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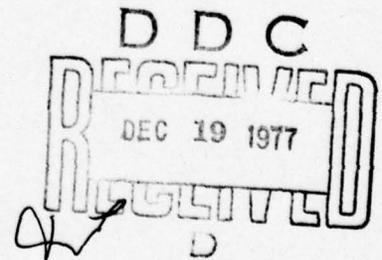
AN INTRODUCTION TO THE AERODYNAMICS OF FLIGHT DYNAMICS

by

H. H. B. M. Thomas

March 1977

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6 AN INTRODUCTION TO THE AERODYNAMICS OF FLIGHT DYNAMICS,

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The solution of problems of flight dynamics requires the aerodynamic forces, which are called into play, to be expressed in a suitable form. In this context a suitable form is one which adequately reflects the nature of the motion being considered and is, at the same time, convenient for the solution of the equations of motion. In the opening sections of this paper formulation in terms of aerodynamic derivatives, and generalizations thereof, are considered. There follows a brief discussion in broad and simple physical terms of how the various motion variables give rise to forces and moments, which within a linearized framework are expressible as force or moment derivatives, specifically for an aeroplane.

Paper to be presented as a contribution to a course of lectures on "Aerodynamic inputs for problems in aircraft dynamics" at the von Karman Institute for Fluid Dynamics, 25-29 April 1977.

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1 INTRODUCTION

In setting up a mathematical model for the calculation of the dynamics of an aircraft, it is necessary to relate the aerodynamic forces to the state variables, that is, those motion and other variables necessary to specify the flight condition. For conditions of steady flight, or conditions of equilibrium, the problem presents no inherent difficulty since these conditions can be assumed to have persisted for a long time. This renders the forces fully determinate in terms of the reduced set of variables, which specify the steady state. Unsteady motion, on the other hand, presents real difficulties and there is no argument by which, *a priori*, we can circumvent the fact that, strictly speaking, the aerodynamic forces and moments acting on an aircraft during unsteady motion are 'functionals', that is, they depend on the whole set of values of the state parameters, past and present.

To a large extent the above difficulty has been ignored in the past development of the subject of flight dynamics and the formulation of the aerodynamic forces and moments has proceeded along essentially intuitive lines of reasoning. Early this century Bryan¹⁻³ introduced a formulation in terms of linearized aerodynamic derivatives, which expressed the aerodynamic force or moment as the sum of the steady-state value and linear terms in the instantaneous values of the disturbance (or perturbation) velocities and motivator angles. Some terms in the first derivative of some of the quantities were added as time went on. This process replaces the 'functional' by a function, in other words, if A be a typical aerodynamic reaction, then in free air (away from the Earth or other large object) we may express the process mathematically as follows, see Fig 1,

$$\begin{aligned} A &= A[u, v, w, p, q, r, \xi, \eta, \zeta], \text{ the functional} \\ &\approx A(u, v, w, p, q, r, \xi, \eta, \zeta, \dot{v}, \dot{w}, \dot{\xi}, \dot{\eta}, \dot{\zeta}) \\ &\approx A_e + A_u u' + A_v v' + \dots \end{aligned}$$

where the suffix e refers to equilibrium conditions of steady flight, for example, with u_e and $w_e \neq 0$, $v_e = 0$, $p = q = r = 0$, $\xi = \zeta = 0$ but $\eta = \eta_e$. The justification for the continued use of these 'quasi-steady' aerodynamic derivatives in the development of more generalized formulations stemming from the original linearized form is to be found in the fact that seemingly they have described adequately aircraft behaviour in most conditions of practical

interest. Furthermore they have received a certain degree of validation through wind-tunnel testing. Nonetheless it must be emphasised that certain aspects of the concept of derivatives lacks rigour. More recently, Tobak⁵ has tried to introduce that rigour by starting with the proposition that the force or moment is a functional. He derives an integral form for an aerodynamic coefficient $C_k(t)$, where k may be X, Y, Z or l, m, n , which contains indicial responses in a functional form. The subsequent development of his formulation hinges on "replacing the functionals by appropriate functions, whose dependence on the past is denoted by a limited number of parameters rather than by continuous functions". We cannot go into the details of his analysis here, but we note that he emerges with formulations akin to those discussed later. However a point worth making is that the analysis is restricted to motions reproducible in a wind tunnel, namely, a rectilinear flight path with constant resultant velocity.

Even if we accept the derivative formulation with all its defects there remain fairly formidable aerodynamic problems in the calculation of the various quantities, so defined. These will be discussed in their context in later sections of this paper. Furthermore there is, and as far as can be foreseen always will be, a need to have available methods suited to the various stages of the design procedure. There is a willingness to sacrifice some accuracy in the interest of ease of application and generality for those methods to be used during the early stages of design. However, the main aim of this introductory paper is to give in broad qualitative terms an account of the source of the various contributions to the forces and moments acting on an aeroplane in unsteady flight. We shall not be concerned with methods of estimation, *per se*.

In order to achieve the maximum convenience and generality it is customary to resort to a non-dimensional form for all the aerodynamic quantities involved. In this way the major effects of speed, size and air density are accounted for automatically. It is unfortunate that schemes for forming these non-dimensional quantities have not evolved in an organized and logic fashion. The result is that the newcomer to the subject is faced with inconsistencies which he finds difficult to comprehend and resolve. In what follows, the relationship between the coefficients and derivatives in the system most commonly used in the United States and that embodied in the documents issued by the International Standards Organisation are explained and detailed. Additionally the corresponding relationship between the latter and the earlier British scheme of notation is also considered.

The use of the coefficient form is essentially intended for treating the forces and moments generated by the relative motion of the aircraft and the ambient air and for this reason the divisors contain the kinetic pressure, $\frac{1}{2}\rho V^2$, as a factor. For other forces, even aerodynamic ones such as the thrust of an engine such divisors are not entirely satisfactory but, since in the dynamic studies it is necessary to be consistent, are used. There is, however, a drawback in this practice as it can be misleading in the process of forming the aerodynamic derivatives in their non-dimensional (or normalized) form. We have to draw a clear distinction between what might be termed coefficient derivatives and 'coefficients of derivatives', which is one way of describing the usual non-dimensional aerodynamic derivatives.

2 CONCEPT OF AN AERODYNAMIC DERIVATIVE AND ITS USES

We have already mentioned in broad terms the concept of a derivative of a force or moment. The reduction of this to its normalized (or non-dimensional) form also needs consideration. Let us return to the equation previously given for a typical aerodynamic reaction in the special case of motion well away from another body that can introduce an interference effect, such as the Earth. We can, without loss of generality, confine our attention to a motion in the longitudinal plane alone, in which case we are concerned with two forces (X,Z) and one moment (M). These are expressible, within the assumption of small perturbation, in the forms,

$$X = X[u, w, q, \eta] \approx X_e(u_e, w_e, \eta_e) + X_u u' + X_w w' + X_q q' + X_\eta \eta' \quad ,$$

$$Z = Z[u, w, q, \eta] \approx Z_e(u_e, w_e, \eta_e) + Z_u u' + Z_w w' + Z_q q' + Z_\eta \eta' + Z_w \dot{w}' \quad ,$$

and

$$M = M[u, w, q, \eta] \approx M_e(u_e, w_e, \eta_e) + M_u u' + M_w w' + M_q q' + M_\eta \eta' + M_w \dot{w}' \quad .$$

In these equations the quantities are dimensional, although it is significant that a merit of the new ISO notation (see also Ref 4) is that the corresponding equations in normalized form have the same form exactly. Hereafter we shall emphasise the difference by either adding the superscript 0 or the subscript 'ord', when we wish to denote a dimensional quantity. The plain symbols are then the aero-normalized derivatives of the ISO system, for example, X_e, X_u, u', w' .

As a basis for this normalizing process we choose divisors formed in the same consistent manner as a system of units derived from three fundamental divisors,

- (1) for lengths, l_0 (a length defined by a significant geometric feature)
- (2) for velocities, the resultant velocity in the datum flight condition, \dot{V}_e (resultant of u_e and w_e in above example) and
- (3) for forces, $\left(\frac{1}{2}\rho_e V_e^2 S\right)_{ord}$.

From this choice of divisors it follows, for example, that

$$z_e = \left(\frac{z_e}{\frac{1}{2}\rho_e V_e^2 S} \right)_{ord},$$

and

$$\begin{aligned} z_{q'} &= \frac{\dot{z}_q \dot{q}'}{\left(\frac{1}{2}\rho_e V_e^2 S\right)_{ord}} \\ &= \left(\frac{z_q}{\frac{1}{2}\rho_e V_e^2 S l_0} \right)_{ord} \left(\frac{q' l_0}{V_e} \right)_{ord} \end{aligned}$$

with, of course,

$$q' = \left(\frac{q' l_0}{V_e} \right)_{ord}.$$

This example illustrates the meaning of the expression 'coefficient of a derivative', since this is precisely what the first of the bracketed terms for $z_{q'}$ is. It is perhaps helpful to set out in full the divisors most commonly needed and this is done in Table 1.

It is implicit in the form of the expressions given above for the forces and moments that the aerodynamic derivative formulation is applicable only to the calculation of small perturbation motion of an aircraft. However, it may be noted here that this refers more specifically to changes in the variables involved in the aerodynamic terms rather than others occurring within the dynamics. In other words the aircraft motions that may justifiably be calculated on this basis can involve appreciable changes in some state variables, for example, the attitude angles. The derivative form for the aerodynamic forces and moments is particularly well suited to the consideration of the stability (in the linearized sense) of a given steady state.

It may be remarked here that the derivatives used in the USA are also essentially of the above type, but there is in this case an inconsistency in the unit of force used to reduce a force and a moment to their respective normalized forms. For the force the divisor is as above $(\frac{1}{2}\rho_e V_e^2 S)_{ord}$ but for the moment it is $(\rho_e V_e^2 S)_{ord} \ell_0$ (a representative length). This usage can be traced back to the early days of aviation when it was the practice to use the factor ρV^2 in forming all coefficients and lift drag and pitching moment coefficients had the following forms,

$$k_L = \frac{L}{\rho V^2 S}, \quad k_D = \frac{D}{\rho V^2 S} \quad \text{and} \quad k_m = \frac{M}{\rho V^2 S \ell_0}.$$

The forms resulting from the retention of this earlier practice for the moments is open to another interpretation since the choice made for the representative (or characteristic) length is $\bar{c}/2$ for the longitudinal quantities and $b/2$, semi-span, for the lateral quantities.

Thus, for example, we may write

$$\left(\rho_e V_e^2 S \frac{\bar{c}}{2} \right)_{ord} = \left(\frac{1}{2} \rho_e V_e^2 S \bar{c} \right)_{ord}$$

when the second variant of the expression can be interpreted as using \bar{c} for ℓ_0 , in the ISO notation. This stratagem merely shifts the inconsistency to the normalized variables, such as, $\left(\frac{q \ell_0}{V_e} \right)_{ord}$ and the time unit, ℓ_0/V_e , which with $\ell_0 = \bar{c}/2$ would not conform with $\ell_0 = \bar{c}$. Nevertheless the two sets of derivatives are easily related one to the other. To illustrate this we have to digress a little and look at the well-known present-day versions of the aerodynamic coefficients. These are defined as follows:

$$C_X = \left(\frac{X}{\frac{1}{2} \rho V^2 S} \right)_{ord}, \quad C_Z = \left(\frac{Z}{\frac{1}{2} \rho V^2 S} \right)_{ord} \quad \text{and} \quad C_m = \left(\frac{M}{\frac{1}{2} \rho V^2 S \bar{c}} \right)_{ord}$$

for the longitudinal forces and moment and

$$C_Y = \left(\frac{Y}{\frac{1}{2} \rho V^2 S} \right)_{ord}, \quad C_\ell = \left(\frac{L}{\frac{1}{2} \rho V^2 S b} \right)_{ord} \quad \text{and} \quad C_n = \left(\frac{N}{\frac{1}{2} \rho V^2 S b} \right)_{ord}$$

for the lateral force and moments, almost universally and in particular in the USA. We should remark here that the International standard does not lay down

any chosen length for l_0 . However, if the choice is made of adopting $l_0 = \bar{c}$ for longitudinal quantities and $l_0 = b$ for the lateral quantities, we note that the normalized forces and moments have a form analogous to the coefficients. Now the product of a derivative and its associated normalized variable represents the contribution of that state variable to the force or moment expressed in its normalized form. If we express this fact mathematically we have, for example,

$$\begin{aligned} (\Delta M)_{\text{due to } w'} &= C_{m_\alpha} \alpha' = C_{m_\alpha} \frac{\dot{w}'}{V_e} \text{ to first order} \\ &= \left(\frac{\Delta M}{\frac{1}{2} \rho_e V_e^2 S \bar{c}} \right)_{\text{ord}} = \left(\frac{M_w}{\frac{1}{2} \rho_e V_e S \bar{c}} \right)_{\text{ord}} \left(\frac{w'}{V_e} \right)_{\text{ord}} \\ &= M_w w', \text{ by definition} \end{aligned}$$

which yields

$$C_{m_\alpha} = M_w .$$

Application of the same argument to the contribution of q' results in

$$\begin{aligned} (\Delta M)_{\text{due to } q'} &= C_{m_q} \left(\frac{q' \bar{c}}{2V_e} \right)_{\text{ord}} \\ &= \left(\frac{\Delta M}{\frac{1}{2} \rho_e V_e^2 S \bar{c}} \right)_{\text{ord}} = \left(\frac{M_q}{\frac{1}{2} \rho_e V_e S \bar{c}^2} \right)_{\text{ord}} \left(\frac{q' \bar{c}}{V_e} \right)_{\text{ord}} \\ &= M_q q', \text{ by definition} \end{aligned}$$

so that

$$C_{m_q} = 2M_q .$$

Likewise,

$$\begin{aligned}
 (\Delta M)_{\text{due to } \dot{\alpha}} &= C_{m_{\dot{\alpha}}} \left(\frac{\dot{\alpha} \bar{c}}{2V_e} \right)_{\text{ord}} = \left(\frac{\Delta M}{\frac{1}{2} \rho_e V_e^2 S \bar{c}} \right)_{\text{ord}} \\
 &= \left(\frac{M_w}{\frac{1}{2} \rho_e S \bar{c}^2} \right)_{\text{ord}} \left(\frac{\dot{w}' \bar{c}}{V_e^2} \right)_{\text{ord}} = M_w \dot{w}'
 \end{aligned}$$

so that

$$C_{m_{\dot{\alpha}}} = 2M_w .$$

Other quantities can be similarly related. Table 2 attempts to set out the more or less complete set of relationships necessary to convert from results in one system of notation to the other. The remarks about the derivatives C_{X_u} and C_{Z_u} should be noted. Also given in Table 2 are the factors to be applied if the semispan, $b/2$, is used in place of b for ℓ_0 .

3 CONCEPT OF A COEFFICIENT DERIVATIVE AND ITS USES

It not infrequently happens that a problem in flight dynamics involves large perturbations of one or more of the state variables. In particular there is a class of problems wherein the speed varies substantially. For such problems it is preferable to exclude the speed from the expansion of the aerodynamic forces and moments by using what we shall term coefficient derivatives, that is, partial derivatives of the appropriate aerodynamic coefficient with respect to a variable. Thus typically we have,

$$X = \frac{1}{2} \rho V^2 S C_X : C_X = C_{X_e} + C_{X\alpha} \alpha' + C_{X\beta} \beta' + C_{Xq} q' + \dots$$

$$M = \frac{1}{2} \rho V^2 S \bar{c} C_m : C_m = C_{m_e} + C_{m\alpha} \alpha' + C_{m\beta} \beta' + C_{mq} q' + \dots$$

where the multipliers of C_X and C_m are dimensional quantities, ρ being the current value of the air density and V the current value of the resultant velocity. In the expansions of the coefficients the primed quantities are aerodynamic normalized perturbation variables. Thus, see Fig 2,

$$\alpha' = \alpha - \alpha_e \quad \text{where } \tan \alpha = w/u ,$$

$$\beta' = \beta = \sin^{-1} \left(\frac{v}{V} \right) ,$$

$$q' = \left(\frac{q \ell_0}{V} \right)_{\text{ord}} \quad \text{etc.,}$$

if suffix e refers to a symmetrical flight condition. It should be noted that in this treatment of the forces and moments the current value of the resultant velocity, V , is used in the definitions and that all the coefficient derivatives are functions of the current Mach number, $M = V/a$. This is in contrast to those of section 2 which are functions of the datum flight Mach number, $M_e = V_e/a_e$. On this account the coefficient derivatives or derivatives of coefficients should not be confused with the usual aerodynamic derivatives used in the USA, which as we have discussed are essentially of a different type in spite of the similarity in notation.

It is possible to deduce the relationships between the derivatives of section 2 and those of the present section⁴. For a general datum flight condition these relationships are complicated, but they simplify considerably when the datum flight condition takes the more usual form of steady, symmetric rectilinear flight. Then for an arbitrary set of body axes we have, for example,

$$Z_u = \left\{ 2C_{Ze} + M_e \left(\frac{\partial C_Z}{\partial M} \right)_e + R_e \left(\frac{\partial C_Z}{\partial R} \right)_e \right\} \cos \alpha_e - (C_{Z\alpha})_e \sin \alpha_e ,$$

$$Z_w = \left\{ 2C_{Ze} + M_e \left(\frac{\partial C_Z}{\partial M} \right)_e + R_e \left(\frac{\partial C_Z}{\partial R} \right)_e \right\} \sin \alpha_e + (C_{Z\alpha})_e \cos \alpha_e ,$$

$$Z_q = (C_{Zq})_e ,$$

$$Z_{\dot{w}} = (C_{Z\dot{\alpha}})_e \cos \alpha_e \quad \text{if } l_1 = l_0 = \bar{c} \text{ (say),}$$

$$Z_\eta = (C_{Z\eta})_e ,$$

where the suffix e indicates that the quantities are evaluated at the datum condition.

Similar relationships exist for the moment derivatives except that the right-hand sides are multiplied by an additional factor, l_1/l_0 , which can be set equal to unity if $l_1 = l_0 = \bar{c}$ (say). For aerodynamic body axes, $\alpha_e = 0$ (Fig 3) and the relationships simplify even further. In addition in that case we can introduce the lift and drag coefficients into the expressions on the right-hand sides (cf section 5.1). If a dependence of C_Z upon the acceleration \dot{V} had been assumed, the expression for $Z_{\dot{w}}$ would have contained a term in $C_{Z\dot{V}}$, but it is normal practice to ignore this term just as we have previously ignored terms involving derivatives with respect to \dot{u} in the small perturbation expressions for forces and moments given in section 2.

4 ALTERNATIVE FORMULATIONS OF AERODYNAMIC FORCES AND MOMENTS FOR OTHER MOTIONS

We have already remarked in the introduction that further generalizations are needed and according to Tobak⁵ are justified. One such generalization is that in which, in addition to substantial changes in the velocity, the changes in the angles of attack and sideslip are sufficiently large as to render the previous formulations of the aerodynamic terms unacceptable. We then incorporate within the first term of the expressions for a force or a moment coefficient the effect of the above two angles as a function. The combination of instantaneous values of the velocity and angles of incidence define a possible steady state, so that we may regard the expressions for the aerodynamic force or moment as representing a disturbance about the said steady state.

In this way we arrive at the formulation illustrated by the following,

$$C_X = C_X(\alpha, \beta) + C_{X\eta}(\alpha, \beta)\eta' + C_{Xq}(\alpha, \beta)q' + \dots$$

and

$$C_m = C_m(\alpha, \beta) + C_{m\eta}(\alpha, \beta)\eta' + C_{mq}(\alpha, \beta)q' + \dots$$

By virtue of what we have already stated the first terms in such formulations are obtained from so-called 'static' tests or calculations. Just as it is sometimes necessary to elaborate this formulation further by including the effect of the motivators (the effect of η in above examples) likewise as a function, it may also be permissible on occasion to omit the sideslip angle from all but the first term. The theoretical arguments advanced by Tobak can take us no further than saying that strictly the derivatives must be determined with reference to the state defined by both angles of incidence. On the other hand too few experiments have been made to allow of a more general statement on empirical grounds.

In dynamic problems which involve high rates of rotation, such as departure conditions, spin entry and established spin motions, further modification of the formulation of the aerodynamic terms is necessary. Again Tobak has put forward arguments in support of a specific form for this case, but we may approach the question from a slightly different angle. If we wish to introduce an angular velocity into the first term of a more general expression than those quoted above, this angular velocity must clearly be about the instantaneous direction of the velocity, or along the tangent to the flight path, see Fig 2.

Let us denote the angular velocity by ω and take as our examples two coefficients which will be strongly affected by the presence of the angular velocity. Two such coefficients are those for the side-force and the rolling moment. These are now written,

$$C_Y = C_Y(\alpha, \beta, \omega) + C_Y(\alpha, \beta, \zeta) + C_{Yp} p' + C_{Yr} r' + \dots$$

and

$$C_\ell = C_\ell(\alpha, \beta, \omega) + C_\ell(\alpha, \beta, \xi) + C_\ell(\alpha, \beta, \zeta) + C_{\ell p} p' + C_{\ell r} r' + \dots$$

By choosing ω such that it has a component p , equal to the current rate of rotation about the x-axis, along that axis we obtain what is, in effect, Tobak's proposed formulation⁵. This would imply that the yaw-rate derivatives are to be determined (experimentally or theoretically) from motions involving oscillations about the coning motion defined by α , β and $\omega = p \sec \sigma$, see Fig 2. It is, however, necessary to note that the above choice for ω is not the only one possible. In any case it is desirable that any formulation of the aerodynamic terms be validated by tests involving motion of the most general kind. It is to be hoped that further light may be cast on the matter by an investigation, presently being undertaken, into the motion of free-flight models during post-stall gyrations, spin entry conditions and in established spins of both the steep and flat type.

5 ON HOW THE FORCES AND MOMENTS ARISE

In this section the aim is to describe in the broadest, and hopefully simplest possible, terms the nature of each contribution to the overall forces and moments and to identify those components of an aeroplane which make significant contributions to the individual derivatives.

It is desirable to work in some specified system of axes and the aerodynamic-body axes are chosen for their convenience and general nature. A definition of this axis system is not amiss, so we note that aerodynamic-body axes are such that in the datum flight condition the x-axis is parallel to the projection on to the plane of symmetry of the velocity of the origin of the axes (usually the centre of gravity of the aeroplane). The x-axis is positive towards the nose of the aeroplane, the z-axis is parallel to the plane of symmetry, positive ventrally. The y-axis is positive to starboard: when the centre of gravity lies in the plane of symmetry, the latter is the zx-plane, see Fig 3.

The choice of axes made here does not imply that these axes are the most suitable for all aircraft configurations or for all experimental techniques. It, therefore, sometimes becomes necessary to be able to convert the values of the aerodynamic derivatives with respect to one system of axes to those appropriate to another system of axes. This matter is discussed in section 6.

To avoid undue complexity we have previously excluded from the discussion any effect location of the aircraft (that is, the position of its centre of gravity and its attitude angles or orientation) may have on the aerodynamic forces acting upon it. This is not always permissible. For example, the proximity of an aeroplane to the ground (Earth's surface) as during landing and take-off introduces changes. Six parameters govern these effects, namely, $x_0, y_0, z_0; \Psi, \Theta, \Phi$, of which the first three define the position of the aeroplane's centre of gravity with respect to an Earth-fixed system of axes and the second trio the aeroplane's orientation relative to these axes. Since it is usually sufficient to simplify to a flat Earth the first trio may be replaced by h , the height of the centre of gravity above the ground. Wind-tunnel test results as well as those from theory have indicated that some of the derivatives already discussed are subject to considerable modification. However, some care needs to be exercised in interpreting results of this sort as they not infrequently refer to a constrained flight condition. In the usual wind-tunnel test the centre of gravity remains fixed, that is, no change in flight path occurs. Orientation of the aeroplane with respect to the incident flow becomes indistinguishable from orientation with respect to the Earth. To illustrate this point it is convenient to refer the motion to a suitable body axis system, see Fig 3, for example, with the x-axis aligned with the no-lift line of the aeroplane. In Fig 4 two special motions of the aeroplane are shown, one in which the attitude in pitch is zero (or constant) and the angle of attack varies the other in which the angle of attack is zero whilst the attitude changes. These and the more general motion shown in Fig 5a may be contrasted with the flight conditions usually examined theoretically and experimentally, Fig 5b. Whether or not these are significant derivatives with respect to the attitude angle Θ has yet to be determined. Similar remarks apply to the other two attitude angles, Ψ and Φ .

The same diagrams serve to illustrate the essential difference between the aerodynamic forces arising from the rate of pitch, q and those due to the rate of change of the angle of attack, $\dot{\alpha}$. Again there is parallel to be drawn for the rate of yaw, r and the rate of change of sideslip angle, $\dot{\beta}$.

5.1 The longitudinal X-force derivatives, X_u and X_w

The derivative X_u represents the contribution to the force along the x-axis due to unit incremental velocity along the x-axis, so that $X_u u'$ is the incremental force due to u' . Likewise $X_w w'$ is the corresponding increment in force due to w' . Since lift and drag coefficient data are more familiar and generally more readily available, we shall express these derivatives X_u and X_w in terms of derivatives of these coefficients.

We may write $X = \frac{1}{2} \rho V^2 S C_X(M, R, \alpha, \eta, q, \delta)$ as the dimensional form of the X-force, from which we have the derivative

$$\dot{X}_u = \left(\frac{\partial \dot{X}}{\partial u} \right)_e = \left[\left\{ \rho V S C_X \frac{\partial V}{\partial u} + \frac{1}{2} \rho V^2 S \sum \left[\frac{\partial C_X}{\partial \gamma} \frac{\partial \gamma}{\partial u} \right]_{\text{ord}} \right\} \right]_e$$

where γ represents each of the parameters within the brackets for the function C_X and the subscript e denotes that the whole is evaluated at datum conditions. Now in general

$$\begin{aligned} V^2 &= u^2 + v^2 + w^2, \\ \tan \alpha &= \frac{w}{u} \\ M &= \frac{V}{a}, \quad R = \frac{\rho l V}{\mu} \end{aligned}$$

where we have for ease of writing omitted the superscript 0 or suffix ord. For small perturbations in u, v, w and in α these can be written, for the chosen axes,

$$\begin{aligned} V &= V_e + u' = u, \\ \alpha &= \alpha' = \frac{w'}{u}, \\ M &= \frac{V_e + u'}{a} \quad \text{and} \quad R = \frac{\rho l}{\mu} (V_e + u') \end{aligned}$$

whence

$$\left(\frac{\partial \alpha}{\partial u} \right) = - \frac{w'}{u^2} = - \frac{\alpha'}{V} \approx - \frac{\alpha'}{V_e}$$

and

$$\left(\frac{\partial \dot{\alpha}}{\partial u} \right) = \frac{\dot{V}}{V^2} \alpha' - \frac{\dot{\alpha}'}{V}, \quad \text{which are zero}$$

in the datum flight conditions. Evaluation of the derivative in its normalized form now gives

$$X_u = \left(\frac{X_u}{\frac{1}{2} \rho_e V_e S} \right)_{\text{ord}} = 2C_{X_e} + M_e \left(\frac{\partial C_X}{\partial M} \right)_e + R_e \left(\frac{\partial C_X}{\partial R} \right)_e .$$

By virtue of the fact that to first order in α' we may write $C_X = -C_L \alpha' - C_D$, the above expression can be expressed in terms of the drag coefficient, thus,

$$X_u = -2C_{D_e} - M_e \left(\frac{\partial C_D}{\partial M} \right)_e - R_e \left(\frac{\partial C_D}{\partial R} \right)_e .$$

The third term, which represents the effect of the Reynolds number, is often ignored, since it is assumed that either the estimation or measurement of the drag has been made at a representative scale. In deriving this result for X_u we have restricted our attention to airframe aerodynamic forces, which necessitates a separate treatment of the thrust contribution to the X-force. This course is advisable since the introduction of a thrust coefficient can be inconvenient and even misleading. If T represent the thrust and i_T the inclination of the thrust line to the x-axis, we have components of thrust along the x-axis and z-axis of $X^T = T \cos i_T$ and $Z^T = T \sin i_T$ respectively. The contribution of the thrust to X_u can be written

$$X_u^T = \left\{ \frac{1}{\frac{1}{2} \rho_e V_e S} \left(\frac{\partial X^T}{\partial u'} \right)_e \right\}_{\text{ord}} = \left[\frac{\cos i_T}{\frac{1}{2} \rho_e V_e S} \left(\frac{\partial T}{\partial u} \right)_e \right]_{\text{ord}} .$$

It is clearly zero in gliding flight since then $T = 0$, otherwise we require to know the dependence of the engine thrust on the aeroplane speed. For example, if the horsepower of the engine is constant, $TV = \text{constant}$, so that $\left(\frac{\partial T}{\partial u} \right)_e = -\frac{T}{V_e}$, which is a reasonable approximation in the case of a piston-engined aeroplane with variable-pitch propellers. There is much less dependence of thrust on speed for jet or rocket engines.

In general

$$\begin{aligned} X_u &= X_u^A + X_u^T \\ &= X_u^T - 2C_{D_e} - M_e \left(\frac{\partial C_D}{\partial M} \right)_e \end{aligned}$$

where the Reynolds number term has been ignored. By application of the same basic relationship of force and coefficient and evaluating the derivative in a similar way we have

$$X_w^A = (C_{X\alpha'})_e$$

which by virtue of the relationship between C_X , C_L and C_D can be rewritten

$$X_w^A = X_w^A = C_{L_e} - (C_{D\alpha'})_e$$

Estimates of the derivatives involved in the above expressions for X_u and X_w can usually be obtained from generalized data and are also available from wind-tunnel test data obtained at an early stage in the design procedure.

5.2 The Z-force derivatives, Z_u , Z_w and Z_q

The following results are readily obtained by applying arguments on the lines of those of section 5.1.

$$Z_u^A = -2C_{L_e} - M_e \left(\frac{\partial C_L}{\partial M} \right)_e,$$

$$Z_u^T = \frac{\sin i_T}{\frac{1}{2}\rho_e V_e^2 S} \left(\frac{\partial T}{\partial u} \right)_e$$

which when combined yield

$$Z_u = Z_u^T - 2C_{L_e} - M_e \left(\frac{\partial C_L}{\partial M} \right)_e$$

of which the first term is usually so small that it is ignored.

For the derivative with respect to w' we have,

$$Z_w = - \left(\frac{\partial C_L}{\partial \alpha} \right)_e - C_{D_e},$$

where this time the second term is usually small compared with the first term.

As for the X force derivatives the information on the lift and drag coefficients is so basic to the design of the aeroplane that the evaluation of the Z force derivatives also presents no problems.

The derivative Z_q is associated with the angular velocity of the aircraft in pitch. We have already seen that motions can be imagined in which there is rotation in pitch without change in the angle of attack as well as motion in which the angle of attack varies without any rotation. Accordingly q and $\dot{\alpha}$ are variables with their separate effects just as Θ and α are. Reference to Figs 6 and 7 shows that the effect of the rotation rate q about the y-axis is equivalent to a decrease in the effective angle of attack of those portions of the aircraft's body and wing lying forward of the y-axis and an increase in the effective local angle of attack of the tail and those portions of the wing and body behind the y-axis. Within the assumptions of linearized theory there is a strict equivalence in the load distribution of the cambered wing in rectilinear flight and the rotating wing. The pressure distribution is proportional to q/V_e for the contributions from the wing, body and tailplane. The integral of these pressure distributions gives the overall force and hence its derivative Z_q . A simple, but approximate, expression for the tailplane contribution can be obtained by ignoring the effect of the additional downwash induced by the additional lift distribution on the wing due to the rate of rotation, q . We further replace the distribution of local effective incremental angle of attack over the tailplane by a mean value, $\frac{q \ell_T}{V_e}$. Here ℓ_T is the tail-arm and is usually taken as equal to the distance from the centre of gravity to the aerodynamic centre of the tailplane, see Fig 7.

The mean kinetic pressure over the tailplane is affected by the wing wake associated with the combined α and q loadings. Let the ratio of this mean to free-stream kinetic pressure be Q_q , then we have approximately,

$$(Z_t)_{\text{due to } q'} = -\frac{1}{2} \rho_e V_e^2 Q_q S_t a_{lt} \left(\frac{q' \ell_T}{V_e} \right)$$

which yields

$$(Z_q)_t = -Q_q \frac{S_t \ell_T}{S \bar{c}} a_{lt}$$

The distribution of effective angle of attack due to the pitch rate also produces a contribution to the rotary derivative Z_q from the body and the wing. That of the body can be estimated using slender-body theory or one of the more refined theoretical methods. Strictly speaking allowance should be made for the mutual interference of wing and body, but this is seldom done, because of the more dominating effect of the tailplane. Likewise the cancelling

effects of reduction of effective angle of attack of that part of the wing forward of the centre of gravity and increase in effective angle of attack of those parts aft of the centre of gravity tend to make the wing contribution small, except for highly-swept wings. However, there is no inherent difficulty in applying any of the available lifting surface theories to the calculation of the derivative Z_q for a wing alone. It is usually accepted that simple summation is adequate for yielding the value of the derivative for the complete aircraft. This view is further justified by the fact that this particular derivative is of little importance in the dynamics of most aeroplanes, apart perhaps for those having very small values of the relative density parameter, μ_1 .

5.3 Moment due to change in velocity component along x-axis - M_u

Just as we expressed the force derivatives X_u and Z_u in terms of the more familiar lift and drag coefficients so we may express the moment derivative, M_u , in terms of a derivative of C_m , at any rate as far as the contribution from the airframe forces is concerned. Accordingly we write

$$M^A = \frac{1}{2} \rho V^2 S \bar{c} C_m(M, R, \alpha, \eta, q, \dot{\alpha})$$

as an equation for the dimensional moment in terms of its coefficient.

From the above relationship we again obtain on differentiation the dimensional derivative

$$\dot{M}_u^A = \left(\frac{\partial \dot{M}^A}{\partial \dot{u}^i} \right)_e = \left[\left\{ \rho V S \bar{c} C_m \frac{\partial V}{\partial u^i} + \frac{1}{2} \rho V^2 S \bar{c} \sum \left\{ \frac{\partial C_m}{\partial \gamma} \frac{\partial \gamma}{\partial u^i} \right\}_{\text{ord}} \right\} \right]_e$$

where as before γ represents each of the parameters within the bracket for the function C_m and the subscripts 'ord' and e denote dimensional value and datum conditions respectively. Evaluation of this expression using the relationships given in section 5.1 yields

$$M_u^A = 2C_{m_e} + \left(M \frac{\partial C_m}{\partial M} + R \frac{\partial C_m}{\partial R} \right)_e$$

If the thrust line passes above the centre of gravity there will be another contribution to M_u . Since the moment arm, d , is usually small in terms of the mean chord the contribution is usually small. We have

$$\dot{M}_u^T = -d \left(\frac{\partial T}{\partial u^T} \right)_e$$

and so

$$M_u^T = -\frac{1}{\frac{1}{2}\rho_e V_e S} \left(\frac{d}{c} \right) \left(\frac{\partial T}{\partial u^T} \right)_e .$$

In the absence of compressibility and Reynolds number effects, $M_u = 0$ for a rigid aircraft in gliding flight.

5.4 Moment derivative due to the change in velocity component along the z-axis - M_w

We can once more approach the estimation of the moment derivative due to the velocity component perturbation, w' via a relationship between it and a derivative of the moment coefficient. It is unnecessary to repeat the argument in detail (cf 5.2) and we content ourselves with merely quoting the result

$$M_w = \left(\frac{\partial C_m}{\partial \alpha^T} \right)_e = \left(\frac{\partial C_m}{\partial \alpha} \right)_e .$$

In an analysis of the pitching moment produced by changes in the angle of attack it is customary to reduce the system of forces acting on the wing/body combination to a moment M_0 about an axis through the aerodynamic centre and forces L_0 (lift) and D_0 (drag) passing through this point. For simplicity we shall ignore the small terms arising from the drag force and the thrust. It then follows that if L_t is the lift on the tailplane the pitching moment about an axis through the centre of gravity can be written,

$$M = M_0 + L_0(h - h_0)\bar{c} - L_t \{ \ell_{a.c} - (h - h_0)\bar{c} \}$$

where $h\bar{c}$ denotes the position of centre of gravity on the mean aerodynamic chord,

$h_0\bar{c}$ the position of the aerodynamic centre,

and $\ell_{a.c}$ the distance between the aerodynamic centre of wing-body combination and that of the tailplane.

But the total lift $L = L_0 + L_t$, so that

$$M = M_0 + L(h - h_0)\bar{c} - L_t \ell_{a.c} .$$

Now

$$L_t = \frac{1}{2} Q_\alpha \rho V^2 S_t a_{1t} \alpha_{t \text{ eff}}$$

$$\text{where } \alpha_{t \text{ eff}} = \alpha \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) + \eta_t .$$

Hence

$$C_m = C_{m_0} + (h - h_0) C_L - Q_\alpha \left(\frac{S_t \ell_{a.c.}}{S_c} \right) a_{1t} \alpha_{t \text{ eff}}$$

and

$$\left(\frac{\partial C_m}{\partial \alpha} \right)_e = (h - h_0) \left(\frac{\partial C_L}{\partial \alpha} \right)_e - a_{1t} Q_\alpha \left(\frac{S_t \ell_{a.c.}}{S_c} \right) \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)_e .$$

Here we have made the usual assumption that the aerodynamic centre position is insensitive to variation in the angle of attack in accord with the findings of linearized theory.

The estimation of the derivative M_w , which determines the increment in pitching moment due to a perturbation velocity, w' , thus reduces to the estimation of the aerodynamic centre of the wing-body combination, the rate of change of the lift coefficient with angle of attack together with the downwash at the tailplane and the lift coefficient derivative for the tailplane, a_{1t} . This last should strictly take account of interference between the tailplane and the rear body of the aircraft.

5.5 Rotary damping in pitch or the moment derivative, M_q

We have already demonstrated that the rate of pitch can be treated as equivalent to an incremental change in the angle of attack of various parts of the airframe. The forces associated with these incremental angles of attack (a longitudinal camber) give rise to moments about the y-axis and hence a derivative M_q .

In dimensional form the moment due to the tailplane is

$$(M_t)_{\text{due to } q} = (Z_t)_{\text{due to } q} \ell_t ,$$

which by virtue of the relationship between the Z-force and q , previously

given, can be expressed in derivative form as

$$(M_q)_t = -Q_q \frac{S_t \ell^2}{S \bar{c}^2} a_{lt} .$$

In contrast to what happens in the case of the force derivative the moment due to an elementary area of the wing ahead of the y-axis adds to that contributed by an elementary area aft of the y-axis, so that the wing contribution to M_q can be important, particularly so for highly swept wings. Similar remarks apply to the contribution of the body and these contributions can be estimated on the same basis as outlined in the discussion of Z_q .

5.6 The \dot{w} or acceleration derivatives

In the sense that the derivatives we have thus far discussed can be identified with forces and moments acting during a steady motion (a given angle of attack or angular velocity) they differ from those that arise from a changing component of velocity along the z-axis, or \dot{w} . To differentiate them they are often termed quasi-steady.

Consider what happens to the pressure distribution over a wing under conditions of varying angle of attack. It takes time to adjust and so differs from the pressure distributions associated with the sequence of angles of attack. The changing conditions are usually accounted for by introducing an acceleration derivative into the force and moment formulation.

It can be demonstrated that this procedure is not wholly satisfactory, in general. In particular if the angle of attack of a three-dimensional wing is suddenly changed from zero to a finite value, the lift generated will vary and, apart from a very short interval of time in the case of subsonic flow, is below (or lags behind) the asymptotic value, which is reached in a finite time supersonically, but after a theoretically infinite time subsonically. Again if we consider a motion in which $q = 0$ and $\dot{\alpha} = \text{constant}$ it can be shown that the lift associated with the changing angle of attack is, according to linearized theory, as shown in Fig 9.

A more usual approach to the determination of the \dot{w} derivatives is via the aerodynamics of wings (or complete aeroplanes) in oscillatory flow retaining only first order terms in the frequency parameter. In experiments with oscillating models the derivatives M_q and $M_{\dot{w}}$ are obtained in combination as the sum $(M_q + M_{\dot{w}})$, but either a theoretically determined value of M_q (or a separate

steady test) enables the M_w derivative to be isolated. The same oscillatory theory can be used, in principle, to evaluate the tailplane contribution to the derivative M_w . However, some insight can be obtained by an approximate approach, which accounts for the lags in the arrival at the tailplane of the downwash due to the loading associated with the angle of attack.

Convection of the vorticity downstream implies that under changing angle-of-attack conditions the change in circulation around the wing is felt at the tailplane after a time lapse of l_t/V_e , or the instantaneous downwash at the tail is that which corresponds not to the current angle of attack, $\alpha(t)$ but to $\alpha\left(t - \frac{l_t}{V_e}\right)$. The associated change in downwash is then, approximately,

$$\begin{aligned}\Delta \epsilon &= - \frac{\partial \epsilon}{\partial \alpha} \frac{l_t}{V_e} \dot{\alpha} \\ &= - \Delta \alpha_t, \quad \text{the change in effective angle} \\ &\quad \text{of attack of the tailplane.}\end{aligned}$$

This generates a force and a pitching moment which can be written

$$Z_t = - \frac{1}{2} Q_\alpha \rho V^2 S_t a_{lt} \Delta \alpha_t,$$

and

$$M_t = - \frac{1}{2} Q_\alpha \rho V^2 S_t l_t a_{lt} \Delta \alpha_t.$$

From which the contributions to the derivatives are easily obtained. They are,

$$(Z_w)_t = - Q_\alpha a_{lt} \frac{S_t l_t}{S \bar{c}} \left(\frac{\partial \epsilon}{\partial \alpha} \right)_e$$

and

$$(M_w)_t = - Q_\alpha a_{lt} \frac{S_t l_t^2}{S c^2} \left(\frac{\partial \epsilon}{\partial \alpha} \right)_e.$$

5.7 Control derivatives - Z_η and M_η

Deflection of the tailplane, or part of it (elevator), with respect to the remainder of the aircraft produces a change in circulation around the tailplane and a force. This force is the source of the derivative Z_η and by the moment it generates around the y-axis, the derivative M_η . We have already

introduced the tailplane derivative a_{1t} . More generally we can write

$$(-C_Z)_t = a_{1t}\alpha_t + a_{2t}\eta$$

so that we have

$$Z_\eta = -Q \frac{S_t}{S} a_{2t}; \quad M_\eta = -Q \frac{S_t}{S} \frac{l_t}{c} a_{2t}.$$

For tailless aircraft the pitch motivator is usually a flap at the wing trailing edge, termed the elevon.

In this case we have for the wing,

$$(-C_Z)_w = a_1\alpha + a_2\eta \quad \text{and} \quad Z_\eta = -a_2.$$

The moment involves a moment arm, which in this case is not easily approximated, but can be obtained from a suitable lifting surface theory.

6 ON HOW THE LATERAL FORCES AND MOMENTS ARISE

In linearized studies of the dynamics of an aeroplane during asymmetric flight the forces and moments are represented by the following derivatives,

Sideslip derivatives Y_v, L_v, N_v ,
 Rotary derivatives L_p, N_p, L_r, N_r ,
 Acceleration derivatives $L_{\dot{v}}$ and $N_{\dot{v}}$,

together with

four motivator derivatives, L_ξ, N_ξ and L_ζ, N_ζ .

6.1 Sideforce derivative - Y_v

This derivative is analogous to Z_w , but in this case there is no large aerodynamic surface like the wing involved. When an aeroplane is in sideslip components of it are at an angle of incidence to the velocity vector (or to the relative incident flow), see Fig 2. It can be readily appreciated that the main contributors to the sideforce derivative are the fin and the fuselage. The first of these is a 'lifting'-surface, which can be accorded an 'effective lift coefficient derivative', a_{1f} in the same way as we introduce a_{1t} in section 5. By following the same line of argument we can easily obtain the result, see Fig 12.

$$Y_{v_f} = - Q_\beta a_{1f} \left(1 - \frac{\partial \sigma}{\partial \beta} \right)_e \frac{S_f}{S} ,$$

where σ is the sidewash induced at the fin by the asymmetric loading distribution present over the wing-fuselage combination. Q_β is a relative kinetic pressure factor, which except in unusual circumstances, large angle of attack or fin immersed in a propeller slipstream, does not differ radically from unity.

6.2 Rolling and yawing moments due to sideslip - L_v and N_v

Three geometric features of the wing influence the rolling moment acting on an aircraft in sideslipping motion. These are its dihedral angle, its position on the body section (high, mid or low wing) and its sweep angle. We consider these in turn.

To examine the effect of dihedral it is convenient to restrict our attention to a planar wing (see Fig 10), in which the port and starboard portions of the wing are inclined at a small constant angle, Γ , to the plane containing the root chord and the y-axis.

As can be seen from the diagram, Fig 10, the velocity normal to the starboard panel is $w_0 + v\Gamma$, whilst that of the port panel is $w_0 - v\Gamma$. This antisymmetric change in the effective angle of attack produces a load distribution giving a negative rolling moment proportional to $v\Gamma$, hence a contribution to the derivative L_v proportional to the dihedral angle.

To explain the way the wing position on the body affects L_v we use the concepts of slender-body theory. This reduces to a consideration of the two-dimensional flow associated with the cross-flow component of velocity alone, that is, the component v , when $\alpha_e = 0$. Fig 11 illustrates the flow around the body and this shows that the body induces velocities which enhance the dihedral effect when the wing is set high on the body. On the other hand a low wing will diminish the dihedral effect.

Any attempt at calculating the rolling moment due to sideslip for a swept wing with zero dihedral presents a number of problems. Nevertheless, the nature of the rolling moment associated with sweep can be appreciated using simple physical arguments. Sideslip (see Fig 12) effectively decreases the sweepback of the starboard panel and increases the sweepback of the port resulting in an increase of lift on the starboard side and decrease on the port side so giving

rise to a rolling moment. Further insight into the form of the rolling moment can be obtained if we consider a large aspect ratio wing of constant chord and sweepback, Λ , see Fig 12.

Suppose V_n represents the component of V_e normal to the leading edge and V_t the component of V_e parallel to the leading edge of the wing. Then

$$V_n = V_e \cos(\Lambda - \beta) \quad \text{and} \quad V_t = V_e \sin(\Lambda - \beta)$$

at some point of the starboard panel. Only the former component produces a lift and on an elementary strip, PQ_1 , this is $\frac{1}{2}\rho V_n^2 a_n c_n \delta s$, where a_n is the lift coefficient derivative of the 'normal' section and c_n , δs are as marked in the diagram.

The velocity normal to the wing panel is

$$w = \alpha_n V_n = u_e \alpha = V_e \alpha$$

where the angles of attack are measured relative to the no-lift line. Only the velocity component V_n produces a lifting force and the lift on an elementary strip normal to the wing leading edge is

$$\begin{aligned} \delta L &= \frac{1}{2}\rho V_e^2 a_n c_n \alpha_n \delta s \\ &= \frac{1}{2}\rho V_e V_e a_n c_n \alpha_n \delta y, \quad \text{see Fig 12.} \end{aligned}$$

But

$$V_n = V_e (\cos \Lambda + \beta \sin \Lambda) \quad \text{to first order in } \beta$$

so that

$$\begin{aligned} \delta L &= \frac{1}{2}\rho V_e^2 a_n (\cos \Lambda + \beta \sin \Lambda) c_n \alpha_n \delta y, \\ &= \frac{1}{2}\rho V_e^2 a (1 + \beta \tan \Lambda) c \alpha \delta y, \end{aligned}$$

where a_n is lift coefficient derivative for 'normal' section and a is the corresponding quantity for a streamwise section.

Since a corresponding section on the port wing experiences a decrease in the lift due to β , a rolling moment is produced which is proportional to $-\alpha\beta$. Hence this particular contribution to the rolling moment due to sideslip cannot

be expressed as a linear derivative L_v except when the angle of attack remains unchanged, as, of course, is the case within the linearized lateral motion.

The moment about the x-axis of the fin sideforce due to sideslip makes another contribution to the derivative L_v . If $-z_{fv}$ denote the z-coordinate of the aerodynamic centre of the load on the fin arising from the sideslipping motion and Y_f the sideforce, the rolling moment is $Y_f z_{fv}$, see Fig 12. By virtue of the relationships already quoted for Y_v we then have

$$L_{vf} = -a_{1f} Q_v \left(1 - \frac{\partial \sigma}{\partial \beta} \right)_e \frac{S_f z_{fv}}{S_b} .$$

The same sideforce also produces a yawing moment and this involves the fin arm, l_f , in place of $-z_{fv}$. It is easily shown that

$$N_{vf} = a_{1f} Q_v \left(1 - \frac{\partial \sigma}{\partial \beta} \right)_e \frac{S_f l_f}{S_b} .$$

This positive contribution to the N_v of the aircraft is opposed by an often large, negative contribution from the body. Inviscid flow theory and, in particular, slender-body theory accounts for such a contribution, but as these theories do not give rise to a sideforce the moment is also in error.

6.3 Rolling and yawing moment derivatives due to the rate of roll - L_p and N_p

Suppose the aircraft rolls with rate of roll, p , about the aerodynamic-body axis, Ox , as shown in Fig 13. This motion clearly induces a downward velocity equal to py for all points of the section of the starboard wing at a distance y from the plane of symmetry. In the presence of the forward velocity V_e this downward velocity can be interpreted as equivalent to an increase in the local angle of attack of py/V_e , see Fig 13. There is an equivalent decrease in the angle of attack of the corresponding section of the port wing. We, therefore, can regard a rolling velocity as equivalent to an antisymmetric twist, which increase in magnitude to a maximum at the wing tips. There is an associated antisymmetric loading and a rolling moment proportional to p . This is the source of the wing contribution to L_p . Smaller contributions come from the tail surfaces. That due to the tailplane is the result of the antisymmetric twist equivalent of the rolling and the changes this effectively produces in the tailplane angle of attack.

As for other fin contributions the L_{pf} can be estimated on the basis of a mean, effective change in the angle of incidence of the fin. This can be approximated as

$$\left\{ -\frac{pz_{fp}}{v_e} + \left(\frac{pb}{v_e}\right) \frac{\partial \sigma}{\partial \left(\frac{pb}{v_e}\right)} \right\} = -\frac{pb}{v_e} \left\{ \frac{z_{fp}}{b} - \frac{\partial \sigma}{\partial \left(\frac{pb}{v_e}\right)} \right\}$$

which gives rise to a sideforce, see Fig 13,

$$Y_f = -\frac{1}{2} \rho v_e Q_p a_{lf} \delta pb \left\{ \frac{z_{fp}}{b} - \frac{\partial \sigma}{\partial \left(\frac{pb}{v_e}\right)} \right\}.$$

From which we can derive the fin contribution to the rolling moment derivative as

$$L_{pf} = -Q_p a_{lf} \frac{S_f z_{fp}}{S_b} \left\{ \frac{z_{fp}}{b} - \frac{\partial \sigma}{\partial \left(\frac{pb}{v_e}\right)} \right\}.$$

Associated with the asymmetric loading that gives rise to the derivative L_p are changes in the forces induced in the xy-plane and these produce a yawing moment proportional to the rate of roll and hence a derivative, N_p .

Again there is a fin contribution and this can easily be shown to be given by the relationship,

$$N_{pf} = Q_p a_{lf} \frac{S_f l_f}{S_b} \left\{ \frac{z_{fp}}{b} - \frac{\partial \sigma}{\partial \left(\frac{pb}{v_e}\right)} \right\}.$$

In both the expression for L_{pf} and N_{pf} the distance z_{fp} represents the distance between the x-axis the aerodynamic centre of the fin loading due to p , that is, due to a linear twist increasing towards the fin tip.

6.4 Rolling and yawing moment derivatives due to rate of yaw, L_r and N_r

When an aircraft experiences a rate of yaw a section of the wing on the starboard side at a distance y from the plane of symmetry has a velocity parallel to the x-axis of $v_e - ry$, whilst the corresponding section of the port side has an increased velocity, $v_e + ry$ (Fig 14). This asymmetric velocity increment decreases the forces on the starboard wing and increases

those acting on the port wing. The resulting moments are proportional to the rate of yaw and account for the wing contributions to L_r and N_r .

There is a significant contribution to L_r from the fin. A rate of yaw r produces an effective increment in the fin angle of incidence, see Fig 14 of

$$\frac{rb}{v_e} \left\{ \frac{l_f}{b} + \frac{\partial \sigma}{\partial \left(\frac{rb}{v_e} \right)} \right\}$$

which gives rise to a sideforce on the fin of

$$(Y_f)_{\text{due to } r} = \frac{1}{2} \rho_e v_e^2 S_f Q_r a_{lf} \frac{rb}{v_e} \left\{ \frac{l_f}{b} + \frac{\partial \sigma}{\partial \left(\frac{rb}{v_e} \right)} \right\}$$

and in turn yields a derivative contribution,

$$L_{rf} = Q_r a_{lf} \frac{S_f z_{fr}}{S_b} \left\{ \frac{l_f}{b} + \frac{\partial \sigma}{\partial \left(\frac{rb}{v_e} \right)} \right\}.$$

Since we are concerned with a mean angle of incidence $z_{fr} \approx z_{fv}$.

There is an even larger contribution to N_r , which can be easily shown to be given approximately by the relationship,

$$N_{rf} = -Q_r a_{lf} \frac{S_f l_f}{S_b} \left\{ \frac{l_f}{b} + \frac{\partial \sigma}{\partial \left(\frac{rb}{v_e} \right)} \right\}.$$

There is a body contribution to N_r , which is analogous to its contribution to M_q .

6.5 Lateral and directional control derivatives

The forces and moments acting on an aircraft due to deflection of its lateral and directional motivators depend upon the form these motivators take. To illustrate the derivatives involved we consider the common flap type motivators, that is, ailerons and rudders.

Deflection of a trailing-edge flap alters the local circulation around the aerofoil. This effect extends the span of motivator and beyond. In general forces and moments are produced by the resulting changes in loading distribution.

For the conventional aileron layout (see Fig 15) the downgoing starboard aileron increases the lift on the outer portions of the starboard wing, whilst on the port side the upgoing aileron decreases local lift. A negative rolling moment is thus produced and associated with the induced effects of the asymmetrical loading and yawing moment. Both these moments are proportional to the mean angle according to linearized theory. The yawing moment is usually positive (that is, in the adverse yaw sense). The moments are represented by $L_{\xi} \xi'$ and $N_{\xi} \xi'$ for an increment ξ' in the aileron angle.

Deflection of the rudder to port (positive sense) results in a positive sideforce and hence a yawing moment (the primary function of the rudder) in the negative sense. The rate of change of this moment with the rudder angle, ζ , defines the derivative N_{ζ} . Because the aerodynamic centre of the loading on the fin is generally off the x-axis a rolling moment is produced at the same time as the yawing moment.

We may take Z_{ζ} to be approximately equal to Z_v , if the chord-ratio of the rudder does not vary much across the fin.

Corresponding to the fin effective lift coefficient slope, a_{1f} , there is a rudder parameter a_{2f} and so we may easily deduce the results

$$N_{\zeta} = - Q_{\alpha} \frac{S_f \ell_f}{Sb} a_{2f}$$

and

$$L_{\zeta} = Q_{\alpha} \frac{S_f z_f}{Sb} a_{2f} .$$

7 FORCE AND MOMENT DERIVATIVES FOR DIFFERENT SYSTEMS OF AXES

We have already remarked that a set of derivatives is particular to the choice of axis system. It is frequently necessary to convert the values of derivatives to correspond to a new choice of axes. The most common requirement is to convert from one body system to another body system having parallel y-axis. Let the two systems of axes be $Oxyz$ and $O_1x_1y_1z_1$. Further let the origin O_1 have coordinates $h, 0, k$ relative to O , in the system $Oxyz$, whilst O has coordinates $-h_1, 0, -k_1$ relative to O_1 in the system $O_1x_1y_1z_1$ (see Fig 16). Then, if we write $c = \cos e$ and $s = \sin e$, where e is the angle between O_1x_1 and Ox , which is taken as positive if clockwise rotation about the y-axis through this angle brings Ox to a position parallel to O_1x_1 .

The following relationships then apply,

$$h_1 = ch - sk ; \quad h = ch_1 + sk_1$$

$$k_1 = ck + sh ; \quad k = ck_1 - sh_1$$

and for the angular velocity components,

$$p_1 = cp - sr ; \quad p = cp_1 + sr_1$$

$$q_1 = q ; \quad q = q_1$$

$$r_1 = cr + sp ; \quad r = cr_1 - sp_1 .$$

If we define u, v, w as the components of the velocity of O along the axes x, y and z whilst u_1, v_1, w_1 are the components in the $0_1x_1y_1z_1$ axis system of the velocity of O_1 , the following further relationships are easily established,

$$u_1 = cu - sw + k_1q ; \quad u = cu_1 + sw_1 - kq$$

$$v_1 = v - kp + hr ; \quad v = v_1 + k_1p_1 - h_1r_1$$

$$w_1 = cw + su - h_1q ; \quad w = cw_1 - su_1 + hq .$$

When the same system of forces is reduced to a force through O and a moment about an axis through O or to a force and moment centred on O_1 , the following relationships exist between the components,

$$X_1 = cX - sZ ; \quad L_1 = cL - sN + k_1Y$$

$$Y_1 = Y ; \quad M_1 = M + hZ - kX$$

$$Z_1 = cZ + sX ; \quad N_1 = cN + sL - h_1Y .$$

Partial differentiation of these equations gives the following conversion equations,

$$X'_{u_1} = c^2X_u - sc(X_w + Z_u) + s^2Z_w ,$$

$$Z'_{u_1} = c^2Z_u + sc(X_u - Z_w) - s^2X_w ,$$

$$X'_{w_1} = c^2X_w + sc(X_u - Z_w) - s^2Z_u ,$$

$$\begin{aligned}
Z'_{w_1} &= c^2 Z_w + sc(X_w + Z_u) + s^2 X_u \\
M'_{u_1} &= cM_u - sM_w + h(cZ_u - sZ_w) + k(sX_w - cX_u) \\
M'_{w_1} &= cM_w + sM_u + h(cZ_w + sZ_u) - k(sX_u + cX_w) \\
M'_{q_1} &= M_q + h(Z_q + M_w) - k(X_q + M_u) + h^2 Z_w - hk(Z_u + X_w) + k^2 X_u \\
Y'_{v_1} &= Y_v \\
L'_{v_1} &= cL_v - sN_v + k_1 Y_v \\
N'_{v_1} &= cN_v + sL_v - h_1 Y_v \\
L'_{p_1} &= c^2 L_p + k_1 \{k_1 Y_v + c(L_v + Y_p)\} + s \{sN_r - c(L_r + N_p) - k_1(Y_r + N_v)\} \\
L'_r &= c^2 L_r + c(k_1 Y_r - h_1 L_v) - h_1 k_1 Y_v + s \{h_1 N_v + k_1 Y_p - sN_p + c(L_p - N_r)\} \\
N'_p &= c^2 N_p + c(k_1 N_v - h_1 Y_p) - h_1 k_1 Y_v + s \{h_1 Y_r + k_1 L_v - sL_r + c(L_p - N_r)\} \\
N'_r &= c^2 N_r + h_1 \{h_1 Y_v - c(Y_r + N_v)\} + s \{sL_p + c(L_r + N_p) - h_1(L_v + Y_p)\} .
\end{aligned}$$

These transformation relationships cover the effect of changing the centre of gravity position ($h \neq 0$, $k \neq 0$, $l = 0$) as well as the change from one body-axis system to another having a common origin, for example, from aero-body axes to a body axes. As such they serve to illustrate the point already made about the complicated nature of the relationship between aero-normalized derivatives and coefficient derivatives for a general body system of axes.

8 CONCLUDING REMARKS

Notwithstanding the undoubted success that has attended the use of aerodynamic derivative formulations of the forces and moments in a variety of flight dynamic problems, it is as well to remind ourselves that it is incorrect to assume that this is always the case. For example, rapid movement of controls (motivators) as gust alleviating devices within an 'active control' system may require the truly unsteady nature of the aerodynamic phenomenon to be represented more exactly. Nevertheless, other requirements place a greater emphasis on the development of formulations valid for large disturbance motion.

To meet this latter need experimental work is in progress using free-flight models and on the extension of the angle-of-attack range of wind-tunnel

oscillatory tests. In addition rotary rigs are being developed. In our efforts in this direction we should not forget that the above more fundamental question remains unanswered, namely, under what conditions, specified quantitatively, is the customary derivative formulation justified.

Table 1

DIVISORS FOR FORMING NORMALIZED QUANTITIES

Quantity	Example	Aero-normalizing divisor
Length		l_0
Time		l_0/v_e
Linear velocity	w	v_e
Linear acceleration	\dot{w}	v_e^2/l_0
Angular velocity	q	v_e/l_0
Force	Z	$\frac{1}{2}\rho_e v_e^2 S$
Moment	M	$\frac{1}{2}\rho_e v_e^2 S l_0$
Force derivatives with respect to:		
linear velocity	Z_w	$\frac{1}{2}\rho_e v_e S$
linear acceleration	$Z_{\dot{w}}$	$\frac{1}{2}\rho_e S l_0$
angular displacement	Z_η	$\frac{1}{2}\rho_e v_e^2 S$
angular velocity	Z_q	$\frac{1}{2}\rho_e v_e S l_0$
Moment derivatives with respect to:		
linear velocity	M_w	$\frac{1}{2}\rho_e v_e S l_0$
linear acceleration	$M_{\dot{w}}$	$\frac{1}{2}\rho_e S l_0^2$
angular displacement	M_η	$\frac{1}{2}\rho_e v_e^2 S l_0$
angular velocity	M_q	$\frac{1}{2}\rho_e v_e S l_0^2$

The value of l_0 can be chosen to suit the needs of the problem being studied. It is not unusual to choose $l_0 = \bar{c}$, the aerodynamic mean chord for the longitudinal derivatives and $l_0 = b$ for the lateral derivatives.

Table 2

(1) US symbol*	(2) ISO symbol	Multiplying factor to obtain (1) from (2)**			
		with $l_1 = \bar{c}$, $l_2 = b$	$l_2 = b/2$		
$C_{X_{\delta e}}$ $C_{X_{\alpha}}$ C_{X_u}	$C_{Z_{\delta e}}$ $C_{Z_{\alpha}}$ C_{Z_u}	X_{η} X_w X_u	Z_{η} Z_w Z_u	1	
$C_{m_{\delta e}}$ C_{m_q} C_{Z_q}	$C_{m_{\alpha}}$ $C_{m_{\dot{\alpha}}}$	M_{η} M_q Z_q	M_w $M_{\dot{w}}$	2	
$C_{Y_{\delta a}}$ $C_{Y_{\beta}}$	$C_{Y_{\delta r}}$	Y_{ξ} Y_v	Y_{ζ}	1	1
$C_{l_{\delta a}}$ $C_{l_{\delta r}}$ $C_{l_{\beta}}$	$C_{n_{\delta a}}$ $C_{n_{\delta r}}$ $C_{n_{\beta}}$	L_{ξ} L_{ζ} L_v	N_{ξ} N_{ζ} N_v	1	$\frac{1}{2}$
C_{Y_p}	C_{Y_r}	Y_p	Y_r		1
C_{l_p} C_{l_r} $C_{l_{\dot{\beta}}}$	C_{n_p} C_{n_r} $C_{n_{\dot{\beta}}}$	L_p L_r $L_{\dot{v}}$	N_p N_r $N_{\dot{v}}$	2	$\frac{1}{2}$

* In this table it has been assumed that C_{X_u} and C_{Z_u} are the non-dimensional forms of \dot{X}_u and \dot{Z}_u , but this is not always the interpretation. For example, Etkin in Ref 3, uses these as the equivalent of the second term in the expressions for X_u and Z_u ,

$$X_u = 2C_{X_e} + \left(\frac{\partial C_X}{\partial u} \right)_e ; \quad Z_u = 2C_{Z_e} + \left(\frac{\partial C_Z}{\partial u} \right)_e$$

** Characteristic length (l_0) of Table 1 taken as equal to l_1 for longitudinal derivatives and equal to l_2 for the lateral derivatives.

Table 3

(1) ISO symbol	(2) Old British symbol (R & M 1801)	Multiplying factor to obtain (1) from (2)
X_u Z_u Y_v X_w Z_w X_η Z_η Y_ζ	x_u z_u y_v x_w z_w x_η z_η y_ζ	2
X_q Z_q $X_{\dot{w}}$ $Z_{\dot{w}}$ M_u M_w M_η	x_q z_q $x_{\dot{w}}$ $z_{\dot{w}}$ m_u m_w m_η	$\frac{2\ell_t}{\ell_1}$
Y_p Y_r $Y_{\dot{v}}$ L_v N_v L_ξ N_ξ L_ζ N_ζ	y_p y_r $y_{\dot{v}}$ ℓ_v n_v ℓ_ξ n_ξ ℓ_ζ n_ζ	$\frac{b}{\ell_2}$
M_q $M_{\dot{w}}$	m_q $\bar{m}_{\dot{w}} = \left(\frac{M_w}{\rho S \ell_t^2} \right)_{ord}$	$\frac{2\ell_t^2}{\ell_1^2}$
L_p N_p L_r N_r $L_{\dot{v}}$ $N_{\dot{v}}$	ℓ_p n_p ℓ_r n_r $\ell_{\dot{v}}$ $n_{\dot{v}}$	$\frac{b^2}{2\ell_2^2}$

In forming the multiplying factor it has been assumed that the normalized derivatives in column (1) are obtained using as characteristic length $\ell_0 = \ell_1$ for longitudinal and $\ell_0 = \ell_2$ for lateral quantities. Furthermore the practice of using the tail-arm ℓ_t was not always followed, so a check on the definition is advisable before using the factors.

LIST OF SYMBOLS

a_1	rate of change of lift coefficient with angle of attack for a lifting surface, eg wing
a_{1f}	value of a_1 for the fin of an aeroplane
a_{1t}	value of a_1 for the tailplane of an aeroplane
b	wing span
\bar{c}	aerodynamic mean chord of the wing
C_D	drag coefficient = $D/\frac{1}{2}\rho V^2 S$
C_L	lift coefficient = $L/\frac{1}{2}\rho V^2 S$
C_{ℓ}	rolling moment coefficient = $L/\frac{1}{2}\rho V^2 S b$
C_m	pitching moment coefficient = $M/\frac{1}{2}\rho V^2 S \bar{c}$
C_n	yawing moment coefficient = $N/\frac{1}{2}\rho V^2 S b$
C_X	coefficient of component of force along x-axis = $X/\frac{1}{2}\rho V^2 S$
C_Y	coefficient of component of force along y-axis = $Y/\frac{1}{2}\rho V^2 S$
C_Z	coefficient of component of force along z-axis = $Z/\frac{1}{2}\rho V^2 S$
D	drag force
h	height above ground of CG of aeroplane
i_T	inclination of thrust line to x-axis
h	position of CG as fraction of \bar{c} from the leading edge of mean chord
h, k	} displacements of the origins of two systems of body axes (section 7)
h_1, k_1	
l_0	general representative length used in normalization of moments etc.
l_1	} particular representative lengths
l_2	
$l_{a.c.}$	
l_f	fin arm - distance from CG to fin a.c
l_t	tailplane arm - distance CG to tailplane a.c
L or \mathcal{L}	rolling moment (moment about x-axis)
L_p	aero-normalized derivative of rolling moment with respect to rate of roll, p
L_r	aero-normalized derivative of rolling moment with respect to rate of yaw, r
L_v	aero-normalized derivative of rolling moment with respect to sideslip velocity, v
L_{ξ}	aero-normalized derivative of rolling moment with respect to aileron angle, ξ
L_{ζ}	aero-normalized derivative of rolling moment with respect to rudder angle, ζ

LIST OF SYMBOLS (continued)

M	Mach number
M	pitching moment (moment about y-axis)
M_q	aero-normalized derivative of pitching moment with respect to rate of pitch
M_u	aero-normalized derivative of pitching moment with respect to velocity u
M_w	aero-normalized derivative of pitching moment with respect to velocity w
$M_{\dot{w}}$	aero-normalized derivative of pitching moment with respect to rate of change of w
N	yawing moment (moment about z-axis)
N_p	aero-normalized derivative of yawing moment with respect to the rate of roll, p
N_r	aero-normalized derivative of yawing moment with respect to the rate of yaw, r
N_v	aero-normalized derivative of yawing moment with respect to the side-slip velocity, v
N_ξ	aero-normalized derivative of yawing moment with respect to aileron angle, ξ
N_ζ	aero-normalized derivative of yawing moment with respect to rudder angle, ζ
p	rate of roll, angular velocity component about x-axis
q	rate of pitch, angular velocity component about y-axis
r	rate of yaw, angular velocity component about z-axis
Q	(with subscript), relative kinetic pressure
S	wing area (more generally a reference area)
S_f	fin area
S_t	tailplane area
t	time
T	thrust force
u	component of V along x-axis
v	component of V along y-axis
w	component of V along z-axis
V	resultant velocity of aeroplane
x, y, z	coordinates in the system of axes
z	(with subscript) fin arm with respect to x-axis
X	component of force along x-axis
X_q	aero-normalized derivative of X-force with respect to the rate of pitch, q

LIST OF SYMBOLS (concluded)

X_u	aero-normalized derivative of X-force with respect to the velocity, u
X_w	aero-normalized derivative of X-force with respect to the velocity, w
Y	component of force along y-axis
Y_p	aero-normalized derivative of Y-force with respect to the rate of roll, p
Y_r	aero-normalized derivative of Y-force with respect to the rate of yaw, r
Y_v	aero-normalized derivative of Y-force with respect to the sideslip velocity, v
Z	component of force along z-axis
Z_q	aero-normalized derivative of Z-force with respect to the rate of pitch, q
Z_u	aero-normalized derivative of Z-force with respect to the velocity, u
Z_w	aero-normalized derivative of Z-force with respect to the velocity, w
Z_η	aero-normalized derivative of Z-force with respect to the pitch motivator angle, η
α	angle of attack
α_e	angle of attack of a body x-axis with respect to equilibrium flight path
α_0	angle of attack of some datum line, eg no lift line
β	angle of sideslip
γ	angle of climb
Γ	dihedral angle
ϵ	downwash angle
η	elevator angle
η_t	tailplane setting angle
Λ	sweep angle
ξ	aileron angle
ρ	air density

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u, v, w Velocity components
 p, q, r Angular velocity components
 X, Y, Z Force components
 L, M, N Moment components.

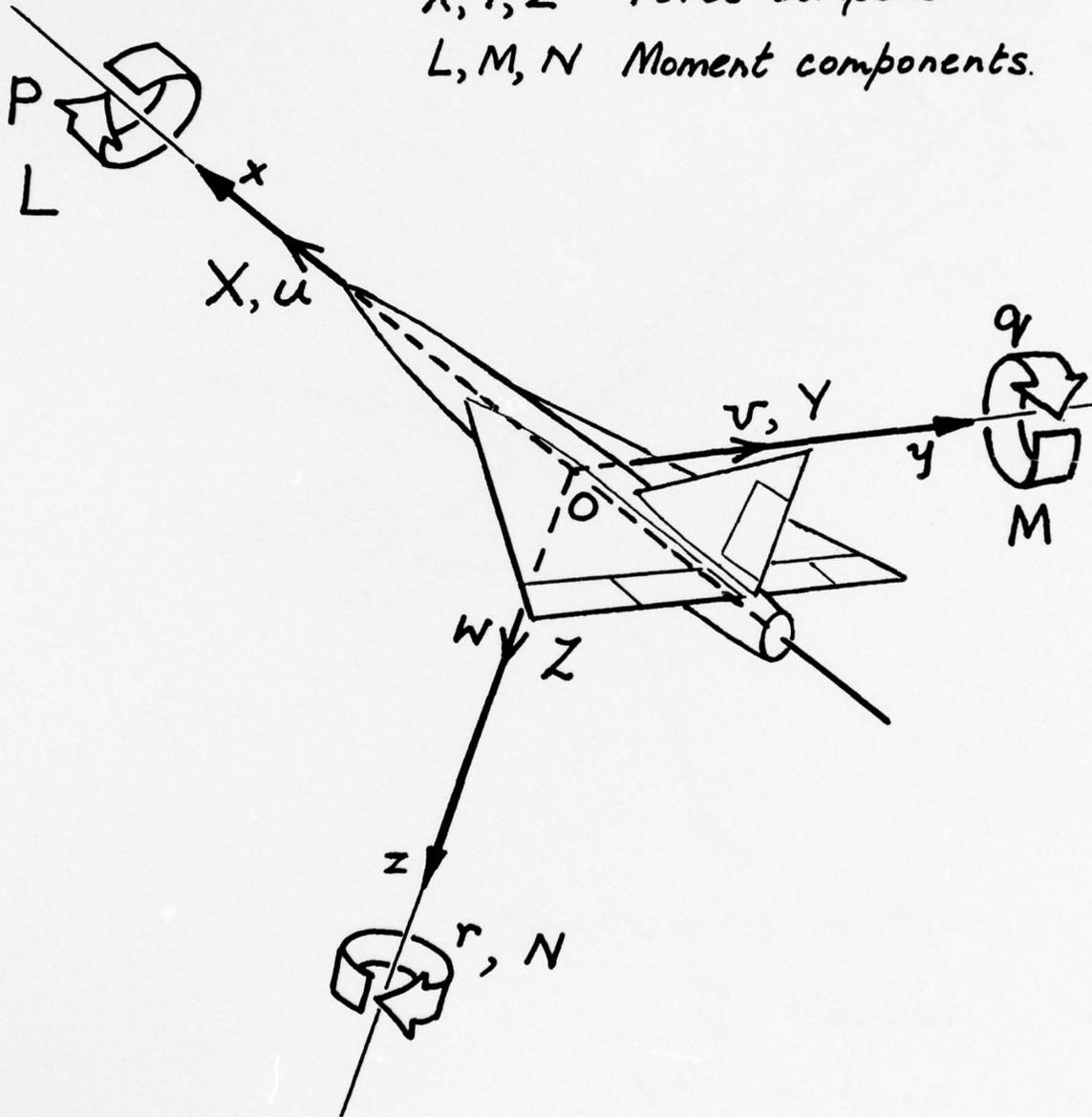
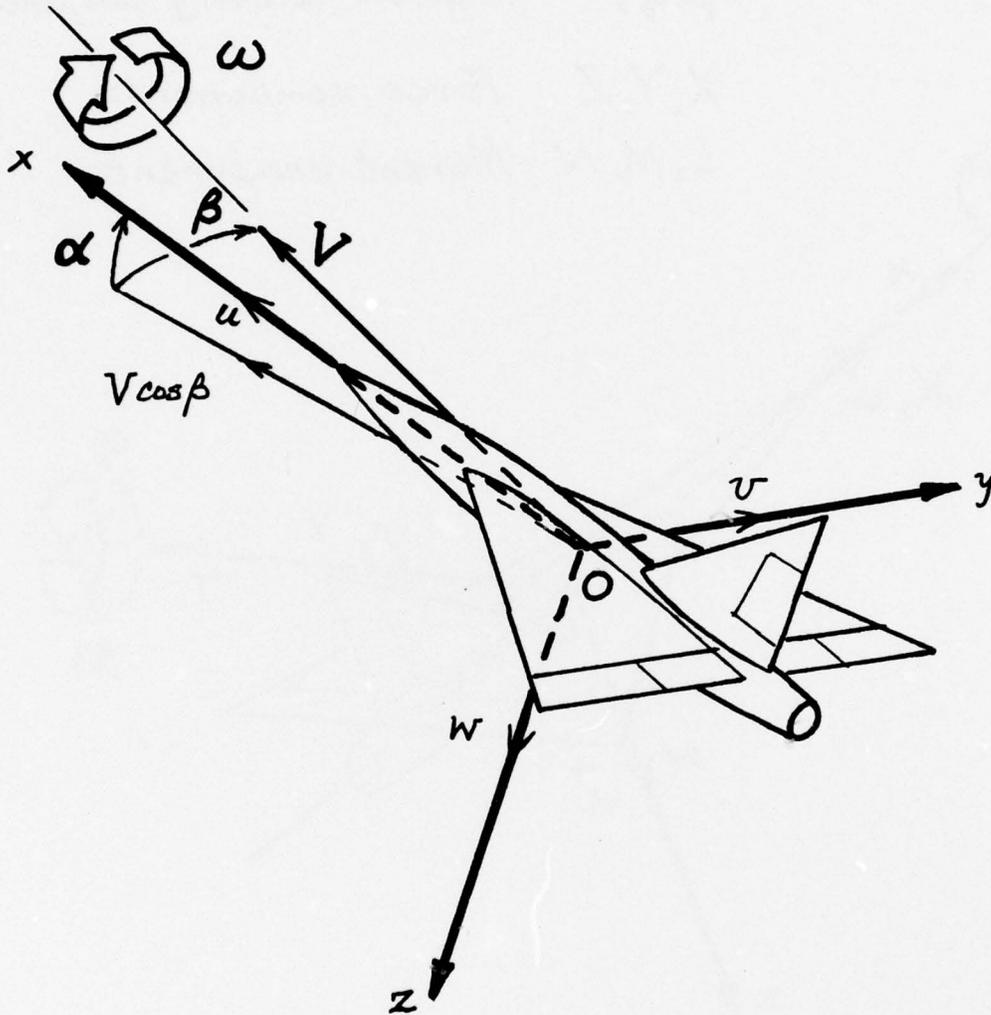


Fig 1 Positive sense of various quantities in a general body system of axes

Fig 2



$$V^2 = u^2 + v^2 + w^2$$

$$\tan \alpha = \frac{w}{u}$$

$$\sin \beta = \frac{v}{V}$$

$$\cos \sigma = \frac{u}{V}$$

Fig 2 Incidence angles

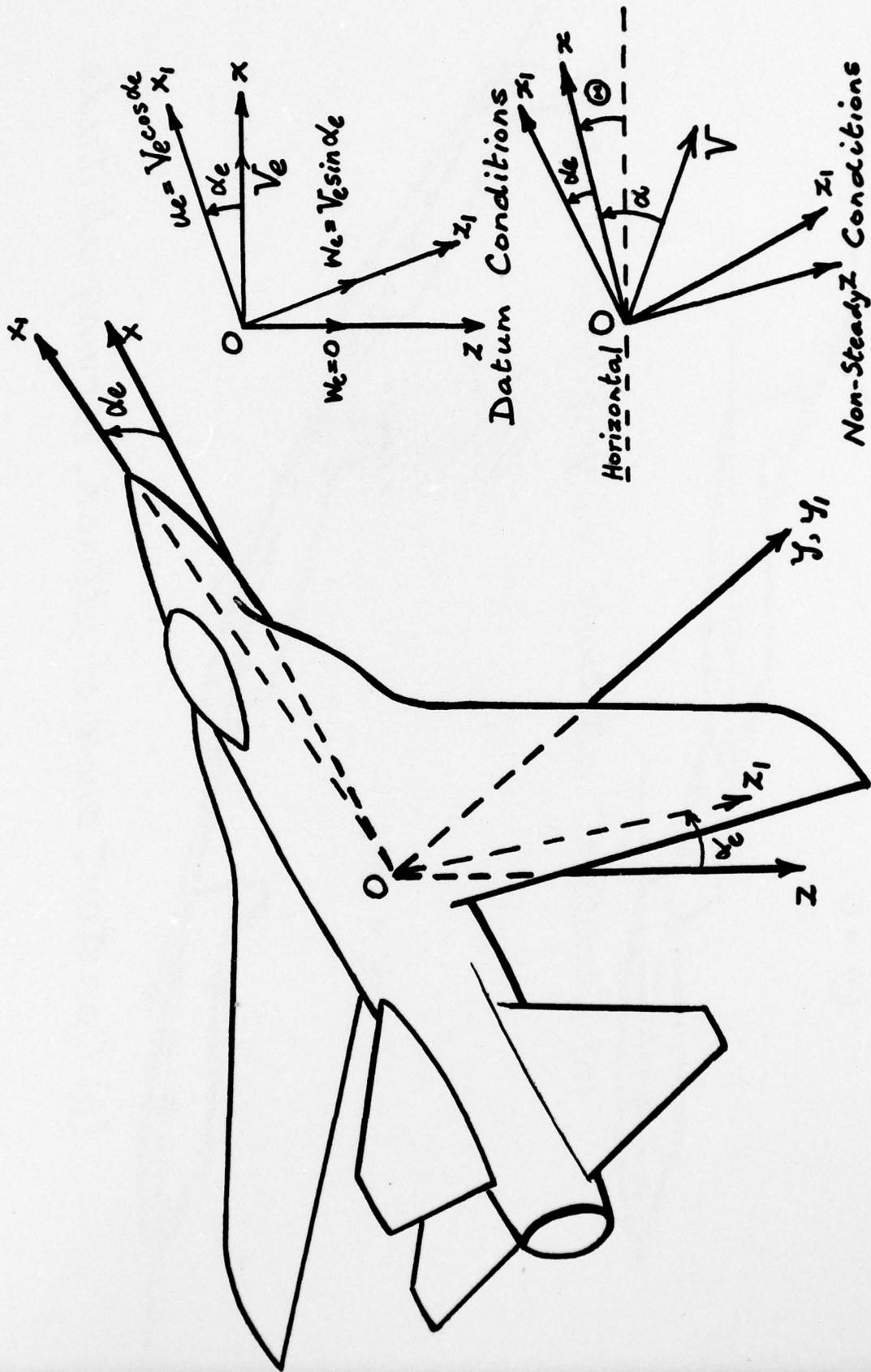


Fig 3 Body and aerodynamic-body axes

Fig 4

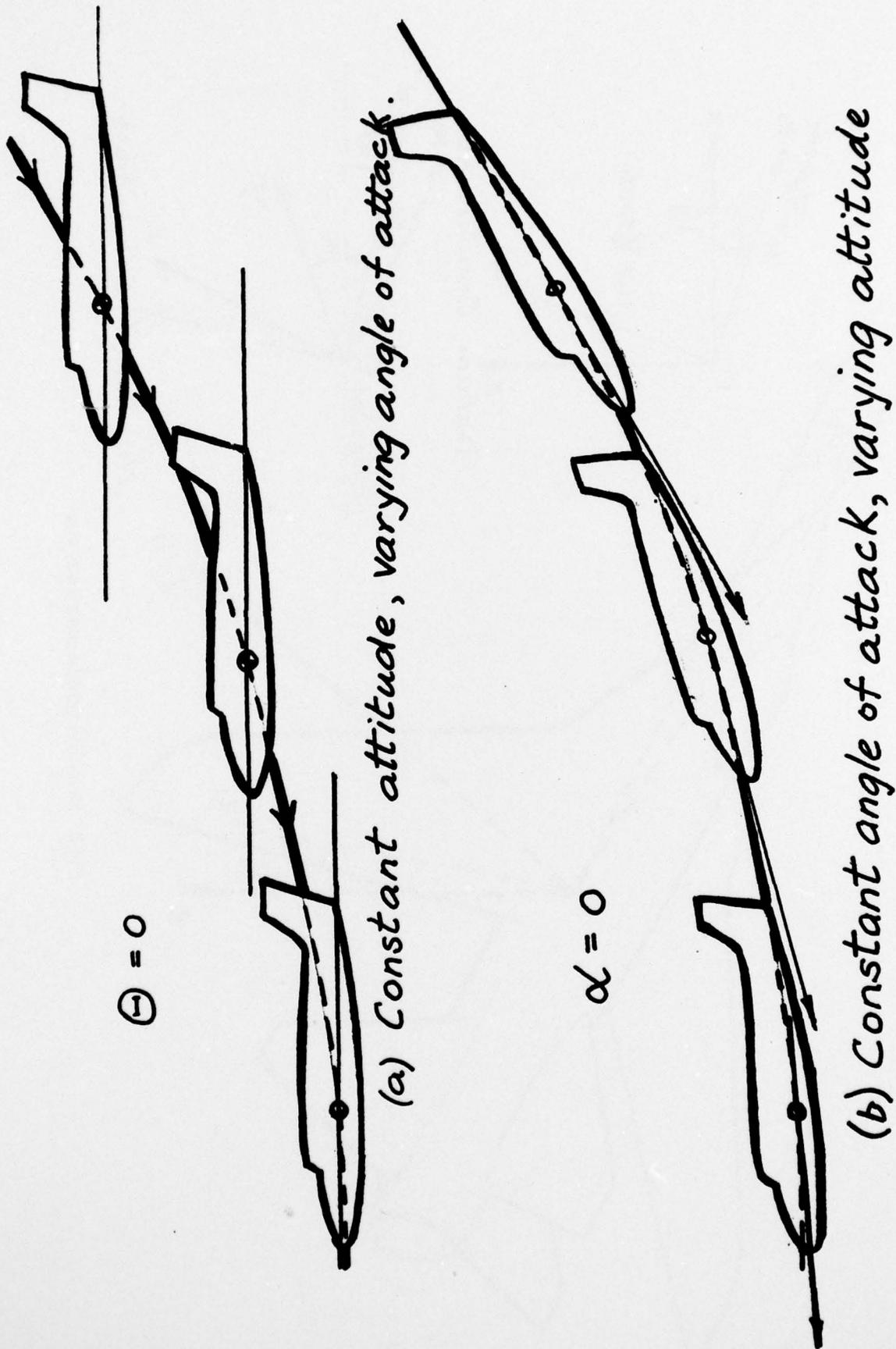
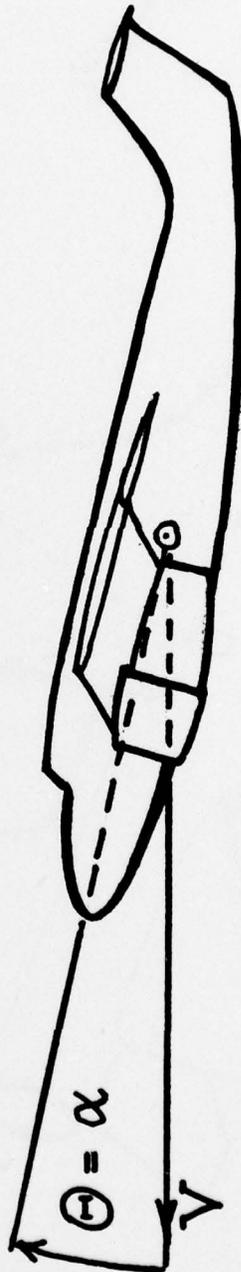


Fig 4 Attitude in pitch and angle of attack



Ground

(a) θ and α independent variables



Ground

(b) Usual wind-tunnel test.

Fig 5

Fig 5 Relationship between angle of attack and attitude angle

Fig 6

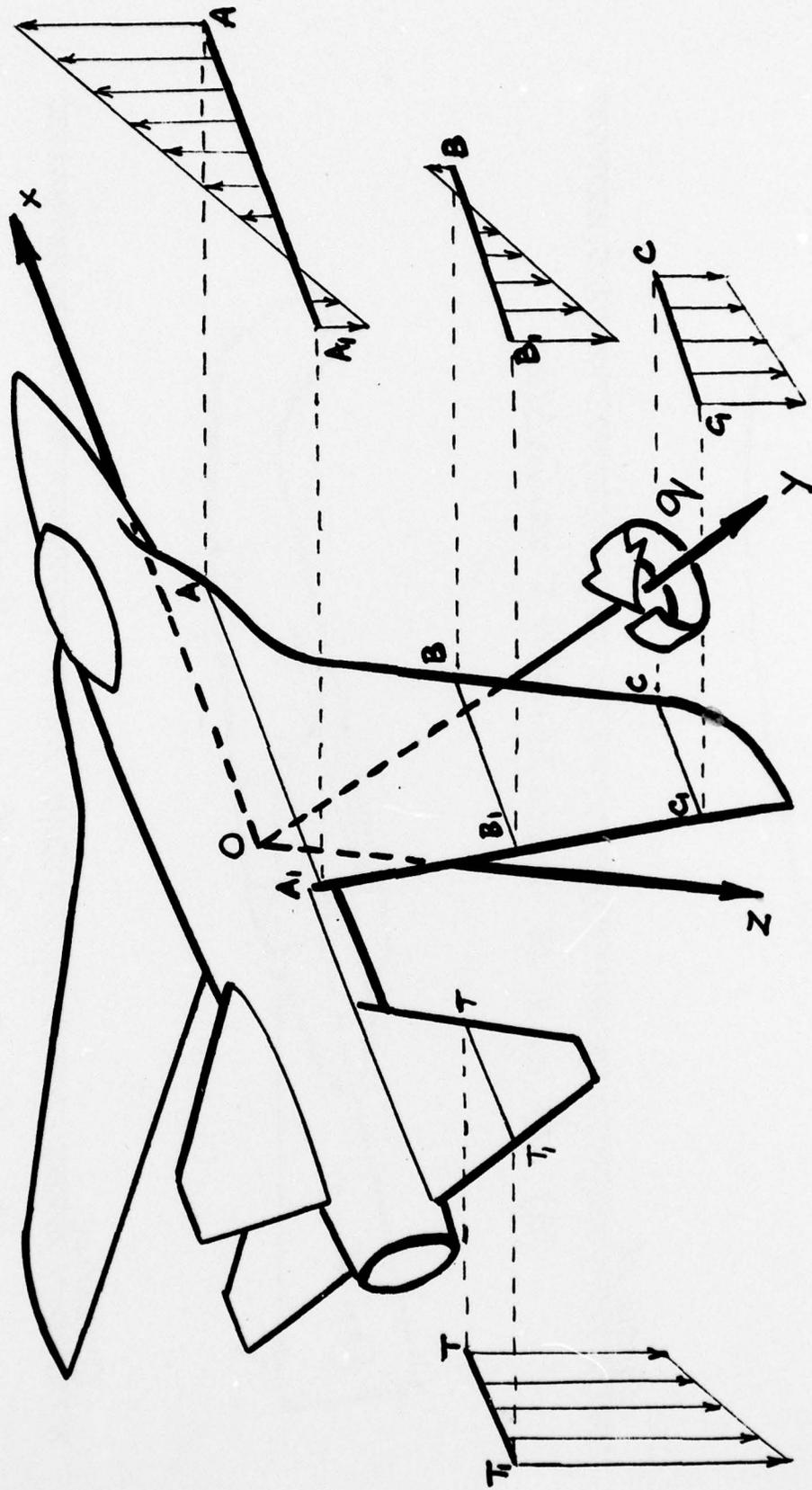


Fig 6 Normal velocities induced by rate of pitch

Fig 7

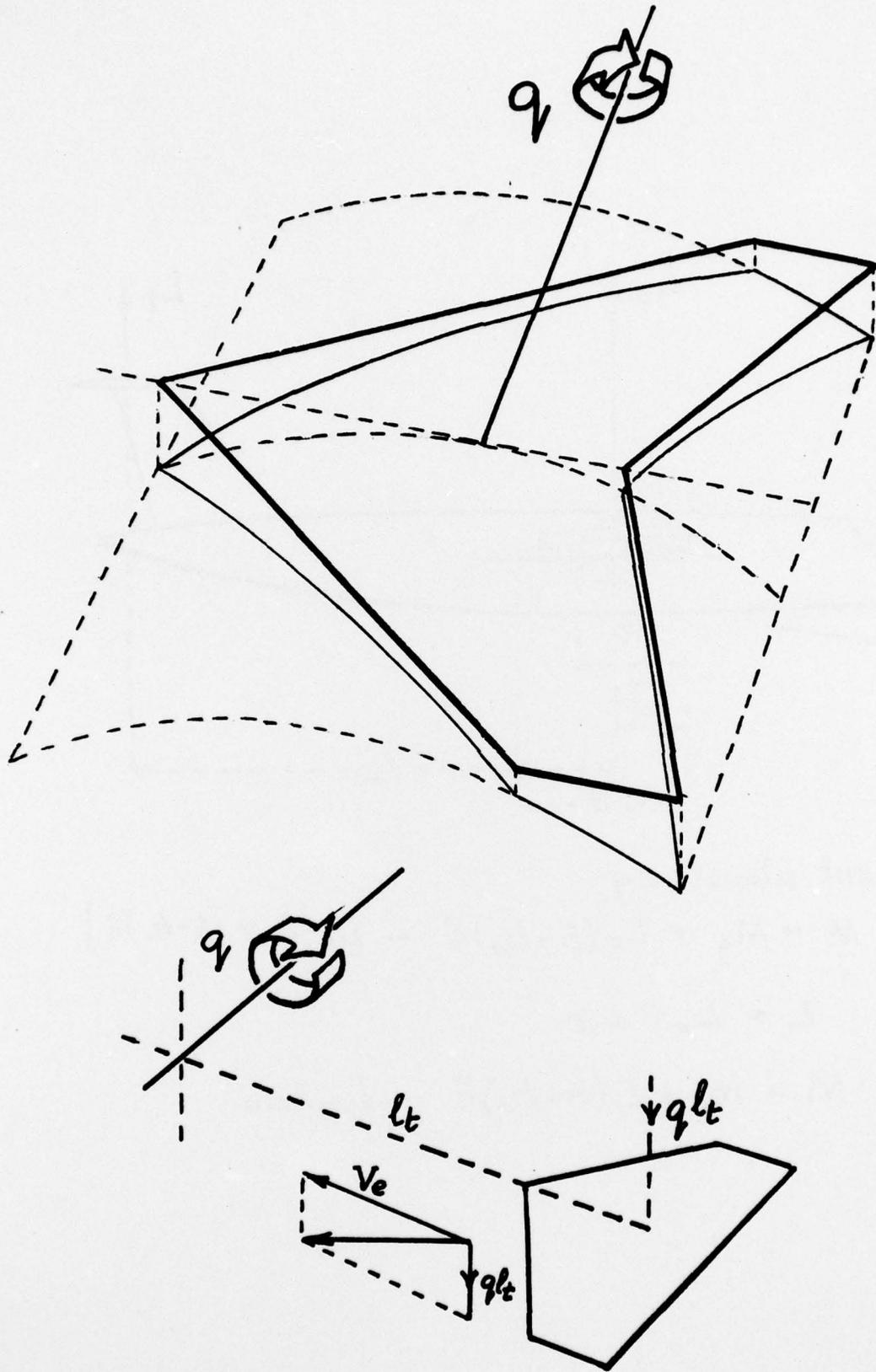
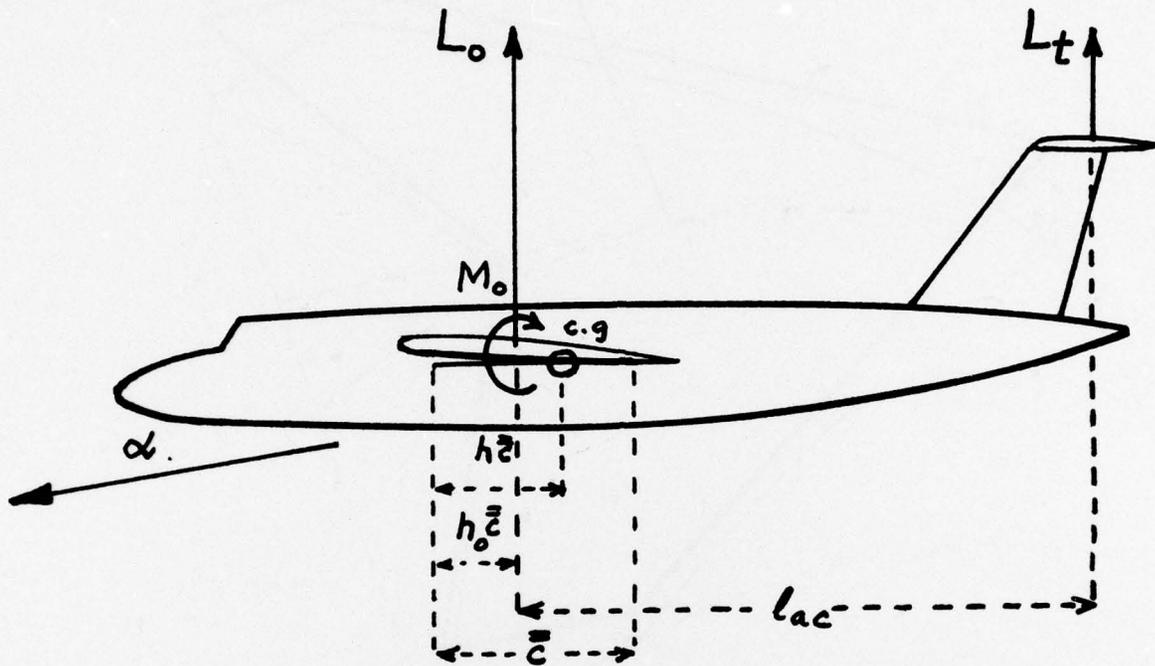


Fig 7 Equivalent cambered wing and tailplane angle of attack

Fig 8



Moment about cg

$$M = M_0 + L_0(h-h_0)\bar{c} - L_t\{l_{a.c.} - (h-h_0)\bar{c}\}$$

But $L = L_0 + L_t$

$$\therefore M = M_0 + L(h-h_0)\bar{c} - L_t l_{a.c.}$$

Fig 8 Forces and moments due to angle of attack

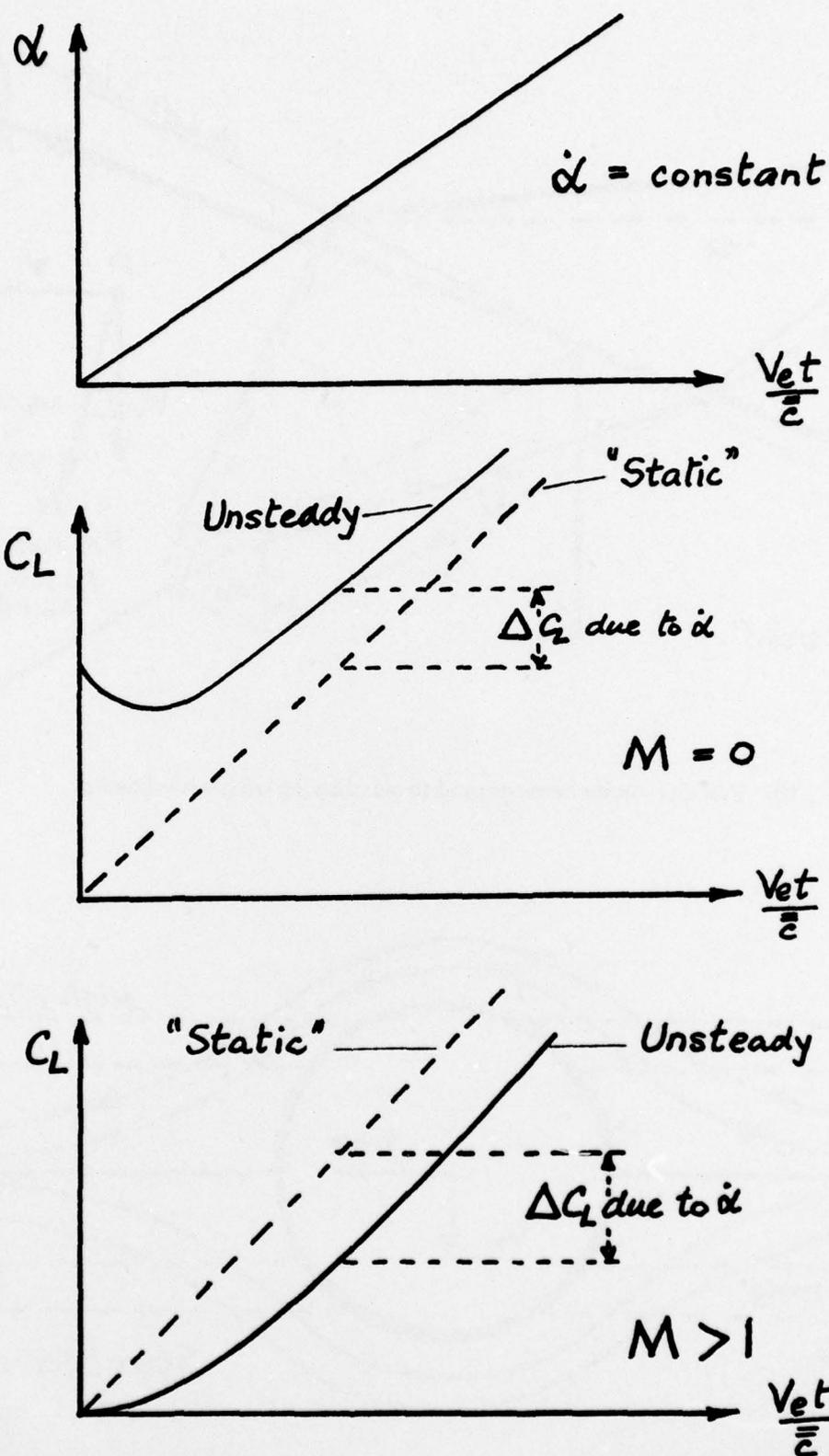


Fig 9 Illustration of shortcoming of derivative formulation of aerodynamic forces (see Ref 3)

Figs 10&11

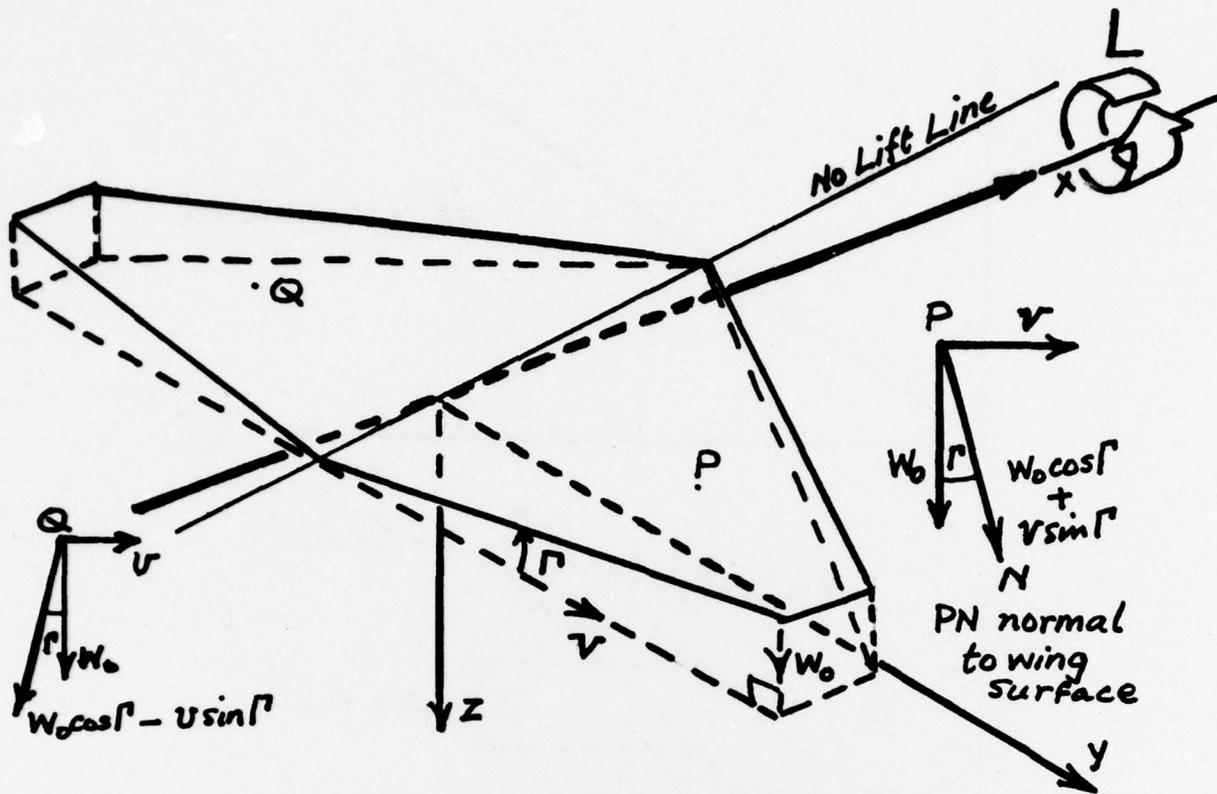


Fig 10 Velocity components normal to sideslipping wing with dihedral

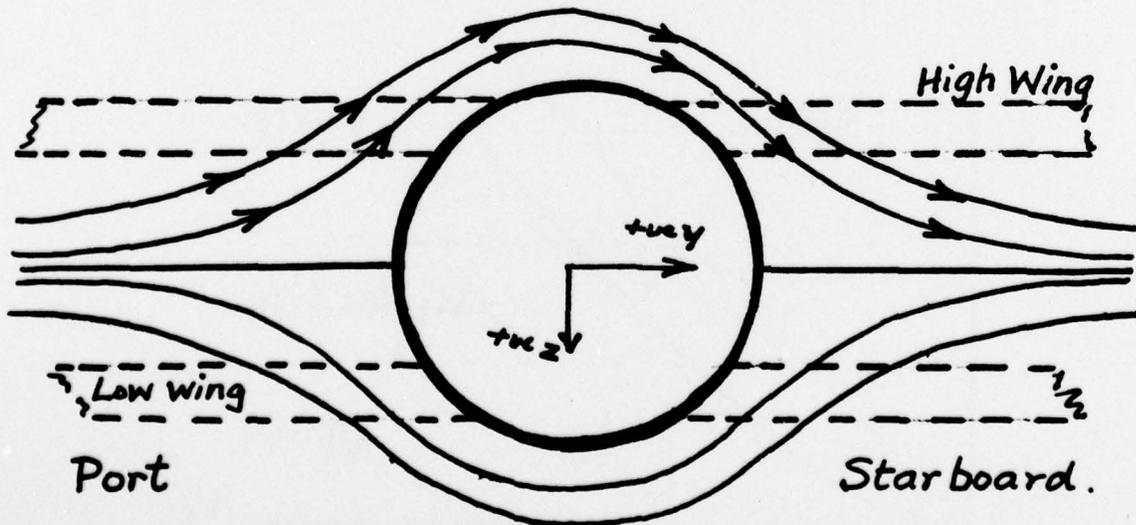


Fig 11 Effect of sideslip flow around body on wing

Fig 13

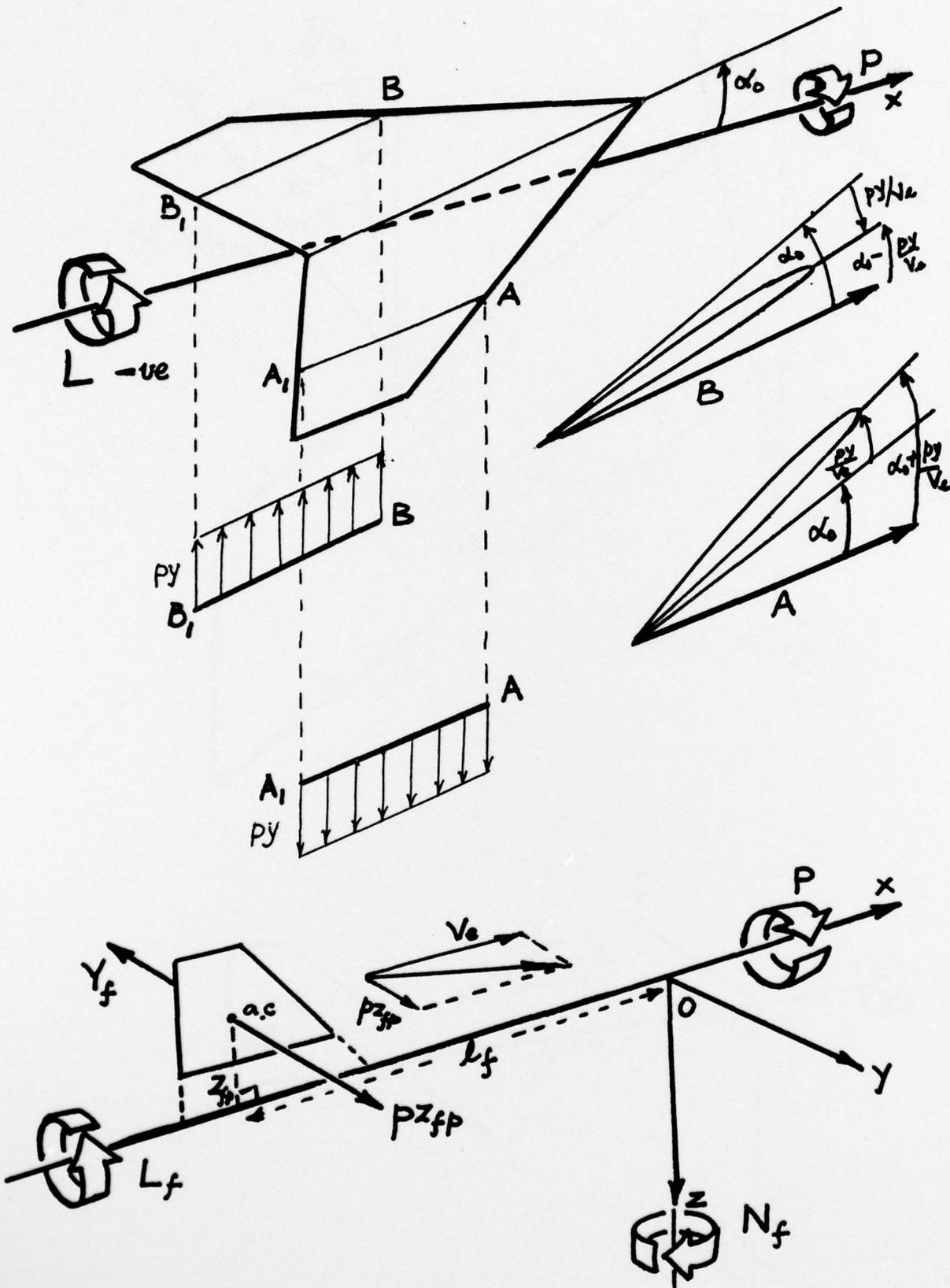


Fig 13 Damping in roll action of wing and fin

Fig 14

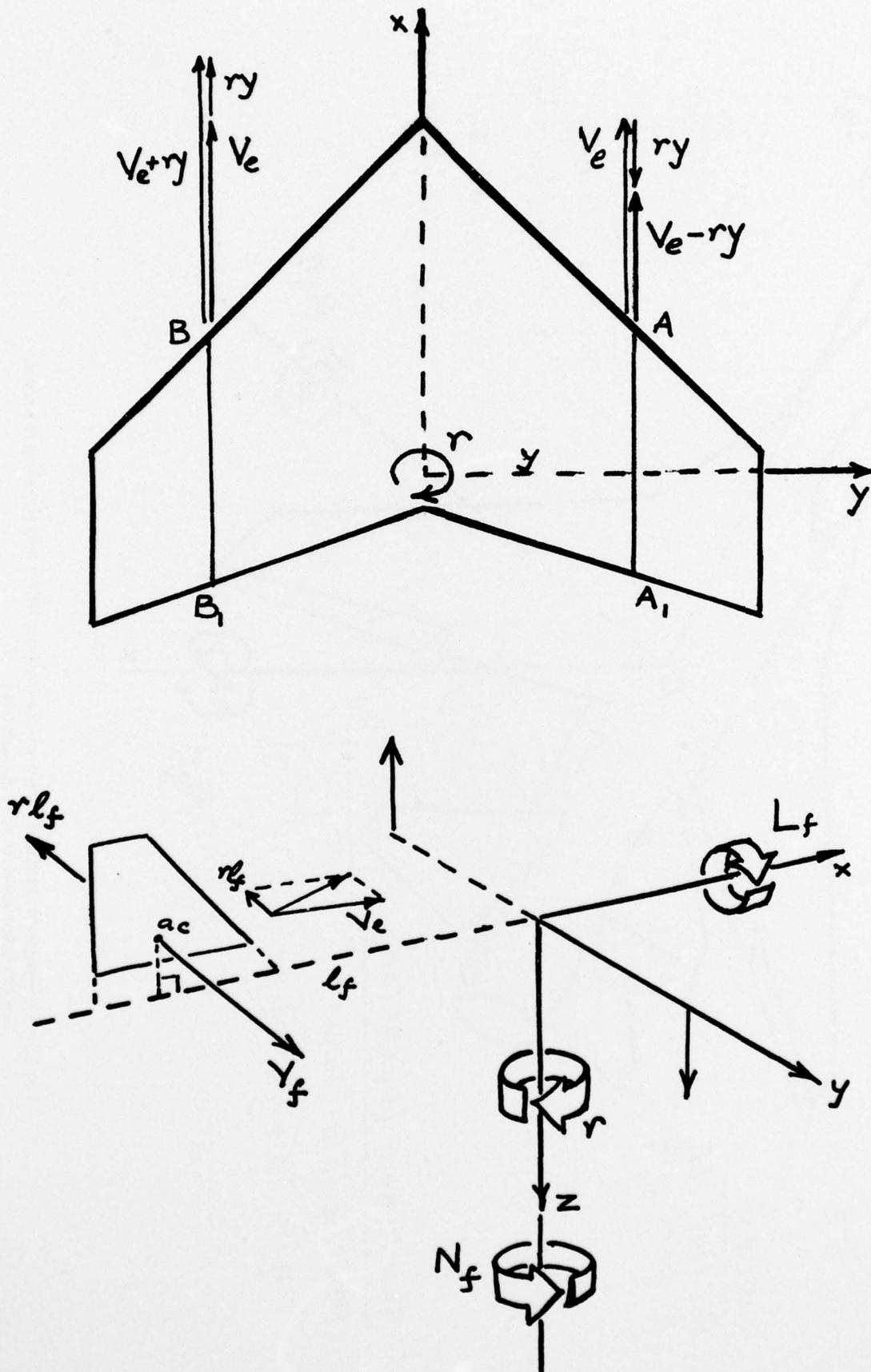


Fig 14 Rolling and yawing moments due to rate of yaw

Fig 15

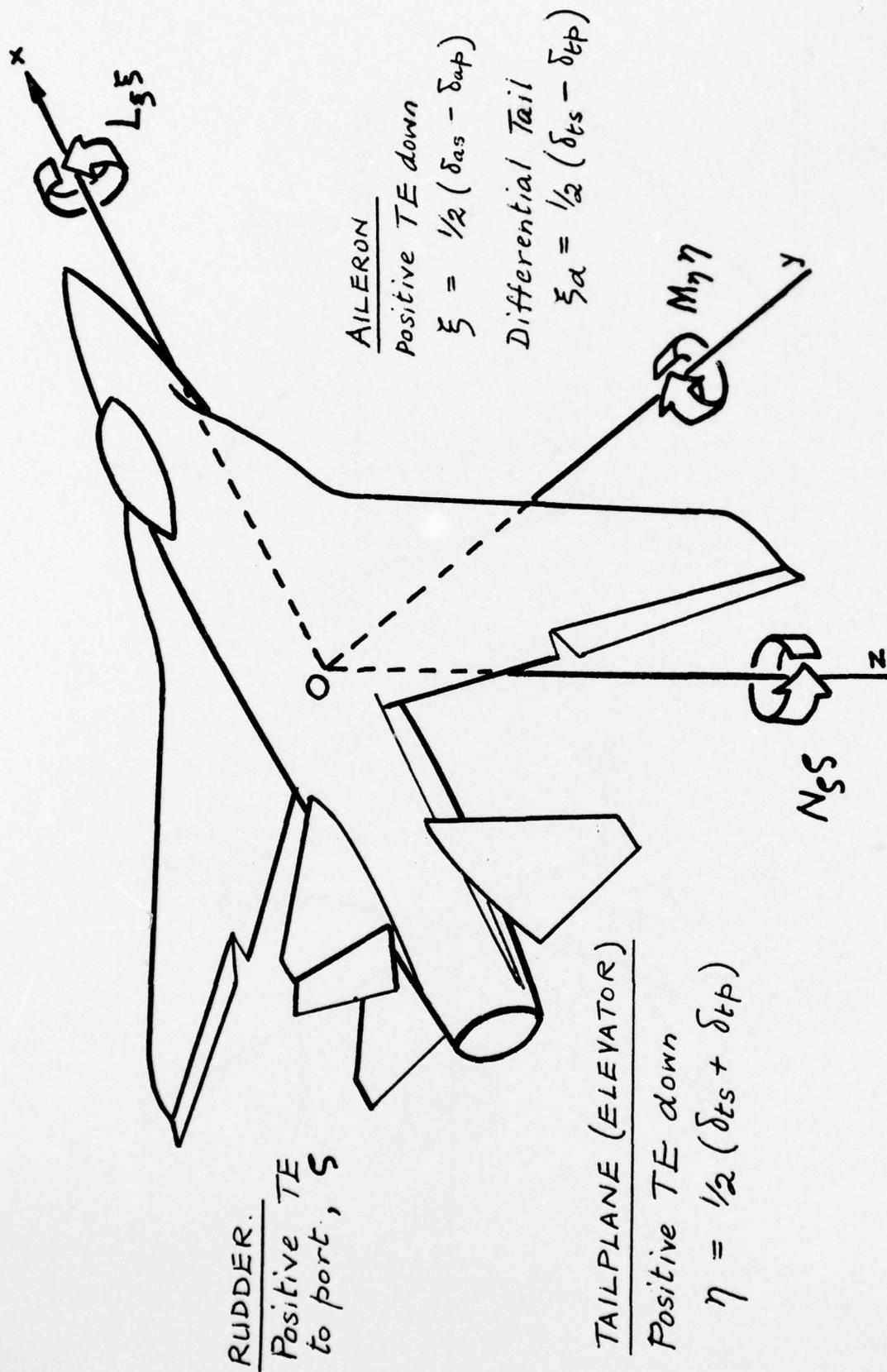


Fig 15 Definition of positive motivator deflections (ISO) and showing the sense of the associated moments (negative)

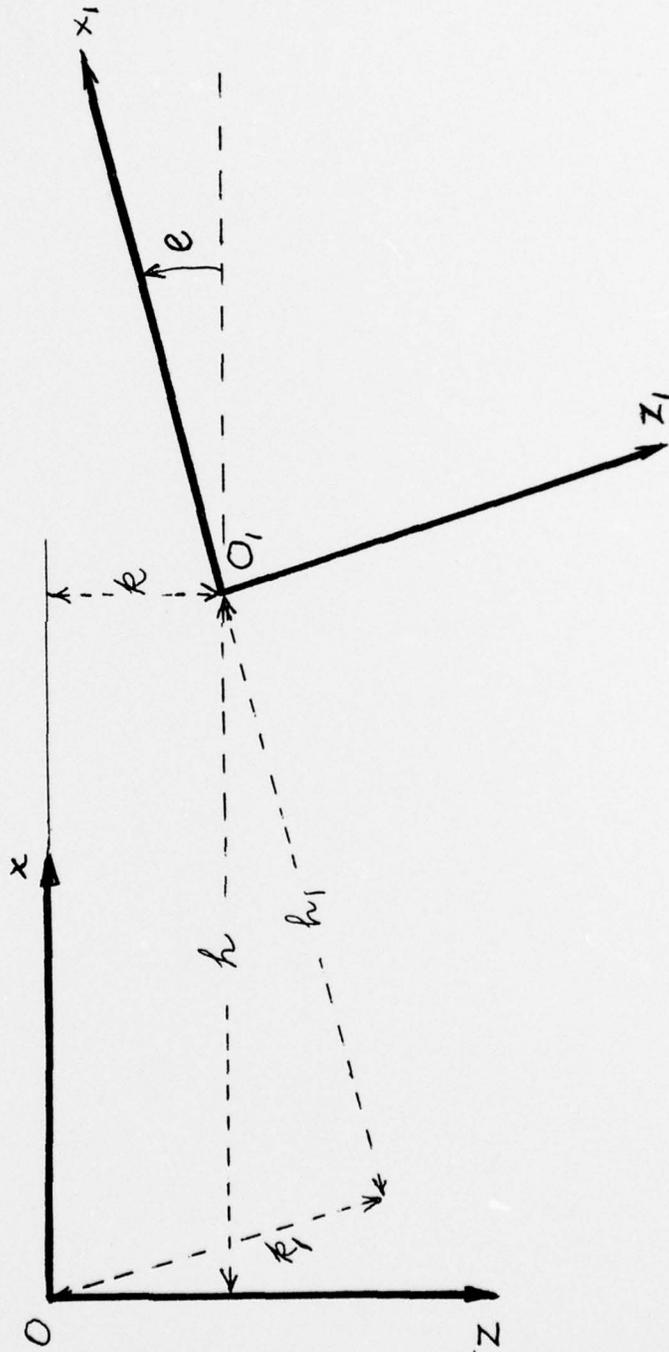


Fig 16 Relative positions of the two systems of body axes involved in transformations of section 7

REPORT DOCUMENTATION PAGE

Overall security classification of this page

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17. Abstract The solution of problems of flight dynamics requires the aerodynamic forces, which are called into play, to be expressed in a suitable form. In this context a suitable form is one which adequately reflects the nature of the motion being considered and is, at the same time, convenient for the solution of the equations of motion. In the opening sections of this paper formulation in terms of aerodynamic derivatives, and generalizations thereof, are considered. There follows a brief discussion in broad and simple physical terms of how the various motion variables give rise to forces and moments, which within a linearized framework are expressible as force or moment derivatives, specifically for an aeroplane.			

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