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**A LIFTING SURFACE THEORY FOR WINGS
EXPERIENCING LEADING-EDGE SEPARATION**

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of the wing and wake. This lifting surface theory program is based on the kernel function formulation, in that the vorticity distribution is described by continuous functions with unknown coefficients. The vortex location is similarly described by functions with unknown coefficients. These unknowns are found by satisfying the downwash condition and the no-force condition on the leading-edge vortex representation. Due to the nonlinear nature of the boundary conditions with respect to the vortex position, the solution is obtained from an iterative scheme based on Newton's method. Results for the delta wing and arrow wing are presented and compared with experiment and other theories. These results indicate that reasonable predictions can be obtained although the computational effort is considerable. Finally, areas of future investigations suggested by the present work are given.



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SUMMARY

This report describes a nonlinear lifting surface theory for a wing with leading-edge vortices in a steady, incompressible flow. A numerical scheme has been developed from this theory and initial runs have been made for the delta wing and arrow wing planforms. A general procedure for other planforms is also described. The present formulation is the result of an extensive modification of the work of Nangia and Hancock, in which a model of the leading-edge vortex is added to a vorticity representation of the wing and wake. This lifting surface theory program is based on the kernel function formulation, in that the vorticity distribution is described by continuous functions with unknown coefficients. The vortex location is similarly described by functions with unknown coefficients. These unknowns are found by satisfying the downwash condition and the no-force condition on the leading-edge vortex representation. Due to the nonlinear nature of the boundary conditions with respect to the vortex position, the solution is obtained from an iterative scheme based on Newton's method. Results for the delta wing and arrow wing are presented and compared with experiment and other theories. These results indicate that reasonable predictions can be obtained although the computational effort is considerable. Finally, areas of future investigations suggested by the present work are given.

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1. Introduction

Supersonic aircraft generally employ highly swept wings with thin leading edges in an effort to reduce drag in their operational environment. This wing design results in leading-edge separation at even low angles of attack, typically about 5° .

Although theoretical predictions are generally excellent for unseparated flow outside the transonic range, the vortex-wing interaction problem has been successfully attacked only recently for general planforms. The difficulty introduced by the separation is two-fold. First, the location of the separated vorticity in a theoretical model is not known a priori. Secondly, due to the large spanwise velocities induced by the presence of the vortex on the wing, the pressure calculations must include non-linear terms as well as the classical linear contribution. Due to the non-linear nature of the boundary condition which is needed to determine the location of the separated vorticity, an iterative procedure must be used to determine the flow field. Details of early efforts to describe, measure, and predict the effects of flow separation are chronicled in Matoi (1975)¹, Smith (1975)², and elsewhere.

The leading-edge separation phenomena has been documented for many planforms, but the delta wing has received the greatest share of attention, due to its inherent simplicity. A description of the flow about a delta wing was given by Örnberg (1954)³, and one of his illustrations is presented in Figure 1, where the separated vortex sheet is seen to feed a primary vortex core, which then induces a secondary separation from the upper surface of the wing.

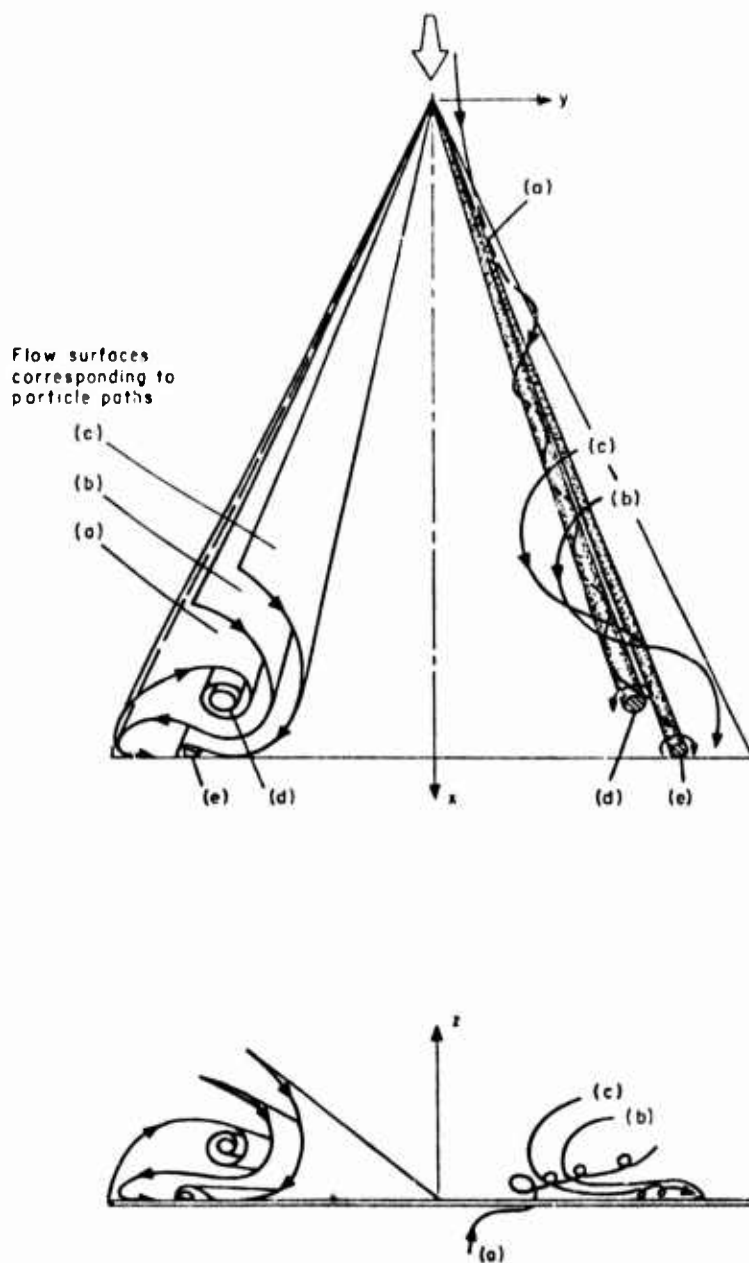


Figure 1. Schematic sketches showing flow on suction side of 70° flat plate delta wing at $\alpha=15^\circ$ [after Örnberg (1954)].

This secondary vortex results from the separation of the viscous boundary layer on the wing, when it encounters the adverse pressure gradient present on the upper surface. Since this line of separation can only be located by a viscous analysis, this additional complexity has been ignored in the following models.

An early effort to theoretically predict this flow field was made by Brown and Michael (1955)⁴. They considered a conical, flat-plate delta wing at moderate angles of attack under the additional restriction of slender-body theory. They modeled the vortex core by a line vortex whose strength increased linearly along its axis. The vortex was fed by a cut, i.e., a feeding sheet from the leading edge which was restricted to the cross-flow plane. This model of the vortex sheet will be referred to as a vortex-cut model.

Smith (1966)⁵ refined the Brown and Michael model to include a representation of the actual force-free vortex sheet as well as the vortex core. In Figure 2 (top) the vortex-sheet and vortex-core location are presented in the cross-flow plane for various extents of the vortex sheet. α designates the angle of attack and λ is the leading-edge sweep angle. The extent of the sheet obviously increases as one increases the fraction (F) of the total shed vorticity which is included in the sheet. These results were obtained by running an amended version of the program provided by Pullin (1973)⁶. Pullin used a representation of the leading-edge vortex sheet similar to the one employed by Smith, but developed a more systematic iteration procedure for finding the stable configuration of the shed vorticity. The case of no sheet ($F = 0$) corresponds to the Brown and Michael model. Increasing the extent of the sheet beyond $F = .19$ results in little change for the parameters

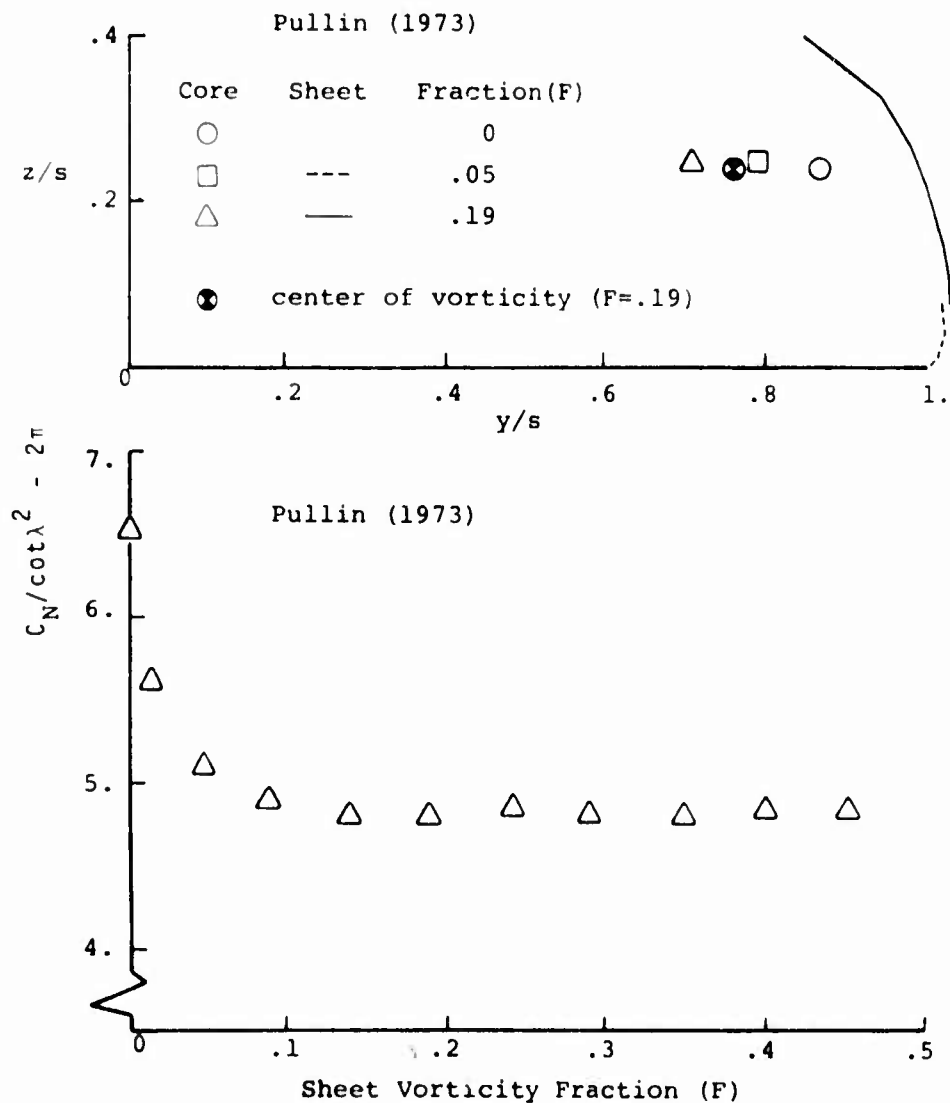


Figure 2. Dependence of vortex-sheet shape and core location (top) and convergence of nonlinear part of normal force (bottom) on the fraction of total shed vorticity contained in the sheet for a delta wing ($\sin \alpha / \cot \lambda = 1$) from slender-body theory.

considered. As can be seen, the effect of introducing the vortex sheet is to move the vortex core inward. It must be noted, that the center of shed vorticity no longer corresponds to the location of the core, and for the case plotted ($F = .19$), the center of vorticity is located at $y/s = .76$, $z/s = .24$. In the lower part of Figure 2, the convergence of the non-linear normal force contribution is presented. This indicates that global quantities may be obtained by considering only a small segment of the vortex sheet.

Recently, the restriction of slender-body theory has been removed, and the general three-dimensional separated problem has now been considered. Lifting-surface theories have been developed along two distinct lines. First, there are the finite-element methods where the wing is replaced by a number of discrete vortex elements and their strengths are determined by satisfying the appropriate boundary conditions. The leading-edge vortex problem has been attacked by finite-element methods by Kandil, Mook, and Nayfeh (1974)⁷ and Brune, Weber, Johnson, Lu, and Rubbert (1975)⁸.

The alternate method is to represent the vorticity distribution on the planform by a set of loading functions whose coefficients are chosen to satisfy the boundary conditions. This method is called the kernel-function method. In Matoi, Covert, and Widnall (1975)⁹, a lifting-surface theory for separated flow based on the kernel-function method was developed for a delta wing. The purpose of this report is to improve and extend the development of that kernel-function procedure.

2. Problem Formulation

The reasons for choosing the kernel-function method over the finite-element method have been detailed in the earlier report by Matoi, et al. (1975)⁹. It was believed that such a procedure could be more easily generalized to include unsteady effects, vortex-breakdown models, and other extensions, and would alleviate some of the difficulties encountered when using discontinuous finite-element procedures. These difficulties, which include "lost" vortices in the line-vortex models and convergence problems as the number of elements is increased, result from the infinite discontinuity in the velocity between discrete panels or at the vortex element. Artifices (such as the introduction of viscosity, finite core radius, or other smoothing procedures) are needed to alleviate this feature of the discrete vortex models. The only other work employing continuous loading functions found in an extensive literature search was by Nangia and Hancock (1968)¹⁰. Many of the symbols and much of the present formulation have their origin in that report.

The coordinate system used in this report is presented in Figure 3. The planform is presently considered to be in the x-y plane. The non-planar problem can also be considered if a spanwise coordinate is used instead of y. The planform can be completely general. The configuration is considered to be symmetric, and the flow field can be described by satisfying the boundary conditions on the right side alone. The boundary conditions on the other side are automatically satisfied by symmetry. However, the asymmetric problem (e.g., the wing at a sideslip angle) can be considered with minor modifications. See Figure 4 for the representation of the wing, leading-edge vortices and wake. It is to be noted that the vortex-cut model

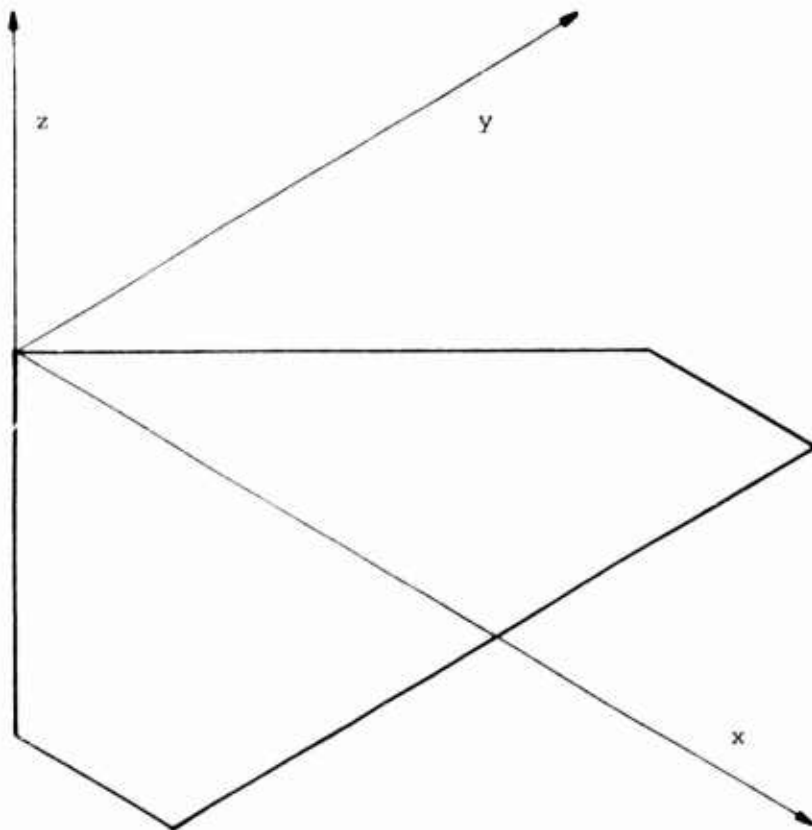


Figure 3. Coordinate system.

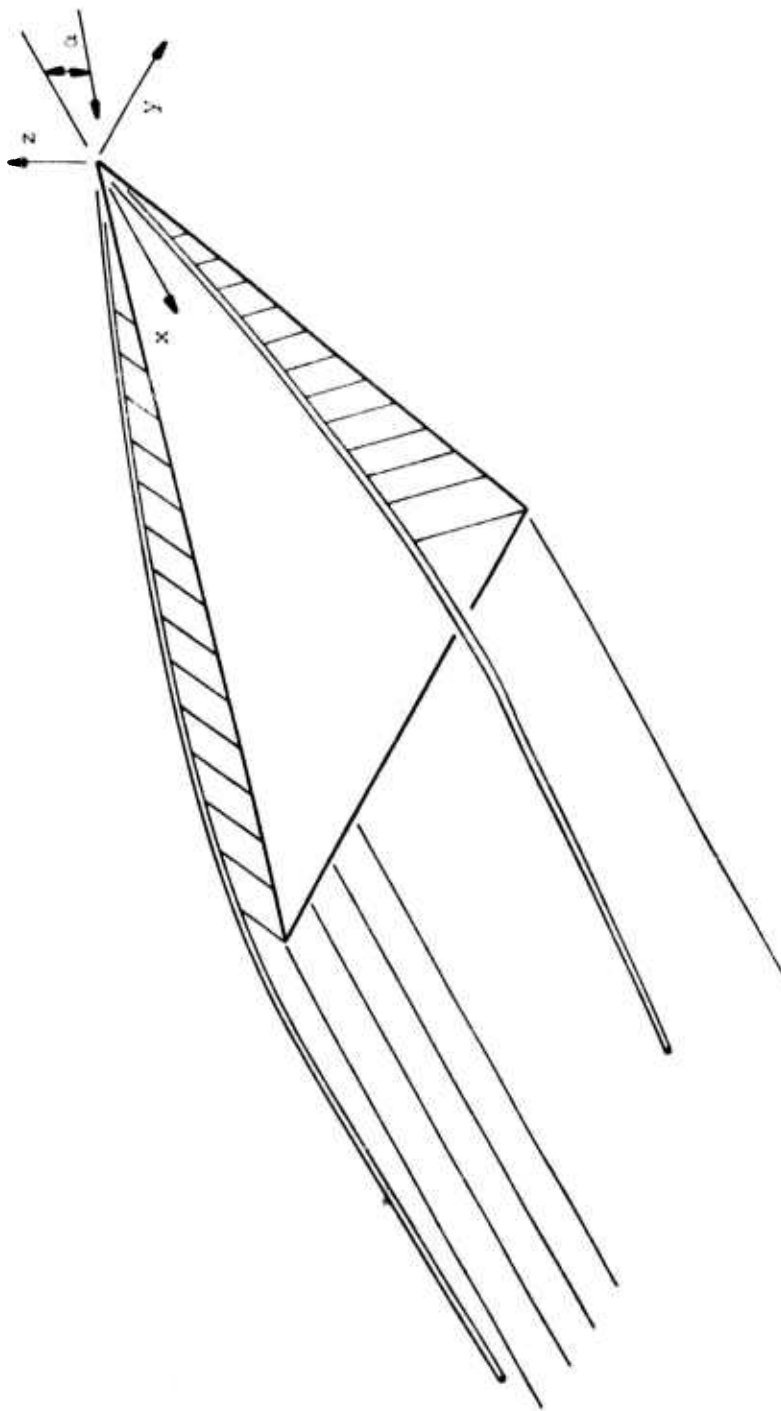


Figure 4. Representation of wing, wake and leading-edge vortices.

of Brown and Michael is presently being used for the reason of simplicity. Later, the more correct representation with some part of the sheet may be included in this type of analysis.

The governing equation in three-dimensional, inviscid, irrotational, steady flow about a wing-body combination is Laplace's equation. The solution can be formulated as an integral equation over the boundary of the aircraft configuration and the regions of shed vorticity. There are several equivalent formulations for the solution, but vortex sheets are used to represent the wing and wake in this report. The velocity distribution can then be given in the following vector form

$$\vec{v}(\vec{r}) = -\frac{1}{4\pi} \int_{S'} \frac{\vec{\gamma}_x(\vec{r}' - \vec{r})}{|\vec{r}' - \vec{r}|^3} dS' \quad (1)$$

where

$$\vec{r}' - \vec{r} = (x' - x)\hat{i} + (y' - y)\hat{j} + (z' - z)\hat{k}$$

$$\vec{\gamma} = \gamma_x \hat{i} + \gamma_y \hat{j} + \gamma_z \hat{k}$$

$$\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$$

S' is the surface of integration, $\vec{\gamma}$ is the vorticity vector, $\vec{v}$ is the perturbation velocity vector, i.e., the velocity minus the uniform free stream; and $\vec{r}$ is the radius vector from the origin. The velocities are nondimensionalized with respect to the free stream, and the distances are nondimensionalized with respect to the maximum chordwise length.

Since the vorticity lies in the plane of the wing and wake, the vorticity representing the wing consists of only two non-zero components γ_x and γ_y . Conservation of vorticity can then be written as

$$\frac{\partial \gamma_y}{\partial y} = - \frac{\partial \gamma_x}{\partial x} \quad (2)$$

In the present formulation, the vorticity in the wing and wake is divided into two parts. First, there is a portion -- represented by subscript 1 -- which behaves like the traditional bound vorticity and only leaves the wing at the trailing edge. Secondly, there is a portion -- represented by subscript 2 -- which feeds the leading-edge vortices.

$$\begin{aligned} \gamma_y &\equiv \gamma = \gamma_1 + \gamma_2 \\ \gamma_x &\equiv \delta = \delta_1 + \delta_2 \end{aligned} \quad (3)$$

The contributions are chosen so that γ_1 and δ_1 fall to zero at the leading edge, while γ_2 and δ_2 are related so that the vorticity is perpendicular to the leading edge. This is necessitated by the Brown and Michael model employing a vortex-cut combination to insure finite velocities at the leading edge. A more complete model employing a leading-edge vortex sheet representation would utilize a general separation angle which would be fixed by the no-load (Kutta) condition at the leading edge.

The functional forms of the wing vorticity are

$$\begin{aligned} \gamma_1(\theta, \eta) &= \frac{8\pi s}{c(\eta)} \sqrt{1-\eta^2} \sum_{m=1}^M \sum_{n=1}^N \frac{4 a_{n,m}}{2^{2n}} U_{2(m-1)}^{(n)} \sin n\theta \\ \gamma_2(x, y) &= \frac{-y}{\sqrt{x^2 + y^2}} \frac{\pi}{2} \sum_{q=1}^Q g_q (2q - 1) \cos \left[\frac{2q-1}{2} - \sqrt{\frac{x^2 + y^2}{1 + s^2}} \right] \end{aligned} \quad (4)$$

where

$$\begin{aligned} x &= 1/2 [x_{TE}(y) + x_{LE}(y)] - 1/2 c(y) \cos \theta \\ y &= s\eta \end{aligned} \quad (5)$$

The distribution for γ_1 was obtained from linearized lifting surface theory and vanishes at the leading and trailing edges. For additional details, see Ashley and Landahl (1965)¹¹. U_m are Chebyshev polynomials of the second kind, x_{LE} and x_{TE} are the location of the leading and trailing edges of the planform, respectively, c is the local chord, and s is the semispan. The form of γ_2 insures that the vorticity feeding the leading-edge vortex will be perpendicular to leading edges, which are formed by rays from the apex. Modes similar to these were developed by Nangia and Hancock (1968)¹⁰ for the three-dimensional delta wing. The coefficients $a_{n,m}$ and g_q are the unknown coefficients of the vorticity functions. The leading-edge vortex strength is defined by

$$\Gamma(x) = \sum_{q=1}^Q g_q \sin [(2q-1)\pi x/2] \quad (6)$$

where the modes have been chosen such that $\frac{d\Gamma}{dx} = 0$ at the trailing edge, i.e., there is no additional feeding of wing vorticity into the leading-edge vortex at the trailing edge. See Figure 5 for a representation of the bound vorticity component γ_2 . Using Equation 2, one can obtain

$$\delta_1(\theta, \eta) = - \frac{4\pi}{\sqrt{1-\eta^2}} \sum_{m=1}^M \sum_{n=1}^N \frac{4 a_{n,m}}{2^{2n}} \left\{ 1/2 \left\{ -[(2m-1)\eta + \frac{(1-\eta^2)}{c(\eta)} \frac{dc}{d\eta}] U_{2(m-1)+}^{(n)} \right. \right.$$

$$\begin{aligned}
& (2m+1) U_{2m-3}(\eta) \} * \left[\frac{\sin(n-1)\theta}{n-1} - \frac{\sin(n+1)\theta}{n+1} \right] \\
& - \frac{2n(1-\eta^2)}{c(\eta)} U_{2(m-1)} \left\{ \left(\frac{dx_{LE}}{d\eta} + 1/2 \frac{dc}{d\eta} \right) \frac{\sin n\theta}{n} \right. \\
& \left. - 1/4 \frac{dc}{d\eta} \left[\frac{\sin(n-1)\theta}{n-1} + \frac{\sin(n+1)\theta}{n+1} \right] \right\} \quad (7)
\end{aligned}$$

where

$$U_{-1}(\eta) \equiv 0$$

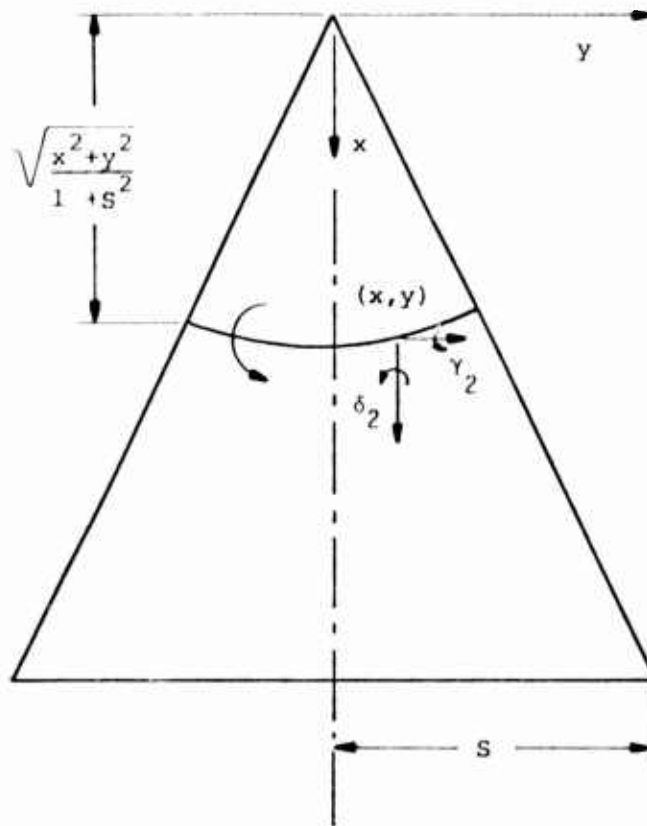
The wake is assumed to be flat and to possess the trailing vorticity distribution imparted at the trailing edge, as in linear lifting surface theory. Consequently, the trailing wake adds no new parameters.

Finally, the location of the leading-edge vortex is defined by the polynomials

$$\begin{aligned}
y_v(x) &= \sum_{\ell=1}^L g_{y_v} T_{2\ell-1}(x) \\
z_v(x) &= \sum_{\ell=1}^L g_{z_v} T_{2\ell-1}(x) \quad (8)
\end{aligned}$$

where T_ℓ are Chebyshev polynomials of the first kind. This introduces the final set of unknowns, g_{y_v} and g_{z_v} .

After the mode shapes have been defined, it is necessary to satisfy the appropriate boundary conditions to determine the unknown coefficients. The applicable boundary conditions are the no-flow condition through the wing and the no-load condition on the free vortex sheet and on the leading-edge vortex-cut combination.



$$\gamma_2(x, y) = \frac{x}{\sqrt{x^2+y^2}} \frac{d\Gamma(x_e)}{dx} \quad x_e = \sqrt{\frac{x^2+y^2}{1+s^2}}$$

$$\delta_2(x, y) = \frac{y}{\sqrt{x^2+y^2}} \frac{d\Gamma(x_e)}{dx} \quad x_e = \sqrt{\frac{x^2+y^2}{1+s^2}}$$

Figure 5. Representation of bound vorticity feeding leading-edge vortex.

The downwash condition becomes

$$w = -\sin\alpha \quad (9)$$

on the wing ($z = 0$), where α is the angle of attack and w is the upwash induced by the vorticity distribution. This requires the evaluation of the w component of the integral in Equation 1 at a set of collocation points. The cosine distribution of Hsu (1957)¹², modified for separated flow, is given in Figure 6 for five chordwise station (NCORD = 5) by five spanwise stations (NSPAN = 5). A large percentage of the computation time is presently consumed by the calculation of the contribution from the wing surface, since the denominator contains a singularity at the collocation points.

A further distinction must now be made between the contributions in Equation 9. Its various components are distinguished by the nature of their contribution to the vertical velocity.

First, since the bound vorticity related to γ_1 , described in Equation 4, only leaves at the trailing edge, horseshoe vortices are used to represent this contribution, which corresponds to an integration in the x direction of Equation 1. Thus, for that contribution, the relevant vorticity component becomes

$$w_1(x,y,z) = \frac{1}{4\pi} \int_{-5}^5 \int_{x_{LE}}^{x_{TE}} \gamma_1 K_w dx' dy' \quad (10)$$

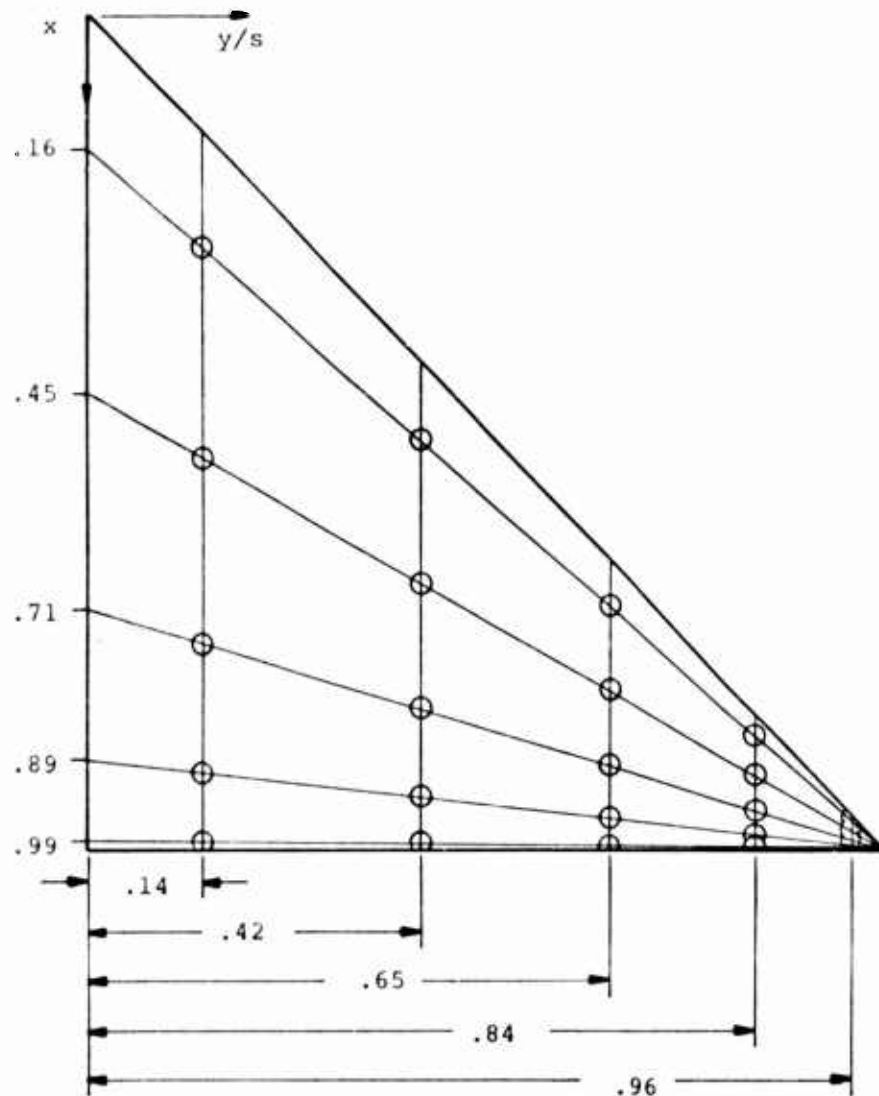


Figure 6. Collocation points for downwash on right half of wing (NSPAN = 5, NCORD = 5).

where

$$K_w = \frac{1}{(y-y')^2 + z^2} \left\{ \left[1 + \frac{x-x'}{\sqrt{(x-x')^2 + (y-y')^2 + z^2}} \right] \cdot \left[1 - \frac{2z^2}{(y-y')^2 + z^2} \right] - \frac{z^2 (x-x')}{[(x-x')^2 + (y-y')^2 + z^2]^{3/2}} \right\}$$

(11)

Thus the singularity for the contribution, γ_1 , is limited to the spanwise direction, when $z = 0$, and the contribution from this term to Equation 8 may be readily evaluated. The four integration regions employed for this surface integration are presented in Figure 7 (top). The actual evaluation was performed using a simplified version of a routine available at M.I.T., which was developed by Widnall (1964)¹³ for the more general problem of the unsteady, incompressible, non-planar wing. This program was based on an extension of the work by Watkins, Runyan, and Woolston (1959)¹⁴. Alternate forms, such as the procedure developed by Hsu (1957)¹² could be used instead.

For the contribution from γ_2 and δ_2 , the evaluation of the integral in Equation 1 is handled in two parts. The integral over the wing surface S_w can be rewritten as

$$w_2(x,y,z) = \frac{1}{4\pi} \iint_{S_w} \frac{(x'-x) \gamma_2(x',y') - (y'-y) \delta_2(x',y')}{[(x-x')^2 + (y-y')^2 + z^2]^{3/2}} dx' dy'$$

(12)

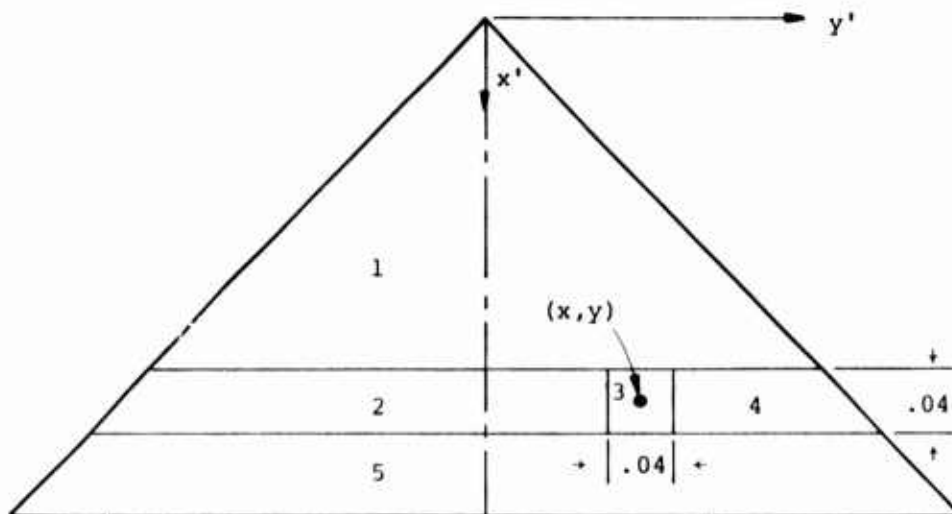
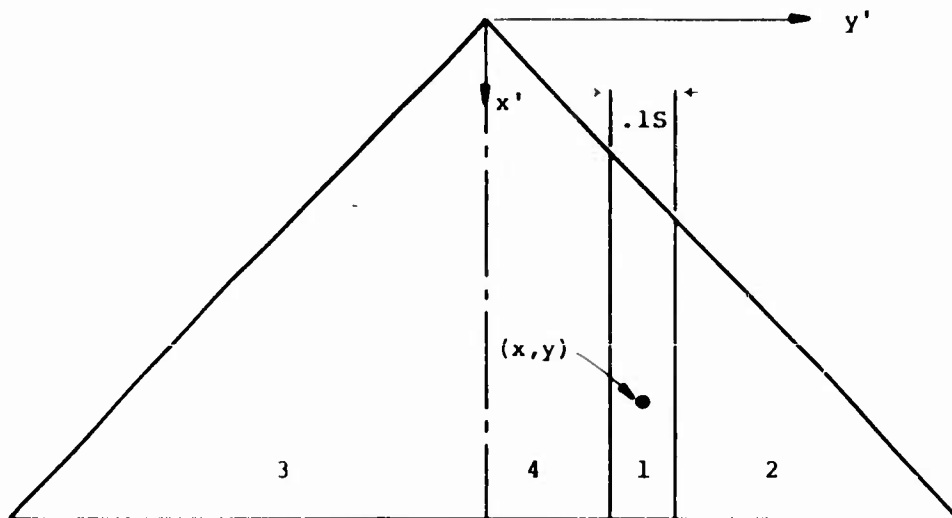


Figure 7. Regions of integration for calculating upwash coefficients at the point (x, y) for the γ_1 contribution (top) and for the γ_2, δ_2 contribution (bottom).

On the wing surface, $z = 0$, the integral may be rewritten to isolate the singularity.

$$\begin{aligned}
 w_2(x,y,0) = & \frac{1}{4\pi} \iint_{S_w} dS \frac{(x'-x)[\gamma_2(x',y') - \gamma_2(x,y)] - (y'-y)[\delta_2(x',y') - \delta_2(x,y)]}{[(x-x')^2 + (y-y')^2]^{3/2}} \\
 & + \frac{1}{4\pi} \gamma_2(x,y) \oint \oint_w \frac{(x'-x) dx' dy'}{[(x-x')^2 + (y-y')^2]^{3/2}} \\
 & + \frac{1}{4\pi} \delta_2(x,y) \oint \oint_w \frac{(y-y') dx' dy'}{[(x-x')^2 + (y-y')^2]^{3/2}} \quad (13)
 \end{aligned}$$

The first term can now be evaluated numerically, while the remaining two terms are evaluated in Appendix A. Figure 7 (bottom) gives the five integration regions employed for this surface integration for the delta wing. Each region is covered by a 24 x 24-point Gaussian quadrature. Fortunately, this computation does not require any iteration and is performed once for a given set of collocation points and wing planform. Since this calculation and a related integral in the no-force condition consume much of the computational effort, significant reductions in this integration would be advantageous.

Additional contributions to the downwash on the wing are obtained from the wake aft of the wing and from the leading-edge vortices.

As described previously in Figure 4, the spanwise component of the vorticity, γ_2 , is assumed to be zero aft of the trailing edge, while the streamwise component, δ_2 , is only a function of the spanwise variable in the wake. Thus, one obtains the following contribution from the wake according to Equation 1.

$$w_T(x,y,z) = -\frac{1}{4\pi} \iint_{S_T} \frac{(y'-y) \delta_2(y') dy' dx'}{[(x'-x)^2 + (y'-y)^2 + z^2]^{3/2}} \quad (14)$$

where S_T represents the wake surface. The streamwise integral may be performed explicitly for a given planform to yield

$$w_T(x,y,z) = -\frac{1}{4\pi} \int_{-S}^S (y'-y) \delta_2(y') I(y'; x,y,z) dy' \quad (15)$$

where

$$I(y'; x,y,z) = \frac{1}{(y'-y)^2 + z^2} \left[1 - \frac{x_{TE}(y') - x}{\sqrt{(x_{TE}(y') - x)^2 + (y'-y)^2 + z^2}} \right] \quad (16)$$

The function $x_{TE}(y)$ describes the location of the trailing edge of the specified planform. On the wing surface, $z = 0$, this reduces to

$$w_T(x,y,0) = -\frac{1}{4\pi} \int_{-S}^S \frac{\delta_2(y')}{y'-y} \left[1 - \frac{x_{TE}(y') - x}{\sqrt{(x_{TE}(y') - x)^2 + (y'-y)^2}} \right] dy' \quad (17)$$

Finally, the contribution from the leading-edge vortices is

$$w_L(x,y,z) = \int_0^\infty \Gamma(x') [f_w(x'; x,y,z) + f_w(x'; x,-y,z)] dx' \quad (18)$$

where

$$f_w(x'; x,y,z) = \frac{1}{4\pi} \frac{(x'-x) \frac{dy_v(x')}{dx'} - (y_v(x')-y)}{[(x'-x)^2 + (y_v(x')-y)^2 + (z_v(x')-z)^2]^{3/2}} \quad (19)$$

This is the upwash velocity induced by two semi-infinite line vortices according to the Biot-Savart law. The contribution on the wing is obtained by calculating this velocity component at the appropriate control point.

Thus, Equation 9 becomes

$$w = w_1 + w_2 + w_T + w_{T'} = - \sin \alpha \quad (20)$$

The no-load condition on the trailing vortex sheet and the Kutta condition at the trailing edge of the wing are essential in distinguishing this fully three-dimensional problem from earlier slender-body and conical models. First, the Kutta condition at the trailing edge requires that the vorticity vector be parallel to the velocity vector

$$\frac{\gamma(x_{TE}(y), y)}{\delta(x_{TE}(y), y)} = \frac{v}{\cos \alpha + u} \quad (21)$$

However, the results of Brune, et al. (1975)⁸ indicate that the spanwise vorticity component, γ , is approximately zero at the trailing edge for the delta wing case. This corresponds to the Kutta condition applied in linear lifting surface theory. Thus, the method employed here was to set the spanwise vorticity component, γ , equal to zero aft of the trailing edge as was previously illustrated in Figure 4., i.e., the linear boundary condition has been applied on the trailing vortex sheet rather than the nonlinear one. The form of γ_1 (Equation 4) insures that this component vanishes smoothly at the trailing edge to satisfy the linear Kutta condition there. However, the contribution from γ_2 (Equation 4) does not

automatically vanish at the trailing edge, and consequently, there is a discontinuous transition in this contribution. The use of the linear boundary condition on the wake seems justified, since the major cause of the nonlinearity in the present problem is the presence of the leading-edge vortices which induce high spanwise velocities on the planform. Kandil, et al. (1974, p. 13)⁷ noted that "Numerical experiments indicate that the wake adjoining the trailing edge does not exert a strong influence on the results."

Finally, the condition of no-load on the vortex-cut combination is formulated on the right-hand vortex as an extension of the Brown and Michael model. The force components per unit length in the y and z directions are given by F_y and F_z , respectively.

$$\begin{aligned} F_y/2\Gamma &= -\frac{dz_v}{dx} - \frac{1}{\Gamma} \frac{d\Gamma}{dx} z_v + w_i + \sin\alpha \\ F_z/2\Gamma &= \frac{dy_v}{dx} + \frac{1}{\Gamma} \frac{d\Gamma}{dx} [y_v - y_{LE}(x)] - v_i \end{aligned} \quad (22)$$

where w_i and v_i are the velocities induced by the vorticity distribution, excluding the contributions of the right-hand vortex on itself due to curvature. The calculations of the velocity component w_i at collocation points along the leading-edge vortex requires the evaluation of terms similar to those developed for Equation 20.

Specifically,

$$w_i = w_1 + w_2 + w_T + w_{\Gamma_L} \quad (23)$$

where

$$w_{\Gamma_L}(x,y,z) = \int_0^\infty \Gamma(x') f_w(x';x,y,z) dx' \quad (24)$$

The integrands no longer possess a singularity, and the integral in Equation 12, for example, is performed by two 24 x 24-point Gaussian quadratures.

Meanwhile, the contributions for v_i can be developed in a parallel manner

$$v_i = v_1 + v_2 + v_T + v_{\Gamma_L} \quad (25)$$

where

$$v_1(x,y,z) = \frac{1}{4\pi} \int_{-S}^S \int_{x_{LE}}^{x_{TE}} \gamma_1 K_v dx' dy' \quad (26)$$

with

$$K_v \equiv \frac{-z(y-y')}{(y-y')^2 + z^2} \left\{ \frac{2}{(y-y')^2 + z^2} \left[1 + \frac{x-x'}{\sqrt{(x-x')^2 + (y-y')^2 + z^2}} \right] + \left[\frac{x-x'}{(x-x')^2 + (y-y')^2 + z^2} \right]^{3/2} \right\} \quad (27)$$

and

$$v_2(x,y,z) = -\frac{z}{4\pi} \iint_{S_w} \frac{\delta_2(x',y') dx' dy'}{[(x'-x)^2 + (y'-y)^2 + z^2]^{3/2}} \quad (28)$$

$$v_T(x,y,z) = -\frac{z}{4\pi} \int_{-S}^S \delta_2(y') I(y';x,y,z) dy' \quad (29)$$

$$v_{\Gamma_L} = -\int_0^\infty \Gamma(x') f_v(x';x,-y,z) dx' \quad (30)$$

where

$$f_v(x'; x, y, z) = \frac{1}{4\pi} \frac{z_v(x') - z - (x' - x) \frac{dz_v(x')}{dx'}}{[(x' - x)^2 + (y_v(x') - y)^2 + (z_v(x') - z)^2]^{3/2}}$$

(31)

The vorticity distribution has again been assumed to depend only on the spanwise variable in the wake. The function $I(y)$ has been defined in Equation 16.

Thus, the original problem of Laplace's equation with its companion boundary conditions has been formulated as a system of nonlinear equations in terms of the unknown vorticity coefficients, $a_{n,m}$ and g_q , and the unknown vortex location coefficients, g_{yv} and g_{zv} . This has the advantage of transforming a set of integro-differential equations (Equations 1, 9 and 22) into a system of algebraic equations which can be solved on a digital computer.

The primary output parameters of vortex location and vorticity distribution on the wing can then be used to obtain the pressure distribution on the wing. One can obtain the lift and the pitching moment by a simple integration of the pressure loading.

The nonlinear pressure difference on the wing is

$$\Delta C_p \equiv C_{p_u} - C_{p_\ell} = -2\Delta u - \Delta(v^2 + u^2)$$

(32)

where the pressure coefficient, C_p , represents the pressure nondimensionalized by the dynamic pressure, the difference symbol, Δ , refers to the difference in the quantity between the upper and lower surfaces, which are represented by the subscripts u and ℓ , respectively. Since the quadratic

term from the chord-wise component of velocity, u^2 , is small compared to the spanwise contribution, v^2 , that term is ignored and the pressure difference can be rewritten in the following form

$$\Delta C_p = -2\gamma + (v_u + v_l) \delta \quad (33)$$

This form has been selected as the vorticity components, γ and δ , can be readily evaluated once the vorticity coefficients have been found. The second term on the right-hand side includes a factor which is twice the local mean spanwise velocity. On the wing the only non-zero contribution comes from the leading-edge vortices. Thus,

$$(v_u + v_l)_{z=0} = 2 \int_0^\infty \Gamma(x') [f_v(x'; x, y, 0) - f_v(x'; x, -y, 0)] dx' \quad (34)$$

This concludes the section on problem formulation. The next section will discuss the actual numerical procedure; and areas of difficulty will be detailed.

3. Numerical Procedure

The procedure to calculate the unknowns is now described. The initial program was written for the delta wing, but it was later generalized to include arrow wings. An iterative scheme to satisfy the system of equations provided by the downwash condition and the no-force condition on the vortex-cut combination must be chosen first. The downwash condition is linear in terms of the vorticity coefficients, while the no-force condition is non-linear in all parameters. Therefore, following the earlier procedure developed by Nangia and Hancock (1968)¹⁰, an attempt was made to satisfy the boundary conditions sequentially.

An initial position for the leading-edge vortex is chosen. For example, for the delta wing, the initial location was obtained from the Brown and Michael model. The number of vorticity modes (M, N , and Q) in Equation 4 must be specified. A set of downwash points greater than or equal to the number of vorticity coefficients must then be chosen. The solution of a set of simultaneous linear equations from Equation 20 then provides a first approximation for the unknown vorticity coefficients. Figure 6 illustrates the choice of collocation points determined to provide adequate resolution for four chordwise vorticity modes ($N = Q = 4$) by five spanwise vorticity modes ($M = 5$) in Equation 4. Adequate resolution was determined by increasing the number of modes and collocation points until the resulting pressure distribution converged to a semblance of the Brown and Michael results (valid for slender wings) for the delta wing ($AR=1$). Some details of this procedure are presented in Matoi, et al. (1975)⁹ for a different set of mode shapes.

An attempt was then made to satisfy the no-force condition in a manner similar to that employed by Nangia and Hancock (1968)¹⁰. The forces were

calculated using Equation 13, at a set of collocation points, typically five, and the vortex was then moved to reduce these forces, according to the following rule.

$$\frac{d}{dx} \Delta y_v = -d \frac{F_z}{(F_y^2 + F_z^2)^{1/2}} \quad (35)$$

$$\frac{d}{dx} \Delta z_v = d \frac{F_y}{(F_y^2 + F_z^2)^{1/2}}$$

where d is chosen small enough to prevent divergence of the procedure, e.g., $d = .01$. Since the forces have been normalized, the vortex movement is restricted to d . Unfortunately, this method seemed to require considerable discretion in the choice of d . Furthermore, it is necessary to select the number of times to apply the no-force condition, before reapplying the downwash condition. The downwash condition must be satisfied again, since the previous set of vorticity coefficients induces a residual downwash on the wing once the vortex is moved.

One could limit the number of modes in Equation 8 to reduce difficulties with oscillation in the vortex position. However, convergence was not obtained using this procedure, and alternatives had to be considered. The form of Equation 22 suggests that Equation 35 is not the optimum manner for moving the vortex as the velocity components, v_i and w_i , also depend on the vortex location. In the slender-body problem of Brown and Michael, one encounters a similar nonlinear problem for finding the vortex location, y_v and z_v , in the cross-flow plane. Brown and Michael (1955)⁴ originally

solved the problem indirectly by assuming values of the vortex location and then satisfying the no-force condition by trial and error. Their problem was greatly simplified in that the downwash condition was automatically satisfied by a conformal transformation, which aligned the two-dimensional flat plate with the flow direction. Later, Pullin (1973)⁸ developed a Newton Raphson iteration scheme for the Smith-type model, which included the Brown and Michael problem as a degenerate case. Trial runs of Pullin's program indicated that the Newton-Raphson procedure "converged" in approximately four iterations for the Brown and Michael model.

The Newton-Raphson procedure has several advantages over the procedure developed by Nangia and Hancock which ignores the effect of the change in the vortex position on the velocity components, v_i and w_i . First, the scheme is amenable to automatic iteration without operator interference. Secondly, the iteration procedure converges whenever the derivatives are locally monotonic. Consequently, although the Nangia and Hancock method appeared to work for the simple case they considered, a Newton's method was developed to locate the new vortex position in an iteration procedure. As a preliminary step, a numerical experiment on the applicability of Newton's method was conducted for the slender body model of Brown and Michael, since it was felt that useful information could be obtained on the force Jacobian more economically in two dimensions than in three dimensions. The numerical experiments were conducted on a slender delta wing of unit aspect ratio ($AR = 1$) at an angle of attack, $\alpha = 14.3^\circ$, which corresponded to the three-dimensional problem being studied. The residual force on the vortex-cut combination was calculated for different vortex locations, which are the unknowns.

In Figure 8 the spanwise force component, F_y , is presented, and in Figure 9 the vertical force component, F_z , is given.

The forces are plotted versus the vortex location, y_v and z_v , at $x = 1$. The triangle symbol represents the point where the lines, $F_y = 0$ and $F_z = 0$, intersect to define the stable location for this flow condition. The two-dimensional results indicate that F_z is a monotonic function of both y_v and z_v . F_y , on the other hand, is monotonic in much of the neighborhood of the stable point, but is poorly behaved near the leading edge. This behavior did not preclude the use of Newton's method in the Brown and Michael model, but should be remembered in the event of difficulties in three dimensions.

Therefore, a Newton's procedure was developed for the three-dimensional case to calculate the new vortex location, based on the forces and the force Jacobian calculated in the preceding iteration. This modification improved the rate of convergence in reducing the forces for a given vorticity distribution. However, when the given vorticity distribution was updated to satisfy the downwash condition, the large changes in the vorticity coefficients resulted in large forces. Various attempts to limit the changes in the vorticity coefficients and the vortex location coefficients were made by only partially reducing the forces and then partially reducing the residual downwash in a sequential procedure. This procedure did not appear to be converging; so a more detailed look was taken of the slender-body problem.

Since the downwash condition can not be automatically satisfied in the three-dimensional case as in the slender-body case, this difference may hide the cause of the convergence difficulties. Therefore, the two-dimensional problem was investigated in a manner parallel to the three-dimensional problem.

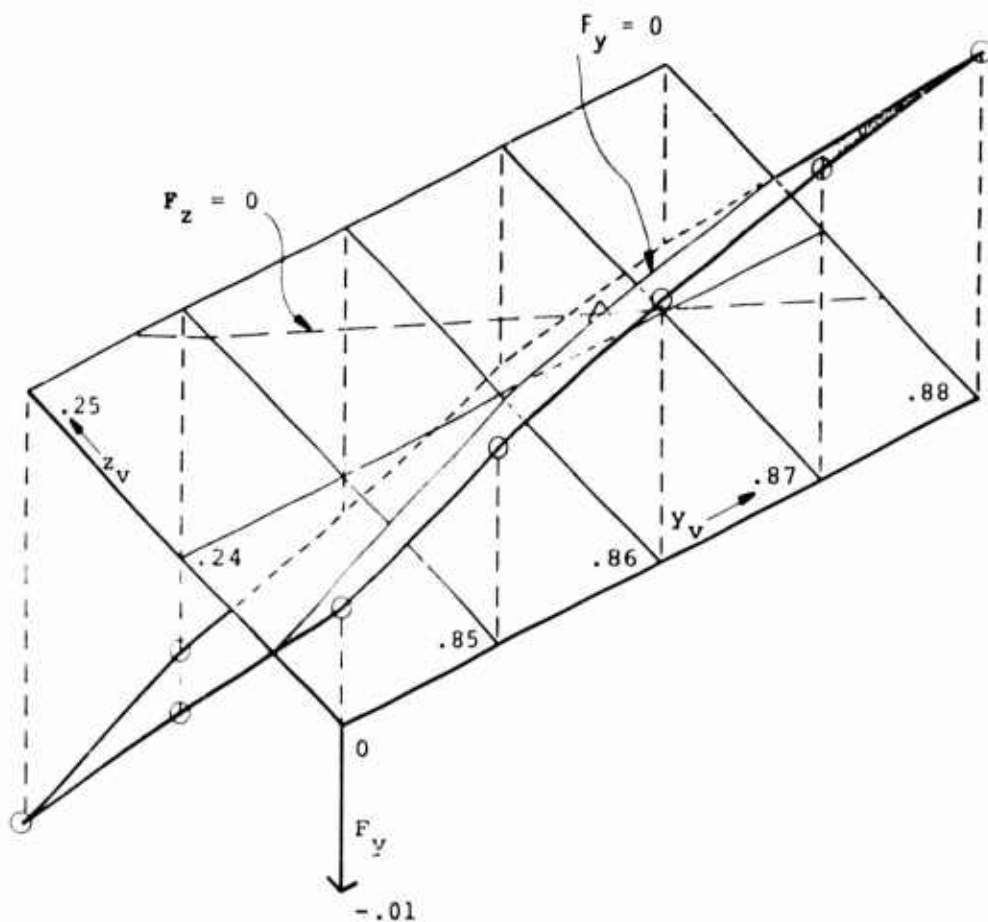


Figure 8. Spanwise force component on leading-edge vortex versus vortex location. Downwash condition satisfied on delta wing ($\sin\alpha/c\cos\lambda = 1$) for Brown and Michael model. Symbol (Δ) represents stable point, $F_z = F_y = 0$.

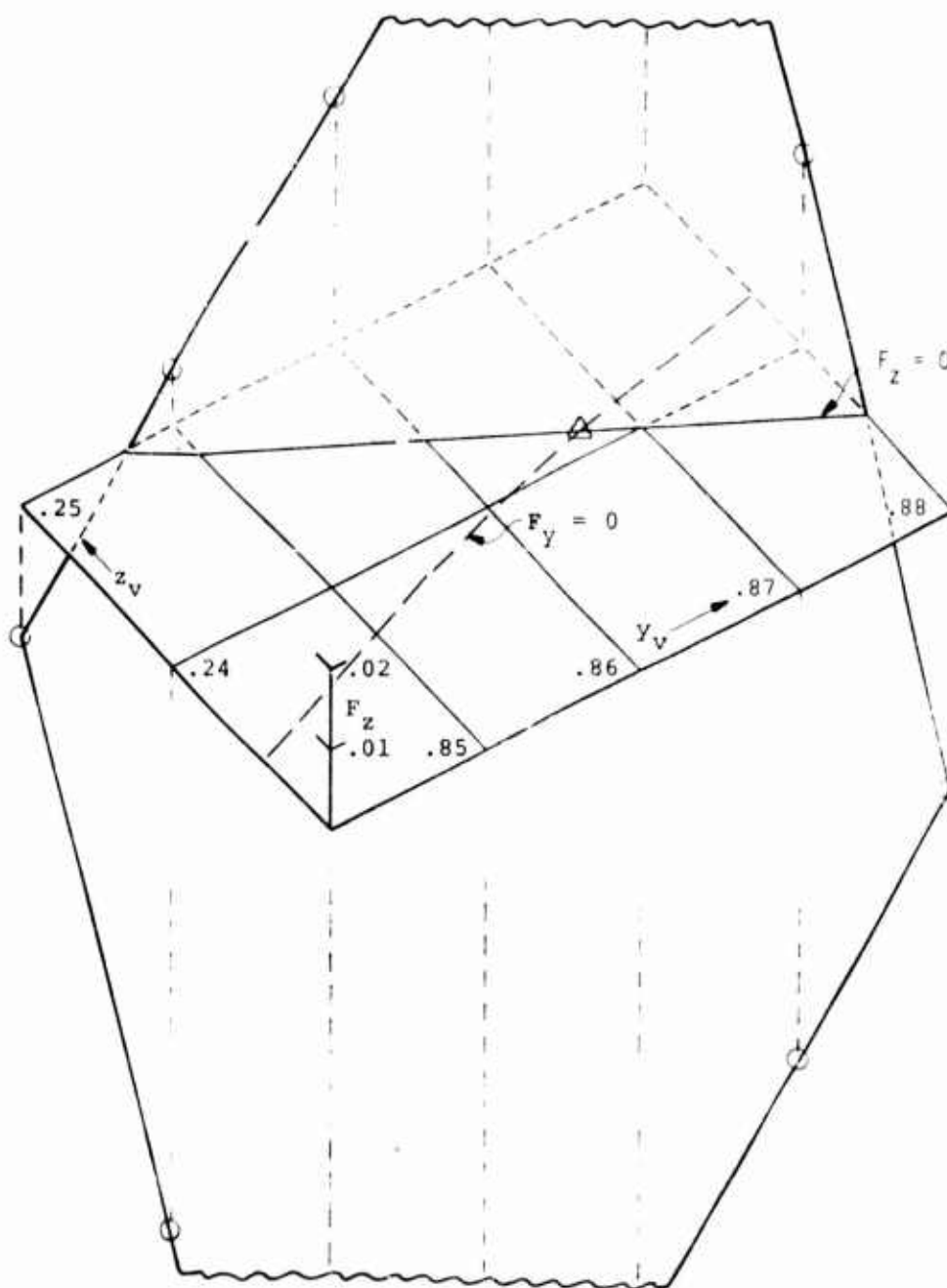


Figure 9. Vertical force component on leading-edge vortex versus vortex location. Downwash condition satisfied on delta wing ($\sin\alpha/\cot\lambda = 1$) for Brown and Michael model. Symbol (Δ) represents stable point, $F_y = F_z = 0$.

The Brown and Michael problem was thus reformulated as a vorticity distribution on the wing with unknown loading coefficients. The leading-edge vortex strength and location were also unknown originally. The downwash and no-force condition were then written in terms of these unknowns in the physical y-z plane.

The known location of the vortex was first used to calculate the vorticity coefficients from the downwash condition. Then the vortex was moved to different points, and the resulting forces are plotted in Figure 10 and Figure 11. Although the vertical force component appears similar to the one obtained previously (cf., Figure 9), the spanwise force component has changed considerably (cf., Figure 8). Especially significant is the fact that the lines, $F_y = 0$ and $F_z = 0$, are nearly coincident, which suggests difficulties in finding their point of intersection. In fact, when an attempt was made to iterate between reducing the downwash residue and the forces on the vortex, the procedure failed to converge. The procedure oscillated between the true solution and a false solution, where the forces were zero, but the downwash condition was not satisfied. Some of the details of these calculations are included in Appendix B.

Therefore, an alternative strategy was developed, whereby the forces and downwash residues were reduced simultaneously, instead of sequentially, by changing the vorticity coefficients as well as the vortex location according to Newton's method. This procedure, although requiring more effort to calculate the derivatives of the downwash terms as well as the derivatives of the force terms with respect to the vorticity coefficients, resulted in smooth convergence to the proper solution. As a typical example, five vorticity modes and six control points were employed for the slender delta

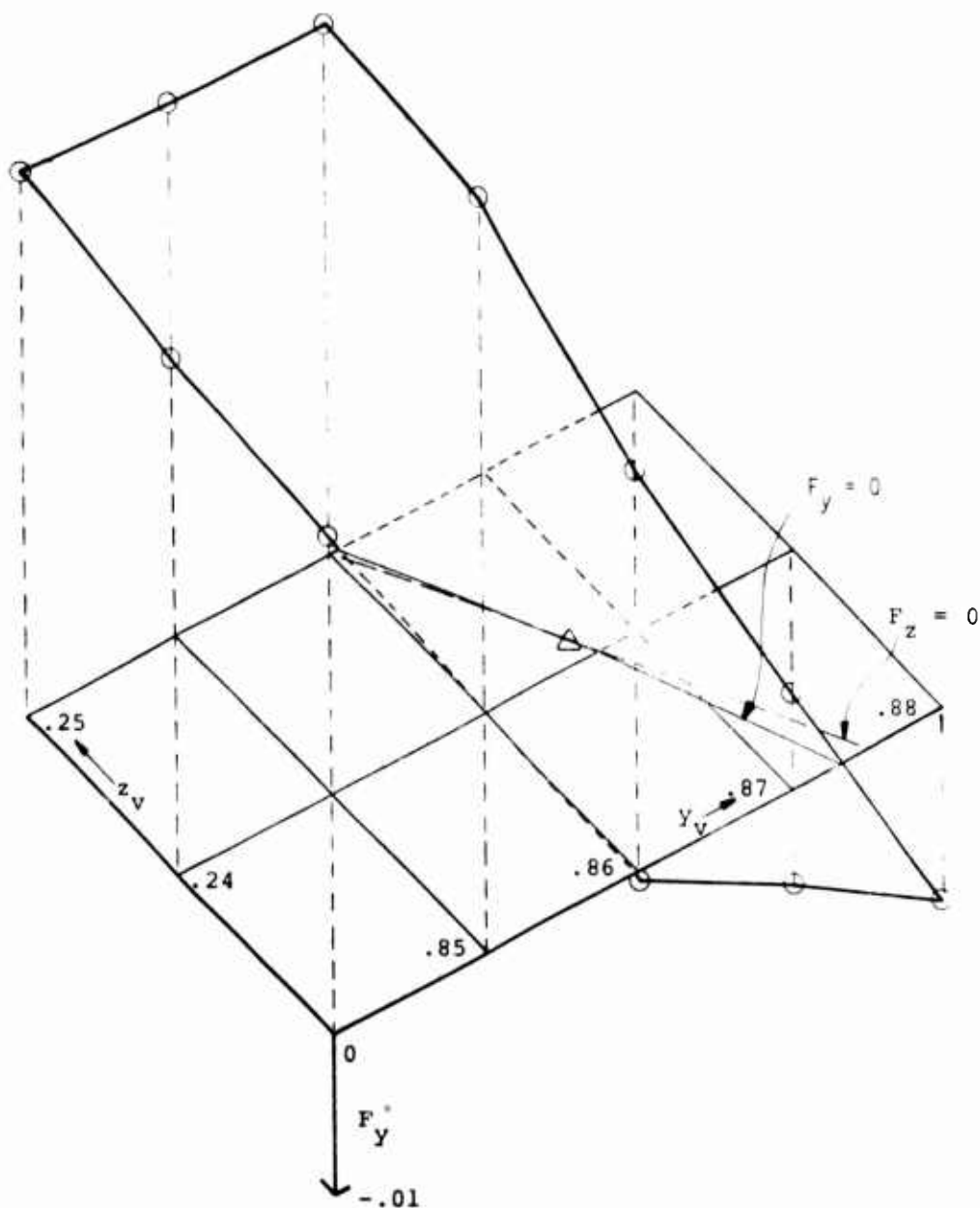


Figure 10. Spanwise force component on leading-edge vortex versus vortex location for modified Brown and Michael model. Vorticity coefficients chosen to satisfy downwash condition at stable point for delta wing ($\sin\alpha/\cot\lambda = 1$). Symbol (Δ) represents stable point, $F_z = F_y = 0$.

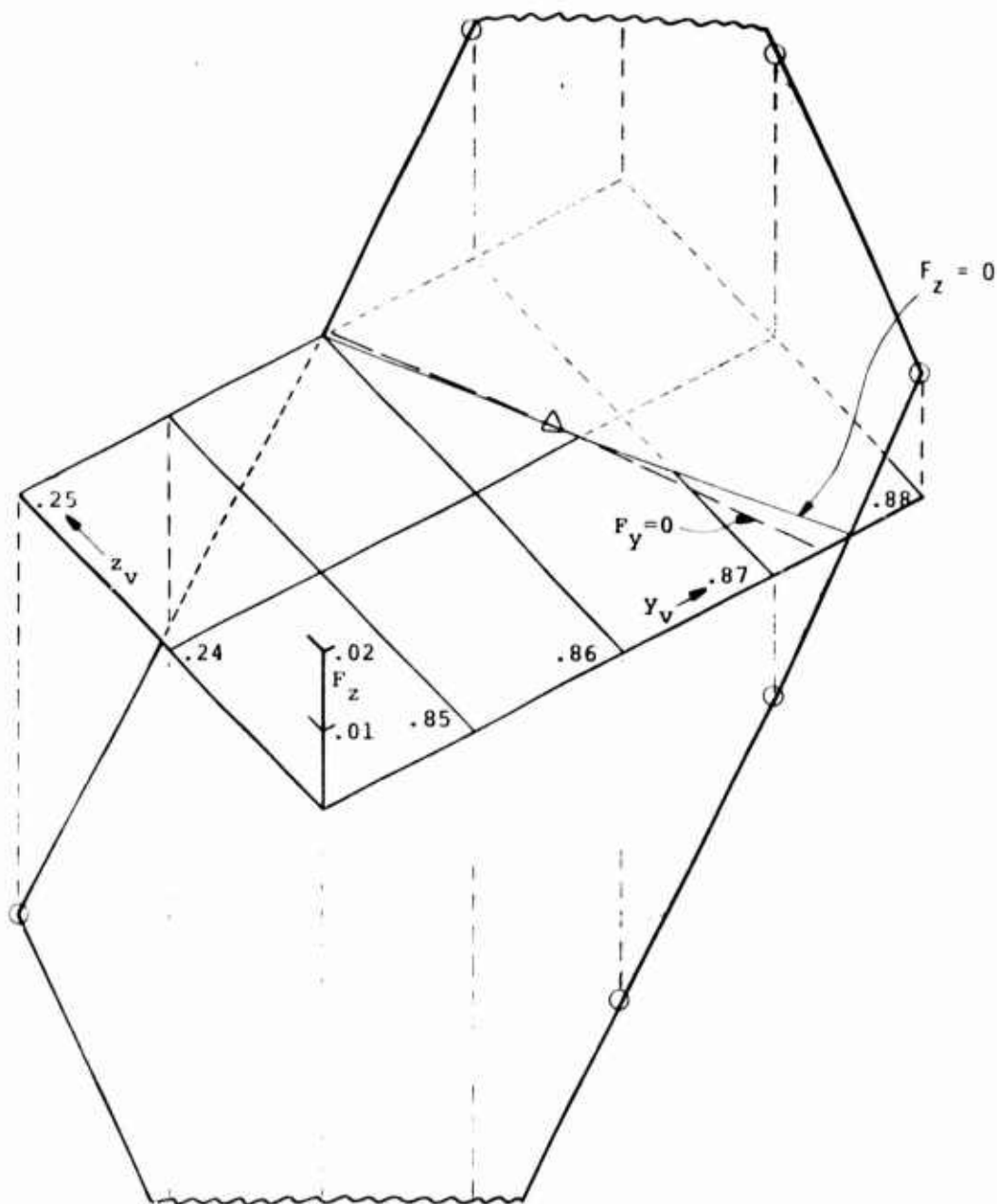


Figure 11. Vertical force component on leading-edge vortex versus vortex location for modified Brown and Michael model. Vorticity coefficients chosen to satisfy downwash condition at stable point for delta wing ($\sin\alpha/\cot\lambda = 1$). Symbol (Δ) represents stable point, $F_y = F_z = 0$.

wing ($AR = 1$, $\alpha = 14.3^\circ$) being considered. The vortex was initially assumed to have a spanwise location of 80 per cent of the semispan and a height of 30 per cent of the semispan ($y_v/s = .8$, $z_v/s = .3$). The iteration procedure converged to a stable point ($y_v/s = .86$, $z_v = .24$) in eight iterations. See Figure 12 for a graphic description of the convergence rate. Thus, the procedure was adapted to the fully three-dimensional case.

The full procedure presently being employed to satisfy the downwash and no-force condition is described next. First, an initial location for the vortex is found. This initial location is used in conjunction with Equation 20 to find an initial distribution of vorticity coefficients. Now, the residual forces and the remaining derivatives for the Jacobian are calculated. The residues and the Jacobian are used to calculate a new set of vorticity coefficients and vortex location coefficients. This last step is iterated until the procedure converges. If the same number of equations and unknowns are employed, then convergence is attained when the residue becomes small compared to the angle of attack. If the number of equations is greater than the number of unknowns, it is generally impossible to satisfy all of the conditions imposed, and convergence is attained when the residue is minimized and further iterations produce no additional change. Although the Jacobian is presently being updated for the force contributions at every iteration, it appears that some savings in computational effort may be obtained by only a partial updating as most of the derivatives change slowly. This modification can be implemented after greater knowledge of the procedure has been acquired.

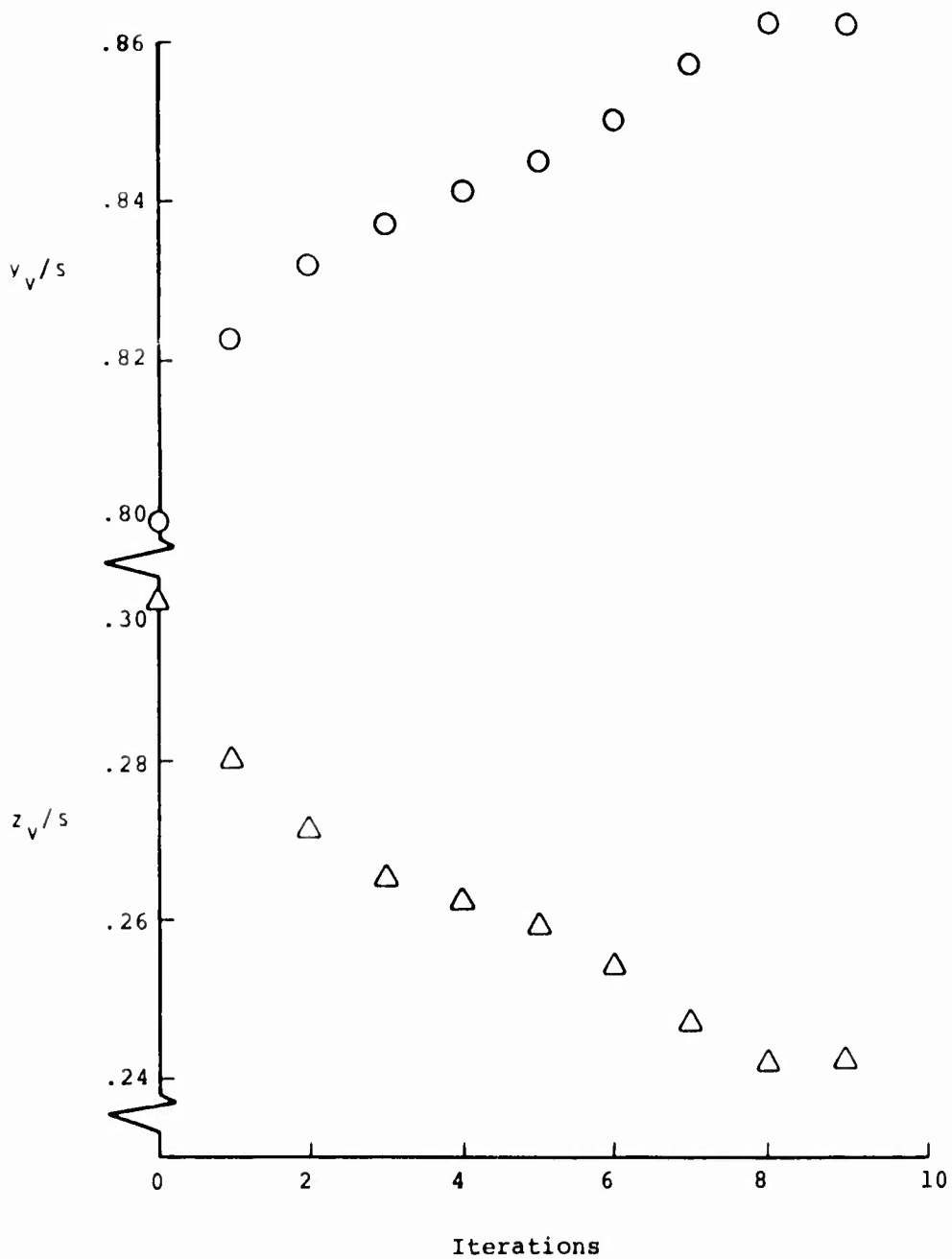


Figure 12. Convergence of vortex location to stable point in cross-flow plane for slender delta wing ($AR = 1$, $\alpha = 14.3^\circ$).

4. Program Description

The actual FORTRAN programs to perform the operations described in the previous section are included in Appendix C and are documented primarily by comment cards. Additionally, the coded symbols are generally similar to their English counterparts to facilitate comprehension. The complete procedure is presently divided into five computer programs: Program I, Program WOW, Program IIIA, Program III Prime, and Program V.

Program I calculates the influence coefficients due to the contributions from γ_2 and δ_2 for the downwash condition at a set of collocation points for the specified number of chordwise modes. The number of chordwise modes, Q and N , in Equation 4 is represented by the FORTRAN variable NOCM. The number of spanwise modes, M , in Equation 4 is represented by the variable NOSM. The number of chordwise collocation stations on the wing surface is given by NCORD and the number of spanwise collocation stations is given by NSPAN, where the product of these numbers (NCORD times NSPAN) must be greater than or equal to the total number of modes (NOCM times (NOSM + 1)) for the system to be completely determined.

Program WOW is the program which evaluates the contribution of γ_1 to the downwash condition according to Equation 10, at the chosen set of collocation points. As mentioned previously, this is a simplified version of the program developed by Widnall (1964)¹³ to calculate the influence coefficients from a distribution of horseshoe vortices.

Program IIIA uses the results of Program I and Program WOW as inputs and, furthermore, calculates the contributions from the leading-edge vortices and wake due to δ_2 . Then it solves a set of simultaneous equations based on the downwash condition (Equation 20), to find the initial values for the

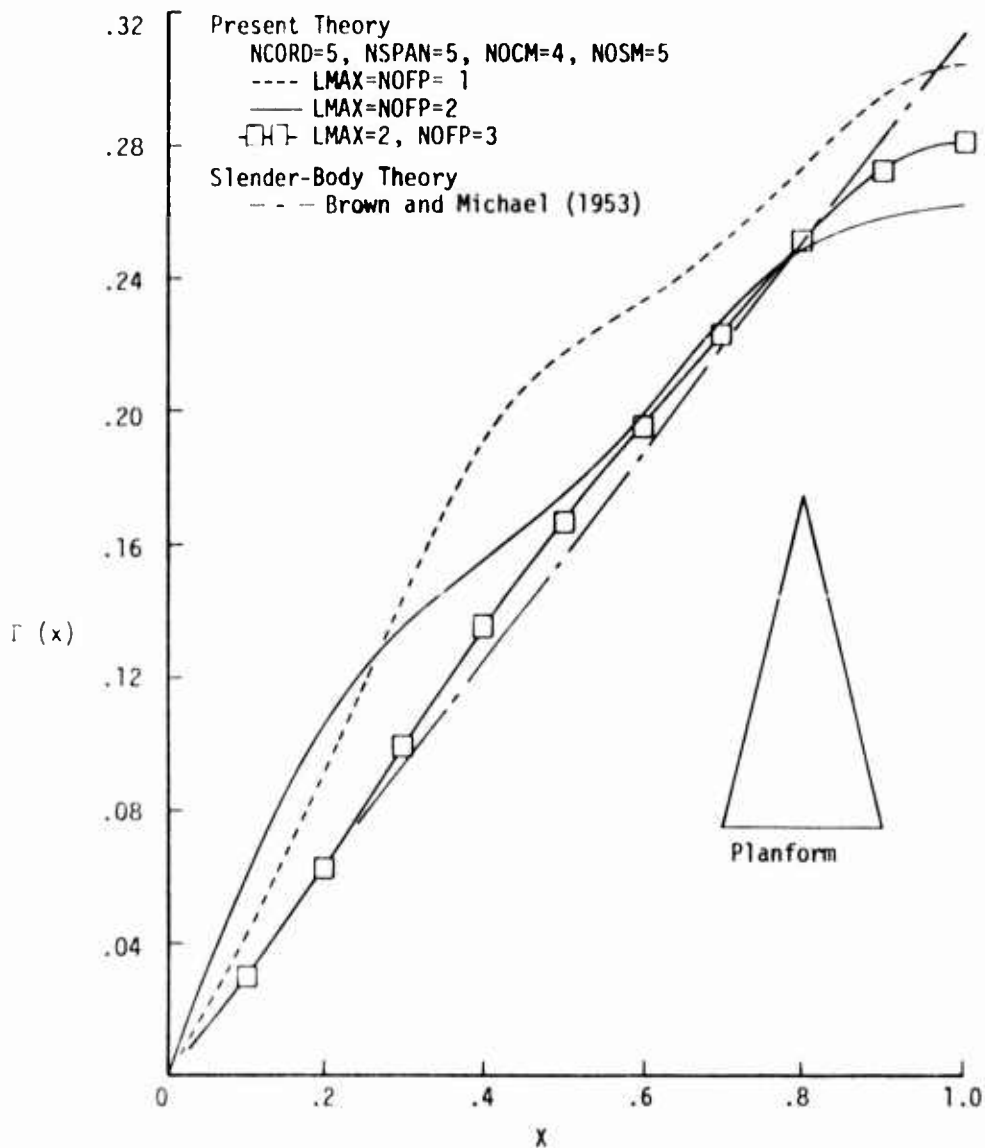


Figure 13. Convergence of leading-edge vortex strength for delta wing ($AR = 1$, $\alpha = 14.3^\circ$).

near the apex. This is in agreement with experiments which generally show that the flow approximately satisfies the Brown and Michael conditions, away from the trailing edge. Three-dimensional effects are apparent in the slope of the leading-edge vortex strength. The modes have been chosen to insure that the slope is zero at the trailing edge, after which no additional vorticity is shed from the leading edge.

Figure 14 illustrates the change in the stable position of the leading-edge vortex on the right half of the wing. The spanwise position from the Brown and Michael model is not included since it is almost coincident with the $NOFP = 1$, $LMAX = 1$ result. The parameter choice, $LMAX = 1$ (see Equation 8 for details of the expansion), corresponds to a linear approximation for the vortex position, while the choice, $LMAX = 2$, represents a cubic fit for the vortex location. As can be seen from Figure 14, the spanwise position changes slightly, although the three-dimensional effect seems to be manifested in an effort to align the vortex with the free stream direction. The same tendency is indicated by the vertical position of the vortex, although convergence is only partly indicated by the bracketing of the final vortex location by the lower order models.

A comparison with the experimental results of Peckham (1958)¹⁵ and with some slender-body models is presented in Figure 15 for the vortex position over the delta wing. The agreement for the vertical position is excellent, while the spanwise position indicates the general limitations of a Brown and Michael model in predicting the vortex location too far outboard. It must be noted that this is not a completely fair test, since the vortex location represents the center of vorticity in the Brown and Michael model. For the Smith-type model,

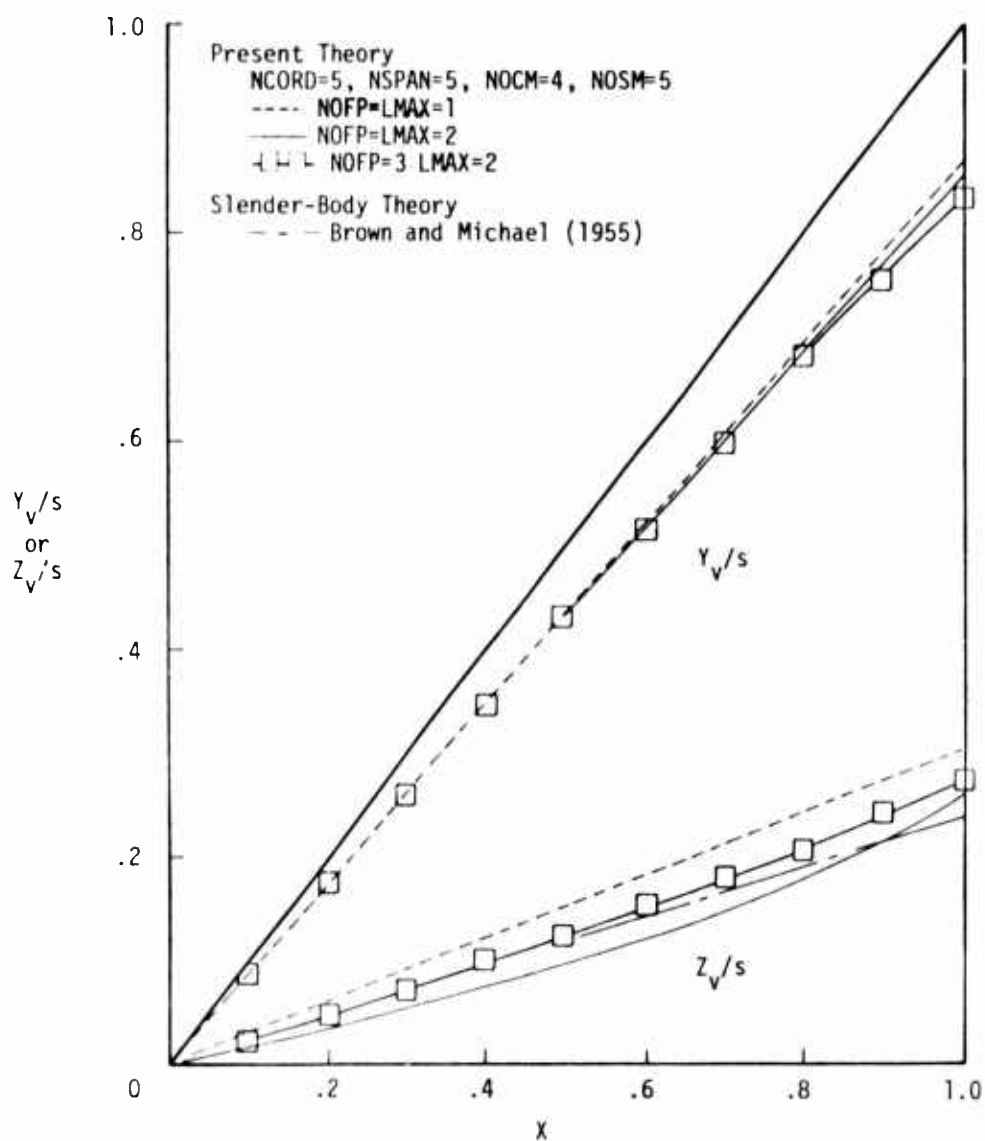


Figure 14. Convergence of vortex position over delta wing ($AR=1, \alpha=14.3^\circ$).

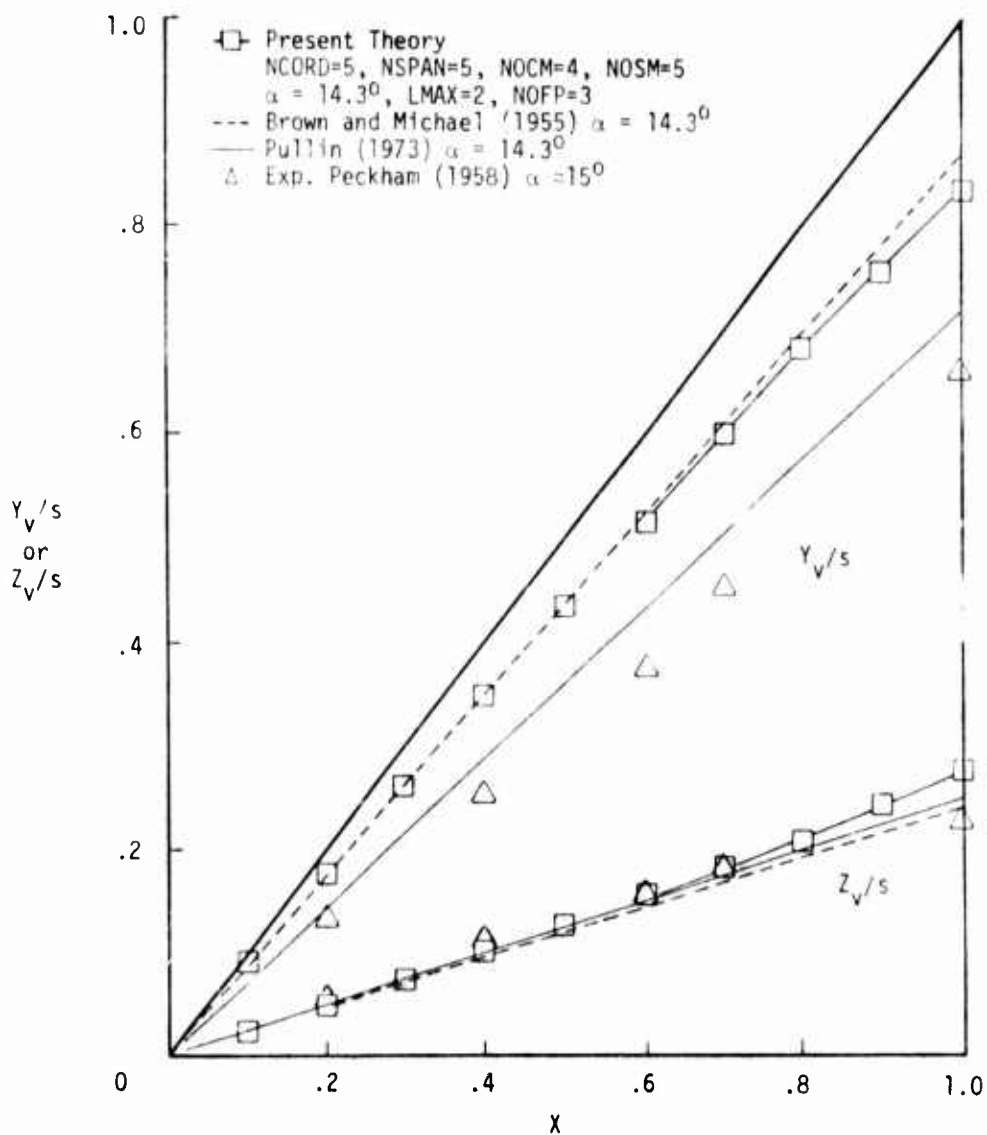


Figure 15. Leading-edge vortex position over right half of delta wing (AR=1).

on the other hand, the spanwise location of the center of vorticity is five per cent of the semispan outboard of the core position. This shift is due to the presence of vorticity in the leading-edge vortex sheet. Thus, it would be reasonable to assume that the spanwise location of the center of vorticity for the experimental data is also outboard of its vortex core location.

Some pressure distributions are presented next. In Figure 16, the pressure distribution calculated by the present procedure is compared with the results of the slender-body models. Again excellent agreement is obtained with the Brown and Michael model for the stations near the apex while the aft stations show the attenuation due to the presence of the trailing edge.

Figure 17 shows a comparison with the experiments of Nangia and Hancock (1969)¹⁶ on a flat plate delta wing. The experimental results are presented as a smooth curve as provided in the referenced report. The general shape and magnitude of the loading have been predicted, but the limitations of a Brown and Michael model are again apparent. The predicted peak is too far outboard and too high.

Figure 18 presents a comparison with some data available for a thick delta wing. Here the thickness to chord ratio is .12, and a comparison of lift curves from Peckham (1958)¹⁵ indicates that the pressure peaks are ten to twenty per cent lower for the thick wing than for the flat plate wing. Also the vortex position is further inboard and higher for the thick wing. These effects have been verified theoretically in the slender-body range by Smith (1971)¹⁷. Thus, detailed comparison between the experimental data of Peckham for thick wings and the theoretical predictions for flat plate delta wings is limited.

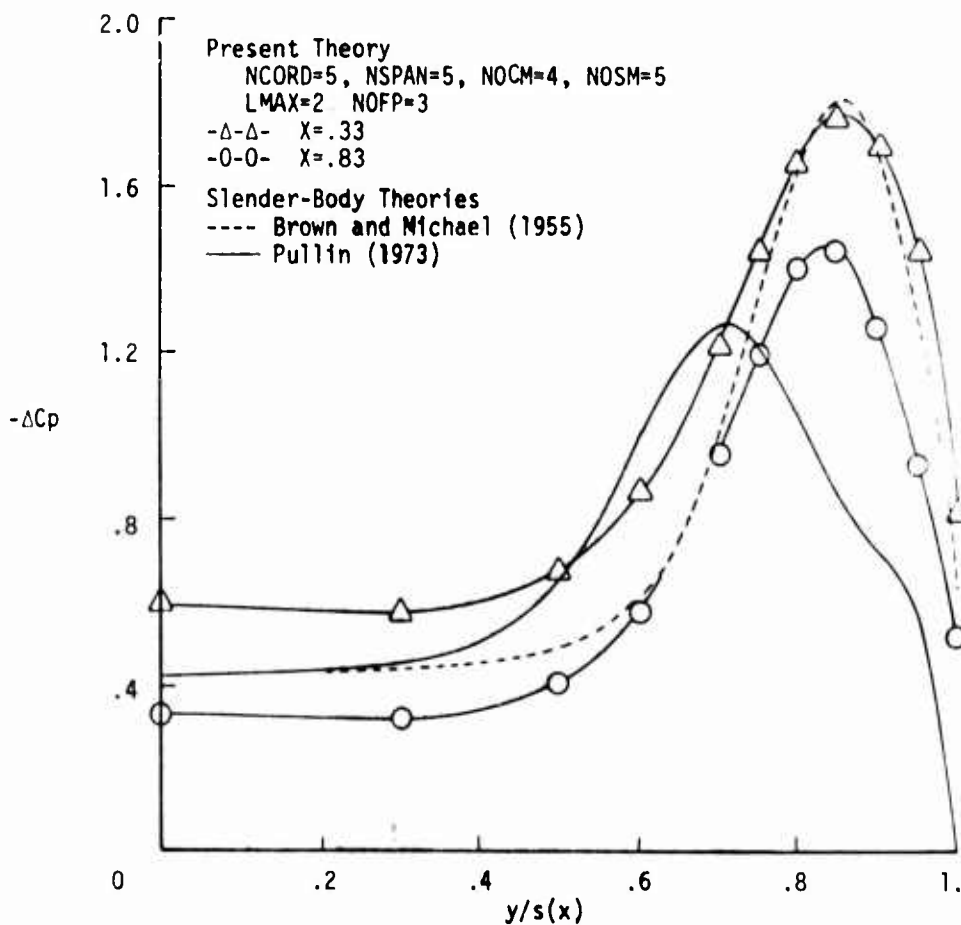


Figure 16. Comparison of theoretical models for calculation of loading on delta wing ($AR=1$, $\alpha = 14.3^\circ$).

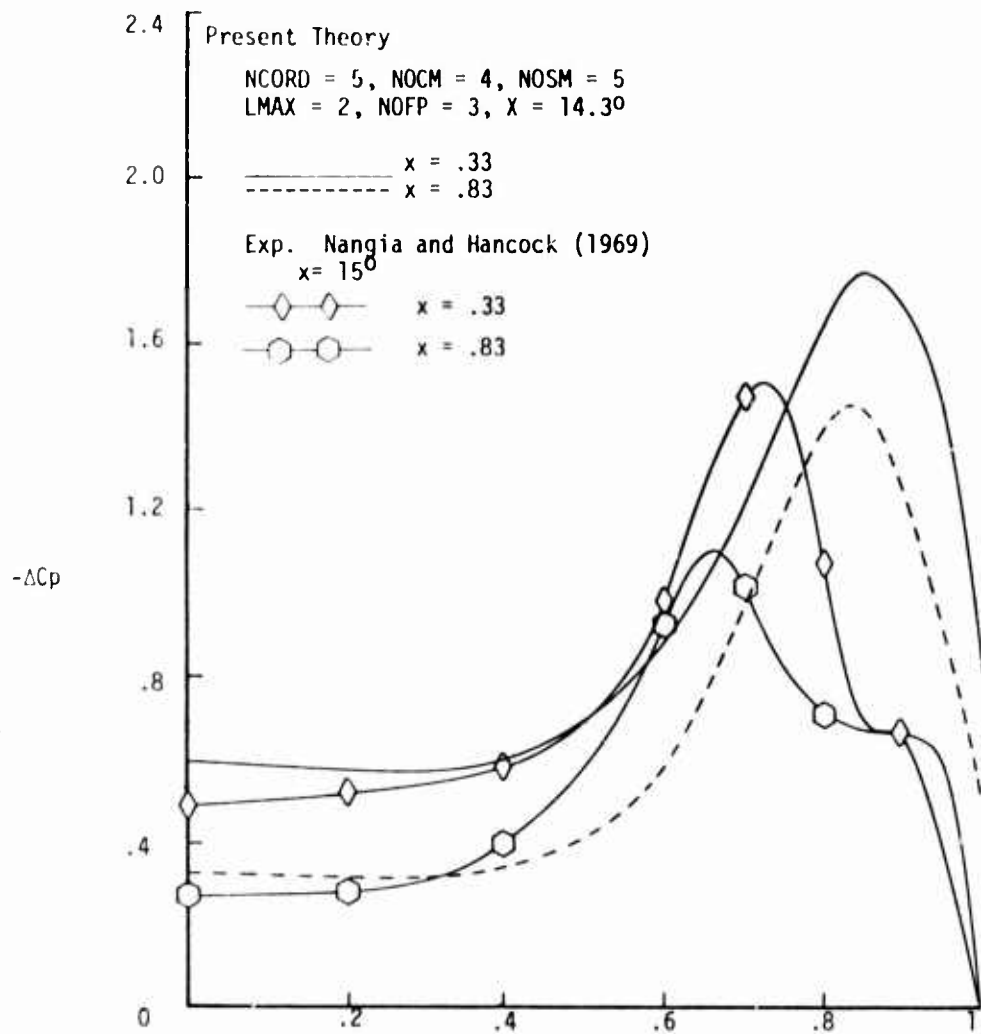


Figure 17. Comparison of experimental and theoretical loading values on delta wing ($AR=1$).

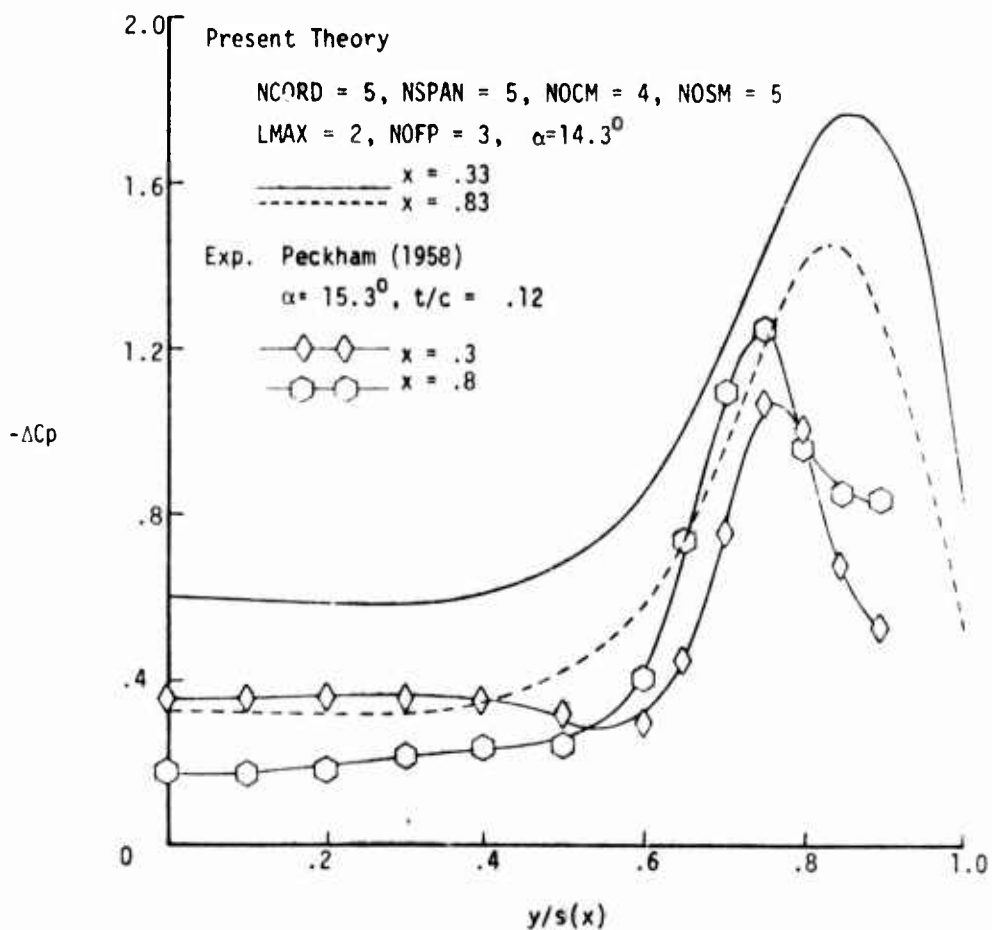


Figure 18. Comparison of experimental and theoretical loading values on delta wing (AR=1).

Finally, a comparison is made with some other lifting surface theories. As mentioned in the Introduction, Brune, et al. (1975)⁸ and Kandil, et al. (1974)⁷ have developed finite-element lifting surface theories with leading-edge separation. A comparison of these theories with the present theory and the experimental results of Peckham (1958)¹⁵ is presented in Figure 19. Both of the other theoretical curves were taken from Kandil, Mook, and Nayfeh (1976)¹⁸, who referenced Weber, Brune, Johnson, Lu, and Rubbert (1975).¹⁹ As expected, the present theory, which is based on a Brown and Michael vortex-cut representation predicts a higher peak loading which is further outboard than the one predicted by the other lifting surface theories for the flat plate delta wing. However, since the experimental curve is for a 12 per cent thick wing, one would expect the experimental pressure peak to be higher and further outboard if the wing were thin, for the reasons presented during the discussion of Figure 18. Thus, although the present theory does not provide solutions identical with those provided by the other theories, the present procedure appears to be competitive in predicting the experimental results compared with the other programs.

In Figure 20, the results for the sectional normal force coefficients are compared with those obtained by Nangla and Hancock (1969).¹⁶ The sectional normal force coefficient is defined as

$$C_N(x) = \int_{-s(x)}^{s(x)} \Delta C_p dy \quad (36)$$

Again, the tendency of the Brown and Michael model to overpredict the magnitude of the loading is apparent. Although the sectional force coefficient calculated by the present method decreases near the trailing

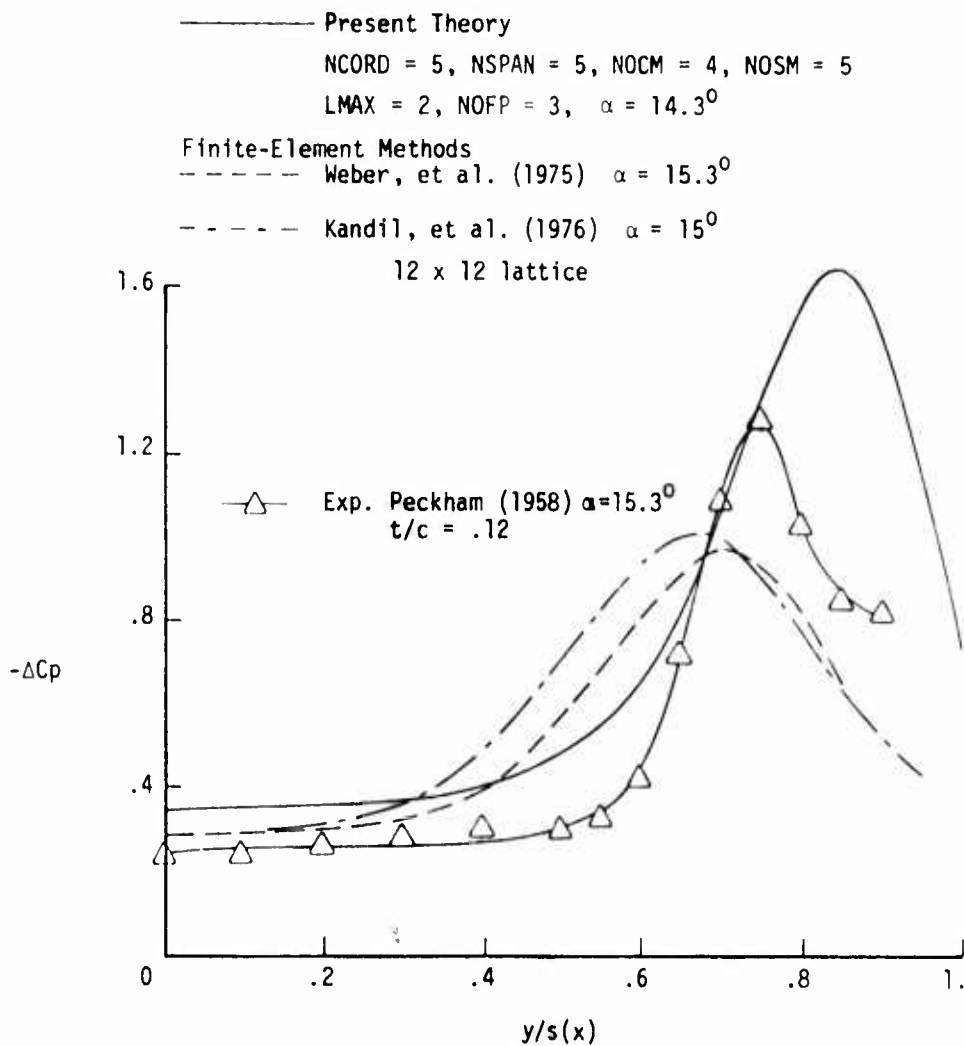


Figure 19. Comparison of lifting surface models for the calculation of loading on delta wing ($AR=1$, $X = .7$).

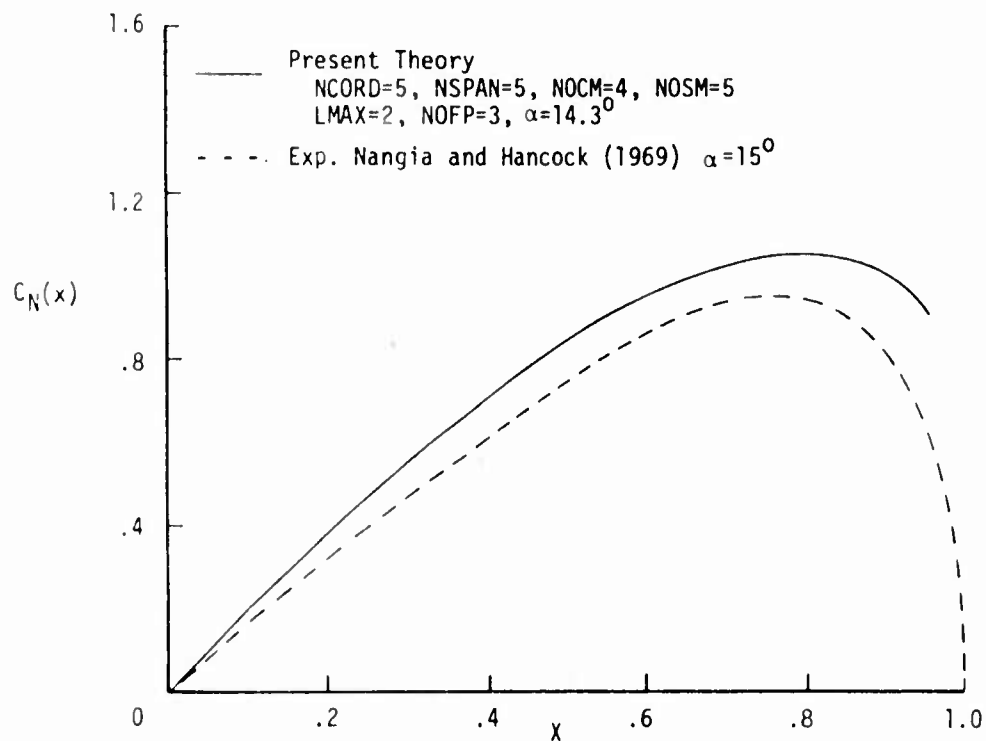


Figure 20. Chordwise distribution of sectional normal force coefficients for delta wing ($AR=1$).

edge, it does not vanish since a modified Kutta condition has been applied which does not require zero loading at the trailing edge.

To demonstrate that this program could be used for planforms other than the delta wing, some runs were made for the arrow wing. However, the problem with this planform and others is that there is little experimental evidence readily available for these planforms.

Figure 21 illustrates the results for the leading-edge vortex strength for an arrow wing whose planform is similar to that of the unit aspect ratio delta wing with the addition of trailing edge sweep ($\lambda = 76^\circ$, $AR = 1.25$) at an angle of attack $\alpha = 14.3^\circ$. The same number of collocation points for the downwash and the same number of vorticity modes were used as for the delta wing ($NCORD = 5$, $NSPAN = 5$, $NOCM = 4$, $NOSM = 5$). The initial approximation for the vortex location was obtained from the delta wing being considered previously. It appears that convergence is more difficult to obtain for the arrow wing than for the delta wing as an extraneous "bump" appears in the curve for the case $NOFP = 2$, $LMAX = 1$. This is smoothed over as an additional constraint is applied. Near the apex, the vortex strength is similar to that for a delta wing of unit aspect ratio, as would be expected away from the trailing edge.

The vortex position for this arrow wing is plotted in Figure 22. The results for the cubic fit ($LMAX = 2$) with three no-force points ($NOFP = 3$), are similar to those obtained for the delta wing, indicating the dominance of the leading-edge sweep in locating the vortex.

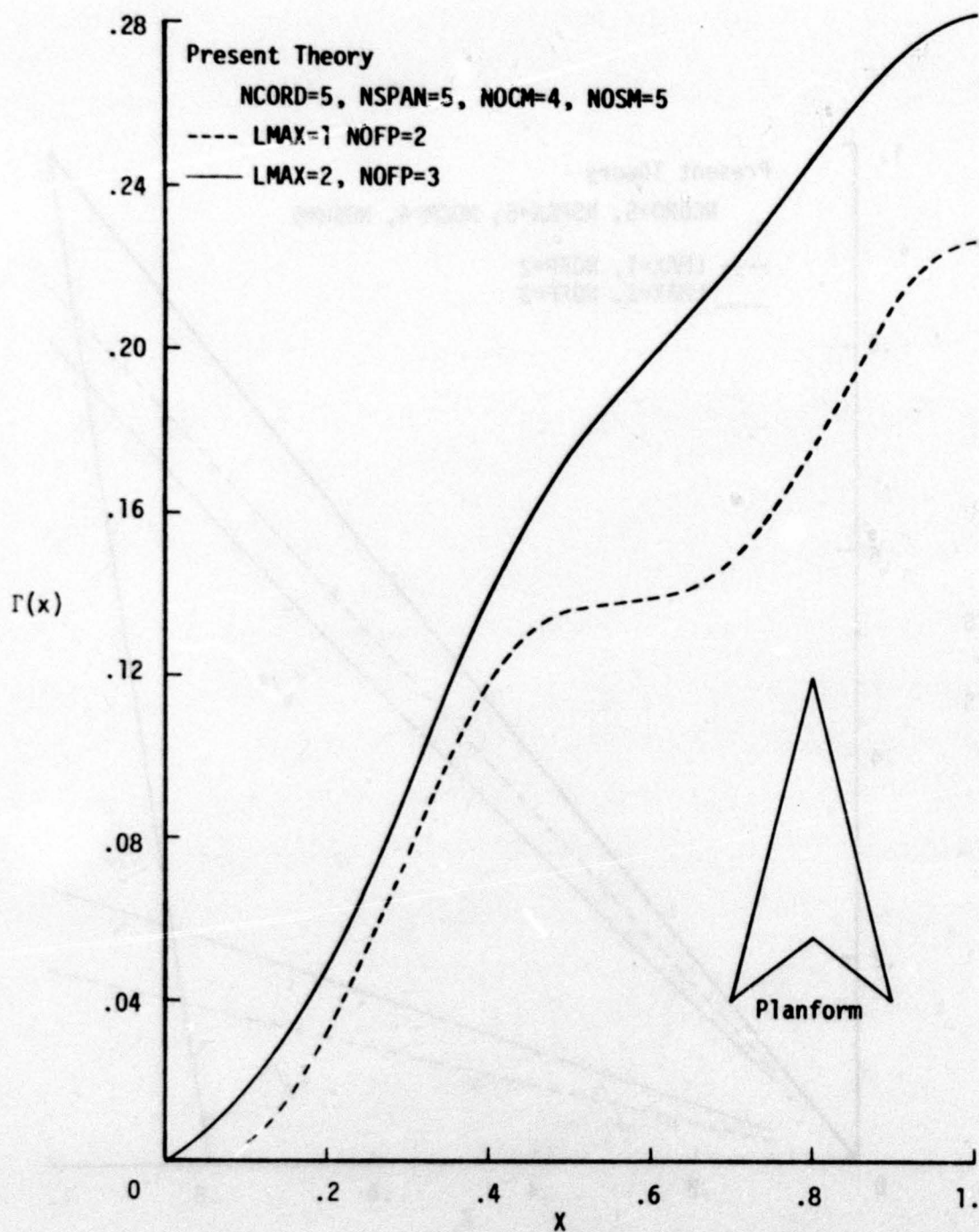


Figure 21. Convergence of leading-edge vortex strength for arrow wing ($AR=1.25$, $\lambda=76^\circ$, $\alpha=14.3^\circ$).

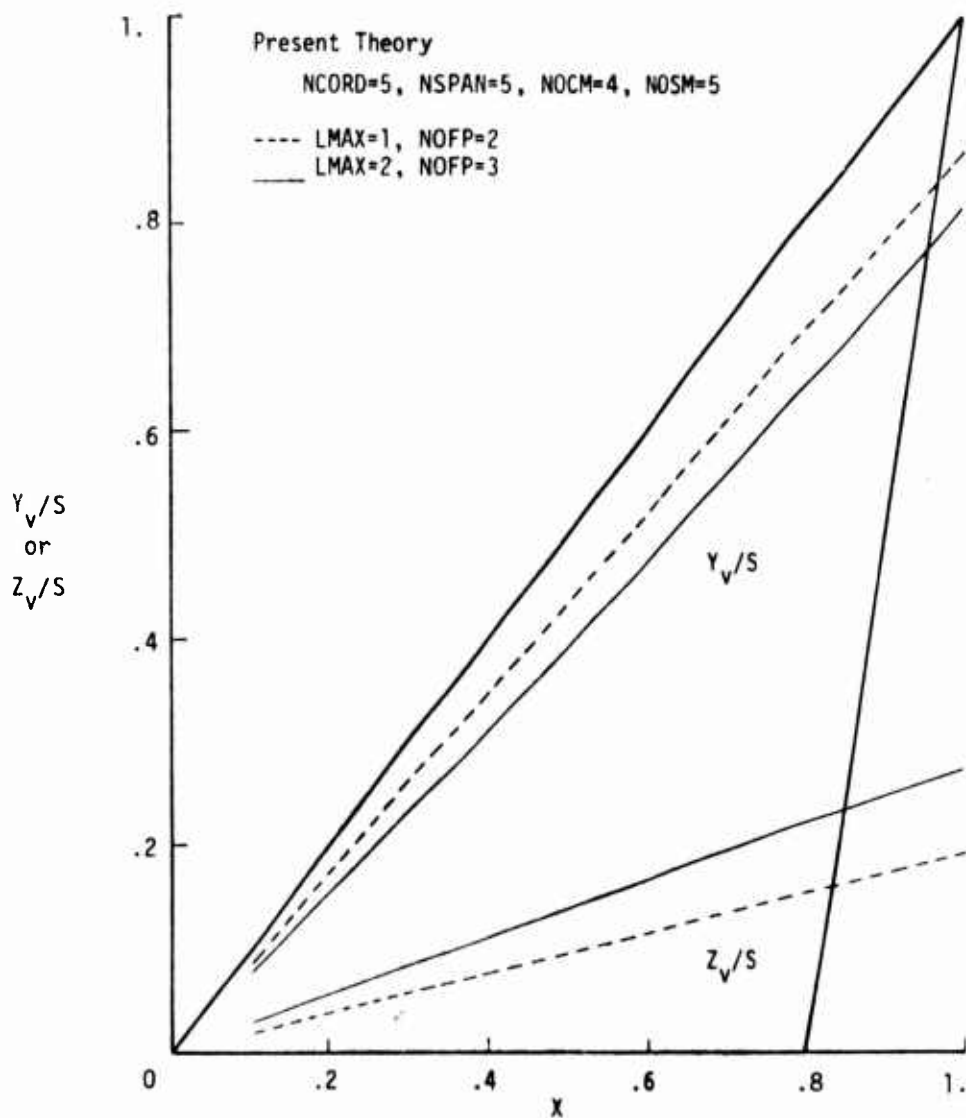


Figure 22. Leading-edge vortex position over right half of arrow wing ($AR=1.25$, $\lambda = 76^\circ$, $\alpha = 14.3^\circ$).

Finally, the pressure distributions at two chordwise stations are plotted in Figure 23. It is to be noted that the station, $x = .833$, is aft of the root chord ($x = .8$) and consequently, the loading should strictly be zero at both the trailing edge ($y/s(x) = .2$) and at the leading edge ($y/s(x) = 1$).

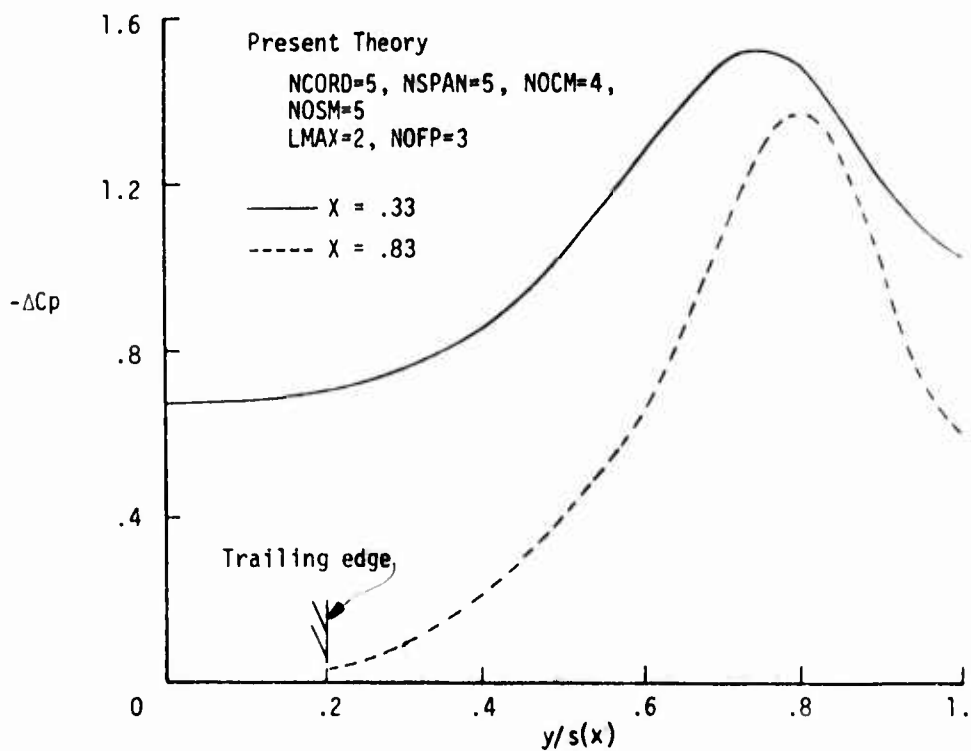


Figure 23. Loading on arrow wing ($AR=1.25$, $\lambda=76^\circ$, $\alpha=14.3^\circ$).

6. Revised Vorticity Modes

The results obtained for the arrow wing in the previous section demonstrated the need for a better representation of the vorticity distribution at the trailing edge for such reentrant surfaces. Problems with convergence in the iteration procedure were encountered as the trailing edge sweep angle was increased. The reason for this difficulty can probably be traced to the form of bound vorticity chosen.

The present model used bound vorticity modes to feed the leading-edge vortices which are circular arcs with their center at the apex of the wing (see Figure 5). However, at the trailing edge, this vorticity was suddenly turned downstream as described in Figure 4. Consequently, there was a discontinuous turning of the vortex lines at the trailing edge. This was not a serious problem for the slender delta wing, where the choice of vorticity modes insured that the spanwise component, γ , was small compared to the chordwise component, δ , at the trailing edge. However, as the sweep angle of the trailing edge was increased, a sharp kink developed in the bound vorticity at the trailing edge due to the nonzero component, γ_2 , which fed the leading-edge vortices. No control points were located at the trailing edge, so no singularities were encountered in the numerical calculations, but such a discontinuity was a potential source of trouble.

An effort was made to develop a better modal description of the bound vorticity which feeds the leading-edge vortices, i.e., γ_2 and δ_2 . The desired conditions to be satisfied by these vorticity components on the right half of the wing are

$$\gamma_2(x_{TE}(y), y) = 0 \quad (37)$$

$$\frac{\delta_2(x_{LE}(y), y)}{\gamma_2(x_{LE}(y), y)} = -s \quad (38)$$

where Equation 37 guarantees that there is no kink at the trailing edge and Equation 38 insures that the vorticity leaves perpendicular to a straight leading edge. It is to be noted that symmetry conditions dictate that γ_2 is even and δ_2 is odd in the spanwise variable.

An attempt was first made to determine vorticity functions which satisfied these conditions in the physical (x, y) plane. However, that approach failed to provide a solution, and the problem was then considered in the transformed (θ, η) plane. (See Equation 5 for the coordinate transformation.) Basically, in the transformed plane, the leading edge of the planform corresponds to the chordwise origin, $\psi = 0$, and the trailing edge corresponds to the chordwise maximum, $\theta = \pi$.

The two vorticity components, γ_2 and δ_2 , can be written in the following form.

$$\begin{aligned} \gamma_2 &= \sum_{q=1}^{NOCM} g_q g_\gamma(\eta, q) \\ \delta_2 &= \sum_{q=1}^{NOCM} g_q g_\delta(\eta, q) \end{aligned} \quad (39)$$

Then, from the continuity of vorticity, Equation 2, the two functions can be related by

$$g_\delta = -\frac{1}{2s} \int_0^\theta \frac{\partial g_\gamma}{\partial \eta} c(\eta) \sin \theta d\theta \quad (40)$$

Assuming a form which satisfies the boundary condition at the trailing edge and is nonzero at the leading edge, let

$$g_Y = \cos \theta/2 f(\eta, q) \quad (41)$$

For the arrow wing planform being considered presently, the local chord on the right-hand side is

$$c(\eta) = c_R (1-\eta)$$

where c_R is the root chord nondimensionalized by the maximum length.

Then Equation 40 becomes

$$\begin{aligned} g_\delta = & \frac{c_R}{6s} \left\{ \frac{\partial f}{\partial \eta} (1-\eta) [3\cos\theta/2 + \cos 3\theta/2] \right. \\ & \left. + \frac{1}{2} f(\eta, q) \left[\frac{3(4-3c_R)}{c_R} \cos\theta/2 + \cos 3\theta/2 \right] \right\} - g(\eta, q) \quad (42) \end{aligned}$$

where the function, $g(\eta, q)$, is a function of integration.

Using the identity

$$\cos \frac{3\theta}{2} = 4\cos^3 \theta/2 - 3\cos \theta/2$$

this can be rewritten as

$$g_\delta = \frac{c_R}{3s} \left\{ 2 \frac{\partial f}{\partial \eta} (1-\eta) \cos^3 \theta/2 + f(\eta, q) \left[\frac{3(1-c_R)}{c_R} \cos \theta/2 + \cos^3 \theta/2 \right] \right\} - g(\eta, q) \quad (43)$$

To determine the function, $f(\eta, q)$, it is necessary to apply the boundary condition at the leading edge, Equation 38.

$$-s = \frac{\frac{c_R}{3s} \left\{ 2 \frac{\partial f}{\partial \eta} (1-\eta) + \frac{3-2c_R}{c_R} f(\eta, q) \right\} - g(\eta, q)}{f(\eta, q)} \quad (44)$$

This provides the following differential equation

$$\frac{\partial f}{\partial \eta} + \frac{1}{2(1-\eta)c_R} \left[3(s^2 + 1) - 2c_R \right] f(\eta, q) = \frac{3s g(\eta, q)}{2(1-\eta)c_R} \quad (45)$$

The solution of this differential equation is

$$f(\eta, q) = C(1-\eta)^{\frac{3(s^2+1)}{2c_R} - 1} + \frac{3s(1-\eta)}{2c_R} \left[\frac{3(s^2+1)}{2c_R} - 1 \right] \int_1^\eta (1-\eta)^{-\frac{3(s^2+1)}{2c_R}} g(\eta, q) d\eta \quad (46)$$

where C is a constant of integration. In order to obtain a general modal description, one must allow $g(\eta, q)$ to be a complete set of functions. The constant, C, is chosen to be zero, while the following form is chosen for $g(\eta, q)$ to simplify the integration

$$g(\eta, q) = \frac{(1-\eta)^q}{3s} \quad (47)$$

Then, integration of Equation 46 yields

$$f(\eta, q) = \frac{-(1-\eta)^q}{2c_R(q+1) - 3(s^2 + 1)} \quad (48)$$

Therefore, the vorticity functions become

$$g_Y = \frac{-(1-\eta)^q}{2c_R(q+1) - 3(s^2+1)} \cos \theta/2$$

$$g_\delta = \frac{1}{3s} (1-\eta)^q \left\{ \frac{(2q-1) c_R \cos^3 \theta/2 + 3(c_R-1) \cos \theta/2}{2c_R(q+1) - 3(s^2+1)} - 1 \right\} \quad (49)$$

These new representations are used to replace the γ_2 , δ_2 contributions in the previous calculations. They have the advantage over the previous functions in that there is no longer a kink in the vortex lines, as γ_2 now vanishes smoothly at the trailing edge.

The programs previously described in Section 4 have been modified to include these new vorticity modes, but time limitations have restricted the investigation with these new modes. After several preliminary runs were made for the unit aspect ratio delta wing to determine convergence and resolution factors, the following parameters were adopted as adequate to describe the bound vorticity modes. Five spanwise and five chordwise stations (NSPAN = 5, NCORD = 5) have been employed, and three chordwise modes (NOCM = 3) and four spanwise modes (NOSM = 4) have been selected.

The planform considered was that of the arrow wing employed by Brune, et al. (1975)⁸ to test their lifting surface theory program, based on finite element panels. The wing has a leading-edge sweep angle, $\lambda = 71.2^\circ$, an aspect ratio, AR = 2.02, and an angle of attack, $\alpha = 15.8^\circ$. Convergence for the leading-edge vortex strength for various numbers of no-force points (NOFP) and degrees of freedom in the vortex location (LMAX) are presented in Figure 24. In comparison with

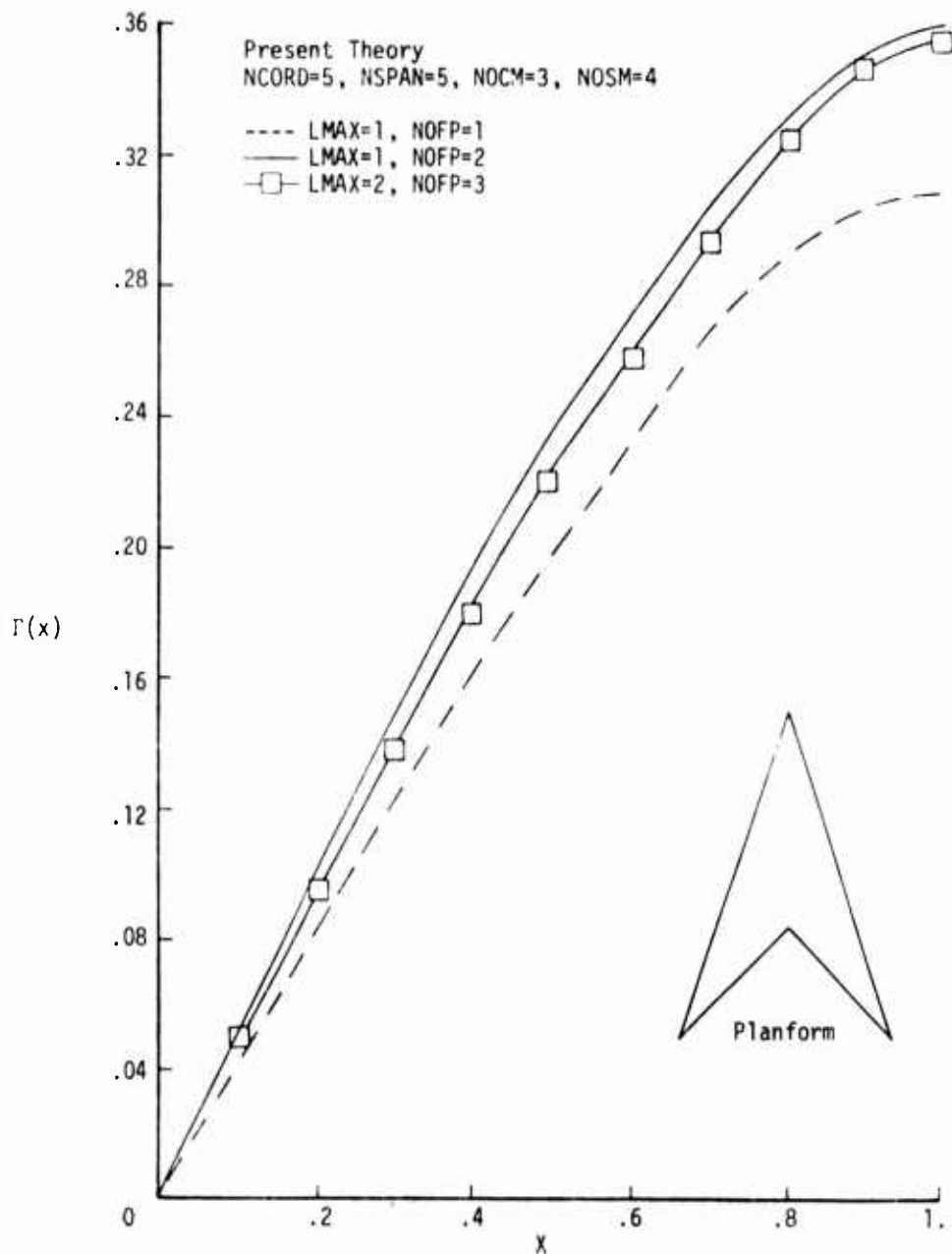


Figure 24. Convergence of leading-edge vortex strength for arrow wing ($AR=2.02$, $\lambda = 71.2^\circ$, $\alpha = 15.8^\circ$)

Figure 21, which provided results for an arrow wing using the earlier vorticity modes, it is apparent that the new vorticity modes result in much smoother convergence. This is true even though a larger trailing-edge sweep angle is being considered now than before. Again the modes have been chosen to provide no additional feeding of vorticity from the leading edge, aft of the trailing edge. Thus, the slope of the circulation strength of the leading-edge vortex vanishes at the trailing edge.

The variation of the vortex position over the right half of the wing is presented in Figure 25 as a function of the number of force points and degrees of freedom in the vortex location. The original forms for the vortex position modes (see Equation 8) have been used and the choice, $L_{MAX} = 1$, corresponds to a linear fit, while the selection, $L_{MAX}=2$, corresponds to a cubic approximation for the vortex position. The vortex position obtained by this numerical procedure appears quite stable even with these few no-force points. For the cubic approximation, the apparent tendency of the leading-edge vortex to align itself with the free stream direction near the trailing edge is noted.

Finally, the pressure distributions predicted by the present program are compared in Figure 26 with the results of Brune, et al. (1975)⁸ at two chordwise stations. Brune, et al. employed 30 wing panels, and 48 free-vortex-sheet panels - each vortex-sheet panel contributed two unknowns since both its strength and orientation were originally unknown - for a total of 126 unknowns. One station has been chosen forward of the

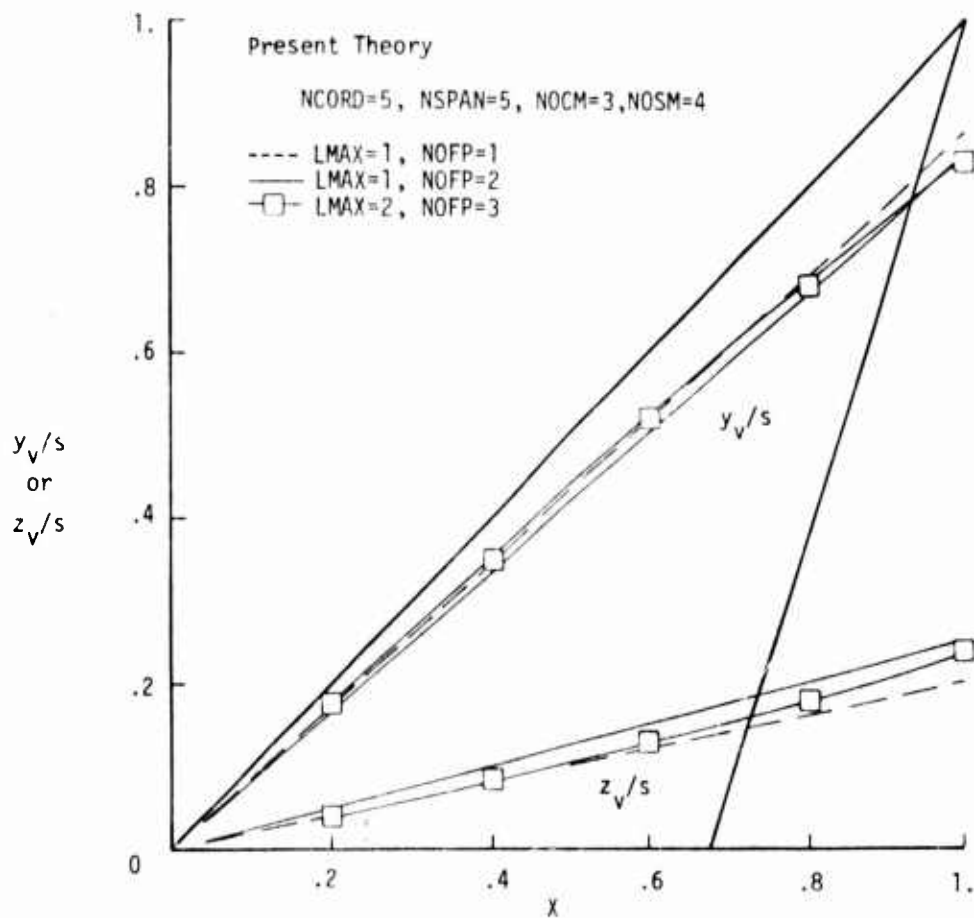


Figure 25. Leading-edge vortex position over right half of arrow wing ($AR=2.02$, $\lambda = 71.2^\circ$, $\alpha = 15.8^\circ$).

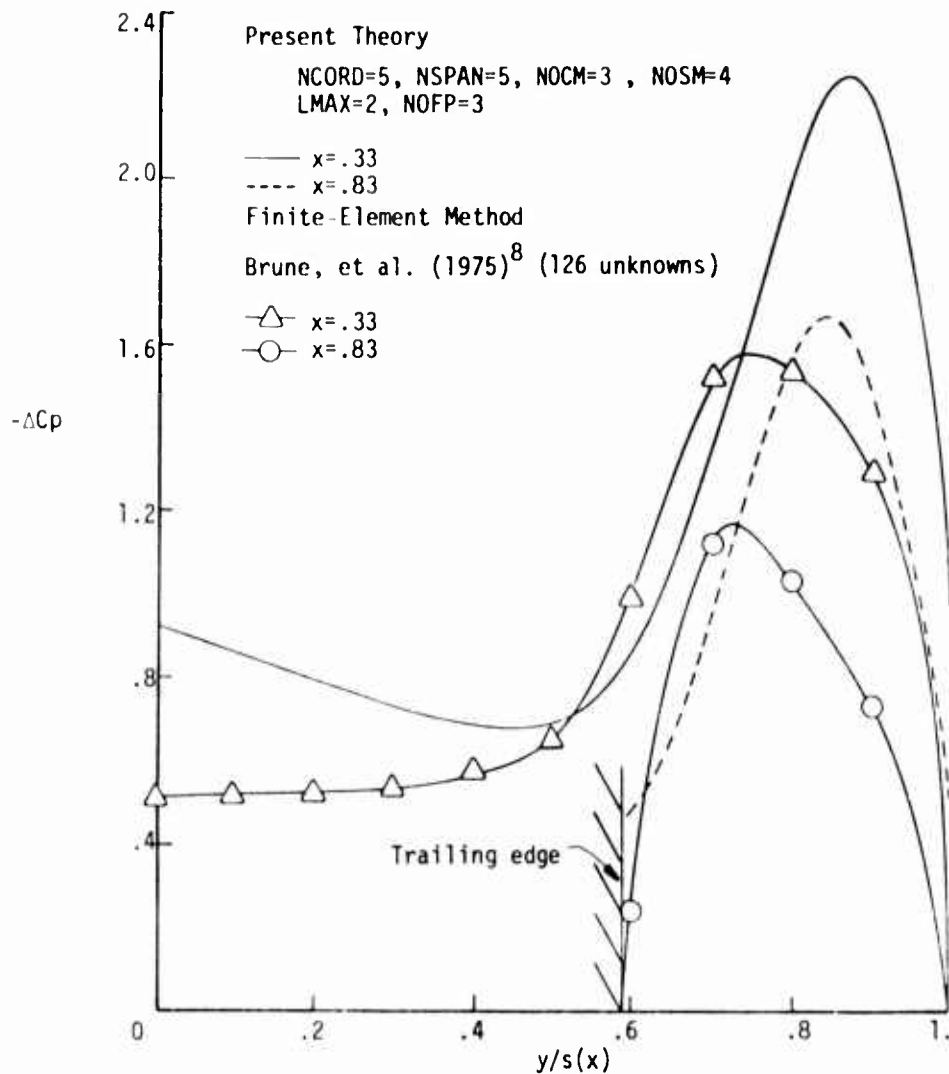


Figure 26. Comparison of theoretical loading models on arrow wing ($AR=2.02$, $\lambda=71.2^\circ$, $\alpha = 15.8^\circ$).

root chord and the other has been selected aft of the trailing edge intrusion to indicate the effect of the Kutta condition at the trailing edge. Again, the different theories predict similar pressure distributions at these two stations. However, the present theory appears to retain the limitations of the Brown and Michael vortex-cut approximation in that the pressure peaks are higher and further outboard than those predicted by the lifting surface theory program of Brune, et al. (1975)⁸ which utilizes a vortex-sheet representation. Also, the present loading predictions satisfy a modified Kutta condition at both the trailing and leading edges, and the pressure is not required to vanish at these points. This does not appear to be a serious problem, since the differences in the pressure distributions from zero contribute only slightly to the total loading on the wing due to the relatively large slopes in the pressure distributions near the wing edges.

This concludes the section on the revised vorticity modes. Preliminary results are promising, but limitations in the present procedure remain.

7. Conclusions and Recommendations

In conclusion, a lifting surface program based on the kernel function procedure has been developed to include leading-edge vortices. The present scheme can be generalized to arbitrary planforms and to include arbitrary sources of free vortices, but its use will probably be restricted by the computational effort.

With the present computer programs, results were first obtained for the delta wing of unit aspect ratio. Comparison with experiments indicate reasonable predictions of the loading with the inherent limitations that a Brown and Michael vortex-cut model imposes. It has been illustrated in the Introduction that a relatively small fraction of the vortex strength (less than 20 per cent) must be incorporated into the sheet to obtain the benefits of the Smith-type models for the slender-body problem.

Results were also obtained for the arrow wing to demonstrate the use of the program for more general planforms. These results emphasized the importance of a better representation of the Kutta condition at the trailing edge for such reentrant surfaces, than was originally employed. A simple bound vorticity model was first used to represent the vorticity feeding the leading-edge vortices, and this vorticity was discontinuously turned parallel to the free stream direction at the trailing edge. Convergence difficulties were encountered as one increased the sweep angle of the trailing edge, and consequently, an alternative bound vorticity distribution was developed to provide a smooth satisfaction of the linear Kutta condition at the trailing edge. Due to time limitations, only a few

runs were made with this revised model, but better convergence has been obtained for the arrow wing case at least. Furthermore, in deriving these new vorticity modes, a general procedure was developed which should provide bound vorticity modes for arbitrary planforms.

In general, the feasibility of the procedure has been demonstrated. Furthermore, an indication of the cause of previous convergence difficulties with programs which had attempted to satisfy the downwash and no-force conditions sequentially was presented, using the simpler slender-body representations. These results suggest that it is necessary to satisfy the boundary conditions simultaneously to obtain convergence.

Much work remains to be done to improve the usefulness of the present lifting surface program. First, it would be advantageous to further reduce the computational effort required to calculate the velocity contributions from the bound vorticity, which feeds the leading-edge vortex. Secondly, it seems that a more accurate prediction of the loading and the vortex position can be obtained by a more complete representation of the leading-edge vortex sheet. This would entail additional degrees of freedom in the orientation of the vorticity leaving at the leading edge. An additional no-force boundary condition on these elements would have to be imposed to determine their orientation. For some purposes, the present vortex-cut model may be adequate, if the results are used in conjunction with slender-body theory corrections. For example, one can use the present procedure to calculate the leading-edge vortex location with the limitation that although the vertical position will be accurate, the spanwise position will, in reality, be further inboard.

Another field of interest would be the application of the lifting surface program to wings of higher aspect ratios. Recently, Nathman, Norton, and Rao(1976)²⁰ have published pressure distributions for less slender delta wings with aspect ratios of three and four and for some related double-delta planforms. One difficulty with such wings is that vortex bursting occurs over the wing at lower angles of attack as the apex angle of the delta wing is increased. Also, at higher angles of attack, the vortex core is not well defined and is replaced by a turbulent core of vorticity.

Additional effort may still be required to model the no-load condition on the trailing vortex sheet. Presently, only the linear, but not the nonlinear, no-load condition is being satisfied on the wake. This does not appear to be too serious in light of the results of Brune, et al. (1975)⁸ and Kandil, et al. (1974)⁷, which indicate that this is a fair representation of the wake. Finally, additional work still needs to be done to develop the new set of vorticity modes presented in this report for other planforms. Questions of resolution and convergence for this modal method remain to be answered, although significant progress has been made for the delta and arrow wing planforms.

The above extensions have been suggested by the present investigation, and their successful implementation would greatly enhance the versatility of this three-dimensional lifting surface program which includes leading-edge vortices.

REFERENCES

1. Matoi, T.K., "On the Development of a Unified Theory for Vortex Flow Phenomena for Aeronautical Applications," M.I.T. Report, AD No. A012399, 1975.
2. Smith, J.H.B., "A Review of Separation in Steady, Three-Dimensional Flow," AGARD Conference Proceedings No. 168 on Flow Separation, pp 31-1 to 31-17, 1975.
3. Ornberg, T., "A Note on the Flow around Delta Wings," KTH Aero TN 38, R.I.T., 1954.
4. Brown, C.E., and Michael, W.H., "On Slender Delta Wings with Leading-Edge Separation," NACA TN 3430, 1955.
5. Smith, J.H.B., "Improved Calculations of Leading-Edge Separation from Slender Delta Wings," RAE TR 66070, 1966.
6. Pullin, D.I., "A Method for Calculating Inviscid Separated Flow about Conical Slender Bodies," ARL/A14, Australian Defense Scientific Service, Rep. 140, 1973.
7. Kandil, O.A., Mook, D.T., and Nayfeh, A.H., "Nonlinear Prediction of the Aerodynamic Loads on Lifting Surfaces," AIAA Paper No. 74-503, 1974.
8. Brune, G.W., Weber, J.A., Johnson, F.T., Lu, P., and Rubbert, P.E., "A Three-Dimensional Solution of Flow over Wings with Leading-Edge Separation; Part I: Engineering Document," NASA CR-132709, 1975.
9. Matoi, T.K., Covert, E.E., and Widnall, S.E., "A Three-Dimensional Lifting Surface Theory with Leading-Edge Vortices," Report ONR-CR215-230-2, 1975.
10. Nangia, R.K., and Hancock, G.J., "A Theoretical Investigation for Delta Wings with Leading-Edge Separation at Low Speeds," ARC CP No. 1086, 1968.
11. Ashley, H. and Landahl, M., Aerodynamics of Wings and Bodies, Addison-Wesley Pub. Co., 1965.
12. Hsu, P-T., "Flutter of Low-Aspect-Ratio Wings; Part I: Calculation of Pressure Distributions for Oscillating Wings of Arbitrary Planforms in Subsonic Flow by the Kernel Function Method," M.I.T. ASRL TR 64-1, 1957.

REFERENCES (continued)

13. Widnall, S.E., "Unsteady Loads on Hydrofoils including Free Surface Effects and Cavitation," M.I.T. Sc.D. Thesis, 1964.
14. Watkins, C.E., Runyan, H.L., and Woolston, D.S., "A Systematic Kernel Function Procedure for Determining Aerodynamic Forces on Oscillating or Steady Finite Wings at Subsonic Speeds," NASA TR R-48, 1959.
15. Peckham, D.H., "Low-Speed Wind-Tunnel Tests on a Series of Uncambered Slender Pointed Wings with Sharp Edges," ARC R&M No. 3186, 1958.
16. Nangia, R.K., and Hancock, G.J., "Delta Wings with Longitudinal Camber at Low Speed," ARC CP No. 1129, 1969.
17. Smith, J.H.B., "Calculations of the Flow over Thick, Conical, Slender Wings with Leading-Edge Separation," ARC R&M No. 3694, 1971.
18. Kandil, O.A., Mook, D.T., and Nayfeh, A.H., "New Convergence Criterion for the Vortex Lattice Models of the Leading-Edge Separation," in Vortex-Lattice Utilization, NASA SP-405, pp 285-300, 1976.
19. Weber, J.A., Brune, G.W., Johnson, F.T., Lu, P., and Rubbert, P.E., "A Three-Dimensional Solution of Flows over Wings with Leading-Edge-Vortex Separation," NASA SP347, pp 1013-1032, 1975.
20. Nathman, J.K., Norton, D.J., and Rao, B.M., "An Experimental Investigation of Incompressible Flow over Delta and Double-Delta Wings," Report ONR-CR215-231-3, 1976.

APPENDIX A

Evaluation of Upwash Integrals

The second and third integrals in Equation 13 will be evaluated explicitly here for the arrow wing configuration. For arbitrary configurations, one integration will be easy to perform, but the second integration may then have to be performed numerically. This should not present any difficulty, as the singularity will appear as a Cauchy Principal Value, which can be handled by a variety of techniques.

The second integral in Equation 13 becomes

$$\begin{aligned}
 B &\equiv \frac{1}{4\pi} \oint \oint_{SW} \frac{(x'-x) dx' dy'}{[(x-x')^2 + (y-y')^2]^{3/2}} \\
 &= \frac{1}{4\pi} \int_{-s}^s \int_{\frac{y'}{s}}^{\frac{y'+s(1-c_r)}{s} + c_r} \frac{(x'-x) dx' dy'}{[(x-x')^2 + (y-y')^2]^{3/2}} \quad (A.1)
 \end{aligned}$$

where s is the semispan and c_r is the root chord, nondimensionalized by the chordwise length. Carrying out the integration and retaining only the finite part of the integral yields

$$B = \frac{1}{4\pi} \left[\frac{s}{\sqrt{s^2 + (1-c_r)^2}} \left[\sinh^{-1} \frac{(1-c_r)(c_r-x) + ys}{|y(1-c_r) - s(c_r-x)|} \right] \right]$$

$$\begin{aligned}
& - \sinh^{-1} \frac{(1-c_r)(1-x) + s(s+y)}{|y(1-c_r) - s(c_r-x)|} - \sinh^{-1} \frac{(1-c_r)(1-x) + s(s-y)}{y(1-c_r) + s(c_r-x)} \\
& + \sinh^{-1} \frac{(1-c_r)(c_r-x) - sy}{y(1-c_r) + s(c_r-x)} \left] + \frac{s}{\sqrt{s^2+1}} \left[\sinh^{-1} \frac{x-sy}{y+sx} \right. \right. \\
& \left. \left. + \sinh^{-1} \frac{s(s+y) + (1-x)}{y+sx} + \sinh^{-1} \frac{s(s-y) + (1-x)}{sx-y} + \sinh^{-1} \frac{x+ys}{sx-y} \right] \right\} \quad (A.2)
\end{aligned}$$

The remaining integral can be evaluated similarly.

$$C = + \frac{1}{4\pi} \iint_{SW} \frac{(y-y') dx' dy'}{[(x-x')^2 + (y-y')^2]^{3/2}} \quad (A.3)$$

$$\begin{aligned}
C = & - \frac{1}{4\pi} \left\{ \frac{1-c_r}{\sqrt{(1-c_r)^2 + s^2}} \left[\sinh^{-1} \frac{sy + (1-c_r)(c_r-x)}{|-y(1-c_r) + s(c_r-x)|} \right. \right. \\
& - \sinh^{-1} \frac{s(s+y) + (1-c_r)(1-x)}{|-y(1-c_r) + s(c_r-x)|} + \sinh^{-1} \frac{s(s-y) + (1-c_r)(1-x)}{y(1-c_r) + s(c_r-x)} \\
& - \sinh^{-1} \frac{(1-c_r)(c_r-x) - sy}{y(1-c_r) + s(c_r-x)} \left. \right] + \frac{1}{\sqrt{1+s^2}} \left[\sinh^{-1} \frac{x-sy}{y+sx} \right. \\
& \left. + \sinh^{-1} \frac{(1-x) + s(s+y)}{y+sx} - \sinh^{-1} \frac{(1-x) + s(s-y)}{sx-y} - \sinh^{-1} \frac{sy+x}{sx-y} \right] \right\} \quad (A.4)
\end{aligned}$$

These results reduce to those for the delta wing case when $c_r = 1$, and are then equivalent with results obtained by Nangia and Hancock (1968)¹⁰, within a few sign errors which appear in their report.

APPENDIX B

Newton's Method for the Slender-Body Problem

This appendix expands the description of the use of Newton's method for the slender-body problem provided in the section on the Numerical Procedure. Originally, Newton's method was used solely to determine the vortex location. Later, it was employed to determine the vorticity distribution on the wing as well.

The flat plate delta wing problem under the restrictions of slender-body theory and conical flow was solved by Brown and Michael (1955)⁴. This problem can be formulated in the complex plane, $\omega = y + iz$ (see Figure B.1) as the complex potential W , due to a flat plate perpendicular to the flow and a pair of vortices.

$$W(\omega) = \frac{-i\Gamma}{2\pi} \ln \frac{\sqrt{\omega^2-1} - \sqrt{\omega_1^2-1}}{\sqrt{\omega^2-1} + \sqrt{\omega_1^2-1}} - i\alpha\sqrt{\omega^2-1} \quad (B.1)$$

where all quantities have been nondimensionalized. ω_1 represents the complex vortex location, $\omega_1 = y_v + iz_v$, and $\bar{\omega}_1$ represents the complex conjugate of ω_1 . Γ is the vortex strength and α is the angle of attack. The Kutta condition of finite velocity at the leading edge can be written as

$$\frac{2\pi\alpha}{\Gamma} = \frac{1}{\sqrt{\omega_1^2-1}} + \frac{1}{\sqrt{\bar{\omega}_1^2-1}} \quad (B.2)$$

This equation can be used to calculate the circulation strength, Γ , in terms of the vortex location. The forces on the vortex-cut combination in the complex plane are

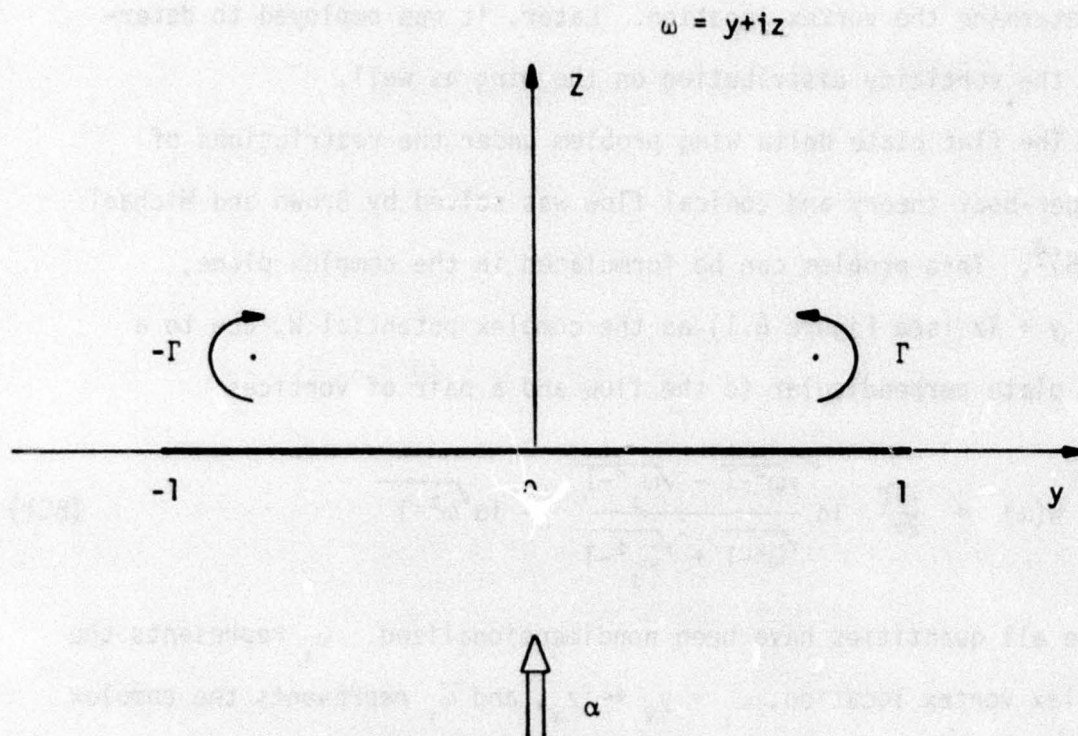


Figure B.1. Coordinate system for Brown and Michael slender-body delta wing problem.

$$F \equiv iF_y + F_z = -\lim_{\omega \rightarrow \omega_1} \left\{ \frac{dW}{d\omega} - \frac{\Gamma}{2\pi i} \frac{1}{\omega - \omega_1} \right\} + 2\bar{\omega}_1 - 1 = 0 \quad (B.3)$$

The details of these derivations can be found in the original paper by Brown and Michael (1955)⁴.

Since the potential specified in Equation B.1 automatically satisfies the downwash condition on the wing, the no-force condition (Equation B.3) provides the two real equations needed to determine the complex vortex position, $\omega_1 = y_v + iz_v$. Unfortunately, Equation B.3 is nonlinear in the vortex position variables; so a Newton's procedure was developed by Pullin (1973)⁶ to solve this problem. A Newton's method is based on a linear extrapolation from some initial approximate solution and can be written in the following manner for this problem.

$$\begin{bmatrix} \Delta y_v \\ \Delta z_v \end{bmatrix} = \begin{bmatrix} \frac{\partial F_y}{\partial y_v} & \frac{\partial F_y}{\partial z_v} \\ \frac{\partial F_z}{\partial y_v} & \frac{\partial F_z}{\partial z_v} \end{bmatrix}^{-1} \begin{bmatrix} -F_y \\ -F_z \end{bmatrix} \quad (B.4)$$

This equation gives an automatic procedure for obtaining an improved solution for the vortex location, if the residues, F_y and F_z , and the Jacobian matrix from the previous iteration are provided. The new vortex location is obtained from

$$\begin{aligned} y_v(\text{new}) &= y_v(\text{old}) + \Delta y_v \\ z_v(\text{new}) &= z_v(\text{old}) + \Delta z_v \end{aligned} \quad (B.5)$$

The derivatives for Equation B.4 can be obtained from

$$\begin{aligned}
\frac{\partial F_y}{\partial y_v} &= \text{Imag} \left[\frac{\partial F}{\partial y_v} \right] \\
\frac{\partial F_z}{\partial y_v} &= \text{Real} \left[\frac{\partial F}{\partial y_v} \right] \\
\frac{\partial F_y}{\partial z_v} &= \text{Imag} \left[\frac{\partial F}{\partial z_v} \right] \\
\frac{\partial F_z}{\partial z_v} &= \text{Real} \left[\frac{\partial F}{\partial z_v} \right]
\end{aligned} \tag{B.6}$$

where

$$\begin{aligned}
\frac{\partial F}{\partial y_v} &= \frac{\partial F}{\partial \omega_1} + \frac{\partial F}{\partial \bar{\omega}_1} \\
\frac{\partial F}{\partial z_v} &= i \left(\frac{\partial F}{\partial \omega_1} - \frac{\partial F}{\partial \bar{\omega}_1} \right)
\end{aligned}$$

This procedure provides convergence to the stable configuration in approximately three iterations if the initial approximation is within 10 per cent of the semispan of the final position. Unfortunately, the three-dimensional problem is more complicated than this, and some unexplained difficulties were encountered when a Newton's procedure was developed to find the vortex location in the lifting surface problem. In an effort to determine the cause of the difficulties, the slender-body problem was developed in a manner analogous to the three dimensional one.

The problem was formulated in the physical y, z plane and the wing was replaced by its bound vorticity representation. The downwash condition was no longer automatically satisfied and could be written as

$$\frac{1}{2\pi} \int_{-1}^1 \frac{\delta(y') dy'}{y - y'} + \alpha + \frac{\Gamma}{2\pi} \text{Real} \left[\frac{1}{y - \omega_1} - \frac{1}{y + \bar{\omega}_1} \right] = 0 \quad (\text{B.7})$$

This equation can be rearranged to yield

$$-\frac{1}{2\pi} \int_{-1}^1 \frac{\delta(y') dy'}{y - y'} = \frac{\Gamma}{2\pi} \text{Real} \left[\frac{1}{y - \omega_1} - \frac{1}{y + \bar{\omega}_1} \right] + \alpha \equiv f(y) \quad (\text{B.8})$$

This was inverted analytically to provide a check for the numerical procedure being developed. The inversion of Equation B.8 yields

$$\delta(y) = \frac{2}{\pi} \sqrt{1-y^2} \int_{-1}^1 \frac{f(y')}{y - y'} \frac{1}{\sqrt{1-y'^2}} dy' \quad (\text{B.9})$$

where $\delta(y)$ is the vorticity distribution on the wing and is equivalent to the difference in the spanwise velocity on the upper and lower surfaces.

For the choice of loading modes,

$$\delta(y) = \sum_{n=1}^N a_n \sqrt{1-y^2} y^{2n-1} \quad (\text{B.10})$$

the integral in Equation B.7 can be done analytically and the unknown a_n 's can be obtained from a simple matrix inversion by choosing more collocation points at which the downwash condition is satisfied on the wing than the number of unknown modal coefficients, once a vortex position has been assumed.

Now the Kutta condition at the leading edge is automatically satisfied by the loading functions. The forces on the vortex become

$$iF_y + F_z = i\alpha + \frac{i\Gamma}{2\pi} \left\{ \int_{-1}^1 \frac{1}{\omega_1 - y'} \frac{\delta/\Gamma dy'}{\omega_1 + \bar{\omega}_1} - \frac{1}{\omega_1 + \bar{\omega}_1} \right\} + 2\bar{\omega}_1 - 1 \quad (B.11)$$

Originally, an attempt was made to satisfy the downwash condition (Equation B.7) and the no-force condition (Equation B.11) sequentially in the manner of Nangia and Hancock (1968)¹⁰. An initial vortex position was assumed and the downwash condition was applied at enough points to find an initial vorticity distribution provided by the a_n 's and Γ . This distribution was then introduced into Equation B.11 which could be used in conjunction with Equation B.4 to obtain a better approximation for the vortex position. After the no-force condition was satisfied by moving the vortices, the downwash condition was no longer satisfied. Thus, the procedure sequentially updated the vorticity coefficients and the vortex position in an effort to satisfy both the downwash and the no-force conditions. However, as mentioned in the section on the Numerical Procedure, this scheme failed to converge as the procedure oscillated between the true solution and a false solution where the forces vanished, but where the downwash condition was not satisfied. As a result, convergence was not obtained.

Therefore, the decision was made to attempt to satisfy the downwash condition and the no-force condition simultaneously by a Newton's procedure. One obtains the following iteration scheme to update the initial approximation.

$$\begin{bmatrix} \Delta A \\ \Delta l' \\ \Delta y_v \\ \Delta z_v \end{bmatrix} = \begin{bmatrix} \frac{\partial w}{\partial A} & \frac{\partial w}{\partial l'} & \frac{\partial w}{\partial y_v} & \frac{\partial w}{\partial z_v} \\ \frac{\partial F}{\partial A} & \frac{\partial F}{\partial l'} & \frac{\partial F}{\partial y_v} & \frac{\partial F}{\partial z_v} \end{bmatrix}^{-1} \begin{bmatrix} -w \\ -F \end{bmatrix} \quad (\text{B.12})$$

where

$$A = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

$$W = \begin{bmatrix} w_1 \\ \vdots \\ w_{n+1} \end{bmatrix}$$

$$F = \begin{bmatrix} F_y \\ F_z \end{bmatrix}$$

This scheme resulted in convergence for the sample case being considered of the unit aspect ratio delta wing in approximately eight iterations for an initial location of the vortex within 10 per cent of the semispan of the final solution. Details of the rate of convergence for this problem were presented in Figure 12.

Due to the success of this procedure in the slender-body problem, a Newton's method has been developed for the lifting surface problem. However, success is not guaranteed as the three-dimensional problem involves a great many more variables than the slender-body problem. This additional complexity will result in slower convergence rates and may cause additional difficulties as well.

APPENDIX C

Listing of FORTRAN Programs

This Appendix consists of the primary programs described in the report. The listings are documented by comment cards. For additional details, see the section titled "Program Description" in this report.

All coding is in FORTRAN IV and the programs were run on the IBM 370/168 at M.I.T. Approximately 20 iterations of Program V can be performed in one minute of CPU time for the following choice of parameters: three no-force points (NOFP = 3), two degrees of freedom in the vortex position (LMAX = 2), five chordwise and five spanwise collocation stations (NCORD = 5 , NSPAN = 5), four chordwise modes (NOCM = 4), and five spanwise modes (NOSM = 5). The choice of these parameters should be dictated by adequate resolution in the final answer.

Duplicate subroutines have not been listed. Duplicate subroutines are generally listed with Program V. The exceptions are the function subprograms B and XLE for Program IIIA which are listed with Program I.

C.1. Program I

The following listing for Program I includes Program I and the subprograms ASINH, A2, A4, B, B1, B2, B6, IGWW, and XINTGR.

Program I calculates the downwash coefficients due to the bound vorticity which feeds the leading-edge vortices. The output from Program 2 is used by Program IIIA to calculate the initial approximation for the vorticity distribution and is used by Program V to calculate the downwash residue on the wing.

C	PROGRAM 1	PGM10001
C		PGM10002
C	CALCULATES DOWNWASH COEFFICIENTS DUE TO BOUND VORTICITY WHICH FEEDS	PGM10003
C	LEADING-EDGE VORTICITIES	PGM10004
C		PGM10005
C	INPUT NCORD, NSPAN, S, CR 2110 2F10.4	PGM10006
C	INPUT NOCM, J1 2110	PGM10007
C		PGM10008
C	PRIMARY OUTPUT GW 5F14.5	PGM10009
C		PGM10010
C	NEED FUNCTIONS A1, A2, A4, A5, B1, B2, B5, B6, GVORT, ASINH, B, XLE	PGM10011
C	NEED SUBROUTINES BLOCK DATA, COLPT, IGW, XINTGR	PGM10012
C		PGM10013
C	DIMENSION XPT(5), YPT(5), COEFF(25,5),	PGM10014
C	CSGW1(5), SGW2(5), SGW3(5), SGW4(5), SGW5(5), SGW6(5)	PGM10015
C	COMMON XPI, YPJ, S, M, MP/PLAN/CR /GAUS/G(24), W(24)/MODES/NOCM	PGM10016
C	EXTERNAL A1, A2, A4, A5, B1, B2, B5, A6, B6	PGM10017
C		PGM10018
C	INITIALIZE VARIABLES	PGM10019
C	DATA COEFF/125*0./	PGM10020
C	DO 150 JDOUMMY = 1, 24	PGM10021
C	W(JDOUMMY) = W(25 - JDOUMMY)	PGM10022
C	150 G(JDOUMMY) = -G(25 - JDOUMMY)	PGM10023
C	PI = 3.141593	PGM10024
C	WRITE(6, 910)	PGM10025
C		PGM10026
C	READ(5, 920) NCORD, NSPAN, S, CR	PGM10027
C		PGM10028
C	NCORD = NO. OF CHORDWISE COLLOCATION POINTS	PGM10029
C	NSPAN = NO. OF SPANWISE COLLOCATION POINTS	PGM10030
C	S = SEMISPAN; NON-D BY MAXIMUM LENGTH	PGM10031
C	CR = ROOT CHORD; NON-D BY MAXIMUM LENGTH	PGM10032
C		PGM10033
C	READ(5, 920) NOCM, J1	PGM10034
C		PGM10035
C	NOCM = NO. OF CHORDWISE MODES	PGM10036
C		
C	J1 IS CONTROL PARAMETER: IF J1=1, NCORD2=NCORD; ELSE NCORD2=NCORD+1	PGM10037
C	WRITE(6, 970) NCORD, NSPAN, S, NOCM, CR	PGM10038
C		PGM10039
C	CALCULATE LOCATION OF COLLOCATION POINTS	PGM10040
C	CALL COLPT(NCORD, NSPAN, XPT, YPT)	PGM10041
C	IF (J1.EQ.1) GO TO 300	PGM10042
C	NCORD2=NCORD+1	PGM10043
C	XPT(NCORD2)=(XPT(NCORD)+XPT(NCORD-1))/2.	PGM10044
C	300 CONTINUE	PGM10045
C	IF (J1.EQ.1) NCORD2=NCORD	PGM10046
C	WRITE(6, 930)	PGM10047
C		PGM10048
C	DEFINE LIMITS OF INTEGRATION IN SPANWISE DIRECTION	PGM10049
C	C'S REFER TO LEFT-HAND SIDE OF RESPECTIVE REGION	PGM10050
C	D'S REFER TO RIGHT-HAND SIDE OF RESPECTIVE REGION	PGM10051
C	C1 = 0.	PGM10052
C	D5 = S	PGM10053
C	C6 = -S	PGM10054
C	D6 = 0.	PGM10055
C		PGM10056
C	CALCULATE COEFFICIENTS AT EACH COLLOCATION POINT	PGM10057
C	DO 400 I=1, NCORD2	PGM10058
C	DO 400 J=1, NSPAN	PGM10059
C	N1=J*(I-1)*NSPAN	PGM10060
C	XPI=XLE(YPT(I))+B(YPT(J))*XPT(I)	PGM10061
C	YPJ=YPT(J)*S	PGM10062
C		PGM10063
C	OUTPUT LOCATION OF COLLOCATION POINTS	PGM10064
C	WRITE(6, 940) N1, XPT(I), YPT(J), XPI	PGM10065
C	ETA = .02	PGM10066
C	IF ((XPI-.02).LT..02) ETA = 1.-XPI	PGM10067
C	D1 = S*(XPI-.02)	PGM10068
C	C2 = 0.	PGM10069
C	IF ((XPI-.02).GT.CR) C2 = S*((XPI-.02)-CR)/(1.-CR)	PGM10070
C	D2 = YPJ-.02	PGM10071
C	C3 = YPJ-.02	PGM10072

	FUNCTION ASINH(Z)	ASIN0001
C		ASIN0002
C	ASINH(Z) CALCULATES INVERSE HYPERBOLIC SINE	ASIN0003
C		ASIN0004
	IF (Z.LT.-10.) GO TO 20	ASIN0005
	ASINH = ALOG(Z+SQRT(1.+Z*Z))	ASIN0006
	RETURN	ASIN0007
C	USE EXPANSION FORM FOR ASINH FOR LARGE NEGATIVE VALUES OF Z	ASIN0008
	20 ASINH = ALOG((1./(4.*Z*Z)-1.)/Z) -.693147	ASIN0009
	RETURN	ASIN0010
	END	ASIN0011

	FUNCTION A2(Y)	0001
C		0002
C	A2(Y) PROVIDES INITIAL INTEGRATION POINT IN X DIRECTION IN REGION 2	0003
C	ARROW WING CONFIGURATION	0004
C		0005
C	ARGUMENT LIST	0006
C	Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	0007
C		0008
	COMMON XPT,YPT,S,M,N	0009
	A2 = XPT-.02	0010
	RETURN	0011
	END	0012
C		0013
C		0014
C		0015
	FUNCTION A4(Y)	0016
C		0017
C	A4(Y) PROVIDES INITIAL INTEGRATION POINT IN X DIRECTION IN REGION 4	0018
C	ARROW WING CONFIGURATION	0019
C		0020
C	ARGUMENT LIST	0021
C	Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	0022
C		0023
	COMMON XPT,YPT,S,M,N	0024
	IF (Y-S*(XPT-.02)) 10,10,20	0025
10	A4 = XPT-.02	0026
	RETURN	0027
20	A4 = Y/S	0028
	RETURN	0029
	END	0030

	FUNCTION BIS)	0001
C		0002
C	BIS) CALCULATES LOCAL CHORD; NON-D BY MAXIMUM LENGTH	0003
C	ARROW WING CONFIGURATION	0004
C		0005
C	ARGUMENT LIST	0006
C	S: SPANWISE COORDINATE; NON-D BY SEMISPAN	0007
C		0008
C	COMMON /PLAN/ CR	0009
C	B = CR*(1.-S)	0010
C	RETURN	0011
C	END	0012
C		0013
C		0014
C	FUNCTION XLE(S)	0015
C		0016
C	XLE(S) CALCULATES LOCATION OF LEADING EDGE; NON-D BY MAXIMUM LENGTH	0017
C	ARROW WING CONFIGURATION	0018
C		0019
C	ARGUMENT LIST	0020
C	S: SPANWISE COORDINATE; NON-D BY SEMISPAN	0021
C		0022
C		0023
C	XLE = S	0024
C	RETURN	0025
C	END	0026

	FUNCTION B1(Y)	0001
C		0002
C	B1(Y) PROVIDES FINAL INTEGRATION POINT IN X DIRECTION IN REGION 1	0003
C	ARROW WING CONFIGURATION	0004
C		0005
C	ARGUMENT LIST	0006
C	Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	0007
C		0008
C	COMMON XPT,YPT,S,M,N /PLAN/CR	0009
C	B = Y*(1.-CR)/S + CR	0010
C	IF (B.GT.(XPT-.02)) B=XPT-.02	0011
C	B1 = B	0012
C	RETURN	0013
C	END	0014
C		0015
C		0016
C	FUNCTION B2(Y)	0017
C		0018
C	B2(Y) PROVIDES FINAL INTEGRATION POINT IN X DIRECTION IN REGION 2	0019
C	ARROW WING CONFIGURATION	0020
C		0021
C	ARGUMENT LIST	0022
C	Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	0023
C		0024
C		0025
C	COMMON XPT,YPT,S,M,N /PLAN/CR,ETA	0026
C	B = Y*(1.-CR)/S + CR	0027
C	IF (B.GT.(XPT+.02)) B = XPT+.02	0028
C	IF (B.GT.1.) B=1.	0029
C	B2 = B	0030
C	RETURN	0031
C	END	0032
C		0033

C.....	0034
C	0035
C	0036
C	0037
C	0038
C	0039
C	0040
C	0041
C	0042
C	0043
C	0044
C	0045
C	0046
C	0047

```

      FUNCTION B6(Y)
      B6(Y) PROVIDES FINAL INTEGRATION POINT IN X DIRECTION IN REGION 6
      ARROW WING CONFIGURATION
      ARGUMENT LIST
      Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH
      COMMON XPT,YPT,S,M,N /PLAN/CR
      B6 = -Y*(1.-CR)/S + CR
      RETURN
      END

```

C	IGWW(C,D,A,B,SGWW)	IGWW0001
C	IGWW CALCULATES DOWNWASH INTEGRAL	IGWW0002
C		IGWW0003
C	ARGUMENT LIST	IGWW0004
C	C: LOWER LIMIT OF INTEGRAL	IGWW0005
C	D: UPPER LIMIT OF INTEGRAL	IGWW0006
C	A: FUNCTION DESCRIBING LOWER LIMIT OF INTEGRAL	IGWW0007
C	B: FUNCTION DESCRIBING UPPER LIMIT OF INTEGRAL	IGWW0008
C	SGWW: INTEGRALS	IGWW0009
C		IGWW0010
C	COMMON XPT,YPT,S,M,DUM,MPOUM	IGWW0011
C	COMMON/GAUS/G(24),W(24)/MODES/NOCM	IGWW0012
C	DIMENSION SGWW(5),ENTGD(5),SUM(5),GVORS(5)	IGWW0013
C		IGWW0014
C	ENTGD(X,Y)=(XDIFF*(X*GVOR2-XPT*GVOR1 1+ CYDIFF*(1-Y*GVOR2+YPT*GVOR1 1)/ATHIRD	IGWW0015
C		IGWW0016
C	PI=3.141593	IGWW0017
C	Z=1.E-9	IGWW0018
C		IGWW0019
C	INITIALIZE SUMMATIONS	IGWW0020
C	DO 10 MQ=1,NOCM	IGWW0021
C	M=MQ-1	IGWW0022
C	ENTGD(MQ)=0.0	IGWW0023
C	SUM(MQ)=0.0	IGWW0024
C	GVORS(MQ)=GVORT(M,XPT,YPT,S)	IGWW0025
C	10 CONTINUE	IGWW0026
C		IGWW0027
C		IGWW0028
C	DO SPANWISE INTEGRAL	IGWW0029
C	DO 200 J=1,24	IGWW0030
C	Y=(1D-C)*G(J1+D+C)/2.	IGWW0031
C	BY=B(Y)	IGWW0032
C	AY=A(Y)	IGWW0033
C	AP=BY-AY	IGWW0034
C	BP=BY+AY	IGWW0035
C		IGWW0036

C.2. Program WOW

The following listing for Program WOW includes Program WOW and the subprograms CHDWS, KERNEL, MNGLR, and PLOT.

Program WOW calculates the downwash coefficients due to the horseshoe vortices on the wing. The output from Program WOW is used by Program IIIA to calculate the initial approximation for the vorticity distribution and is used by Program V to calculate the downwash residue on the wing.

```

C
C
C      PROGRAM WOV
C
C CALCULATES COEFFICIENTS OF CHOSEN MODES OF THE VORTICITY DISTRIBUTION
C AS A NUMERICAL SOLUTION OF THE INTEGRAL EQUATION RELATING
C THE STRENGTH OF HORSTSHOE VORTICES AND DOWNWASH
C
C      INPUT  NOST,N1(1),SPAN      215  F10.4
C      INPUT  NOCP,NOLT,NCP,MP,J1,J2,CSR      415,215,F14.5
C      INPUT  XOC,SOS,ETA,N1(2),N1(4)      3F10.4  3I2
C
C      PRIMARY OUTPUT DWR 5E14.5
C
C      NEED SUBPROGRAMS B,XLE,CHDWS,FUNCTN,KERNL,MNGLR,PLOT,BLOCK DATA
C
C      DIMENSION N1(4),XVECT(25),YVECT(25),S(4),W(4),DWR(25,25),F(5)
C      COMMON AR(4,5,5),ALS(5,10),CR(5),TKR(10),XOC,SOS,Y,Z,
C      YMN,ZMZ,RSQR,ETA,GAUSS(10),PDZ,NOLT,NCP,MP,N,X,GZ,
C      J1,J2,GS,YMN2,ZMZ,CSR /PLAN/CXMAX
C      COMMON /GAUN/GN(10,10),WN(10,10) /MODES/NOST,NINC
C
C INITIALIZE VARIABLES
C      DATA S(3),S(4),W(3)/-1.0,0.,1.0/
C      PDZ=1.5707963
C      NINC=-1
C
C      READ 100, NOST,N1(1),SPAN
C
C      NOST =NO. OF SPANWISE LOADING MODES
C      N1(1) =NO. OF LEGENDRE-GAUSS POINTS IN SPANWISE INTEGRATION
C      IN REGION J
C      SPAN =SPAN
C      JS=4
C
C      JS=NO. OF INTEGRATION REGIONS IN SPANWISE DIRECTION
C
C      5 READ 501, NOCP,NOLT,NCP,MP,J1,J2, CSR

```

```

PW0W0001
PW0W0002
PW0W0003
PW0W0004
PW0W0005
PW0W0006
PW0W0007
PW0W0008
PW0W0009
PW0W0010
PW0W0011
PW0W0012
PW0W0013
PW0W0014
PW0W0015
PW0W0016
PW0W0017
PW0W0018
PW0W0019
PW0W0020
PW0W0021
PW0W0022
PW0W0023
PW0W0024
PW0W0025
PW0W0026
PW0W0027
PW0W0028
PW0W0029
PW0W0030
PW0W0031
PW0W0032
PW0W0033
PW0W0034
PW0W0035
PW0W0036

```

```

C
C      NOCP =NO. OF COLLOCATION POINTS IN HALF WING
C      NOLT =NO. OF CHORDWISE LOADING MODES
C      MP =NO. OF CHORDWISE LEGENDRE-GAUSS QUADRATURE POINTS IN MNGLR
C      NCP =NO. OF CHORDWISE LEGENDRE-GAUSS QUADRATURE POINTS IN CHDWS
C      J1,J2 CONTROL OUTPUT; NORMALLY 0
C      CSR =CHORD TO SPAN RATIO
C      CXMAX = CSK*SPAN
C      WRITE(6,930)
C      NMODE=NOST*NOLT
C      WRITE(6,911) SPAN, NOLT,NOST,NOCP,CSR
C      WRITE(6,909) N1(3),NCP,4P
C
C CALCULATE COEFFICIENTS AT THE COLLOCATION POINTS
C DO 90 L=1,NOCP
C
C      READ 6,XOC,SOS,ETA,N,N1(2),N1(4)
C
C      XOC =FRACTION OF CHORD; 0 AT LE, 1.0 AT TE
C      SOS =FRACTION OF SEMISPAN; 0 AT ROOT, 1.0 AT TIP
C      ETA =ZETA=SMALL INTEGRATION REGION ABOUT SINGULARITY
C      N =SECTION NO. INDICATOR
C      X=XLEIN,SOS)+2.*RIN,SOS)=XOC
C
C      XLE =LOCATION OF LEADING EDGE; REF: ROOT SEMICHORD
C      B =1/2*LENGTH OF LOCAL SEMICHORD; REF: ROOT SEMICHORD
C      Y=SOS
C      XVECT(1) = X
C      YVECT(1) = Y
C      IF(SOS=ETA,LT.1.) GO TO 30
C      ETA=1.-SOS
C
C NO REGION 2
C      N1(2)=0
C      30 CONTINUE
C      IF(SOS=ETA,GT.0.0) GO TO 40
C      ETA=SOS

```

```

PW0W0037
PW0W0038
PW0W0039
PW0W0040
PW0W0041
PW0W0042
PW0W0043
PW0W0044
PW0W0045
PW0W0046
PW0W0047
PW0W0048
PW0W0049
PW0W0050
PW0W0051
PW0W0052
PW0W0053
PW0W0054
PW0W0055
PW0W0056
PW0W0057
PW0W0058
PW0W0059
PW0W0060
PW0W0061
PW0W0062
PW0W0063
PW0W0064
PW0W0065
PW0W0066
PW0W0067
PW0W0068
PW0W0069
PW0W0070
PW0W0071
PW0W0072

```

C NO REGION 4	PW00073
NI(4)=0	PW00074
40 S(2)=SOS*ETA	PW00075
C	PW00076
C OUTPUT COLLOCATION POINT AND RELATED DATA	PW00077
WRITE(6,910) L,XOC,SOS,ETA,NI(2),NI(4),X	PW00078
W(2)=1.0-S(2)	PW00079
W(4)=SOS-ETA	PW00080
C	PW00081
C S =LEFT-HAND LIMIT OF INTERVAL	PW00082
C W =LENGTH OF INTERVAL	PW00083
C	PW00084
C INITIALIZATION OF INTEGRALS FOR EACH INTEGRATION REGION	PW00085
DO 41 I=1,JS	PW00086
DO 41 NI=1,NOLT	PW00087
DO 41 N2=1,NOST	PW00088
AR(1,NI,N2)=0.0	PW00089
41 CONTINUE	PW00090
C	PW00091
C DO INTEGRALS IN REGIONS WITH NO SINGULARITY BY GAUSSIAN QUADRATURE	PW00092
DO 500 I=2,JS	PW00093
411 NSIP=NI(1)	PW00094
C	PW00095
C NSIP=NO. OF INTEGRAL POINTS	PW00096
IF(NSIP.EQ.0) GO TO 500	PW00097
DO 50 J=1,NSIP	PW00098
GS=S(1)-(GN(IJ,NSIP)-1.0)/2.*W(1)	PW00099
C	PW00100
C GNI(J,NSIP) =JTH ABSCISSA OF LEGENDRE-GAUSS QUADRATURE OF ORDER NSIP	PW00101
GY=GS	PW00102
YMN=Y-GY	PW00103
YMN2=YMN*YMN	PW00104
RSQR=YMN2	PW00105
WT=WNI(J,NSIP)* W(1)/(2.*RSQR)	PW00106
C	PW00107
C WNI(J,NSIP) =JTH WTG. FUNCTION OF LEGENDRE-GAUSS QUADRATURE	PW00108
C	
C CALCULATE VORTICITY STRENGTH	PW00109
CALL FUNCTNINUST,GS,F1	PW00110
C	PW00111
C DO CHORDWISE INTEGRATION	PW00112
CALL CHOWS	PW00113
DO 45 M=1,NOLT	PW00114
DO 45 NSF=1,NOST	PW00115
AR(1,M,NSF)=AR(1,M,NSF)+CR(M)*F(NSF)*WT	PW00116
C	PW00117
C AR(1,M,NSF) = SURFACE INTEGRAL IN REGION 1	PW00118
45 CONTINUE	PW00119
50 CONTINUE	PW00120
500 CONTINUE	PW00121
C	PW00122
C DO INTEGRAL OF HANGLER-TYPE SINGULARITY	PW00123
CALL MNGLR(NOST)	PW00124
DO 60 I=1,NMODE	PW00125
DWRIL(I)=0.0	PW00126
60 CONTINUE	PW00127
C	PW00128
C SUM INTEGRALS OVER ALL REGIONS OF INTEGRATION	PW00129
DO 70 I=1,NOLT	PW00130
DO 70 J=1,NOST	PW00131
K=I+NOLT*(J-1)	PW00132
DO 70 MS=1,JS	PW00133
DWRIL(K)=DWRIL(K)+AR(MS,I,J)	PW00134
C	PW00135
C DWR =REAL PART OF GENERALIZED AFRODYNAMIC INFLUENCE COEFFICIENTS	PW00136
70 CONTINUE	PW00137
90 CONTINUE	PW00138
NOCD=(NMODE+4)/5	PW00139
C	PW00140
C OUTPUT AFRODYNAMIC INFLUENCE COEFFICIENTS	PW00141
DO 110 L=1,NDCP	PW00142
DO 110 K=1,NDCD	PW00143
	PW00144

```

JMAX=500
JMIN=JMAX-4
WRITE(6,91) (DWR(L,JC),JC=JMIN,JMAX),L,K
WRITE(7,920) (DWR(L,JC),JC=JMIN,JMAX),L,K
110 CONTINUE
C
C PLOT PLANFORM AND COLLOCATION POINTS
CALL PLOT(XVCT,YVCT,NOCPI)
6 FORMAT(1F10.4,1I2)
81 FORMAT(5E15.7,2X,'DWR',12,1X,12)
91 FORMAT(1NO,'SPAN=',F8.3,3X,
          'NO. OF CHOWS L
          COADING MODE=',13,3X,'NO. OF SPMS LOADING MODF=',14,7,1X,'NO. OF
          COLLOCATION PTS=',14,3X,'CHORD TO SPAN RATIO=',F10.6,/)
100 FORMAT(215,F10.4)
501 FORMAT(
          415,211, F14.5)
900 FORMAT(1H,'INTEGRATION PTS. IN REGION 3=',15,9X,'IN CHOWS=',15,
          3X,'IN MNGLR=',15,/)
910 FORMAT(1H,'COLLOCATION PT.',14,3X,'XOC=',F7.4,3X,'SOS=',F7.4,3X,
          3X,'ETA=',F7.4,3X,'NI(2)=' ,13,3X,'NI(4)=' ,13,3X,'X=' ,F7.4)
920 FORMAT(5E14.6,2X,'DWR',12,1X,12)
930 FORMAT(1' NOW: CALCULATES INFLUENCE COEFFICIENTS FOR DOWNWASH ON W
          CING: APRIL 27,1977',/)
STOP
END

```

PWDW0145
 PWDW0146
 PWDW0147
 PWDW0148
 PWDW0149
 PWDW0150
 PWDW0151
 PWDW0152
 PWDW0153
 PWDW0154
 PWDW0155
 PWDW0156
 PWDW0157
 PWDW0158
 PWDW0159
 PWDW0160
 PWDW0161
 PWDW0162
 PWDW0163
 PWDW0164
 PWDW0165
 PWDW0166
 PWDW0167
 PWDW0168

```

SUBROUTINE CHOWS
C
C CHOWS: EVALUATION OF CHORDWISE INTEGRAL USING GAUSSIAN QUADRATURE
C
C DIMENSION RETAI(10),THETA(10),GX(10),AL(5)
COMMON AR(4,5,5),ALS(5,10),CR(5),TKR(10),XOC,SOS,Y,Z,
C YMN,ZM,ZSCR,ETA,GAUSK(10),POZ,NOLT,NCP,MP,N,X,GZ,J1,J2,GS,YMN2,
C ZM2,CSR /GAUN/GH(10,10),WH(10,10) /MODES/NDST,NINC
C
C SEMICD=BIN,GS)
ELE=XLE(N,GS)
C
C INITIALIZE SUMMATIONS
DO 1 I=1,NOLT
CR(I)=0.0
1 CONTINUE
C
C IF(RSOR-.1)>0, THE INTEGRAL IS EVALUATED AS A SINGLE INTEGRAL
IF (RSOR-0.1) 21,3,3
C
C NINC INSURFS THAT ALS IS ONLY CALCULATED ONCE
3 IF (NINC) 4,4,7
4 NINC=7
DO 5 I=1,NCP
BETA(I)=(1.-GN(I,NCP))*POZ
GX(I)=-COS(BETA(I))
DO 5 J=1,NOLT
ALS(J,I)=SIN(BETA(I))*FLOAT(IJ)/FLOAT(12*J)*4.
C
C ALS(I,J) = LOADING FUNCTIONS. REF: ASHLEY AND LANDAHL
5 CONTINUE
7 DO 6 I=1,NCP
6 GAUSK(I)=X-(ELE*SEMICD*11.*GX(I))
CALL KERNL
WGHT=POZ
DO 20 I=1,NCP

```

CHDW0001
 CHDW0002
 CHDW0003
 CHDW0004
 CHDW0005
 CHDW0006
 CHDW0007
 CHDW0008
 CHDW0009
 CHDW0010
 CHDW0011
 CHDW0012
 CHDW0013
 CHDW0014
 CHDW0015
 CHDW0016
 CHDW0017
 CHDW0018
 CHDW0019
 CHDW0020
 CHDW0021
 CHDW0022
 CHDW0023
 CHDW0024
 CHDW0025
 CHDW0026
 CHDW0027
 CHDW0028
 CHDW0029
 CHDW0030
 CHDW0031
 CHDW0032
 CHDW0033
 CHDW0034
 CHDW0035
 CHDW0036

CW=WN(I,NCP)*SIN(THETA(I))*WGHT	CHDW0017
DO 20 J=1,NOLT	CHDW0018
CR(J)=ALS(J,I)*CW*TKR(I)*CR(I)	CHDW0019
20 CONTINUE	CHDW0040
GO TO 50	CHDW0041
C	CHDW0042
C FOR RSQR=1.0, THE CHORDWISE INTEGRAL IS COMPUTED BY TWO GAUSSIAN	CHDW0043
C QUADRATURES TO HANDLE THE FINITE JUMP IN KERNEL AT X=XI=Y=YI=0	CHDW0044
21 IF(X-ELE) 3,3,220	CHDW0045
220 IF(X-ELE+2.*SEMICD) 22,3,3	CHDW0046
22 THRD=ARCOS((ELE+SEMICD-XI)/SEMICD)	CHDW0047
K=-1	CHDW0048
WGHT=THRD/2.	CHDW0049
DO 23 I=1,NCP	CHDW0050
THETA(I)=1.-GN(I,NCP)*THRD/2.	CHDW0051
23 GAUSX(I)=X-(ELE+SEMICD*(1.-COS(THETA(I))))	CHDW0052
GO TO 35	CHDW0053
24 WGHT=PD2-WGHT	CHDW0054
K=1	CHDW0055
DO 25 I=1,NCP	CHDW0056
THETA(I)=THRD*(1.-GN(I,NCP))*(PD2-THRD/2.)	CHDW0057
25 GAUSX(I)=X-(ELE+SEMICD*(1.0-COS(THETA(I))))	CHDW0058
35 CALL KERNL	CHDW0059
DO 40 I=1,NCP	CHDW0060
CW=WN(I,NCP)*SIN(THETA(I))*WGHT	CHDW0061
DO 40 J=1,NOLT	CHDW0062
AL(J)=SIN(THETA(I))*FLOAT(I)/FLOAT(2*(2+J))*.4.	CHDW0063
CR(J)=AL(J)*CW*TKR(I)*CR(I)	CHDW0064
C	CHDW0065
C CR = CHORDWISE INTEGRAL	CHDW0066
40 CONTINUE	CHDW0067
IF(K) 24,50,50	CHDW0068
50 CONTINUE	CHDW0069
60 RETURN	CHDW0070
END	CHDW0071

SUBROUTINE KERNL		KERN0001
C		KERN0002
C KERNL: EVALUATION OF KERNEL FUNCTION FROM STEADY, NON-PLANAR,		KERN0003
C INCOMPRESSIBLE LIFTING SURFACE THEORY. REF: ASHLEY AND		KERN0004
C LANDAHL, CH. 5		KERN0005
C		KERN0006
COMMON AR(4,5,5),ALS(5,10),CR(5),TKR(10),XDC,SDS,Y,Z,		KERN0007
C YMN,ZM2,RSQR,ETA,GAUSX(10),PD2,NOLT,NCP,MP,N,X,CZ,		KERN0008
C J1,J2,CS,YMN2,ZM2,CSR		KERN0009
C		KERN0010
C GAUSX(I) = X-XI		KERN0011
C YMN = Y-YI		KERN0012
5 DO 10 I=1,NCP		KERN0013
XMF=GAUSX(I)*CSR		KERN0014
XMF2=XMF*XMF		KERN0015
R=SQRT(RSQR+XMF2)		KERN0016
G=1.0+XMF/R		KERN0017
TKR(I)=-G		KERN0018
C		KERN0019
C TKR = REAL PART OF KERNEL		KERN0020
10 CONTINUE		KERN0021
15 RETURN		KERN0022
END		KERN0023

```

SUBROUTINE MNGLR(NOST)
C
C MNGLR: COMPUTES PRINCIPAL VALUE OF A HANGLER INTEGRAL WHICH
C INVOLVES A SINGULARITY AT Y=YL. REF: WATKINS, NASA TN R-48
C
C ARGUMENT LIST
C NOST: NO. OF SPANWISE MODES
C
C DIMENSION D(6),SW(6),CRTE( 5),AL(5),F(5)
C COMMON AR(4,9,5),ALS(5,10),CR(5),TKR(10),XDC,SOS,Y,Z,
C YMN,IMZ,RSQR,ETA,GAUSX(10),PDZ,NOLT,NCP,MP,N,X,GZ,
C J1,J2,GS,YMN2,ZM2,CSR /GAUN/GN(10,10),WN(10,10)
C
C DO 1 I=1,NOLT
C CRTE(I)=0.0
1 CONTINUE
SKR=-2.0
DATA SW/11.,72.,495.,495.,72.,13./
C
C SW =WEIGHTING COEFFICIENTS AT THE RESPECTIVE INTEGRATION POINTS
C D(1)=ETA
D(2)=ETA*2./3.
D(3)=ETA/3.
D(4)=-D(3)
D(5)=-D(2)
D(6)=-D(1)
C
C D(1) = LOCATION OF INTEGRATION STATIONS W.R.T. Y WITHIN INTERVAL
C DO LOOP 50 COMPUTES F1 THROUGH F7, EXCEPT F4
DO 50 J=1,6
GS=SOS*D(J)
GY=GS
CALL FUNCTN(NOST,GS,F)
YMN=Y-GY
YMN2=YMN*YMN
RSQR=YMN2
C
C CALL CHOWS
DO 40 L=1,NOLT
DO 40 K=1,NOST
W=SW(J)
AR(1,L,K)=CR(L)*F(K)*W*AR(1,L,K)
C AR = REAL PART OF SURFACE INTEGRAL
40 CONTINUE
50 CONTINUE
CALL FUNCTN(NOST,SOS,F)
THMAX=ARCS(1,0-XDC*2.0)
C
C DO LOOP 100 COMPUTES F4 AT Y= Y1
DO 100 I=1,MP
THETA=(1.-GN(I,MP))/2.*THMAX
CW=WN(I,MP)*SIN(THETA)*THMAX/2.0
C
C CW = WEIGHTING TERM
C
C AL(1) = THE TWO-D CHORDWISE LOADING FUNCTIONS
DO 70 L=1,NOLT
AL(1)=SIN(FLOAT(L)*THETA)/FLOAT(2*(2*L)+4.
CRTE(L)=AL(1)*CW*SKR*CRTE(L)
70 CONTINUE
C
C CRTE = CHORDWISE INTEGRAL
100 CONTINUE
103 FCTR = 100.*ETA
DO 105 L=1,NOLT
DO 105 K=1,NOST
AR(1,L,K)=(-1360.0*CRTE(L)*F(K)*AR(1,L,K))/FCTR
C
C AR: FINAL VALUE OF THE SURFACE INTEGRAL
105 CONTINUE
RETURN
END

```

```

MNGL0001
MNGL0002
MNGL0003
MNGL0004
MNGL0005
MNGL0006
MNGL0007
MNGL0008
MNGL0009
MNGL0010
MNGL0011
MNGL0012
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MNGL0014
MNGL0015
MNGL0016
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MNGL0020
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MNGL0043
MNGL0044
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MNGL0050
MNGL0051
MNGL0052
MNGL0053
MNGL0054
MNGL0055
MNGL0056
MNGL0057
MNGL0058
MNGL0059
MNGL0060
MNGL0061
MNGL0062
MNGL0063
MNGL0064
MNGL0065
MNGL0066
MNGL0067
MNGL0068
MNGL0069
MNGL0070
MNGL0071

```


C.3. Program IIIA

The following listing for Program IIIA includes Program IIIA and subprograms XGWGMW, AXA, DETERM, and PRESS.

Program IIIA calculates the initial approximation for the vorticity distributed from the leading-edge vortex location and the outputs of Program I and Program WOW. The output of Program IIIA is used to provide the initial vorticity distribution for Program V.

```

C          PROGRAM IITA
C
C IITA CALCULATES VORTICITY COEFFICIENTS FROM VORTEX LOCATION AND
C OUTPUTS OF PROGRAMS WOV AND I
C
C NOTE: DOWNWASH POSITIVE FOR WIDHALL      UPWASH POSITIVE HERE
C
C      INPUT  J1,J2,J3,J4      415
C      INPUT  NCOORD,NSPAN,S,NOCM,NOSM,CR      2110,F10.4,2110,F10.4
C      INPUT  AWW,GWW      5E14.5
C      INPUT  LMAX,ALFA      15 F10.6
C      INPUT  GVVOR,GZVOR      5E14.5
C
C      PRIMARY OUTPUT VORTICITY COEFFICIENTS:
C      FIRST NOCM*NOSM MODES ARE HGRSF5HDE VORTEX MODES:
C      REMAINING ARE LEADING-EDGE VORTEX MODES      5E14.5
C
C      NEED FUNCTIONS  B,FW,XGWT,XI,XGWMW,XLE
C      NEED SUBROUTINES AXA,COLPT,DETERM,DFNCT,FNCTN,GAUSID,PRESS
C
C      EXTERNAL XGWMW,XGWT
C      DIMENSION XPT(5),YPT(5),VK(25),ATVR(25),PR(25,25),AWW(25,20),
C      C GWW(25,5),COEFF(25,25)
C      COMMON XPT,YPJ,S,M,MP/GVOR/GVVOR(5),GZVOR(5)/PLAN/CR /SEC/ZPT
C      COMMON/MODES/NOCM,NOSM/CONTR/J2,J3/VLOC/LMAX /GAUS/G(24),W(24)
C
C      DO 100 JDUMMY = 13,24
C      W(JDUMMY) = W(25-JDUMMY)
C 100 G(JDUMMY) = -G(25-JDUMMY)
C      DATA COEFF/625*0.0/
C      PI=3.141593
C
C ON WING 2 = 0
C      IPT=0.
C      WRITE(6,860)
C
C      READ(5,950) J1,J2,J3,J4
C
C
C      J1 = CONTROL PARAMETER. IF J1=1, NCOORD2=NCOORD; ELSE, NCOORD2=NCOORD+1
C      J2,J3 ARE CONTROLS; J2=1 CALLS DETERM; J3 > 1 CALLS ITERATION PROCEDURE
C      J4 CONTROLS OUTPUT; IF J4.EQ.1, OUTPUTS INTERMEDIATE RESULTS
C      WRITE(6,950) J1,J2,J3,J4
C
C      READ(5,880) NCOORD,NSPAN,S,NOCM,NOSM,CR
C
C      NCOORD = NO. OF CHORDWISE COLLOCATION POINTS
C      NSPAN = NO. OF SPANWISE COLLOCATION POINTS
C      S = SEMISPAN
C      NOCM = NO. OF CHORDWISE MODES
C      NOSM = NO. OF SPANWISE MODES
C      CR = ROOT CHORD DIVIDED BY MAXIMUM LENGTH
C      WRITE(6,870) NCOORD,NSPAN,S,NOCM,NOSM,CR
C      NCOORD2=NCOORD+1
C      IF(J1.EQ.1) NCOORD2=NCOORD
C      NGCP= NCOORD2 *NSPAN
C      NMOD=NOSM*NOCM
C      NMODT=NMOD*NOCM
C
C      NGCP = NO. OF COLLOCATION POINTS
C      NMOD = NO. OF HOFSESHOE VORTEX MODES
C      NMODT = TOTAL NUMBER OF MODES
C      DO 200 I=1,NGCP
C
C      READ (5,900) (AWW(I,J),J=1,NMOD)
C
C      AWW = DOWNWASH INFLUENCE COEFFICIENTS FROM PROGRAM WOV
C      DO 200 J=1,NMOD
C      COEF(I,J)=AWW(I,J)
C 200 CONTINUE
C      DO 250 I=1,NGCP
C
C      READ(5,920) (GWW(I,J),J=1,NOCM)
C

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PG3A0001
 PG3A0002
 PG3A0003
 PG3A0004
 PG3A0005
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 PG3A0012
 PG3A0013
 PG3A0014
 PG3A0015
 PG3A0016
 PG3A0017
 PG3A0018
 PG3A0019
 PG3A0020
 PG3A0021
 PG3A0022
 PG3A0023
 PG3A0024
 PG3A0025
 PG3A0026
 PG3A0027
 PG3A0028
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 PG3A0032
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 PG3A0034
 PG3A0035
 PG3A0036

PG3A0037
 PG3A0038
 PG3A0039
 PG3A0040
 PG3A0041
 PG3A0042
 PG3A0043
 PG3A0044
 PG3A0045
 PG3A0046
 PG3A0047
 PG3A0048
 PG3A0049
 PG3A0050
 PG3A0051
 PG3A0052
 PG3A0053
 PG3A0054
 PG3A0055
 PG3A0056
 PG3A0057
 PG3A0058
 PG3A0059
 PG3A0060
 PG3A0061
 PG3A0062
 PG3A0063
 PG3A0064
 PG3A0065
 PG3A0066
 PG3A0067
 PG3A0068
 PG3A0069
 PG3A0070
 PG3A0071
 PG3A0072

```

C GWM = DOWNWASH INFLUENCE COEFFICIENTS FROM PROGRAM I
250 CONTINUE
C
  READ(5,910) LMAX,ALFA
C
C LMAX = ORDER OF VORTEX APPROXIMATION
C ALFA = ANGLE OF ATTACK (IN RADIANS)
  SINALF = SIN(ALFA)
  WRITE(6,960) LMAX,ALFA
C
C CALCULATE COLLOCATION POINTS
  CALL COLPTNC(ORD,NSPAN, XPT,YPT)
  IF(J1.EQ.1) GO TO 300
  XPT(NCORD2)=(XPT(NCORD)+XPT(NCORD-1))/2.
300 CONTINUE
C
  READ(5,920) GYVOR,GZVOR
C
C GYVOR ARE COEFFICIENTS OF VORTEX SPANWISE LOCATION
C GZVOR ARE COEFFICIENTS OF VORTEX VERTICAL LOCATION
  WRITE(6,870) GYVOR,GZVOR
C
C CALCULATES LOCATION OF VORTEX AT X =1.
  CALL FNCTN(GYVOR,LMAX,1.,YVOR)
  CALL FNCTN(GZVOR,LMAX,1.,ZVOR)
C
C CALCULATE CONTRIBUTION FROM LEADING-EDGE VORTEX TO DOWNWASH
  DO 400 I=1,NCORD2
  DO 400 J=1,NSPAN
  N1=J+(I-1)*NSPAN
  XPI=XLE(YPT(J))+B(YPT(J))*XPT(I)
  YPJ=YPT(J)+S
C
C OUTPUT LOCATION OF COLLOCATION POINTS
  WRITE(6,940) N1,XPT(I),YPT(J),XPI
  YDIFF=YVOR -YPJ

  YSUM=YVOR +YPJ
  XDIF=1.-XPI
  YDIFFSQ=YDIFF*YDIFF
  YSUMSQ=YSUM*YSUM
  ZSQ=ZVOR *ZVOR
  TERM1=YDIFFSQ+ZSQ
  TERM2=YSUMSQ+ZSQ
C
C CALCULATE CONTRIBUTION OF VORTEX AFT OF X =1.
  GWMW2=-(YDIFF/TERM1+1.-XDIF/SQRT(TERM1+XDIF*XDIF))
  1+YSUM/TERM2*(1.-XDIF/SQRT(TERM2+XDIF*XDIF)))/(4.*PI)
  DO 450 MQ=1,NCCM
  M=MQ-1
C
C OBTAIN CONTRIBUTION FROM WAKE AND VORTEX SEGMENT BEFORE X =1.
  CALL GAUSID1(0.0,S,GWT,XGWT)
  CALL GAUSID1(0.0,.13,GH,XGWMW1)
  GWMW1=GH
  CALL GAUSID1(.13,.25,GH,XGWMW1)
  GWMW1=GWMW1+GH
  CALL GAUSID1(.25,.57,GH,XGWMW1)
  GWMW1=GWMW1+GH
  CALL GAUSID1(.57,1.0,GH,XGWMW1)
  GWMW1=GWMW1+GH
  COEFF(N1,NPOD*MQ)=GWMW1*MQ+G1*GWMW1+GWMW2
  GWMW2=-1.*GWMW2
450 CONTINUE
400 CONTINUE
  IF(J4.NE.1) GO TO 510
  NOCD=(NMOUT+4)/5
C
C OUTPUT TOTAL DOWNWASH INFLUENCE COEFFICIENTS IF DESIRED
  DO 500 I=1,NCCP
  DO 500 K=1,NCCD
  JMAX=5*K
  JMIN=JMAX-4

```

```

PG130073
PG140074
PG150075
PG160076
PG170077
PG180078
PG190079
PG200080
PG210081
PG220082
PG230083
PG240084
PG250085
PG260086
PG270087
PG280088
PG290089
PG300090
PG310091
PG320092
PG330093
PG340094
PG350095
PG360096
PG370097
PG380098
PG390099
PG400100
PG410101
PG420102
PG430103
PG440104
PG450105
PG460106
PG470107
PG480108
PG490109
PG500110
PG510111
PG520112
PG530113
PG540114
PG550115
PG560116
PG570117
PG580118
PG590119
PG600120
PG610121
PG620122
PG630123
PG640124
PG650125
PG660126
PG670127
PG680128
PG690129
PG700130
PG710131
PG720132
PG730133
PG740134
PG750135
PG760136
PG770137
PG780138
PG790139
PG800140
PG810141
PG820142
PG830143
PG840144

```

WRITE(6,940) (COEFF(I,J),J=JMIN,JMAX),I,K	PG3A0145
WRITE(7,940) (COEFF(I,J),J=JMIN,JMAX),I,K	PG3A0146
500 CONTINUE	PG3A0147
510 CONTINUE	PG3A0148
C	PG3A0149
C CALCULATES VORTICITY COEFFICIENTS A,GQ FROM BOUNDARY CONDITION	PG3A0150
C	PG3A0151
C SQUARE MATRIX BY FORMING A TRANSPOSE*A	PG3A0152
CALL AXAT(COEFF,PR,NOCP,NMODT)	PG3A0153
IF(J4.NE.1) GO TO 160	PG3A0154
C	PG3A0155
C OUTPUT A TRANSPOSE*A , IF DESIRED	PG3A0156
DO 150 L=1,NMODT	PG3A0157
WRITE(6,920) (PR(L,K),K=1,NMODT)	PG3A0158
150 CONTINUE	PG3A0159
160 CONTINUE	PG3A0160
C	PG3A0161
C FIX DOWNWASH, VR(1), ON WING	PG3A0162
DO 530 I=1,NOCP	PG3A0163
VR(I) = -SINALF	PG3A0164
530 CONTINUE	PG3A0165
C	PG3A0166
C FORM A TRANSPOSE*DOWNWASH VECTOR	PG3A0167
DO 140 K=1,NMODT	PG3A0168
ATVR(K)=0.0	PG3A0169
DO 140 L=1,NOCP	PG3A0170
ATVR(K)=ATVR(K)+COEFF(L,K)*VR(L)	PG3A0171
140 CONTINUE	PG3A0172
IF(J4.NE.1) GO TO 145	PG3A0173
C	PG3A0174
C OUTPUT A TRANSPOSE*DOWNWASH, IF DESIRED	PG3A0175
WRITE(6,920) (ATVR(I),I=1,NMODT)	PG3A0176
145 CONTINUE	PG3A0177
IF(NMODT.NE.NOCP) GO TO 130	PG3A0178
C	PG3A0179
C SOLVE SIMULTANEOUS LINEAR EQUATIONS, A X = B, FOR X	PG3A0180
CALL PRESS (COEFF,VR,NMODT,J4)	PG3A0181
C	PG3A0182
C SOLVE EQUATION ATA X = AT B FOR X	PG3A0183
130 CALL PRESS(IPR,ATVR,NMODT,J4)	PG3A0184
860 FORMAT(' PROGRAM III A CALCULATES A,GQ: GIVEN AMW,GMW:',5X,	PG3A0185
C'UPDATED APRIL 27,1977',/)	PG3A0186
870 FORMAT(' CHOW COLL PTS=',I3,3X,'SPNWS COLL PTS=',I3,3X,	PG3A0187
C'SEMISPAN=',F10.4,3X,'CHOWS MODES=',I3,3X,'SPNWS MODES=',I3,	PG3A0188
C 3X,'CR=',F7.4,/))	PG3A0189
880 FORMAT(2I10,F10.4,2I10,F10.4)	PG3A0190
890 FORMAT(' THE VALUES OF GYVOR,GZVOR ARE'/(5E14.5))	PG3A0191
900 FORMAT(5E14.6)	PG3A0192
910 FORMAT(15,F10.6)	PG3A0193
920 FORMAT (5E14.5)	PG3A0194
940 FORMAT(' COLLOCATION POINT',I3,2F12.4,3X,'LOCAL X=', F12.4)	PG3A0195
950 FORMAT(4I5)	PG3A0196
960 FORMAT(1H0,I3,' DEGREES OF FREEDOM IN VORTEX LOCATION',5X,	PG3A0197
C 'ANGLE OF ATTACK =',F7.4,/))	PG3A0198
980 FORMAT (5E14.5,2X,'COF',I2,1X,I2)	PG3A0199
STOP	PG3A0200
END	PG3A0201

```

      FUNCTION XGMMW(X)
C
C   XGMMW CALCULATES DOWNWASH CONTRIBUTION FROM BOTH VORTICES
C
C   ARGUMENT LIST
C     X:   INTEGRATION POINT
C
C   COMMON XPT,YPT,S,M,N
C   PI=3.141593
C   GGAM=SIN(FLDAT(2*M+1)/2.*PI*X)
C   XGMMW=GGAM*(FW(X,YPT)+FW(X,-YPT))
C   RETURN
C   END

```

```

XG=00001
XG=00002
XG=00003
XG=00004
XG=00005
XG=00006
XG=00007
XG=00008
XG=00009
XG=00010
XG=00011
XG=00012
XG=00013

```

```

      SUBROUTINE AXA(A,B,NROW,NCOL)
C
C   AXA CALCULATES B = A TRANSPOSE*A
C
C   ARGUMENT LIST
C     A:   INPUT MATRIX
C     B:   OUTPUT MATRIX
C     NROW: NO. OF ROWS IN A TO BE PROCESSED
C     NCOL: NO. OF COLUMNS IN A TO BE PROCESSED
C
C   DIMENSION A(25,25),B(25,25)
C   DO 10 I=1,NCOL
C   DO 10 J=1,NROW
C   B(I,J)=0.0
C   DO 10 N=1,NROW
C   B(I,J)=A(N,I)*A(N,J)+B(I,J)
C 10 CONTINUE
C   RETURN
C   END

```

```

AXA 0001
AXA 0002
AXA 0003
AXA 0004
AXA 0005
AXA 0006
AXA 0007
AXA 0008
AXA 0009
AXA 0010
AXA 0011
AXA 0012
AXA 0013
AXA 0014
AXA 0015
AXA 0016
AXA 0017
AXA 0018
AXA 0019
AXA 0020

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C		DETE0001
C	DETERM CALCULATES DETERMINANT OF COFACTORS AS MATRIX INDICATOR	DETE0002
C		DETE0003
C	ARGUMENT LIST	DETE0004
C	A: MATRIX TO BE TESTED	DETE0005
C	N: ORDER OF MATRIX OF INTEREST	DETE0006
C		DETE0007
C	DIMENSION A(25,25),DUMMY(25,25),IPER(25),DET(25,25)	DETE0008
C	IFR=0	DETE0009
C		DETE0010
C	USE DUMMY TO PRESERVE ORIGINAL MATRIX AS SOLUTION PROCEDURE IS DESTRUCTIVE	DETE0011
C	DO 100 I=1,N	DETE0012
C	DO 100 J=1,N	DETE0013
C	100 DUMMY(I,J)=A(I,J)	DETE0014
C	WRITE(6,940) ((DUMMY(I,J),J=1,N),I=1,N)	DETE0015
C	940 FORMAT(' THE VALUES OF THE MATRIX ELEMENTS ARE'/15E14.5))	DETE0016
C	N1=N-1	DETE0017
C		DETE0018
C	NOW DEVELOP PROCESS FOR COFACTORS	DETE0019
C	DO 800 I=1,N	DETE0020
C	DO 800 J=1,N	DETE0021
C	DO 300 I1=1,N	DETE0022
C	I2=I1	DETE0023
C	IF(I1.GT.I) I2=I1-1	DETE0024
C	DO 300 J1=1,N	DETE0025
C	J2=J1	DETE0026
C	IF(J1.GT.J) J2=J1-1	DETE0027
C	DUMMY(I2,J2)=A(I1,J1)	DETE0028
C	300 CONTINUE	DETE0029
C	CALL MFG(DUMMY,25,N1,IPER,IS,IER)	DETE0030
C		DETE0031
C	MFG IS IBM SLMATH SUBROUTINE TO PERFORM LU DECOMPOSITION OF MATRIX	DETE0032
C		DETE0033
C	ARGUMENT LIST MFG(A,N,N,IPER,IS,IER)	DETE0034
C		DETE0035
C		DETE0036
C		
C	A: INPUT MATRIX TO BE FACTORED	DETE0037
C	M: ORDER OF MATRIX IN DIMENSION STATEMENT	DETE0038
C	N: NUMBER OF SIMULTANEOUS EQUATIONS	DETE0039
C	IPER: PERMUTATION VECTOR GENERATED FOR MSG	DETE0040
C	IS: SIGN OF DETERMINANT	DETE0041
C	IER: ERROR INDICATOR	DETE0042
C		DETE0043
C	DET(I,J) = FLOATE(15)	DETE0044
C	DO 400 K=1,N1	DETE0045
C	DET(I,J)= DUMMY(I,K)*DET(I,J)	DETE0046
C	400 CONTINUE	DETE0047
C	800 CONTINUE	DETE0048
C	DO 600 I=1,N	DETE0049
C	600 WRITE(6,930) I,(DET(I,J),J=1,N1)	DETE0050
C	930 FORMAT(' ROW', 15/(5E14.5))	DETE0051
C	RETURN	DETE0052
C	END	DETE0053

C	SUBROUTINE PRESS(COEFF,SOLN,NMODT,J4)	PRE50001
C	PRESS CALCULATES VORTICITY COEFFICIENTS BY SOLUTION OF SIMULTANEOUS	PRE50002
C	EQUATIONS AND CALCULATES LEADING-EDGE VORTEX STRENGTH	PRE50003
C		PRE50004
C	ARGUMENT LIST	PRE50005
C	COEFF: A MATRIX IN A X = B	PRE50006
C	SOLN: B VECTOR IN A X = B	PRE50007
C	NMODT: NUMBER OF SIMULTANEOUS EQUATIONS	PRE50008
C	J4: CONTROLS PRINTING OF INTERMEDIATE RESULTS	PRE50009
C		PRE50010
C	REAL*8 COEFF(25,25),DSOLN(25)	PRE50011
C	DIMENSION XDUM(25),BPRIM(25),ADUM(25),COEFF(25,25),	PRE50012
C	C SOLN(25),IPER(25),GAMMA(10)	PRE50013
C	COMMON/MODES/NOCM,HOSM/CONTRL/J2,J3	PRE50014
C	IER=0	PRE50015
C	PI=3.141593	PRE50016
C		PRE50017
C	INITIALIZE VORTICITY COEFFICIENTS XDUM(I)	PRE50018
C	DO 10 I=1,NMODT	PRE50019
C	XDUM(I)=0.0	PRE50020
C	DO 10 J=1,NMODT	PRE50021
C	10 DCOEFF(I,J) = DBLE(COEFF(I,J))	PRE50022
C	IF(IJ2.NF.1) GO TO 40	PRE50023
C		PRE50024
C	DETERM CAN BE CALLED TO TEST FOR ILL-CONDITIONED MATRICES	PRE50025
C	CALL DTERM(COEFF,NMODT)	PRE50026
C	40 CONTINUE	PRE50027
C	CALL DMFG(COEFF,25,NMODT,IPER,IS,IER)	PRE50028
C		PRE50029
C	DMFG IS IBM SLMATH SUBROUTINE TO PERFORM LU DECOMPOSITION OF MATRIX	PRE50030
C		PRE50031
C	ARGUMENT LIST DMFG(A,M,N,IPER,IS,IER)	PRE50032
C		PRE50033
C	A: INPUT MATRIX TO BE FACTORED	PRE50034
C	M: ORDER OF MATRIX IN DIMENSION STATEMENT	PRE50035
C		PRE50036
C		
C	N: NUMBER OF SIMULTANEOUS EQUATIONS	PRE50037
C	IPER: PERMUTATION VECTOR GENERATED FOR DMFG	PRE50038
C	IS: SIGN OF DETERMINANT	PRE50039
C	IER: ERROR INDICATOR	PRE50040
C		PRE50041
C	DET = FLOAT(IS)	PRE50042
C	DO 200 I=1,NMODT	PRE50043
C	DET = DET*SGN(DCOEFF(I,I))	PRE50044
C	200 CONTINUE	PRE50045
C		PRE50046
C	OUTPUT DETERMINANT OF MATRIX	PRE50047
C	WRITE(6,950) NMODT,DET	PRE50048
C	IC=0	PRE50049
C		PRE50050
C	IC = CONTROL: COUNTS NUMBER OF ITERATIONS ALLOWED TO ELIMINATE	PRE50051
C	RESIDUE FROM AX = B SOLUTION	PRE50052
C	IF(IER.EQ.0) GO TO 15	PRE50053
C		PRE50054
C	IER IS CONDITION PARAMETER PRODUCED BY DMFG. IER=0 IS BAD	PRE50055
C	RETURN	PRE50056
C	15 CONTINUE	PRE50057
C	IC=IC+1	PRE50058
C	DO 20 I=1,NMODT	PRE50059
C	DSOLN(I) = SOLN(I)	PRE50060
C	20 BDUM(I)=SOLN(I)	PRE50061
C	CALL DMSG(COEFF,25,IPER,NMODT,0,DSOLN,IER)	PRE50062
C		PRE50063
C	DMSG IS IBM SLMATH SUBROUTINE TO SOLVE SIMULTANEOUS LINEAR EQUATIONS	PRE50064
C	GIVEN LU DECOMPOSITION	PRE50065
C		PRE50066
C	ARGUMENT LIST DMSG(A,M,IPER,N,J,R,IER)	PRE50067
C		PRE50068
C	A: OUTPUT FROM DMFG	PRE50069
C	M: ORDER OF MATRIX IN DIMENSION STATEMENT	PRE50070
C	IPER: OUTPUT FROM DMFG	PRE50071
C	N: NUMBER OF SIMULTANEOUS EQUATIONS	PRE50072

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C          J:  NORMALLY 0, TO OUTPUT X
C          B:  INPUT B; OUTPUTS X IN A X = B
C          IER: ERROR INDICATOR
C          DO 100 I=1,NMODT
100 SOLN(I) = SINGL(DSOLN(I))
      IF (IER.EQ.0) GO TO 70
C
C      IER IS MATRIX CONDITION PARAMETER PRODUCED BY DMSG
C      RETURN
70 CONTINUE
      IF (J4.NE.1) GO TO 120
C
C      OUTPUT INTERMEDIATE RESULTS IF DESIRED
      WRITE(6,940) ( SOLN(I),K=1,NMODT)
120 CONTINUE
      NMOD=NOCM*NOSH
C
C      CALCULATE VORTICITY COEFFICIENT VECTOR XDUM(I)
      DO 65 I=1,NMODT
65 XDUM(I)=XDUM(I)+SOLN(I)
C
C      CALCULATE LEADING-EDGE VORTEX STRENGTH, GAMMA(X)
      DO 90 I=1,10
      GAMMA(I) = 0.
      X = .1*FLOAT(I)
      DO 80 J=1,NOCM
      J=I*NMOD
      GAMMA(I) = XDUM(J)*SIN(FLOAT(2*I-1)/2.*PI*X) + GAMMA(I)
80 CONTINUE
90 CONTINUE
C
C      OUTPUT LEADING-EDGE VORTEX STRENGTH
      WRITE(6,940) GAMMA
C
C      OUTPUT VORTICITY COEFFICIENTS
      WRITE(6,920) ( XDUM(I),J=1,NMODT)

```

PRES0073
PRES0074
PRES0075
PRES0076
PRES0077
PRES0078
PRES0079
PRES0080
PRES0081
PRES0082
PRES0083
PRES0084
PRES0085
PRES0086
PRES0087
PRES0088
PRES0089
PRES0090
PRES0091
PRES0092
PRES0093
PRES0094
PRES0095
PRES0096
PRES0097
PRES0098
PRES0099
PRES0100
PRES0101
PRES0102
PRES0103
PRES0104
PRES0105
PRES0106
PRES0107
PRES0108


```

C
C      CHECK NUMERICAL PROCEDURE BY CALCULATING B, FROM A*X
      DO 50 I=1,NMODT
      BPR(I)=0.0
      DO 50 J=1,NMODT
50 BPRIM(I) = BPRIM(I) + COEFF(I,J) *SOLN(J)
C
C      CHECK DIFFERENCE BETWEEN INITIAL B VECTOR AND CALCULATED B VECTOR
      DO 55 I=1,NMODT
55 SOLN(I)=BPRIM(I)-XDUM(I)
      IF (J4.NE.1) GO TO 250
C
C      OUTPUT CALCULATED B VECTOR, IF DESIRED
      WRITE(6,940) (BPRIM(I),I=1,NMODT)
250 CONTINUE
C
C      OUTPUT DELTA B
      WRITE(6,970) (SOLN(I),I=1,NMODT)
      IF (IC.LT.J3) GO TO 15
C
C      PUNCH VORTICITY COEFFICIENTS
C
C      FIRST NOCM*NOSH MODES ARE HORSESHOE MODES; REMAINING MODES ARE
C      LEADING-EDGE VORTEX MODES
C
      WRITE(7,940) (XDUM(K),K=1,NMODT)
920 FORMAT('0 LEADING COEFFICIENTS ARE',/,(10E13.5))
930 FORMAT(15F14.5,4X,'SOLN')
940 FORMAT('0 GAMMA AT X = .1, .2, .3, ..., 1.0',/,(10F10.6))
950 FORMAT(' DETERMINANT OF MATRIX OF ORDER',I5,2X,'IS',E12.5)
960 FORMAT(' DEL X IS',/,(5F14.5))
970 FORMAT(' DEL B IS',/,(5F14.5))
980 FORMAT(' THE CALCULATED B VECTOR IS',/,(5F14.5))
      RETURN
      END

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PRES0109
PRES0110
PRES0111
PRES0112
PRES0113
PRES0114
PRES0115
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PRES0118
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PRES0123
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PRES0125
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PRES0137
PRES0138
PRES0139
PRES0140
PRES0141
PRES0142
PRES0143

C.4. Program III Prime

The following listing for Program III Prime includes Program III Prime and subprograms XGVGM, ADEL, and AGAM.

Program III Prime calculates the loading on the wing for a given vorticity distribution and vortex location.

```

C          PROGRAM III PRIME
C
C III PRIME CALCULATES LOADING ON WING; GIVEN VORTICITY
C
C NEED FUNCTIONS P,FV,GVORT,XGVGM
C NEED SUBROUTINES ADEL,AGAM,DFNCT,FNCTN,GAUS1D
C
C      INPUT  LMAX,J4      215
C      INPUT  GVVOR,GZVOR      SE14.5
C      INPUT  NCORD,S,NOCM,NOSM,CR      110,F10.4,2110,F10.4
C      INPUT  XPI      SF10.6
C      INPUT  GVGAM      SE14.5      IF J4=1
C      INPUT  A,GO      SE14.5
C
C      EXTERNAL XGVGM
C      DIMENSION XPI(5),      YTEST(11),ADELT(5,5),AGAMM(5,5),
C      A(5,5),GO(5),GVGAM(66,5)
C      COMMON XPI,YPI,S,M,MP/WING/CSR/VLOC/LMAX/PLAN/CR
C      COMMON/GVOR/GVOR(5),GZVOR(5)/SEC/ZVORT /GAUS/G(24),W(24)
C
C      DATA YTEST/0.0,.3,.5,.6,.7,.75,.8,.85,.9,.95,1.0/,GVGAM/330*0./
C      PI=3.141593
C      ZVORT=0.0
C      DO 150 JDUMMY =13,24
C      W(JDUMMY) = W(25-JDUMMY)
C      150 G(JDUMMY) = -G(25-JDUMMY)
C
C      WRITE(6,890)
C
C      READ(5,910) LMAX,J4
C
C      LMAX IS DEGREES OF FREEDOM IN VORTEX POSITION
C      J4 IS A CONTROL PARAMETER. IF(J4.EQ.1) INPUT GVGAM; ELSE CALCULATE
C
C      READ(5,920) GVVOR,GZVOR
C
C
C      GVVOR: COEFFICIENTS FOR VORTEX SPANWISE POSITION
C      GZVOR: COEFFICIENTS FOR VORTEX VERTICAL POSITION
C      WRITE(6,930) GVVOR,GZVOR
C
C      CALCULATE VORTEX POSITION AT X=1
C      CALL FNCTN(GVVOR,LMAX,1.,YVOR)
C      CALL FNCTN(GZVOR,LMAX,1.,ZVOR)
C
C      READ(5,880) NCORD,      S,NOCM,NOSM,CR
C
C      NCORD: NO. OF CHORDWISE POINTS
C      S: SEMISPAN; NON-D BY MAXIMUM LENGTH
C      NOCM: NO. OF CHORDWISE MODES
C      NOSM: NO. OF SPANWISE MODES
C      CR: ROOT CHORD; NON-D BY MAXIMUM LENGTH
C      WRITE(6,870) NCORD,      S,NOCM,NOSM,CR
C
C      CALCULATE CHORD TO SPAN RATIO, CSR
C      CSR = CR/(2.*S)
C
C      INPUT CHORDWISE POINTS OF INTEREST
C
C      READ(5,940) XPI
C      NC = 11*NCORD
C      IF(J4.NE.1) GO TO 110
C      DO 100 I=1,NC
C
C      READ(5,920) (GVGAM(I,MQ),MQ=1,5)
C      100 CONTINUE
C      110 CONTINUE
C
C      READ(5,920) ((A(I,J),I=1,NOCM),J=1,NOSM),(GO(K),K=1,NOCM)
C
C      A: HORSPOF VORTEX COEFFICIENTS
C      GO: LEADING-EDGE VORTEX COEFFICIENTS
C      WRITE(6,990)((A(I,J),I=1,NOCM),J=1,NOSM),(GO(K),K=1,NOCM)

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PG1P0001
PG1P0002
PG1P0003
PG1P0004
PG1P0005
PG1P0006
PG1P0007
PG1P0008
PG1P0009
PG1P0010
PG1P0011
PG1P0012
PG1P0013
PG1P0014
PG1P0015
PG1P0016
PG1P0017
PG1P0018
PG1P0019
PG1P0020
PG1P0021
PG1P0022
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PG1P0031
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PG1P0062
PG1P0063
PG1P0064
PG1P0065
PG1P0066
PG1P0067
PG1P0068
PG1P0069
PG1P0070
PG1P0071
PG1P0072

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C
WRITE(6,940)
C
C CALCULATE PRESSURES ALONG LINES X = XP(1)
DO 400 I=1,NCORD
  XP=XP(I)
  DO 400 J=1,11
    NI=J*(11-1)*11
    IF (XP1.GT.CR) GO TO 120
    YPJ=YTEST(IJ)*XP1*5
    GO TO 130
  120 CONTINUE
    YTE = S*(XP1-CR)/(1.-CR)
    YLE = S*XP1
    YPJ = YTE+YTEST(IJ)*(YLE-YTE)
  130 ETA = YPJ/S
C
C ASSUMED ARROW WING FOR THETA
THT = ARCCOS((2.-CR)*ETA -2.*XP1*CR)/(CR*(1.-ETA+.00001))
C
C INITIALIZE SUMMATION VARIABLES
GAMMA=0.0
DELTA=0.0
VGM=0.0
C
C CALCULATE LEADING-EDGE VORTEX CONTRIBUTIONS TO WING VORTICITY
DO 350 MQ=1,NOCM
  M=MQ-1
  SGAMMA=GG(MC)*XP1*GVORTIM,XP1,YPJ,S)
  GAMMA=SGAMMA+GAMMA
  SDELTA=-GG(MQ)*YPJ*GVORT(M,XP1,YPJ,S)
  350 DELTA = DELTA + SDELTA
  IF(J.EQ.11) GO TO 260
C
C CALCULATE HORSESHOE VORTEX CONTRIBUTIONS TO WING VORTICITY
CALL ADEL (ADELT,THT,ETA,NOCM,NCSM)

CALL AGAM (AGAMP,THT,ETA,NOCM,NCSM)
DO 300 MQ=1,NOCM
  DO 300 MPP=1,NCSM
    SGAMMA=A(MQ,MPP)*AGAMP(MQ,MPP)
    GAMMA=SGAMMA+GAMMA
    SDELTA=A(MQ,MPP)*ADELT(MQ,MPP)
  300 DELTA = DELTA + SDELTA
  260 CONTINUE
  IF(J4.EQ.1) GO TO 365
C
C CALCULATE CONTRIBUTION TO SPANWISE VELOCITY FROM LEADING-EDGE
C VORTEX AFT OF X =1.
C CALCULATE GVGM2
XDIFF=1.-XP1
YDIFF=YVOR -YPJ
YSUM=YVOR +YPJ
TERM1=YDIFF*YDIFF+ZVOR *ZVOR
TERM2=YSUM*YSUM+ZVOR *ZVOR
GVGM2=ZVOR *(11.-XDIFF/SQRT(TERM1*XDIFF*XDIFF))/TERM1-
111.-XDIFF/SQRT(TERM2*XDIFF*XDIFF)/TERM2)/(4.*PI)
C
C CALCULATE V
DO 360 MQ=1,NOCM
  M=MQ-1
C
C CALCULATE CONTRIBUTION FROM VORTEX FORWARD OF X=1
CALL GAUSID1 0.0,.13,GVGM5,XGVGM )
GVGM1=GVGM5
CALL GAUSID1 .17,.25,GVGM5,XGVGM )
GVGM1=GVGM5+GVGM1
CALL GAUSID1 .25,.57,GVGM5,XGVGM )
GVGM1=GVGM5+GVGM1
CALL GAUSID1 .57,1.0,GVGM5,XGVGM )
GVGM1=GVGM5+GVGM1
GVGM1(M1,MQ)=GVGM1+GVGM2
  360 GVGM2=-1.*GVGM2

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PG1P0073
 PG1P0074
 PG1P0075
 PG1P0076
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 PG1P0080
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 PG1P0141
 PG1P0142
 PG1P0143
 PG1P0144

365 CONTINUE	PG3P0145
DO 370 MQ=1,NOCM	PG3P0146
VGM=GOINQI*GVGAMINI,MQ)*VGM	PG3P0147
370 CONTINUE	PG3P0148
C	PG3P0149
C CALCULATE PRESSURE DIFFERENCE COMPONENTS DUE TO SPANWISE AND	PG3P0150
C CHORDWISE COMPONENTS	PG3P0151
PV=2.*VGM*DELTA	PG3P0152
PU=-2.*GAMMA	PG3P0153
DELT=PU*PV	PG3P0154
C	PG3P0155
C OUTPUT PRESSURE DIFFERENCE	PG3P0156
WRITE(6,910) XPI,YTEST(J),ETA,GAMMA,DELTA,PU,PV,DELT	PG3P0157
400 CONTINUE	PG3P0158
C	PG3P0159
C OUTPUT VELOCITY CONTRIBUTIONS	PG3P0160
WRITE(6,900) (GVGAMI(J),J=1,NOCM),I=1,NC	PG3P0161
C	PG3P0162
870 FORMAT('CHORDS PT' =',I3,3X,' SEMISPAN =',	PG3P0163
C F6.3,3X,'CHORDS MODES=',I3,3X,'SPNWS MODES=',I3,3X,'CR=',F7.4,/)	PG3P0164
880 FORMAT(110,F10.4,2110,F10.4)	PG3P0165
890 FORMAT(' III PRIME; UPDATED APRIL 28,1977',/)	PG3P0166
900 FORMAT('OTHE VALUES OF GVGAM ARE'/(10E13.4))	PG3P0167
910 FORMAT(215)	PG3P0168
920 FORMAT(15E14.5)	PG3P0169
930 FORMAT(3F7.4,5E14.5)	PG3P0170
940 FORMAT(1H0,T4,'XPI',T11,'YPI',T18,'ETA',T28,'GAMMA',T42,'DELTA',	PG3P0171
C T57,'PU',T71,'PV',T84,'DELT'//)	PG3P0172
950 FORMAT('O THE VALUES OF GYVCR, GZVOR ARE'/(5E14.5))	PG3P0173
960 FORMAT(5F10.6)	PG3P0174
970 FORMAT('CHORDS MODES =',I5,3X,'SPNWS MODES =',I5)	PG3P0175
990 FORMAT('OTHE VALUES OF A,GQ ARE'/(5E14.5))	PG3P0176
STOP	PG3P0177
END	PG3P0178
FUNCTION XGVGMIXI	
C	XGVG0001
C XGVGM CALCULATES CONTRIBUTION TO TG SPANWISE VELOCITY FROM VORTEX	XGVG0002
C	XGVG0003
C ARGUMENT LIST	XGVG0004
C	XGVG0005
C X: CHORDWISE COORDINATE; NON-D BY MAXIMUM LENGTH	XGVG0006
C	XGVG0007
C	XGVG0008
C COMMON XPT,YPT,S,M,N	XGVG0009
PI=3.141593	XGVG0010
C	XGVG0011
C CALCULATE LEADING-EDGE VORTEX STRENGTH	XGVG0012
GGAM=SIGN(FLOAT(2*M+1)/2.*PI*X)	XGVG0013
C	XGVG0014
C CALCULATE SPANWISE VELOCITY CONTRIBUTION FROM LEADING-EDGE VORTICES	XGVG0015
XGVGM=GGAM*(FV(X,YPT,XPT)-FV(X,-YPT,XPT))	XGVG0016
RETURN	XGVG0017
END	XGVG0018

SUBROUTINE ADEL (ADELT,THT,ETA,NOCM,NOSM)		ADEL0001
C	GAUSS CALCULATES ADEL: CONTRIBUTION TO CHORDWISE VORTICITY	ADEL0002
C	FROM HOSES:HOE VORTICES	ADEL0003
C	ARGUMENT LIST	ADEL0004
C	ADEL: CHORDWISE VORTICITY CONTRIBUTION	ADEL0005
C	THT: ANGULAR CHORDWISE LOCATION	ADEL0006
C	ETA: SPANWISE COORDINATE; NON-D BY SEMISPAN	ADEL0007
C	NOCM: NO. OF CHORDWISE MODES	ADEL0008
C	NOSM: NO. OF SPANWISE MODES	ADEL0009
C	COMMON /PLAN/CR	ADEL0010
C	DIMENSION CHDMOD(6),CHEBY2(10),ADELT(5,5)	ADEL0011
C	PI=3.141593	ADEL0012
C	CF = 2.*(2.-CR)/CR	ADEL0013
C	IF(ETA.GT..000001) GO TO 150	ADEL0014
C	DO 140 ICM=1,NOCM	ADEL0015
C	DO 140 JSM=1,NOSM	ADEL0016
C	FOR POINTS NEAR CENTERLINE, ZERO STRENGTH	ADEL0017
C	ADEL(ICM,JSM)=0.0	ADEL0018
C	140 CONTINUE	ADEL0019
C	GO TO 800	ADEL0020
C	150 CONTINUE	ADEL0021
C	THETA=THT	ADEL0022
C	USE CHEBYSHEV POLYNOMIALS FOR SPANWISE LOADING FUNCTIONS	ADEL0023
C	CALCULATE CHEBYSHEV POLYNOMIALS	ADEL0024
C	CHEBY2(1)=1.0	ADEL0025
C	CHEBY2(2)=2.*ETA	ADEL0026
C	NOSM2=2*NOSM-1	ADEL0027
C	IF(NOSM2.LT.3) GO TO 40	ADEL0028
C		ADEL0029
C		ADEL0030
C		ADEL0031
C		ADEL0032
C		ADEL0033
C		ADEL0034
C		ADEL0035
C		ADEL0036
C		ADEL0037
C		ADEL0038
C		ADEL0039
C		ADEL0040
C		ADEL0041
C		ADEL0042
C		ADEL0043
C		ADEL0044
C		ADEL0045
C		ADEL0046
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C	</	

	SUBROUTINE AGAM (AGAM,TH,ETA,NOCM,NOSM)	AGAM0001
C		AGAM0002
C	GAUSSIO CALCULATES SPANWISE VORTICITY CONTRIBUTION FROM	AGAM0003
C	MORSETHSIE VORTICES	AGAM0004
C		AGAM0005
C	ARGUMENT LIST	AGAM0006
C		AGAM0007
C	AGAM: SPANWISE VORTICITY CONTRIBUTION	AGAM0008
C	TH: ANGULAR CHORDWISE LOCATION	AGAM0009
C	ETA: SPANWISE COORDINATE; NON-D BY SEMISPAN	AGAM0010
C	NOCM: NO. OF CHORDWISE MODES	AGAM0011
C	NOSM: NO. OF SPANWISE MODES	AGAM0012
C		AGAM0013
C	COMMON/WMG/CSR	AGAM0014
C	DIMENSION AGAM(15,5),CHEBYZ(15),CHOMOD(15)	AGAM0015
C	PI=3.141593	AGAM0016
C		AGAM0017
C	CALCULATE CHERYSHOV POLYNOMIALS	AGAM0018
C	CHEBYZ(1)=1.0	AGAM0019
C	CHEBYZ(2)=4.*ETA*ETA-1.	AGAM0020
C	IF(NOSM.LT.3) GO TO 40	AGAM0021
C	CSQ=CH*BYZ(2)-1.	AGAM0022
C	DO 30 J=3,NOSM	AGAM0023
C	CHEBYZ(J)=CSQ*CHEBYZ(J-1)-CHEBYZ(J-2)	AGAM0024
C	30 CONTINUE	AGAM0025
C	40 CONTINUE	AGAM0026
C		AGAM0027
C	PREVENT BLOWING UP	AGAM0028
C	IF(ETA.GE.1.) ETA = .999	AGAM0029
C	THETA=TH	AGAM0030
C		AGAM0031
C	CALCULATE INTERMEDIATE FACTORS	AGAM0032
C	FACTOR = 16.*PI/(CSR*B(1,ETA))*SORT(1.-ETA*ETA)	AGAM0033
C		AGAM0034
C	DO 70 ICM=1,NOCM	AGAM0035
		AGAM0036
	CHOMOD(ICM)=SIN(FLOAT(ICM)*THETA)/FLOA(12*(2*(ICM)))	AGAM0037
	70 CONTINUE	AGAM0038
	DO 60 JCM=1,NOCM	AGAM0039
	DO 60 JSM=1,NOSM	AGAM0040
	AGAM(I,ICM,JSM)=FACTOR*CHOMOD(ICM)*CHEBYZ(JSM)	AGAM0041
	60 CONTINUE	AGAM0042
C		AGAM0043
C	PRIMARY OUTPUT AGAM PASSED THROUGH ARGUMENT LIST	AGAM0044
C	TO CALLING PROGRAM	AGAM0045
C		AGAM0046
C	RETURN	AGAM0047
	END	AGAM0048

C.5. Program V

The following listing of Program V includes Program V and subprograms BLOCK DATA, A1, A5, B, XLE, B5, B7, DIDY, DIDZ, FV, FW, GVORT, XGVL, XGVT, XGWL, XGWT, CHDWS, COLPT, DFNCT, DGWGM, DGWV, FNCTN, FUNCTN, GAUS1D, GVCTR, GWVD, KERNEL, TUCHEB, VORINT, WOV, and WPDW.

Program V calculates the new vortex position and vorticity distribution for a given initial solution given the outputs of Program I, Program WOW, and Program IIIA.

C	WRITE(6,950) GYVCR,GZVCR	PGM50071
C		PGM50074
C	READ(5,940) ((A(I,J),I=1,NOCM),J=1,NOSM),(GO(K),K=1,NOCM)	PGM50075
C		PGM50076
C	A: HORSESHOE VORTICITY COEFFICIENTS	PGM50077
C	GO: LEADING-EDGE VORTICITY COEFFICIENTS	PGM50078
C	WRITE(6,920) ((A(I,J),I=1,NOCM),J=1,NOSM),(GO(K),K=1,NOCM)	PGM50079
C		PGM50080
C	NMOD=NOSM*NOCM	PGM50081
C	NMODT=NMOD+NOCM	PGM50082
C	NOCF = NMODT	PGM50083
C	NPTS = 2*NOCF + NOCF	PGM50084
C	NMOD2 = NMODT + 2*LMAX	PGM50085
C		PGM50086
C	NMOD = NO. OF HORSESHOE VORTEX MODES	PGM50087
C	NMODT = TOTAL NO. OF VORTICITY MODES	PGM50088
C	NOCF = NO. OF COLLOCATION POINTS ON WING	PGM50089
C	NPTS = TOTAL NO. OF CONTROL POINTS	PGM50090
C	NMOD2 = TOTAL NO. OF MODES	PGM50091
C	DO 200 I=1,NOCF	PGM50092
C		PGM50093
C	READ(5,940) (COEFF(I,J),J=1,NMOD)	PGM50094
C		PGM50095
C	(COEFF(I,J),J=1,NMOD): OUTPUT FROM PROGRAM WOV	PGM50096
C	DO 200 J=1,NMOD	PGM50097
C	200 COEFF(I,J) = -COEFF(I,J)	PGM50098
C	DO 250 I=1,NOCF	PGM50099
C		PGM50100
C	250 READ(5,940) (GWM(I,J),J=1,NOCM)	PGM50101
C		PGM50102
C	GWM: OUTPUT FROM PROGRAM I	PGM50103
C		PGM50104
C	NINC=-1	PGM50105
C		PGM50106
C	NINC IS A CONTROL PARAMETER TO LIMIT REPETITIONS IN CHOWS	PGM50107
C		PGM50108
C		
C	JS=4	PGM50109
C	JS=NO. OF INTEGRATION REGIONS IN SPANWISE DIRECTION	PGM50110
C		PGM50111
C	5 READ 501, NI(1),NI(2),NI(3),NI(4),NCP,J1,J2,ETA	PGM50112
C		PGM50113
C	NI(J)=NO. OF LEGENDRE-GAUSS POINTS IN SPANWISE INTEGRATION	PGM50114
C	IN REGION J	PGM50115
C	NCP=NO. OF CHORDWISE LEGENDRE-GAUSS QUADRATURE POINTS IN CHOWS	PGM50116
C	J1,J2 CONTROL OUTPUT; NORMALLY 0	PGM50117
C	ETA=ZETA+SMALL INTEGRATION REGION ABOUT SINGULARITY	PGM50118
C	WRITE(6,970) NI(3),NCP,NI(1)	PGM50119
C	SPAN = 2.05	PGM50120
C	CSR = CMAX/SPAN	PGM50121
C		PGM50122
C	SPAN = WING SPAN; NON-0 BY MAXIMUM LENGTH	PGM50123
C	CSR =CHORD TO SPAN RATIO	PGM50124
C	N=1	PGM50125
C		PGM50126
C	N=SECTION NO. INDICATOR	PGM50127
C		PGM50128
C	FORM SCALES TO NORMALIZE EQUATIONS	PGM50129
C	F1 = 2.*ALFA/(PI*PI)	PGM50130
C	F2 = ALFA*PI*5	PGM50131
C	F3 = 5	PGM50132
C	F4 = ALFA/4.	PGM50133
C		PGM50134
C	ITERMAX = MAXIMUM NO. OF ITERATIONS ALLOWED TO CONVERGE	PGM50135
C	ITERMAX = 15	PGM50136
C		PGM50137
C	LOOP TO SATISFY DOWNWASH AND NO-FORCE CONDITIONS	PGM50138
C	DO 800 ITH=1,ITERMAX	PGM50139
C		PGM50140
C	CALCULATE VORTEX LOCATION AT X = 1.	PGM50141
C	CALL FNCTNGYVOR,ITH,1.,GYVCR	PGM50142
C	CALL FNCTNGZVOR,ITH,1.,ZVCR	PGM50143
C		PGM50144

C	PGM50145
C FORMULATE DOWNWASH CONDITIONS ON WING	PGM50146
CALL WDWIA(GO,NOCF,ALF)	PGM50147
C	PGM50148
C ADD CONTRIBUTIONS FROM DOWNWASH CONDITION TO RESIDUE VECTOR	PGM50149
DO 60 I=1,NOCF	PGM50150
60 PARAM(I) = -VR(I)	PGM50151
N1= NOCF	PGM50152
C	PGM50153
C FORMULATE NO-FORCE CONDITION ON RIGHT-HAND VORTEX	PGM50154
DO 400 L=1,NCF	PGM50155
C	PGM50156
C FIND CONTROL POINT ON VORTEX	PGM50157
XPI = XVECT(I,NCF)	PGM50158
CALL FNCIN(GVOR,LMAX,XPI,YVORT)	PGM50159
CALL FNCIN(GZVR,LMAX,XPI,ZVORT)	PGM50160
C	PGM50161
C XPI = CHORDWISE COORDINATE; NON-D BY MAXIMUM LENGTH	PGM50162
C YVORT = SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	PGM50163
C ZVORT = VERTICAL COORDINATE; NON-D BY MAXIMUM LENGTH	PGM50164
XOC = XPI/CXMAX	PGM50165
SOS = YVORT/S	PGM50166
Z = ZVORT/S	PGM50167
C	PGM50168
C XOC = CHORDWISE POSITION; NON-D BY ROOT CHORD	PGM50169
C SOS = SPANWISE POSITION; NON-D BY SEMISPAN	PGM50170
C Z = VERTICAL POSITION; NON-D BY SEMISPAN	PGM50171
C	PGM50172
C CALCULATE CONTRIBUTIONS TO FORCES AND RELATED DERIVATIVES FROM	PGM50173
HORSESHOE VORTICES	PGM50174
CALL HSHV(I)	PGM50175
C	PGM50176
C CALCULATE FORCES AND REMAINING DERIVATIVES FOR JACOBIAN	PGM50177
CALL GVCTR(NOCF)	PGM50178
400 CONTINUE	PGM50179
C	PGM50180
C ADD CONTRIBUTIONS FROM FORCE CONDITION TO RESIDUE VECTOR	PGM50181
DO 500 J=1,NOCF	PGM50182
PARAM(J+NOCF) = - FSUBY(J)	PGM50183
PARAM(J+NOCF+NOCF) = -FSUBZ(J)	PGM50184
500 CONTINUE	PGM50185
C	PGM50186
C CALCULATE MAGNITUDE OF RESIDUE	PGM50187
DMAG = 0.	PGM50188
C	PGM50189
C SCALES DEPENDENT VARIABLES TO ORDER 1	PGM50190
DO 560 IA = 1,NOPTS	PGM50191
DMAG = DMAG + PARAM(IA)*PARAM(IA)	PGM50192
DO 420 N2 = 1,NMOD	PGM50193
420 XACOB(IA,N2) = XACOB(IA,N2)*F1	PGM50194
DO 430 N2 = 1,NOCF	PGM50195
430 XACOB(IA,N2+NMOD) = XACOB(IA,N2+NMOD)*F2	PGM50196
DO 450 N2 = 1,LMAX	PGM50197
440 XACOB(IA,N2+NMOD1) = XACOB(IA,N2+NMOD1)*F3	PGM50198
450 XACOB(IA,N2+LMAX+NMOD1) = XACOB(IA,N2+LMAX+NMOD1)*F4	PGM50199
560 CONTINUE	PGM50200
C	PGM50201
C FORM MATRIX A TRANSPOSE*A	PGM50202
C OBTAIN A TRANSPOSE * A MATRIX FOR STABILITY REASONS	PGM50203
C	PGM50204
C SAVE DACOB IN ATA SINCE SOLUTION PROCEDURE IS DESTRUCTIVE	PGM50205
DO 360 I=1,NMOD2	PGM50206
DO 340 J=1,NMOD2	PGM50207
DACOB(I,J) = 0.00	PGM50208
DO 340 K= 1,NOPTS	PGM50209
DACOB(I,J) = DACOB(I,J) + DBLE(XACOB(K,I)*XACOB(K,J))	PGM50210
340 CONTINUE	PGM50211
360 ATAI(J) = SNGLDACOB(I,J)	PGM50212
C	PGM50213
C PERFORM LU DECOMPOSITION OF MATRIX DACOB	PGM50214
530 CALL DMFGIDACOB,35,NMOD2,IPER,IS,IF4)	PGM50215
C	PGM50216

C DMSG IS IBM SLMATH SUBROUTINE	PGM50217
C	PGM50218
C CALCULATE DETERMINANT OF ATA MATRIX	PGM50219
DET = FLOAT(15)	PGM50220
C	PGM50221
C CALCULATE ATA R VECTOR	PGM50222
DO 320 I=1,NMOD2	PGM50223
DET = DET*SNGL(DACOB(I,1))	PGM50224
DARAM(I) = 0.00	PGM50225
DO 320 J=1,ACPTS	PGM50226
320 DARAM(I) = DARAM(I) + DALE(KACOB(J,1)*PARAM(J))	PGM50227
C	PGM50228
C OUTPUT DETERMINANT OF ATA MATRIX	PGM50229
WRITE(16,860) DET	PGM50230
C	PGM50231
C AS CHECK FOR SOLUTION PROCEDURE, COMPARE DARAM WITH FMINS	PGM50232
C	PGM50233
C OUTPUT A TRANSPOSE * B VECTOR	PGM50234
WRITE(16,940) (DARAM(1L),1L =1,NMOD2)	PGM50235
C	PGM50236
C SOLVE SIMULTANEOUS LINEAR EQUATIONS	PGM50237
CALL DMSG(DACOB,35,1PER,NMOD2,0,DARAM,1ER)	PGM50238
C	PGM50239
C SUBROUTINE DMSG IS IBM SLMATH SUBROUTINE	PGM50240
C	PGM50241
C CALCULATE A TRANSPOSE A * X VECTOR	PGM50242
DO 570 IB =1,NMOD2	PGM50243
FMINS(1B) = 0.	PGM50244
DO 570 JB =1,NMOD2	PGM50245
570 FMINS(1B) = FMINS(1B) + ATA (1B,JB)*SNGL(DARAM(JB))	PGM50246
C	PGM50247
C OUTPUT CALCULATED A TRANSPOSE A * X VECTOR	PGM50248
WRITE(16,940) (FMINS(1D),1D=1,NMOD2)	PGM50249
C	PGM50250
C OUTPUT PREDICTED CHANGES IN X IN AT A X = AT B	PGM50251
WRITE(16,940) (DARAM(1E),1E=1,NMOD2)	PGM50252
C	
C CALCULATE RELATIVE MAGNITUDE OF PREDICTED CHANGE IN VORTEX POSITION	PGM50253
DEL1=FACTOR/SNGL(DSORT(DARAM(1+NMODT)**2*DARAM(1+LMAX+NMODT)**2))	PGM50254
IF(DEL1.GT.1.) DEL1=1.	PGM50255
C	PGM50256
C CALCULATE RELATIVE MAGNITUDE OF PREDICTED CHANGES IN HORSESHOE	PGM50257
C VORTICITY COEFFICIENTS	PGM50258
ANUM=0.	PGM50259
DNUM=0.	PGM50260
DO 620 IK=1,NOCM	PGM50261
DO 620 JK=1,NOSM	PGM50262
ANUM= ANUM+A1(IK,JK)*A1(IK,JK)	PGM50263
DNUM = DNUM+ SNGL(DARAM(IK+(JK-1)*NOCM))**2	PGM50264
620 CONTINUE	PGM50265
DEL2 = S2*SQRT(ANUM/DNUM)/F1	PGM50266
C	PGM50267
C USE SMALLER OF THE TWO SCALES	PGM50268
IF (DEL1.GT.DEL2) DEL1= DEL2	PGM50269
C	PGM50270
C OUTPUT SCALES	PGM50271
WRITE(16,930) S2,DEL1	PGM50272
C	PGM50273
C CALCULATE NEW VORTICITY COEFFICIENTS	PGM50274
DO 600 I=1,NOCM	PGM50275
GQ(I) = GQ(I) + DEL1*SNGL(DARAM(NMOD+1))*F2	PGM50276
DO 600 J=1,NOSM	PGM50277
A1(I,J) = A1(I,J) + DEL1*SNGL(DARAM(1+(J-1)*NOCM)) *F1	PGM50278
600 CONTINUE	PGM50279
C	PGM50280
C CALCULATE NEW VORTEX LOCATION COEFFICIENTS	PGM50281
DO 550 I=1,LMAX	PGM50282
GVVOR(I) = GVVOR(I)+DEL1*SNGL(DARAM(1+NMODT))*F3	PGM50283
550 GVVOR(I) = GVVOR(I)+DEL1*SNGL(DARAM(1+LMAX+NMODT))*F4	PGM50284
C	PGM50285
C OUTPUT NEW VORTICITY COEFFICIENTS	PGM50286
WRITE(16,920) (A1(I,J),I=1,NOCM,J=1,NOSM),(GQ(I),I=1,NOCM)	PGM50287
	PGM50288

```

C OUTPUT NEW VORTEX LOCATION COEFFICIENTS
  WRITE(6,950) GYVCR,GZVCR
  DMAG = SORT(DMAG)

C
C OUTPUT ITERATION NO. AND RESIDUE
  WRITE(6,960) ITR,DMAG

C
C CHECK FOR CONVERGENCE
  IF (DMAG.LT..001) GO TO B10
  800 CONTINUE
  810 CONTINUE

C
C OUTPUT JACOBIAN
C
  WRITE(6,910) ((JACOBI(I,G),J=1,NMOD2),IG=1,NOPTS)

C
C PUNCH NEW VORTEX LOCATION COEFFICIENTS
  WRITE(7,840) GYVCR
  WRITE(7,850) GZVCR

C
C PUNCH NEW VORTICITY COEFFICIENTS
  WRITE(7,940) ((A(I,J),I=1,NOCM),J=1,NOSH),(GQ(K),K=1,NOCM)

C
  501 FORMAT(5I5,2I2,F10.4)
  830 FORMAT(' PROGRAM V CALCULATES VORTICITY COEFFICIENTS AND VORTEX LO-
    CATION FROM INITIAL GUESS APRIL 29,1977',/)
  840 FORMAT(5E14.5,1X,'GY( 1- 5)')
  850 FORMAT(5E14.5,1X,'GZ( 1- 5)')
  860 FORMAT(' DETERMINANT OF THE JACOBIAN IS',E14.5)
  870 FORMAT(' CHOWS PTS =',I3,3X,'SPNWS PTS =',I3,3X,' SEMISPAN =',
    CF6.3,3X,' CHOWS MODES =',I3,3X,' SPNWS MODES =',I3,3X,' CR =',F7.4)
  880 FORMAT(2I10,F10.4,2I10,F10.4)
  890 FORMAT(2I5,3F10.6)
  900 FORMAT ('O CHIEF OF VORTEX APPROXIMATION IS',I3,3X,'FACTOR=',
    CF10.6,3X,'ANGLE OF ATTACK=',F10.6,3X,'NOFP=',I3)

910 FORMAT(10 E13.4)
920 FORMAT('THE VALUES OF A,GQ ARE',/(5E14.5))
930 FORMAT('SCALE FROM CHANGE IN LOADING COEFFICIENTS=',F10.4,5X,
  C 'CFI'=(F10.5,/)
940 FORMAT(5E14.5)
950 FORMAT('THE VALUES OF GYVCR,GZVCR ARE',/(5E14.5))
970 FORMAT('H , INTEGRATION PTS. IN REGION 3=',I5,5X,'IN CHOWS=',I5,
  5X,'IN REGION 1=',I5)
980 FORMA ('O AFTER',I3,' ITERATIONS, RESIDUE IS =',F10.6,/)
  STOP
  END

```

C	BLOCK DATA	BLK00001
C	GAUSSIAN QUADRATURE ABSCISSA AND WEIGHTS	BLK00002
C		BLK00003
C	COMMON/GAUN/GN(10,10),WN(10,10) /GAUS/ G(24),W(24)	BLK00004
C		BLK00005
	DATA GN/40*0.,	BLK00006
	1-.9061798,-.5184693,0.0, .5384643, .9061798,45*0.,	BLK00007
	2-.9719065,-.8650614,-.6794096,-.4333954,-.1498741,	BLK00008
	3-.1488743, .4311954, .6794096, .4650614, .9719065/,WN/40*0.,	BLK00009
	4-.2369269, .4786247, .5688889, .4786247, .2369267,45*0.,	BLK00010
	5-.0666713, .1494513, .2199864, .2692667, .2955242,	BLK00011
	6-.2955242, .2692667, .2199864, .1494513, .0666713/	BLK00012
	DATA G/-.9951872,-.9747286,-.9382746,-.8864155,-.8200020,-.7401242	BLK00013
	1,-.6480936,-.5454215,-.4337935,-.3150427,-.1911149,-.0640569,12*0.	BLK00014
	2/,W/.0123412,.0285314,.0442774,.0592986,.0733465,.0861902,	BLK00015
	3,.0976186,.1074443,.1155057,.1216705,.1258374,.1279382,12*0.0/	BLK00016
	END	BLK00017
		BLK00018

C	FUNCTION ALI(Y)	0001
C		0002
C	ALI(Y) PROVIDES LOWER LIMIT FOR SURFACE INTEGRAL	0003
C	ARROW WING CONFIGURATION	0004
C		0005
C	ARGUMENT LIST	0006
C	Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	0007
C		0008
C	COMMON XPT,YPT,S,M,N	0009
C	A1 = ABS(Y)/S	0010
C	RETURN	0011
C	END	0012
C		0013
C		0014
C	FUNCTION ASI(Y)	0015
C		0016
C	ASI(Y) PROVIDES LOWER LIMIT FOR SURFACE INTEGRAL	0017
C	ARROW WING CONFIGURATION	0018
C		0019
C	ARGUMENT LIST	0020
C	Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	0021
C		0022
C	COMMON XPT,YPT,S,M,N	0023
C	Y1 = ABS(Y)/S	0024
C	IF (Y1.GT.(XPT+.02)) GO TO 20	0025
C	10 AS = XPT+.02	0026
C	RETURN	0027
C	20 AS = Y1	0028
C	RETURN	0029
C	END	0030
		0031

```

C          FUNCTION MIN(S)                                0001
C  2: SEMICORD NONDIMENSIONALIZED BY ROOT SEMICORD      0002
C  ARROW WING CONFIGURATION                               0003
C                                                         0004
C          ARGUMENT LIST                                   0005
C          N: SECTION NO. INDICATOR                       0006
C          S: SPANWISE COORDINATE; NON-D BY SEMISPAN      0007
C                                                         0008
C          B=1.-ABS(S)                                     0009
C          RETURN                                           0010
C          END                                              0011
C                                                         0012
C .....                                                  0013
C                                                         0014
C          FUNCTION XLE(N,S)                               0015
C  XLE: DESCRIBES LEADING EDGE OF WING; NON-D BY WING ROOT SEMICORD 0016
C  ARROW WING CONFIGURATION                               0017
C                                                         0018
C          ARGUMENT LIST                                   0019
C          N: SECTION NO. INDICATOR                       0020
C          S: SPANWISE COORDINATE; NON-D BY SEMISPAN      0021
C                                                         0022
C          COMMON/PLAN/CR                                   0023
C          XLE = -1.*2.*ABS(S)/CR                          0024
C          RETURN                                           0025
C          END                                              0026
C                                                         0027
C                                                         0028

```

```

C          FUNCTION BS(Y)                                  0001
C  B5 PROVIDES PLANFORM LIMITS TO INTEGRATION ROUTINE     0002
C  ARROW WING CONFIGURATION                               0003
C                                                         0004
C          ARGUMENT LIST                                   0005
C          Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH 0006
C                                                         0007
C          COMMON XPT,YPT,S,M,N /PLAN/CR                 0008
C          B5 = ABS(Y)*(1.-CR)/S *CR                      0009
C          RETURN                                           0010
C          END                                              0011
C                                                         0012
C .....                                                  0013
C                                                         0014
C          FUNCTION B7(Y)                                  0015
C  B7 PROVIDES PLANFORM LIMITS FOR INTEGRATION ROUTINE    0016
C  ARROW WING CONFIGURATION                               0017
C                                                         0018
C          ARGUMENT LIST                                   0019
C          Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH 0020
C                                                         0021
C          COMMON XPT,YPT,S,M,N /PLAN/CR                 0022
C          IF (Y.GT.S*(XPT+.02-CR)/(1.-CR)) GO TO 20      0023
C          B7 = ABS(Y)*(1.-CR)/S *CR                      0024
C          RETURN                                           0025
C          20 B7 = XPT+.02                                  0026
C          RETURN                                           0027
C          END                                              0028
C                                                         0029
C                                                         0030

```

```

      FUNCTION DIDY(Y,VVORT)
C
C DIDY PROVIDES DERIVATIVE FOR JACOBIAN
C
C ARGUMENT LIST
C      Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH
C      VVORT: VORTEX SPANWISE POSITION; NON-D BY MAX. LENGTH
C
C FACTOR OF T(2*L-1) OUTSIDE OF FUNCTION
COMMON XPI,YDUM,S,M,MP /SEC/ ZVORT /PLAN/CR
YDIFF=Y-VVORT
XEDGE = CR*Y*(1.-CR)/S
XDIFF = XEDGE-XPI
TERM1=YDIFF*YDIFF+ZVORT*ZVORT
TERM3=TERM1+XDIFF*XDIFF
DIDY = YDIFF/TERM1*(2./TERM1*(1.-XDIFF/SQRT(TERM3)))
C -XDIFF/(TERM3*SQRT(TERM3))
RETURN
END

```

DIDY0001
DIDY0002
DIDY0003
DIDY0004
DIDY0005
DIDY0006
DIDY0007
DIDY0008
DIDY0009
DIDY0010
DIDY0011
DIDY0012
DIDY0013
DIDY0014
DIDY0015
DIDY0016
DIDY0017
DIDY0018
DIDY0019

```

      FUNCTION DIDZ(Y,VVORT)
C
C DIDZ PROVIDES DERIVATIVE FOR JACOBIAN
C      ARROW WING CONFIGURATION
C
C ARGUMENT LIST
C      Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH
C      VVORT: VORTEX SPANWISE POSITION; NON-D BY MAX. LENGTH
C
C FACTOR OF T(2*L-1) OUTSIDE OF FUNCTION
C
C USE MULTIPLE DEFINITION TO REDUCE TRUNCATION ERRORS
REAL*8 YDIFF,XDIFF,TERM1,TERM2,A
COMMON XPI,YPT,S,M,MP /PLAN/CR /SEC/ZVORT
YDIFF = DBLE(Y-VVORT)
XEDGE = CR*Y*(1.-CR)/S
XDIFF = DBLE(XEDGE-XPI)
TERM1 = YDIFF*YDIFF+DBLE(ZVORT*ZVORT)
A=TERM1/(XDIFF*XDIFF)
IF(A.LE..00500) GO TO 100
TERM2=TERM1+XDIFF*XDIFF
DIDZ = ZVORT*SNGL((1.-DO/TERM1*(1.-XDIFF/DSQRT(TERM2)))*
C XDIFF/(TERM2*DSQRT(TERM2)))/TERM1)
RETURN
100 DIDZ = ZVORT/4.*SNGL((1.-3.DO+5.DO*A)/XDIFF**4)
RETURN
END

```

DIDZ0001
DIDZ0002
DIDZ0003
DIDZ0004
DIDZ0005
DIDZ0006
DIDZ0007
DIDZ0008
DIDZ0009
DIDZ0010
DIDZ0011
DIDZ0012
DIDZ0013
DIDZ0014
DIDZ0015
DIDZ0016
DIDZ0017
DIDZ0018
DIDZ0019
DIDZ0020
DIDZ0021
DIDZ0022
DIDZ0023
DIDZ0024
DIDZ0025
DIDZ0026
DIDZ0027

C	FUNCTION FV(X,YPT,XPT)	FV	0001
C	FV GIVES CONTRIBUTION FROM LEADING-EDGE VORTEX TO V	FV	0002
C		FV	0003
C	ARGUMENT LIST	FV	0004
C	X: CHORDWISE INTEGRATION POINT; NON-D BY MAXIMUM LENGTH	FV	0005
C	YPT: SPANWISE LOCATION OF CONTROL POINT	FV	0006
C	XPT: CHORDWISE CONTROL POINT; NON-D BY MAXIMUM LENGTH	FV	0007
C	LET XPT=0. TO USE ON WING	FV	0008
C	COMMON/GVOR/ GYVOR(5),GVOR(5) /SEC/ZPT /VLDC/LMAX	FV	0009
C	PI=3.141593	FV	0010
C		FV	0011
C	CALCULATE LOCATION OF VORTEX	FV	0012
C	CALL FNCIN(GVOR,LMAX,X,YVORT)	FV	0013
C	CALL FNCIN(GVOR,LMAX,X,ZVORT)	FV	0014
C	CALL DFNC(GVOR,LMAX,X,DZVORT)	FV	0015
C	XDIFF=X-XPT	FV	0016
C	YDIFF=YVORT-YPT	FV	0017
C	ZDIFF=ZVORT-ZPT	FV	0018
C	FV = (ZDIFF-XDIFF*DZVORT)/(4.*PI* (XDIFF*XDIFF+YDIFF*YDIFF+ZDIFF*ZDIFF)**1.5)	FV	0019
C	RETURN	FV	0020
C	END	FV	0021
		FV	0022
		FV	0023

C	FUNCTION FW(X,YPT)	FW	0001
C	FW GIVES CONTRIBUTION OF LEADING-EDGE VORTEX TO W	FW	0002
C		FW	0003
C	ARGUMENT LIST	FW	0004
C	X: CHORDWISE INTEGRATION POINT; NON-D BY MAXIMUM LENGTH	FW	0005
C	YPT: SPANWISE CONTROL POINT; NON-D BY MAXIMUM LENGTH	FW	0006
C		FW	0007
C	TO USE ON WING, LET ZPT=0.0	FW	0008
C	COMMON XPT,YDUM,S,M,N /SEC/ZPT /VLDC/LMAX	FW	0009
C	COMMON/GVOR /GYVOR(5),GVOR(5)	FW	0010
C	PI=3.141593	FW	0011
C		FW	0012
C	CALCULATE LOCATION OF LEADING-EDGE VORTEX	FW	0013
C	CALL FNCIN(GVOR,LMAX,X,YVORT)	FW	0014
C	CALL FNCIN(GVOR,LMAX,X,ZVORT)	FW	0015
C	CALL DFNC(GVOR,LMAX,X,DZVORT)	FW	0016
C	XDIFF=X-XPT	FW	0017
C	YDIFF=YVORT-YPT	FW	0018
C	ZDIFF=ZVORT-ZPT	FW	0019
C	FW = (XDIFF*DYVORT-YDIFF)/(4.*PI* (XDIFF*XDIFF+YDIFF*YDIFF+ZDIFF*ZDIFF)**1.5)	FW	0020
C	RETURN	FW	0021
C	END	FW	0022
		FW	0023
		FW	0024

C	FUNCTION GVORTH(X,Y,S)	GVOR0001
C	GVORT CALCULATES VORTICITY STRENGTH ON WING DUE TO LEADING-EDGE	GVOR0002
C	VORTICES	GVOR0003
C	ARGUMENT LIST	GVOR0004
C	M: MODAL SPECIFICATION PARAMETER	GVOR0005
C	X: CHORDWISE POINT OF INTEREST: NON-D BY MAX. LENGTH	GVOR0006
C	Y: SPANWISE POINT OF INTEREST: NON-D BY MAX. LENGTH	GVOR0007
C	S: SEMISPAN: NON-D BY MAXIMUM LENGTH	GVOR0008
C		GVOR0009
C		GVOR0010
C		GVOR0011
C	PI=3.141593	GVOR0012
C	CONST = PI*FLOAT(2*M+1)/2.	GVOR0013
C	X*Y2=SQRT(X*X+Y*Y)	GVOR0014
C	XEDGE=X*Y2/SQRT(1.+S*S)	GVOR0015
C	GVORT=CONST*COS(CONST*XEDGE)/X*Y2	GVOR0016
C	RETURN	GVOR0017
C	END	GVOR0018

C	FUNCTION XGVL(X)	0001
C	XGVL CALCULATES CONTRIBUTION TO V FROM LEFT-HAND VORTEX	0002
C	ARGUMENT LIST	0003
C	X: CHORDWISE INTEGRATION POINT: NON-D BY MAX. LENGTH	0004
C		0005
C	COMMON XPT,YPT,S,M,MP	0006
C	PI=3.141593	0007
C	CONST=FLOAT(2*M+1)/2.*PI	0008
C	XGVL =-SIN(CONST*X)*FV(X,-YPT,XPT)	0009
C	RETURN	0010
C	END	0011
C		0012
C		0013
C	FUNCTION XGVT(Y)	0014
C	XGVT GIVES CONTRIBUTION OF WAKE VORTICITY TO SPANWISE VELOCITY	0015
C	ARROW WING CONFIGURATION	0016
C	ARGUMENT LIST	0017
C	Y: SPANWISE COORDINATE: NON-D BY MAXIMUM LENGTH	0018
C		0019
C	COMMON XPT,YPT,S,M,MP/SEC/ZPT /PLAN/CR	0020
C	PI=3.141593	0021
C	CONST=PI*FLOAT(2*M+1)/2.	0022
C	XEDGE = CR*(Y*(1.-CR))/S	0023
C	X*Y2 = SQRT(XEDGE**2+Y*Y)	0024
C	XEDGE=X*Y2/SQRT(1.+S*S)	0025
C	GDELTA=-CONST*COS(CONST*XEDGE)	0026
C	XGVT =-ZPT*GDELTA*(X(Y,YPT)-X(Y,-YPT))/(4.*PI)	0027
C	RETURN	0028
C	END	0029

C	FUNCTION XGWL(X)	0001
C	XGWL CALCULATES CONTRIBUTION TO W FROM LEFT-HAND VORTEX	0002
C		0003
C	ARGUMENT LIST	0004
C	X: CHORDWISE INTEGRATION POINT; NON-D BY MAX. LENGTH	0005
C		0006
C	COMMON XPT,YPT,S,M,N	0007
C	PI=3.141593	0008
C	CONST=FLOAT(2*M+1)/2.*PI	0009
C	XGWL =SIN(CONST*X)*FW(X,-YPT)	0010
C	RETURN	0011
C	END	0012
C		0013
C		0014
C		0015
C	FUNCTION XGWT(Y)	0016
C		0017
C	XGWT GIVES CONTRIBUTION OF WAKE VORTICITY TO DOWNWASH	0018
C		0019
C	ARGUMENT LIST	0020
C	Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	0021
C		0022
C	TO USE ON WING, LET ZPT=0.0	0023
C	COMMON XPT,YPT,S,M,MOUM /PLAN/CR	0024
C	PI=3.141593	0025
C	CONST=PI*FLOAT(2*M+1)/2.	0026
C	XEDGE = CR+Y*(1.-CR)/S	0027
C	X2Y2 = SQRT(XEDGE**2+Y*Y)	0028
C	XEDGE=X2Y2/SQRT(1.+S*S)	0029
C	GDEL1=-CONST*COS(CONST*XEDGE)	0030
C	XGWT =-GDEL1*(Y-YPT)*X11(Y,YPT)*(Y+YPT)*X1(Y,-YPT)/(4.*PI)	0031
C	RETURN	0032
C	END	0033
		0034

C	FUNCTION X1(Y,YPT)	X1 0001
C	X1 GIVES CONTRIBUTION FROM WAKE VORTICITY	X1 0002
C	ARROW WING CONFIGURATION	X1 0003
C		X1 0004
C	ARGUMENT LIST	X1 0005
C	Y: SPANWISE COORDINATE; NON-D BY MAXIMUM LENGTH	X1 0006
C	YPT: VORTEX SPANWISE LOCATION	X1 0007
C		X1 0008
C		X1 0009
C	COMMON XPT,YOUM,S,N,M /PLAN/CR /SEC/ZPT	X1 0010
C	A=(Y-YPT)*(Y-YPT)+ZPT*ZPT	X1 0011
C	XEDGE = CR+Y*(1.-CR)/S	X1 0012
C	B = XEDGE -XPT	X1 0013
C	X1=(1.-B/SQRT(A+B*B))/A	X1 0014
C	RETURN	X1 0015
C	END	X1 0016

```

SUBROUTINE CHOWS
C
C CHOWS: EVALUATION OF CHORDWISE INTEGRAL USING LEGENDRE-GAUSS
C QUADRATURE FOR CONTRIBUTION TO VELOCITY ON VORTEX
C
C DIMENSION BETA(10),THETA(10),CX(10),AL(5)
C COMMON /JACOBY/ DKDY(10),DKDZ(10),DKVDY(10),XDWDY(5),XDWDZ(5),
C /XOYV(5)/ GAUN/GN(10,10),WN(10,10)/POBTS/MUCH,NCSM
C /XOYZ(ARI4,5,5),ALS(5,5),NINC,CH(5),TKR(10),XUC,SOS,Y,Z,YMN,ZM2,
C RSOR,ETA,GAUSX(10),PO2,NCP,MP,N,X,GZ,J1,J2,GS,YMN2,ZM2,CSR
C COMMON /SOWSH/ TKVR(10),AVR(4,5,5),CVR(5)
C
C INITIALIZE SUMMATION VARIABLES
DO 1 I=1,NCP
  CR(I)=0.0
  CVR(I)=0.0
  XDWDY(I)=0.0
  XDWDZ(I)=0.0
  XDVDY(I)=0.0
1 CONTINUE
C
C CALCULATE LOCATION OF LEADING EDGE AND LOCAL SEMICHORD
C NON-D BY ROOT SEMICHORD
ELE=XLEIN,GS)
SEMICD=BIN,GS)
C
C IF RSOR=.11>0, THE INTEGRAL IS EVALUATED AS A SINGLE INTEGRAL
C IF (RSOR=0.1) 21,3,3
3 IF (NINC) 4,4,7
4 NINC=2
DO 5 I=1,NCP
C
C CALCULATE ANGULAR SPACING FOR INTEGRAL
BETA(I)=(1.-GN(I,NCP))*PO2
GX(I)=-COS(BETA(I))
DO 5 J=1,NOCM
  ALS(J,I)=SIN(BETA(I)*FLOAT(J))/FLOAT(2*(I2+J1))*4.
C
C ALS(J,I) = LOADING FUNCTIONS. REF: ASHLEY AND LANDAHL
5 CONTINUE
7 DO 6 I=1,NCP
6 GAUSX(I)=X-(ELE+SEMICD*(1.+GX(I)))
C
C GAUSX = X-X1: NON-D BY ROOT SEMICHORD
C
C CALCULATE KERNELS FOR SURFACE INTEGRALS
CALL KERNEL
WGHT=PO2
DO 20 I=1,NCP
  CW=WN(I,NCP)*SIN(BETA(I))*WGHT
  DO 20 J=1,NOCM
    CR(J)=ALS(J,I)*CW*TKR(I)*CR(J)
    CVR(J)=ALS(J,I)*CW*TKVR(I)*CVR(J)
    XDWDY(I)=XDWDY(I)+DKDY(I)*ALS(J,I)*CW
    XDWDZ(I)=XDWDZ(I)+DKDZ(I)*ALS(J,I)*CW
    XDVDY(I)=XDVDY(I)+DKVDY(I)*ALS(J,I)*CW
20 CONTINUE
GO TO 50
C
C FOR RSOR=.1<0, THE CHORDWISE INTEGRAL IS COMPUTED BY 2 LEGENDRE-
C GAUSS QUADRATURES TO HANDLE FINITE JUMP IN KERNEL AT X=X1=Y=Y1=0
C
C IF X IS OFF WING AT Y, USE SINGLE INTEGRAL
21 IF (X-ELE) 3,3,220
220 IF (X-(ELE+2.*SEMICD)) 22,3,3
22 THBD=AKCOS((ELE+SEMICD-X)/SEMICD)
  K=-1
  WGHT=THBD/2.
  DO 23 I=1,NCP
    THETA(I)=(1.-GN(I,NCP))*THBD/2.
    GAUSX(I)=X-(ELE+SEMICD*(1.-COS(THETA(I))))
23 GO TO 35

```

```

CHOW0001
CHOW0002
CHOW0003
CHOW0004
CHOW0005
CHOW0006
CHOW0007
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CHOW0065
CHOW0066
CHOW0067
CHOW0068
CHOW0069
CHOW0070
CHOW0071
CHOW0072

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24	WGHT=PO2-WGHT	CHDW0073
	K=1	CHDW0074
	DO 25 I=1,NCP	CHDW0075
	THETA(I)=THD0+I*1.-GN(I,NCP)*(PO2-THD0/2.)	CHDW0076
25	GAUSX(I)=X-TELE+SEMICD*(1.0-COS(THETA(I)))	CHDW0077
C		CHDW0078
C	GAUSX = X-X1	CHDW0079
C		CHDW0080
C	CALCULATE KERNELS FOR SURFACE INTEGRALS	CHDW0081
35	CALL KERN	CHDW0082
C		CHDW0083
C	DO CHORDWISE INTEGRALS	CHDW0084
	DO 40 I=1,NCP	CHDW0085
	CW=MN(I,NCP)*SIN(THETA(I))*WGHT	CHDW0086
	DO 40 J=1,NCCM	CHDW0087
	AL(IJ)=SIN(THETA(I)*FLOAT(IJ))/FLOAT(2*(2*J)) *4.	CHDW0088
	CN(IJ)=AL(IJ)*CW*KN(IJ)*CR(IJ)	CHDW0089
	CVR(IJ)=AL(IJ)*CW*TVR(IJ)*CVR(IJ)	CHDW0090
	XDWDY(IJ)=XDWDY(IJ)+ DKDY(IJ)*AL(IJ)*CW	CHDW0091
	XDWDZ(IJ)=XDWDZ(IJ)+ DKDZ(IJ)*AL(IJ)*CW	CHDW0092
	XDV DY(IJ)=XDV DY(IJ)+DKVDY(IJ)*AL(IJ)*CW	CHDW0093
C		CHDW0094
C	CR,CVR,XDWDY,XDWDZ,XDV DY ARE THE CHORDWISE INTEGRALS	CHDW0095
40	CONTINUE	CHDW0096
C		CHDW0097
C	LOOP FOR SECOND INTEGRAL	CHDW0098
	IFIX= 24,50,50	CHDW0099
50	CONTINUE	CHDW0100
C		CHDW0101
C	PRIMARY OUTPUT CR,CVR,XDWDY,XDWDZ,XDV DY ARE RETURNED THROUGH	CHDW0102
C	COMMON BLOCK TO CALLING PROGRAM	CHDW0103
C		CHDW0104
60	RETURN	CHDW0105
	END	CHDW0106

	SUBROUTINE COLPT(NCORD,NSPAN,XPT,YPT)	COLP0001
C		COLP0002
C	COLPT CALCULATES COLLOCATION POINTS ON PLANFORM	COLP0003
C		COLP0004
C	ARGUMENT LIST	COLP0005
C	NCORD: NO. OF CHORDWISE POINTS	COLP0006
C	NSPAN: NO. OF SPANWISE POINTS	COLP0007
C	XPT: CHORDWISE POINT; NORMALIZED BY 1	COLP0008
C	YPT: SPANWISE POINT; NORMALIZED BY 1	COLP0009
C		COLP0010
	DIMENSION XPT(5),YPT(5)	COLP0011
	PI = 3.141593	COLP0012
	DO 10 I=1,NCORD	COLP0013
10	XPT(I) = COS(PI*FLOAT(2*I-1)/FLOAT(4*NCORD))	COLP0014
	DO 20 J=1,NSPAN	COLP0015
20	YPT(J) = COS(FLOAT(IJ)*PI/FLOAT(2*NSPAN+1))	COLP0016
	RETURN	COLP0017
	END	COLP0018

C		DFNC0001
C		DFNC0002
C	DFNCTN CALCULATES DERIVATIVES OF CHEBYSHEV POLYNOMIALS	DFNC0003
C		DFNC0004
C	ARGUMENT LIST	DFNC0005
C	GF: COEFFICIENTS OF POLYNOMIAL APPROXIMATION	DFNC0006
C	M: ORDER OF POLYNOMIAL APPROXIMATION	DFNC0007
C	X: CHORDWISE ARGUMENT	DFNC0008
C	DFN: VALUE OF DERIVATIVE OF FUNCTION BEING APPROXIMATED	DFNC0009
C		DFNC0010
C	DIMENSION GF(5),CHEBY2(5),DCHEBY(5)	DFNC0011
C	CHEBY2(1)=1.	DFNC0012
C	CHEBY2(2)=4.*X*X-1.	DFNC0013
C	IF(M.LT.3) GO TO 80	DFNC0014
C	CSQ=CHEBY2(2)-1.	DFNC0015
C	DO 60 I=1,M	DFNC0016
C	CHEBY2(I)=CSQ*CHEBY2(I-1)-CHEBY2(I-2)	DFNC0017
C	60 CONTINUE	DFNC0018
C	80 CONTINUE	DFNC0019
C	DO 100 I=1,M	DFNC0020
C	DCHEBY(I)=CHEBY2(I)*FLOAT(2*I-1)	DFNC0021
C	100 CONTINUE	DFNC0022
C	DFN=0.0	DFNC0023
C	DO 500 I=1,M	DFNC0024
C	500 DFN=DFN+GF(I)*DCHEBY(I)	DFNC0025
C	RETURN	DFNC0026
C	END	DFNC0027

C		DGWC0001
C		DGWC0002
C	DGWMH CALCULATES CONTRIBUTION TO W FROM LEADING-EDGE VORTICES	DGWC0003
C		DGWC0004
C	ARGUMENT LIST	DGWC0005
C	DGWM2: CONTRIBUTION TO Y DERIVATIVE	DGWC0006
C	DGWM4: CONTRIBUTION TO Z DERIVATIVE	DGWC0007
C	GWGM1: CONTRIBUTION TO W VELOCITY	DGWC0008
C		DGWC0009
C	DIMENSION TCHEBY(5),UCHEBY(5),SUM2(5,5),SUM4(5,5),A(4),B(4),	DGWC0010
C	DGWM2(5,5),DGWM4(5,5),GWGM1(5),SUM(5),DFWDY(5),DFWDZ(5)	DGWC0011
C	COMMON XPI,YPI,S,MDUM,MPDUM /GAUS/G(24),W(24)	DGWC0012
C	COMMON/ GVCR/GYVOR(5),GZVOR(5) /VLOC/LMAX/MODES/NOCM,NOSM	DGWC0013
C		DGWC0014
C	PI= 3.141593	DGWC0015
C	CONST2= 1./ (8.*PI)	DGWC0016
C		DGWC0017
C	CONST2 = 1/(4.*PI) * 1/2 TO SCALE INTEGRAL	DGWC0018
C		DGWC0019
C	INITIALIZE SUMMATION VARIABLES	DGWC0020
C	DO 100 I=1,5	DGWC0021
C	SUM(I)=0.	DGWC0022
C	DO 100 J=1,5	DGWC0023
C	SUM2(I,J) = 0.	DGWC0024
C	SUM4(I,J) = 0.	DGWC0025
C	100 CONTINUE	DGWC0026
C		DGWC0027
C	WANT FOUR CALLS TO INTEGRATION ROUTINE	DGWC0028
C	DATA A,B/0.,.125,.25,.5,.125,.25,.5,1./	DGWC0029
C	DO 300 IC=1,4	DGWC0030
C	BMA = B(IC)-A(1C)	DGWC0031
C	BPA=B(1C)+ A(1C)	DGWC0032
C		DGWC0033
C	DO CHORDWISE INTEGRALS BY 24-POINT GAUSS. QUAD.	DGWC0034
C	DO 200 J=1,24	DGWC0035
C		DGWC0036

C CALCULATE INTERMEDIATE FACTORS	DGWC0017
$X = (RM3*G13)*PPA)/Z$	DGWC0018
C	DGWC0019
C CALCULATE CHERYSHEV POLYNOMIALS	DGWC0040
CALL TCHERBY(LMAX,X,TCHBY,UCHEBY)	DGWC0041
C	DGWC0042
C CALCULATE VORTEX POSITION AND DERIVATIVES	DGWC0043
CALL FNCT1(GVDR,LMAX,X,VVORT)	DGWC0044
CALL FNCT1(GZDR,LMAX,X,ZVORT)	DGWC0045
CALL FNCT1(GVDR,LMAX,X,DVVORT)	DGWC0046
C	DGWC0047
C CALCULATE INTERMEDIATE FACTORS	DGWC0048
XDIFF = X-XPT	DGWC0049
YDIFF = YVORT-YPT	DGWC0050
YSUM = YVORT*YPT	DGWC0051
TERM1 = XDIF*XDIF + YDIFF*YDIFF + ZVORT*ZVORT	DGWC0052
TERM2 = XDIF*XDIF + YSUM*YSUM + ZVORT*ZVORT	DGWC0053
TERM3 = TERM1*SQRT(TERM1)	DGWC0054
TERM4 = TERM2*SQRT(TERM2)	DGWC0055
C	DGWC0056
C CONTRIBUTION TO DOWNWASH FROM LEADING-EDGE VORTEX	DGWC0057
FW = (XDIF*DYVORT-YDIFF)/TERM3 + (XDIF*DYVORT-YSUM)/TERM4	DGWC0058
DO 140 L=1,LMAX	DGWC0059
C	DGWC0060
C CHANGE IN DOWNWASH CONTRIBUTION DUE TO CHANGE IN VORTEX POSITION	DGWC0061
DFWZ(L) = (XDIF*FLOAT(2*L-1)*UCHEBY(L)-TCHBY(L)	DGWC0062
-3.0*(XDIF*DYVORT-YDIFF)*YDIFF*TCHBY(L)/TERM3	DGWC0063
+ (XDIF*FLOAT(2*L-1) * UCHEBY(L)-TCHBY(L)	DGWC0064
-3.0*(XDIF*DYVORT-YSUM)*YSUM*TCHBY(L)/TERM2)/TERM4	DGWC0065
DFWZ(L) = ZVORT* TCHBY(L)*((XDIF*DYVORT-YDIFF)/	DGWC0066
C (TERM1*TERM3) + (XDIF*DYVORT-YSUM)/(TERM2*TERM4))	DGWC0067
140 CONTINUE	DGWC0068
DO 150 MQ=1,NOCM	DGWC0069
M=MQ-1	DGWC0070
CONST = FLOAT(2*M+1)/2.*PI	DGWC0071
GGAM = SIN(CONST*X)	DGWC0072
C	DGWC0073
C GGAM = LEADING-EDGE VORTEX STRENGTH	DGWC0074
SUM(MQ) = GGAM*FW*W(L) + SUM(MQ)	DGWC0075
DO 150 L=1,LMAX	DGWC0076
SUM2(MQ,L) = GGAM*DFWZ(L)*W(L) + SUM2(MQ,L)	DGWC0077
SUM4(MQ,L) = GGAM*DFWZ(L)*W(L) + SUM4(MQ,L)	DGWC0078
150 CONTINUE	DGWC0079
200 CONTINUE	DGWC0080
C DETERMINE WEIGHTING FACTOR	DGWC0081
CONST=1.	DGWC0082
IF (IC.GT.1) CONST=.5	DGWC0083
DO 400 MQ=1,NOCM	DGWC0084
SUM(MQ) = CONST*SUM(MQ)	DGWC0085
DO 400 L=1,LMAX	DGWC0086
SUM2(MQ,L) = CONST*SUM2(MQ,L)	DGWC0087
SUM4(MQ,L) = CONST*SUM4(MQ,L)	DGWC0088
400 CONTINUE	DGWC0089
300 CONTINUE	DGWC0090
DO 500 MQ=1,NOCM	DGWC0091
GGW1(MQ) = CONST2*SUM(MQ)	DGWC0092
DO 500 L=1,LMAX	DGWC0093
DGWC2(MQ,L) = CONST2*SUM2(MQ,L)	DGWC0094
DGWC4(MQ,L) = -3.0 CONST2*SUM4(MQ,L)	DGWC0095
500 CONTINUE	DGWC0096
C	DGWC0097
C PRIMARY OUTPUTS. DGWGM1,DGWCMP2,DGWCMP4 PASSED THROUGH	DGWC0098
C ARGUMENT LIST TO CALLING PROGRAM	DGWC0099
RETURN	DGWC0100
END	DGWC0101

```

C SUBROUTINE DGMVIDGWTY,DGVTY,DGWTZ,DGVTZ)
C
C DGMV PROVIDES CONTRIBUTION TO JACOBIAN FROM WAKE
C
C ARGUMENT LIST
C DGWTY: CHANGE IN GWT FROM CHANGE IN YV
C DGVTY: CHANGE IN GVT FROM CHANGE IN YV
C DGWTZ: CHANGE IN GWT FROM CHANGE IN ZV
C DGVTZ: CHANGE IN GVT FROM CHANGE IN ZV
C
C FACTOR OF  $(1/2 \cdot L - 1)$  OUTSIDE OF SUBROUTINE
C
C COMMON XPI,YVORT,S,MDUM,MPDUM/SEC/ZVORT/GAUS/GI24,WE24)
C /MODES/NOCM,NOCM /PLAN/CR
C DIMENSION SUM(5,4),DGWTY(5),DGVTY(5),DGWTZ(5),DGVTZ(5)
C
C PI=3.141593
C
C INITIALIZE SUMMATION VARIABLES
C DATA SUM/20*0.0/
C
C DO SPANWISE INTEGRAL FROM 0. TO S
C DO 200 J=1,24
C
C CALCULATE ABSCISSAS FOR GAUSSIAN QUADRATURE
C Y=S*(1.+GJ11)/2.
C
C CALCULATE INTERMEDIATE FACTORS
C DIPYD=DIDY(Y,YVORT)
C DIMYD=-DIDY(Y,-YVORT)
C DIPDZ=DIDZ(Y,YVORT)
C DIMDZ=DIDZ(Y,-YVORT)
C YDIFF=Y-YVORT
C YSUM=Y+YVORT
C XEDGE = CR*Y*(1.-CR)/S
C X2Y2 = SQRT(XEDGE**2+Y*Y)
C
C
C XEDGE=X2Y2/SQRT(1.+S*S)
C XIP=XI(Y,YVORT)
C XIM=XI(Y,-YVORT)
C
C DO FOR ALL MODES
C DO 100 MQ=1,NOCM
C M=MQ-1
C CONST=PI*FLOAT(2*M+1)/2.
C GDELTA=-CONST*COS(CONST*XEDGE)
C YDGWTY=GDELTA*(XIP-XIM-YDIFF*DIPYD-YSUM*DIMYD)
C YDGVTY=GDELTA*(DIPYD-DIMYD)
C YDGWTZ=GDELTA*(YDIFF*DIPDZ+YSUM*DIMDZ)
C YDGVTZ=GDELTA*(ZVORT*(DIPDZ-DIMDZ)+XIP-XIM)
C SUM(MQ,1)=SUM(MQ,1)+YDGWTY*W(J)
C SUM(MQ,2)=SUM(MQ,2)+YDGVTY*W(J)
C SUM(MQ,3)=SUM(MQ,3)+YDGWTZ*W(J)
C SUM(MQ,4)=SUM(MQ,4)+YDGVTZ*W(J)
C 100 CONTINUE
C 200 CONTINUE
C
C CONST FROM S/2. * 1/(4.*PI)
C CONST=S/(8.*PI)
C DO 300 K=1,NOCM
C DGWTY(MQ)=CONST*SUM(MQ,1)
C DGVTY(MQ)=-ZVORT*CONST*SUM(MQ,2)
C DGWTZ(MQ)=-CONST*SUM(MQ,3)
C DGVTZ(MQ)=-CONST*SUM(MQ,4)
C DO 300 K=1,4
C SUM(MQ,K)=0.0
C 300 CONTINUE
C
C RESULTS PASSED TO GVCYR THROUGH ARGUMENT LIST
C RETURN
C END

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DGWV0001
DGWV0002
DGWV0003
DGWV0004
DGWV0005
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DGWV0068
DGWV0069
DGWV0070

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C
C SUBROUTINE FNCTN(GF,M,X,FN)
C
C FNCTN EVALUATES CHEBYSHEV POLYNOMIALS
C
C ARGUMENT LIST
C   GF: COEFFICIENTS OF POLYNOMIAL APPROXIMATION
C   M: ORDER OF POLYNOMIAL APPROXIMATION
C   X: CHORDWISE POINT OF INTEREST
C   FN: VALUE OF FUNCTION
C
C DIMENSION GF(5),CHERY(5)
C USE CHEBYSHEV POLYNOMIALS OF THE FIRST KIND
C   CHERY(1)=X
C   CSQ=4.*X*X-2.
C   CHERY(2)=(CSQ-1.)*X
C   IF(M.LT.3) GO TO 80
C   DO 60 I=1,M
C   CHERY(I)=CSQ*CHERY(I-1)-CHERY(I-2)
C   60 CONTINUE
C CALCULATE FUNCTION FROM POLYNOMIAL CONTRIBUTIONS
C   80 FN=0.0
C   DO 500 I=1,M
C   FN=FN+GF(I)*CHERY(I)
C   RETURN
C   END

```

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FNCT0001
FNCT0002
FNCT0003
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FNCT0020
FNCT0021
FNCT0022
FNCT0023
FNCT0024
FNCT0025

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C
C SUBROUTINE FUNCTN(NOSM,S,F)
C
C FUNCTN: SPANWISE LOADING FUNCTIONS
C
C ARGUMENT LIST
C   NOSM: NO. OF SPANWISE HORSESHOE VORTEX MODES
C   S: SPANWISE COORDINATE; NON-D BY SEMISPAN
C   F: VALUE OF FUNCTION
C
C DIMENSION F(5)
C
C USE CHEBYSHEV POLYNOMIALS AS LOADING FUNCTIONS
C   SQ=S*S
C   R=SQ*(1.0-SQ)
C   F(1)=R
C   F(2)=4.*(4.*SQ-1.)*R
C   C=4.*SQ-2.
C   DO 20 J=3,NOSM
C   F(J)=C*(F(J-1)-F(J-2))
C   RETURN
C   END

```

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FUNC0001
FUNC0002
FUNC0003
FUNC0004
FUNC0005
FUNC0006
FUNC0007
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FUNC0019
FUNC0020
FUNC0021

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      SUBROUTINE GAUSID(C,D,ENTGL,F)
C
C   GAUSID PERFORMS 1-D INTEGRATION BY 24-POINT GAUSSIAN QUADRATURE
C
C   ARGUMENT LIST
C       C: LOWER LIMIT OF INTEGRAL
C       D: UPPER LIMIT OF INTEGRAL
C       ENTGL: VALUE OF INTEGRAL
C       F: FUNCTION TO BE INTEGRATED
C
C   COMMON /GAUS/G(24),W(24)
C
C   INITIALIZE SUMMATION VARIABLES
      SUM=0.0
      DC=D-C
      DAC=D+C
      DO 200 J=1,24
      XNEW=(DC*G(J)+DAC)/2
      SUM = SUM+F(XNEW)*W(J)
200 CONTINUE
      ENTGL = DC*SUM/2.
      RETURN
      END

```

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GAUS0001
GAUS0002
GAUS0003
GAUS0004
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GAUS0006
GAUS0007
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GAUS0010
GAUS0011
GAUS0012
GAUS0013
GAUS0014
GAUS0015
GAUS0016
GAUS0017
GAUS0018
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GAUS0020
GAUS0021
GAUS0022
GAUS0023

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      SUBROUTINE GVCTR(NOCPI)
C
C   GVCTR CALCULATES FORCE ON VORTEX AND CORRESPONDING DERIVATIVES
C
C   ARGUMENT LIST
C       NOCP: NO. OF COLLOCATION POINTS
C
C   COMMON/MDDES/NOCP,NOSM/VLOC/LMAX/GVOR/GVORIS,GZVORIS)
C /GVIC/ A(5,5),G(15),V1,FSUBY(5),FSUBZ(5),PI,SIN(PI/NOCP)
C /VORT/YVOR,ZVOR/ /XPI,YVORT,S,M,MP/ /SIC/ZVORT
C /YACGB/XACGB(35,35),SAWH(5,5),SAVM(5,5),
C DAWDY(5,5),DAWDZ(5,5),DAVDY(5,5),DAVDZ(5,5)
C DIMENSION SGVV(5),SGWV(5),DGWDY(5),DGWDZ(5),
C CDGVDY(5),CDGVDZ(5),DGWY(5),DGWZ(5),DGVZ(5),TCHEBY(5),
C CUCHERY(5),DGVGM2(5,5),DGVGM2(5,5),DGVGM4(5,5),DGVGM4(5,5)
C EXTERNAL XGWT, XCVT,XGWL,XGVL
C
C   NI=NI+1
C   NMODT= NOCP*NOSM+NOCP
C
C   NMODT = TOTAL NO. OF VORTICITY MODES
C
C   CALCULATE VORTEX LOCATION AND DERIVATIVES
      CALL FNCTNIGVOR,LMAX,XPI,YVORT)
      CALL FNCTNIGVOR,LMAX,XPI,ZVORT)
      CALL DFNCT(GVOR,LMAX,XPI,DYVORT)
      CALL DFNCT(GVOR,LMAX,XPI,DZVORT)
C
C   OUTPUT CONTROL POINT LOCATION
      WRITE(6,910) XPI,YVORT,ZVORT
C
C   CALCULATE LEADING-EDGE VORTEX STRENGTH AND DERIVATIVE
C   CALCULATE GAMMA AND DGAMMA/DX
      DGAMM = 0.
      GAMMA=0.0
      DO 600 MQ=1,NOCP

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GVCT0001
GVCT0002
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GVCT0035
GVCT0036

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GVC100117
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GVC10106
GVC10107
GVC10108

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221 FSURY(1)-NCCP ) = FY
    FSUBZ(1)-NCCP ) = FZ
C
C CALCULATE DERIVATIVES W.R.T. VORTEX POSITION COEFFICIENTS
C
C CALCULATE CHERYSEV POLYNOMIALS
    CALL TUCHEB (LMAX,XPI,TCHFBY,UCHEBY)
C
C CONTRIBUION FROM LEADING-EDGE VORTEX
    CALL VORINT(DGWM2,DGVM2,DGWM4,DGVM4)
C
C CONTRIBUION FROM WAKE
    CALL DGWIDGWTY,DGVTY,DGWTZ,DGVTZ)
    TERM5=TERM4*HLOT4
    TERM6=1.-XDIFF/ROOT4
C
C CONTRIBUION FROM VORTEX AFT OF X = L.
    DGWG1=1./TERM2*( TERM6 *(-1.+2.*YSUM*YSUM/TERM2)
    C=YSUM*YSUM*XDIFF/ TERM5 1/(4.*PI)
    DGVG1=ZDIFF*YSUM/TERM2*12./TERM2* TERM6
    C=XDIFF/ TERM5 1/(4.*PI)
    DGWG1=DGVG1
    DGVG1=1./TERM2*( TERM6 *(-1.+2.*ZDIFF*ZDIFF/TERM2)
    C=ZDIFF*XDIFF*ZDIFF/TERM5 1/(4.*PI)
C
C CONTRIBUION FOR ALL MODES
    DO 220 LDUM=1,LMAX
    L=LDUM
C
C INITIALIZE SUMMATION VARIABLES
    DWIDGY=0.0
    DVIDGY=0.0
    DWIDGZ=0.0
    DVIDGZ=0.0
C
C CALCULATE CONTRIBUION FROM LEADING VORTICES AFT OF WING

    DGWM1=DWG1*(1.+TCHBY(L))
    DGVM1=DVG1*(1.+TCHBY(L))
    DGWM3=DWG3*(1.-TCHBY(L))
    DGVM3=DVG3*(1.-TCHBY(L))
    DO 310 MO=1,NOCM
C
C CALCULATE CONTRIBUION FROM WAKE
    DGWTDY=TCHBY(L)*DGWTY(MQ)
    DGVTDY=TCHBY(L)*DGVTY(MQ)
    DGWTDZ=DGWTZ(MQ)*TCHBY(L)
    DGVTZ=DGVTZ(MQ)*TCHBY(L)
C
C CALCULATE CONTRIBUION FROM LEADING-EDGE VORTEX
    DGWMY=DGWM1+DGWM2(MQ,L)
    DGVMY=DVG1+DVG2(MQ,L)
    DGWMZ=DGWM3+DGWM4(MQ,L)
    DGVMZ=DVG3+DVG4(MQ,L)
C
C CALCULATE CONTRIBUION FROM WING
    DSGWDY=DGWDY(MQ)*TCHBY(L)
    DSGVDY=DGVDY(MQ)*TCHBY(L)
    DSGWZ=DGWZ(MQ)*TCHBY(L)
    DSGVZ=DGVZ(MQ)*TCHBY(L)
C
C SUM CONTRIBUIONS FOR COEFFICIENTS
    DWIDGY=DWIDGY+GQ(MQ)*(DGWTDY+DGWMY+DSGWDY)
    DVIDGY=DVIDGY+GQ(MQ)*(DGVTDY+DGVY+DSGVDY)
    DWIDGZ=DWIDGZ+GQ(MQ)*(DGWTDZ+DGWMZ+DSGWZ)
    DVIDGZ=DVIDGZ+GQ(MQ)*(DGVTZ+DGVZ+DSGVZ)
C
    DGWM1=-DGWM1
    DGVM1=-DGVM1
    DGWM3=-DGWM3
    DGVM3=-DGVM3
C
C CONTRIBUION FROM HORSESHOE VORTICITY

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GVC10109
 GVC10110
 GVC10111
 GVC10112
 GVC10113
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 GVC10172
 GVC10173
 GVC10174
 GVC10175
 GVC10176
 GVC10177
 GVC10178
 GVC10179
 GVC10180

```

DO 310 MPP=1,NOSM
  DV1DGY = DV1DGY + A(MQ,MPP)*DAVDY(MQ,MPP)*TCHBY(L)
  DV1DGY = DV1DGY + A(MQ,MPP)*DAVDY(MQ,MPP)*TCHBY(L)
  DV1DGY = DV1DGY + A(MQ,MPP)*DAVDY(MQ,MPP)*TCHBY(L)
  DV1DGY = DV1DGY + A(MQ,MPP)*DAVDY(MQ,MPP)*TCHBY(L)
  DV1DGY = DV1DGY + A(MQ,MPP)*DAVDY(MQ,MPP)*TCHBY(L)
310 CONTINUE

C
C CALCULATE CONTRIBUTION TO JACOBIAN
C CALCULATE DFV/DGYV DFZ/DGYV DFV/DGZV DFZ/DGZV
C
  DFYDGY = DV1DGY
  DFYDGY = (FLCAT(2*L-1)*UCHEBY(L)-DV1DGY2)-DGAMM*TCHBY(L)/GAMMA
  DFZDGY = (FLCAT(2*L-1)*UCHEBY(L)-DV1DGY)*DGAMM*TCHBY(L)/GAMMA
  DFZDGY = DV1DGY2
  N2 = L*NYDDT
  XACOB(N1,N2) = DFYDGY
  XACOB(N1,NCFP,N2) = DFZDGY
  N2 = L*NYDDT
  XACOB(N1,N2) = DFYDGY
  XACOB(N1,NCFP,N2) = DFZDGY
220 CONTINUE

C
C OUTPUTS PASSED TO MAIN PROGRAM THROUGH COMMON STATEMENTS
C
910 FORMAT('X =',F10.4,5X,'YV(X) =',F10.4,5X,'ZV(X) =',F10.4)
930 FORMAT('GAMMA =',E12.4,5X,'V =',E12.4,5X,'W =',E12.4,5X,'FY =',
  E12.4,5X,'FZ =',E12.4)
  RETURN
  END

```

GVC10181
 GVC10182
 GVC10183
 GVC10184
 GVC10185
 GVC10186
 GVC10187
 GVC10188
 GVC10189
 GVC10190
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 GVC10192
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 GVC10199
 GVC10200
 GVC10201
 GVC10202
 GVC10203
 GVC10204
 GVC10205
 GVC10206
 GVC10207
 GVC10208

```

SUBROUTINE GWVDISGVV,SGWV,DGVGY,DGVZ,DGWGY,DGWZ
C
C CALCULATES SGWV,SGWV AND THEIR DERIVATIVES FOR PROGRAM V
C
C ARGUMENT LIST
C SGWV: CONTRIBUTION TO V FROM WING VORTICITY
C SGWV: CONTRIBUTION TO W FROM WING VORTICITY
C DGVGY: CHANGE IN GVV DUE TO CHANGE DGY
C DGVZ: CHANGE IN GVV DUE TO CHANGE DGZ
C DGWGY: CHANGE IN GVV DUE TO CHANGE DGY
C DGWZ: CHANGE IN GVV DUE TO CHANGE DGZ
C
C COMMON XPT,YPT,S,MDUM,MPDUM /GAUS/G124),W124) /PLAN/CR
COMMON/SEC/ZPT /MODES/NOCH,NOSM
DIMENSION SGWV(5),DGWGY(5),DGVZ(5),DGWZ(5),
C SGWV(5),INTGD(5,6),SUM(5,6)
C
  PI=3.141593
C
C INITIALIZE SUMMATION VARIABLES
  DO 10 I=1,5
  DO 10 J=1,6
    INTGD(I,J) = 0.
    SUM(I,J) = 0.
  10 CONTINUE
C
C DIVIDE WING INTO TWO SECTION ABOUT XPLUS
  XPLUS = XPT+.02
  IF (XPLUS.GT..98) XPLUS = .98
  IF (XPLUS.LT..CR) GO TO 50
C
C ESTABLISH LIMITS FOR SPANWISE INTEGRATION
  CPM1 = S*(1.+XPLUS)/2.
  DPM1 = S*(1.-XPLUS)/2.
  CPM2 = S*(1.-XPLUS)/(1.-CR)*2.
  DPM2 = S*(1.-2.*CR*XPLUS)/(1.-CR)*2.

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GWV00001
 GWV00002
 GWV00003
 GWV00004
 GWV00005
 GWV00006
 GWV00007
 GWV00008
 GWV00009
 GWV00010
 GWV00011
 GWV00012
 GWV00013
 GWV00014
 GWV00015
 GWV00016
 GWV00017
 GWV00018
 GWV00019
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 GWV00031
 GWV00032
 GWV00033
 GWV00034
 GWV00035
 GWV00036

30 CONTINUE	GWV00037
C	GWV00038
C DO SURFACE INTEGRAL IN TWO 24X24 GAUSS. QUADRATURES	GWV00039
DO 600 ICGUAT = 1,2	GWV00040
C	GWV00041
C DO SPANWISE INTEGRAL	GWV00042
DO 200 J=1,24	GWV00043
IF (ICOUNT.EQ.2) GO TO 30	GWV00044
IF (XPLUS.GE.CR) GO TO 25	GWV00045
20 Y = S*XPLUS*G(IJ)	GWV00046
B = XPLUS	GWV00047
GO TO 27	GWV00048
25 Y = CPRIM1*G(IJ) + DPRIM1	GWV00049
B = B7(Y)	GWV00050
27 A = A7(Y)	GWV00051
GO TO 40	GWV00052
30 IF (XPLUS.GE.CR) GO TO 35	GWV00053
Y = S*G(IJ)	GWV00054
GO TO 37	GWV00055
35 Y = CPRIM2*G(IJ) + DPRIM2	GWV00056
37 A = A5(Y)	GWV00057
B = B5(Y)	GWV00058
40 CONTINUE	GWV00059
AP = B-A	GWV00060
BP = B-A	GWV00061
C	GWV00062
C DO CHORDWISE INTEGRAL	GWV00063
DO 100 I=1,24	GWV00064
X = (AP*G(I) + BP)/2.	GWV00065
C	GWV00066
C CALCULATE INTERMEDIATE QUANTITIES	GWV00067
XDIFF = X - XPT	GWV00068
YDIFF = Y - YPT	GWV00069
TERM1 = YDIFF*YDIFF + ZPT*ZPT + XDIFF*XDIFF	GWV00070
ROOT1 = SQRT(TERM1)	GWV00071
TERM2 = TERM1*ROOT1	GWV00072
TERM3 = (XDIFF*X + YDIFF*Y) / TERM1	GWV00073
C	GWV00074
C DO FOR ALL MODES	GWV00075
DO 400 MQ=1,NOCM	GWV00076
M=MQ-1	GWV00077
GVORS=GVORT(M,X,Y,S)	GWV00078
GDELT=-Y*GVORS	GWV00079
XGWM=GDELT/TERM2	GWV00080
XGWM=GVORS*TERM3	GWV00081
ENTGD(MQ,4)=ENTGD(MQ,4)+XGWM *W(I)/ROOT1	GWV00082
ENTGD(MQ,1)=ENTGD(MQ,1)+XGWM *W(I)	GWV00083
XDGWDY=(GDELT+3.*XGWM*YDIFF)/TERM2	GWV00084
ENTGD(MQ,5)=ENTGD(MQ,5)+XDGWDY*W(I)	GWV00085
XDGVDY=XGWM*YDIFF/TERM1	GWV00086
ENTGD(MQ,2)=ENTGD(MQ,2)+XDGVDY*W(I)	GWV00087
XDGVDZ=XGWM*(1.-3.*ZPT*ZPT /TERM1)	GWV00088
ENTGD(MQ,3)=ENTGD(MQ,3)+XDGVDZ*W(I)	GWV00089
ENTGD(MQ,6) = ENTGD(MQ,6) + W(I)*XGWM/(ROOT1*TERM1)	GWV00090
400 CONTINUE	GWV00091
100 CONTINUE	GWV00092
DO 300 MQ=1,NOCM	GWV00093
DO 300 MC=1,6	GWV00094
SUM(MQ,MC)=SUM(MQ,MC)+ENTGD(MQ,MC)*W(IJ)*AP	GWV00095
ENTGD(MQ,MC)=0.0	GWV00096
300 CONTINUE	GWV00097
200 CONTINUE	GWV00098
C	GWV00099
C SELECT PROPER MULTIPLYING FACTOR	GWV00100
IF (ICOUNT.EQ.2) GO TO 130	GWV00101
IF (XPLUS.GE.CR) GO TO 125	GWV00102
CONST = XPLUS	GWV00103
GO TO 140	GWV00104
125 CONST = (1.+XPLUS)*(1.-CR)/(CR*(1.-XPLUS))	GWV00105
GO TO 140	GWV00106
130 CONST = 5/18.*PI	GWV00107
IF (XPLUS.LT.CR) GO TO 140	GWV00108

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      CONST = CONST*CR*(1.-XPLUS)/(1.-CR)/2.
140 CONTINUE
      DO 500 MQ=1,NOCM
C
C   CALCULATE CONTRIBUTION TO W,V VELOCITY AT VORTEX FROM WING VORTICITY
C   WHICH FEEDS LEADING-EDGE VORTICES
      SGVVMQ1=CONST*SUM(MQ,4)
      SUM(MQ,4) = SGVVMQ1
      SUM(MQ,1) = SUM(MQ,1) +CONST
      SGVVMQ1=-ZPT*SUM(MQ,1)
C
C   CALCULATE DERIVATIVES
      DGWDY(MQ)=CONST*SUM(MQ,5)
      SUM(MQ,5) = DGWDY(MQ)
      SUM(MQ,2) = SUM(MQ,2)+CONST
      DGVDY(MQ)=-3.*ZPT *SUM(MQ,2)
      SUM(MQ,3) = SUM(MQ,3)+CONST
      DGVDZ(MQ)=- SUM(MQ,3)
      SUM(MQ,6) = SUM(MQ,6)+CONST
      DGWDZ(MQ) = -3.*ZPT*SUM(MQ,6)
500 CONTINUE
600 CONTINUE
C
C   PRIMARY OUTPUT  SGVV,SGWC,DGVDY,DGVDZ,DGWDY,DGWDZ
C   PASSED TO CALLING PROGRAM THROUGH ARGUMENT LIST
C
      RETURN
      END

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GWV00109
GWV00110
GWV00111
GWV00112
GWV00113
GWV00114
GWV00115
GWV00116
GWV00117
GWV00118
GWV00119
GWV00120
GWV00121
GWV00122
GWV00123
GWV00124
GWV00125
GWV00126
GWV00127
GWV00128
GWV00129
GWV00130
GWV00131
GWV00132
GWV00133
GWV00134
GWV00135
GWV00136

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      SUBROUTINE KERNL
C
C   KERNL: EVALUATION OF KERNEL FUNCTIONS FROM STEADY,NON-PLANAR,
C   INCOMPRESSIBLE LIFTING SURFACE THEORY. REF: ASHLEY AND LANDAHL
C
      COMMON /JACGB/DKDY(10),DKDZ(10),DKVDY(10),XWDY(5),XWDZ(5),
C   XVDY(5) /SDWSH/ TKVR(10),AVR(4,5,5),CVR(5)
C   /KDVZ/AR(4,5,5),ALS(5,5),NINC,CR(5),TKR(10),XOC,SOS,Y,Z,YMN,ZM2,
C   RSRQ,ETA,GAUSX(10),PO2,NCP,MP,N,X,C2,J1,J2,GS,YMN2,ZM22,CSR
C
      5 DO 10 I=1,NCP
C   NON-D X-XL BY SEMISPAN
      XME=GAUSX(1)*CSR
      XME2=XME*XME
      R2=RSQR+XME2
      R=SQRT(R2)
      G=1.0+XME/R
      C=XME/(R2*R)
      D=4.*G/(R*SCR+RSQR)
      E=2./HSCR+3./P2
      H=2./HSCR+G+C
      F=G-ZM22*H
C
C   CALCULATE KERNELS FOR SIDEWASH,DOWNWASH, AND DERIVATIVE INTEGRALS
      TKV(1)=ZM2*YMN*H
      TKR(1)= F
      DKDY(1)=YMN*(1-2.*F/RSQR+ZM22*D+C*(-1.+F*ZM22))
      DKVDY(1)= (2.*YMN2/RSQR-1.)*ZM2*H +ZM2*YMN2*(D+C*E)
      10 DKDZ(1)=ZM2*(1-2.*F/RSQR-2.*H+ZM22*D+C*(-1.+ZM22*E))
C
C   PRIMARY OUTPUT  TKVR,TKR,DKDY,DKVDY,DKDZ
C   PASSED THROUGH COMMON BLOCK TO CALLING PROGRAM
C
      15 RETURN
      END

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      SUBROUTINE TUCHB(LMAX,X,TCHBY,UCHBY)
C
C CALCULATES U(2*L-1) T(2*L-1) FOR SUBROUTINE VORINT
C
C      ARGUMENT LIST
C      LMAX: ORDER OF POLYNOMIAL APPROXIMATION
C      X: CHORDWISE POINT OF INTEREST
C      TCHBY: CHERYSHIEV POLYNOMIAL OF FIRST KIND
C      UCHBY: CHERYSHIEV POLYNOMIAL OF SECOND KIND
C
      DIMENSION TCHBY(5),UCHBY(5)
      TCHBY(1)=X
      UCHBY(1)=1.
      CSQ=4.*X*X-2.
      UCHBY(2)=CSQ+1.
      TCHBY(2)=TCSQ-1.)*X
      IF(LMAX.LT.3) GO TO 80
      DO 60 L=3,LMAX
      TCHBY(L)=CSQ*TCHBY(L-1)-TCHBY(L-2)
      UCHBY(L)=CSQ*UCHBY(L-1)-UCHBY(L-2)
60 CONTINUE
80 RETURN
      END

```

TUCH0001
 TUCH0002
 TUCH0003
 TUCH0004
 TUCH0005
 TUCH0006
 TUCH0007
 TUCH0008
 TUCH0009
 TUCH0010
 TUCH0011
 TUCH0012
 TUCH0013
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 TUCH0015
 TUCH0016
 TUCH0017
 TUCH0018
 TUCH0019
 TUCH0020
 TUCH0021
 TUCH0022
 TUCH0023

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      SUBROUTINE VORINT(DGVGM2,DGVGM2,DGVGM4,DGVGM4)
C
C VORINT CALCULATES EFFECT OF VORTEX CONTRIBUTIONS DUE TO CHANGE IN
C VORTEX LOCATION
C
C      ARGUMENT LIST
C      DGVGM2: CHANGE IN W CONTRIBUTION DUE TO CHANGE IN GYV
C      DGVGM2: CHANGE IN V CONTRIBUTION DUE TO CHANGE IN GYV
C      DGVGM4: CHANGE IN W CONTRIBUTION DUE TO CHANGE IN GZV
C      DGVGM4: CHANGE IN V CONTRIBUTION DUE TO CHANGE IN GZV
C
      COMMON XPI,YPI,S,PDUM,MPDUM/SEC/ZPT /CAUS/G(24),V(24)
C /GVOR/GYVOR(5),GVOR(5)/MDES/MCOSM,MODM/VLNC/LMAX
      DIMENSION TCHBY(5),UCHBY(5),CHEBY(5),SUM(5,5,4),
C DGVGM2(5,5),DGVGM2(5,5),DGVGM4(5,5),DGVGM4(5,5)
C ,DFVDY(5),DFVDY(5),DFVU2(5),DFVU2(5)
C
C INITIALIZE SUMMATION VARIABLES
      DATA SUM /100*0.0/
      PI=3.141593
C
C ALL INTEGRALS FROM 0 TO 1 DONE IN 1 24X24 LGOP
      CALL TUCHB(LMAX,XPI,CHEBY1,UCHBY1)
C
C DO CHORDWISE INTEGRAL FROM 0 TO 1
      DO 200 J=1,24
      X=(G1J)*1.1/2.
C
C CALCULATE ARGUMENTS
      CALL TUCHB(LMAX,X,TCHBY,UCHBY)
C
C CALCULATE VORTEX LOCATION
      CALL FNCT1(GYVOR,LMAX,X,YVORT)
      CALL FNCT1(GVOR,LMAX,X,ZVORT)
      CALL DFNCT1(GYVOR,LMAX,X,DYVORT)
      CALL DFNCT1(GZVOR,LMAX,X,DZVORT)

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VOR10001
 VOR10002
 VOR10003
 VOR10004
 VOR10005
 VOR10006
 VOR10007
 VOR10008
 VOR10009
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 VOR10034
 VOR10035
 VOR10036

C		VOR10017
C	CALCULATE INTERMEDIATE FACTORS	VOR10018
	XDIF=K-XPI	VOR10019
	YSUM=YVORT*YPI	VOR10040
	ZDIF=ZVORT-ZPI	VOR10041
	TERM1=XDIF*XDIF+YSUM*YSUM+ZDIF*ZDIF	VOR10042
	TERM2=TERM1*1%URT(TERM1)	VOR10043
	TERM3=TERM2*TERM1	VOR10044
	DO 140 L=1,LMAX	VOR10045
	CHEBS=ICHERY(L)+ICHERY(L)	VOR10046
	CHEBD=ICHERY(L)-ICHERY(L)	VOR10047
C		VOR10048
C	CALCULATE INTEGRANDS	VOR10049
	DFWY(L)=(XDIF*FLOAT(2*L-1)*UCHERY(L)-CHEBS-3.*(XDIF*DYVORT-	VOR10050
	CYSUM)*YSUM*CHEBS/TERM1)/TERM2	VOR10051
	DFVY(L)=(ZDIF-XDIF*DYVORT)*YSUM*CHEBS/TERM3	VOR10052
	DFWZ(L)=(XDIF*DYVORT-YSUM)*ZDIF*CHEBD/TERM3	VOR10053
	DFVZ(L)=(CHEBD-XDIF*FLOAT(2*L-1)*UCHERY(L)-3.*(ZDIF-XDIF	VOR10054
	C*DYVORT)*ZDIF*CHEBD/TERM1)/TERM2	VOR10055
	140 CONTINUE	VOR10056
C		VOR10057
C	DO FOR ALL MODES	VOR10058
	DO 150 MQ=1,NOCM	VOR10059
	M=MQ-1	VOR10060
	CONST=FLOAT(2*M+1)/2.*PI	VOR10061
	GGAM=SIN(CONST*X)	VOR10062
C		VOR10063
C	GGAM = LEADING-EDGE VORTEX STRENGTH	VOR10064
	DO 150 L=1,LMAX	VOR10065
	SUM(MQ,L,1)=SUM(MQ,L,1)+GGAM*DFWY(L)*W(J)	VOR10066
	SUM(MQ,L,2)=SUM(MQ,L,2)+GGAM*DFVY(L)*W(J)	VOR10067
	SUM(MQ,L,3)=SUM(MQ,L,3)+GGAM*DFWZ(L)*W(J)	VOR10068
	SUM(MQ,L,4)=SUM(MQ,L,4)+GGAM*DFVZ(L)*W(J)	VOR10069
	150 CONTINUE	VOR10070
	200 CONTINUE	VOR10071
	CONST=1./(9.*PI)	VOR10072
C		
C	MULTIPLY SUMMATIONS BY APPROPRIATE CONSTANTS	VOR10073
	DO 300 MQ=1,NOCM	VOR10074
	DO 300 L=1,LMAX	VOR10075
	DGWM2(MQ,L)=SUM(MQ,L,1)*CONST	VOR10076
	DGVGM2(MQ,L)=3.*SUM(MQ,L,2)*CONST	VOR10077
	DGWM4(MQ,L)=-3.*SUM(MQ,L,3)*CONST	VOR10078
	DGVGM4(MQ,L)=-SUM(MQ,L,4)*CONST	VOR10079
	DO 300 K=1,4	VOR10080
	SUM(MQ,L,K)=0.0	VOR10081
	300 CONTINUE	VOR10082
C		VOR10083
C	PRIMARY OUTPUTS DGWM2,DGVGM2,DGWM4,DGVGM4	VOR10084
C	PASSED THROUGH ARGUMENT LIST TO CALLING PROGRAM	VOR10085
C		VOR10086
C	RETURN	VOR10087
	END	VOR10088
		VOR10089

```

SUBROUTINE MOV11
C
C   MOV CALCULATES VELOCITY INFLUENCE COEFFICIENTS ON VORTEX FROM
C   MORSESHOE VORTICITY
C
C   ARGUMENT LIST
C   L:  NO. OF FORCE POINT
C
C   DIMENSION S(4),W(4),F(5),YDWDY(4,5,5),YDWDZ(4,5,5),YDWDY(4,5,5)
C   COMMON /SDWSH/ TKVR(10),AVR(4,5,5),CVR(5)
C   COMMON /JACCD/DKDY(10),DKDZ(10),DKVDY(10),XDWDY(5),
C   XDWDZ(5),XDVDY(5), /GAUN/GN(10,10),WN(10,10) /MDES/NOCM,NOSM
C   /WDV2/AR(4,5,5),ALS(5,5),NINC,CR(5),TKR(10),XCC,SOS,Y,Z,YMN,ZMZ,
C   RSQR,ETA,GAUSK(10),PD2,NCP,MP,N,K,GZ,J1,J2,GS,YMN2,ZM2,CSR
C   /WOW1/ JS,N(14) /YACCD/XACCD(35,35),SAWW(5,5),SAVW(5,5),
C   DAWDY(5,5),DAWDZ(5,5),DAVDY(5,5),DAVDZ(5,5) /PLAN/CRMAX
C
C   DATA S(3),S(4),W(3)/ -1.,0.,1./
C
C   S = LEFT-HAND LIMIT OF INTERVAL
C   W = LENGTH OF INTERVAL
C   PD2=1.5707963
C
C   TRANSFORM NON-D BY CHORD TO NON-D BY SEMICHORD
C   X=-1.*Z.*XCC
C   Y=SOS
C   ZMZ=Z
C
C   Y = VORTEX SPANWISE POSITION; NON-D BY SEMISPAN
C   ZMZ = VORTEX VERTICAL POSITION; NON-D BY SEMISPAN
C
C   CHECK IF POINT IS CLOSE TO WING TIP
C   IF(SOS+ETA.LT.1.) GO TO 30
C   ETA=1.-SOS
C   NO REGION 2
C   NI(2)=0
C
C   30 CONTINUE
C
C   CHECK IF POINT IS CLOSE TO CENTER LINE
C   IF(SOS-ETA.GT.0.0) GO TO 40
C   ETA=SOS
C   NO REGION 4
C   NI(4)=0
C   40 S(2)=SOS+ETA
C
C   OUTPUT LOCATION OF CONTROL POINT AND OTHER INFO
C   WRITE(6,910) L,XCC,SOS,ETA,NI(2),NI(4),X,Z
C   W(2)=1.0-S(2)
C   W(4)=SOS-ETA
C   S(1)=SOS-ETA
C   W(1)=2.*ETA
C
C   INITIALIZE SUMMATION VARIABLES
C   DO 41 I=1,JS
C   DO 41 NI=1,NOCM
C   DO 41 N2=1,NOSM
C   AR(1,N1,N2)=0.0
C   AVR(1,N1,N2)=0.0
C   YDWDY(1,N1,N2)=0.0
C   YDWDZ(1,N1,N2)=0.0
C   YDWDY(1,N1,N2)=0.0
C   41 CONTINUE
C
C   DO INTEGRALS OVER FOUR SPANWISE REGIONS
C   DO 500 I=1,JS
C   411 NSIP=NI(1)
C   NSIP=NO. OF INTEGRAL POINTS
C   IF(NSIP.EQ.0) GO TO 500
C   DO SPANWISE INTEGRAL
C   DO 50 J=1,NSIP

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      GS=SI(I)-(GNE(J,NSIP)-1.0)/2.*W(I)
C
C  GNE(J,NSIP) = JTH ARCSISSA OF LEGENDRE-GAUSS QUADRATURE OF ORDER NSIP
      GY=GS
      YMN=Y-GY
      YPN2=YPN*YPN
      ZM2=ZM2*ZM2
      RSCR=YMN2+ZM2
      WT=WN(J,NSIP)* W(I)/(2.*RSCR)
C
C  WN(J,NSIP) = JTH WTG. FUNCTION OF LEGENDRE-GAUSS QUADRATURE
C
C  CALCULATE SPANWISE VORTICITY MODE
      CALL FUNCTINDSM,GS,F)
C
C  DO CHORDWISE INTEGRAL
      CALL CHDWS
      DO 45 M=1,NGCM
      DO 45 NSF=1,NGSM
        AR(I,M,NSF)=AR(I,M,NSF)+CR(M)*F(NSF)*WT
        AVR(I,M,NSF)=AVR(I,M,NSF)+CVR(M)*F(NSF)*WT
        YDWY(I,M,NSF)=YDWY(I,M,NSF)+XDWY(M)*F(NSF)*WT
        YDWDZ(I,M,NSF)=YDWDZ(I,M,NSF)+XDWDZ(M)*F(NSF)*WT
        YDVOY(I,M,NSF)=YDVOY(I,M,NSF)+XDVOY(M)*F(NSF)*WT
C
C  AR,AVR,ETC. ARE SURFACE INTEGRALS
      45  CONTINUE
      50  CONTINUE
      500 CONTINUE
C
C  INITIALIZE SUMMATION VARIABLES
      DO 60 I=1,NGCM
      DO 60 J=1,NGSM
        SAWW(I,J)=0.
        SAVW(I,J)=0.
        DAWDY(I,J)=0.
        DAWDZ(I,J)=0.
        DAVDY(I,J)=0.
      60  CONTINUE
C
C  SUM OVER ALL INTEGRATION REGIONS
      DO 70 I=1,NGCM
      DO 70 J=1,NGSM
      DO 65 MS=1,JS
        SAWW(I,J) = SAWW(I,J) + AR(MS,I,J)
        SAVW(I,J) = SAVW(I,J) + AVR(MS,I,J)
        DAWDZ(I,J) = DAWDZ(I,J) + YDWDZ(MS,I,J)*2.*CSR/CXMAX
        DAWDY(I,J) = DAWDY(I,J) + YDWY(MS,I,J)*2.*CSR/CXMAX
        DAVDY(I,J) = DAVDY(I,J) + YDVOY(MS,I,J)*2.*CSR/CXMAX
      65  CONTINUE
        DAWDZ(I,J) = DAWDY(I,J)
      70  CONTINUE
C
C  RESULTS ARE PASSED THROUGH COMMON STATEMENT
C
      910 FORMAT(IH0,'COLLOCATION PT.',I4,3X,'XOC=',F7.4,3X,'SDS=',F7.4,3X,
      C'ETA=',F7.4,3X,'N1121=',I3,3X,'N114=',I3,3X,'X=',F7.4,3X,'Z=',
      C'F7.4)
      RETURN
      END

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SUBROUTINE WPMW(A,GO,NOC,P,ALFA)
C WPMW CALCULATES DOWNWASH AND PROVIDES CONTRIBUTION TO JACOBIAN
C
C ARGUMENT LIST
C A: COEFFICIENTS FOR HORSESHOE VORTEX MODES
C GO: COEFFICIENTS FOR LEADING-EDGE VORTEX MODES
C NOC: NO. OF COLLOCATION POINTS
C ALFA: ANGLE OF ATTACK (IN RADIANS)
C
C EXTERNAL XGMT
C DIMENSION XPT(5),YPT(5), A(5,5),GC(5)
C ,GWGMW(5),DGWMW(5,5),DGWMW(5,5),DWDGY(5),DWDGZ(5)
C COMMON XPI,YPJ,S,M,MP/GVOR(5),GVOR(5)/SEC/ZPT/PLAN/CR
C /MODES/NOCM,NOSH /CONTRZ/J1,J4 /VLGC/LMAX /VACCB/XACCB(35,35),
C /SAWH(5,5),SAVW(5,5),DAWDY(5,5),DAWDZ(5,5),DAVDY(5,5),DAVDZ(5,5)
C COMMON /CWPMW/ NCORD,NSPAN,COEFF(25,25),GWM(25,5),VR(25)
C
C PI=3.141593
C SINLF = SIN(ALFA)
C
C FOR POINTS ON WING, Z = 0.
C ZPT=0.
C NCORD2=NCORD+1
C IF(J3.EQ.1) NCORD2=NCORD
C
C NMOD=NOSH*NOCM
C NMOT=NMOD+NOCM
C
C NMOD = NO. OF HORSESHOE VORTEX MODES
C NMOT = TOTAL NO. OF VORTICITY MODES
C
C CALCULATE LOCATION OF COLLOCATION POINTS
C CALL COLPT(NCORD,NSPAN, XPT,YPT)
C IF(J3.EQ.1) GO TO 300
C XPT(NCORD2)=(XPT(NCORD)+XPT(NCORD-1))/2.
C
C 300 CONTINUE
C
C CALCULATE LOCATION OF VORTEX AT X = 1.
C CALL FNCTN(GVOR,LMAX,1.,VVCR)
C CALL FNCTN(GVOR,LMAX,1.,ZVCR)
C
C CALCULATE DOWNWASH RESIDUE AT COLLOCATION POINTS
C DO 400 I=1,NCORD2
C DO 400 J=1,NSPAN
C
C CALCULATE FIRST INDEX FOR MATRICES, NI
C NI=J*(I-1)+NSPAN
C IF(NI.GT.NOCM) GO TO 400
C XPI=(XLC(1,YPT(J))+1.)/2. + B(1,YPT(J))*XPT(I)*CR
C YPJ=YPT(J)*S
C
C CHORDWISE POINT: NGN=D BY MAXIMUM LENGTH
C YPJ: SPANWISE POINT: NON-D BY MAXIMUM LENGTH
C
C FORM INTERMEDIATE FACTORS
C YDIFF=VVCR -VPJ
C YSUM=VVOR +VPJ
C XDIF=1.-XPI
C YDIFSO=YDIFF*YDIFF
C YSUMSO=YSUM*YSUM
C ZSO=ZVOR +ZVOR
C TERM1=YDIFF*ZSO
C TERM2=YSUM*ZSO
C TERM3=TERM1+ XDIF*XDIF
C TERM4 = TERM2*XDIF*XDIF
C TERM5 = SQRT(TERM3)
C TERM6 = SQRT(TERM4)
C
C CALCULATE CONTRIBUTIONS FROM LEADING-EDGE VORTICES AT OF X = 1.
C GWGMW2=-(YDIFF/TERM1)*(1.-XDIF/TERM5)
C 1+YSUM/TERM2*(1.-XDIF/ TERM6 )/(1.+PI)
C DGWMW1= - 1.*((1.-2.*YDIFSO/TERM1)*(1.-XDIF/TERM5)

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C *YDIFF50*(XDIFF/(TERM1+TERM5))/TERM1*(11.-2.*YSUM50/TERM2)
C *11.-XDIFF/TERM4) + YSUM50*(XDIFF/(TERM4+TERM6))/TERM2/(4.*PI)
DGMGM1 = -2*YDIFF/TERM1*(XDIFF/(TERM1+TERM5))
C -2./TERM1*(11.-XDIFF/TERM5) + YSUM/TERM2*(XDIFF/TERM4
C *TERM6) -2./TERM2*(11.-XDIFF/TERM6))/4.*PI)
C
C CALCULATE CONTRIBUTIONS FROM LEADING-EDGE VORTICES FORWARD OF X=1.
CALL DGMGM(DGMGM2,DGMGM4,GWGM1)
C
C INITIALIZE SUMMATION VARIABLES
DO 320 L1=1,LMAX
DWDGY(L1) = 0.
DWDGZ(L1) = 0.
320 CONTINUE
C
DO 450 MQ=1,NOCM
M=MQ-1
C
C CALCULATE CONTRIBUTIONS TO DOWNWASH COEFFICIENT FROM WAKE
CALL GAUSIDF(0.0,5,GMT,XGMT)
C
C CALCULATE DOWNWASH INFLUENCE COEFFICIENTS
COEFF(N1,NMOD+MQ)=GWGM1(MQ)+GMT+GWGM2+GWGM1(MQ)
GWGM2=-1.*GWGM2
C
C CALCULATE DERIVATIVES FOR JACOBIAN
DO 330 L1=1,LMAX
DWDGY(L1) = DWDGY(L1) + GQ(MQ)*(DGMGM2(MQ,L1)+ DGMGM1)
DWDGZ(L1) = DWDGZ(L1) + GQ(MQ)*(DGMGM4(MQ,L1) + DGMGM3)
330 CONTINUE
DGMGM1 = -1.*DGMGM1
DGMGM3 = -1.*DGMGM3
450 CONTINUE
DO 430 L1=1,LMAX
XACOB(N1,NMOD+L1) = DWDGY(L1)
XACOB(N1,NMOD+LMAX+L1) = DWDGZ(L1)

430 CONTINUE
440 CONTINUE
IF(IJ4.NE.1) GO TO 510
DO 500 I=1,NOCF
C
C OUTPUT DOWNWASH INFLUENCE COEFFICIENTS, IF DESIRED
WRITE(6,940) (COEFF(I,J),J=1,NMOD)
500 CONTINUE
510 CONTINUE
C
C CALCULATE RESIDUE FROM DOWNWASH CONDITION
DO 140 I=1,NOCF
V(I)= S1*ALF
DO 140 J=1,NOCM
V(I) = V(I) + COEFF(I,NMOD+J)*GQ(J)
XACOB(I,NMOD+J)= COEFF(I,NMOD+J)
DO 140 K=1,NOSM
N2=J+NOCM*(K-1)
V(I) = V(I) + COEFF(I,N2)*A(J,K)
XACOB(I,N2) = COEFF(I,N2)
140 CONTINUE
C
C OUTPUT RESIDUE FROM DOWNWASH CONDITION
WRITE(6,930) (V(I),I=1,NOCF)
C
C PRIMARY OUTPUTS V(I),XACOB PASSED TO CALLING PROGRAM
C THROUGH COMMON
C
930 FORMAT(' DOWNWASH ON WING',(I5E14.5))
940 FORMAT(' COLLOCATION POINT',I3,2F12.4,3X,'LOCAL X=',F12.4)
980 FORMAT(10F13.5)
RETURN
END

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WPDW0071
WPDW0074
WPDW0075
WPDW0076
WPDW0077
WPDW0078
WPDW0079
WPDW0080
WPDW0081
WPDW0082
WPDW0083
WPDW0084
WPDW0085
WPDW0086
WPDW0087
WPDW0088
WPDW0089
WPDW0090
WPDW0091
WPDW0092
WPDW0093
WPDW0094
WPDW0095
WPDW0096
WPDW0097
WPDW0098
WPDW0099
WPDW0100
WPDW0101
WPDW0102
WPDW0103
WPDW0104
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WPDW0107
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WPDW0109
WPDW0110
WPDW0111
WPDW0112
WPDW0113
WPDW0114
WPDW0115
WPDW0116
WPDW0117
WPDW0118
WPDW0119
WPDW0120
WPDW0121
WPDW0122
WPDW0123
WPDW0124
WPDW0125
WPDW0126
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WPDW0134
WPDW0135
WPDW0136
WPDW0137
WPDW0138
WPDW0139
WPDW0140
WPDW0141

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SYMBOLS

a	vorticity coefficient
AR	aspect ratio
c	chord
C_N	normal force coefficient
C_p	pressure coefficient
F	force on right-hand vortex
i	unit vector in x-direction
j	unit vector in y-direction
K	kernel function for surface integral
k	unit vector in z-direction
l	dummy index
m	dummy index
n	dummy index
q	dummy index
r	radius vector from origin
S	surface of integration
s	semispan
t	thickness
U	free stream velocity
u	perturbation velocity in x-direction
v	perturbation in y-direction; vector v is total perturbation velocity
w	perturbation velocity in z-direction
x	chordwise coordinate
y	spanwise coordinate; with subscript v, represents leading-edge vortex spanwise location
z	vertical coordinate; with subscript v, represents leading-edge vortex vertical location

SYMBOLS (cont'd.)

α	angle of attack
Γ	leading-edge vortex strength
γ	spanwise vorticity component; vector γ is total vorticity
δ	chordwise vorticity component
η	spanwise coordinate nondimensionalized by semispan
θ	chordwise azimuthal coordinate
λ	leading-edge sweep angle
z	complex plane

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