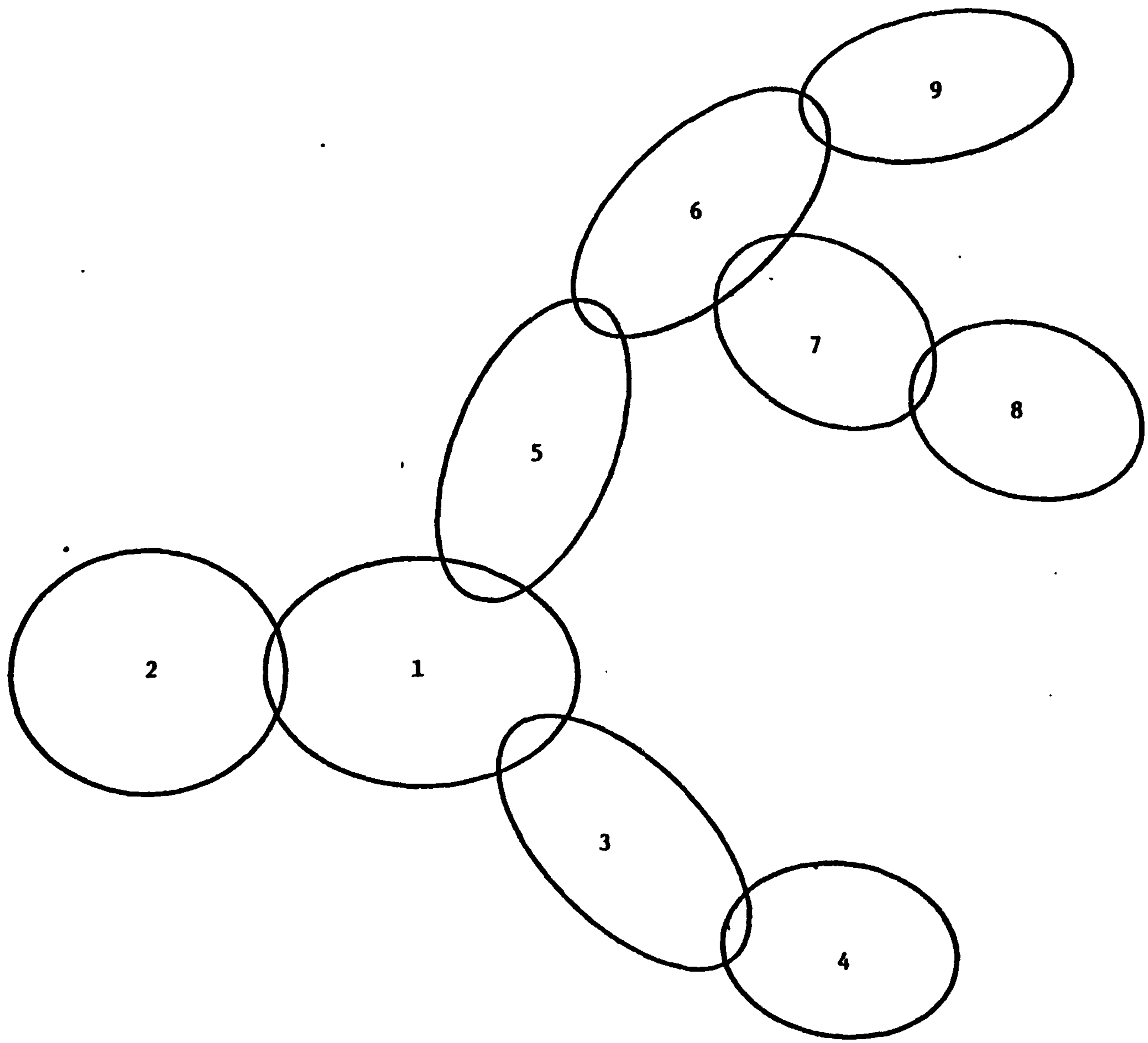

Figure 1. A General Chain System

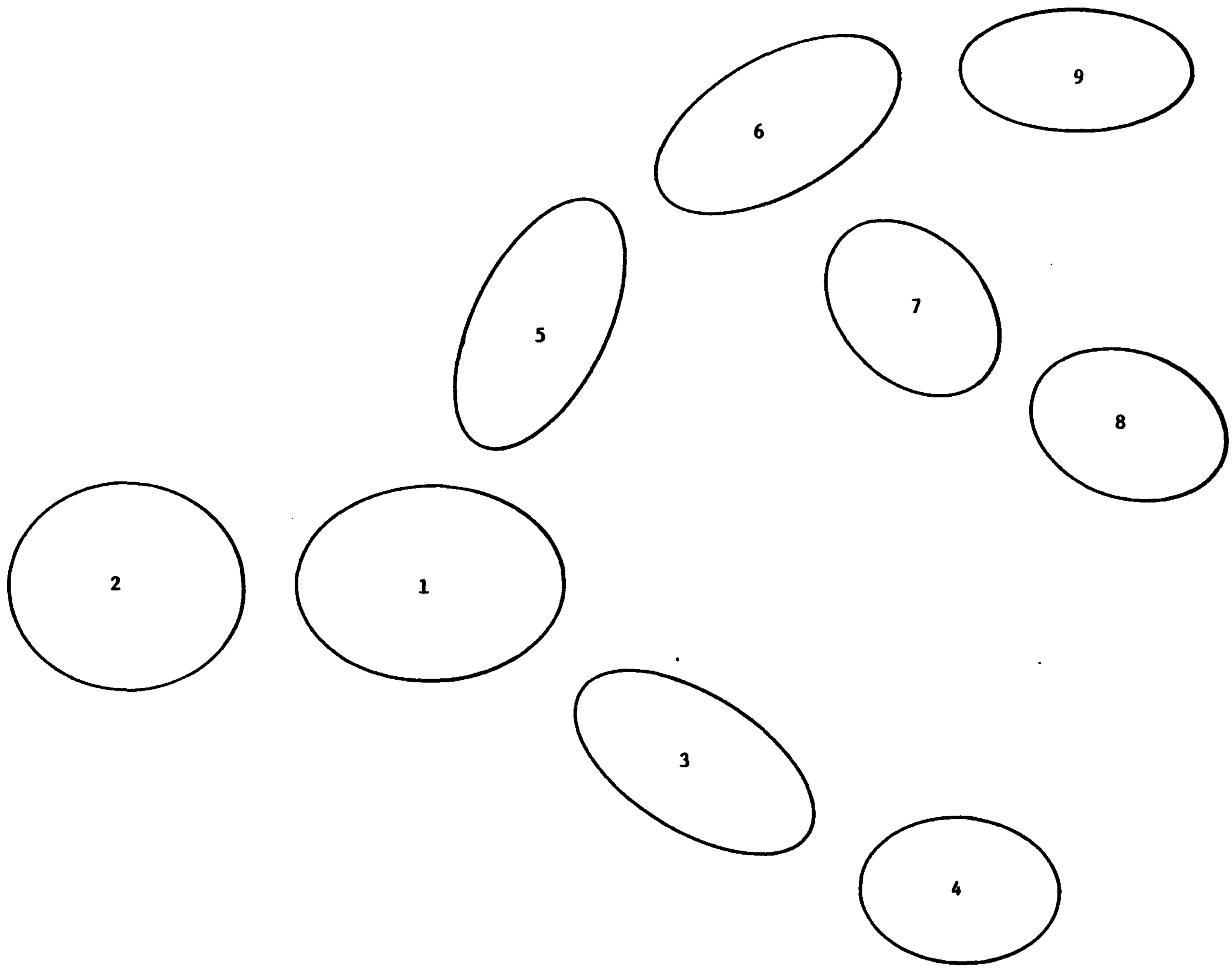

Figure 2. A General Chain System with Translation Between the Bodies

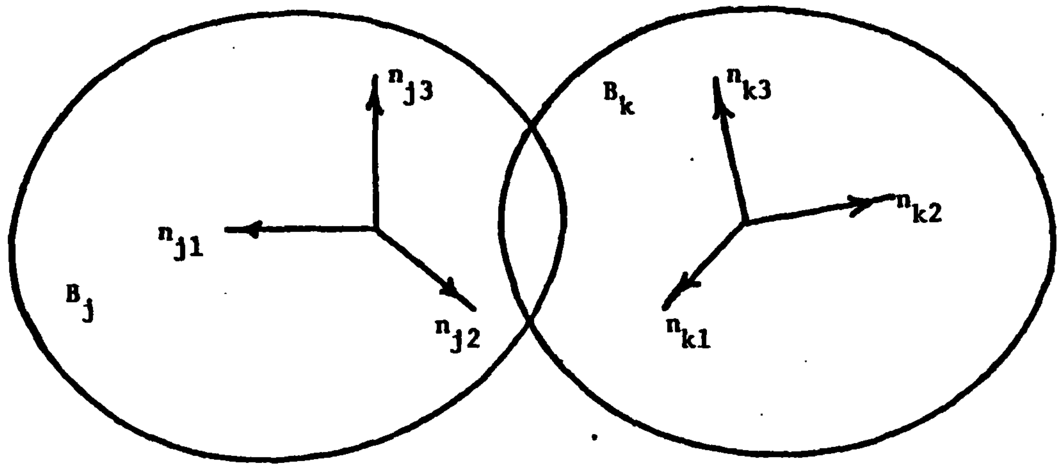

Figure 3. Two Typical Adjoining Bodies

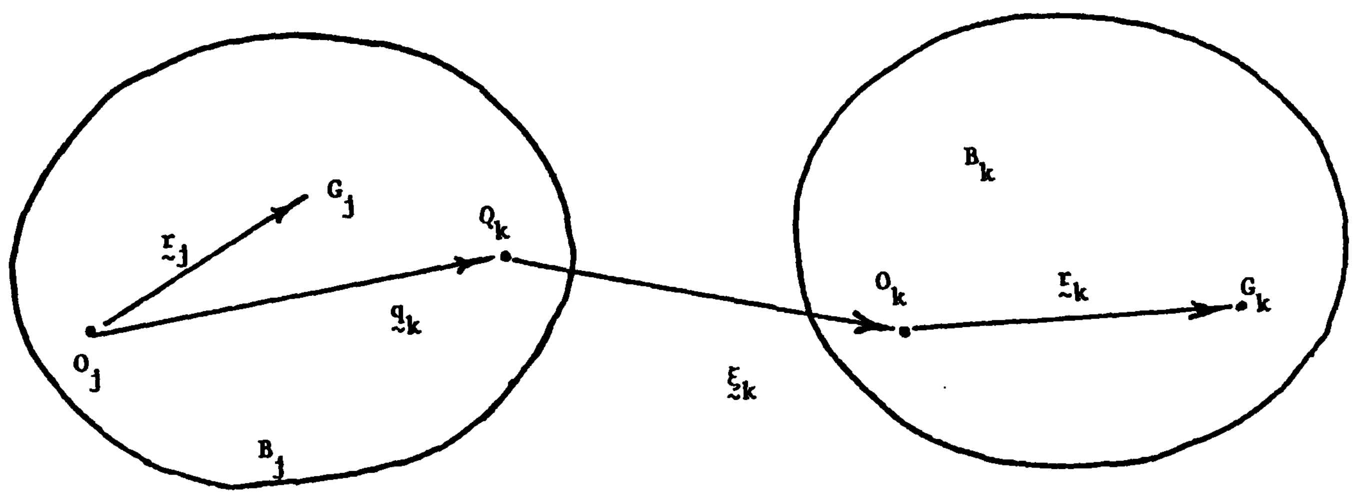

Figure 4. Reference Points and Position Vectors of Two Typical Adjoining Bodies

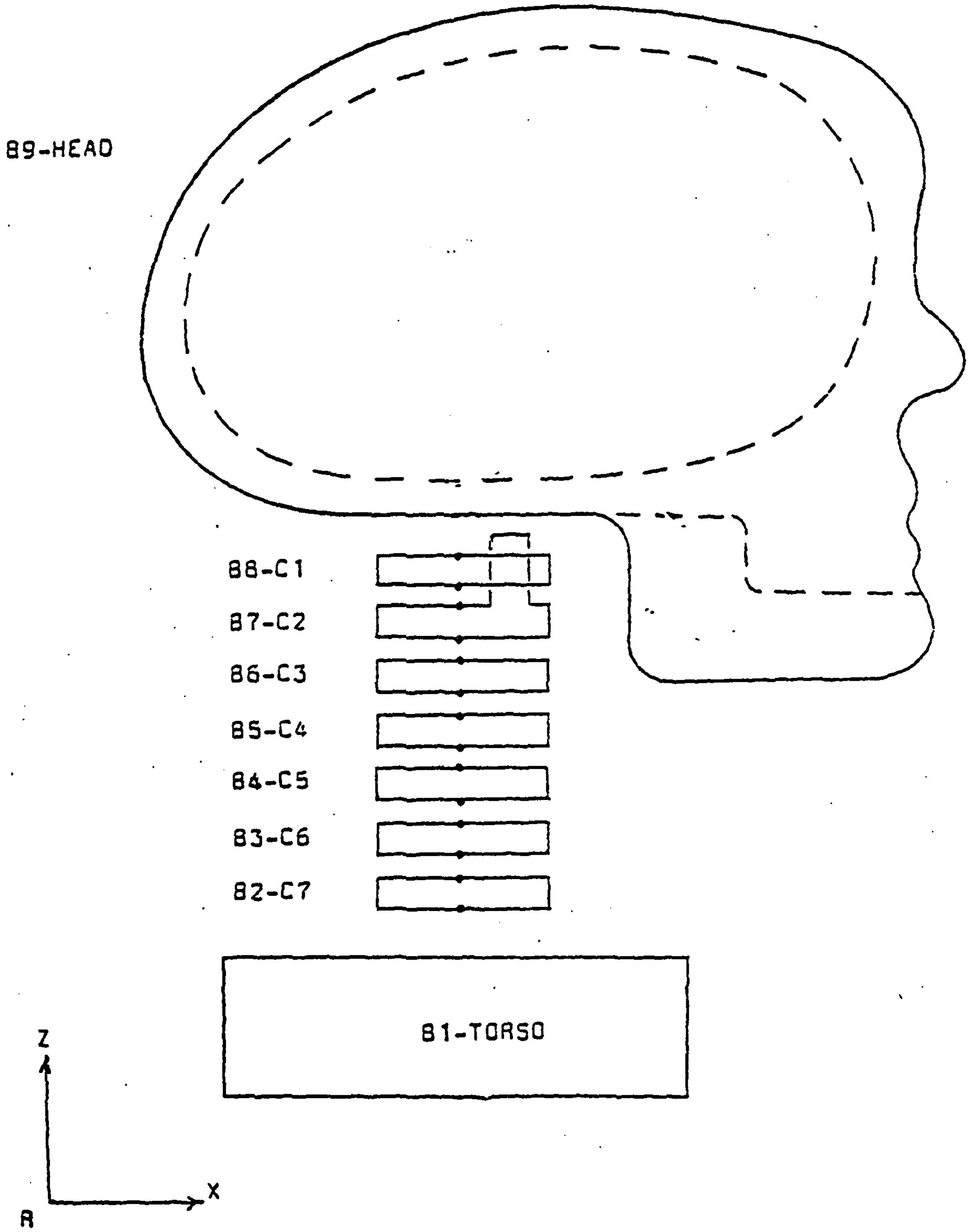

Figure 5. The Head-Neck Model

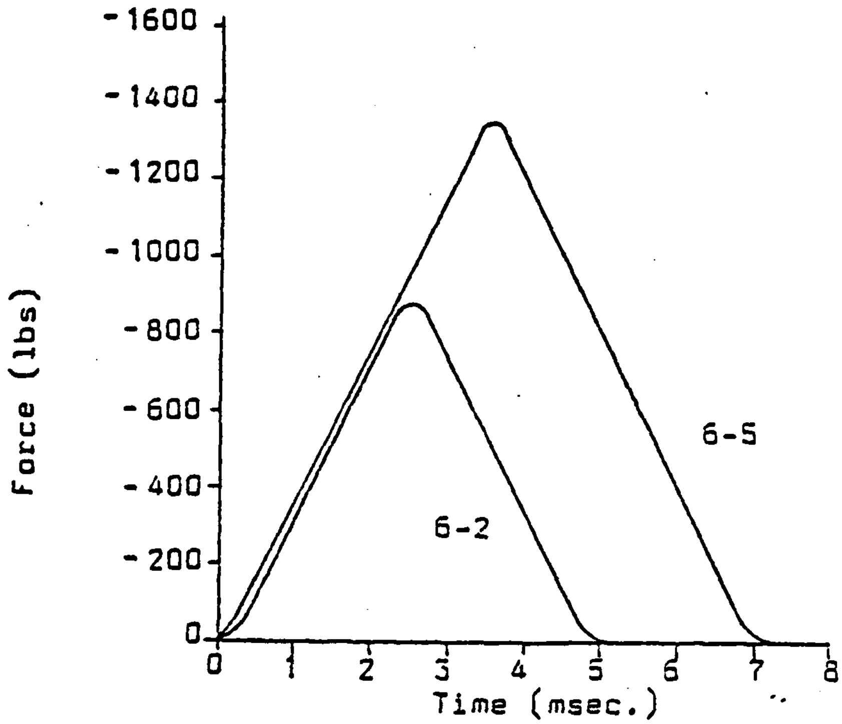

Figure 6. Frontal Impact Force

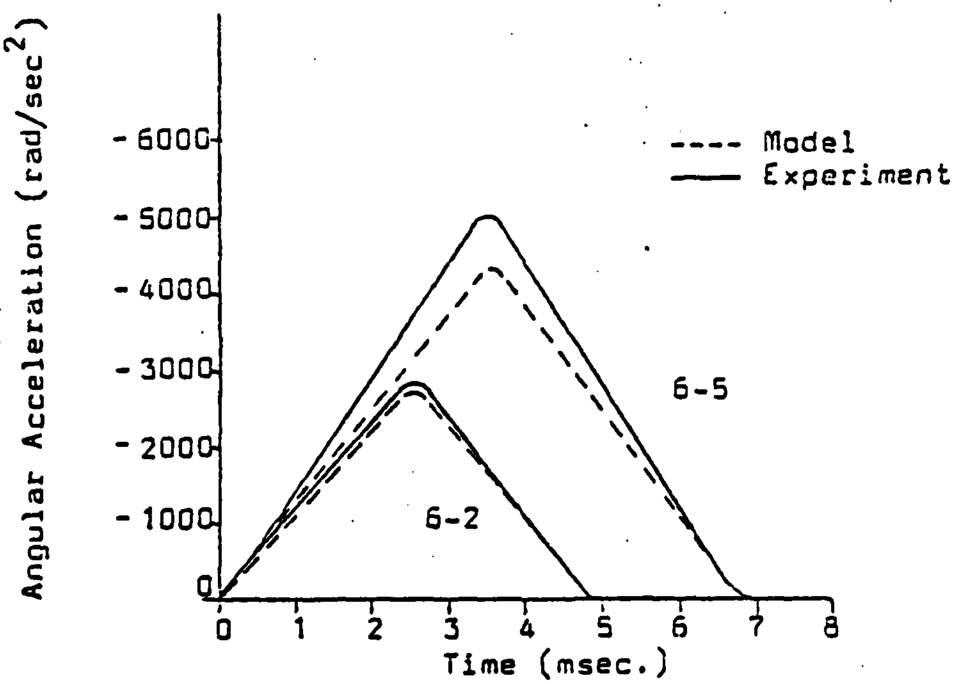

Figure 7. Comparison of Model and Experiment for Angular Acceleration of the Head

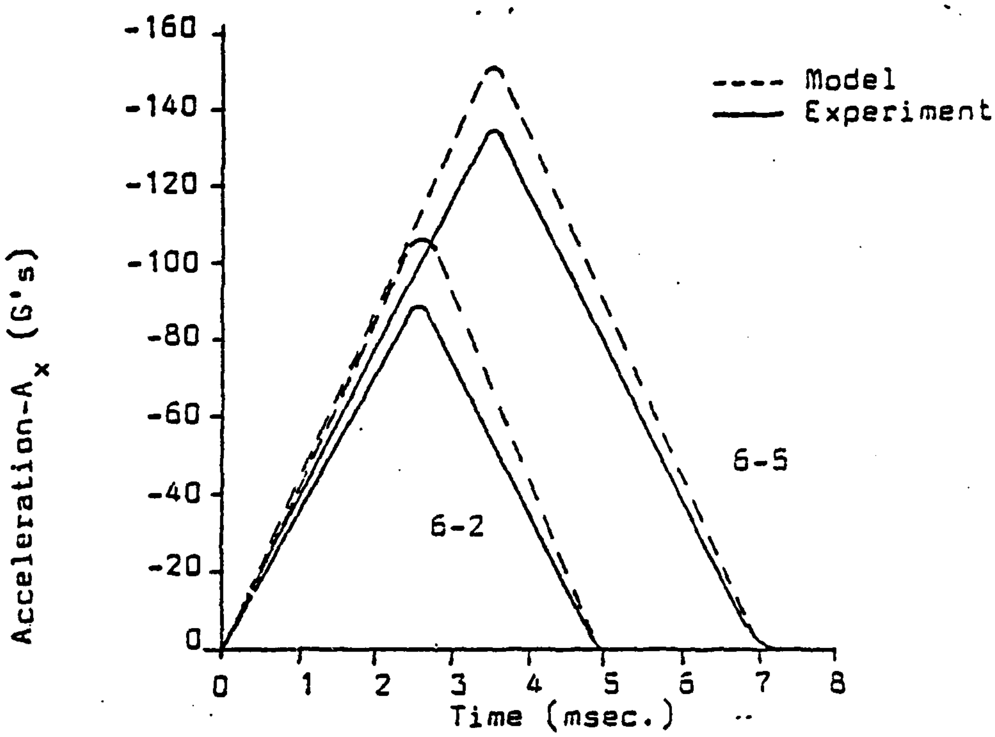

Figure 8. **Comparison of Model and Experiment for Forward Acceleration**

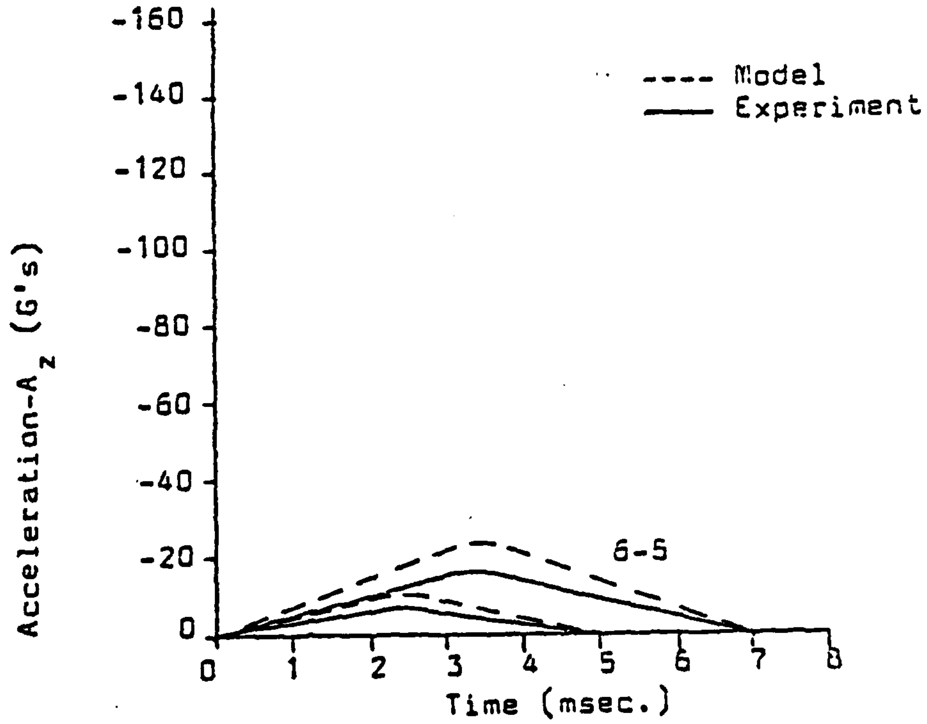

Figure 9. **Comparison of Model and Experiment for Vertical Acceleration**

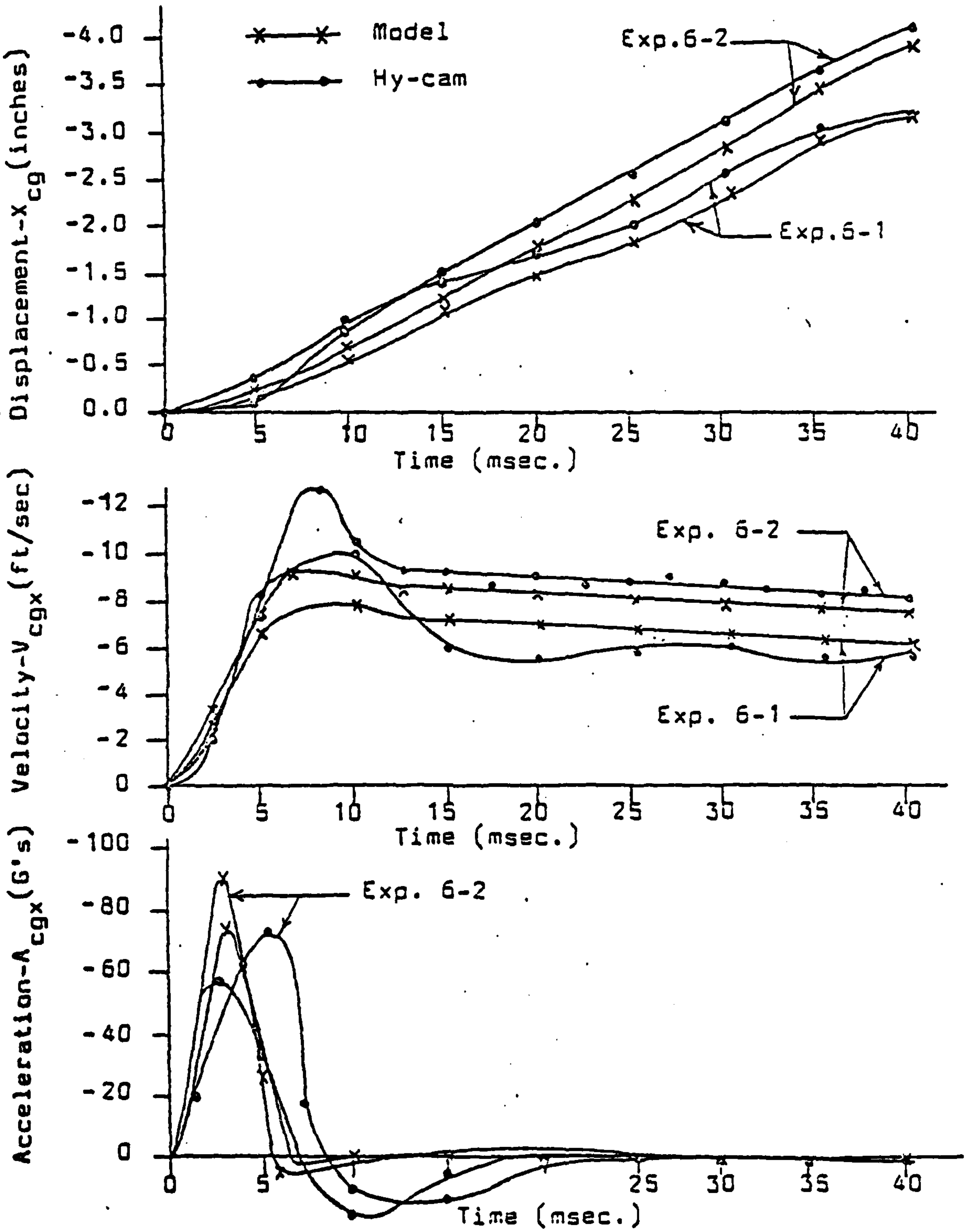

Figure 10. Comparison of Model and Experiment for Head Mass-Center Displacement, Velocity, and Acceleration

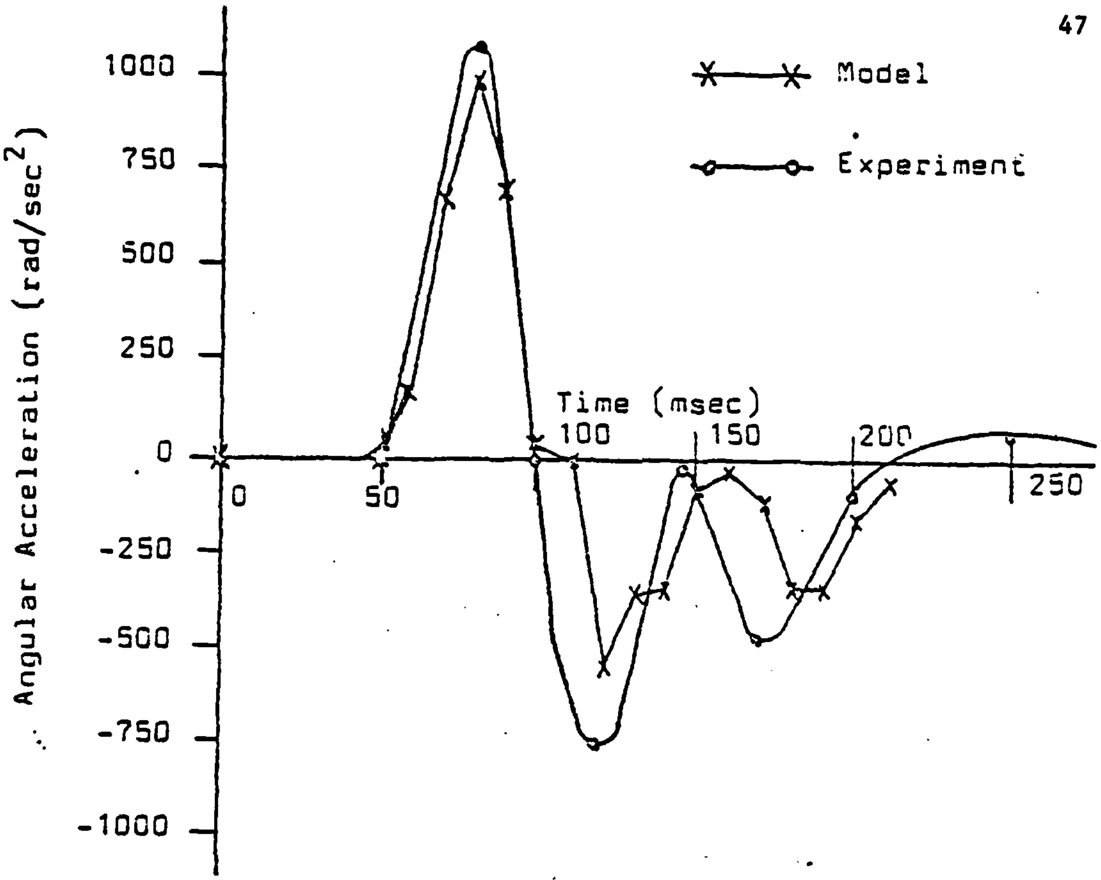

Figure 11. Comparison of Model and Ewing Experiment for Head Angular Acceleration

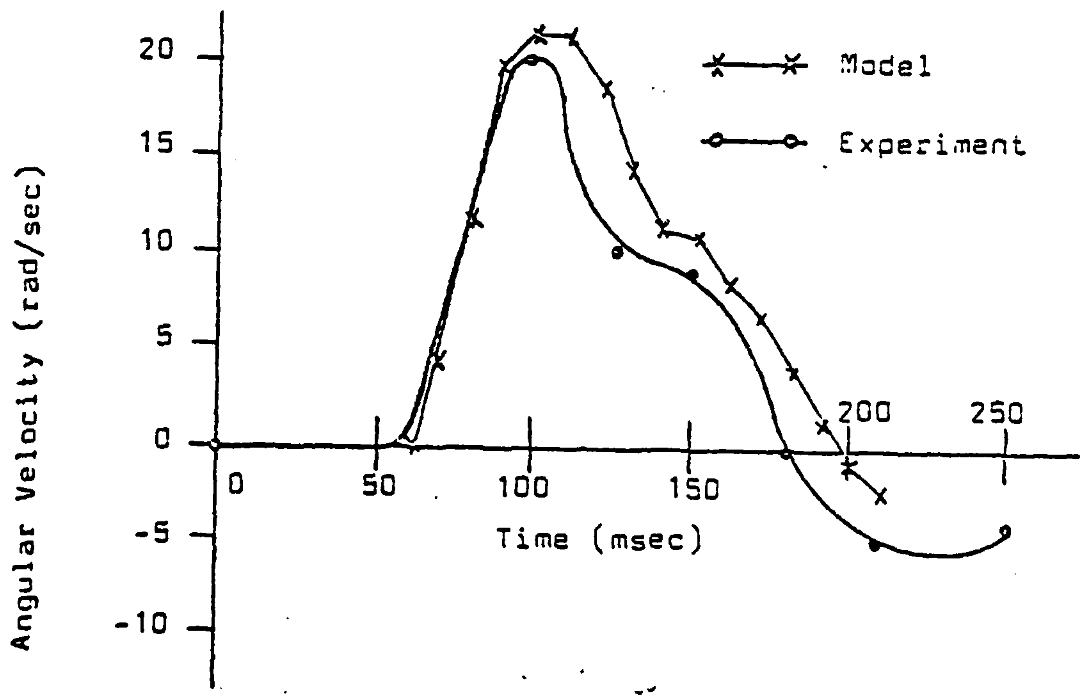

Figure 12. Comparison of Model and Ewing Experiment for Head Angular Velocity

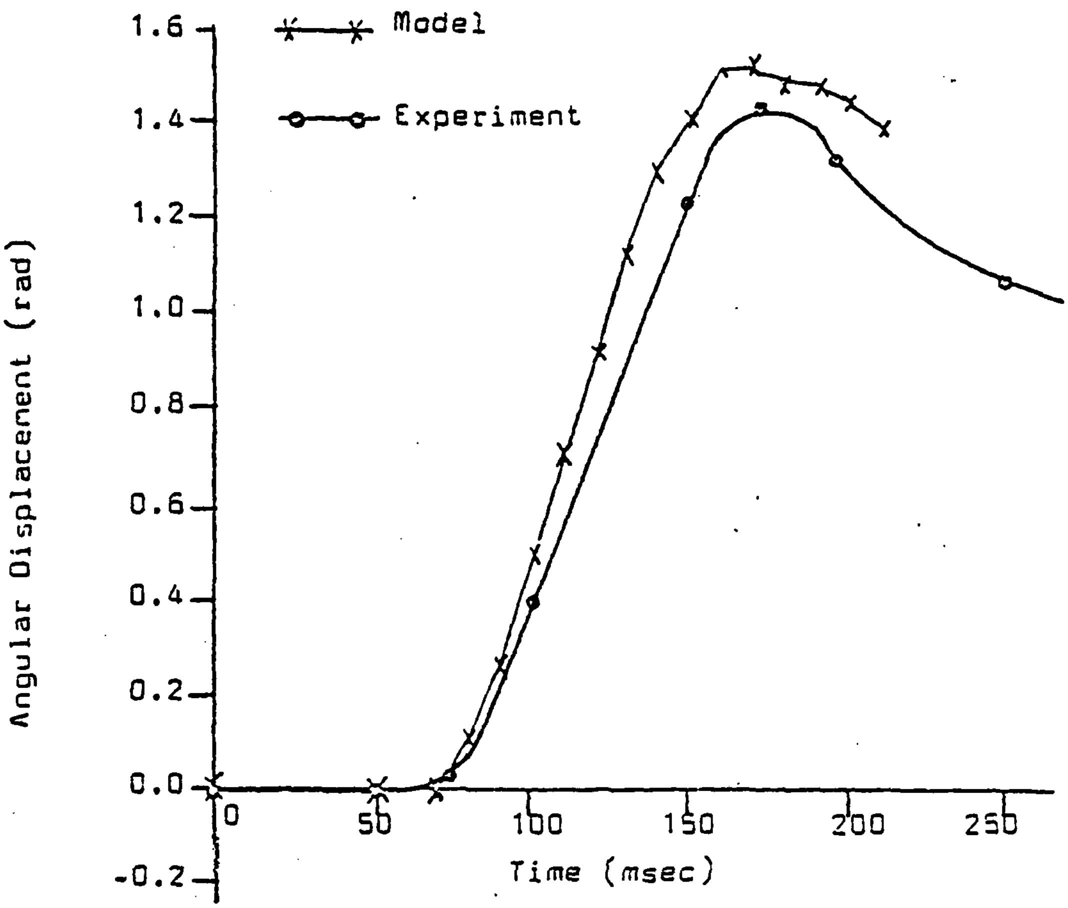

Figure 13. Comparison of Model and Ewing Experiment for Head Angular Displacement

$\omega_{k\ell m}$

B_k \ x_ℓ	1,2,3	4,5,6	7,8,9	10,11,12	13,14,15	16,17,18	19,20,21	22,23,24	25,26,27
1	I	0	0	0	0	0	0	0	0
2	I	S01	0	0	0	0	0	0	0
3	I	0	S01	0	0	0	0	0	0
4	I	0	S01	S03	0	0	0	0	0
5	I	0	0	0	S01	0	0	0	0
6	I	0	0	0	S01	S05	0	0	0
7	I	0	0	0	S01	S05	S06	0	0
8	I	0	0	0	S01	S05	S06	S07	0
9	I	0	0	0	S01	S05	0	0	S06

Table 1. $\omega_{k\ell m}$ for the System of Figure 1.

$\dot{\omega}_{k\ell m}$

B_k \ x_ℓ	1,2,3	4,5,6	7,8,9	10,11,12	13,14,15	16,17,18	19,20,21	22,23,24	25,26,27
1	0	0	0	0	0	0	0	0	0
2	0	$S\dot{0}1$	0	0	0	0	0	0	0
3	0	0	$S\dot{0}1$	0	0	0	0	0	0
4	0	0	$S\dot{0}1$	$S\dot{0}3$	0	0	0	0	0
5	0	0	0	0	$S\dot{0}1$	0	0	0	0
6	0	0	0	0	$S\dot{0}1$	$S\dot{0}5$	0	0	0
7	0	0	0	0	$S\dot{0}1$	$S\dot{0}5$	$S\dot{0}6$	0	0
8	0	0	0	0	$S\dot{0}1$	$S\dot{0}5$	$S\dot{0}6$	$S\dot{0}7$	0
9	0	0	0	0	$S\dot{0}1$	$S\dot{0}5$	0	0	$S\dot{0}6$

Table 2. $\dot{\omega}_{k\ell m}$ for the System of Figure 1.

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This report presents new and recently developed concepts which are useful for obtaining and solving equations of motion of multibody mechanical systems with translation between the respective bodies of the system. These concepts are then applied in the study of human/neck systems in high acceleration configurations.

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The developed concepts include the use of Euler parameters, Lagrange's form of d'Alembert's principle, quasi-coordinates, relative coordinates, and body connection arrays. This leads to the development of efficient computer algorithms for the coefficients of the equations of motion. The developed procedures are applicable to "chain-link" systems such as finite-segment cable models, mechanisms, manipulators, robots, and human body models.

The application with human head/neck models consists of a 54 degree of freedom, three-dimensional system representing the head, the vertebrae, and the connecting discs, muscles, and ligaments. The computer results for the system in a high acceleration configuration agree very closely with available experimental data.

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