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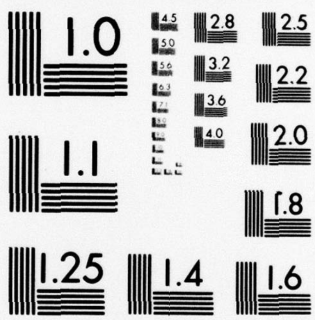

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Curve-of-Growth Function for a Random Array of Voigt Lines

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Interim Report

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This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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The accuracy of two approximations to the curve-of-growth function V(a, x) for a random array of Voigt lines with exponential line strength distribution is examined. An heuristic extension of the Rodgers and Williams approximation for isolated Voigt lines is accurate to better than 9% for all x and a. A modified version of the approximation used in the NASA plume radiation code is accurate to better than 4% for all x and a.

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CURVE-OF-GROWTH FUNCTION FOR A RANDOM ARRAY OF VOIGT LINES

The curve-of-growth function for a random array of Voigt lines whose strengths are distributed exponentially is[1]

$$V(a,x) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} \frac{xK(a,y)}{1 + xK(a,y)}\, dy \qquad (1)$$

where

$$K(a,y) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-u^2}}{a^2 + (y-u)^2}\, du$$

$$a = \sqrt{\ell n 2}\; \bar{\gamma}_L / \bar{\gamma}_D$$

$$x = \sqrt{\frac{\ell n 2}{\pi}}\; \frac{\bar{S}u}{\bar{\gamma}_D}$$

$\bar{S}$, $\bar{\gamma}_L$, and $\bar{\gamma}_D$ are, respectively, the mean line strength, Lorentz (collision) halfwidth, and Doppler halfwidth of the lines comprising the array, and u is the optical depth of the medium.

Gille and Ellingson[1] evaluated V(a, x) by numerical quadrature and presented their results in the form

$$V(a,x) = \acute{C}(x,a)\, F(a,x) \qquad (2)$$

F(a, x) is the curve-of-growth function that would result if the lines were purely Lorentzian

$$F(a,x) = \frac{x}{\sqrt{1 + 2(x/2a\sqrt{\pi})}} \tag{3}$$

C(x, a) is tabulated in Table 1 of Ref. 1. The connection between the present notation and that of Ref. 1 is $x \equiv R$ and $a \equiv d/2$. The accuracy of the two following approximations is determined by comparison to this solution.

Rodgers and Williams[2] devised an approximation for the curve of growth of an isolated Voigt line by combining the curves of growth for pure Lorentz and Doppler lines. An heuristic extension of their method to an array of Voigt lines yields

$$V(a,x) = \sqrt{F^2(a,x) + E^2(x) - \left[\frac{F(a,x)\,E(x)}{x}\right]^2} \tag{4}$$

where F(a, x) is given by Eq. (3), and E(x) is the curve-of-growth function for an array of Doppler lines[3]

$$E(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{xe^{-u^2}}{[1 + x\exp(-u^2)]}\,du \tag{5}$$

The isolated line version of Eq. (4) displays a maximum error of ~ 8%. It occurs near the point a = 0.4, x = 1 and along the curve $ax \simeq 15$ for $a \lesssim 0.0008$ (see Fig. 3 of Ref. 2). For an array of lines, the error topography is qualitatively similar, but the error is more near 9% around the point and only ~ 4% along the curve.

The NASA radiation code[4] employs a model that also combines the curves of growth for purely Lorentz and Doppler arrays in order to approximate the curve of growth for a Voigt array. In a form consistent with Eqs. (2) and (4), it is

$$V(a, x) = x\sqrt{1 - 1/\sqrt{Y(a, x)}} \tag{6}$$

$$Y(a, x) = \left[1 - \left(\frac{F(a, x)}{x}\right)^2\right]^{-2} + \left[1 - \left(\frac{E(x)}{x}\right)^2\right]^{-2} - 1$$

Two forms of this approximation are considered. In the first form, E(x) is taken as

$$E(x) = 0.62714\sqrt{\ell n\left[1 + \left(\frac{x}{0.62714}\right)^2\right]} \tag{7}$$

which is consistent with the actual function prescribed in the NASA code. In this case, errors up to 18% occur over the (x, a) plane and fall below 10% only for $a \gtrsim 0.3$. The explanation for large errors below $a \simeq 0.3$ is that Eq. (7) is derived from an approximation to the isolated line curve-of-growth function D(x) rather than to E(x) itself. If the proper expression [Eq. (5)] is used for E(x) in Eq. (6), the maximum error falls to less than 4% and occurs only for $a \lesssim 0.0005$ and $x \gtrsim 3 \times 10^4$.

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