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Twenty publications were prepared or published during this year of research. The areas of research include developing numerical methods for control and identification of delay systems, optimal control of diffusion-reaction systems, qualitative theory for delay systems, theory for nonlinear oscillations and and bifurcation, and stability conditions for discrete control systems.

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CONTROL OF DYNAMICAL SYSTEMS

for the period

September 1, 1977 - August 31, 1978

Brown University
Lefschetz Center for Dynamical Systems
Division of Applied Matheamtics
Providence, Rhode Island 02912

Principal Investigators

H.T. Banks
J.K. Hale
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October 22, 1978

Report prepared by

H.T. Banks

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I. Numerical Methods for Control and Identification of Delay Systems (H.T. Banks)

1. Spline-based techniques

In the past year Banks and Kappel (Universitat Graz, Austria) have developed ideas involving the use of spline-based approximations for computation of solutions of systems of equations with delays. Through their theoretical and numerical investigations, they have strong evidence to support the use of these approximations as an attractive alternative to the "averaging" type approximations (see [1]) in optimal control and parameter identification problems. The basic conceptual framework for the spline methods along with a summary of numerical results for integration of linear delay systems can be found in a just completed manuscript [2]. In these efforts, numerical experiments using both piecewise linear and piecewise cubic spline bases were carried out with excellent results. Pursuing these ideas, Banks developed software packages to use the spline methods in optimal control problems. Numerical experiments were made with test examples and again it was found that the spline-based methods appear to offer significant advantage over the "averaging" methods for control problems. Details of some of these numerical findings can be found in [3]. Efforts (joint with J. Burns and E. Cliff, an aerospace engineer at VPI) have now begun to use these ideas in the context of parameter estimation schemes. The early results, both theoretical and numerical, are extremely promising.

2. Approximation of nonlinear systems

In [4] Banks developed a theory based on the averaging approximations to treat nonlinear delay system optimal control problems. Numerical results to demonstrate the feasibility of these methods were also given. Banks has now made some progress in extending these ideas to allow for more general nonlinear systems and also to allow for spline and other type of approximation schemes as well as the "averaging" schemes. The theory, when completed, will hopefully also be applicable to nonlinear parameter estimation problems. Efforts on both theoretical and numerical aspects of the work is continuing.

3. Difference equation type techniques

D. Reber, under the direction of Banks, has completed his Ph.D. thesis [5] on modifications (of the Banks-Burns theoretical framework [1]) which result in difference equation approximations (as opposed to the finite systems of ordinary differential equations in the Banks-Burns approach). Both theoretical and computational efforts to study general linear nonautonomous (time-varying) system control problems (via the "averaging" type approximation scheme) were made. The results (which are summarized in the recent manuscript [6]) indicate that while the methods developed by Reber offer a viable alternative, one cannot expect to make significant gains here unless one employs something better than a simple first order differencing scheme on derivatives in the problems. Banks, and another graduate student, Rosen, are now trying to develop ideas to handle higher-order differencing approximations. When combined with the higher-order spline based schemes, these ideas should yield methods considerably more efficient than those developed to date.

II. Optimal Control of Diffusion-Reaction Systems (H.T. Banks)

Banks, J.P. Kernevez and M. Duban (an assistant to Kernevez at Université de Technologie de Compiégne) are continuing their efforts on methods for optimization of diffusion-reaction problems

$$\frac{\partial s}{\partial t} - \frac{\partial^2 s}{\partial x^2} + \frac{a}{1+a} v(s) = 0$$

$$\frac{\partial a}{\partial t} - \alpha \frac{\partial^2 a}{\partial x^2} = 0$$

through boundary controls. (Here v is a nonlinear reaction velocity approximation determined by the specific reactions considered.) Some progress in the theoretical aspects of these investigations has been made and is partially summarized in [7]. Additional theoretical results have been obtained which reveal to some extent how "singularities" arise in the standard numerical procedures (i.e. conjugate-gradient methods) when applied to these problems even though the problem itself may be theoretically well-posed. Current efforts are centered on comparison of several alternative procedures for efficiently finding numerical solutions.

III. Qualitative Theory of Delay Systems (J.K. Hale)

1. Effects of variations in the delays

In many problems in the applications, one encounters differential difference equations

$$(1) \qquad \dot{x}(t) = f(x(t), x(t-r_1), \ldots, x(t-r_n))$$

and difference equations

$$(2) \qquad y(t) = g(y(t-r_1), \ldots, y(t-r_n))$$

where $0 < r_1 < r_2 < \ldots < r_n$. The vector parameter $r = (r_1, \ldots, r_n)$ is a physical parameter which is known only to belong to some interval. It is, therefore, important to know how properties of the solutions depend upon r.

Even for the difference equation (2), this problem must be investigated for initial data from a function space $\mathcal{B}$ of functions on $[-b,0]$ where $b \geq r_n$. Hale and some of his students have been investigating this question for some time. The basic difficulty centers around the fact that the solution $x(\phi,r)$ through an initial function ϕ can never be smooth in r regardless of the function space $\mathcal{B}$. The overall objective is to determine a qualitative theory even though these smoothness properties in r do not hold.

Specific results obtained in the past year are the following. For a linear difference equation

$$(3) \qquad y(t) = \sum_{j=1}^{n} A_j y(t-r_j)$$

where each A_j is an $N \times N$ constant matrix, the asymptotic behavior of the solutions is given by the set $Z(r) = \{\mathrm{Re}\ \lambda : \det[I - \Sigma A_j e^{-\lambda r_j}] = 0\}$. Hale and Avellar have proved the following:

<u>Theorem</u>. There exist continuous functions $\sigma(r), \rho(r)$ such that

$$\mathrm{cl}\ Z(r) \subseteq [\rho(r), \sigma(r)]$$

and this is the minimal interval containing $\mathrm{cl}\ Z(r)$ if the components of r are rationally independent.

A method is also given for computing $\rho(r), \sigma(r)$. If $T_0(t,r)$ is the semigroup generated by (3) on $C_0 = \{\psi \in C([-b,0], \mathbb{R}^n)$: $\psi(0) = \Sigma A_j\psi(-r_j)\}$ defined by $(T_0(t,r)\psi)(\theta) = y(\psi,r)(t+\theta)$, $-r \leq \theta \leq 0$, and $y(\psi,r)$ is the solution of (3) through ψ, then they also prove that $\gamma(T_0(t,r)) = r_e(T_0(t,r)) \leq \exp(\sigma(r)t)$ with equality holding for the components of r rationally independent. Here γ is the spectral radius and r_e is the radius of the essential spectrum.

The implications of this result for stability are clear. Hale and Avellar also show how this result can be used to obtain continuous dependence on r of solutions of the neutral equation

$$\frac{d}{dt}[x(t) - \Sigma A_j x(t-r_j)] = B_0 x(t) + \Sigma B_j x(t-r_j).$$

These results have appeared in the thesis of Avellar and are being prepared for publication.

In an attempt to discover the essential elements of a qualitative theory for (1) as a function of r, Hale has discovered how to prove that a smooth Hopf bifurcation with respect to r

occurs for (1) even though the solutions are not differentiable in r ([8]). This has important implications to the existence of periodic solutions for equations with several delays. The methods should be applicable to other problems.

2. Dissipative systems

Cooperman [9], a student of Hale, has completed his dissertation on α-contractions and dissipative processes. The results have implications to the asymptotic behavior of the solutions for retarded and neutral functional differential equations. In addition, he has shown an intimate connection between dissipative systems and Liapunov functions.

IV. Nonlinear Oscillations and Bifurcation Theory (J.K. Hale)

1. Multiple parameter bifurcation problems

Hale and Táboas [10, 11] have completed their work on harmonic and subharmonic bifurcation in the equation

$$\ddot{x} + G(x) = \lambda\dot{x} + \mu g(t)$$

where λ,μ are small independent parameters, $g(t+1) = g(t)$ and the nonlinear equation $\ddot{x} + G(x) = 0$ has a nonconstant periodic solution of period k, an integer. The methods have new applications to homoclinic orbits, but the results are not complete at this time.

2. Reaction-diffusion equations

Alikakos [12, 13] a student of Hale and Mallet-Paret, has completed his dissertation on asymptotic behavior in parabolic equations. He obtained results on a scalar parabolic equation with nonlinear boundary conditions, generalizing results of Aronson, Peletier and Ball. He also obtained results on systems of equations with food pyramid conditions, generalizing results of Chow and Williams. In addition to showing that the methods of dynamical systems are more powerful than maximum principle arguments, he was able to obtain uniform bounds on the solutions rather than L^p bounds with $1 \leq p < \infty$.

V. Control Systems (J.P. LaSalle)

1. Stable feedback generators

An initial effort on this problem by LaSalle and Palmer has been discontinued. Some progress was made on attempting to extend the concept of solutions of differential equations with discontinuous right-hand sides. Earlier work of Fillipov, Hermes, Hajek, and others was extended. However, recent work by Aizerman and one of his colleagues at the Institute of Control Sciences, Moscow, in this same direction indicates it does not provide the necessary mathematical framework for the problem of investigating stable feedback generators.

2. Learning, identification and adaptation

The concept of eventual stability for nonautonomous ordinary differential equations was introduced by LaSalle around 1958 and studied by R. Rath in his Notre Dame Ph.D. dissertation, 1962. An application was made in 1963 (R. Rath and J.P. LaSalle, "Eventual Stability", Proceedings of Second IFAC Congress, Basel, 1963) to prove the convergence of a scheme for adaptive control. It is clear today that the concept of eventual stability is, and should be, related to the limiting equations of nonautonomous systems. Last fall, LaSalle and Zvi Artstein, Weizmann Institute of Science, Israel, began a joint investigation of the relationship of questions of stability, adaptation, identification, and learning to the limiting equations of control systems. This research is into its initial stages, and it is hoped that Artstein can visit Brown next spring to continue these efforts.

3. Stability

a. functional difference equations

Under the direction of LaSalle, Palmer has completed his thesis [14] on an extension of Liapunov's direct method for the study of the stability of nonautonomous functional (delay-differential) equations.

b. stability of nonautonomous difference equations

M. Latina, R.I. Jr. College, under the direction of LaSalle has completed an extension of the invariance principle and Liapunov's direct method for nonautonomous discrete systems (time-varying difference equations). LaSalle gave, in outline, the development of this theory in his lectures at an NSF-CBMS Regional Conference on Applied Mathematics, Mississippi State, August 1975. Improvements in the theory have been made that extend the range of applications.

4. Stability and control of discrete processes

LaSalle spent most of his time this summer working on a two-volume exposition on the control and stability of discrete-time processes. A final draft of Volume I on linear systems is almost completed. Volume II, which will be directed towards the more general nonlinear theory of both stability and control, will be completed by the end of this year. Much of our knowledge of discrete systems appears within many different contexts (numerical analysis, engineers' studies of sampled data systems, probability and statistics, etc.), and this exposition provides through the use of the language and concepts of dynamical system both a unity and extension of the theory and is a good preparation for the study of continuous-time systems (ordinary and functional differential equations, etc.).

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ABSTRACTS OF PAPERS WRITTEN OR PUBLISHED

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September 1, 1977 - August 31, 1978

SPLINE APPROXIMATIONS
FOR
FUNCTIONAL DIFFERENTIAL EQUATIONS

H. T. Banks and F. Kappel

Abstract: We develop an approximation framework for linear hereditary systems which includes as special cases approximation schemes employing splines of arbitrary order. Numerical results for first and third order spline based methods are presented and compared with results obtained using a previously developed scheme based on averaging ideas.

APPROXIMATION OF DELAY SYSTEMS WITH APPLICATIONS TO CONTROL AND IDENTIFICATION

H. T. Banks

Abstract: We discuss approximation ideas for functional differential equations and how these ideas can be employed in optimal control and parameter estimation problems. Two specific schemes are described, one based on integral averages of the function being approximated, the other based on best L_2 spline approximations. An example illustrating numerical behavior of these schemes applied to an optimal control problem is presented.

OPTIMAL CONTROL OF DIFFUSION-REACTION SYSTEMS

by

H. T. Banks, M. C. Duban and J. P. Kernevez

Abstract: We discuss several formulations of optimization problems which arise in a natural way in the investigation of transport properties of artificial membranes and general diffusion-reaction media. Nonlinear reaction velocity approximations dictated by reactions of interest to biochemists place the problems in a class to which one cannot apply the usual computational techniques (e.g. gradient, conjugate-gradient) in a straightforward manner. The inherent difficulties, how one might circumvent them, and some of our initial efforts towards development of feasible computational schemes are discussed.

APPROXIMATION TECHNIQUES FOR CONTROL SYSTEMS WITH DELAYS

by

H. T. Banks and J. A. Burns

Abstract

We present a theoretical framework for approximation techniques for nonlinear system optimal control problems. Two particular approximation schemes that may be used in the context of this framework are discussed and typical numerical results for two examples to which we have applied these schemes are given. We conclude with a brief survey of related investigations.

HEREDITARY CONTROL PROBLEMS: NUMERICAL METHODS BASED ON AVERAGING APPROXIMATIONS

by

H. T. Banks and J. A. Burns

Abstract: An approximation scheme involving approximation of linear functional differential equations by systems of high order ordinary differential equations is formulated and convergence is established in the context of known results from linear semigroup theory. Applications to optimal control problems are discussed and a summary of numerical results is given. The paper is concluded with a brief survey of previous literature on this class of approximations for systems with delays.

Abstract of

α-CONDENSING MAPS AND DISSIPATIVE SYSTEMS

by Gene David Cooperman, Ph.D., Brown University, June 1978

Dissipative systems are a generalization of differential equations with Lyapunov functions. The main idea is that solutions should enter a given bounded set and stay there for all further time. It was first studied by N. Levinson in 1944. The concept is intimately connected with the existence of a maximal, compact, invariant set. This is important, since any maximal, compact, invariant set must contain all equilibrium solutions, periodic solutions, and almost periodic solutions. Its role is analogous to the role of the maximal, invariant set in the LaSalle Invariance Principle.

In Euclidean space, the various possible definitions of a dissipative system are degenerate. In a Banach space, this is not so. Since the natural setting for delay differential equations is in a Banach space, it is important to identify which definition of dissipative systems should be used in a particular application. Counterexamples are given to indicate where the best possible result has been obtained. Applications include Lyapunov and stability theory, a continuous dependence result for invariant sets, and a demonstration of the use of these methods in facilitating work with skew product flows.

In Chapter II, a theorem is proven which allows us to extend the results on α-contractions to α-condensing maps. This is important, since α-condensing maps are very general solution operators of delay differential equations, which include α-contractions. Additional applications of the theorem include a generalization of a fixed point theorem, and of a continuous dependence theorem for fixed points, to the α-condensing case.

NONLINEAR OSCILLATIONS IN EQUATIONS WITH DELAYS

Jack K. Hale

Abstract: These lectures are concerned only with some aspects of bifurcation theory in the local theory of nonlinear oscillations in equations with delays; that is, behavior of solutions near an equilibrium. In particular, we study how the qualitative behavior of solutions change as parameters vary. A detailed study of the local theory is important in order to know the types of solutions to expect in a global problem. Of course, there is no reason to only study local theory near an equilibrium. One should study how the qualitative behavior changes near any invariant set - for example, behavior near a periodic orbit, behavior near an orbit which connects a saddle point to itself, etc. More complicated behavior is expected near these large invariant sets. One can obtain invariant torii, homoclines points which exhibit a chaotic behavior, etc. We restrict ourselves in these lectures to behavior near equilibrium.

The simplest type of smooth bifurcation is from an equilibrium to a periodic orbit - the so-called Hopf bifurcation. In Section 2, we discuss the Hopf bifurcation in equations with finite delays permitting the bifurcation parameters to be the delays themselves. At first glance, such a result does not seem possible because the vector field in the equation is not differentiable in the parameters. The theorem does require some new ideas and, for this reason, the proof is given in some detail. Several examples are given in Section 3.

In Section 4, similar results are presented for equations with infinite delays. In Section 5, we give an example in two dimensions for which stable Hopf bifurcation occurs with decreasing delay. In Section 6, we give an introduction to some of the methods available for nonautonomous equaitons.

RETARDED EQUATIONS WITH INFINITE DELAYS

Jack K. Hale

Abstract: It is the purpose of these notes to describe the theory of Hale and Kato for functional differential equations based on a space of initial data which satisfy some very reasonable axioms. We also indicate some recent results of Naito showing how extensive the theory of linear systems can be developed in an abstract setting in particular, the characterization of the spectrum of the infinitesimal generator together with the decomposition theory and exponential estimates of solutions.

SOME RECENT RESULTS ON DISSIPATIVE PROCESSES

Jack K. Hale

Abstract: A detailed study of dissipative processes has obvious implications in the asymptotic behavior of solutions of differential equations. The properties of such maps has been extensively studied over the past years with the base space being a Banach space rather than $\mathbb{R}^n$. The results on asymptotic behavior pertain to the properties of solutions of functional differential equations of retarded and neutral type as well as certain types of partial differential equations. It is the purpose of this lecture to present some recent developments in this theory, especially the ones due to Coooperman. A brief discussion of the history of the subject may be found in Chapter 4 of Theory of Functional Differential Equations by J.K. Hale.

TOPICS IN LOCAL BIFURCATION THEORY

by

Jack K. Hale

Abstract

Suppose Λ, X, Z are Banach spaces, $M: \Lambda \times X \to Z$ is a mapping continuous together with derivatives up through some order r. A bifurcation surface for the equation (1) $M(\lambda, x) = 0$ is a surface in parameter space Λ for which the number of solutions x of (1) changes as λ crosses this surface. Under certain generic hypotheses on M, the author and his colleagues have shown that one can systematically determine the bifurcation surfaces by elementary scaling techniques and the implicit function theorem. This talk gives a summary of these results for the case of bifurcation near an isolated solution or families of solutions of the equation $M(\lambda_0, x) = 0$. The results have applications to the buckling theory of plates and shells under the effect of external forces, imperfections, curvature and variations in shape. The results on bifurcation near families has applications in nonlinear oscillations and the theory of homoclinic orbits.

BIFURCATION NEAR DEGENERATE FAMILIES

by

Jack K. Hale and Placido Táboas

Abstract: Suppose X, Λ, Z are Banach space, $M: X \times \Lambda \to Z$ is a smooth function and the equation

$$M(x,\lambda) = 0, \tag{1}$$

for $\lambda = 0$, has a one parameter family of smooth solutions $p(s)$, $0 \leq s \leq 1$, $p(0) = p(1)$ with $dp(s)/ds \neq 0$. If $\Gamma = \{p(s), 0 \leq s \leq 1\}$, the objective is to discuss the solutions of (1) near Γ for λ in a neighborhood of zero. Assuming $A(s) = \partial M(p(s),0)/\partial s$ is Fredholm of index zero and $\dim \mathcal{N}(A(s)) = 1$, for $0 \leq s \leq 1$, the authors have previously given a solution to this problem under natural hypotheses on M. In this paper, the case where $\dim \mathcal{N}(A(s)) = 2$ is considered. Applications are given to the effect of small damping and small forcing on the existence of periodic solutions near a periodic solution of an autonomous Hamiltonian system.

BIFURCATION NEAR FAMILIES OF SOLUTIONS

Jack K. Hale

Abstract: Many investigations in bifurcation theory are concerned with the following problem. If $M(0,0) = 0$ and $\partial M(0,0)/\partial x$ has a nontrivial null space, find all solutions of the equation

$$M(x,\lambda) = 0 \tag{1.1}$$

for (x,λ) in a neighborhood of $(0,0) \in X \times \Lambda$.

If $\dim \Lambda = 1$; that is, there is only one parameter involved then the existence of more than one solution in a neighborhood of zero can be proved by making assumptions only about $\partial M(0,0)/\partial x$ and $\partial M(0,0)/\partial x \partial \lambda$. However, if $\dim \Lambda \geq 2$, then the problem is much more difficult and more detailed information is needed about the function M. A careful examination of the existing literature for $\dim \Lambda \geq 2$ reveals that the additional conditions imposed on M imply, in particular, that the solution $x = 0$ of the equation

$$M(x,0) = 0 \tag{1.2}$$

is isolated (see, for example, the papers on catastrophe theory). These hypotheses eliminate the possibility that Equation (1.2) has a family of solutions containing $x = 0$. Such a situation occurs, for example, for $M(x,\lambda) = Ax + N(x,\lambda)$, where A is linear with a nontrivial null space and $N(x,0) = 0$ for all x. There also are interesting applications where Equation (1.2) is nonlinear and there exists a family of solutions. For example, Equation (1.2) could be an autonomous ordinary differential equation with a nonconstant periodic orbit of period 2π with the family of solutions being obtained by a phase shift. When the differential equation in the latter situation is a Hamiltonian system, the parameters (λ_1, λ_2) could correspond to a small damping term and a small forcing term of period 2Π. To the author's knowledge, the first complete investigations of special problems of each of these latter types are contained in papers by Hale, Táboas and Rodrigues.

It is the purpose of this paper to begin the investigation of the abstract problem for Equation (1.1), especially to extend the results in the paper by Hale and Táboas.

INTERACTION OF DAMPING AND FORCING IN A SECOND ORDER EQUATION

by

Jack K. Hale and Plácido Táboas

Synopsis. Suppose λ,μ are real parameters, f is a scalar function which is 2π-periodic, $xg(x) > 0$ for $x \neq 0$ and consider the equation

$$\ddot{x} + g(x) = -\lambda\dot{x} + \mu f(t). \tag{1}$$

For $\lambda = \mu = 0$, every solution has the form $x(t) = \phi(\omega(a)t + \alpha, a)$ for some constants a,α and $\phi(\theta+2\pi,a) = \phi(\theta,a)$. If there is an a_0 such that $\omega(a_0) = 1$ (i.e., there is a 2π-periodic orbit Γ in $(x,\dot{x})$-space) and $\omega'(a_0) \neq 0$, the problem is to characterize the number of 2π-periodic solutions of Equation (1) which lie in a neighborhood of Γ for (λ,μ) in a small neighborhood of $(0,0)$. A complete solution of this problem is given under the hypothesis that the function $h(\alpha) = \int_0^{2\pi}[\partial\phi(t,a_0)/\partial t]f(t-\alpha)dt \Big/ \int_0^{2\pi}[\partial\phi(t,a_0)/\partial t]^2 dt$ has a nonzero derivative except at a finite number of points α_j and $h''(\alpha_j) \neq 0$. The bifurcation curves in (λ,μ)-space are determined by the α_j and are tangent to the straight lines $\lambda = h(\alpha_j)\mu$ at $(\lambda,\mu) = (0,0)$. In general, the 2π-periodic solutions of (1) are not continuous at $(\lambda,\mu) = (0,0)$. The nature of this discontinuity is discussed in detail. It is also shown that a necessary and sufficient condition for a 2π-periodic solution $x(\lambda,\mu)$ to be continuous at $(\lambda,\mu) = (0,0)$ is that $\lambda/\mu \to$ constant as $\lambda \to 0$, $\mu \to 0$.

GENERIC BIFURCATION WITH APPLICATIONS

J. K. Hale

Abstract: This paper is a set of lecture notes on generic bifurcation and its applications with the emphasis on equations involving more than one independent parameter. The general theory is discussed for problems which are degenerate to order one or two. Applications are given primarily to the buckling of plates and shells with the parameters representing external forces, loading, imperfections, curvature and dimension.

RESTRICTED GENERIC BIFURCATION

Jack K. Hale

Abstract: In the past few years, there has been considerable attention devoted to the existence of bifurcation for one-parameter families of mappings. Concurrent with this development has been the extensive theory of universal unfolding of mappings or generic bifurcation for families of mappings which depend on a sufficiently large number of parameters. The purpose of this paper is to discuss methods for determining the nature of bifurcation when the family of mappings has $k \geq 1$ parameters but k is generally smaller than the number of parameters necessary to describe the universal unfolding.

NEW STABILITY RESULTS FOR NONAUTONOMOUS SYSTEMS

by

J.P. LaSalle

Abstract

The new invariance properties that have been established for nonautonomous ordinary differential equations greatly extend the range and power of Liapunov's direct method for the study of the stability of time-varying systems. An essential feature of the method is the establishment of a relationship between Liapunov functions and the location of the positive limit sets of solutions. The principal contribution of this paper is a theorem connecting Liapunov functions and positive limit sets of sufficient generality to close a gap in the present theory.

STABILITY THEORY FOR DIFFERENCE EQUATIONS

by

J. P. LaSalle

Abstract: This article is designed to give through the study of difference equation (discrete dynamical systems) a view of and an introduction to the general theory of the stability of dynamical systems in its most modern aspect. Much of what is presented here is known, although not perhaps as well known as it should be, and there are some things that are new. One of these has to do with a connectedness property of the positive limit sets of the solutions of difference equations which provides a means through the use of Liapunov functions of establishing the existence of equilibrium points (fixed points) and oscillations (periodic points). Another is the generalization of the usual concept of a vector Liapunov function, and this leads to a possible method of designing control systems where the measure of the error or the performance criterion is a vector rather than a scalar. Applications of the theory are illustrated by simple examples.

The article was written for undergraduate teachers of mathematics but it should also serve as a good introduction for engineers and scientists to the latest results in the theory of the stability of dynamical systems.

LIAPUNOV STABILITY THEORY FOR NONAUTONOMOUS FUNCTIONAL DIFFERENTIAL EQUATIONS

John William Palmer

Abstract

It is a standard technique in all areas of mathematics to reduce a more complicated problem to one that has already been solved. Aside from providing a unity to mathematics, this idea presents us with new approaches to unsolved problems. The area of differential equations has many interesting applications of this method, and it is the purpose of this paper to expose an opportunity to use this idea to extend Liapunov's direct method for studying stability. Specifically, we shall be interested in extending the LaSalle Invariance Principle [32] to the context of nonautonomous functional differential equations (FDE).

We begin our discussion by giving the historical development of invariance principles in Chapter II. Then, in Chapter III, we construct a semiflow from the solutions of nonautonomous functional differential equations. From this semiflow an invariance principle can be obtained. In Chapters IV and V we weaken the requirements on the vector field necessary for obtaining an invariance property. First, in Chapter IV, we use an extension of the concept of differential equation due to Kurzweil [31]. The use of Kurzweil's concept of a differential equation allows us to consider the stability problem for a class of FDE larger than that discussed in Chapter III. In Chapter V the invariance property is obtained directly from a continuous dependence result without formally constructing a semiflow. This allows us to consider equations with nonunique solutions. Also, in Chapter V, we introduce the integral-like operator equations of Neustadt [48]. These

operator equations allow for yet another enlargement of the class of FDE for which an invariance principle can be obtained. Chapter VI contains results on uniform asymptotic stability, total stability, and eventual stability. All of these results are shown to be intimately connected to the stability properties of the limiting equations of FDE. Examples illustrating the stability theorems are given. Finally, in Chapter VII we indicate the extension of the above results to FDE with infinite delay and to neutral FDE. Appendix A and B summarize results from topology and topological dynamics, respectively, that are of importance for the body of the text.

Throughout, we shall be concerned with working directly with the differential equations. Dafermos [11.5] has studied the stability problem by working directly with the solutions.

A comment on notation is in order. We abuse two symbols. The symbol ω variously stands for the ω-limit set in dynamical systems and for the maximal right hand endpoint of the domain of definition of a solution of a differential equation. The symbol $^-$ variously stands for the closure operator in topology and for the operation of taking a one sided limit from below. In either case the particular usage of the symbol in the text eliminates any ambiguity.

A FINITE DIFFERENCE TECHNIQUE FOR SOLVING OPTIMIZATION PROBLEMS GOVERNED BY LINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

by

Douglas C. Reber

Abstract: Aspects of the approximation and optimal control of systems governed by linear retarded nonautonomous functional differential equations (FDE) are considered. First, certain FDE are shown to be equivalent to corresponding abstract ordinary differential equations (ODE). Next, it is demonstrated that these abstract ODE may be approximated by difference equations in finite dimensional spaces. The optimal control problem for systems governed by FDE is then reduced to a sequence of mathematical programming problems. Finally, numerical results for two examples are presented and discussed.

Abstract of APPROXIMATION AND OPTIMAL CONTROL OF LINEAR HEREDITARY SYSTEMS

by Douglas Christian Reber, Ph.D., Brown University, June 1978.

Our concern in this investigation is with the approximation and optimal control of systems governed by linear retarded functional differential equations (FDE). In chapter I we establish the existence, uniqueness and continuous dependence of solutions of FDE. We further demonstrate that certain FDE are equivalent to corresponding abstract ordinary differential equations (ODE). Such ODE are also known as abstract evolution equations (AEE). Chapter II details the manner in which these AEE may be approximated by difference equations in finite-dimensional spaces.

The optimal control problem for systems governed by FDE is then reduced to a sequence of mathematical programming problems in chapter III. In chapter IV we discuss numerical results for two systems, having applied standard techniques of numerical analysis to compute the solutions of the approximating problems. The first of these systems was chosen for its simplicity, so that an analytical solution would be readily available; the second system is associated with a biochemical process.