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6 QUANTITATIVE PREDICTIONS OF LENGTH IN

THE Mueller-LYER ILLUSION AS PERCEIVED

BY THE HUMAN VISUAL SYSTEM.

THESIS

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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Graduate Electrical Engineering

March 1978

Preface

There are many theories suggested by psychologist to explain visual illusions. However, these theories tend to be very limited in applicability and scope and are unacceptable for making quantitative predictions of the perception of visual illusions by human subjects. Biological research has measured a number of parameters which describe the performance of the visual system. Likewise, pyschophysical research has supported the biological research as well as providing methods to make a quantitative measurement of the perception of a human subject to a simple geometric visual illusion. This thesis represents an attempt to use the biologically derived parameters of the visual system in a model to make quantitative predictions of length of a simple geometric visual illusion which correlates with the response measured in a psychological experiment with human subjects to the same visual illusion. This research is an extension of a portion of the research done by Arthur Ginsburg into the human visual system applying results from both biological and psychophysical research.

I gratefully acknowledge the stimulating environment provided by Dr. Matthew Kabrisky which allowed my entrance into this new area of research. His advice, encouragement

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Abstract

The biologically derived bandwidth characteristics of the human visual system were used to determine the shape parameters of a filter. This filter was used, as a model of the visual system, to produce quantitative predictions of the Müller-Lyer visual illusion. These predictions were compared to judgments of the length of the shaft of the illusion by human subjects.

The best agreement between the subject data and the predictions of the model occurred when the filter had a double exponential shape, a bandwidth of ± 1.0 octave and a center spatial frequency between 2 and 4 cycles per object size.

This is the first experiment to show that the filter model could predict similar quantitative distortions of length of the Müller-Lyer visual illusion as reported by the human subjects. These results support the theory, advocated by Arthur Ginsburg, that the bandwidth limitations of the human visual system are responsible for geometric visual illusions as well as other visual phenomena such as the Gestalt principles of similarity, proximity and closure.

Quantitative Predictions of Length in

the Müller-Lyer Illusion as Perceived

by the Human Visual System

I. Introduction

All physical information processing systems are inher-
ently band-limited. This fact must be applied to the pro-
cessing by the human visual system. Neurophysiological and
psycholophysical evidence shows that the human visual system
discards a large amount of information it receives at the
early stages of processing. This reduction of the spatial
data can be quantified in terms of the system's filtering
characteristic. Therefore, investigating the filtering char-
acteristics of the human visual system may give insight into
certain operational characteristics of the visual system.

Quantifying many aspects of the visual system has re-
vealed the presence of numerous narrow-band, quasi-indepen-
dent mechanisms called "channels", which are combined to pro-
duce the overall filter characteristic of the visual system
(Ref 1, 11). Single channels refer to single bandwidth
mechanisms in the visual system that are tuned to different
regions of spatial frequency. It has been suggested that
perception may be the result of the extraction of spatial

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information from a hierarchy of filtered images (Refs 29, 31).

This filtering of the spatial information by the visual system must have some consequence on the perception of that information. Clearly some loss of intensity as well as some spatial distortion from filtering can be expected. The perceived distortion of shape, length, or orientation of simple geometric objects consisting of lines and angles by the human visual system results in visual illusions. An understanding of visual illusions is important because they demonstrate instances where the visual system fails at seemingly simple tasks.

At one time or another, most visual scientists have attempted to explain visual illusions. Thus, there seems to be as many theories of visual illusions as there are illusions themselves. Furthermore, each illusion has numerous variations that are used to provide examples and counterexamples to validate or dispute each particular theory. Contemporary theories range from distortions due to optical properties of the eye (Ref 15) to physiological mechanisms such as inhibition (Refs 2, 23), finally to cognitive processes (Ref 17, 36). It should be noted that very few of these competing theories, each of which explains a class of visual illusions, is general enough to explain the majority of the illusions. Also, even

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fewer theories have attempted to establish their effect in the general perception of spatial information.

In contrast to those theories, the concept of a band limited visual system with quantifiable filtering characteristics provides an overall theoretical and quantitative structure in which to study the phenomena of spatial perception (Ref 31). The relevant mathematics and analysis techniques for handling bandwidth limited systems have been developed in other areas of engineering. Ginsburg has applied the filter theory to explain many general concepts of perception such as the Gestalt principles of proximity, similarity, and closure (Refs 26, 27). Further, the concept of filtering has been used to visually demonstrate the distortion of geometric illusions is as predicted (Fig 1). From this very general theory, the explanation of visual illusions is an inadvertent consequence of the filtering of the spatial information processed by the human visual system.

This paper provides an initial attempt to determine the ability of the spatial filter concept to predict geometric distortions of a common visual illusion; the Müller-Lyer illusion. The psychophysical response of subjects to the visual illusion will be compared to computer generated responses using a model of the biologically determined filtering

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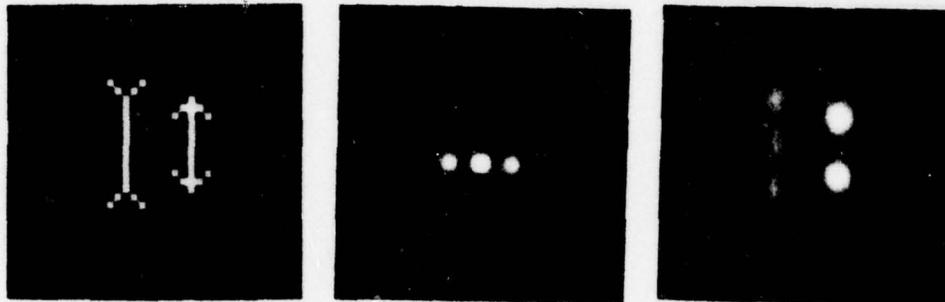
Figure 1. Hypothetical Example of Channel
Filtering (Ref 32:1297)

characteristics of the human visual system.

A pilot's perception of the information presented to
him by his instruments directly affects his ability to fly
an aircraft. Many instruments use arrows as pointers to in-
dicate the important information. Any distortion in these
pointers may affect the acquisition of information by the
pilot. For this reason, research into the operation of the
human visual system and the illusions that it perceives is

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important. This thesis is but part of a continuing research effort by researchers in the U.S. Air Force to attempt to relate basic research about vision to operational problems (Refs 30, 32).

This thesis will briefly cover some background material about the human visual system and about the visual illusion used in this experiment. Next, a description of the experiment will be provided. Last, the results will be discussed along with the conclusions reached.

II. Survey of Previous Visual Research

All realizable systems that transmit information are band-limited. It is unrealistic to believe that the human visual system with its neural structure has been exempted from this fact of nature. A study of the research on mammalian visual systems supports the contention that they are band-limited systems (Ref 4). The filter characteristics of the visual system would seem to be relevant in order to describe the behavior of the visual system.

To study the human visual system, it is useful to consider the system as a series of cascaded elements, each with its own transmission properties. This technique, borrowed from systems engineering, results in the following pathway: object; image; ganglion cells; lateral geniculate body; visual cortex.

First step is the optical properties of the eye. Campbell and Green (Ref 7) and later, Campbell and Gubisch (Ref 8) studied the transmission properties of the dioptics of the eye. They concluded the resolution of the image on the retina is very good for normal light levels and is limited mainly by the subsequent processing.

The next convenient spot for detecting signals due to spatial stimuli is the ganglion cells following the retina.

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Enroth-Cugell and Robson (Ref 20) used sinusoidal gratings to study the response of these cells in cats. They found one class of cells that responded in a linear manner to the spatial summation of gratings. They also found that each cell responded to only a limited range of spatial frequencies. The lateral geniculate body was found to respond in a similar manner in studies by Cooper and Robson (Ref 16) and by Campbell, Cooper, and Enroth-Cugell (Ref 5). These are the first indication of mechanisms in the visual system having limited bandwidths.

A number of studies have been made using microelectrodes placed in the striate visual cortex to determine the response of visual neurons to a visual stimuli (Refs 39, 40, 41, 42, 43). Studies have been conducted on both cats and monkeys by Cooper and Robson (Ref 16); by Campbell, Cooper, and Enroth-Cugell (Ref 5); and Maffei and Fiorentini (Ref 50). These studies have shown that cortical cells respond to only limited bands of spatial frequencies. The combination of the cells responding to different bands provides coverage of the total spatial frequency spectrum.

Up to this point, the neurophysiological findings of both cats and monkeys have suggested that the spatial frequency content of the image is extracted and transmitted by

the visual system to the brain. Now let us look at the psycho-physical studies that were being conducted.

It is interesting to note that as early as 1955, Schade used what neurophysiological and psychophysical data that was available to construct an electronic model of the human visual system for evaluation of television quality (Ref 64). He used many techniques as an engineer such as Fourier analysis and filter characteristics, which were later being re-discovered by the vision research community.

In 1964, Campbell and Robson suggested that the Fourier analysis techniques used in engineering might be applied to psychophysical studies of the visual system (Ref 11). They subsequently reported a series of experiments that measured the contrast sensitivity threshold of the human visual system using sine-wave gratings to predict the contrast threshold of different complex waveforms (Ref 12). They found that the threshold was determined by the amplitude of the funda-mental Fourier component of the waveform and that higher har-monic components did not contribute to the threshold unless they were above their own threshold. These findings led them to suggest there must be a number of "channels" in the human visual system each tuned to different spatial frequencies.

To investigate these channels, Campbell and Kulikowski

(Ref 9), with later improvements by Gilinsky (Ref 25), developed a technique of masking a low contrast grating with a high contrast grating. The contrast sensitivity threshold was raised by the adaptation effect. Pantle and Sekuler (Ref 55) using squarewave gratings and Blakemore and Campbell (Ref 1) using sinewave gratings employed the masking technique to accurately measure the bandwidth of these channels. The bandwidth of these "channels" measured by the adaptation effect was about ±1 octave. These experiments also demonstrated the existence of many channels.

The frequency response of the Blakemore-Campbell adaption experiment has a resemblence to the neural response of the cat in earlier experiments (Fig 2). The equation which results from fitting a curve to this data is:

$$n = (e^{-f^2} - e^{-(2f)^2})3$$

where n is contrast sensitivity and f is the spatial frequency (Ref 4:98).

To determine the independence of each channel, Sachs, Nachmias and Robson (Ref 63) tested subject with a mixture of two different spatial frequencies. Their findings show that even when the frequencies are quite close in period, they add according to the laws of probability summation.

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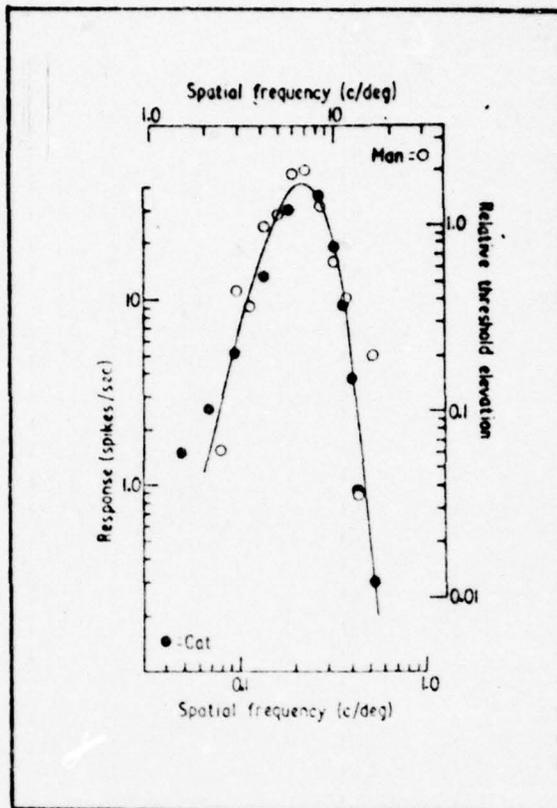
Figure 2.   Graphic Comparison Between the Response of a Cat
Neuron and the Psychophysical Adaptation
Response of a Human (Ref 4:98)

This indicates the channels are functionally independent

and possibly their bandwidth is even narrower than ±1 octave.

One further question, now that we have stepped through

the pathway of the human visual system, is whether the spatial

frequency information that the system has extracted is suf-

ficient to produce the cognitive procedures of which the

brain is clearly capable, such as pattern recognition.
Kabrisky (Refs 45, 46, 47) following the neurophysiological
research of Dusser de Baronne and McCullough (Ref 19) on
the cortical interconnectivity, suggested that human pattern
recognition might be accomplished with a two dimensional
cross-correlation technique. Further evolution of the cross-
correlation mathematics by Kabrisky resulted in the use of
Fourier analysis techniques in the spatial frequency domain.
Machine pattern recognition techniques based on a filtered
spatial frequency domain properties have been shown to be
quite successful by Radoy (Ref 61), Tallman (Ref 65), Gill
(Ref 24), Carl (Ref 13), and Mahaffey (Ref 51). Ginsburg
(Refs 26, 27, 28, 29) has shown that the Gestalt principles
of proximity, similiarity and closure can be explained in
terms of a filtered spatial frequency domain. Also, Ginsburg
has shown that individuals do not need high spatial frequencies
to perform a learning task using alphabet letters. Clearly,
the filtered spatial frequency domain is a powerful domain
in which to investigate the most basic of all visual percep-
tion problems, the recognition of shapes.

Recent attempts to apply the neurophysiological and
psychophysiological data about the filter characteristics
of the human visual system to the way the human visual system

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reacts to various stimuli has been conducted by Ginsburg (Ref 31). The effect upon perception which could result from the filtering of the spatial frequency data before it reaches the brain has been used to explain many visual phenomena. Of particular interest for this paper is the results caused by filtering the geometric illusions which results in certain spatial distortions. These distortions are similar to our perceptions of visual illusions (Ref 27).

III. <u>Visual</u> <u>Illusions</u>

Visual illusions were studied before the discipline of psychology was even recognized. As early as 1856, a book describing many different illusions was written by von Helmholtz entitled <u>Handbuck</u> <u>der</u> <u>Physiologischen</u> <u>Optik</u> (Ref 38). Since then, there has been a steady increase in the number of papers written on visual illusions. These papers offer explanations for particular illusions, examples to support a favored explanation, or counter-examples to attack an unfavored explanation. This abundance of competing theories has defied classification and organization except to the most persistent researchers such as Luckiesch (Ref 48), Tolanski (Ref 65), and Robinson (Ref 62).

This paper will use a visual illusion from the class of geometrical optical illusions. The term "geometrical optical illusion" is a translation of the German pharse "geometrish optische Tauschung" coined in 1855 by Oppel (Ref 54). These illusions are the apparent distortion of shape, length, or orientation by the human visual system of very simple non-semantic geometric objects, consisting of lines and angles. In this paper, the terms geometrical optical illusion, visual illusion and illusion will be used interchangeably.

For this class of visual illusions, there have been

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numerous explanation proposed. The most important con-
temporary theories cover many areas of research. First,
the involuntary eye-movements have been considered by
Carr (Ref 14) and by Virsu (Ref 67) as a source of illu-
sion. Next, distortions due to the optical properties of
the eye have led to two theories by Motokawa (Ref 52) and
Chiang (Ref 15). A number of theories which involve some
physiological mechanism such as lateral inhibition proposed
by Ganz (Ref 23) and by Blakemore, Carpenter and Georgeson
(Ref 2) or adaptation levels proposed by Green and Hoyle
(Ref 33) and by Green and Stacey (Ref 34). Finally, a num-
ber of cognitive explanations such as Gregory's (Refs 36,
37) and Day's (Refs 17, 18) constancy scaling theory,
Pressey's (Refs 56, 57, 58, 60) assimilation theory or
Eriksson's (Ref 21) field theory have been proposed.

It should be noted that despite all these different ex-
planations, no one explanation has risen as a unifying theory
to encompass the majority of the visual illusion phenomena
that have been observed. A detailed discussion of each of
these theories is beyond the scope of this paper, however,
an excellent review of each theory as well as its strengths
and weaknesses are available in Robinson's book, The
Psychology of Visual Illusion (Ref 62).

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Ginsburg's approach, however, was to consider the visual system as having band-limited mechanisms and investigate the results of spatial filtering of the object by such mechanisms (Ref 31). Since this filter concept of the visual system has been strongly supported by biological data from many different areas, successful explanation regarding visual illusions using the filtering properties of the visual system cannot help but unify our understanding of vision. It is contended that the majority of the visual illusion phenomena reported are directly attributable to the filtering characteristics of the bandwidth limited visual system as discussed in the preceding chapter (Ref 31).

The specific geometrical optical illusion chosen for this study was the Müller-Lyer illusion (Ref 52). The classical Müller-Lyer illusion consists of two equal length shafts, one with outward pointing fins, and the other with inward pointing fins (Fig 3). The shaft having the inward pointing fins appears shorter than the other shaft.

The Müller-Lyer illusion was chosen for several reasons. First, it produces a change in line length, which was the variable of interest for this study. Second, it was easy to draw in a quantized version suitable for both digital processing and displaying to subjects. Finally, it was an

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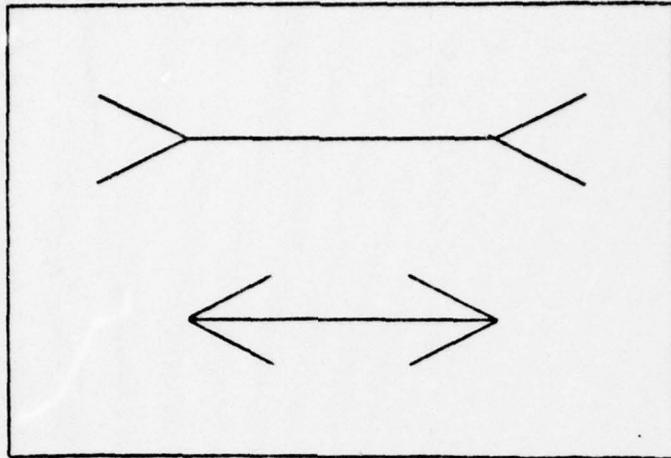
Figure 3.   Classical Müller-Lyer Illusion.

illusion that has been used in many studies which aids in
the comparison of results.

Previous studies using the Müller-Lyer illusion have
shown that the length of the shaft changes from being under-
estimated to overestimated as the distance between the end
of the shaft and the apex of the fins is increased (Refs 44,
68). However, as the distance between the end of the shaft
and the fins is increased beyond a certain point, the over-
estimation begins to decrease. A plot of the perceived
length of the shaft of the illusion results in an inverted
U-shaped curve (Fig 4) as shown in studies by Pressey and
Bross (Ref 59) and by Fellows (Ref 22).

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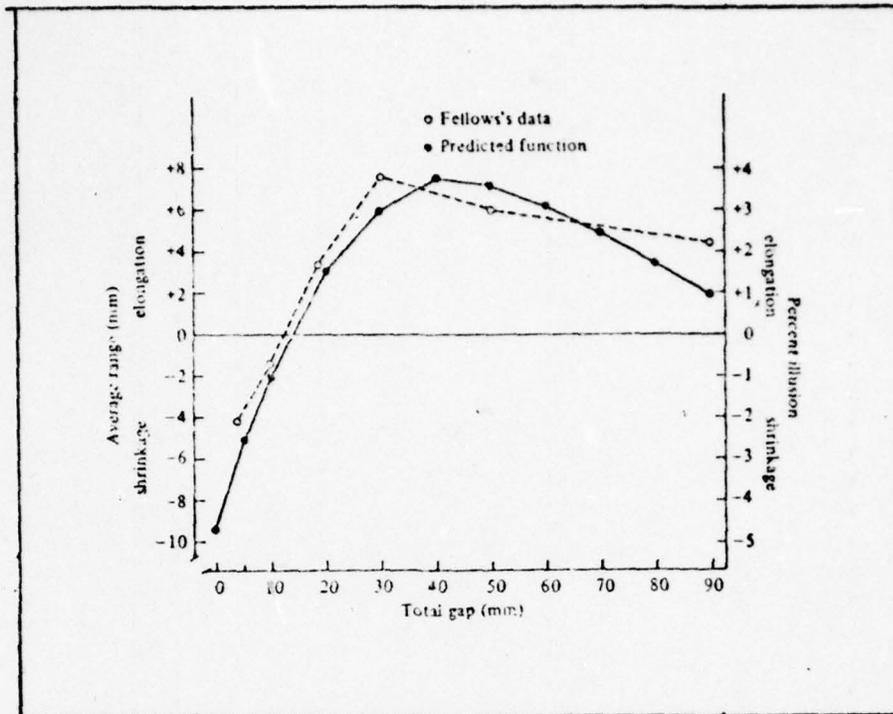
Figure 4. The Inverted U-Shaped Curve (Ref 59:212)

Pressey and Bross used Pressey's assimilation theory (Refs 56, 57) to attempt to explain the occurrence of the inverted U-shaped curve. An ideal curve formed by the assimilation theory equation was aligned by Pressey and Bross (by arbitrarily rescaling the axes) to Fellow's data. Pressey and Bross did not, however, try to match a curve to the data that they subsequently produced. In fact, they used different values than Fellow for their shaft length, fin length, fin angle, and gap sizes. This prevents a straight forward comparison to Fellow's data, since the assimilation theory is based on the relationship of the

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intra-fin length to shaft length and fin distance from the center of the illusion.

Assimilation theory evolved from the observation that the range of size of surrounding objects (typically line figures) can affect the perception of an individual object by causing it to be perceived as closer to the mean of the surrounding objects. Thus, a larger object is underestimated in the presence of smaller objects and conversely a smaller object is overestimated in the presence of larger objects. Also, it has been observed that this effect decreases as the distance between the surrounding objects and the individual object of interest is increased. Therefore, the total affect on the perception of an individual object is the summation of the differences between the individual object and each surrounding object divided by a weighting factor representing the distance between these objects. These observations were the basis for an empirically derived equation to predict the perception of individual objects in a group of objects.

Applying this empirical equation to a simple geometric figure such as the Müller-Lyer illusion is straight forward. The object to be perceived is the length of the shaft. The surrounding objects are the lengths between the endpoints

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of the fins and between the apices of the fins. The distances involved are from the center of the shaft to the endpoint of the fins and from the center of the shaft to the apex of the fins. The summation of the differences between the shaft length and the intra-fin and intra-apex lengths each inversely weighted by their distance from the center of the shaft should predict the magnitude of the illusion.

The assimilation theory can be criticized on several points. Of lesser importance is the lack of procedures to determine which surrounding values should be considered as an influence in a complex figure. Even with a simple geometric figure like the Müller-Lyer illusion, there are other length measurements which could be considered such as the fin length or the length of the projection of the fins on the shaft. Further, the process of summing all the permutations from the possible values in a complex scene quickly becomes prohibitive in terms of time and complexity. However, the strongest criticism is that the theory is based on an empirically derived equation without any supporting arguments as to why the observed effects from the surrounding values occur or how these effects are mechanized in the human visual system. Assimilation theory merely changes the problem

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from explaining how the illusion occurs to one of explaining why the theory works.

The filter concept of the human visual system does not suffer from these criticisms. First, it has been shown to have biological evidence to support both its structure and its implementation. This is a factor in many theories which try to expand from a single class of illusions to a broad principle. Third, the concept is computationally feasible and implementable for future testing. Many existing theories have not been completely explained nor do they have tools to implement or compute their predictions. Filtering has been applied in optical and electrical engineering to analyze complex systems and, hence, relevant mathematics are available for use by the human vision research scientists.

The existence of quantifiable data produced by Pressey and Bross (Fig 5) makes this an excellent place to begin an investigation of the predictive capability of the filtered spatial frequency concept.

The experiment that is explained in the next chapter is similar to the Pressey and Bross experiment. Due to the apparatus used, the dimensions of the illusion and other details have been changed. However, the metric used in the following experiment is still the perceived length of the

PRESSEY AND BROSS DATA

SHORT FINS

LONG FINS

PERCENT CHANGE IN SHAFT LENGTH

GAPS AS PERCENT OF SHAFT LENGTH
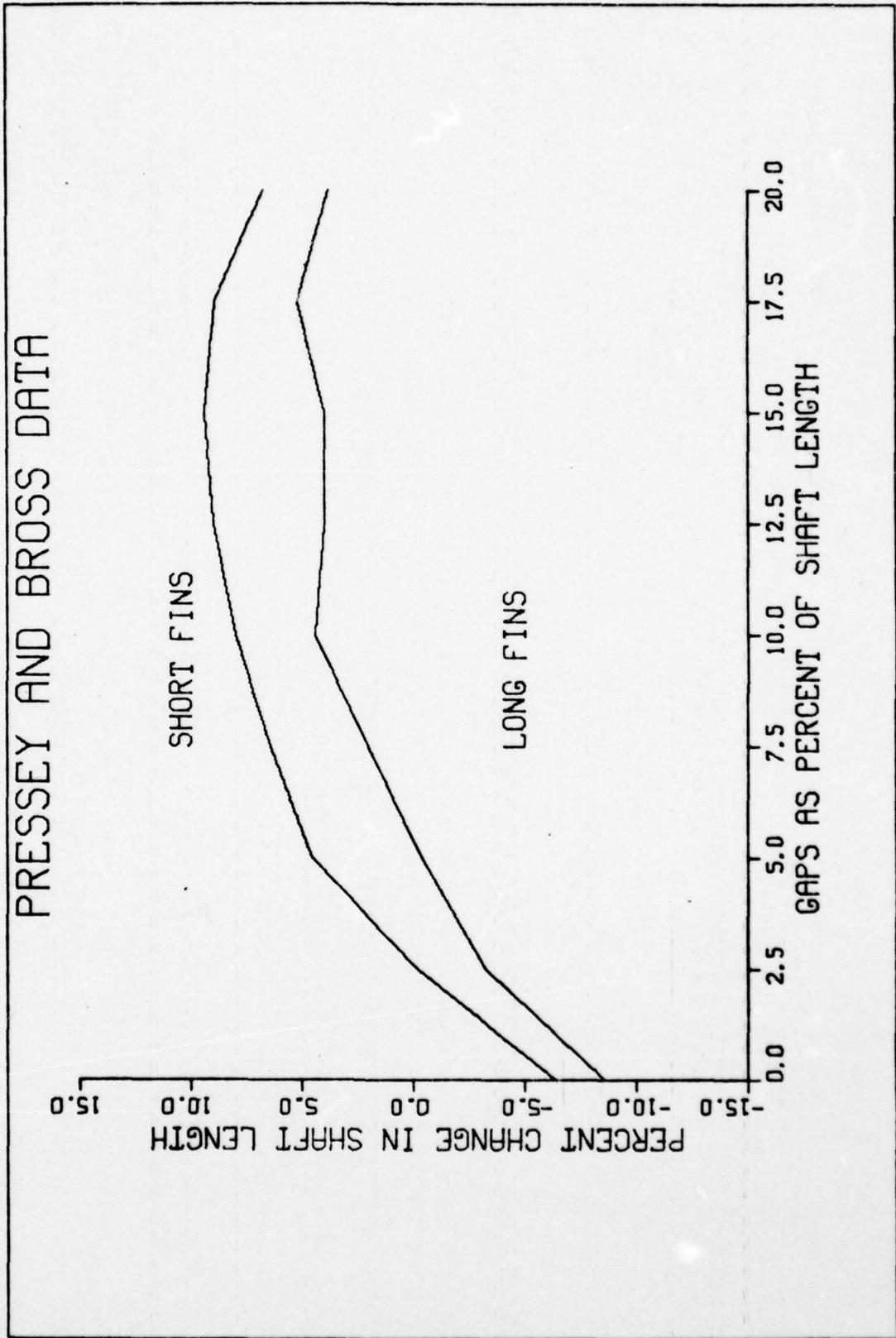
Figure 5. Pressey and Bross

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shaft of the Müller-Lyer illusion when the fins are moved

away from the shaft. However, the theory that is being

investigated is quite different. The explanation of the

illusions existence is based on the bandwidth limitations

of the human visual system.

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IV.   The Müller-Lyer Shaft Length Experiment

Experiment

This experiment required the measuring of the response

of a subject to a set of stimuli. The stimuli used in this

experiment were variations of the Müller-Lyer illusion with

a constant shaft length and a variable gap between the fins

and the shaft. The subject's response was to adjust a line

to be equal to the perception of the length of the shaft in

the illusion. A device was needed to display the stimuli

and make repeared measurements of the responses of the sub-

jects. Since measuring the subject's responses would be

more difficult than displaying a visual stimulus, it was con-

sidered first.

The techniques for measuring the response of a subject

to a quantitative metric such as length can, in general, take

three forms: the method of forced choice, the method of

staircase approximation or the method of adjustment. The

method of adjustment was chosen because of the following rea-

sons: 1) the other methods require many more trials to ob-

tain a single value than does the method of adjustment; 2)

the method of adjustment provides a more exact value as opposed

to measurements to the nearest artificially quantized level

used by the other methods; and 3) the first two methods

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require a display device capable of quickly changing the display stimulus for an experimental session of reasonable length and such a device was not available. Therefore, a device utilizing the method of adjustment to measure the subject's response to the stimuli was created.

A cathode-ray tube (CRT) device with a computer to drive it that would allow rapid stimulus changes and automated data collection was desired but was not readily available. In favor of simplicity, availability and time of construction and set-up, the apparatus described in the next section was chosen.

Apparatus

The apparatus used in this experiment consisted of an adjustable 1m x 1m display board and an adjustable .3m x 1m line adjustment and measurement board (Fig 6A). Both boards were mounted to an upright 2.5m frame which allowed adjustments in both height above the ground and separation between the boards.

The display board was used to support the cards containing the stimuli. The display area was covered with white contact paper to eliminate any visual clues of position within the display area. The stimuli were attached to the display board with double sided tape. A high intensity light was
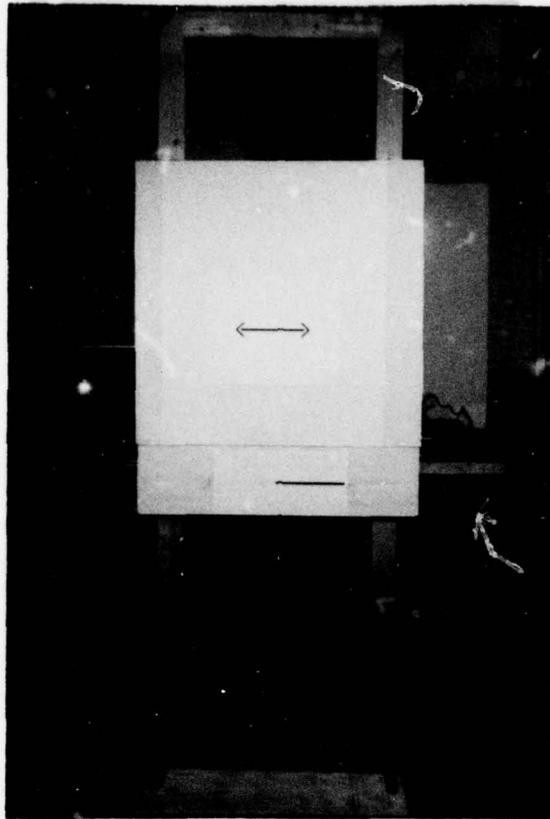
24

Figure 6A.   Line Display and Measurement Apparatus

used to supply a uniform level of light (400 foot-Lamberts

measured with a spot spectrometer) for the display board.

The line measurement board was used by the subject to

indicate his perception of the length of the line by ad-

justing a comparison line on this board.   The length of the

comparison line was adjusted by the subject from his seated

position by use of a rope and pulley system.   The ropes

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Figure 6B. Line Adjustment Knob.

were connected to the line adjustment knob at the subject's

chair to allow easy line adjustment (Fig 6B). The adjust-

able comparison line consisted of a long black line drawn

on a clear plastic background. The plastic background strip

with the line was terminated at one end by a slit through

which the plastic strip passes. The movement of the plastic

background strip through the slit produced a line that could be adjusted to any length. A standard millimeter ruler with a pointer controlled by the plastic background strip was mounted on the back of the board to indicate to the experimenter the line length to which the subject had adjusted the comparison line.

The stimuli placed on the display board for viewing by the subjects consisted of a set of Müller-Lyer illusions previously discussed (a subset of the stimuli is shown in Figure 7). Each stimulus was drawn in black ink on white 54cm x 60cm cardboard sheets. The illusions were drawn in their quantized versions so that an identical replica of the stimuli could be used in the computer analysis. The digital stimuli were created on a 128 x 128 grid with each element equal to 2.5cm. The shaft was 20cm long and the fins were 3.5cm long making a 45 degree angle with the shaft. The fins were moved away from the shaft in 0.5cm steps from having no gap to having a gap of 4.0cm. Included in the stimuli set was one reference shaft without any fins.
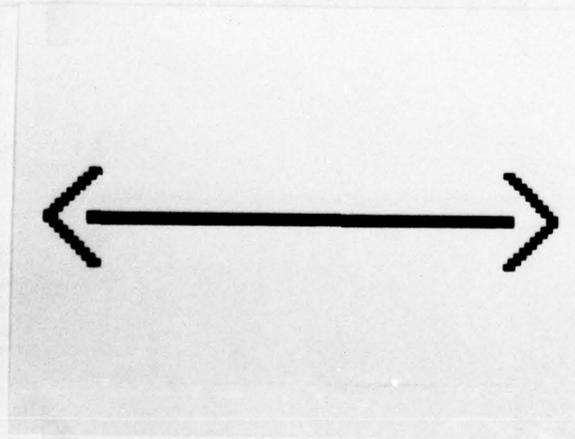
After construction of the apparatus and stimuli, it was necessary to determine how well the apparatus worked and what external factors needed to be controlled. The first three factors that required controlling were immediately obvious
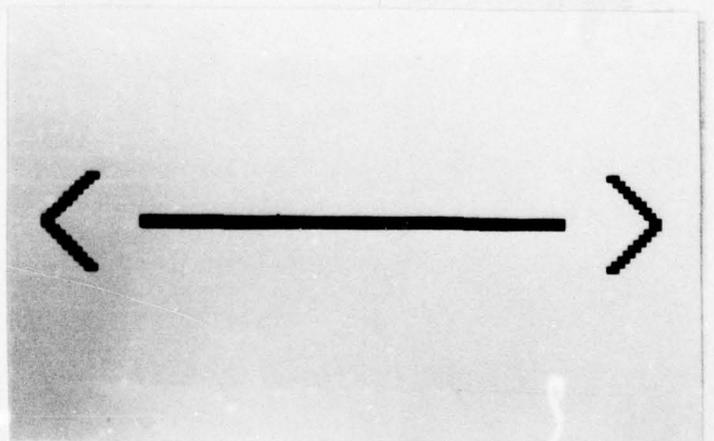
27
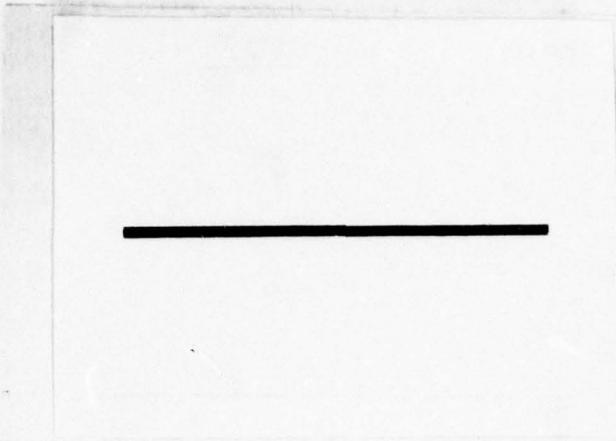
(Reference)
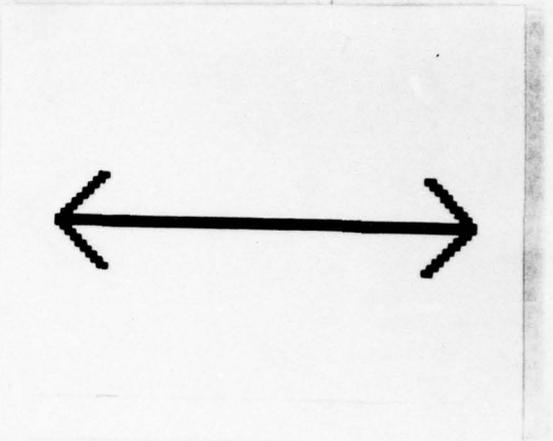
(0%)

(10%)

(20%)

Figure 7.   A Subset of the Muller-Lyer Stimuli

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after the first few trial runs. First, the precision with
which the subject could position the comparison line was
poor. This was corrected by the line adjustment knob that
was previously described since it provided a constant level
of tension on the ropes. Next, the subjects were able to
detect a difference in the contrast between the comparison
line and the stimuli. This was overcome by drawing the com-
parison line on paper of the same texture as that used for
the stimuli using the same ink and attaching this line to
the plastic background strip with double-sided tape. The
same texture cardboard used for the stimuli was also used
behind the comparison line and the clear plastic strip. The
third problem to be solved was the visual clues, such as the
wood grain pattern along the top of the line adjustment board,
which the subjects reported were influencing their judgments
of length of the stimuli. This problem was eliminated by
covering both the display board and the line adjustment board
with white contact paper to remove those clues.

Procedure

Pilot studies were needed to determine if certain ex-
perimental factors affected the measurements.

First, the distance between the subject and the display
board was varied from 1m to 6m. The results obtained during

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these runs were substantially the same except that the subject felt less confident of his responses at 1m. (By an analysis of variance test, changes in distance were not significant: $F = 0.59$; d.f. = 2, 14; $p > 0.25$). Therefore, a value of 4m was chosen since it was the most convenient distance to use in the laboratory. This results in a visual subtended angle of 2.8 degrees for the shaft of the illusion. The adjustment ropes connecting the chair of the subject to the display board provided an excellent method for accurately establishing this distance for each experimental session.

Next, the effects of the distance between the display board and the line adjustment board were investigated. Trials were run at the extremes of the board's position adjustments which were touching to approximately 2m separation. The subject's data was similar in both cases. (By an analysis of variance test, changes in separation were not significant: $F = 0.01$; d.f. = 1, 9; $p > 0.25$). To decrease any difficulty of the task, the two boards were placed next to each other with their junction even with the line of sight of the subject while seated.

Finally, the framing effect of the display board was considered as a possible variable. A brief study was done to determine if a framing effect was noticeable using two

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different size cardboard backgrounds for the illusions. Sizes of 20cm x 28cm and 40cm x 50cm were tested and showed little difference. (By an analysis of variance test, changes in background size were not significant: $F = 0.04$; d.f. = 1, 17; $p > 0.25$). To be on the conservative side, the larger dimensions were used.

During these pilot studies, it was determined that a number of repetitions should be made with each stimulus and averaged together for a single measurement. The number of repetitions necessary was determined by looking at the change in variance versus the number of repetitions used. Eight repetitions caused minimum change in the variance. Ten repetitions of the line length measurements for each stimulus was chosen as a conservative number for the subsequent experiment.

The subjects were initially informed to adjust the comparison line in only one direction per measurement. The starting points of the test line alternated between being shorter and longer then that of the stimulus line. This procedure was stopped when the subjects continually forgot where the next starting point should be. Also, the subjects expressed a lack of confidence in their measurements when constrained to that procedure. The subjects tended to counter-

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balance their own measurements by occasionally adjusting the comparison line too long and reversing direction of their measurement process during a single measurement.

The testing procedure was throughly briefed to each subject indicating the desired response to the stimuli and clearly indicating the central shaft of the illusion prior to the beginning of each experimental session. The subjects adjusted the comparison line to be the same length as the perceived length of the shaft of the Müller-Lyer illusion. The value of the measurement was recorded by the experimenter and the comparison line was reset to 3cm (minimum size possible with the apparatus). Ten repetitions were made with each test condition, the stimulus was then changed by the experimenter and the procedure was continued.

The subjects used in this experiment were all graduate engineering students attending a sensory nervous systems seminar. All subjects had normal or corrected vision.

Computer Model

A digital computer program was written to model the biologically derived characteristics of channel filtering. This program was used to filter the same Müller-Lyer illusion stimuli that was viewed by the subjects during the data

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collection. The output of this program was used to obtain
a predicted length of the shaft of the illusion for compari-
son with the data from the subjects.

The input to the computer program was a binary valued
128 x 128 array which contained the illusion stimuli. This
array was transformed by a two-dimensional Fourier transform
into the spatial frequency domain. The image in the spatial
frequency domain was multiplied by the chosen filter and
transformed back to the space domain using the inverse Fourier
transform.

The filters used consisted of an ideal band-pass filter
and the double exponential filter described in the section
on the human visual system (Figs 8A, 8B, and 9). Three vari-
ations of each filter were used with corresponding bandwidths
of $\pm 0.5, \pm 1.0, \pm 1.5$ octaves. Each variation of the filters
was used at center frequencies of 1, 2, 4, 8, 16, 32, 64,
and 128 cycles per object size.

In filtering the image of an object, a reference for the
spatial frequencies must be found. Ginsburg has used some
overall size dimensions of the object as the fundamental unit
of spatial frequency (Ref 31). Since the illusion used in
this study is composed of separate parts, it was not known
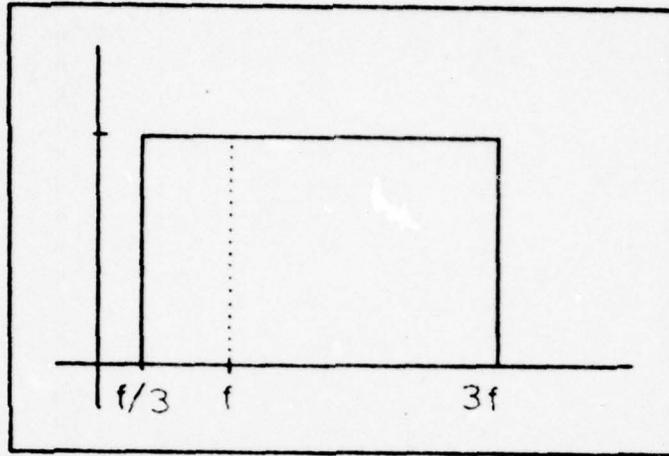whether the fundamental size dimension should be just the
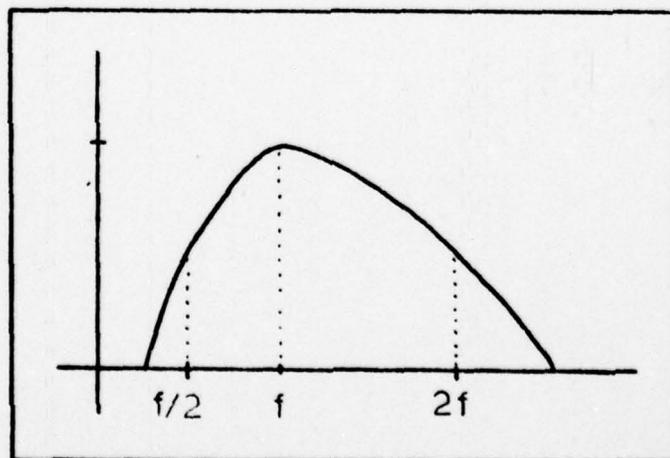
Figure 8A. Ideal Low-pass Filter (±1.5 octaves)

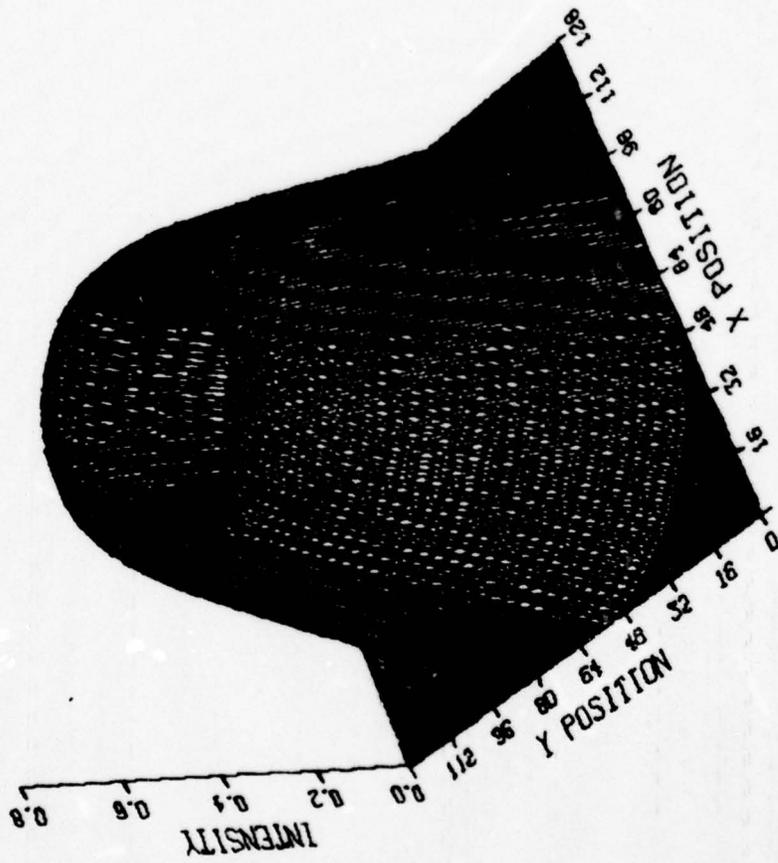Figure 8B. Double Exponential Filter (±1.0 octaves)

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Figure 9. 3-D Plot of Double Exponential Filter

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shaft or the shaft and the fins. Therefore, both approaches for calculating the spatial frequency were tried.

The length of the shaft of the filtered illusion was measured to obtain predictions of the computer model. The algorithm used to determine the length of the shaft computes the distance between certain intensity values that occur along the middle row of array (hypothetically illustrated in Fig 10). The intensity values used to determine the end-points of the shaft were 100%, 75%, 50%, and 25% of the peak intensity. As can be seen in the hypothetical situation (Fig 10), there may exist two points on each side of the peak which have intensity values that are 75%, 50% or 25% of the peak intensity. To resolve this ambiguity, the length of the shaft was always measured between the intensity values which occur outside the peak intensity values. The length of the shaft obtained from this computer model was used to predict the perceived length of the shaft of the illusion that the subject viewed.
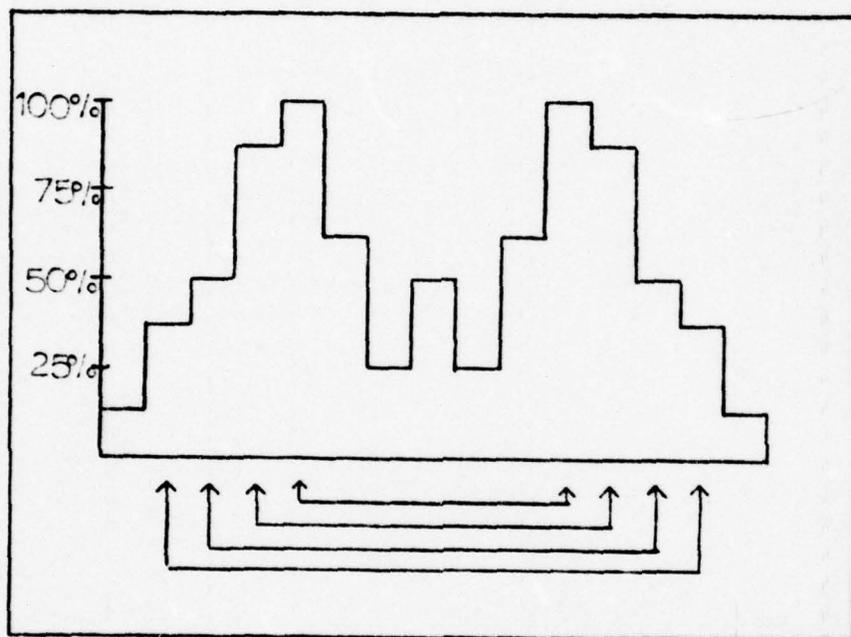
Figure 10. Illustration of Shaft Length Determination.

V. Results

The apparatus and procedure described in tha last chapter
were used to collect data from eight subjects. Each subject
completed the stimulus set three times for a total of twenty-
four data sets for each stimuli.

The three data sets corresponding to a single stimuli
for an individual subject were examined for consistency by
overlaying the curves produced by each data set. This review
identified two classes of data. The first class consisted of
six subjects and showed of the expected inverted U-shaped
curve. The second class consisted of two subjects and showed
large fluctuations.

After the experiment, the subjects were asked to describe,
in their own words, how they accomplished the line comparison
task. Their explanations fell into two general categories
with slight individual variations. The first category consisted
of subjects who observed the length of the shaft to obtain a
perception of its length and adjusted the comparison line to
that length. They finished the task with fine adjustments
made after iteratively shifting their center of attention
between the illusion and the comparison line. The other tech-
nique consisted of adjusting the comparison line to construct
an imaginary parallelogram. The parallelogram was constructed

using the shaft of the illusion as the top line and the comparison line as the bottom line. The side lines were the imaginary lines connecting the endpoints of the comparison line and the shaft of the illusion. The length of the comparison line was adjusted until the imaginary lines appeared to be parallel.

The differences in technique used by each subject were recorded by the experimenter. These observations were given greater significance after the data reduction indicated a correlation between the subjects performance and the technique used. The six subjects that used the first technique (subjects 2, 3, 5 - 8) produced the inverted U-shaped curve as expected. The two subjects that used the second method (subjects 1, 4) produced data that was fluctuating in a somewhat random manner.

These different results from the two techniques appear to be due to the subjects performing a task either directly or indirectly. Those subjects that chose to use the direct judgment of length to perform the experiment produced the expected results while those who chose to use the indirect judgment of parallelness were unable to achieve the same performance. Further research in this area is needed before a definite explanation of why and how differences in techniques have yielded such different results. For the present, the

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data from those two subjects was removed from further consideration on the basis of an analysis of variance test which showed they differed significantly from the remaining subjects ($F = 2.60$; d.f. = 7, 71; $p < 0.05$).

The three data sets from each subject were plotted in Appendix A. The curves of subjects 1 and 4 will not be used for further data manipulations as explained above. The data from the remaining six subjects was combined to form the composite curve shown in Figure 11.

The data from the six subjects was checked for homegneity by use of an analysis of variance test. An analysis of variance test was used since it permitted the comparison of a number of treatments with repeditive measurements simultaneously. It also allows the statistical testing of the significance of any variations between the treatments of the individuals using an F distribution test. The variations between the subjects were found to be insignificant ($F = 1.88$; d.f. = 5, 53; $p > 0.10$). The variations due to the stimuli was found to be significant ($F = 16.06$; d.f. = 8, 40; $p < 0.01$). This data was used for comparison with the theoretical predications generated by the computer program.

As a cross check of the experimental data collected from the subjects, the composite curve was plotted with the data
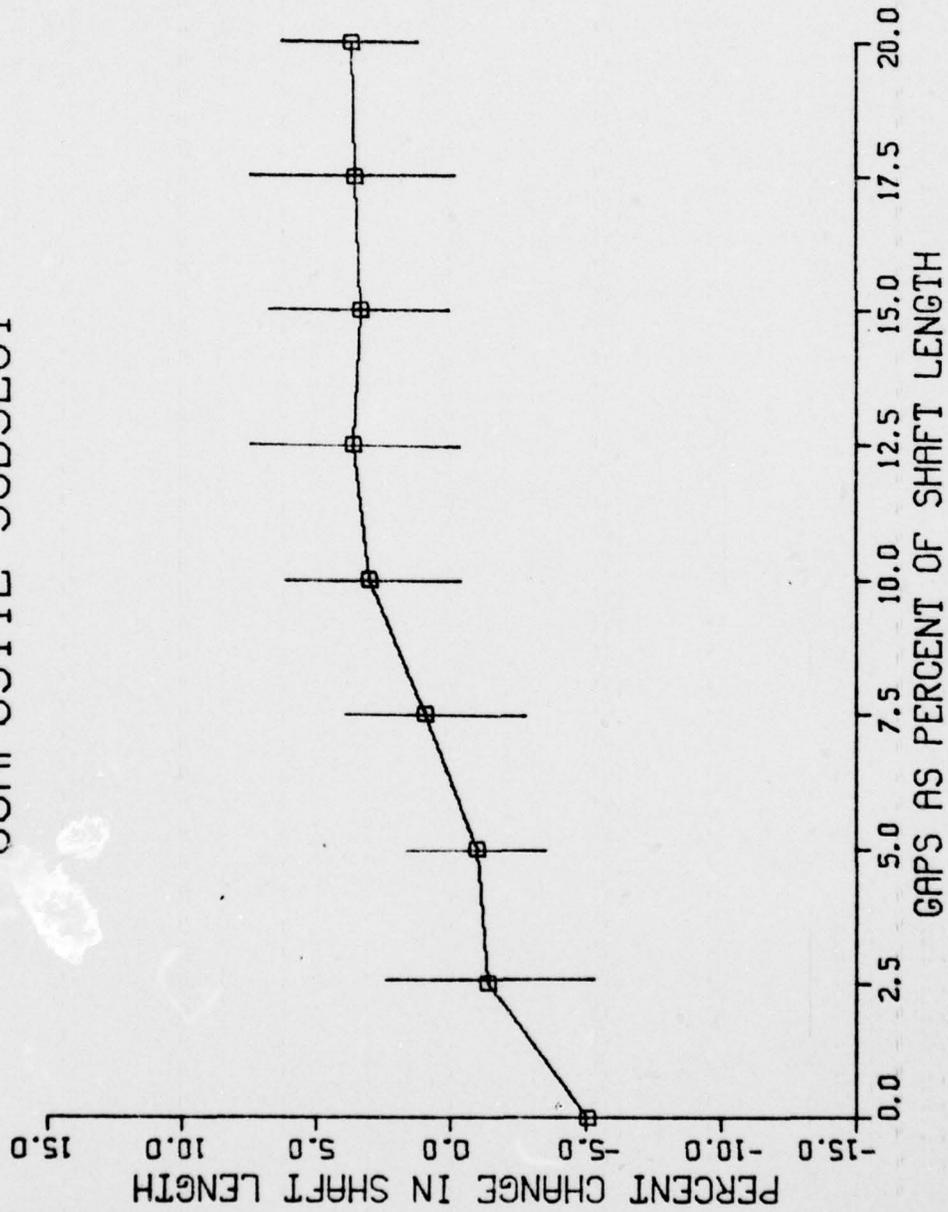
Figure 11. Composite Subject Curve

collected by Pressey and Bross (Ref 59) in Figure 12. The curves can be seen to be in general agreement, therefore, providing verification of the experimental techniques used.

The theoretical curves predicted by the computer model were plotted in Appendix B. A family of curves for the four endpoint intensity values of 25%, 50%, 75% and 100% of the peak intensity along the illusion was calculated for each combination of filter type, bandwidth, and center frequency. Since all the filters with center frequencies above 8 cycles per object size only produced a straight line along the zero axis and, therefore, predicted no variations in length for the illusion, these curves were not plotted. This resulted in twenty-four different prediction curves.

The fundamental reference for calculating spatial frequencies that was based on the length of the shaft alone provided a family of prediction curves that differed greatly from the data collected from the subject in magnitude but was similar in shape. Basing the fundamental reference on the overall size of the illusion, that is including the fins with the shaft, yielded a better agreement between the magnitudes of the predicting curves and the subjects data. Hence, the prediction curves will use, as their fundamental reference for determining spatial frequencies, the length of the whole illusion (i.e., the shaft with the fins) for each stimuli.
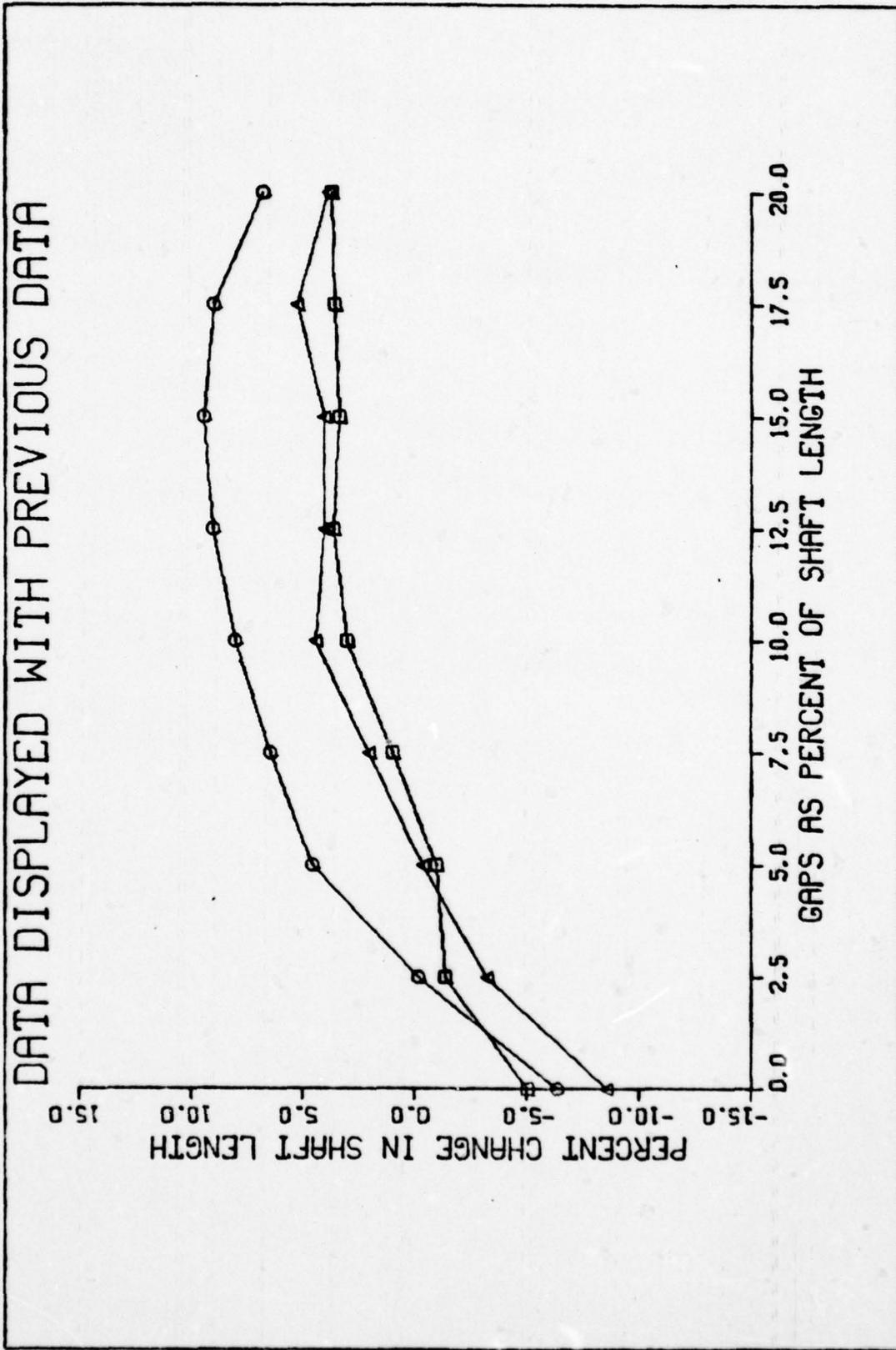
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Figure 12. Pressey and Bross Curves Compared to Composite Subject Curve.

Now to select the best matches to the subjects data. To decrease the number of candidate curves for predicting the subjective data, all curves that had no resemblance to an inverted U-shape curve were eliminated from consideration. Next, all curves which greatly exceeded the maximum and minimum values of the subjects data were also eliminated. Finally, the position of the peak value of the subjects curves was considered. This resulted in two curves being retained as possible predictors of the subjects data (Figs 13 and 14). These prediction curves are from the double exponential filter using ±1 octave bandwidth and a center frequency of either 2 or 4 cycles per object size.
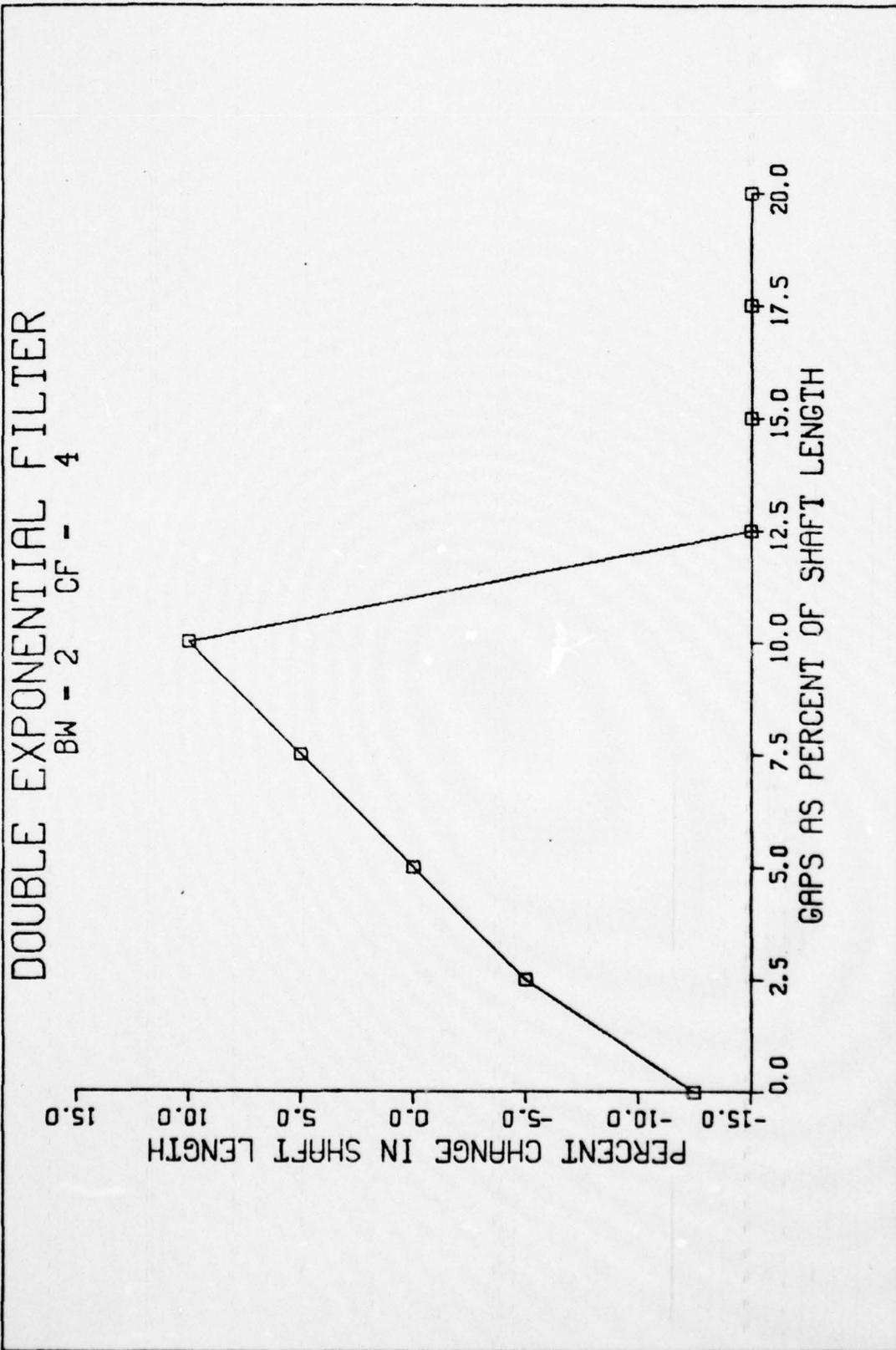
Figure 13. Theoretical Prediction Curve Number 1
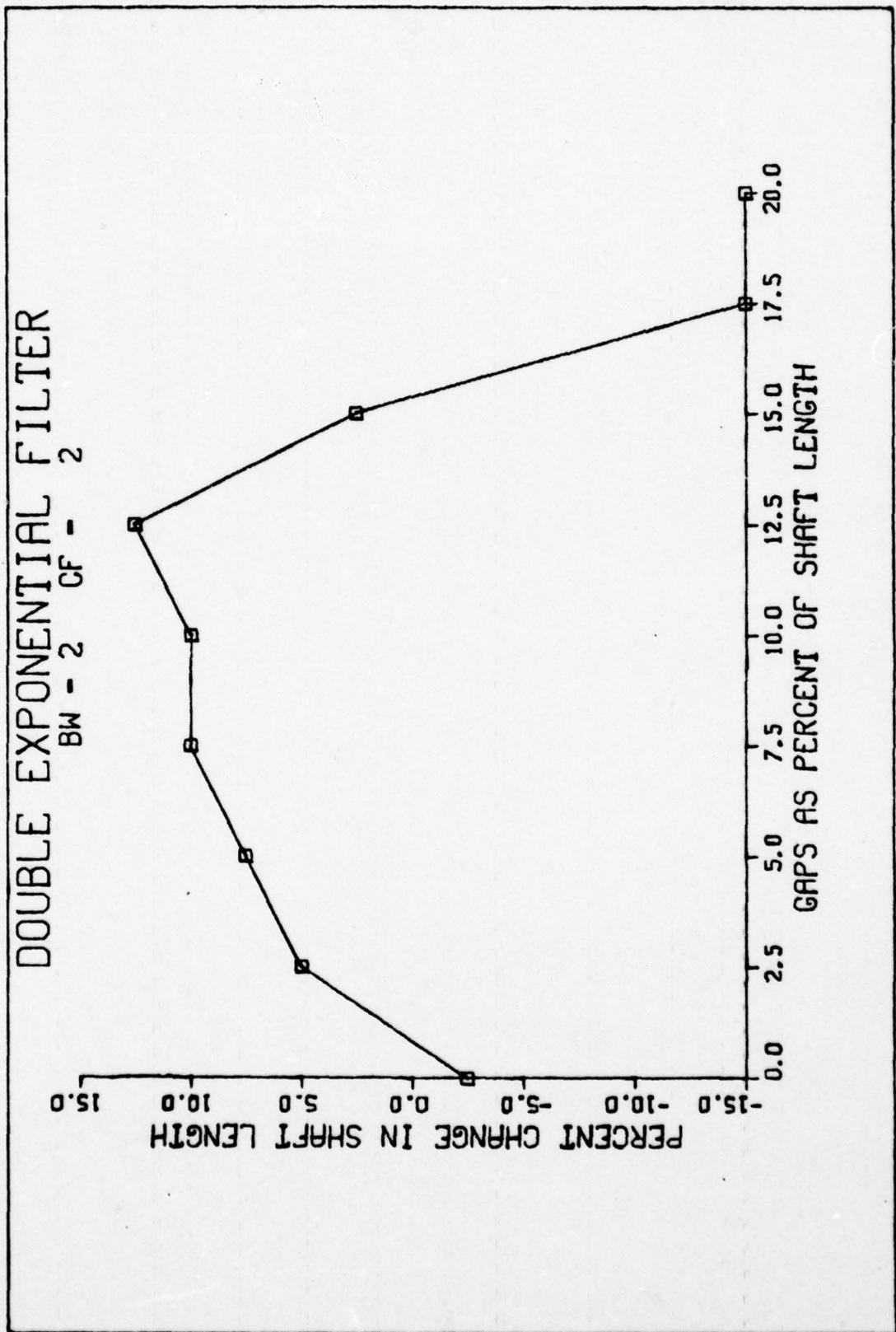
Figure 14. Theoretical Prediction Curve Number 2

VI. Discussion

In order to compare the theoretical curves generated by the computer model and the data curves from the subjects, it is necessary to look at the points of agreement between the curves. First, the general shape of the curves are similar. Each curve shows an inverted U-shape that has been demonstrated by Fellow (Ref 22) and Pressey and Bross (Ref 59) in their studies on this same illusion. Secondly, the theoretical curves predict that the length of the shaft will be initially underestimated, will increase to a peak value where the length of the shaft is overestimated and then will start decreasing. The curves of the subjects clearly follow these predictions. Third, the theoretical curves predicts the underestimated and overestimated lengths to be less than ±15 percent of the length of the shaft. All of the subjects data was also less than ±15 percent of the length of the shaft. Fourth, the location of the peak value of overestimation is predicted by the theoretical curves to occur with a gap size which is approximately 10.0 to 12.5 percent of the length of the shaft. The composite subjects curves also show a peak value at that location.

The previous agreements account for six out of the nine data points collected that comprise the experimental and

theoretical data. In fact, the two prediction curves bracket that the data collected by this experiment and that collected by Pressey and Bross for the first half of those curves (Fig 15). The only minor shape factor in the data from the subjects that is not predicted by the theoretical curves is the gradual rather than abrupt decrease in the perceived length of the shaft of the illusion after the occurence of the peak value.

The algorithm used to implement the length determination subroutine in the computer model might have contributed to the prediction of an abrupt decrease in the perceived length of the shaft of the illusion. The endpoints of the shaft was determined to the nearest array position (which corresponds to 2.5mm steps), however, no attempt was made to obtain a more precise length by interpolating between adjacent array positions. This might explain the abruptness of the prediction curves, however, it does not explain the decrease in the perceived length. The decrease in the predicted length of the shaft of the illusion might result from a harmonic interaction between the size of the gap and the size of the array used to contain the illusion, however, this remains to be shown. At this time, the algorithms used in the computer model cannot be said to cause the abrupt decrease observed
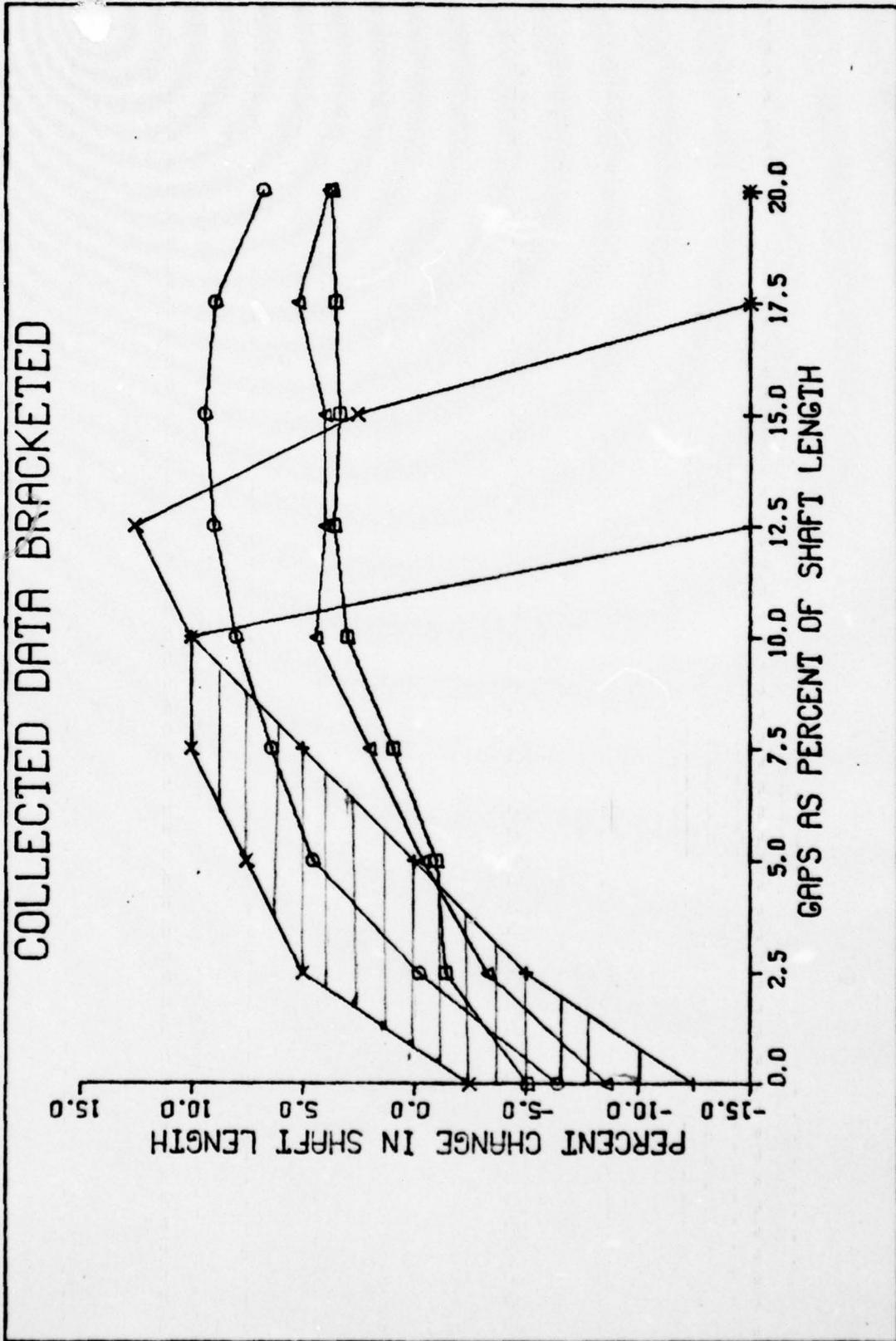
Figure 15. Pyschophysical Curves Bracketed by the Theoretical Prediction Curves.

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in the theoretical data.

Next, the assumptions upon which the filter was con-
structed may explain the difference between the experimental
and theoretical data. Since the computer model was based on
the use of a single channel to determine the length of the
shaft, the fact that the model could not predict all the
data could be attributed to this assumption. Experimental
data has provided the shape of the individual "channels" but
not how these channels are combined to accomplish the task
of length estimation. To prevent an abrupt change when
switching between channels, it might be expected that the
spatial frequency channels on each side of a selected channel
would contribute to the total image perceived by the visual
system. The combining of channels was not considered in this
experiment as it was investigating the simplest theory, that
a single channel was totally responsible for the observed
phenomena.

Therefore, an explanation of the theoretical prediction
of an abrupt decrease in the estimated line length might be
attributable to either a defect in the line length determina-
tion algorithm, the use by the visual system of multiple
channels which are combined to produce the length estimate,
or a combination of these explanations.

However, this model of a single channel did successfully predict the largest portion of the data curves obtained from the subjects. These results support the contention that the human visual system not only consists of bandwidth limited channels as shown by Blakemore and Campbell (Ref 1) but uses spatial data from each channel to determine, at least, the length of the object (Ref 31). These results also suggest that the determination of length and, hence, form and shape, could possibly be accomplished by the visual system from low-pass filtered images. This agrees with the results of experiments in pattern recognition which have successfully used the low spatial frequency information for shape discrimination (Ref 61, 65). Conversely, the results show that the high spatial frequencies (edge information) are not required to determine length and, in fact, will not produce the observed distortions in length perception.

The method which worked the best in the computer model for determining the endpoints of the shaft used the peak intensity of a low spatial frequency channel. This result is intuitively pleasing since an algorithm to find a peak intensity value is probably the simplest detection algorithm possible.

The results of this experiment would seem to be very

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important to visual researchers. This is the first time that the filtering characteristics of spatial frequency channels, derived from biological data, have been used to produce quantitative predictions of human judgments of spatial size in a complex object. Changes in perceived length in geometric illusions generally have been considered to be the result of cognitive processes. This experiment shows that the spatially limited mechanisms in the human visual system can produce the same qualitative and quantitative distortions in length that are reported by subjects in a psychophysical experiment.

VII. Conclusions

The following conclusions have been reached for this experiment:

1. The determination of length in a complex object appears to be based upon spatial information in low spatial frequency channels in the human visual system as predicted by Ginsburg (Ref 31).

2. The determination of length may not be completely based upon spatial information from a single channel although much predictive power has come from a single channel with filter characteristics of about ±1 octave bandwidth and a center frequency between 2 and 4 cycles per object size.

VIII. Recommendations

This research suggests many new experiments to clarify and expand the information that was obtained. These will be listed in order of importance.

First, the method used by the human visual system to combine the outputs of the individual channels for determining length should be investigated. One experiment to do this would be to mask different regions of the spatial frequency spectrum of an object. This technique could be used to selectively decrease the contribution of different spatial frequencies in channels around the 2 - 4 cycles per object size channel. Then, by repeating the experiment described in this thesis, it is anticipated that there will be little change in the judgment of length by the subject for the first six stimuli used in the experiment and that the length judgments of the subjects will abruptly drop, like the single channel prediction curves, for the remaining three stimuli. This experiment might allow the channel combination algorithm used by the visual system, if one exists, to be studied.

While the retina is non-homegeneous, especially outside the foveal region, it may be considered homegeneous for low spatial frequencies. Since the data from this study indicates that the low spatial frequencies are used to determine length,

measuring the perceived length of a stimuli placed in the periphery of the subject's vision should be possible. This experiment, besides demonstrating whether length judgments may be made by the periphery using the low spatial frequency information, might provide insight into the function and organization of the pera-foveal region.

Lastly, other geometric illusions should be investigated using the channel filter concept. This will provide additional data to better evaluate the channel filter concept as a quantitative tool with which to predict the performance of the human visual system.

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APPENDIX A

Subjects

Curves
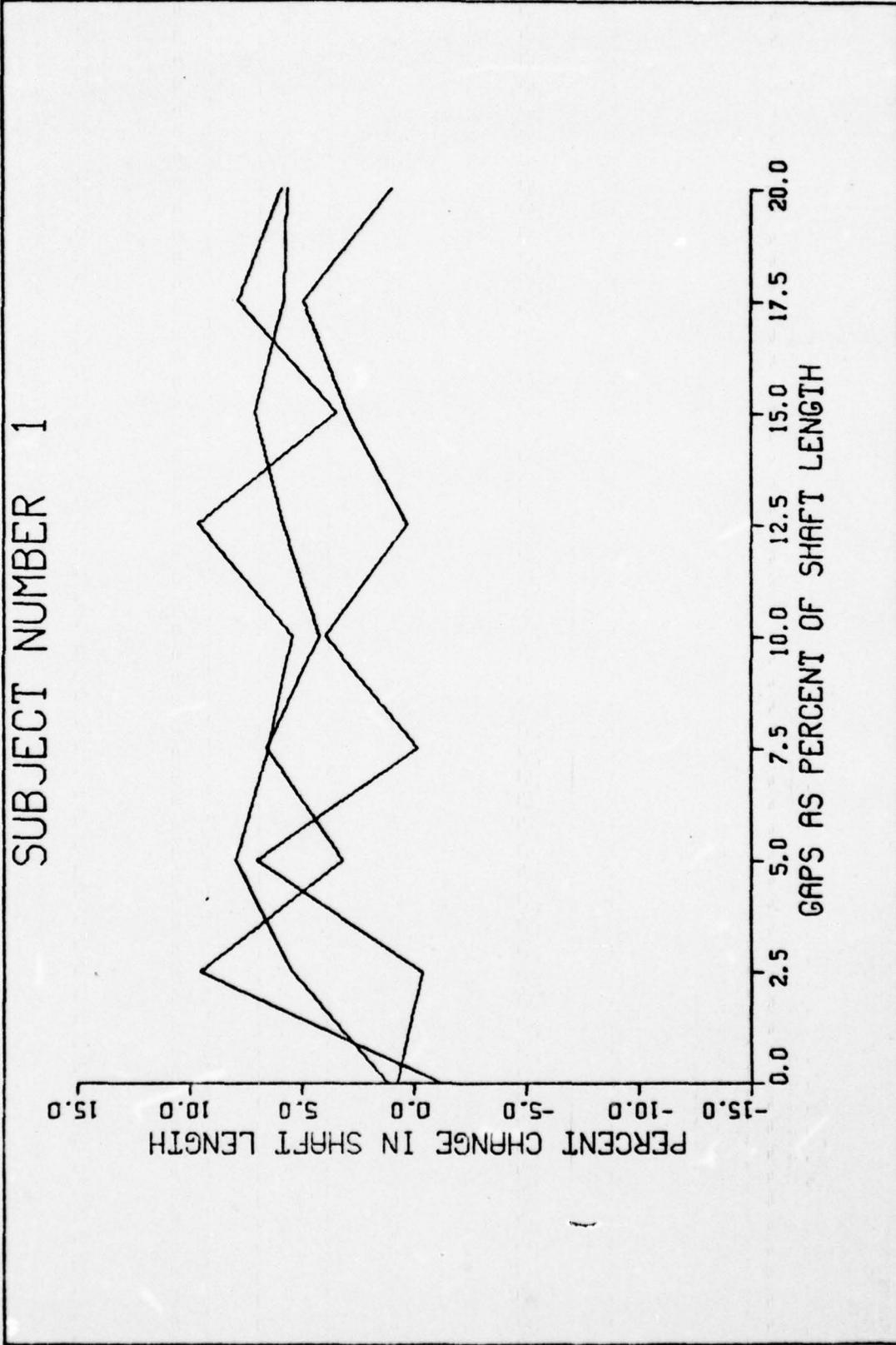
Figure A1. Data Curve - Subject Number 1

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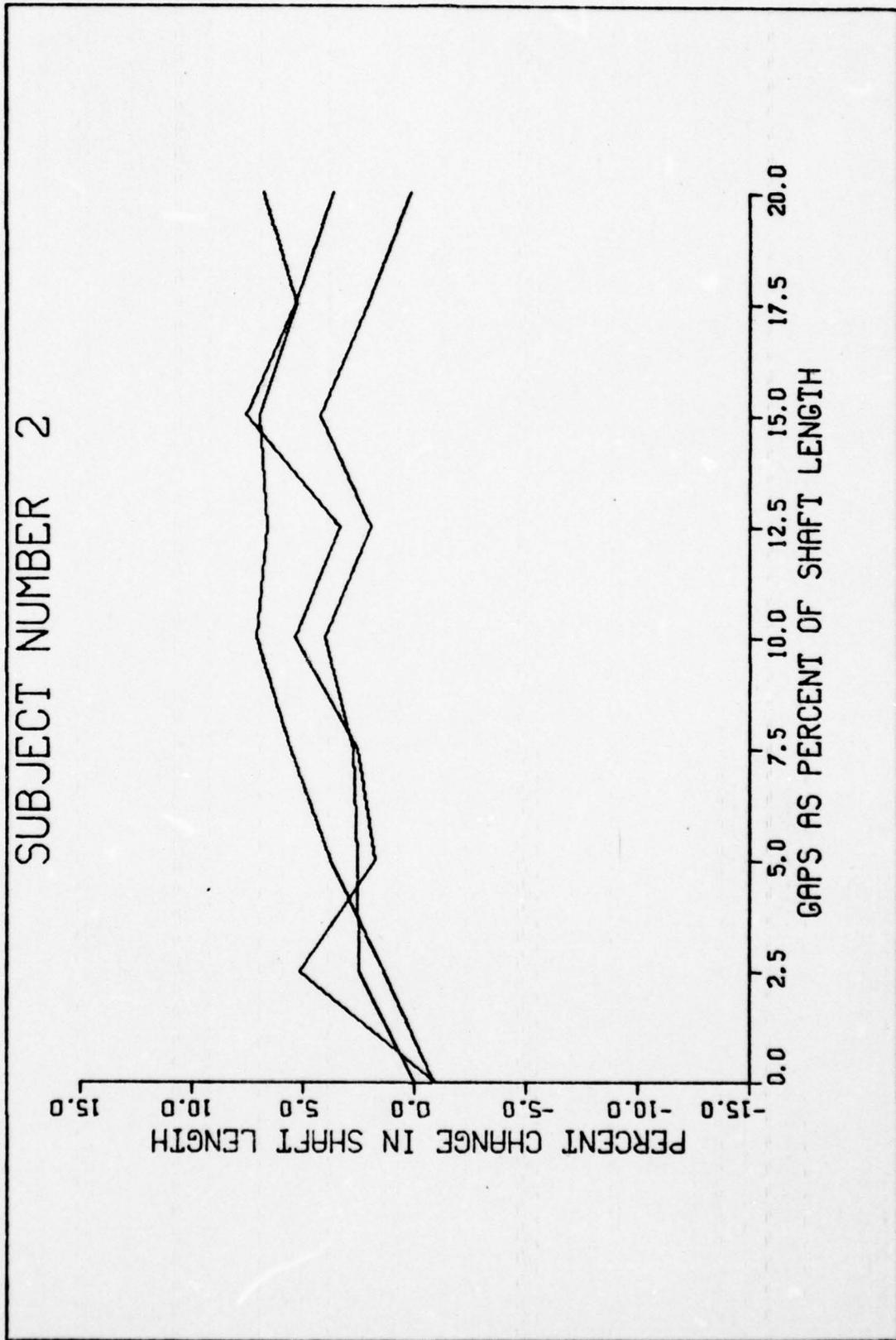
Figure A2. Data Curve – Subject Number 2

64
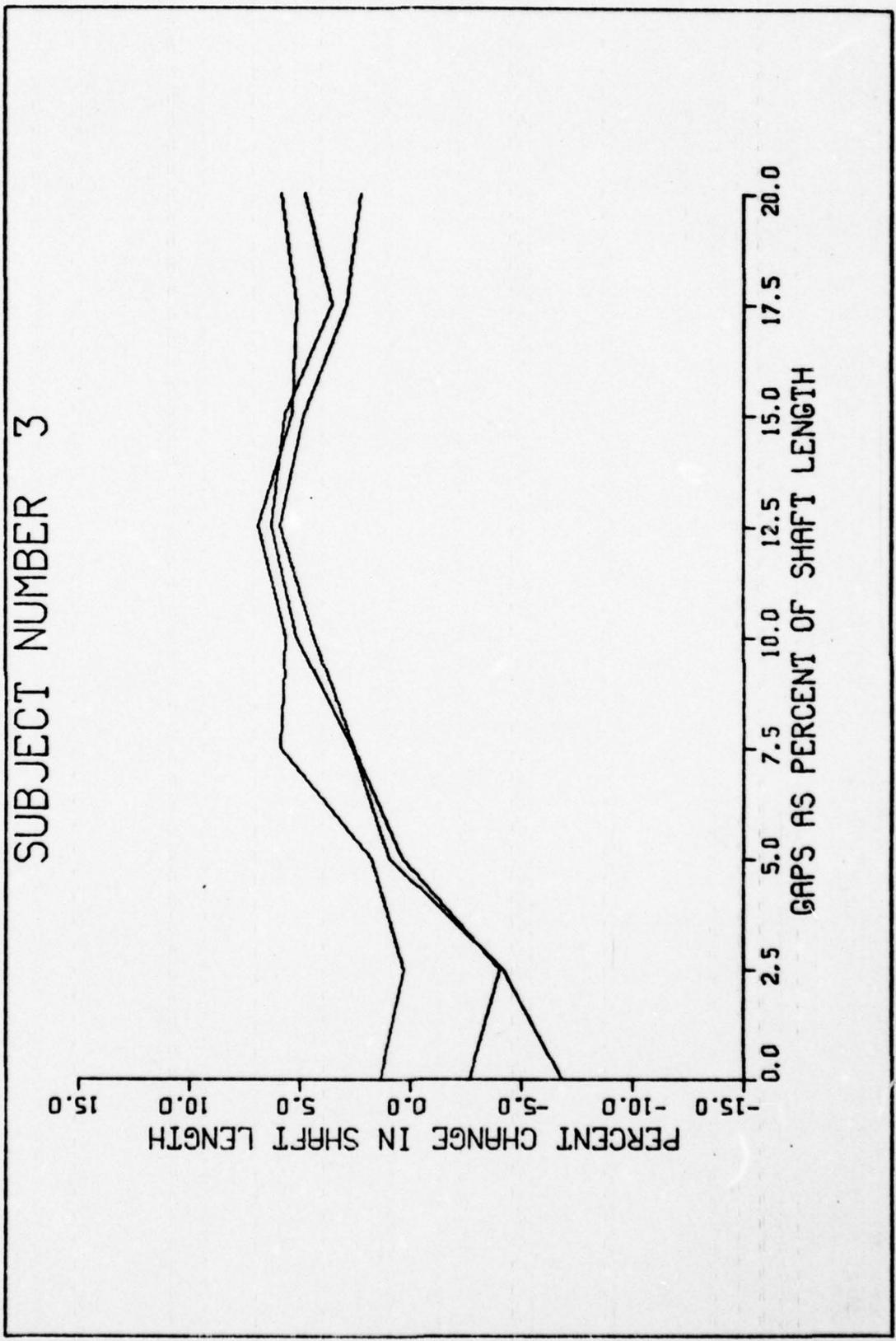
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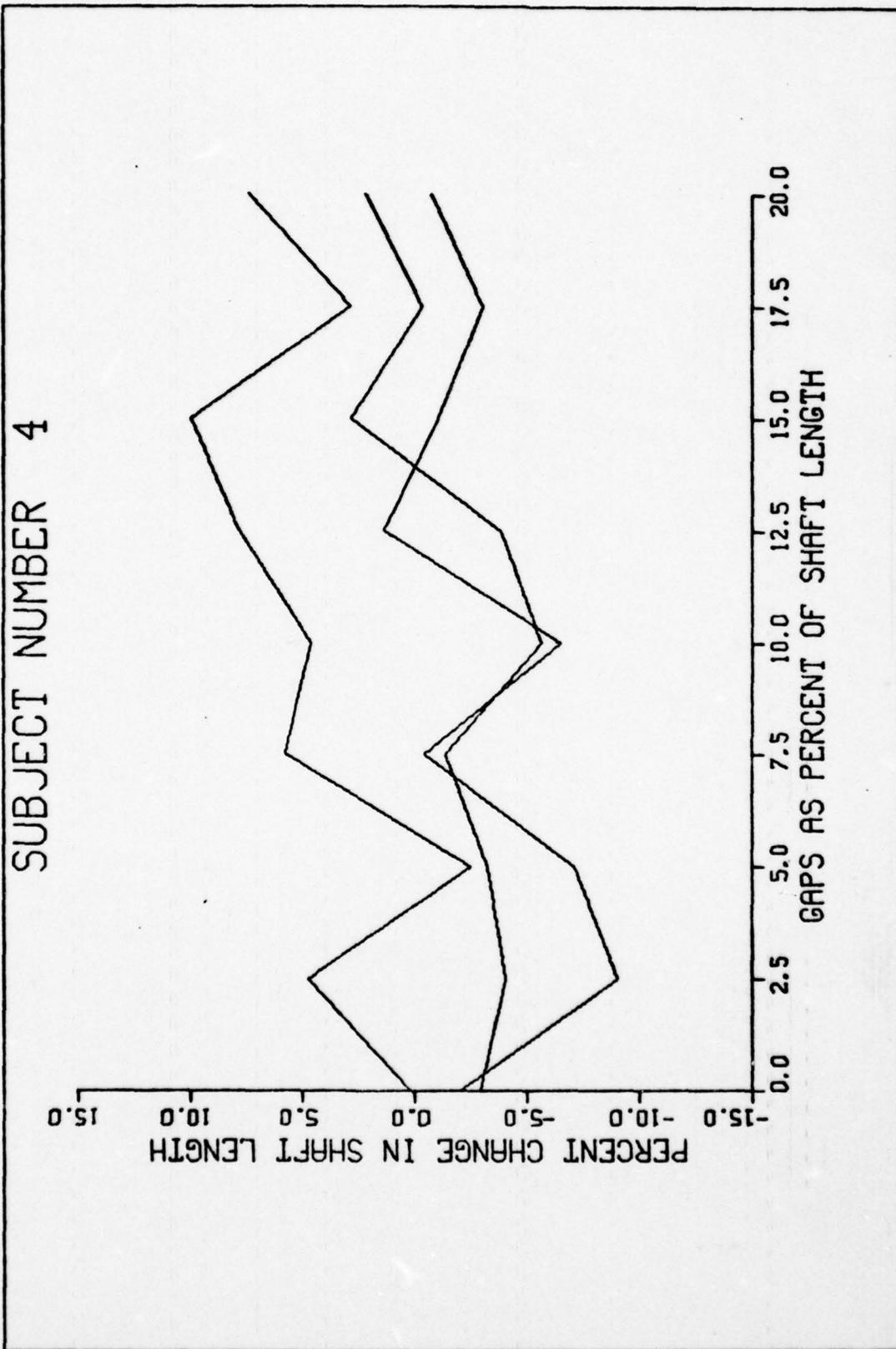
Figure A4. Data Curve - Subject Number 4
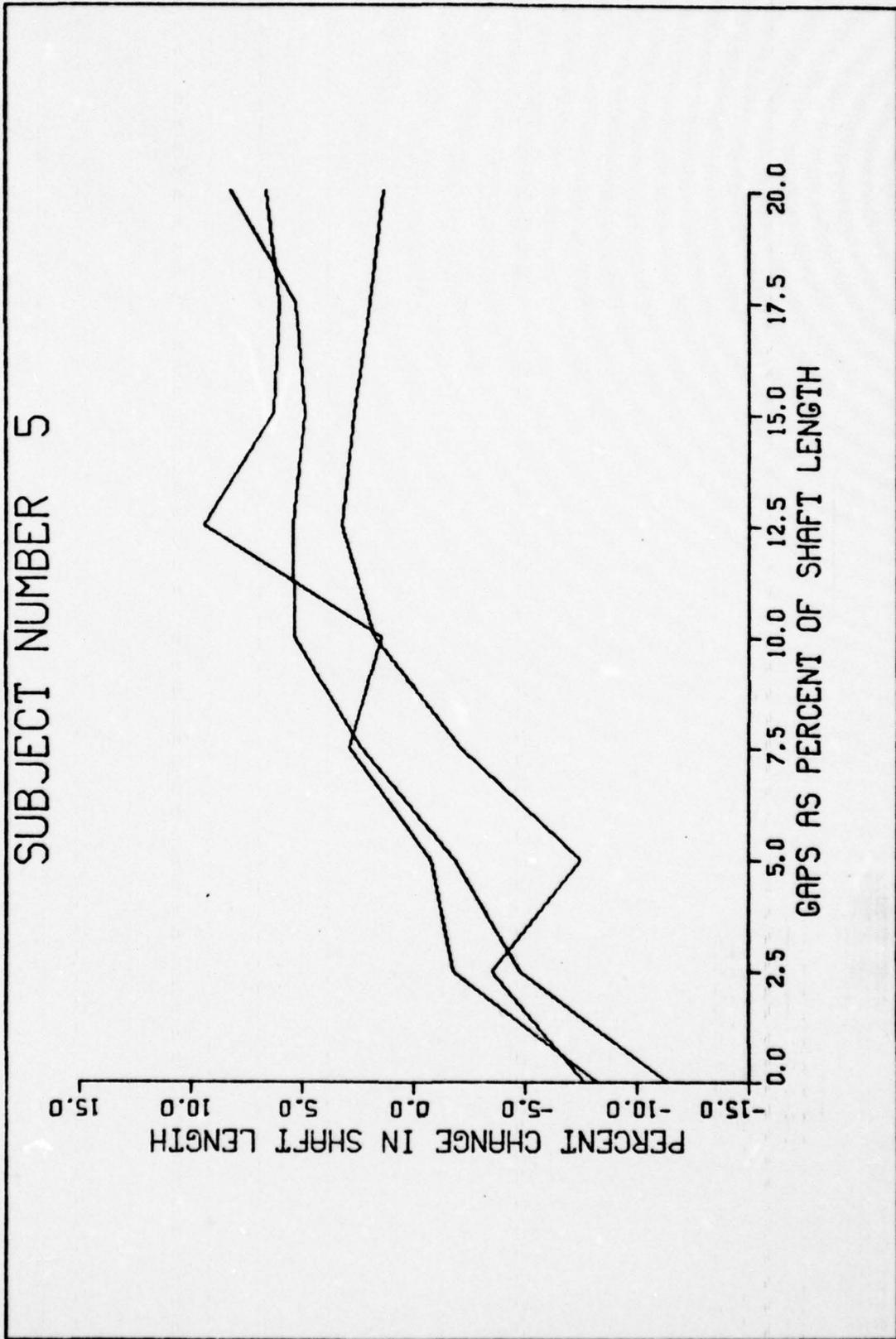
Figure A5. Data Curve - Subject Number 5

67
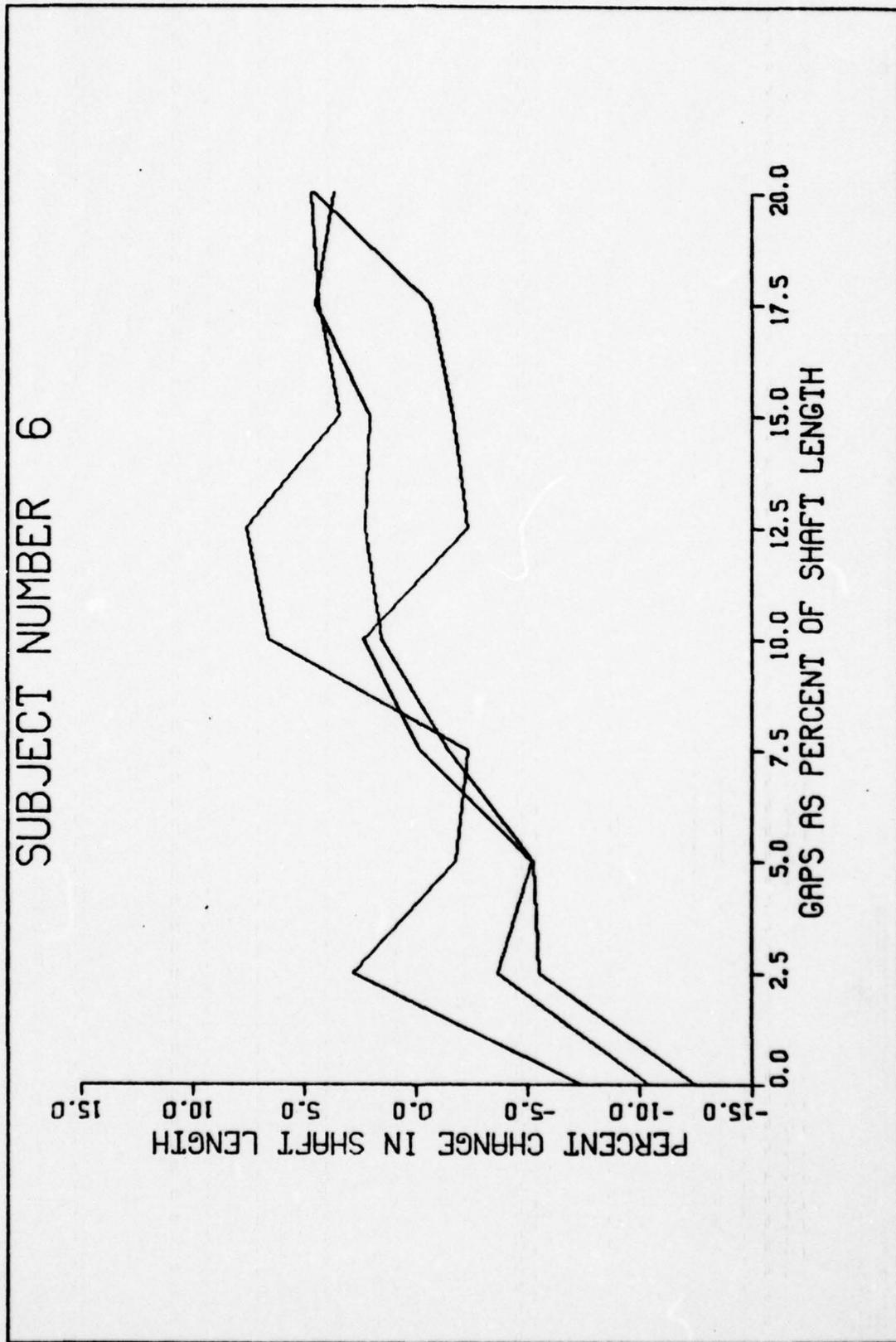
Figure A6. Data Curve – Subject Number 6

68
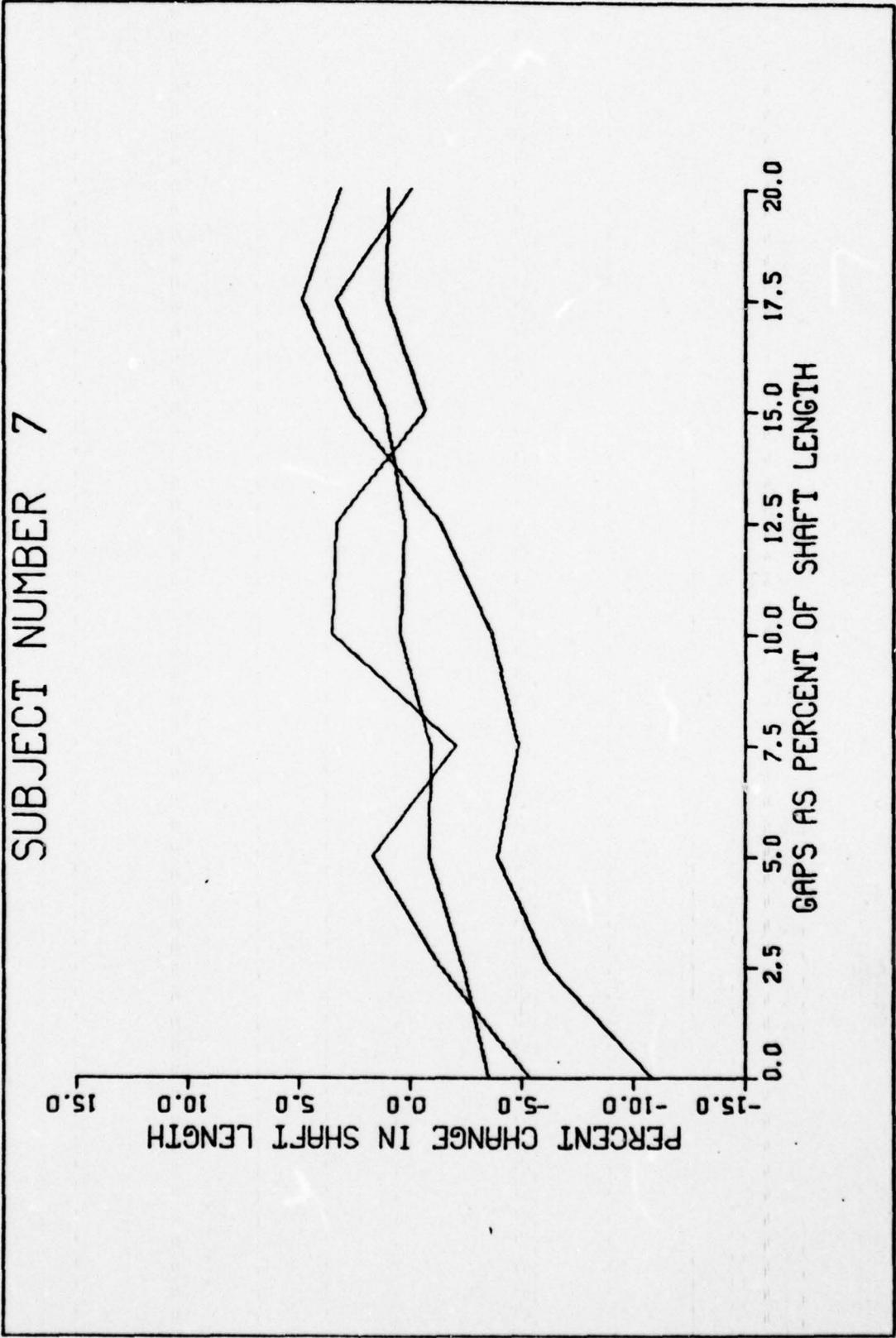
Figure A7. Data Curve - Subject Number 7
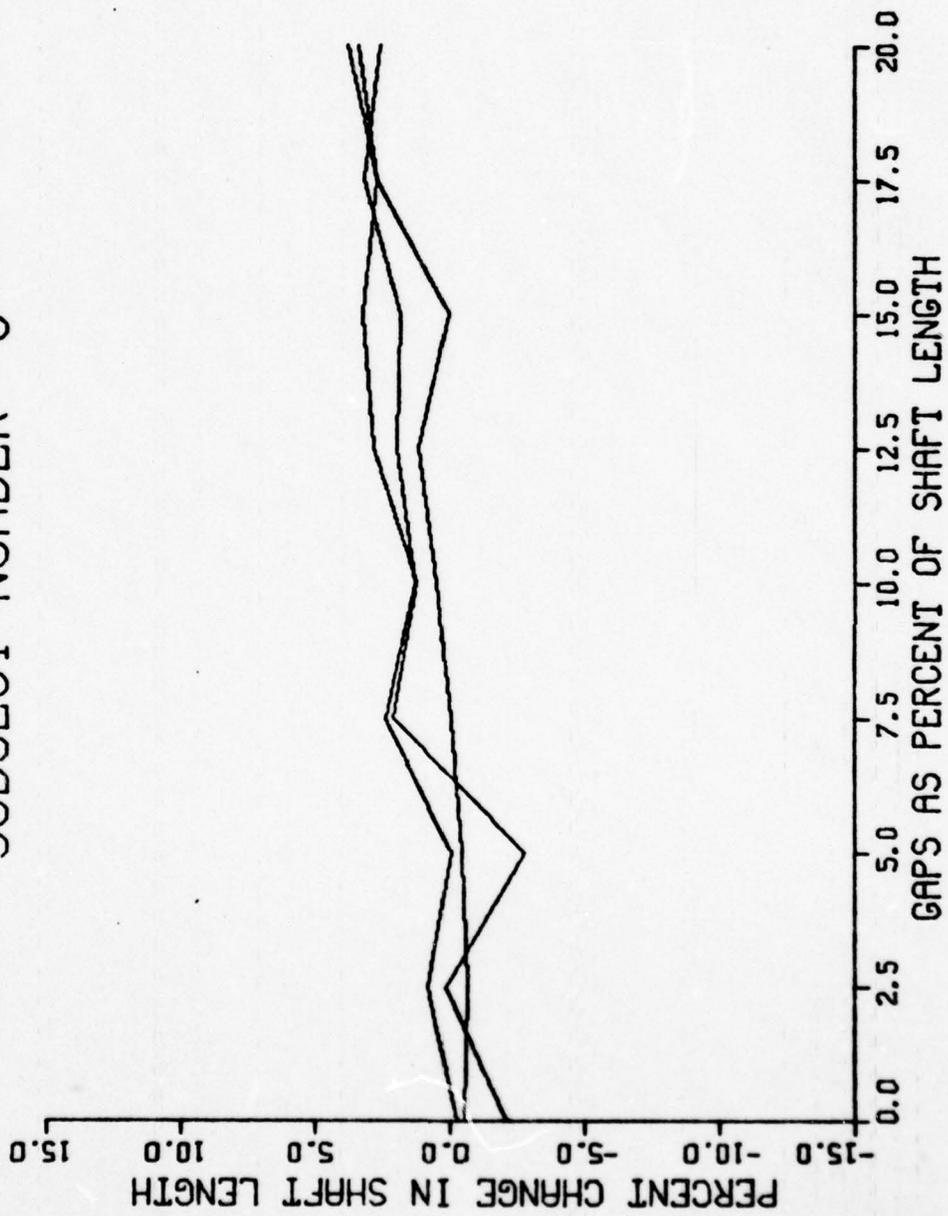
Figure A8. Data Curve – Subject Number 8

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APPENDIX B

Theoretical Prediction Curves

The four curves on each plot represent the predicted length using the 100% (□), 75% (○), 50% (△) and 25% (+) value of the peak. Bandwidth (BW) is specified in octaves and the center frequency (CF) is specified in cycles per object size.
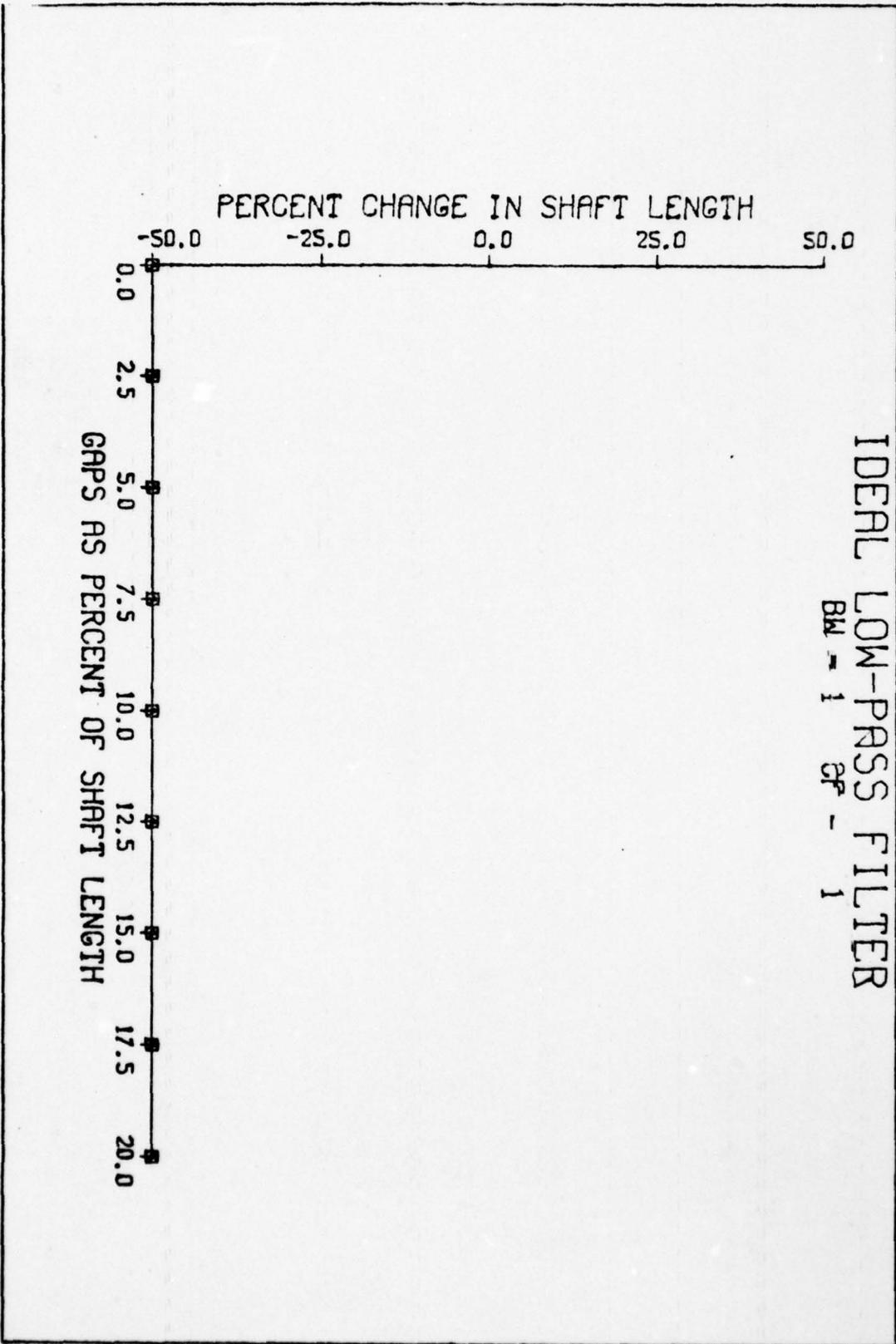
Figure B1.   Prediction Curve   Ideal Low Pass   BW=1   CF=1
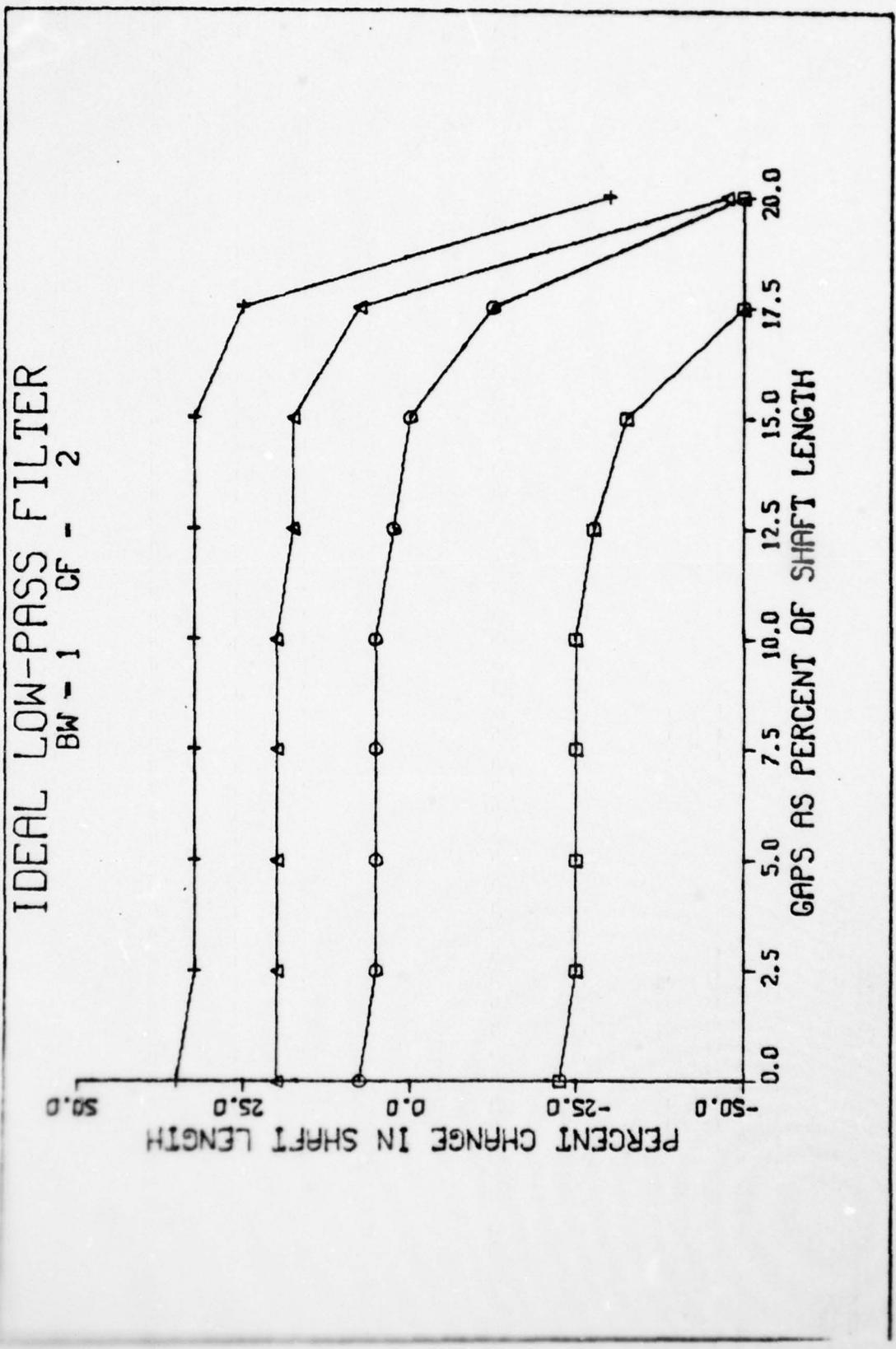
Figure B2.   Prediction Curve - Ideal Low Pass   BW=1   CF=2
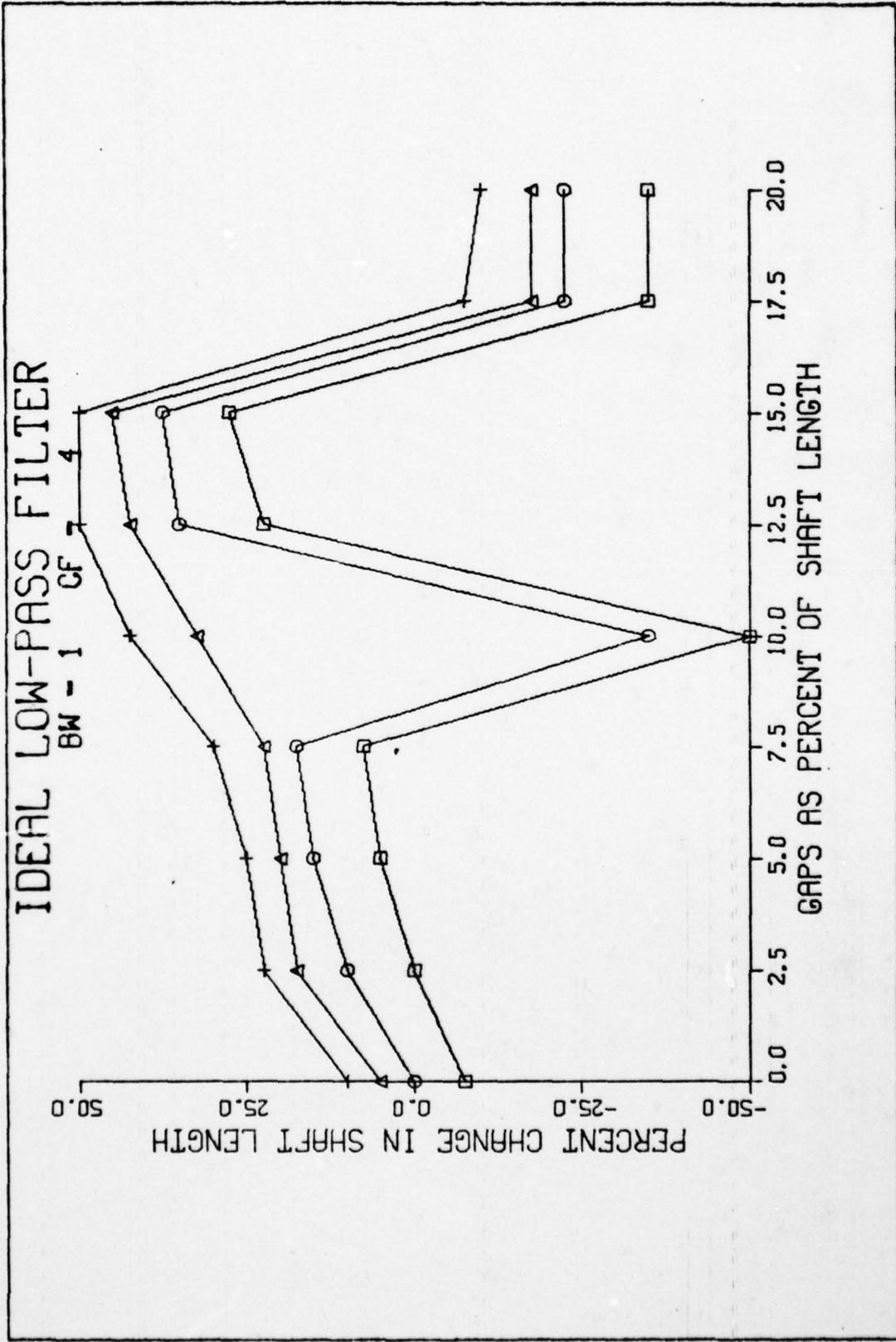
Figure B3. Prediction Curve – Ideal Low Pass BW=1 CF=4
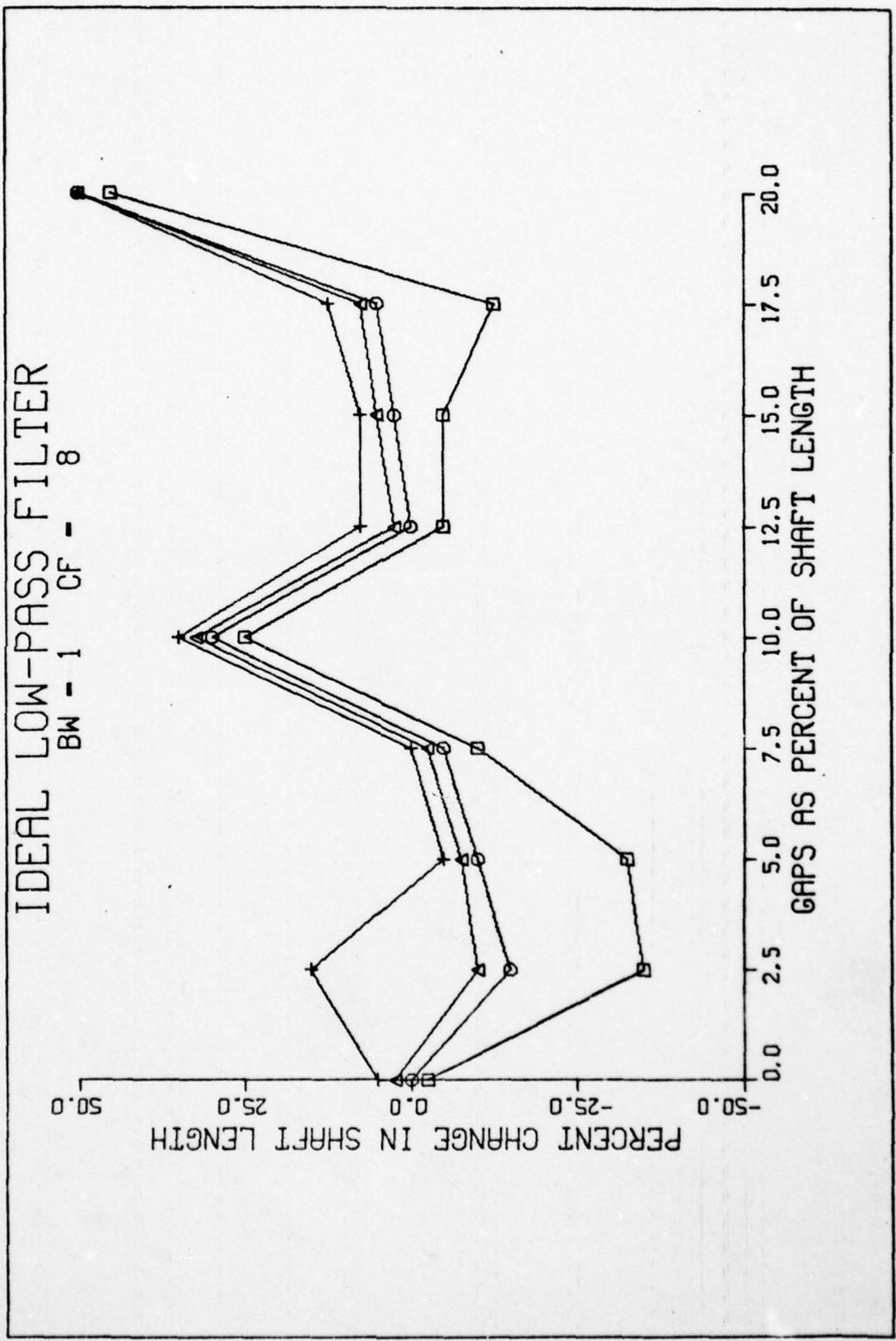
Figure B4.   Prediction Curve – Ideal Low Pass   BW=1   CF=1
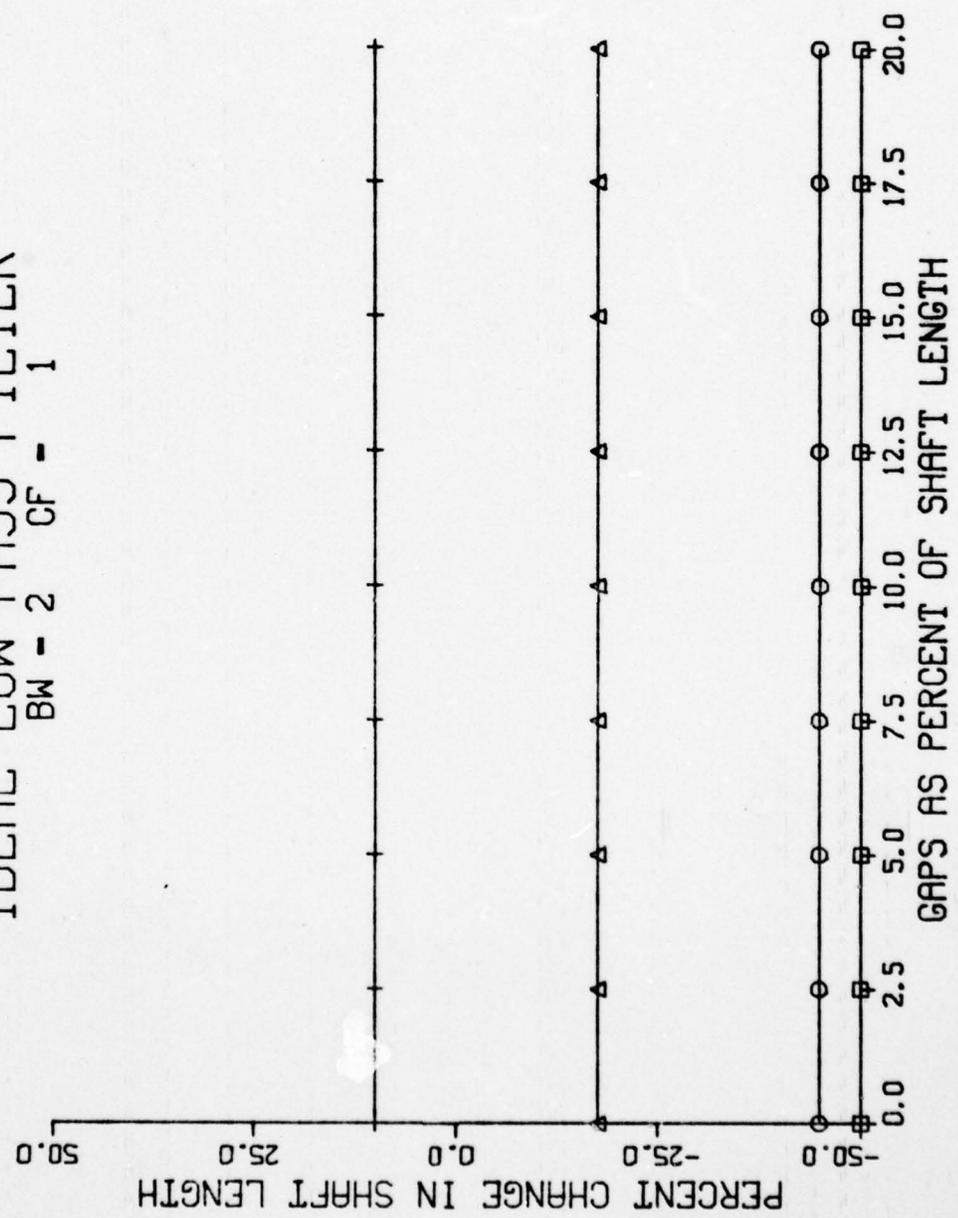
Figure B5. Prediction Curve – Ideal Low Pass BW=2 CF=1
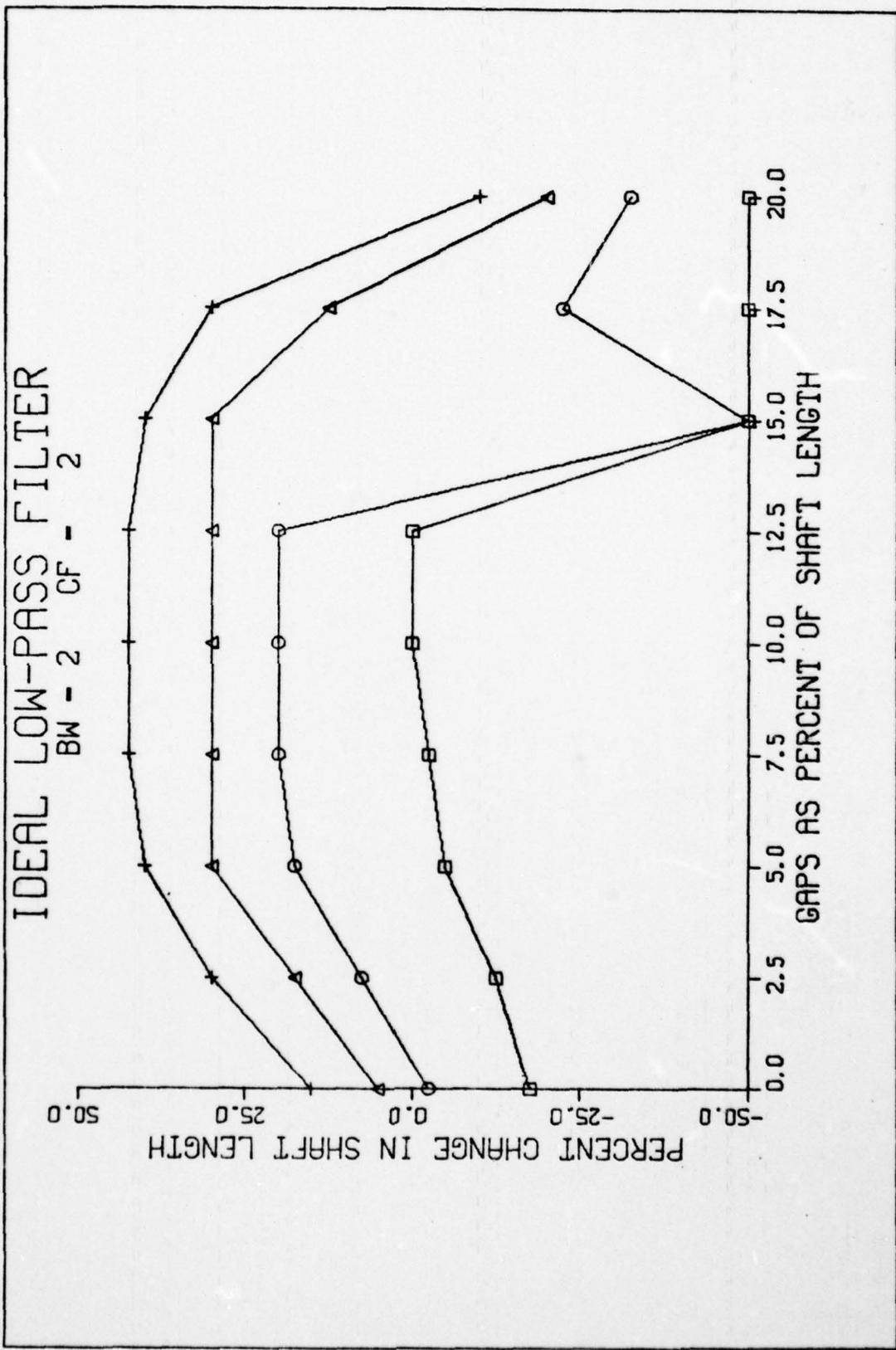
Figure B6. Prediction Curve – Ideal Low Pass BW=2 CF=2
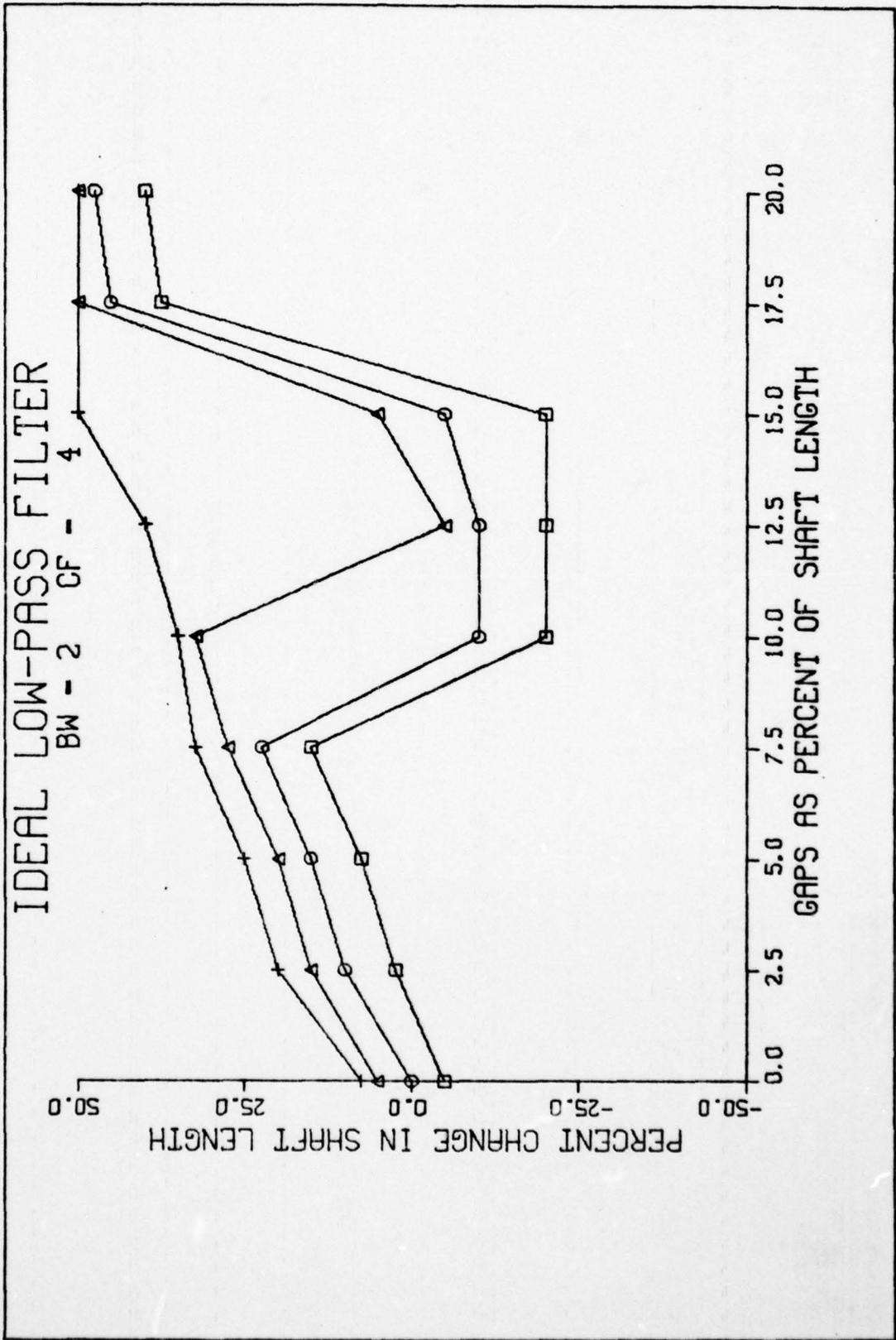
Figure B7. Prediction Curve - Ideal Low Pass BW=2 CF=4
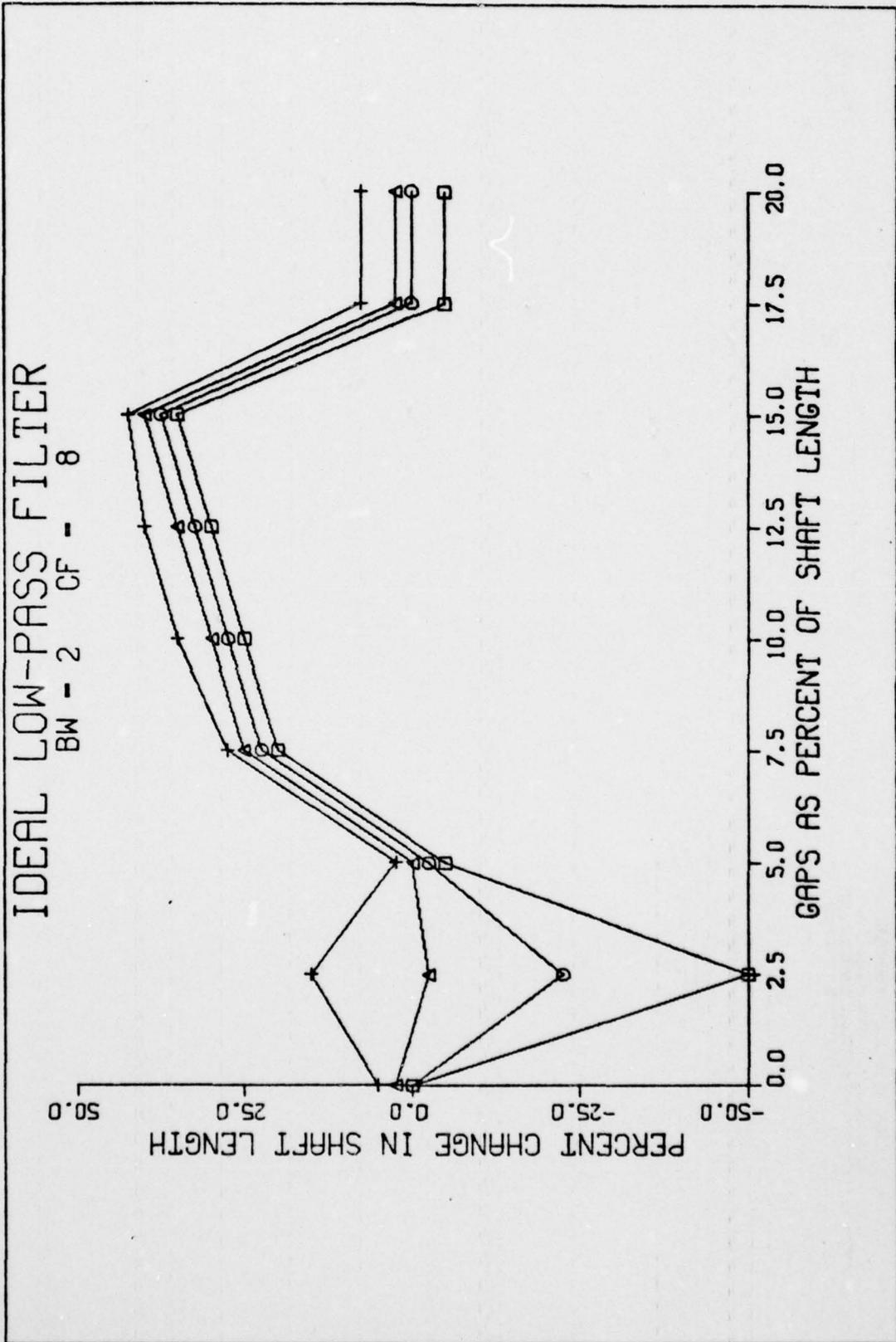
Figure B8. Prediction Curve - Ideal Low Pass BW=2   CF=8
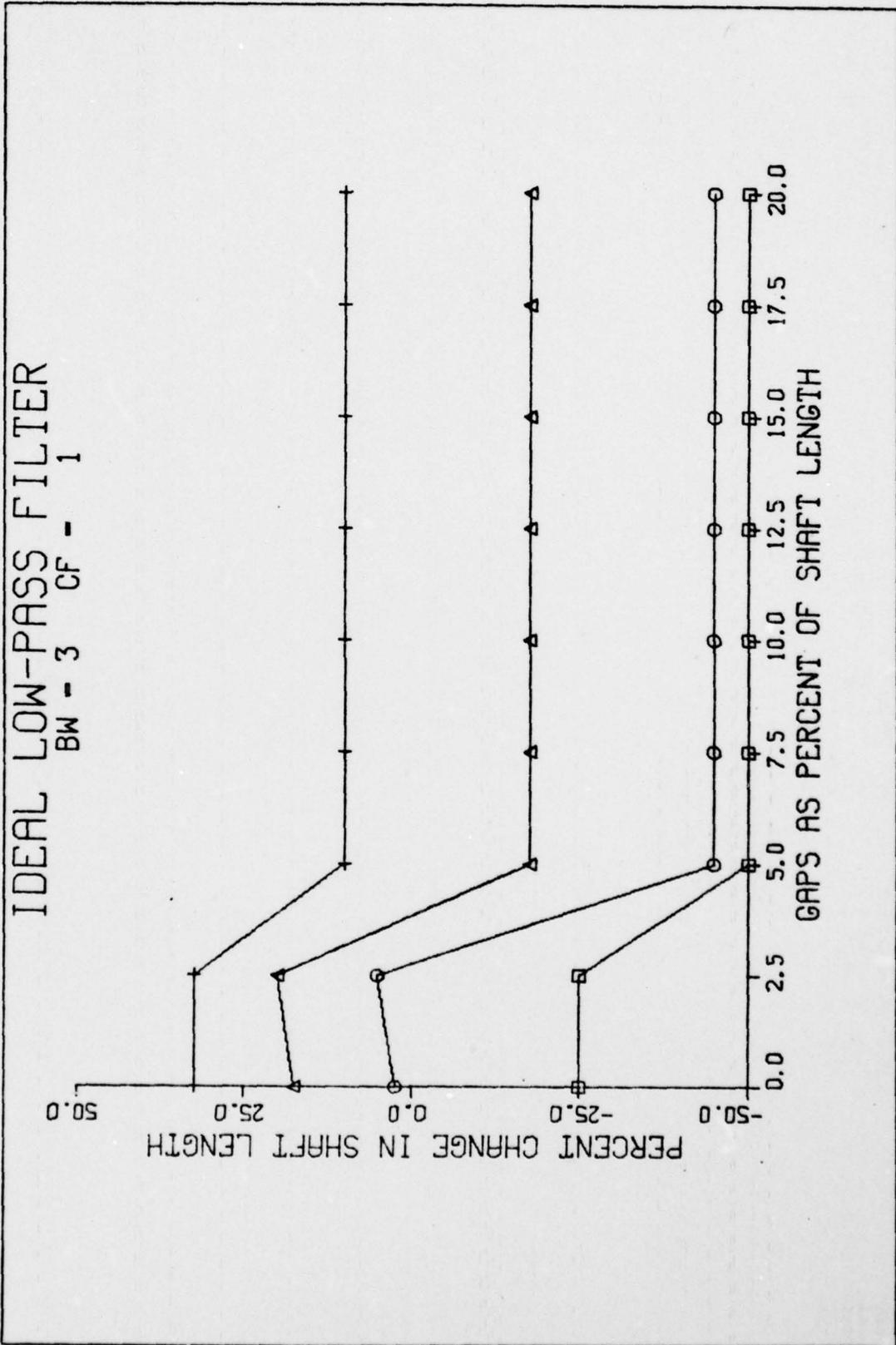
Figure B9.   Prediction Curve – Ideal Low Pass BW=3   CF=1
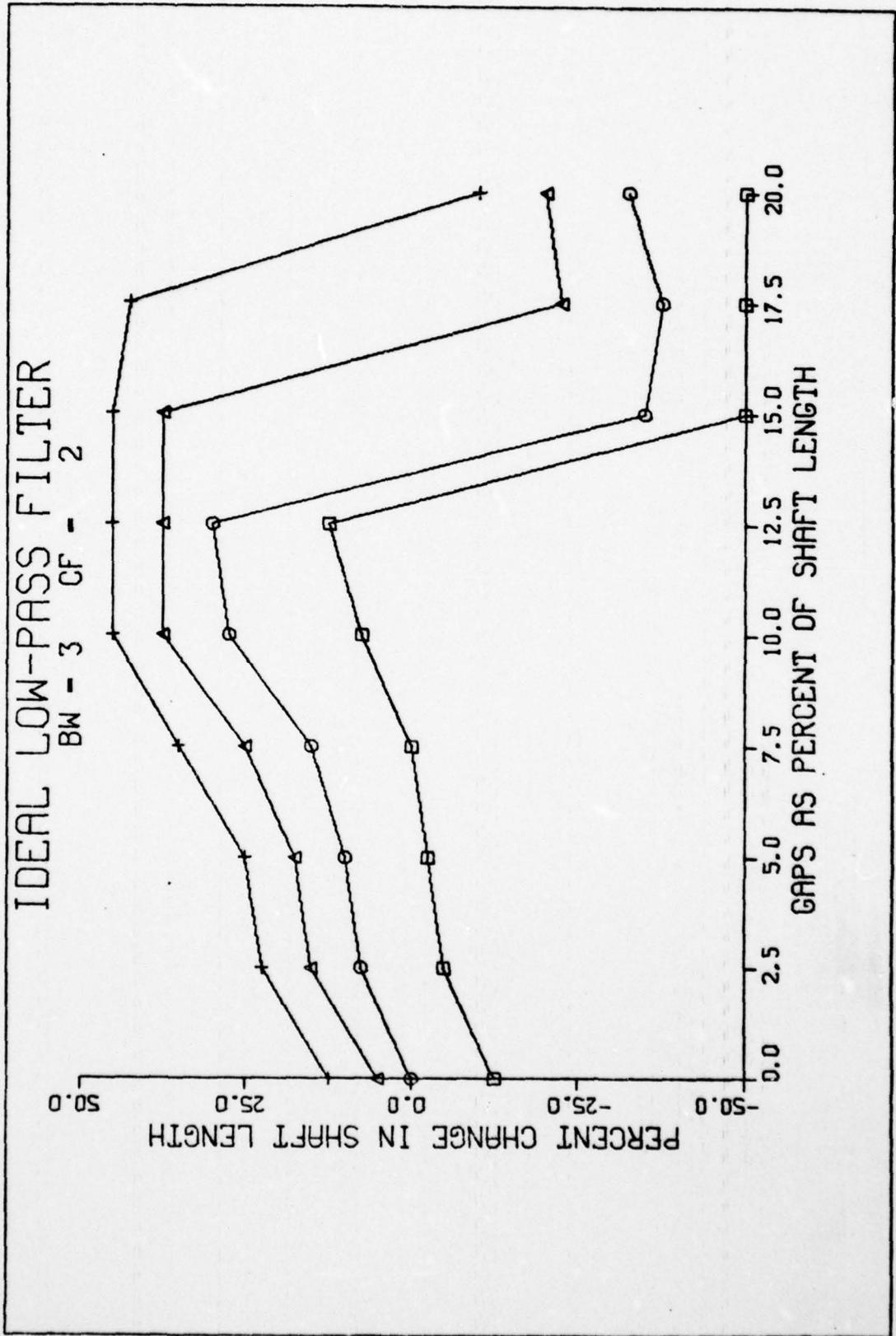
Figure B10.   Prediction Curve - Ideal Low Pass BW=3   CF=2
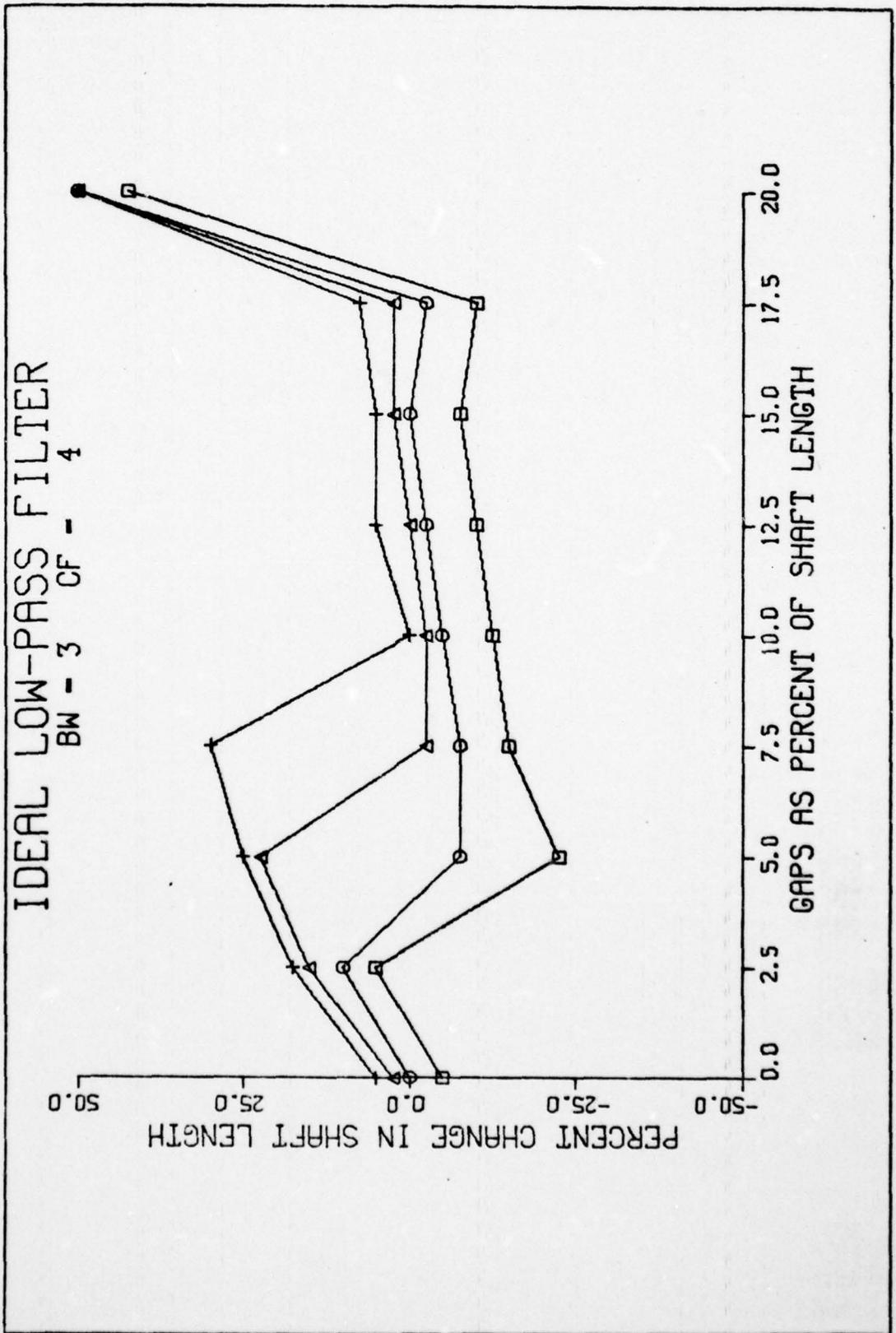
Figure B11. Prediction Curve - Ideal Low Pass BW=3 CF=4
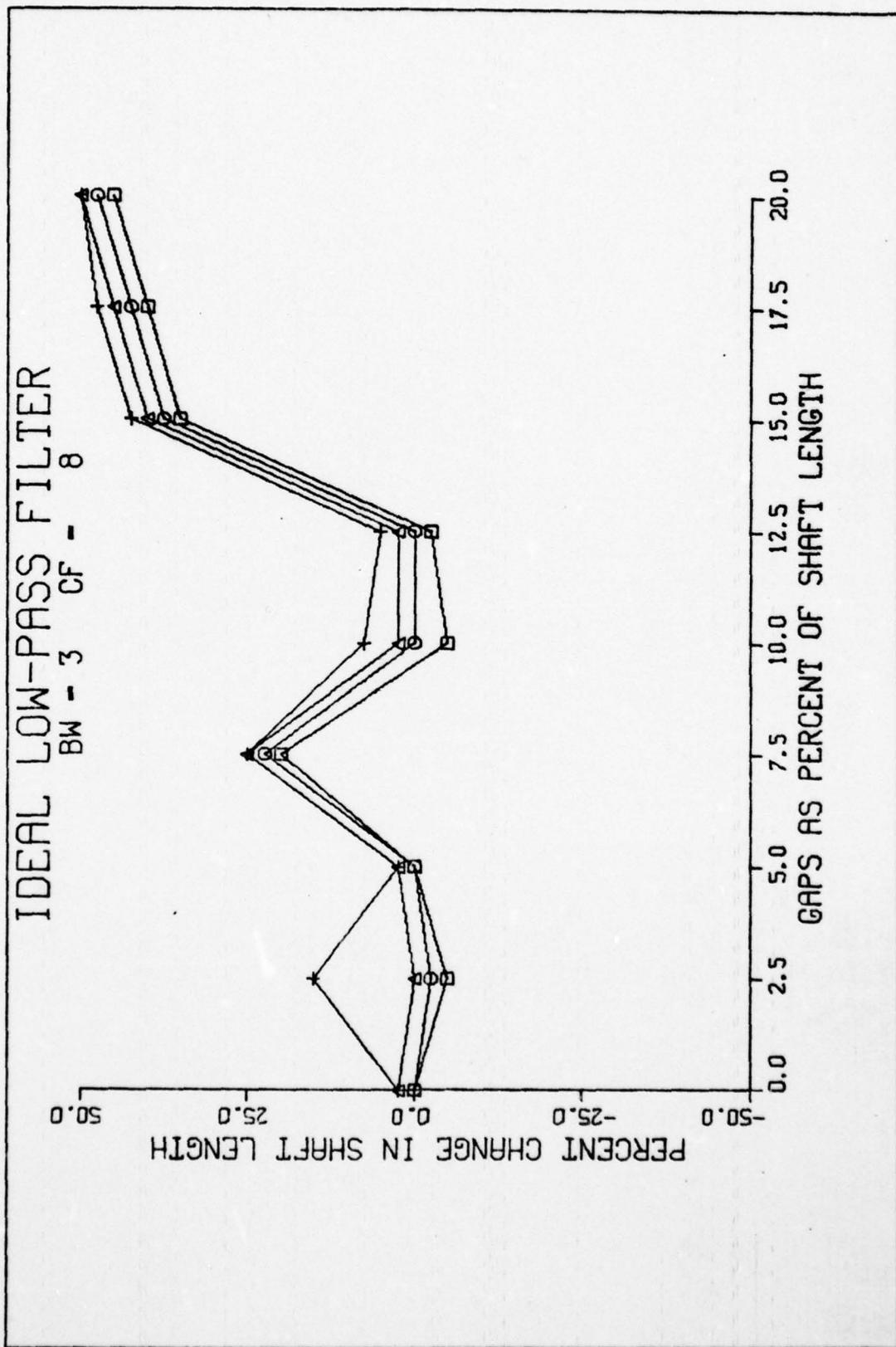
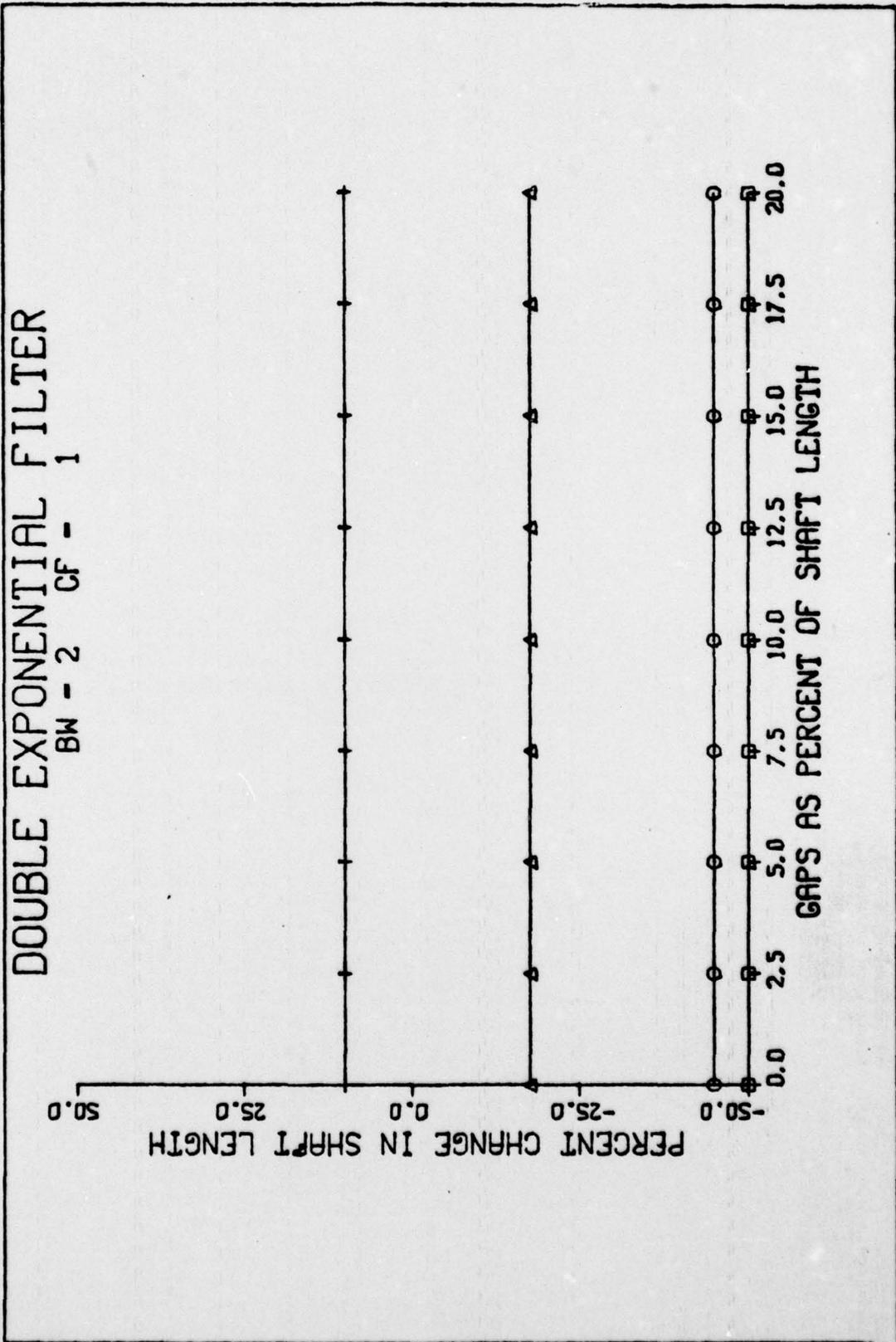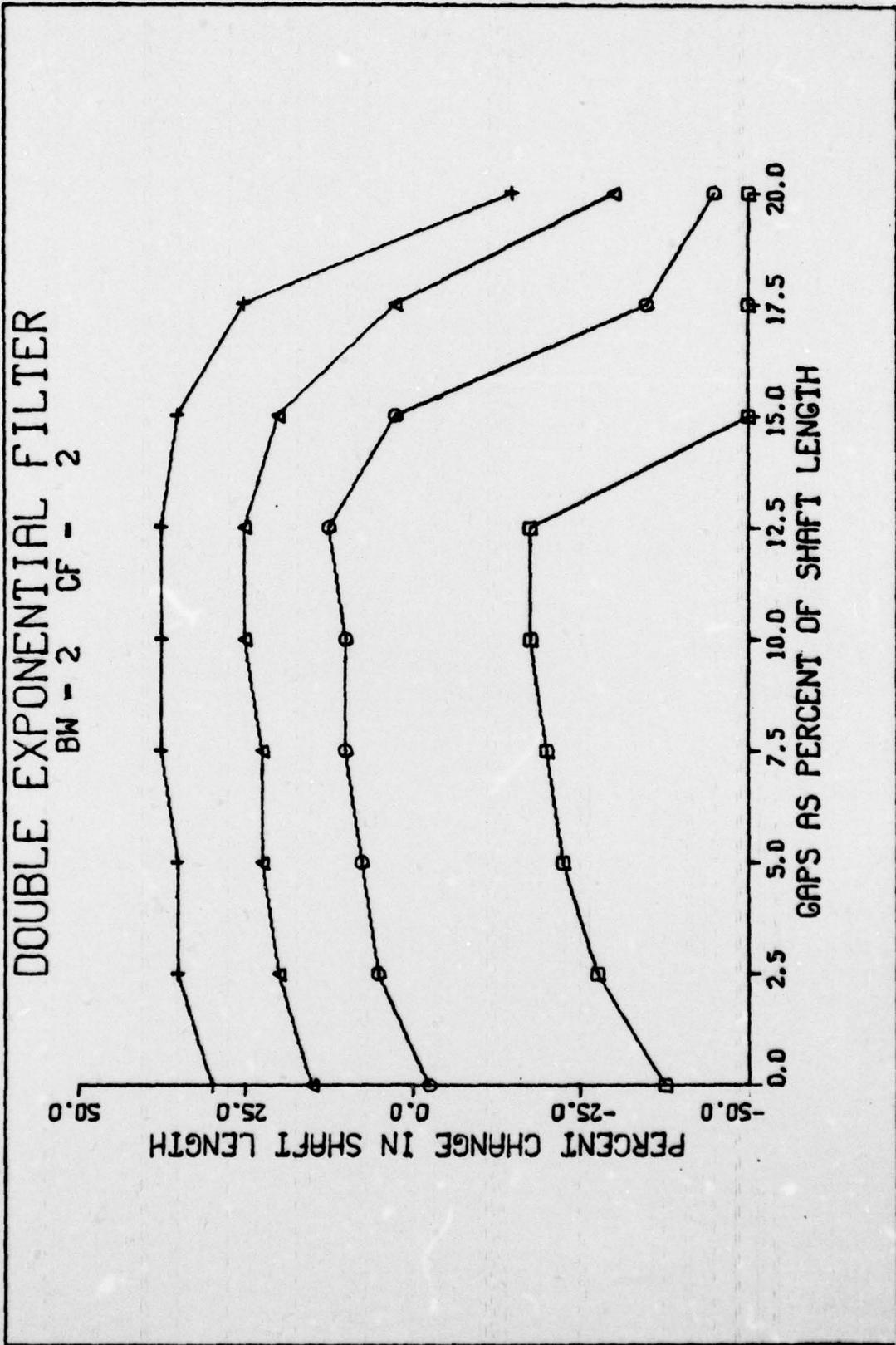Figure B12. Prediction Curve - Ideal Low Pass   BW=3   CF=8
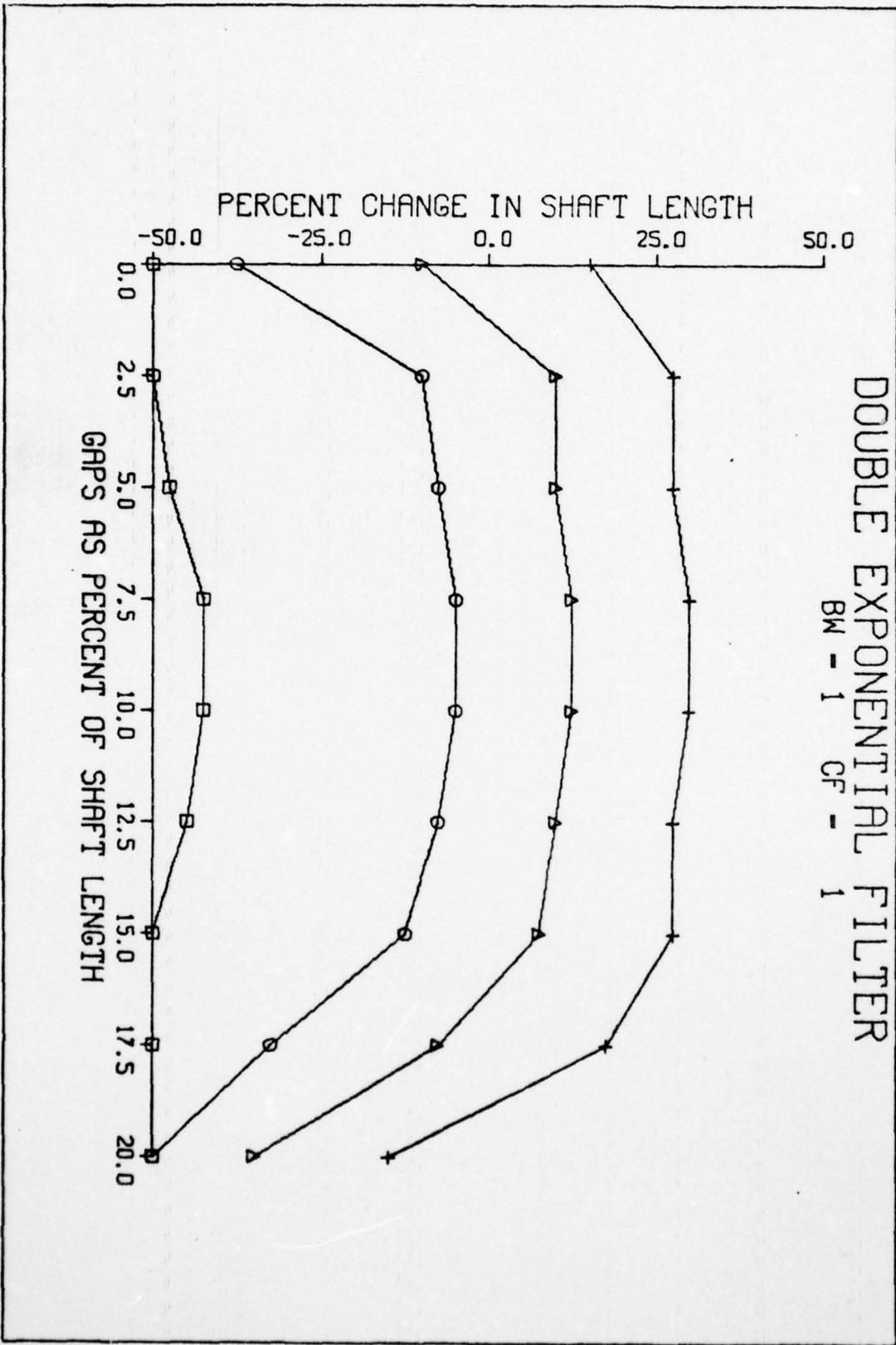
Figure B13. Prediction Curve Double Exponential BW=1 CF=1
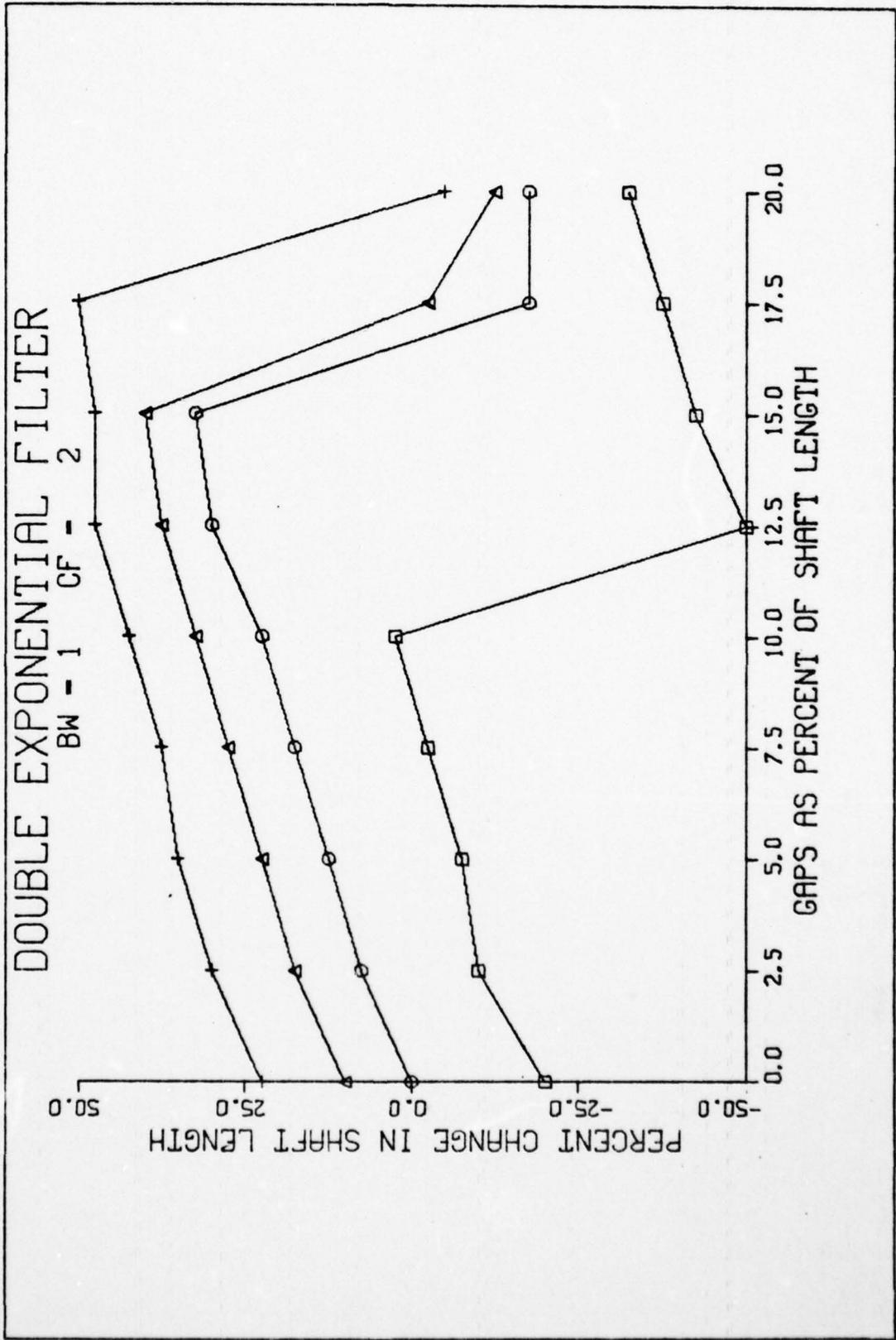
Figure B14.   Prediction Curve - Double Exponential   BW=1   CF=2
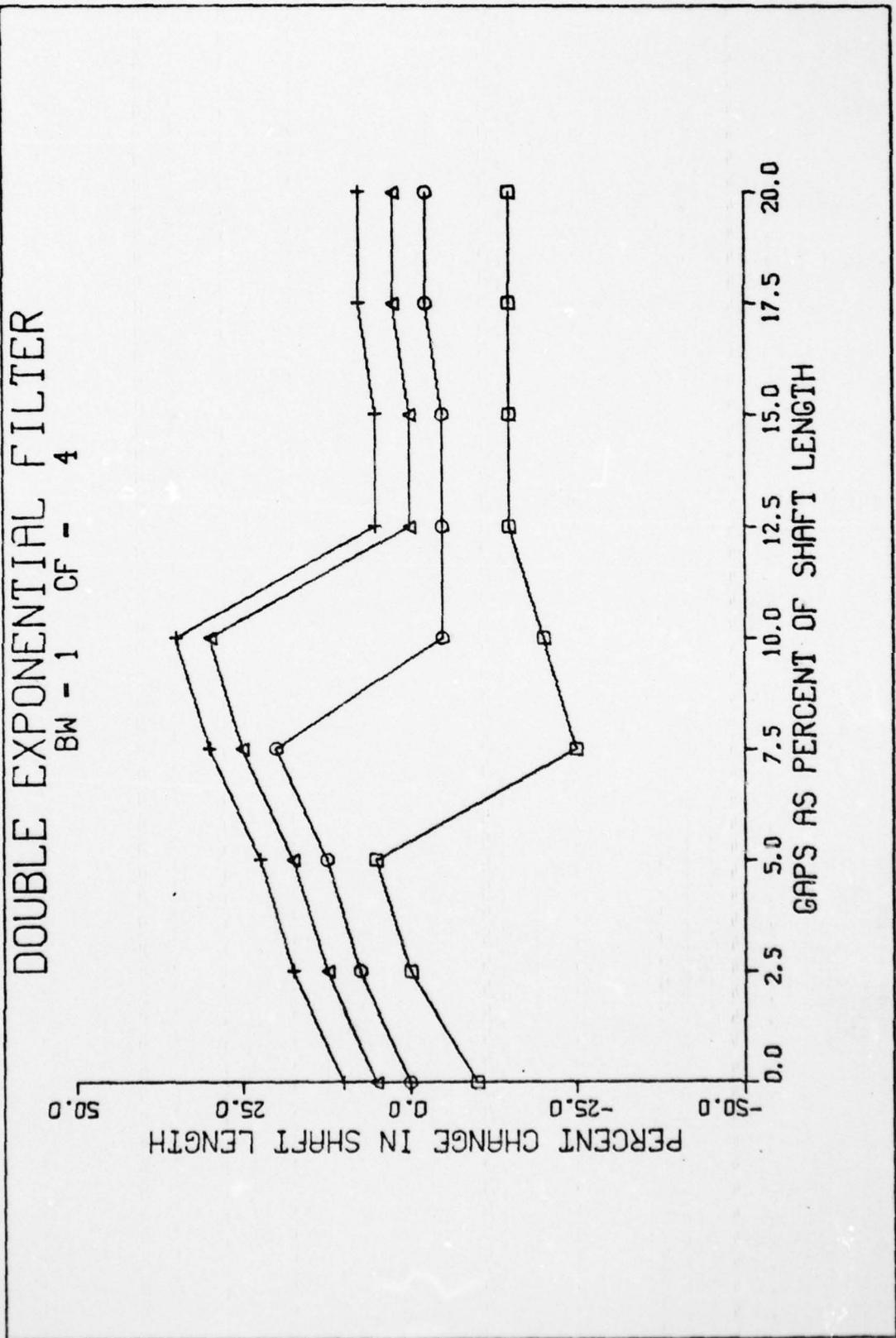
Figure B15.   Prediction Curve – Double Exponential   BW=1   CF=4

1·0   2·8   2·5
3·15 2·2
3·5
1·1   4·0   2·0
4·5
1·8
1·25 1·4   1·6
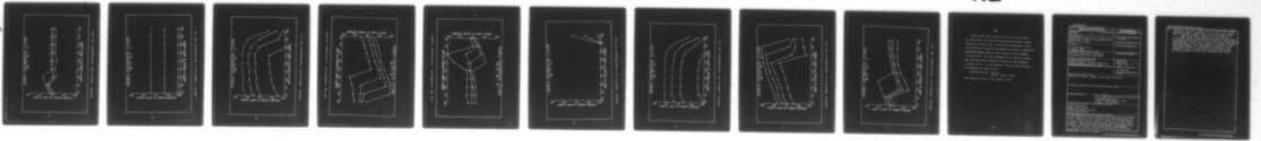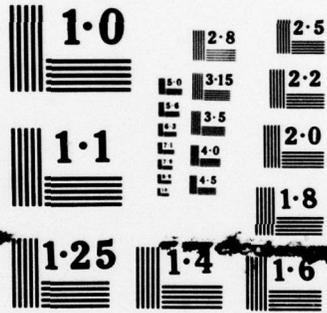
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MICROCOPY RESOLUTION TEST CHART
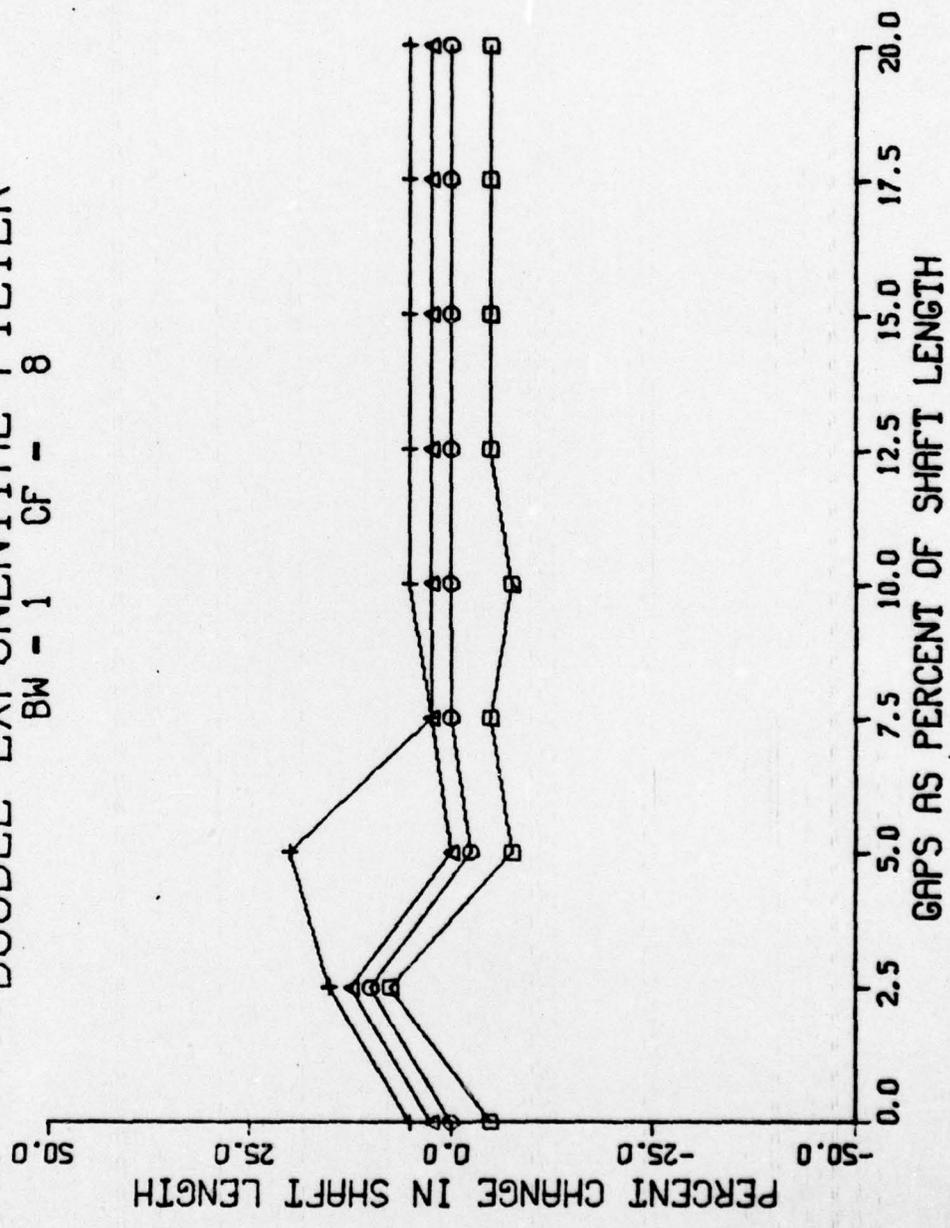
Figure B16. Prediction Curve - Double Exponential BW=1 CF=8

Figure B17. Prediction Curve - Double Exponential BW=2 CF=1

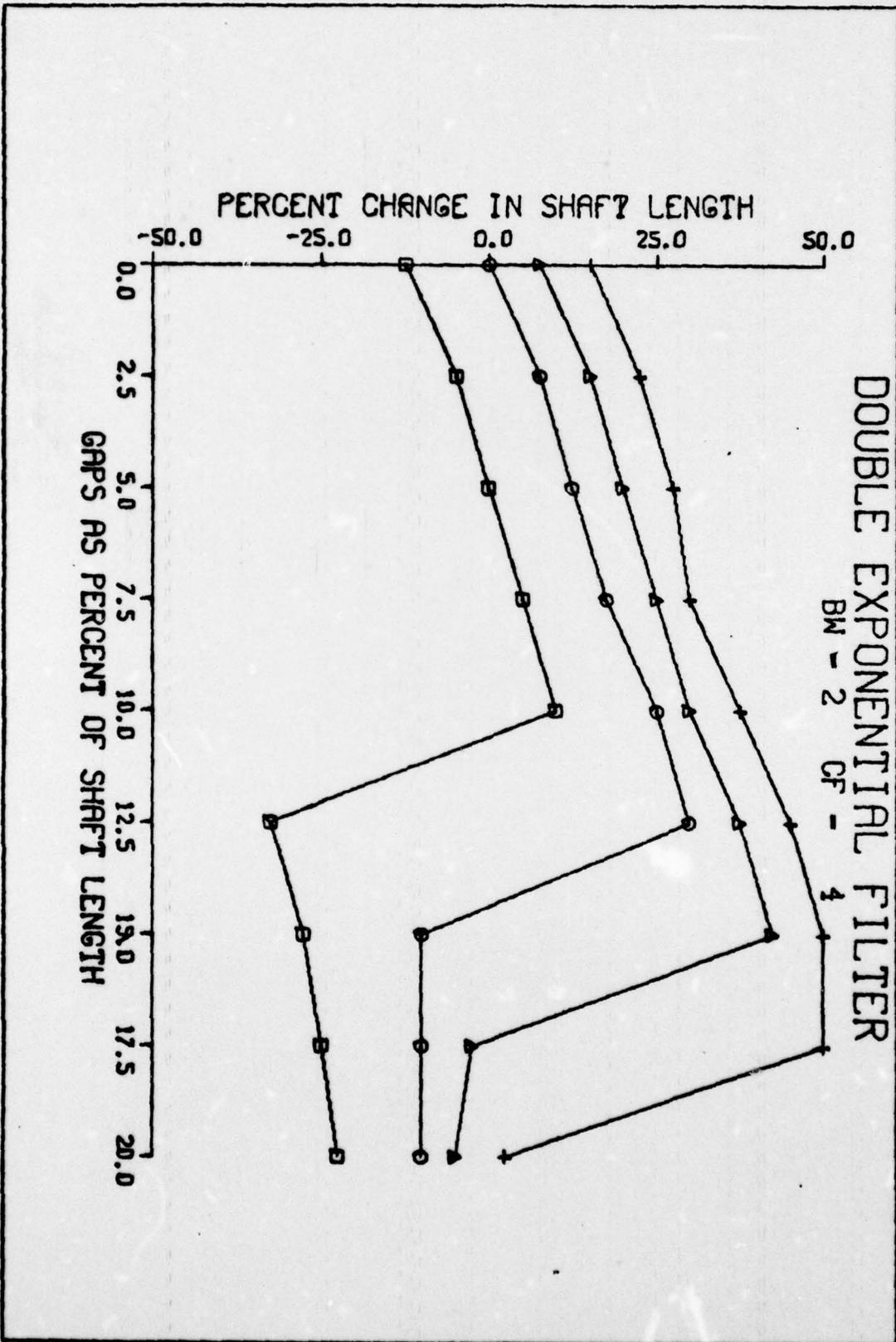Figure B18. Prediction Curve - Double Exponential BW=2 CF=2

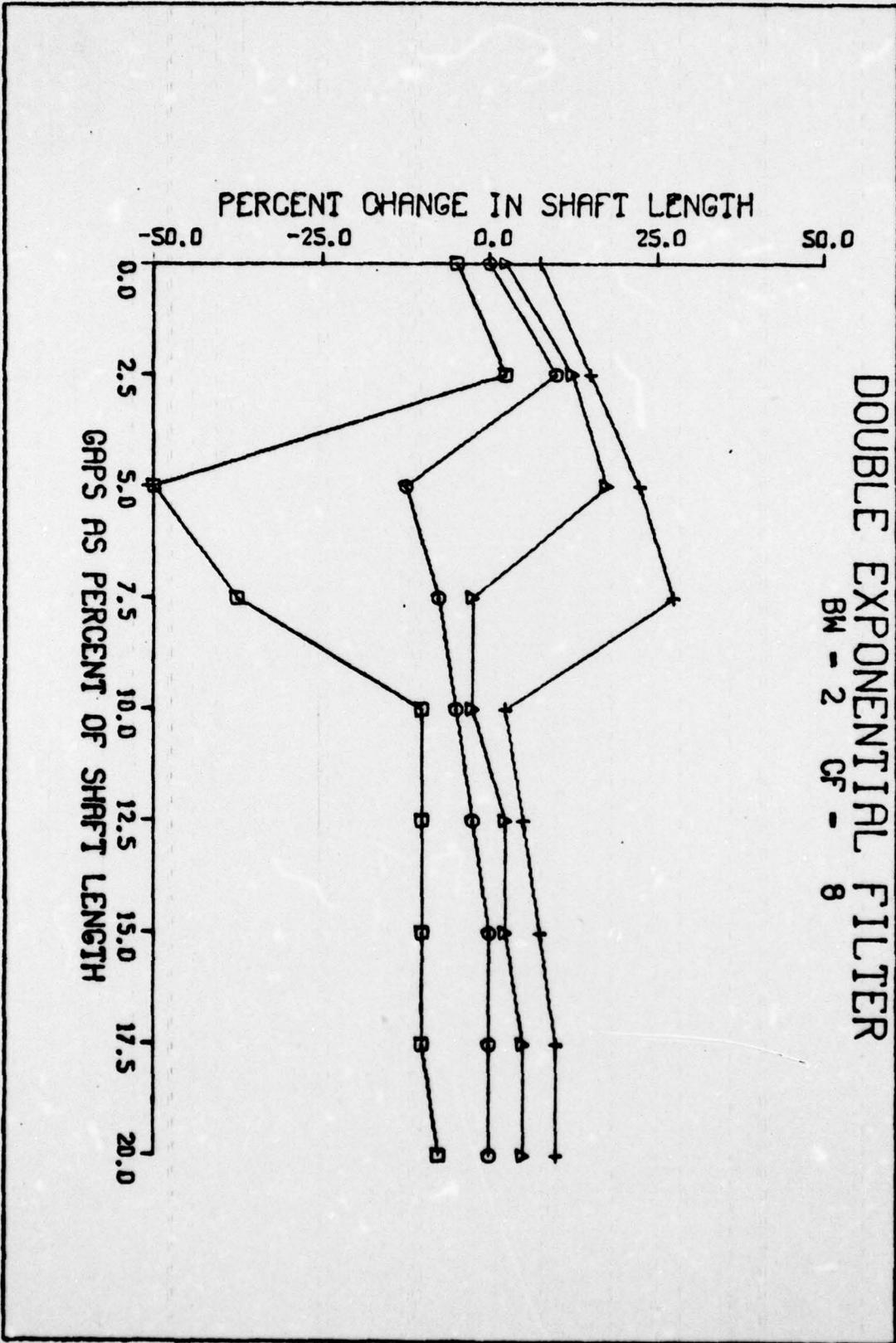Figure B19. Prediction Curve - Double Exponential BW=2 CF=4

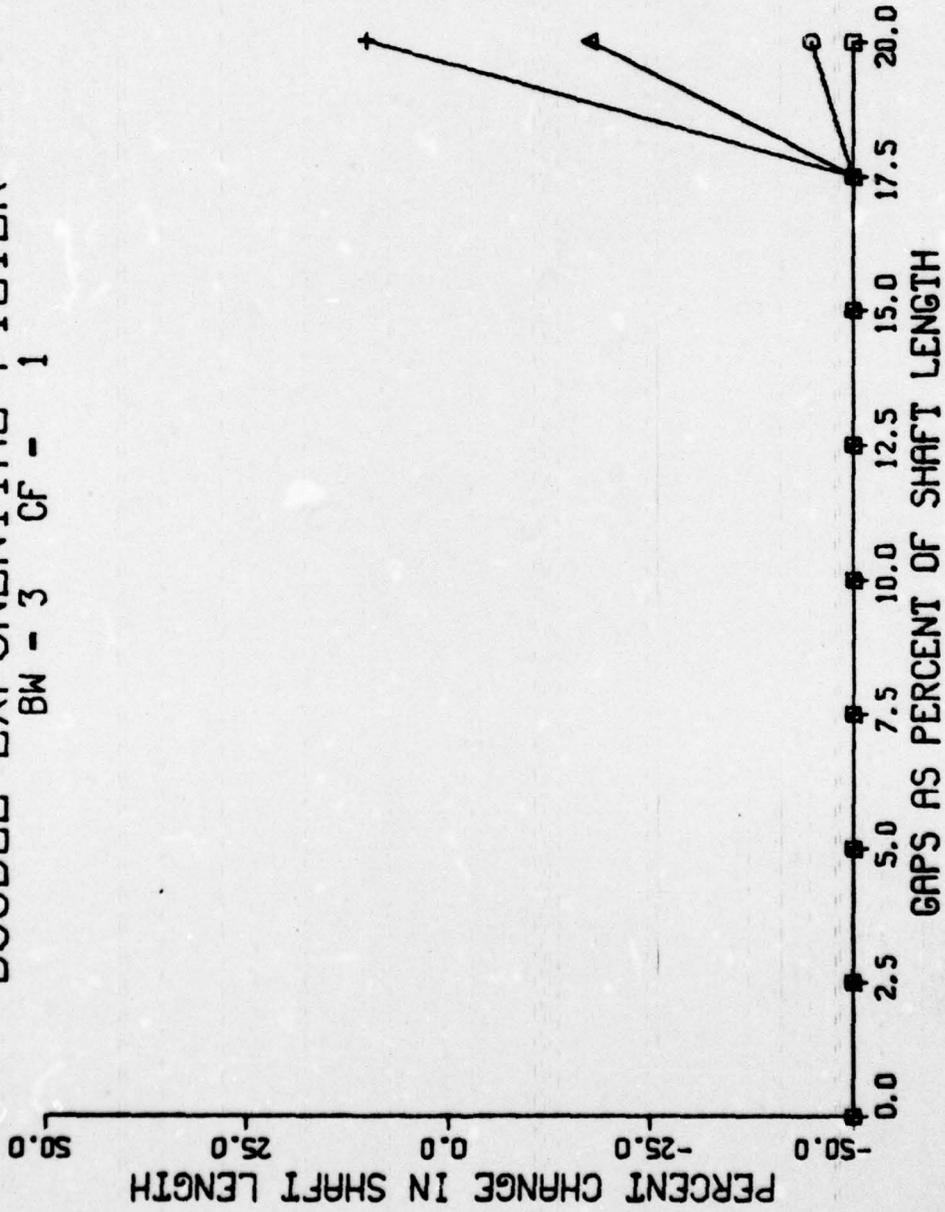Figure B20. Prediction Curve ◯ Double Exponential BW=2 CF=8

Figure B21. Prediction Curve – Double Exponential BW=3   CF=1

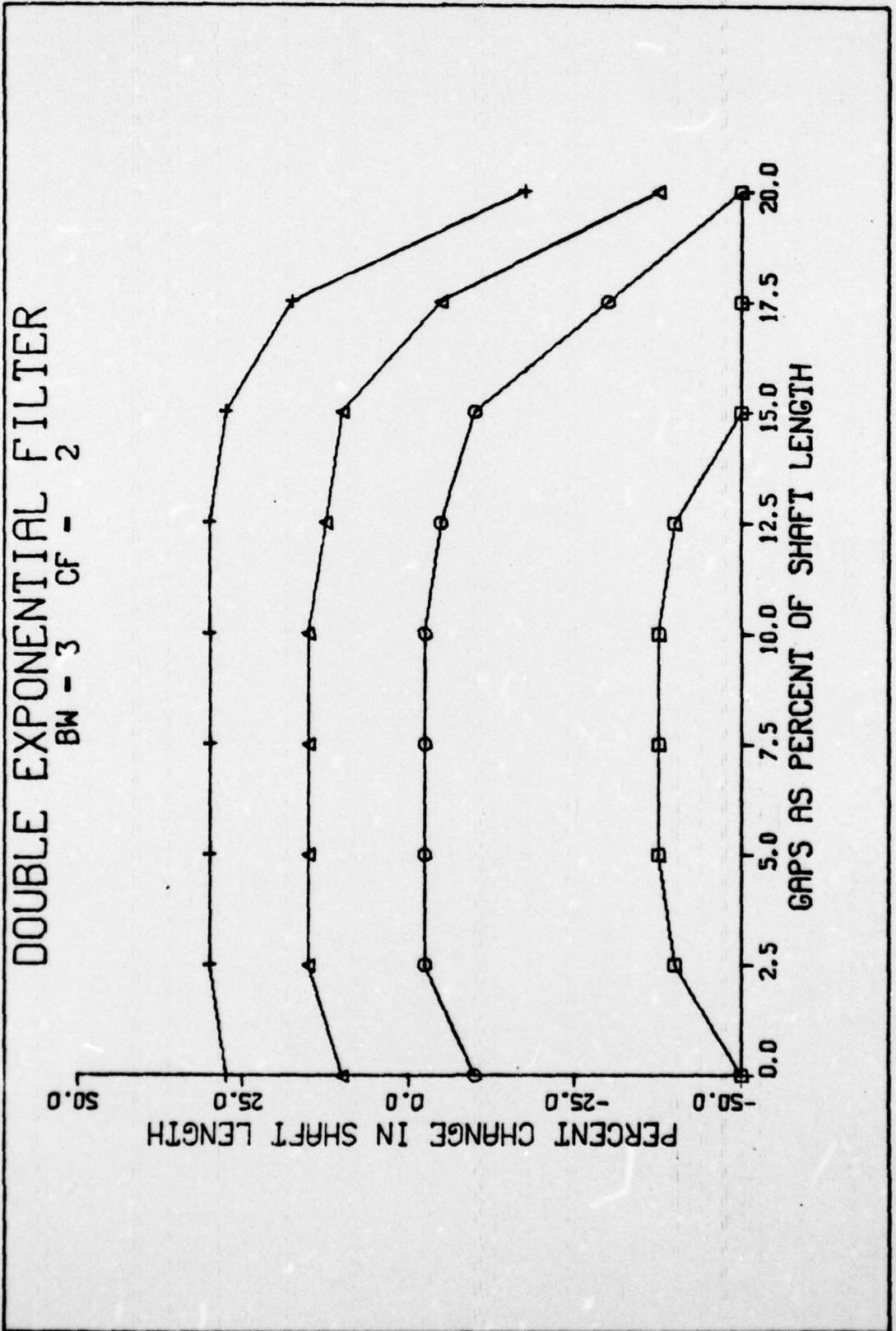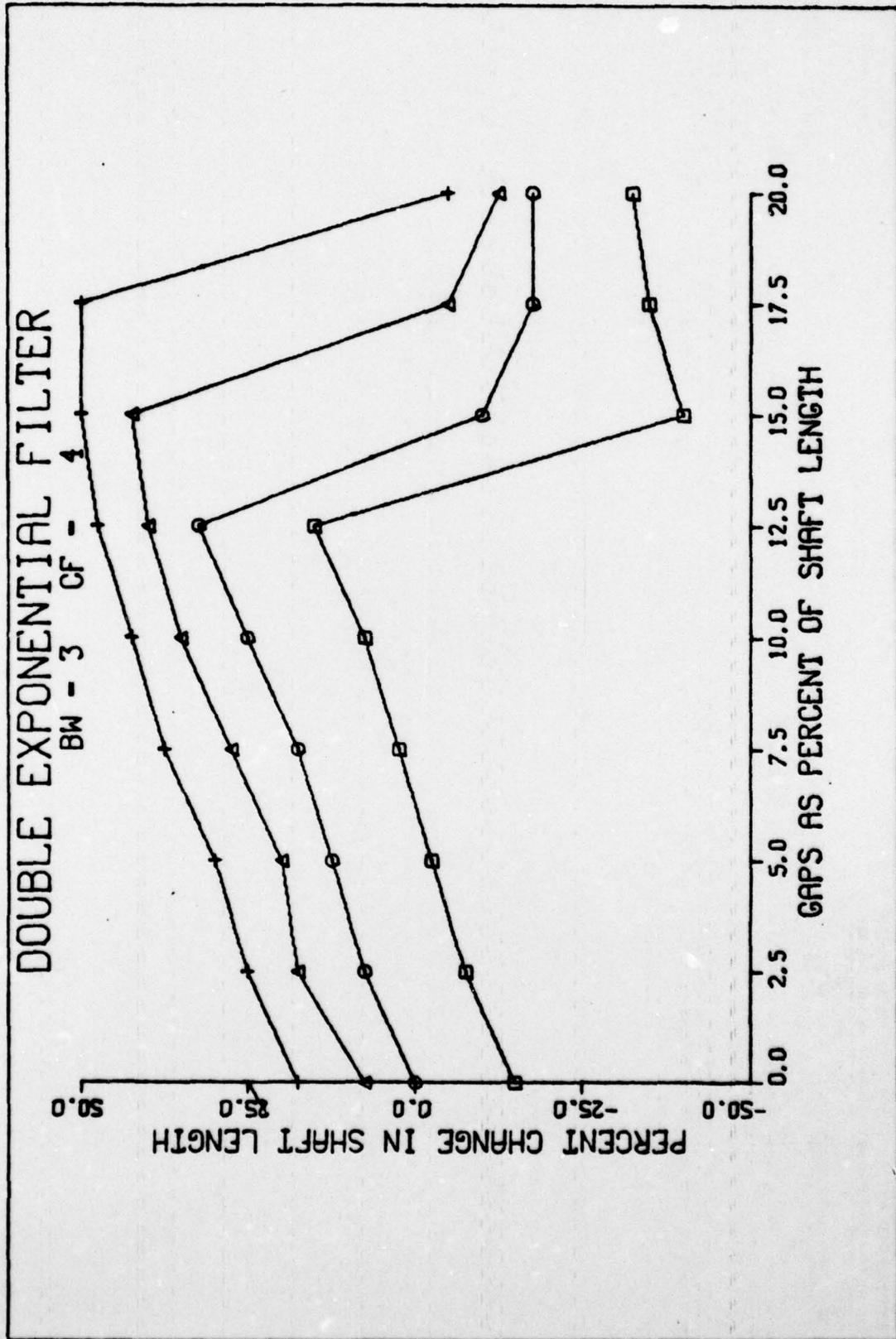Figure B22. Prediction Curve - Double Exponential BW=3   CF=2

Figure B23. Prediction Curve - Double Exponential BW=3   CF=4
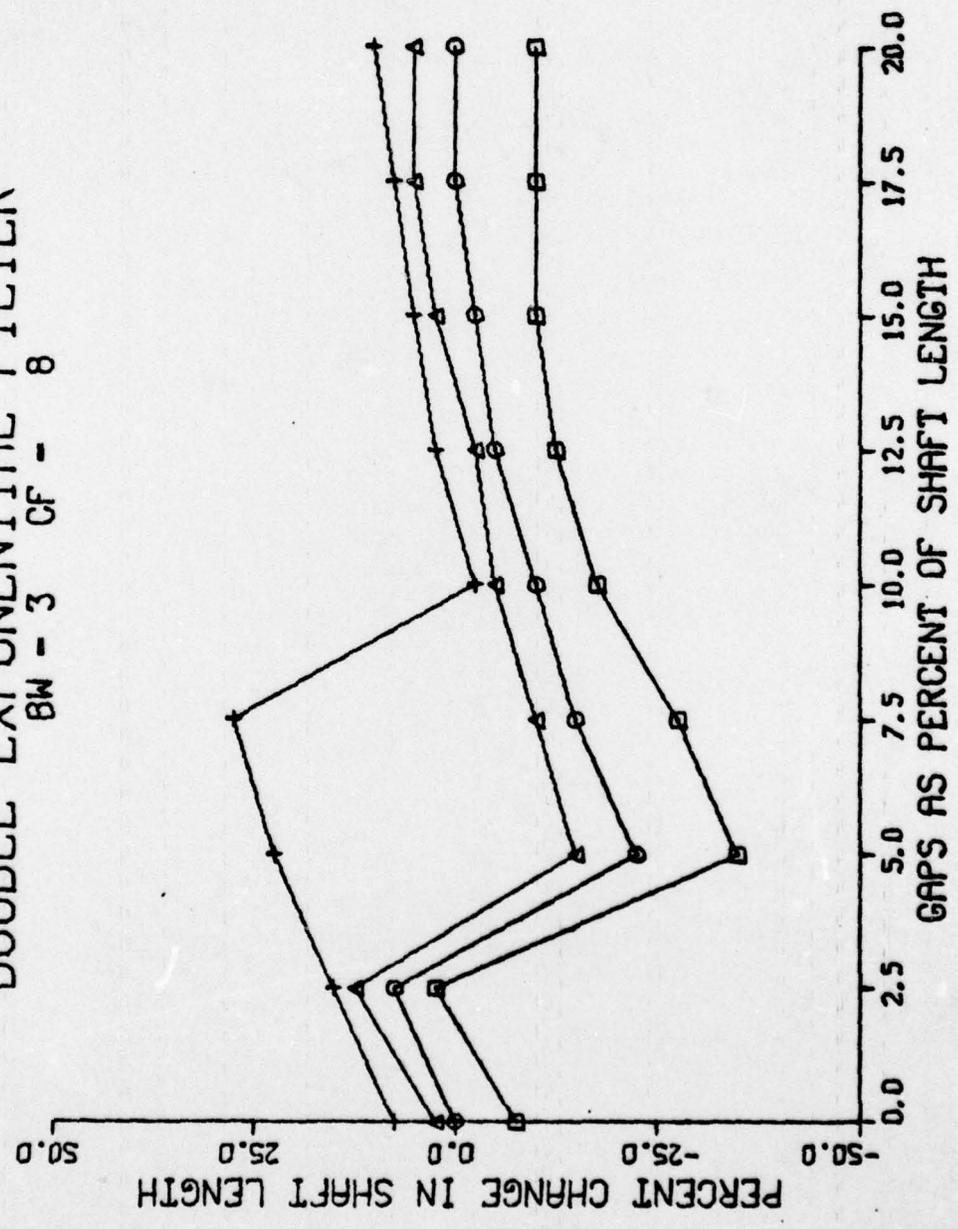
Figure B24. Prediction Curve - Double Exponential   BW=3   CF=8

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The biologically derived bandwidth characteristics of the human visual system were used to determine the shape parameters of a filter. This filter was used, as a model of the visual system, to produce quantitative predictions of the Müller-Lyer visual illusion. These predictions were compared to judgements of the length of the shaft of the illusion by the human subjects.

The best agreement between the subject data and the pre-

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20. dictions of the model occurred when the filter had a double exponential shape, a bandwidth of ± 1.0 octave and a center spatial frequency between 2 and 4 cycles per object size.

This is the first experiment to show that the filter model could predict similar quantitative distoritions of length of the Muller-Lyer visual illusion as reported by the human subjects. These results support the theory, advocated by Arthur Ginsburg, that the bandwidth limitations of the human visual system are responsible for geometric visual illusions as well as other visual phenomena such as the Gestalt principles of similarity, proximity and closure.