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MULTIPLE-COUPLED RANDOM ACCESS TECHNIQUES
FOR PACKET RADIO NETWORKS

Lawrence Charles Siegel

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Electronic Systems Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASSACHUSETTS 02139

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LAWRENCE CHARLES SIEGEL

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MULTIPLE-COUPLED RANDOM ACCESS TECHNIQUES

FOR PACKET RADIO NETWORKS

by

LAWRENCE CHARLES SIEGEL

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Computer Science on May 23, 1978 in partial fulfillment
of the requirements for the Degree of Bachelor of Science.

ABSTRACT

The random accessing problem for packet radio networks
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problem, is developed. The deadlock-free property is
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the network is presented.

Thesis Supervisor: James L. Massey,
                        Professor of Electrical Engineering

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My deepest thanks go to my parents for all of their
love and support.

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I. INTRODUCTION

I.A. Multiple Access and Random Access

The need for systems to provide data communication
capabilities for a network of terminals and computers has
posed some new and difficult problems. A central problem
is the choice of the organizing principle to be employed
to determine when a user may access the communication
channel(s) of the system. In the case of a single channel
shared by many users, this multiple access problem may
be solved by means of time-division multiple accessing (TDMA)
or frequency-division multiple accessing (FDMA). These
techniques consist of dividing the channel capacity equally
among all users and allowing each user to use his portion
of the channel when he wishes to. They are equivalent to
having a separate independent channel for each user. Such
a scheme may not be efficient for a system of terminals
which have "bursty" communication demands, that is, terminals
that operate at low duty cycles. In order to make efficient
use of communication facilities, traffic from many terminals
must be truly merged for transmission over one channel.
This shared channel has capacity large enough for the
average demand of the user population rather than for the
sum of the peak demands. Thus, the shared channel provides
each user with high bandwidth communication capability

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but the overall bandwidth requirement for the system is
low.   The central problem in such a random access situation
is the scheme by which the accessing is accomplished.

I.B.   Packet Radio Networks

A packet radio network consists of geographically
distributed, and possibly mobile, terminals, repeaters,
and stations which communicate via the broadcast medium.
Messages are sent in fixed length segments called packets
on a shared radio channel.

The random access schemes proposed in the past are
appropriate only for one-hop packet radio systems. In
such a system, all terminals transmit on a common channel
directly to a single station, as shown in Figure 1. The
assumption is usually made that if two or more terminals
transmit simultaneously, the packets will collide, and
the station will not receive any good data. The random
access problem deals with how to coordinate transmissions
from the terminals to get messages to the station. The
two basic random access methods which have been proposed
for one-hop systems are the "classical" Aloha technique
of Abramson [1] and the recent tree algorithm of Capetanakis [2].
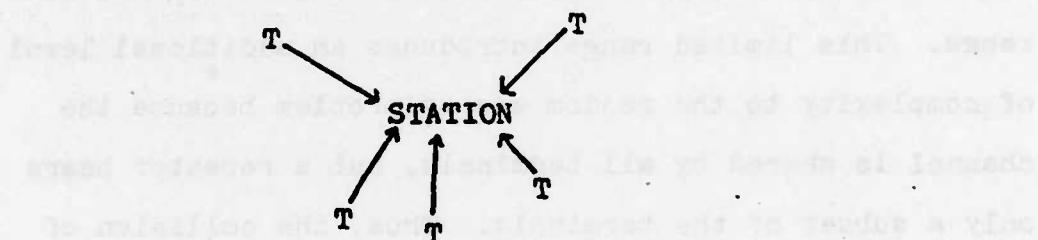
FIGURE 1.   A one-hop packet radio system.

I.C. Multiple-Coupled Random Access

In a multiple hop packet radio system, terminals
transmit to repeaters which forward the messages to other
repeaters, and eventually to the destination. Repeaters
are necessary because terminals have limited transmission
range. This limited range introduces an additional level
of complexity to the random access problem because the
channel is shared by all terminals, but a repeater hears
only a subset of the terminals. Thus, the collision of
two or more packets is more complex in such a system.
By moving from a single hop system in which terminal
interference is always total to a multiple hop system
with terminal interference which may be only partial,
the usual single random access problem becomes a multiple-
coupled random access problem. The presence of more than
one repeater makes the problem multiple random access,
and the possibility of geometries in which more than one
repeater may hear a single terminal provides the coupled
aspect. Figure 2 shows a typical multiple hop system.
In considering the multiple-coupled random access problem,
we must address the network aspect which is absent in the
single random access problem.

A model for the multiple-coupled random access problem
is developed in section II. Algorithms for the multiple-

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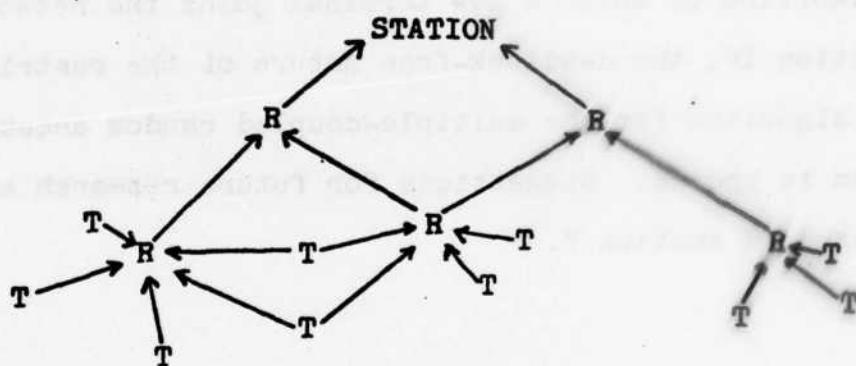
FIGURE 2.   A multiple-hop packet radio system.

coupled random access problem are considered in section III. Deadlock is defined and a deadlock-free algorithm called "the restricted-entry algorithm for the multiple-coupled random access problem" is presented. The process by which a new terminal may join the network is stated algorithmically. The two major results of this thesis are the deadlock-free algorithm for random accessing and the algorithm by which a new terminal joins the network. In section IV, the deadlock-free nature of the restricted-entry algorithm for the multiple-coupled random access problem is proved. Suggestions for future research are discussed in section V.

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II. A MODEL FOR THE MULTIPLE-COUPLED RANDOM ACCESS PROBLEM

The model we shall adopt to study the multiple-coupled random access problem consists of the following:

1) a main data channel which is slotted in time.

2) a set of repeaters which receive packets.

3) a set of terminals which send packets.

A terminal is "connected" to a repeater if the repeater receives the packet when this terminal transmits a packet and no other terminal transmits in the same slot. Note that we consider all types of connectivity, that is, we allow a terminal to be connected to any subset of the repeaters.

4) perfect feedback channels from repeaters to terminals. After each slot, a teminal learns from each repeater to which it is connected whether the repeater heard

a) no packets,

b) 1 packet (which was thus correctly received),

or c) 2 or more packets,

in the previous slot.

5) service channels used for initial entry of terminals into the system.

6) information within each packet identifying the

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<u>intended</u> repeater.

The sending terminal regards the packet as received correctly if and only if it is received by the intended repeater. It must retransmit the packet if it is correctly received but only by unintended repeaters. If we envision the repeaters forwarding the intended packets which they receive, this assumption seems reasonable because it implies that multiple copies of a packet will not be forwarded.

In Figure 3, we show an example of a network corresponding to our model. We remark that what the repeaters do with packets which have been successfully received (such as forwarding them to other repeaters, etc.) is not at issue in this thesis. We wish to study the random access problem in as much isolation as possible.
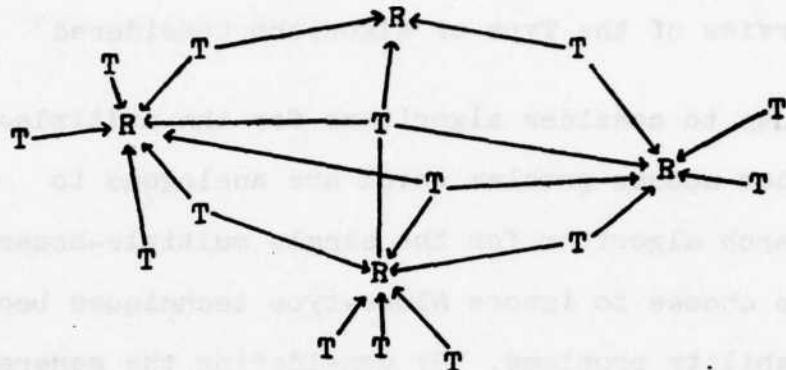
FIGURE 3. An example of a packet radio network corresponding
to our model. There are many repeaters and there are
various forms of connectivity of the terminals.

III. DEADLOCK-FREE ALGORITHMS FOR THE MULTIPLE-COUPLED
RANDOM ACCESS PROBLEM

III.A. Overview of the Type of Algorithm Considered

We choose to consider algorithms for the multiple-
coupled random access problem which are analogous to
the tree search algorithm for the single multiple-access
problem. We choose to ignore Aloha-type techniques because
of their stability problems. By considering the general
features of a tree search contention resolution algorithm,
we will be led to the problem of deadlock for the model
we are considering. We will see that deadlock occurs when
the tree algorithm for the single random access problem
is naively generalized to a tree algorithm for the multiple-
coupled random access problem. The main result of this
thesis is the development of a deterministic tree search
algorithm which is deadlock-free.

III.B. Tree Search Contention Resolution Algorithm

A tree search contention resolution algorithm is used
for each repeater. Each terminal has a fixed binary address
for each repeater. No two terminals have the same address
on the same repeater. Thus, for each repeater, the terminals'
addresses are distinct leaves on a binary tree. The "state"
of the algorithm for a repeater consists of an i bit address,
S, corresponding to a node in the tree.

III.B.1. Updating the State of a Repeater

After a slot, the algorithm for determining the new
state proceeds as follows:

(1)   If a collision has occurred, then

(2)   if i<m (the maximum address length)

(3)   i←i+1   (go one more level in tree)

(4)   S(i)←0   (consider first branch)

(5)   exit

(6)   if i=m

(7)   no change in i or S

(8)   exit

(9)   If a packet gets through, then

(10)   if S(i)=0

(11)   S(i)←1   (now consider second branch)

(12)   exit

(13)   if S(i)=1

(14)   i←i-1   (back up)

(15)   if i=0, exit

(16)   go to (10)

(17) If there is no transmission, then

(18)   if S(i)=0

(19)   S(i)←1   (drop down to second)

(20)   exit

(21)   if S(i)=1

(22)   go to (14)   (back up)

The state cannot exceed the maximum length of m bits. Steps (6), (7), and (8) accomplish a truncation of the state. That is, if a collision occurs and the state is already at its maximum length, then, rather than extend the state by one bit as is normally done after a collision, the state is simply left unchanged. The approach taken here is a deterministic one with finite length states and addresses. If a nondeterministic scheme were used, such as a random address method, then a state could theoretically become infinite in length. The inability to represent an infinite state in any physical system implies that some sort of state truncation procedure must actually exist. This procedure may introduce deadlock because the nondeterministic behavior, which would guarantee no deadlocks, is actually not present.

III.B.2. Repeaters "Call for" Terminals

A repeater "calls for" a terminal if the i bits of the state of the repeater are identical to the first i bits of the address of the terminal. We may treat the state of a repeater as a binary fraction b, $0 \leq b < 1$, and a length i. If we also treat the state of a terminal as a binary fraction b^T, then the repeater calls for the terminal when $b^T \in [b, b+2^{-i})$. For example, the state 110 corresponds to calling for terminal b^T when $b^T \in [3/4, 7/8)$.

III.B.3. Rule for Transmission

A rule must be employed as to when a terminal shall transmit. For example, one possible rule would be that a terminal transmit only when it is called for by <u>all</u> repeaters to which it is connected. Another possible rule is that a terminal transmit only when it is called for by <u>at</u> <u>least</u> <u>one</u> repeater to which it is connected. In choosing a rule, we must be wary of deadlocks that may lurk therein.

III.C. Deadlock

Let T_i be the ith terminal

R_k by the kth repeater

$S_k(t)$ be the "state" of repeater R_k at time t

$T^A(t) = \left\{ T_i : \; T_i \text{ has a packet to send at time } t \right\}$

The system is deadlocked if

1) $T^A(t) \neq \emptyset$,

2) During the interval t to $t+T$ (where $T \geq 1$ slot) no new packets arrive to be sent,

3) $T^A(t+T) = T^A(t)$, that is, no packets are successfully transmitted in the interval,

and 4) $S_k(t+T) = S_k(t)$ for all k, that is, return to previous state.

In words, the system is deadlocked if for an interval in which no new packets arrive, all repeaters return simultaneously to their initial states and no packet has gotten through in the interval.

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III.D.   The Rule, "Send When All Repeaters Call", Deadlocks

The example of figure 4 illustrates that if the rule
is adopted that a terminal transmit only when all repeaters
to which it is connected call for it, deadlock may result.
Thus, this rule is too restrictive and leads to deadlock.

III.E.   The Rule, "Send When Any Repeater Calls", Deadlocks

Under the rule that a terminal shall send when it
is called for by any repeater to which it is connected,
it is possible for a repeater's state to be such that $i=m$
(that is, the state is at a final node in the tree) and
for a collision to occur. We shall consider two different
methods of handling this situation:

Method 1: Simply retransmit in the next slot.

> This is the method prescribed by the tree
> algorithm as we have written it. Deadlock may
> result as shown in the example of figure 5.

Method 2: Continue as if there had been no trasmission.

> That is, we modify the tree search algorithm as
> follows:

>> change line (7) to   go to (10)

> This method may deadlock, as the example of
> figure 6 indicates.
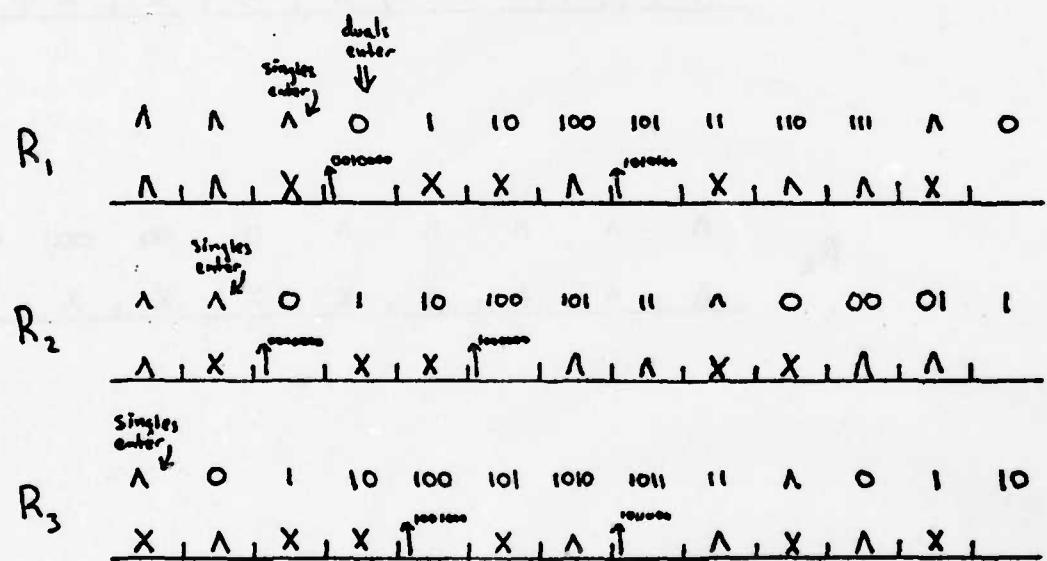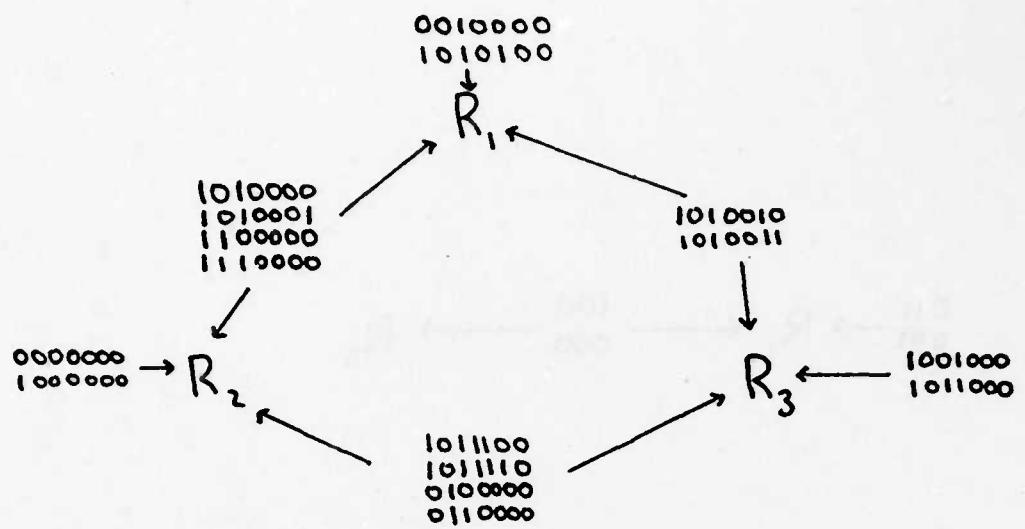
FIGURE 4. An example of the rule "send when all repeaters call" which demonstrates deadlock.
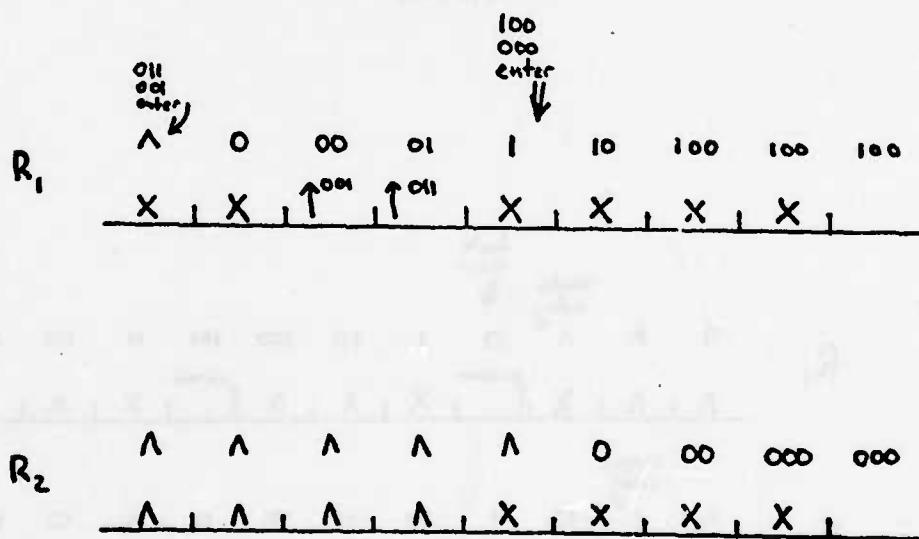
FIGURE 5. An example of the first version of the rule
"send when any repeater calls" which demonstrates
deadlock.

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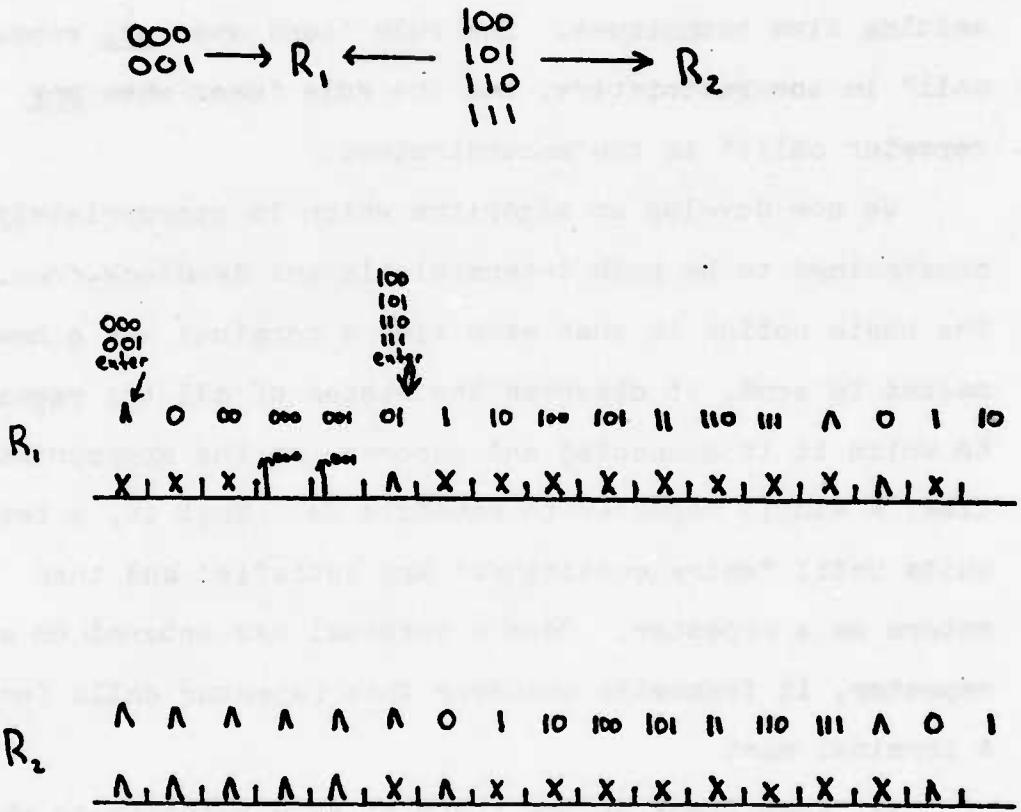
FIGURE 6. Example of the rule "Send when either says to,
and if collision occurs when at end of address, just
continue (packet gets skipped)." We observe deadlock.

III.F.   The Restricted-Entry Algorithm for the Multiple-
Coupled Random Access Problem

We want a scheme which does not deadlock and which
is deterministic, that is, which does not employ any random
waiting time techniques. The rule "send when all repeaters
call" is too restrictive, and the rule "send when any
repeater calls" is too unconstrained.

We now develop an algorithm which is appropriately
constrained to be both deterministic and deadlock-free.
The basic notion is that each time a terminal has a new
packet to send, it observes the states of all the repeaters
to which it is connected and chooses, at the appropriate
time, a single repeater to schedule on. That is, a terminal
waits until "entry conditions" are satisfied and then
enters on a repeater. Once a terminal has entered on a
repeater, it transmits whenever this repeater calls for it.
A terminal must

- a) keep track of the state of each repeater to which
      it is connected,
- b) decide when to enter on a repeater if it has a
      packet and has not yet entered on a repeater,

and c) transmit when appropriate.

We may consider the actions a terminal must take in a time
sequence for ease of discussion (Figure 7).

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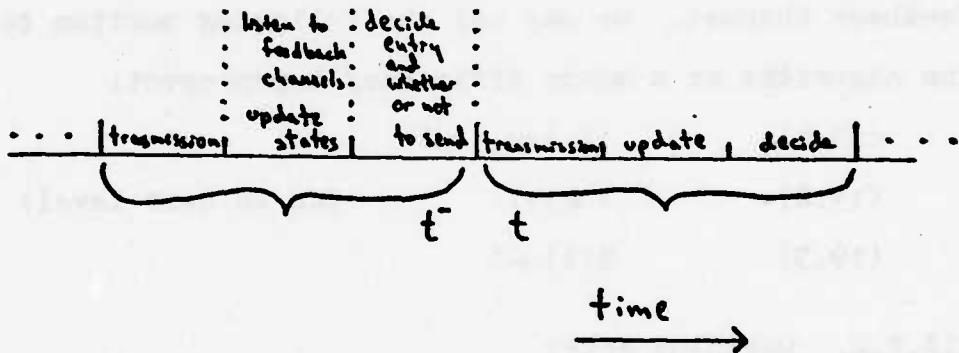
FIGURE 7. The time sequence for the actions of a terminal.

III.F.1. Updating of State

After each transmission, a terminal must update the state for each repeater to which it is connected. The algorithm for updating a state is given in III.B.1. The terminal learns whether a repeater heard no transmission, a good transmission, or a collision by listening to the feedback channel. We may add the following portion to the algorithm as a minor efficiency improvement:

$$(19.1) \qquad \text{if } i=m, \text{ exit}$$

$$(19.2) \qquad i \leftarrow i+1 \qquad (\text{go to next level})$$

$$(19.3) \qquad S(i) \leftarrow 0$$

III.F.2. Deciding Entry

After updating the states for each repeater to which a terminal is connected, the terminal, if it has a packet to send and has not yet scheduled, must determine whether or not to enter on a repeater.

III.F.2.a. Notation

First, let us establish some notation:

Let T_i be the ith terminal

R_k be the kth repeater

$A_k^{T_i}$ be the address of terminal T_i on repeater R_k

$S_k(t)$ be the state of repeater R_k at time t

Connectivity

$R_i^C = \left\{ R_k: \ R_k \text{ hears } T_i \right\}$ all repeaters hearing T_i

$T_k^C = \left\{ T_i: \ T_i \text{ is heard by } R_k \right\}$ all terminals which R_k hears

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Activity

$$T_k^E(t) = \left\{ T_i: \quad T_i \text{ enters on } R_k \text{ at time } t \right\} \quad \text{entering}$$

terminals

$$T^E(t) = \bigcup_k T_k^E(t)$$

$$R^A(t) = \left\{ R_k: \quad S_k(t) \neq \Lambda \text{ (that is, } i \neq 0) \text{ or } T_k^E(t) \neq \emptyset \right\}$$

active repeaters

$$R^D(t) = \left(R^A(t) \right)^c \qquad \text{dormant repeaters}$$

Age

$$R_i^Y(t^-) = \left\{ R_k: \quad R_k \, (R_i^C \quad R^A(t^-)) \text{ and } S_k(t^-) \leq A_k^{T_1} \right\}$$

young repeaters

The set $R_i^Y(t^-)$ consists of all repeaters that
T_i is connected to that are running tree
resolutions and have states such that T_i will
be called for during the current epoch (that is,
before the state of the repeate; returns to null).

Note that the inequalities to compare stæ s and addresses
may be interpreted by using the binary fraction representation
of the states and addresses.

III.F.2.b. The Threshold $\tilde{S}_{1,k}^{T_i}$

The threshold on R_1 for T_i entering on R_k, $\tilde{S}_{1,k}^{T_i}$,
is such that

for each T_m

if for all 1 such that $R_1 \in (R_m^C \cap R_i^C)$

$$1 \neq k$$

we have $A_1^{T_m} \geq \tilde{S}_{1,k}^{T_i}$

then $R_m^C \subset R_i^C$

and $R_k \notin R_m^C$

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III.F.2.c. Entry

If

T_i has a packet waiting at time t^-

and $R_i^Y(t^-) \neq \emptyset$

and for some k

$R_k \in R_i^Y(t^-)$

and $S_l \tilde{S}_{l,k}^{T_i}$ for all l: $R_l \in R_i^C - \{R_k\} - R^D(t^-)$

then

T_i will enter on R_k at time t

(that is, $T_i \in T_k^E(t)$)

If

T_i has a packet waiting at time t^-

and $R_i^Y(t^-) = \emptyset$

and $R_i^C \cap R^D(t^-) \neq \emptyset$

and for the minimum value of k such that $R_k \in R_i^C \cap R^D(t^-)$,

$S_l \gtrless \tilde{S}_{l,k}^{T_i}$ for all l: $R_l \in R_i^C - R^D(t^-)$

then

T_i will enter on R_k at time t

(that is, $T_i \in T_k^E(t)$)

The fundamental concept of the entry rule is that a terminal, T_i, will not enter on a repeater, R_k, when T_i fears that T_i will cause interference at another repeater R_l which might call for a terminal T_j that will either cause interference at R_k, or cause interference at some third repeater to which T_i is not connected. The thresholds

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are chosen such that when a terminal observes that the state of a repeater is beyond the threshold, the terminal knows that no terminal will be called for by the repeater which would cause such undesired interference. Figures 8 and 9 give examples of the entry restrictions.

III.F.3. Deciding Whether Or Not To Send

A terminal transmits in a slot when it is called for by the repeater it is scheduled on.

III.G. Appearance of a New Terminal in the Network

In the above discussion, we have assumed that a terminal knows its address for each repeater and the thresholds for entry on each repeater. We now address the question of how a terminal may initially obtain these addresses and thresholds. The procedure is as follows:

1) New terminal appears.

2) Terminal finds out which repeaters it is connected to through use of service channel.

3) Terminal describes (using service channel) its connectivity to each repeater to which it is connected.

4) Each repeater supplies an address for terminal.

5) Each repeater supplies the thresholds.

6) Each repeater tells its current state to the terminal to enable the terminal to thereafter
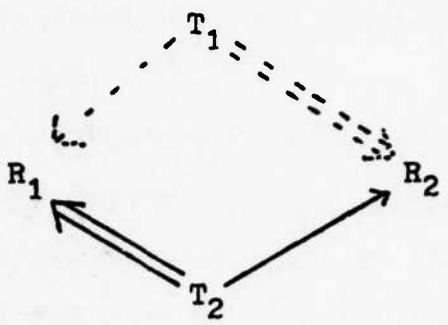
FIGURE 8. Example of restriction on entry. T_1 does not
enter on R_2 because it fears that, with T_2 running
on R_1, deadlock may result.
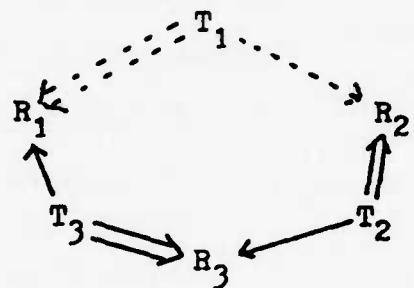
FIGURE 9. Example of restriction on entry. T_1 will not
enter on R_1 because it fears that, with T_2 running
on R_2 and causing interference at a repeater to which
T_1 is not connected (T_1 cannot know the state of R_3),
there might be T_3 running on R_3 and causing interference
at R_1. Fear of circular-type deadlock prevents
entry.

compute the subsequent states by listening to

the feedback channels and using the state updating

algorithm.

We note that this exchange of information between a terminal

and the repeaters occurs only once for a terminal, that

is, when a terminal first appears in the network. The

service channel can thus be of quite low capacity.

III.G.1. Repeater Supplies Address for Terminal

We shall consider a specific algorithm for assigning

addresses. Note that alternative schemes might be employed.

The algorithm presented should help clarify the process

by which addresses and thresholds may be assigned in

general.

Consider N repeaters

$$R_1, R_2, \ldots, R_N$$

The binary address on R_k for a terminal T_1 has 3 components:

$$\underbrace{}_{\text{group}} \underbrace{}_{\text{subgroup}} \underbrace{}_{\text{leaf}}$$

The length of group is $\log_2 N$ bits.

Let $w = \#$ of _other_ repeaters T_1 is connected to

The length of the subgroup is $\log_2 \binom{N-1}{w}$ bits.

group $\leftarrow$ comp(w)

where comp takes the bitwise complement.

We note that as the group increases, the level of connectivity

diminishes.

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Let $b^k = b_{k+1}, b_{k+2}, \ldots, b_N, b_1, b_2, \ldots, b_{k-1}$

where

$$b_j = \begin{cases} 1 & \text{if } T_j \text{ is connected to } R_j \\ 0 & \text{if } T_j \text{ is not connected to } R_j \end{cases}$$

subgroup $\leftarrow$ comp($i(b^k)$)

where $i(b^k)$ is the ordinal number for the bit sequence b^k which is of length N-1 and weight w. The algorithm to compute the function i and its inverse, i^{-1}, is presented in [3].

The group specifies the level of connectivity, and the subgroup splits up terminals of the same level of connectivity according to which set of repeaters they are connected to. The leaf provides the final resolution to separate the terminals of identical connectivity. The leaf is chosen to allow quick separation when the binary tree is searched. For each group,subgroup , a counter, n, is maintained. Every time a new terminal appears in this group,subgroup, the leaf is given as

comp(rev(n)$_2$)

and the counter, n, is incremented. rev(n)$_2$ specifies bit reversal. For example, rev(01100001)$_2$=10000110. The notion underlying this bit reversal scheme is that for quick separation in the tree, we want the most significant bits to have the greatest variability. But a counter has the most variability in the least significant bits; hence, we use bit reversal. As an example, consider the order in which leaves are assigned for a 4 bit leaf as shown in Figure 10.

| n	comp(rev(n)$_2$)
0000	1111
0001	0111
0010	1011
0011	0011
0100	1101
0101	0101
0110	1001
0111	0001
1000	1110
1001	0110
1010	1010
1011	0010
1100	1100
1101	0100
1110	1000
1111	0000
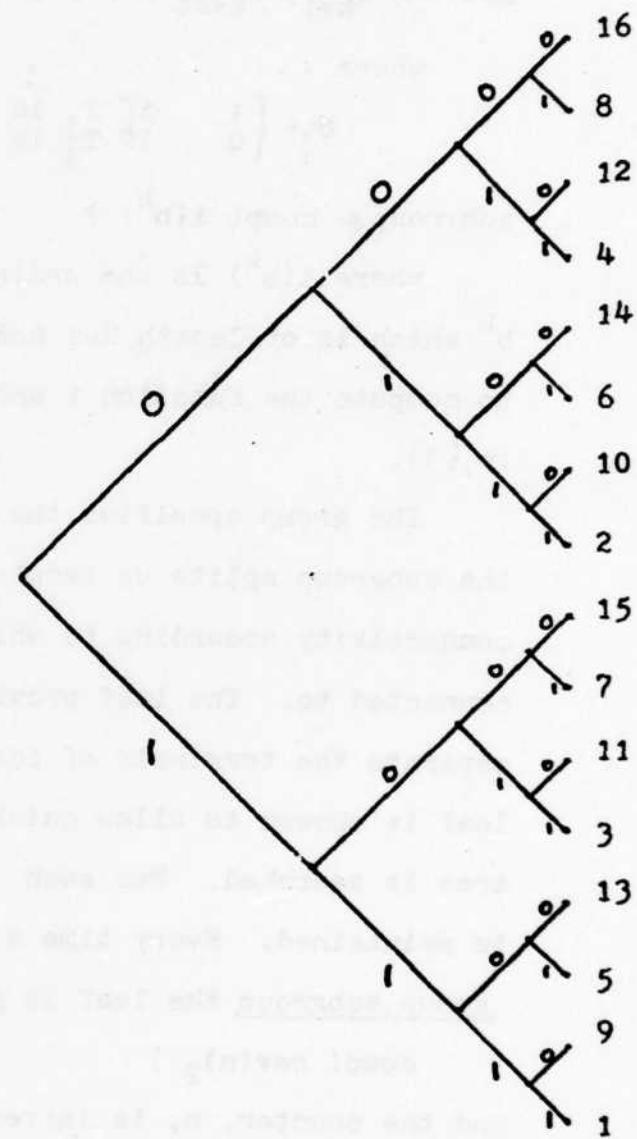
FIGURE 10. An example indicating the order in which 4
bit leaves are issued to terminals.

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We have used the bit-wise complement throughout to make any unused portion of the tree occupy the low (early) address space. This scheme will tend to make the state of a repeater get to a higher value more quickly thus providing better information with regard to making comparisons with thresholds. Alternatively, we could eliminate the use of the complements in assigning the addresses and simply complement the state by appropriately modifying the state update algorithm.

It is worthwhile to note that the bit reversal scheme for generating the leaves might also be used advantageously in the single-hop random access problem where the leaf is the complete address.

III.G.2. Repeater Supplies Thresholds for Terminal

A repeater, R_k, must supply a terminal, T_i, with thresholds for each repeater R_1 ($1 \neq k$) to which T_i is connected. A threshold is chosen to satisfy the requirements as specified in III.F.2.b. A repeater must know the connectivity of the terminal and the structures of the address spaces at each repeater in order to construct a minimally restrictive threshold.

A scheme for finding a threshold, $\tilde{S}^{T_i}_{1,k}$, when the address spaces have been assigned as described above in III.G.1 is given below.

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Knowing the algorithm for assigning addresses, a
repeater is able to consider group, subgroup, possibilities
and determine the corresponding connectivity. A repeater
is thus able to determine the minimum value threshold which
has the connectivity properties specified in III.F.2.b.

$d \leftarrow b^k$

$$\text{(that is,} \quad d_0, d_1, \ldots, d_{N-2} = b_{k+1}, b_{k+2}, \ldots,$$
$$b_N, b_1, b_2, \ldots, b_{k+1}$$

so $\quad d_0 = b_{k+1}$

$\qquad d_1 = b_{k+2}$

$\qquad \vdots$

$\qquad \qquad)$

$d_i \leftarrow d_{i+1} \quad$ for $i = ((1-(k+1))_N$ to $N-1 \qquad$ (remove $((1-(k+1))_{N-1})$

$d_{N-2} \leftarrow 0$

$d \leftarrow comp(d) \qquad\qquad$ (complement)

$c_j \leftarrow d_{((j-(k-1)))_{N-1}} \qquad$ for $j=0$ to $N-2 \qquad$ (rotate circularly to

the right $k-1$ places)

The bit string c is a bit mask which we can compare to b^1
to determine if there is a terminal of undesired connectivity.

$group_1 \leftarrow comp(000)$

$group_2 \leftarrow comp(001)$

$subgroup_1 \leftarrow comp(000)$

$subgroup_2 \leftarrow comp(000)$

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```
        ↓
    w ←comp(group₂)                    (weight)
    b ←i⁻¹(comp(subgroup₂))            (use inverse ordinal
                                        function to get N-1 bit
                                        string of weight w which
                                        describes connectivity)

    if b  bit-wise and  c  is 0
        group₁ ←group₂
        subgroup₁ ←subgroup₂
        subgroup₂ ←subgroup₂-1
        if subgroup₂=comp( (N-1 w) )
            group₂ ←group₂-1
            subgroup₂ ←comp(000)
        go to ──┐
```

$$w \leftarrow comp(group_2) \qquad \text{(weight)}$$
$$b \leftarrow i^{-1}(comp(subgroup_2)) \qquad \text{(use inverse ordinal function to get N-1 bit string of weight w which describes connectivity)}$$

if b bit-wise and c is 0

$$group_1 \leftarrow group_2$$
$$subgroup_1 \leftarrow subgroup_2$$
$$subgroup_2 \leftarrow subgroup_2 - 1$$
if $subgroup_2 = comp\left(\binom{N-1}{w} \right)$
$$group_2 \leftarrow group_2 - 1$$
$$subgroup_2 \leftarrow comp(000)$$

go to ──┐

$$threshgroup \leftarrow group_1$$
$$threshsubgroup \leftarrow subgroup_1$$

threshold

$$\bar{s}^{T_1}_{1,k} \leftarrow \underline{threshgroup, threshsubgroup}$$

III.H. Example Network

III.H.1. Addresses and Thresholds

The example of Figure 11 is a typical three repeater system. The addresses and thresholds are given in Figure 12 as they would be assigned by the algorithms described above.
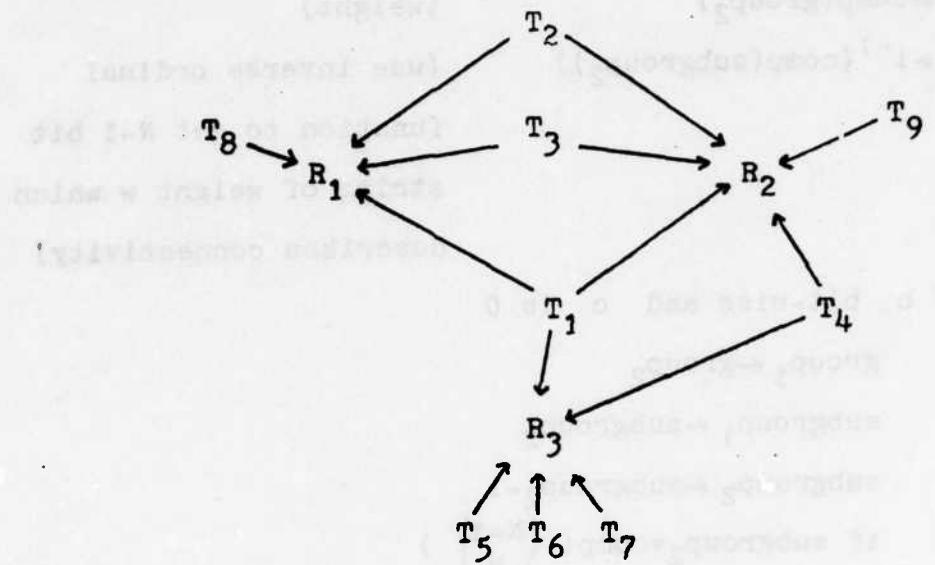
FIGURE 11. A typical three repeater system.

T_i	$S_1^{T_i}$	$S_2^{T_i}$	$S_3^{T_i}$	$S_{2,1}^{T_i}$	$S_{3,1}^{T_i}$	$S_{1,2}^{T_i}$	$S_{3,2}^{T_i}$	$S_{1,3}^{T_i}$	$S_{2,3}^{T_i}$
T_1	0111	0111	0111	1100	1010	1010	1100	1100	1010
T_2	1001	1011		1100		1100			
T_3	1000	1010		1100		1100			
T_4		1001	1011				1100		1100
T_5			1111						
T_6			1101						
T_7			1110						
T_8	1111								
T_9		1111							

FIGURE 12. The addresses and thresholds for the example network of Figure 11.

III.H.2. Transmission Sequences

Example transmission sequences are given in Figures 13, 14, and 15.
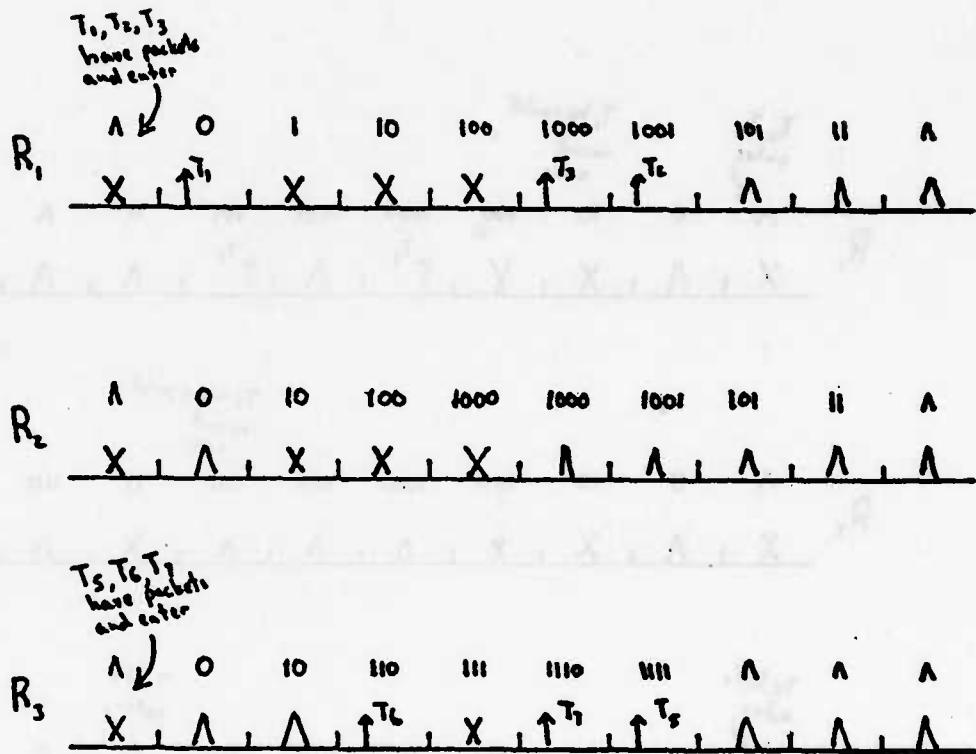
FIGURE 13. An example transmission sequence for the example network described in Figures 11 and 12.
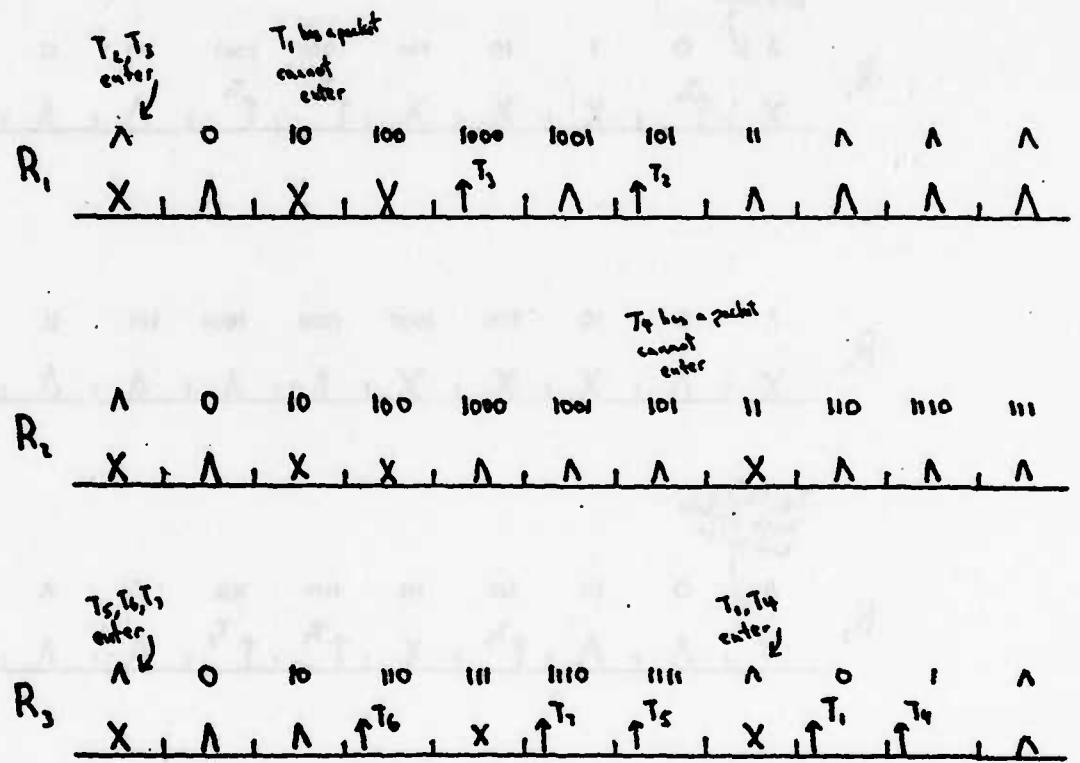
FIGURE 14. An example transmission sequence for the example network described in Figures 11 and 12.
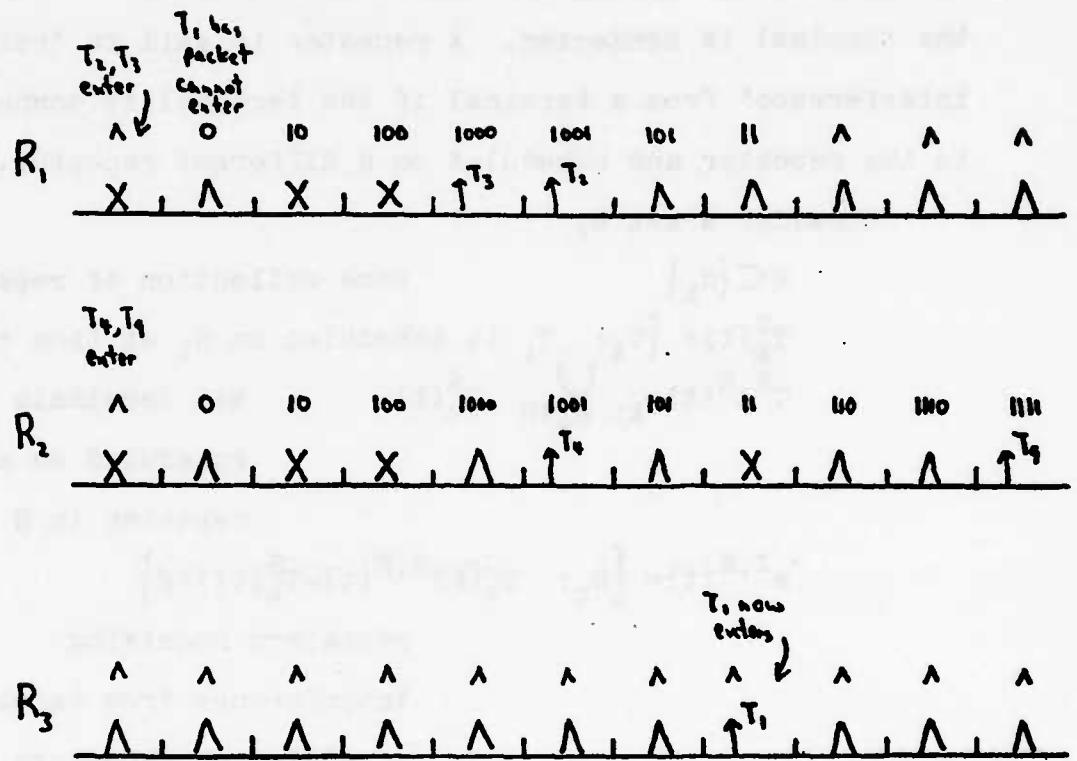
FIGURE 15. A further example transmission sequence. Note
how entry is held back, thus avoiding possible deadlock
situation with, for example, T_1 scheduled on R_3, T_2
and T_3 on R_1 and T_4 on R_2.

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IV. PROOF THAT THE RESTRICTED-ENTRY ALGORITHM IS DEADLOCK-FREE

A terminal that is scheduled on a repeater is said to "cause interference" at any other repeaters to which the terminal is connected. A repeater is said to "receive interference" from a terminal if the terminal is connected to the repeater and scheduled on a different repeater.

Consider a set R,

$$R \subset \{R_k\} \qquad \text{some collection of repeaters}$$

$$T_k^S(t) = \{T_i: \ T_i \text{ is scheduled on } R_k \text{ at time } t\}$$

$$T^{S,R}(t) = \bigcup_{k: \ R_k \in R} T_k^S(t) \qquad \text{all terminals}$$

scheduled on any

repeater in R

$$R^{I,R}(t) = \{R_k: \ T_k^C \cap (T^{S,R}(t) - T_k^S(t)) \neq \emptyset\}$$

repeaters receiving

interference from terminals

scheduled on repeaters in R

We will show that

$$R^{I,R}(t) \cap R \neq R$$

That is, take any set of repeaters. The terminals scheduled on these repeaters may cause interference at repeaters. Not all of these repeaters receive interference from these terminals.

If we take the entire set of repeaters in the system, then we see that the above statement implies that there must always be at least one repeater which receives no

interference. After a slot, the state of a repeater which
receives no interference will increase in binary fraction
value, or stay the same with an increase in length, or
return to Λ. This property is readily seen by writing
the state update algorithm of III.B.1 using the binary
fraction notation.

$$(4,5) \qquad\qquad (b,i) \rightarrow (b,i+1)$$

$$(11) \qquad\qquad (b,i) \rightarrow (b+2^{-i},i)$$

$$(14,15,16,21,22) \qquad (b,i) \rightarrow (b+2^{-i},i-n) \quad \text{where } n \text{ is}$$
$$\text{the largest integer}$$
$$\text{such that } 2^n | b2^i$$

$$(19.2,19.3) \qquad\qquad (b,i) \rightarrow (b+2^{-i},i+1)$$

We observe that it is impossible for there to be a collision
at a repeater which receives no interference when i=m
because at most one terminal will be called for. Thus,
(6,7) are of no concern.

When the state of a repeater returns to Λ, we know
that all terminals which had scheduled on the repeater
must have gotten their packets through. This statement
is true because, (a) no terminal is allowed to enter on
a repeater that would not call for the terminal during
the current epoch, and (b) the binary fraction value of
the state of a repeater will not exceed the value of the
address of a scheduled terminal until the terminal gets
its packet through.

There is always at least one repeater whose state increases in binary fraction, or stays the same with an increase in length, or returns to Λ; thus, eventually, an epoch must end. An epoch ending implies that all packets scheduled on the repeater were successfully transmitted. It is impossible for deadlock to occur because at least one epoch must finish before the states can simultaneously return to previous values and hence at least one packet must be successfully transmitted.

We shall now prove that

$$R^{I,R}(t) \bigcap R \neq R$$

Proof by induction in time.

Basis:

We know that the theorem is true for $t = 0^-$ (before any terminal has a packet) because

$$T_k^S(0^-) = \emptyset \qquad \text{for all } k$$

For any R, such that $R \subset \{R_k\}$ and $R \neq \emptyset$,

$$T^{S,R}(t^-) = \bigcup_{k: R_k \in R} T_k^S(t^-)$$

$$T^{S,R}(0^-) = \bigcup_{k: R_k \in R} \emptyset$$

$$T^{S,R}(0^-) = \emptyset$$

$$R^{I,R}(t^-) = \left\{ R_k : \quad T_k^C \bigcap (T^{S,R}(t^-) - T_k^S(t^-)) \neq \emptyset \right\}$$

$$R^{I,R}(0^-) = \emptyset$$

Now,

$$\emptyset \bigcap R = \emptyset$$

and

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$$\emptyset \neq R$$

So,

$$R^{I,R}(0^-) \bigcap R \neq R$$

Thus, the theorem is true at the beginning of all time.

Induction:

Suppose the theorem is true at time t^-. We shall prove that the theorem is true at time t. The interval from t^- to t is the only time period during which new terminals schedule. Thus, this interval is the only interval in which new interference is introduced and hence, it is the only interval we need consider.

$$R^{I,R}(t^-) \bigcap R \neq R \qquad \text{assume true.}$$

If $T^E(t) = \emptyset$, then

$$T^S(t) = T^S(t^-) \bigcup T^E(t)$$

$$T^S(t) = T^S(t^-)$$

$$T_k^E(t) \subset T^E(t)$$

$$T_k^E(t) = \emptyset$$

$$T_k^S(t) = T_k^S(t^-) \bigcup T_k^E(t)$$

$$T_k^S(t) = T_k^S(t^-)$$

$$R^{I,R}(t) = R^{I,R}(t^-)$$

$$R^{I,R}(t^-) \bigcap R \neq R$$

and thus,

$$R^{I,R}(t) \bigcap R \neq R$$

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If $T^E(t) \neq \emptyset$, then

consider a set R such that $R_k \varepsilon R$ where $T_k^E(t) \neq \emptyset$ for some k. We shall add to the set R, repeaters which receive interference from terminals scheduled on repeaters that are members of R. We shall see that it is impossible to construct a set such that all repeaters in the set receive interference from terminals scheduled on repeaters in the set.

Let $R^1 = \{R_k\}$

$R^2 = R^{I,R^1}(t)$ repeaters receiving interference from terminals scheduled on repeater R_k

Let us break R^2 into three mutually exclusive, collectively exhaustive sets, R^{2A}, R^{2D}, and R^{2DA}.

$R^{2A} = R^2 \bigcap R^A(t^-)$

$R^{2D} = R^2 \bigcap R^D(t)$

$R^{2DA} = R^2 \bigcap R^D(t^-) \bigcap R^E(t)$

where $R^E(t) = \{R_k: \quad T_k^E(t) \neq \emptyset\}$

We shall consider the terminals which might be scheduled on the repeaters in these three sets. We examine the interference that these terminals cause. Let us restate the entry rules for more ready use in the proof:

(1) If

(2) T_i has a packet waiting at time t^-

(3)   and $R_i^Y(t^-) \neq \emptyset$

(4)   and for some k

(5) $R_k \in R_i^Y(t^-)$

(6)   and

(7) $S_l \geq \tilde{S}_{l,k}^{T_i}$ for all l: $R_l \in R_i^C - \{R_k\} - R^D(t^-)$

(8) then

(9) $T_i \in T_k^E(t)$

(10) If

(11) T_i has a packet waiting at time t^-

(12)   and $R_i^Y(t^-) = \emptyset$

(13)   and $R_i^C \cap R^D(t^-) \neq \emptyset$

(14)   and for the minimum value of k

(15)   such that $R_k \in R_i^C \cap R^D(t^-)$

(16) $S_l \geq \tilde{S}_{l,k}^{T_i}$ for all l: $R_l \in R_i^C - R^D(t^-)$

(17) then

(18) $T_i \in T_k^E(t)$

(19) For each T_m

(20)   if for all l such that $R_l \in (R_m^C \cap R_i^C)$
                                        $l \neq k$

(21)   we have $S_l^{T_m} \geq \tilde{S}_{l,k}^{T_i}$

(22)   then $R_m^C \subset R_i^C$

(23)   and

(24) $R_k \not\in R_m^C$

Consider R^{2A}

$T_k^E(t) \neq \emptyset$; thus, the entry condition must have been satisfied for some $T_1 \in T_k^E(t)$. Hence, either (7) or (20,21) must have been satisfied, implying (22) and (24). Thus,

$$R_k \notin R^{I,R^{2A}}(t)$$

$$R^{I,R^{2A}}(t) \subset R^2$$

Let us add R^{2A} to our set R.

Consider R^{2D}

$$T^{S,R^{2D}}(t) = \emptyset$$

$$R^{I,R^{2D}}(t) = \emptyset$$

Thus,

$$R_k \notin R^{I,R^{2D}}(t)$$

$$R^{I,R^{2D}}(t) \subset R^2$$

Let us add R^{2D} to our set R.

Consider R^{2DA}

Consider two cases

Case 1) $R_k \in R^A(t^-)$

$$R_k \notin R^{I,R^{2DA}}(t)$$

because entry condition (16) cannot be satisfied due to (24)

Case 2) $R_k \in R^D(t^-)$

k must be the minimum value of l such that $R_1 \in R_k \cup R^{2DA}(t)$ by (14). Suppose we had some T_1 such that

$T_i \in T^{S,R^{2DA}}(t)$ and $T_i \in T_l^E(t)$ and $R_k \in R_l^C$. We must have $k<l$ because (14) must have been true for entry of $T_k^E(t)$. But such a situation violates (14) for T_i. Thus, no such T_i is possible.

$$R_k \notin R^{I,R^{2DA}}(t)$$
$$k<l \text{ for all } l: R_l \in R^{I,R^{2DA}}(t)$$

Thus,
$$R_k \notin R^{I,R^{2DA}}(t)$$

Let us add R^{2DA} to our set R.

Now consider repeaters that might receive interference from terminals scheduled on repeaters we just added to R. That is, we consider

$$R^3 = R^{I,R^{2DA}}(t)$$

Let us break R^3 into three mutually exclusive, collectively exhaustive sets, R^{3A}, R^{3D}, R^{3DA}:

$$R^{3A} = R^3 \bigcap R^A(t^-)$$
$$R^{3D} = R^3 \bigcap R^D(t)$$
$$R^{3DA} = R^3 \bigcap R^D(t^-) \bigcap R^E(t)$$

Consider R^{3A}

$T_j^E(t) \neq \emptyset$ for some j: $R_j \in R^{2DA}$. Thus, the entry condition must have been satisfied for some $T_i \in T_j^E(t)$. Hence, (20,21) must have been satisfied, implying (22) and (24).

Thus,

$$R^{I,R^{3A}}(t) \subset R^3$$

$$R^{I,R^{3A}}(t) \subset R^{I,R^{2DA}}(t)$$

$$R_k \not\subset R^{I,R^{2DA}}(t)$$

Hence,

$$R_k \not\subset R^{I,R^{3A}}(t)$$

Let us add R^{3A} to our set R.

Consider R^{3D}

$$T^{S,R^{3D}}(t) = \emptyset$$

$$R^{I,R^{3D}}(t) = \emptyset$$

Thus,

$$R_k \not\subset R^{I,R^{3D}}(t)$$

$$R^{I,R^{3D}}(t) \subset R^3$$

Let us add R^{3D} to our set R.

Consider R^{3DA}

Consider two cases

Case 1) $R_k \in R^A(t^-)$

Consider some T_1 such that $T_1 \in T_m^E(t)$ where $R_m \in R^{3DA}$.

$R_m \in R^{3DA}$; thus, there must be some T_j such that $T_j \in T_1^E(t)$ and $R_m \in R_j^C$. Entry rule (14) for this T_j requires that $1 < m$. Thus $R_1 \not\subset R_1^C$ because (14) would require entry on

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R_1 rather than R_m otherwise. But we know that $R_1 \epsilon R^{2DA}$; thus, there must be a T_p such that $T_p \epsilon T_k^S(t)$ and $R_1 \epsilon R_p^C$. Entry rules (20,21,24) imply that

$$R_k \not\epsilon R_1^C$$

and hence

$$R_k \not\epsilon R^{I,R^{3DA}}(t)$$

Case 2) $R_k \epsilon R^D(t^-)$

Suppose we have some T_1 such that $T_1 \epsilon T_1^E(t)$ and $R_1 \epsilon R^{2DA}(t)$ and $T_m^C \cap T_1 \neq \emptyset$ where $R_m \epsilon R^{3DA}(t)$. Then we must have $l < m$ from (14) for T_1. But we already know $k < l$. Thus, $k < l < m$. There is no T_1 such that $T_1 \epsilon T_m^E(t)$ and $T_1 \epsilon T_k^C$ because such a T_1 violates entry condition (14) since $k < m$. Thus,

$$R_k \not\epsilon R^{I,R^{3DA}}(t)$$

We may now consider $R^4 = R^{I,R^{3DA}}(t)$. The above arguments may be repeated here with the notational change that 3 becomes 4 and 2 becomes 3. Again the argument yield no interference at R_k. We may add the repeaters of R^4 to the set and continue to apply the above arguments repeatedly until we consider some set R^N which is empty, at which point we may stop. After this procedure, there is still no repeater in the set R which causes interference at R_k. This complete argument could be made for each repeater that is an element of $R^E(t)$. Thus we cannot construct

a set R such that

$$R^{I,R}(t) \bigcap R = R$$

And thus we have proved that

$$R^{I,R}(t) \bigcap R \neq R \qquad \text{for all } R,t$$

The system is deadlock-free. QED

V. SUGGESTIONS FOR FURTHER RESEARCH

Issues that are important in any multiple access
scheme are throughput, delay, stability, and deadlock.
A desirable system would be stable and deadlock-free, with
high throughput and low delay. The scheme developed in
this thesis is deadlock-free. The tree search contention
resolution algorithm was chosen as the basis of the work
here because of its excellent stability, high throughput,
and low delay for a single-hop system. The throughput
and delay properties of the restricted-entry algorithm
for the multiple-coupled random access problem developed
in this thesis must be analyzed. Along with the issue
of delay are the questions of priority and fairness.
Terminals with different connectivities may experience
different delays both in scheduling on repeaters and in
getting packets successfully transmitted, once scheduled.

The parameters of the algorithm that may be
manipulated are the addresses and the thresholds. Perhaps
choices may be made in assigning addresses and thresholds
to achieve the desired throughput, delay, priority, and
fairness properties. The scheme for providing addresses
and thresholds of section III.G uses symmetry with respect
to connectivity. An asymmetric scheme might be used to
obtain different performance. The thresholding scheme

of section III.G.2 was chosen to require the minimum constraint on entry. Perhaps even more constraining thresholds would result in more desirable performance. If we have additional information available, such as some topological connectivity constraints, we might exploit the information in our address and threshold scheme to enhance performance.

This thesis has provided a deadlock-free algorithm for the multiple-coupled random access problem. The performance parameters need to be studied, and consideration must be given to how these parameters may be adjusted through selection of the addresses and thresholds.

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BIOGRAPHICAL NOTE

Lawrence Charles Siegel was born in Cleveland, Ohio on July 5, 1957. He had a happy childhood and was strongly influenced by his loving mother, father, and brother. Throughout his early years, Mr. Siegel proved to be a bright and eager student. He studied electrical engineering as an undergraduate at the Massachusetts Institute of Technology and is a member of Tau Beta Pi and Eta Kappa Nu. His research interests have included biomedical engineering and computer communications. He plans to pursue graduate studies in electrical engineering at MIT with a fellowship from the National Science Foundation. Mr. Siegel's other interests include reading, squash, sailing, cooking, spending time with friends, and being a good guy. He plans to continue to develop his professional and personal interests in the future.

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