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Quasi-Static Range Propagation Equations For the Approximate Fields Within a Conducting Slab

Peter R. Bannister René L. Dube Submarine Electromagnetic Systems Department



Newport,Rhode Island • New London,Connecticut

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PREFACE

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INTRODUCTION

During the last several years, considerable interest has developed in determining the quasi-static field components of antennas located above, or buried within, the earth's surface. Quasi-static range is defined as the range of transmission where the measurement distance is much less than a free-space wavelength. Quasi-static range results are useful for submarine radio communication and detection; they also have applications in locating buried miners and help geophysicists determine the electrical properties of the earth.

For the semi-infinite conducting medium case (i.e., air, single layered earth), some work has been done to determine the quasi-static fields produced by various subsurface sources when the measurement distance is comparable to the earth skin depth.¹⁻⁶ (Most of these results are summarized by Kraichman.⁷) However, the field strength expressions are very complex because they involve products of modified Bessel functions of different argument. Recently, by using finitely conducting earth-image theory techniques, we derived approximate expressions for the general quasi-static range electromagnetic fields (in air and in earth) produced by various subsurface antennas.⁸ Some numerical calculations have also been provided.⁹

These investigations revealed that when the source is not buried too far from the surface of the conducting half-space, the resultant electromagnetic field is significantly different from that obtained in an infinite conducting medium. Therefore, if the conductor is two-layered, the lower interface must likewise affect the field (especially if the upper layer in which the source is located is not too deep). For example, if the transmitting antenna is situated in a shallow sea, the theory for a uniform conducting half-space would not accurately describe the field at the sea bottom. This observation is also true at the surface of the sea if the sea depth (l_1) is less than approximately one skin depth.

Weaver has evaluated the exact Sommerfeld integral expressions and obtained numerical results for the quasi-static fields produced by horizontal and vertical electric dipole (HED and VED) antennas located in the upper layer of a two-layer conducting half-space.⁶ Numerical results for the quasistatic fields produced in the sea by a vertical magnetic dipole (VMD) for various values of sea bottom conductivity have been obtained by Coggon and Morrison.¹⁰

This report employs finitely conducting earth-image theory techniques to derive approximate expressions for the general ac quasi-static range electromagnetic fields produced by various subsurface dipole antennas, which include the HED, VED, VMD, and horizontal magnetic dipole (HMD) types, each of which is located in the upper layer of a two-layer conducting earth. For mathematical convenience, consider the conductivity of the bottom layer as being equal to zero. Thus, the problem is reduced to calculating the fields in a conducting slab. Treating the bottom layer conductivity as equal to zero is not

that restrictive; in many practical cases, the conductivity of the upper layer (σ_1) is much greater than the conductivity of the bottom layer (σ_2) , particularly in the sea-to-sea bed case. Therefore, this assumption limits the results to measurement distances (ρ) of approximately $\delta_2/5$, where δ_2 is the skin depth in the bottom layer. For example, at a frequency of 1 Hz, if $\sigma_1 = 4$ S/m (sea) and $\sigma_2 = 10^{-2}$ S/m (sea bed), then $\delta_1 \sim 250$ m and $\delta_2 \sim 5$ km. Therefore, for this example, the results should be valid to a measurement distance of approximately 1 km. Furthermore, if the conductivity of the sea bed is 10^{-4} S/m ($\delta_2 \sim 50$ km), the results should be valid to a measurement distance of approximately 10 km.

For the purpose of this report, all four sources are located at depth h ($h \ge 0$) with respect to a cylindrical coordinate system (ρ, ϕ, z) and are assumed to carry a constant current, I. The VED and HED antennas (of infinitesimal length £) are oriented in the z and x directions, respectively. The axes of the VMD and HMD antennas (of infinitesimal area A) are oriented in the z and y directions, respectively. Free space occupies the regions z < 0 and $z > \pounds_1$, whereas the conducting slab occupies the region $0 < z < \pounds_1$ (see figure 1).* Displacement currents are neglected in both the slab and the air. The magnetic permeability of the conducting slab is assumed to equal μ_0 , the permeability of free space. Meter-kilogram-second (mks) units are employed and a suppressed time factor of exp (iwt) is assumed.

MODIFIED FINITELY-CONDUCTING, EARTH-IMAGE THEORY TECHNIQUES

In the semi-infinite conducting medium, where both the source and receiving antennas are located above the earth's surface (z,h < 0) (see figure 1), the quasi-static range $(\gamma_0 \sim 0)$ integrals are of the type that can not be evaluated analytically throughout the quasi-static range,¹¹ that is

$$I_{1} \sim \int_{0}^{\infty} \left(\frac{u-\lambda}{u+\lambda}\right) e^{\lambda(z+h)} J_{0}(\lambda \rho) d\lambda, \qquad (1)$$

where

$$u = (\lambda^{2} + \gamma^{2})^{1/2} \sim 0 \text{ (air),}$$

$$u = (\lambda^{2} + \gamma^{2})^{1/2} \text{ (earth),}$$

 $J(\lambda \rho)$ = Bessel function of the first kind, order zero, and argument $\lambda \rho$.

*Figures 1 through 6 are presented at the end of the text.

Physically, the essence of the finitely conducting earth-image theory technique is to replace the finitely conducting earth with a perfectly conducting earth located at the (complex) depth d/2, where $d = 2/\gamma = \delta(1-i)$. Analytically, this corresponds to replacing the algebraic reflection coefficient $(u-\lambda)/(u+\lambda)$ in the exact integral equations by exp $(-\lambda d)$, where λ is the variable of integration. Once this is accomplished, (1) can be readily evaluated. For antennas located at, or above, the earth's surface, the general image theory approximation is valid throughout the quasi-static range.^{11,12}

When the source and receiving antennas are located below the earth's surface (z, h > 0) (see figure 1), the quasi-static range integrals to be evaluated are of the form¹⁻⁶,⁸

$$I_2 \sim \int_{\Omega} \left(\frac{u-\lambda}{u+\lambda} \right) e^{-u(z+h)} J_0(\lambda \rho) d\lambda . \qquad (2)$$

Integrals of this type can, and have, been evaluated analytically throughout the quasi-static range.¹⁻⁶ However, the resulting expressions are very complex because they involve products of modified Bessel functions of different argument. Therefore, we let⁸

$$e^{-u(z+h)} \sim e^{-\gamma a(z+h)} e^{-\gamma b(z+h)}$$
, (3)

where

a = 0 and b = 1 for $R_1/\delta \ll 1 \left[R_1 = \sqrt{\rho^2 + (z + h)^2} \right]$, a = 0.4 and b = 0.96 for R_1/δ less than approximately 1, a = 0.96 and b = 0.4 for R_1/δ between approximately 1 and 10, and a = 1 and b = 0 for $\rho > 3(z+h)$.

Substituting (3) into (2) and substituting $exp(-\lambda d)$ for $(u-\lambda)/(u+\lambda)$ results in

$$I_2 \sim e^{-\gamma a(z+h)} \int_0^{\infty} e^{-\lambda} \left[d + b(z+h) \right] J_0(\lambda \rho) d\lambda, \qquad (4)$$

which can be readily evaluated throughout the quasi-static range.

DERIVATION OF THE CONDUCTING SLAB FIELD-COMPONENT EXPRESSIONS FOR THE GENERAL QUASI-STATIC RANGE

Because we have already derived the general quasi-static range subsurface-to-subsurface field-component expressions for the semi-infinite

medium case, we can employ these results and the method of images to derive approximate expressions for the ac quasi-static fields within a conducting slab.⁸ In this situation, the multiple image pattern is an infinite array that supplements the original dipole exactly as the images of a physical object located between two plane mirrors appear to be an infinite array of that object.

HORIZONTAL ELECTRIC DIPOLE (HED)

The HED equations can easily be derived from equations (64) through (69) of an earlier report.⁸ They are

$$\begin{split} E_{\rho} \sim \frac{I\epsilon \cos \phi}{4\pi\sigma} \sum_{n=0}^{\infty} \epsilon_{n} \left\{ \frac{e^{-\gamma R_{ON}}}{R_{ON}^{3}} \left[\left(\frac{3^{2}_{\rho}}{R_{ON}^{2}} - 1 \right) \left(1 + \gamma R_{ON} \right) - \gamma^{2} x_{1}^{2} \right] \\ - \frac{e^{-\gamma R_{IN}}}{R_{IN}^{3}} \left[\left(\frac{3^{2}_{\rho}}{R_{IN}^{2}} - 1 \right) \left(1 + \gamma R_{IN} \right) - \gamma^{2} x_{2}^{2} \right] \quad (5) \\ + \frac{2e^{-\gamma a X_{2}}}{A_{IN}^{3}} \left[1 + b - \frac{3b^{3} x_{2}^{2}}{A_{IN}^{2}} - \gamma a b x_{2} \right] \right\}, \\ E_{\phi} \sim \frac{1\epsilon \sin \phi}{4\pi\sigma} \sum_{n=0}^{\infty} \epsilon_{n} \left\{ \frac{e^{-\gamma R_{ON}}}{R_{ON}^{3}} \left[1 + \gamma R_{ON} + \gamma^{2} R_{ON}^{2} \right] \right] \\ - \frac{e^{-\gamma R_{IN}}}{R_{IN}^{3}} \left[1 + \gamma R_{IN} + \gamma^{2} R_{ON}^{2} \right] \\ + \frac{2e^{-\gamma a X_{2}}}{R_{IN}^{3}} \left[1 + \gamma R_{IN} + \gamma^{2} R_{IN}^{2} \right] \\ + \frac{2e^{-\gamma a X_{2}}}{R_{IN}^{3}} \left[1 + \frac{2A^{2}_{IN}}{A^{2}_{IN}} \left(1 - \frac{A_{IN}}{A_{2N}} \right) \right] \right\}, \quad (6) \\ E_{Z} \sim \frac{1\epsilon \cos \phi}{4\pi\sigma} \sum_{n=0}^{\infty} \rho \epsilon_{n} \left[\frac{x_{1}}{R_{ON}^{5}} \left(3 + 3\gamma R_{ON} + \gamma^{2} R_{ON}^{2} \right) e^{-\gamma R_{ON}} \\ + \frac{x_{2}}{R_{IN}^{5}} \left(3 + 3\gamma R_{IN} + \gamma^{2} R_{IN}^{2} \right) e^{-\gamma R_{IN}} \right], \quad (7) \end{split}$$

$$H_{\rho} \sim -\frac{I \ell \sin \phi}{4\pi} \sum_{n=0}^{\infty} \epsilon_n \left\{ \left[\frac{d + b X_2}{\rho^2 A_{2N}} - \frac{b X_2}{\rho^2 A_{1N}} \right] e^{-\gamma a X_2} \right\}$$

$$+\frac{X_{1}}{R_{ON}^{3}}\left(1+\gamma R_{ON}\right)e^{-\gamma R_{ON}}+\frac{(d+bX_{2})}{A_{2N}^{3}}e^{-\gamma aX_{2}}$$
(8)

$$H_{\phi} \sim - \frac{I\ell \cos \phi}{4\pi} \sum_{n=0}^{\infty} \epsilon_n \begin{cases} \frac{X_1}{R_{ON}^3} \left(1 + \gamma R_{ON}\right) e^{-\gamma R_{ON}} \end{cases}$$

$$+\frac{x_{2}}{R_{1N}^{3}}\left(1+YR_{1N}\right)e^{-YR_{1N}}-\frac{e^{-YaX_{2}}}{\rho^{2}}\left[\frac{d+bx_{2}}{A_{2N}}-\frac{bx_{2}}{A_{1N}}\right]\right\},$$
 (9)

$$H_{Z} \sim \frac{I \epsilon \sin \phi}{4\pi} \sum_{n=0}^{\infty} \rho \epsilon_{n} \left\{ \frac{e^{-\gamma R} O N}{R_{ON}^{3}} \left(1 + \gamma R_{ON} \right) - \frac{e^{-\gamma R} I N}{R_{IN}^{3}} \left(1 + \gamma R_{IN} \right) + e^{-\gamma a X_{2}} \left[\frac{1}{A_{IN}^{3}} - \frac{1}{A_{2N}^{3}} \right] \right\},$$

$$(10)$$

where

$$x_{1} = 2n\ell_{1} + z - h,$$

$$x_{2} = 2n\ell_{1} + z + h,$$

$$R_{ON}^{2} = \rho^{2} + x_{1}^{2},$$

 $R_{1N}^{2} = \rho^{2} + x_{2}^{2},$ $A_{1N}^{2} = \rho^{2} + b^{2}x_{2}^{2},$ $A_{2N}^{2} = \rho^{2} + (d + bx_{2})^{2}, \text{ and}$ $\varepsilon_{0} = 1, \varepsilon_{n} = 2, n = 1, 2, 3, \dots$

HORIZONTAL MAGNETIC DIPOLE (HMD)

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The HMD equations can be derived from equations (76) through (81) of an earlier report. 8 They are

$$E_{\rho} \sim -\frac{i\omega\mu_{0}IA \cos\phi}{4\pi} \sum_{n=0}^{\infty} \epsilon_{n} \left\{ \frac{X_{1}}{R_{ON}^{3}} - \left(1 + \gamma R_{ON}\right) - e^{-\gamma R_{ON}} - \frac{X_{2}}{R_{IN}^{2}} \left(1 + \gamma R_{IN}\right) e^{-\gamma R_{IN}} + \frac{e^{-\gamma a X_{2}}}{\rho^{2}} \left[\frac{d + b X_{2}}{A_{2N}} - \frac{b X_{2}}{A_{1N}^{2}}\right] \right\}, \quad (11)$$

$$E_{\phi} \sim -\frac{i\omega\mu_{0}IA \sin\phi}{4\pi} - \sum_{n=0}^{\infty} \epsilon_{n} \left\{ -\frac{X_{1}}{R_{ON}^{3}} \left(1 + \gamma R_{ON}\right) - e^{-\gamma R_{ON}} + \frac{(d + b X_{2})}{A_{2N}^{3}} - e^{-\gamma a X_{2}} + \frac{e^{-\gamma a X_{2}}}{\rho^{2}} \left[\frac{d + b X_{2}}{A_{2N}} - \frac{b X_{2}}{A_{1N}}\right] \right\}, \quad (12)$$

$$E_{z} \sim \frac{i\omega\mu_{0}IA \cos\phi}{4\pi} - \sum_{n=0}^{\infty} \rho \epsilon_{n} \left[\frac{e^{-\gamma R_{ON}}}{R_{ON}^{3}} - \left(1 + \gamma R_{ON}\right) \right]$$

$$-\frac{e^{-YR}IN}{R_{1N}^{3}}\left(1+YR_{1N}\right)\right],$$
(13)

$$\begin{split} H_{\rho} &\sim - \frac{IA \sin \phi}{4\pi} \sum_{n=0}^{\infty} \epsilon_{n} \left\{ \frac{e^{-\gamma R_{ON}}}{R_{ON}^{3}} \left[\left(1 - \frac{3^{2}_{\rho}}{R_{ON}^{2}} \right) 1 + \gamma R_{ON} + \gamma^{2} \chi_{1}^{2} \right] \right. \\ &- \frac{e^{-\gamma R_{1N}}}{R_{1N}^{3}} \left[\left(1 - \frac{3^{2}_{\rho}}{R_{1N}^{2}} \right) (1 + \gamma R_{1N}) + \gamma^{2} \chi_{2}^{2} \right] \\ &+ e^{-\gamma a \chi_{2}} \left[\frac{1}{A_{1N}^{3}} \left(1 - \frac{3^{2}_{\rho}}{R_{1N}^{2}} \right) + \frac{1}{A_{2N}^{3}} \left(1 - \frac{3^{2}_{\rho}}{R_{2N}^{2}} \right) \right] \right] , \end{split}$$
(14)
$$H_{\phi} \sim - \frac{IA \cos \phi}{4\pi} \sum_{n=0}^{\infty} \epsilon_{n} \left[\frac{e^{-\gamma R_{ON}}}{R_{ON}^{3}} \left(1 + \gamma R_{ON} + \gamma^{2} R_{ON}^{2} \right) \right] \end{split}$$

$$-\frac{e^{-\gamma R} N}{R_{1N}^{3}} \left(1 + \gamma R_{1N} + \gamma^{2} R_{1N}^{2}\right) + e^{-\gamma a X_{2}} \left(\frac{1}{A_{1N}^{3}} + \frac{1}{A_{2N}^{3}}\right) \right], \quad (15)$$

and

$$H_{z} \sim \frac{IA \sin \phi}{4\pi} \sum_{n=0}^{\infty} \rho \epsilon_{n} \left[-\frac{X_{1}}{R_{ON}^{5}} \left(3 + 3 \gamma R_{ON} + \gamma^{2} R_{ON}^{2} \right) e^{-\gamma R_{ON}} + \frac{3(d + bX_{2})}{A_{2N}^{5}} e^{-\gamma aX_{2}} \right].$$
(16)

VERTICAL ELECTRIC DIPOLE (VED)

The VED equations can be derived from equations (73) through (75) of an earlier report. 8 They are

$$E_{\rho} \sim \frac{I\ell}{4\pi\sigma} \sum_{n=0}^{\infty} \rho \varepsilon_{n} \left[\frac{X_{1}}{R_{ON}^{5}} \left(3 + 3\gamma R_{ON} + \gamma^{2} R_{ON}^{2} \right) e^{-\gamma R_{ON}} \right]$$

$$-\frac{x_{2}}{R_{1N}^{5}}\left(3+3YR_{1N}+\gamma^{2}R_{1N}^{2}\right)e^{-YR_{1N}}\right],$$
 (17)

$$E_{z} \sim -\frac{I\epsilon}{4\pi\sigma} \sum_{n=0}^{\infty} \epsilon_{n} \left\{ \frac{e^{-\gamma R_{ON}}}{R_{ON}^{3}} \left[\left(1 - \frac{3x_{1}^{2}}{R_{ON}^{2}} \right) \left(1 + \gamma R_{ON} \right) + \gamma^{2} \rho^{2} \right] - \frac{e^{-\gamma R_{1N}}}{R_{1N}^{3}} \left[\left(1 - \frac{3x_{2}^{2}}{R_{1N}^{2}} \right) \left(1 + \gamma R_{1N} \right) + \gamma^{2} \rho^{2} \right] \right\}, \quad (18)$$

and

$$H_{\phi} \sim \frac{I\ell}{4\pi} \sum_{n=0}^{\infty} \rho \epsilon_{n} \left[\frac{e^{-\gamma R}_{ON}}{R_{ON}^{3}} \left(1 + \gamma R_{ON} \right) - \frac{e^{-\gamma R}_{1N}}{R_{1N}^{3}} \left(1 + \gamma R_{1N} \right) \right].$$
(19)

It should be noted that (17), (18), and (19) are identical to Weaver's results. 6

VERTICAL MAGNETIC DIPOLE (VMD) ANTENNA

The VMD equations can be derived from equations (70), (71), and (72) of an earlier report.⁸ They are

$$E_{\phi} \sim - \frac{i\omega\mu_{0} IA}{4\pi} \sum_{n=0}^{\infty} \rho \varepsilon_{n} \left\{ \frac{e^{-\gamma R} ON}{R_{ON}^{3}} \left(1 + \gamma R_{ON}\right) - \frac{e^{-\gamma R} IN}{R_{1N}^{3}} \left(1 + \gamma R_{1N}\right) \right\}$$

$$+ e^{-\gamma a X_{2}} \left[\frac{1}{A_{1N}^{3}} - \frac{1}{A_{2N}^{3}} \right] \right\} , \qquad (20)$$

$$H_{\rho} \sim -\frac{1A}{4\pi} \sum_{n=0}^{\infty} \rho \varepsilon_{n} \left[\frac{X_{1}}{R_{ON}^{5}} \left(3 + 3YR_{ON} + Y^{2}R_{ON}^{2} \right) e^{-YR_{ON}} + \frac{3(d + bX_{2})}{A_{2N}^{5}} e^{-YaX_{2}} \right], \qquad (21)$$

and

$$H_{z} \sim -\frac{1A}{4\pi} \sum_{n=0}^{\infty} \epsilon_{n} \left[\frac{e^{-\gamma R_{ON}}}{R_{ON}^{3}} \left\{ \left[1 - \frac{3x_{1}^{2}}{R_{ON}^{2}} \right] \left(1 + \gamma R_{ON} \right) + \gamma^{2} \rho^{2} \right\} - \frac{e^{-\gamma R_{1N}}}{R_{1N}^{3}} \left\{ \left[1 - \frac{3x_{2}^{2}}{R_{1N}^{2}} \right] \left(1 + \gamma R_{1N} \right) + \gamma^{2} \rho^{2} \right\} + e^{-\gamma A_{2N}^{2}} \left\{ \frac{1}{A_{1N}^{3}} \left[1 - \frac{3b^{2} x_{2}^{2}}{R_{1N}^{2}} \right] - \frac{1}{A_{2N}^{2}} \left[1 - \frac{3(d - bx_{2})^{2}}{R_{2N}^{2}} \right] \right\} \right].$$
(22)

DERIVATION OF THE CONDUCTING SLAB FIELD-COMPONENT EXPRESSIONS FOR THE QUASI-NEAR RANGE

Quasi-near range is defined as the asymptotic part of the quasi-static range, i.e., where the measurement distance (ρ) is much greater than a skin depth in the conducting medium and much greater than the depth of burial of the transmitting and receiving antennas. Generally, ρ must be greater than 3δ and >3(z + h). For the conducting slab case, ρ should also be greater than $3\ell_1$. However, as we shall see later, the requirement that $\rho \ge 2\delta$ and $\rho \ge 2\ell_1$ may be sufficient for most cases.

The quasi-near range approximation is setting the function $u = \sqrt{\lambda^2 + \gamma^2}$ in the exact integral expressions equal to γ , which is the propagation constant in the conducting medium.

LONG HORIZONTAL LINE SOURCE/ANTENNA

For a long horizontal line source antenna, oriented in the x-direction, the electric field within a conducting slab may be written exactly as (for z > h)

$$E_{\chi} = -\frac{i\omega\mu_0 I}{2\pi} \int_0^{\infty} \frac{F(\lambda)}{u} \cos \lambda y \, d\lambda, \qquad (23)$$

where

$$\frac{F(\lambda)}{u} = \frac{\left(e^{uh} + R_g e^{-uh}\right)\left(e^{-uz} + R_g e^{-u(2\ell_1 - z)}\right)}{u\left(1 - R_g^2 e^{-2u\ell_1}\right)},$$
 (24)

$$R_g = \frac{u - u_o}{u + u_o}$$
, and

 $u_0 = \sqrt{\lambda^2 + \gamma_0^2} \sim \lambda$ for the quasi-static range.

The magnetic fields within the conducting slab may be determined from

$$H_{Y} = -\frac{1}{i\omega\mu_{0}} \frac{\partial E_{X}}{\partial z} \text{ and } H_{Z} = \frac{1}{i\omega\mu_{0}} \frac{\partial E_{X}}{\partial y}.$$
 (26)

(25)

Application of the quasi-static ($\gamma_0 \sim 0$) and quasi-near ($u \sim \gamma$) approximations to (24) and (25) results in

$$\frac{F(\lambda)}{u} \sim \frac{2\left[\cosh \gamma h + \frac{\lambda}{\gamma} \sinh \gamma h\right] \left[\cosh \gamma (\ell_1 - z) + \frac{\lambda}{\gamma} \sinh \gamma (\ell_1 - z)\right]}{\left(\gamma \sinh \gamma \ell_1 \left[1 + \frac{2\lambda}{\gamma} \coth \gamma \ell_1\right]\right)} (27)$$

If the slab is not too thin, the denominator of (27) may be approximated as

$$\frac{1}{\gamma \sinh \gamma \ell_1 \left[1 + \frac{2\lambda}{\gamma} \coth \gamma \ell_1\right]} \sim \frac{1}{\gamma \sinh \gamma \ell_1} \left[1 - \frac{2\lambda}{\gamma} \coth \gamma \ell_1\right]. \quad (28)$$

Substituting (27) and (28) into (23) and (26), and noting that¹³

$$\int_{0}^{\infty} \cos \lambda y \, d\lambda = 0$$
 (29)

and

$$\int_{0}^{\infty} \lambda \cos \lambda y \, d\lambda = -\frac{1}{y^{2}}$$
(30)

results in

$$E_{\chi} \sim \left[-\frac{I}{\pi \sigma y^2} \right] \left[Q^2 \right] \left[\Lambda(z,h) \right] , \qquad (31)$$

$$H_{Y} \sim \left[\frac{I}{\pi \gamma y^{2}}\right] \left[Q\right] \left[B(z,h)\right], \qquad (32)$$

and

$$H_{Z} \sim \left[\frac{2I}{\pi \gamma y}\right] \left[Q^{2}\right] \left[A(z,h)\right] .$$
(33)

Equations (31), (32), and (33) are each divided into three parts. The first is the quasi-near range field-component expression valid at the surface of a semi-infinite conducting half-space.⁷ The second is the familiar planewave correction factor employed to account for the presence of stratification in the earth.¹⁴ For $\sigma_1 >> \sigma_2$,

$$Q \sim \operatorname{coth} \gamma \ell_1$$
 (34)

The third part accounts for the burial depth of the transmitting and receiving antennas. For the conducting slab situation when z is greater than h,

$$A(z,h) \sim \frac{2 \cosh \gamma h \cosh \gamma (\ell_1 - z)}{\cosh \gamma \ell_1} - \frac{\sinh \gamma (\ell_1 + h - z)}{\sinh \gamma \ell_1 \cosh^2 \gamma \ell_1}$$
(35)

and

$$B(z,h) \sim \frac{\cosh \gamma(\ell_1 + h - z)}{\cosh \gamma \ell_1} - \frac{2 \cosh \gamma h \sinh \gamma(\ell_1 - z)}{\sinh \gamma \ell_1} . \quad (36)$$

When h is greater than z, the resulting expressions are

$$A(z,h) \sim \frac{2 \cosh \gamma z \cosh \gamma (\ell_1 - h)}{\cosh \gamma \ell_1} - \frac{\sinh \gamma (\ell_1 + z - h)}{\sinh \gamma \ell_1 \coth^2 \gamma \ell_1}$$
(37)

and

$$B(z,h) \sim \frac{2 \sinh \gamma z \cosh \gamma (\ell_1 - h)}{\sinh \gamma \ell_1} - \frac{\cosh \gamma (\ell_1 + z - h)}{\cosh \gamma \ell_1} . \quad (38)$$

It should be noted that

$$\frac{\partial}{\partial z}$$
 A(z,h) $\sim \frac{\gamma}{Q}$ B(z,h) (39)

and

$$\frac{\partial}{\partial z} B(z,h) \sim \gamma Q A(z,h)$$
 (40)

When z = h = 0 or ℓ_1 ,

$$A(z,h) \sim 2 - \tanh^2 \gamma \ell_1 \sim 1 \text{ for } |\gamma \ell_1| > 1$$
(41)

and $B(z,h) \sim \pm 1$. Furthermore, when $|\gamma \ell_1| > 2\sqrt{2} (\ell_1/\delta > 2)$ and $\ell_1 > 2(z + h)$,

 $A(z,h) \sim B(z,h) \sim e^{-Y(z+h)}$. (42)

If h = 0 and $z = l_1$,

$$A(z,h) \sim 2B(z,h) \sim \frac{2}{\cosh \gamma \ell_1}$$
, (43)

which is identical to von Aulock's result.¹⁵ (See chapter 4, p. 14 of Kraichman⁷ and Bannister.¹⁶) Furthermore, if h = 0 and $\ell_1/\delta > 2$, then

$$A(z,h) \sim 2e^{-\gamma k_1} \left[2 \cosh \gamma (k_1 - z) - \sinh \gamma (k_1 - z) \right]$$
(44)

and

$$B(z,h) \sim 2e^{-\gamma \ell_1} \left[\cosh \gamma(\ell_1 - z) - 2 \sinh \gamma(\ell_1 - z) \right], \quad (45)$$

which is identical to von Aulock's result.¹⁵ (Also, see chapter 4, p. 13 in Kraichman.⁷)

HORIZONTAL ELECTRIC DIPOLE (HED) ANTENNA

By following the same procedure outlined in the derivation of the equations for the long horizontal line source antenna, the quasi-near range HED antenna field-component expressions, which are valid within the conducting slab, may readily be determined. They are

$$E_{\rho} \sim \frac{I \ell \cos \phi}{2 \pi \sigma \rho^3} Q^2 A(z,h) , \qquad (46)$$

$$E_{\phi} \sim \frac{I\ell \sin \phi}{\pi \sigma \rho^3} Q^2 A(z,h) , \qquad (47)$$

$$E_Z \sim 0$$
, (48)

$$H_{\rho} \sim \frac{I\ell \sin \phi}{\pi \gamma \rho^3} Q B(z,h) , \qquad (49)$$

$$H_{\phi} \sim - \frac{I\ell \cos \phi}{2\pi v^{0}} Q B(z,h) , \qquad (50)$$

and

$$H_{Z} \sim \frac{3Il \sin \phi}{2\pi \gamma} Q^2 A(z,h) .$$
 (51)

When h = 0 and $z = \ell_1$, the HED antenna expressions reduce to the results previously derived by Bannister.¹⁷

HORIZONTAL MAGNETIC DIPOLE (HMD) ANTENNA

By employing Maxwell's equations and the reciprocity theorem, we see that the quasi-near range HMD antenna field component expressions, which are valid within the conducting slab, may readily be determined from the HED antenna expressions. They are

$$E_{\rho} \sim \frac{\frac{i\omega\mu}{0} IA \cos \phi}{2\pi\gamma\rho^{3}} Q B(h,z) , \qquad (52)$$

$$E_{\phi} \sim \frac{i\omega\mu IA \sin \phi}{\pi\gamma\rho^3} Q B(h,z) , \qquad (53)$$

$$E_z \sim 0$$
, (54)

$$H_{\rho} \sim \frac{IA \sin \phi}{\pi o^{3}} \frac{Q}{\gamma} \frac{\partial B(h,z)}{\partial z}, \qquad (55)$$

$$H_{\phi} \sim - \frac{IA \cos \phi}{2\pi \sigma^3} \frac{Q}{\gamma} \frac{\partial B(h,z)}{\partial z}, \qquad (56)$$

and

$$H_z \sim \frac{3IA \sin \phi}{2\pi \gamma \rho} Q B(h,z) , \qquad (57)$$

where B(h,z) = B(z,h), with h and z interchanged. That is,

$$B(h,z) \sim \frac{\cosh \gamma(\ell_1 + z - h)}{\cosh \gamma \ell_1} - \frac{2 \cosh \gamma z \sinh \gamma(\ell_1 - h)}{\sinh \gamma \ell_1} .$$
(58)

Furthermore,

$$\frac{Q}{\gamma} \frac{\partial B(h,z)}{\partial z} \sim \frac{\sinh \gamma(\ell_1 + z - h)}{\sinh \gamma\ell_1} - \frac{2 \sinh \gamma z \sinh \gamma(\ell_1 - h)}{\sinh \gamma\ell_1} .$$
(59)

VERTICAL MAGNETIC DIPOLE (VMD) ANTENNA

By employing Maxwell's equations and the reciprocity theorem, we see that the quasi-near range VMD antenna field-component expressions, which are

valid within the conducting slab, may easily be determined from the HED antenna expressions. They are

$$E_{\phi} \sim -\frac{3IA}{2\pi\sigma\rho} Q^2 A(h,z)$$
 (60)

$$H_{\rho} \sim -\frac{3IA}{2\pi\gamma\rho^4} Q B(h,z)$$
, (61)

and

$$H_z \sim -\frac{9IA}{2\pi\gamma} Q^2 A(h,z)$$
, (62)

where A(h,z) = A(z,h), with z and h interchanged. That is,

$$A(h,z) \sim \frac{2 \cosh \gamma z \cosh(\ell_1 - h)}{\cosh \gamma \ell_1} - \frac{\sinh \gamma(\ell_1 + z - h)}{\sinh \gamma \ell_1 \cosh^2 \gamma \ell_1} .$$
(63)

DISCUSSION

It would be of interest to compare the results derived in this report with some known results. Figures 2 through 6 show comparisons of modified image theory, quasi-near (asymptotic) theory, and Weaver's numerical integration results⁶ for the electric and magnetic fields at the surface (z = 0) of a one-skin-depth-thick ($\ell_1 = \delta$) conducting slab produced by an HED located in the middle of the slab ($h/\delta = 0.5$). The normalized amplitude of each component (E' or H') is given by

$$H' = \frac{4\pi\delta^2 H}{I\ell \begin{pmatrix} \sin\phi\\ \cos\phi \end{pmatrix}} \text{ and } E' = \frac{4\pi\sigma\delta^3 E}{I\ell \begin{pmatrix} \sin\phi\\ \cos\phi \end{pmatrix}}.$$
 (64)

Two values of a and b are considered for the modified image theory plots. The a = 0.4 and b = 0.96 results should be valid in situations close to the source, and the a = 0.96 and b = 0.4 results should be valid at further distances. In each of these cases, only five terms of the infinite summation were needed for 1 percent accuracy.

As Weaver has indicated,⁶ this particular model possesses symmetry about the plane z = h. When z = h, all components that vary as B(z,h) equal zero. Furthermore, if z = 0 or l_1 , all components that vary as A(z,h) are equal, whereas all components that vary as B(z,h) are equal and opposite.

Figures 2 and 3 show horizontal electric field comparisons of the modified image theory, asymptotic theory, and numerical integration results. We can see from figure 2 that, for the E δ component, the modified image theory a = 0.4 and b = 0.96 curve agrees well with the numerical integration results throughout the range of ρ/δ considered ($0 < \rho/\delta < 3$). However, we observe that beyond approximately 1.25 skin depths, the (simple form) asymptotic theory agrees more closely.

We can see from figure 3 that, for the E ϕ component, the modified image theory a = 0.4 and b = 0.96 curve agrees well with the numerical integration results for $0 \le \rho/\delta \le 0.75$, while the a = 0.96 and b = 0.4 curve more closely agrees for $0.75 \le \rho/\delta \le 3$. Beyond approximately 2 skin depths, the asymptotic theory provides the better fit to the numerical integration data.

Figures 4, 5, and 6 show the magnetic field comparisons of the modified image theory, asymptotic theory, and numerical integration results. We see from figure 4 that, for the Ho component, the a = 0.4 and b = 0.96 curve agrees very closely with the numerical integration results for $\rho/\delta \leq 0.5$, but only moderately so for $\rho/\delta > 0.5$. The a = 0.96 and b = 0.4 curve is in good agreement beyond 1.5 skin depths, while the asymptotic theory curve provides the best fit beyond 2.5 skin depths.

In figure 5, we can see that, for the H ϕ component, the modified image theory a = 0.4 and b = 0.96 curve agrees very closely with the numerical integration results to $\rho/\delta \sim 2$. Beyond that, the asymptotic theory curve provides the best fit.

In figure 6, we can see that, for the Hz component, the modified image theory, a = 0.4 and b = 0.96 curve agrees very well with the numerical integration results for $\rho/\delta < 1$. Beyond $\rho/\delta = 1$, the a = 0.96 and b = 0.4 curve agrees more closely. For $\rho/\delta > 2$, the asymptotic theory curve also agrees well with the numerical integration results.

Thus, it appears that the (simple form) asymptotic theory will provide results of sufficient accuracy when the measurement distance is greater than 2 skin depths and greater than twice the slab depth (i.e., $\rho/\delta > 2$ and $\rho/\ell_1 > 2$).

CONCLUSIONS

Approximate expressions for both the general ac quasi-static and quasinear fields produced by electric and magnetic dipole antennas located within a conducting slab have been derived by employing finitely conducting earthimage theory techniques and by applying the quasi-near approximation to the basic Sommerfeld integrals.

We have demonstrated that the resultant approximations very closely agree with previously derived numerical integration results. In particular, it appears that the (simple form) asymptotic theory will provide sufficiently accurate results when the measurement distance is greater than 2 skin depths and greater than twice the slab depth.

Although displacement currents in the conducting slab have been ignored in the analysis, they can be included by simply replacing σ with σ + iwe in the field strength equations, providing $|\gamma^2| >> |\gamma_0^2|$. The resultant expressions are applicable to short range electromagnetic propagation in a shallow sea.

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gure 1. Comparison of Modified Image Theory, Asymptotic Theory, and Meaver's Seistic for the P. Component







Figure 3. Comparison of Modified Image Theory, Asymptotic Theory, and Weaver's Result for the E_{ϕ} Component









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Figure 5. Comparison of Modified Image Theory, Asymptotic Theory, and Weaver's Result for the H_{ϕ}^{*} Component

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Figure 6. Comparison of Modified Image Theory, Asymptotic Theory, and Weaver's Result for the H^{*}_z Component

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