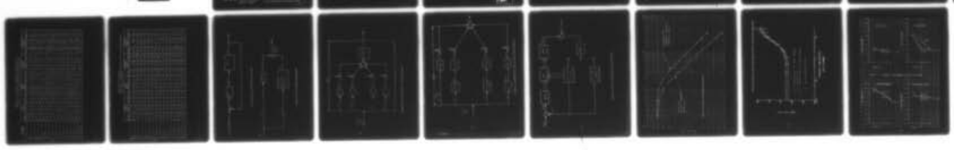
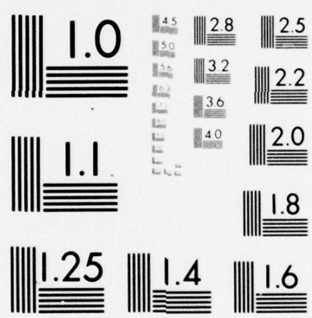
AD-A041 856 AEROSPACE MEDICAL RESEARCH LAB WRIGHT-PATTERSON AFB OHIO F/G 5/8
STUDY OF IDENTIFICATION METHODS AND STRUCTURAL MODELING TECHNIQ--ETC(U)
JUL 77 D W REPPERGER, A M JUNKER
UNCLASSIFIED AMRL-TR-77-45 NL

1 OF 1
ADA041856



END
DATE
FILMED
8 - 77


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0

ADA041856

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) AMRL-TR-77-45	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) STUDY OF IDENTIFICATION METHODS AND STRUCTURAL MODELING TECHNIQUES ON EMPIRICAL DATA FROM A MOTION STUDY		5. TYPE OF REPORT & PERIOD COVERED Proceedings
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) D. W. Repperger, A. M. Junker		8. CONTRACT OR GRANT NUMBER(s) (16) (17) 04
9. PERFORMING ORGANIZATION NAME AND ADDRESS Aerospace Medical Research Laboratory, Aerospacr Medical Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62202F, 6893-04-34
11. CONTROLLING OFFICE NAME AND ADDRESS (12) 21p.		12. REPORT DATE (11) July 1977
		13. NUMBER OF PAGES 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

DDC JUL 21 1977

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Prepared for presentation at the IEEE International Conference on Systems Man and Cybernetics, Washington, D. C., 19-21 September 1977, and publication in the proceedings.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Motion
Model order tests
Canonical model structure
input-output time series
Man-machine interaction

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A study of three different model order tests is conducted on empirical data from a motion experiment. The parameter estimates obtained from thise identification scheme are compared to spectra estimates of the same data. The approach presented here utilizes a simple canonical model that has application in the study of man-machine systems.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

009 850

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

STUDY OF IDENTIFICATION METHODS AND STRUCTURAL MODELING TECHNIQUES ON EMPIRICAL DATA FROM A MOTION STUDY

D. W. Repperger and A. M. Junker
Aerospace Medical Research Laboratory
Wright-Patterson Air Force Base, Ohio 45433

Abstract

A study of three different model order tests is conducted on empirical data from a motion experiment. The parameter estimates obtained from this identification scheme are compared to spectra estimates of the same data.

The approach presented here utilizes a simple canonical model which has application in the study of man-machine systems.

Introduction

In the application of identification methods to empirical data [1,2], two problems of primary importance are the choice of a proper canonical model structure and the method to verify the appropriate system order. If the number of parameters necessary to describe an input-output time series is incorrectly specified, problems of identifiability occur. Using Fisher's definition of the amount of information about a parameter θ contained in an observation $y(t_i)$, the information obtained from the identification procedure varies inversely with the variance of the parameter estimate [3]. Thus an ill posed guess on the correct model order and/or structure manifests itself by parameter estimates which are inconsistent and the information contained in the measured sample of data is essentially lost. This important problem of identifiability has been studied in the control literature [4,5] and also in the Biosciences [6] with the related compartmental modeling problems that occur [7].

This paper will consider the problem of identification and structural modeling of empirical data from a motion study that has been discussed in the manual control literature [8,9] and has also been studied in other contexts [10,11]. In order to proceed with an accurate identification analysis, the four important steps [12] of experimental design, model structure determination, parameter estimation, and model validation (with the consideration of parameter bias) will be conducted.

The experimental design of the empirical data presented here involves a man-machine interaction with motion from a simulator at the Aerospace Medical Research Laboratory. The input forcing function to the closed loop man-machine problem consists of a sum of sine waves appropriately spaced [11] to appear random to the human. Such an input is persistently exciting [13] within the modes of interest and also provides the additional advantage of having available spectra estimates from Fast Fourier transforms which will be used in the sequel as a comparison to the parametric estimation procedure conducted here.

The model structure determination problem applied to empirical data provides the main contribution in this paper. It is first necessary to discuss the identification algorithm and the apriori choice of model structure. Of the many possible choices [14] of identification algorithms, the type of identification procedure considered here is an output error least squares, Newton-Raphson convergence algorithm with a canonical form based on parametric estimates. A parametric model is chosen of a canonical form based on apriori knowledge of model structures commonly occuring in man-machine systems. This approach has advantages in the study of identification methods as discussed in [15,16]. The canonical models chosen here satisfy the required properties of controllability, observability, and identifiability for each value of n (the system order). In addition, the canonical model is derived such that the parameters obtained are linear combinations of the possible sensory inputs to the human such as the displayed error signal, its derivatives, and a possible integration of this error signal. This may be termed a PID or proportional, integral, derivative algorithm as discussed in [8,9]. By determining which parameters are consistent (and eliminating those parameters not consistent through a model order test), knowledge of which inputs the human is using in the tracking task can be obtained. The model order test can, therefore, be used to study the effect of a parameter (or sensory input) on this identification procedure which gives insight as to which sensory input may be used by man. The formulation of the modeling problem as it fits in the context of manual control is next discussed.

Formulation of the Modeling Problem

In the study of manual control problems in which data has already been collected there exists several advantages in studying data when model parameters can be expressed in a PID formulation. The primary motivation for such a representation of a human is in the quantification of the ability of the human to differentiate (generate lead).

Figure (1a) illustrates the man-in-the-loop problem considered here. For the purposes of analysis, the time series variables that are available for modeling are illustrated in Figure (1b). The displayed error signal e(t) which is the input to the man is also the input time series to the computer model. The output time series of the man is denoted as $X_s(t)$. The computer model has an output $\hat{X}_s(t)$ based on the structure of the model assumed and the available data e(t) and $X_s(t)$. This type of modeling is termed output error because the difference between two time series outputs are considered. The modeling error is denoted as $e_M(t)$ and it satisfies:

$$e_M(t) = X_s(t) - \hat{X}_s(t) \quad (1)$$

The objective of this identification procedure is to choose a transfer function H(s) to minimize the output error loss functional J denoted as:

$$J = \frac{1}{N} \sum_{i=1}^{N} [e_M(t_i)]^2 \quad (2)$$

where N is the number of samples of data.

The choice of the canonical model H(s) is the primary motivation for the approach presented in this paper. If H(s) can be chosen in a manner such that the human can be characterized by parameters which quantify the amount of differentiation or lead generation in a tracking task, then this model has application in the study of manual control problems. First the canonical model will be specified as follows:

$$H(s) = \frac{a_o + a_1 s^1 + a_2 s^2 + \frac{a_3}{s}}{(1 + s/\alpha)^3} \qquad (3)$$

where s is the Laplace transform variable. Equation (3) is an ideal representation of the man-in-the-loop for several reasons. The coefficients a_o, a_1, and a_2 represent differentiation (or lead generation) in the tracking task. Therefore, instead of giving heuristic arguments as to whether the describing function of the man has more lead in one experimental condition as compared to another, the coefficients a_o, a_1, and a_2 will quantitatively indicate this fact. Also, the coefficient a_3 allows the consideration of precognitive effects in a quantitative manner. The coefficient α in equation (3) is used to generate a third order pole for some value of α greater than 10 radians. This allows the transfer function H(s) to have a denominator with a higher order polynomial of s than the numerator and hence can be realized using state variables. The form of equation (3) allows the transfer function H(s) to have any amount of lead (including up to double differentiation) for frequencies from 0 to α radians. The amount of lead generation will depend on the numerical values of the coefficients a_o, a_1, and a_2.

Another interpretation of the transfer function H(s) in equation (3) can be seen in Figure (2). In Figure (2) the man is replaced by a parallel processing channel which describes the input signal e(t) and the output signal $X_s(t)$. The coefficients a_o, a_1, a_2 and a_3 indicate with what importance this particular time signal is converted or processed into the stick signal $X_s(t)$. If $a_2 >> a_o$ and $a_2 >> a_1$ then one would expect the signal $X_s(t)$ to be dominated by double differentiation of e(t). On a Bode plot of $\frac{X_s(s)}{E(s)}$ one would expect to see second order lead characteristics. A description of the Newton-Raphson identification algorithm is next presented to illustrate the estimation scheme for the determination of parameters.

The Newton-Raphson Output Error Identification Algorithm

With reference to Figure (1b), the system input-output description can be written

$$\dot{\bar{X}}_s(t) = A\,\bar{X}_s(t) + B\,e(t) + \xi(t) \qquad (4a)$$

$$y(t) = X_s(t) = \bar{X}_s(t) + \eta(t) \qquad (4b)$$

where $\xi(t)$ and $\eta(t)$ represent, respectively, noise vectors associated with human induced randomness and uncertainty in measurements of the data. The objective of this identification procedure is to choosen an estimate $\hat{X}_s(t)$ in such a way to minimize the output error loss functional J of equation (2). Let θ be a vector of unknown parameters. We wish to choose θ^* such that $J(\theta^*)$ is minimized. First rewrite $J(\theta)$ as follows:

$$J[\theta] = \sum_{i=1}^{N} [y(t_i) - \hat{X}_s(t_i)]^2 W \qquad (5)$$

where W is a positive weighting coefficient which may be considered unity without loss of generality. The similarity of this approach to the Newton-Raphson procedure will now be demonstrated (see Bellman for a description of the appropriate Newton-Raphson approach [17]). Assume the response variable $\hat{X}_s(t)$ can be linearized with respect to the unknown parameter vector θ:

$$\hat{X}_{si} = \hat{X}_{si}o + \nabla_\theta \hat{X}_{si}\,(\theta - \theta_o) \qquad (6)$$

where i is the iteration number, $\hat{X}_{si}o$ is the nominal response due to the parameter θ_o and $\nabla_\theta \hat{X}_{si}$ is the gradient of $\hat{X}_{si}$ with respect to θ. Substituting equation (6) into (5) and solving for the value of θ which minimizes $J(\theta)$ yields:

$$\hat{\theta} = \theta_o + [\sum_{i=1}^{N} (\nabla_\theta \hat{X}_{si})^T W (\nabla_\theta \hat{X}_{si})]^{-1} \cdot$$

$$[\sum_{i=1}^{N} (\nabla_\theta \hat{X}_{si})^T W (y(t_i) - \hat{X}_{si}(t_i))] \qquad (7)$$

If $\hat{\theta}$ is updated iteratively with respect to the unknown parameter vector, the value of θ^* will result which minimizes $J(\theta)$ of equation (5). This result can be seen to be a modified form of Newton-Raphson [18] by writing it as:

$$\theta_{k+1} = \theta_k + [\nabla_\theta^2 J_k]^{-1} [\nabla_\theta J_k] \qquad (8)$$

where

$$\nabla_\theta J_k = -2 \sum_{i=1}^{N} \nabla_\theta \hat{X}_{is}^T W[y(t_i) - \hat{X}_{si}] \qquad (9a)$$

$$\nabla_\theta^2 J_k = 2 \sum_{i=1}^{N} \nabla_\theta \hat{X}_{is}^T W \nabla_\theta \hat{X}_{is} + 2 \sum_{i=1}^{N} \nabla_\theta^2 \hat{X}_{is}^T W \cdot$$

$$[y(t_i) - \hat{X}_{is}(t_i)] \qquad (9b)$$

The second term of (9b) approaches zero as $\hat{X}_s(t)$ approaches y(t) and this modified Newton-Raphson method is identical to Newton-Raphson (or quasilinearization) if this term approaches zero. Therefore, the initial starting points may be the only apparent source of difficulty because the Newton-Raphson method [17] can be shown to have quadratic convergence. In any event, the expression given in equation (8) provides a minimum of $J(\theta)$ specified in equation (5) which is the objective of this least squares approach.

The time series implementation of this identification procedure will be discussed next. The digital implementation of the time series is presented in the next section. Appendix A describes the discretization procedure and the method of obtaining the necessary gradients.

Implementation of This Identification Procedure

This paper will illustrate a simple manner of implementing a PID type model and, in addition, provide several ways to validate such a model. Figure (3) illustrates the implementation procedure used in this paper which is equivalent to the diagram in Figure (2). The first step in the implementation procedure is to determine the prefiltered variables $\hat{e}(t)$, $\dot{\hat{e}}(t)$, $\ddot{\hat{e}}(t)$, and $\int_o^t e(\tau)d\tau$ by the following realizable transfer functions (capital letters indicate Laplace Transform variables):

$$\hat{E}(s) = E(s) \qquad (10a)$$

$$\frac{\dot{\hat{E}}(s)}{E(s)} = \frac{s}{(1 + s/\alpha)^2} \quad (10b)$$

$$\frac{\ddot{\hat{E}}(s)}{E(s)} = \frac{s^2}{(1 + s/\alpha)^2} \quad (10c)$$

$$\frac{\int_o^t \hat{E}(\tau)d\tau}{E(s)} = 1/s \quad (10d)$$

Equations (10a-d) are easily implemented by using digital filter techniques. The identification stage of this implementation procedure requires the choosing of state variables so that a_o, a_1, a_2, and a_3 can be obtained. In the identification procedure, the following relationship holds:

$$X_s(t) = X_{s1}(t) + X_{s2}(t) + X_{s3}(t) + X_{s4}(t) + \xi_1 + \xi_2 + \xi_3 + \xi_4$$

To implement these equations choose state variables:

$$X_1 \triangleq X_{s1}(t) \quad (11a)$$

$$X_2 \triangleq X_{s2}(t) \quad (11b)$$

$$X_3 \triangleq X_{s3}(t) \quad (11c)$$

$$X_4 \triangleq X_{s4}(t) \quad (11d)$$

Then

$$\frac{X_1(s)}{\hat{E}(s)} = \frac{a_o}{1 + s/\alpha} \quad (12a)$$

$$\frac{X_2(s)}{\dot{\hat{E}}(s)} = \frac{a_1}{1 + s/\alpha} \quad (12b)$$

$$\frac{X_3(s)}{\ddot{\hat{E}}(s)} = \frac{a_2}{1 + s/\alpha} \quad (12c)$$

$$\frac{X_4(s)}{\int_o^t \hat{E}(\tau)d\tau} = \frac{a_3}{1 + s/\alpha} \quad (12d)$$

The implementation of equations (12a-d) proceeds as follows:

$$\text{Col } [\dot{X}_1, \dot{X}_2, \dot{X}_3, \dot{X}_4] = [-\alpha] \cdot \text{Col } [X_1, X_2, X_3, X_4] + [\alpha a_o, \alpha a_1, \alpha a_2, \alpha a_3] \cdot \text{Col } [\hat{e}(t), \dot{\hat{e}}(t), \ddot{\hat{e}}(t), \textstyle\int_o^t e(\tau)d\tau] \quad (13a)$$

$$y(t) = X_s(t) = [1, 1, 1, 1] \cdot \text{Col } [X_1, X_2, X_3, X_4] + \sum_{i=1}^{4} \xi_i(t) \quad (13b)$$

Therefore, the variable α is just the prefiltering variable, the unknowns are αa_o, αa_1, αa_2, and αa_3 which are determined from a least squares identification algorithm. The PID identification algorithm is determined by identifying α, a_o, a_1, a_2, and a_3.

In this implementation, the time series e(t) was delayed by 0.20 seconds, $\dot{e}(t)$ was delayed by 0.12 seconds, and $\ddot{e}(t)$ was delayed by 0.04 seconds. The manner of achieving these delays was accomplished by shifting the real time series by an integral multiple of the sampling rate (.04 seconds). The assumption of different delays on the perceptual variables is perhaps a better assumption than a single, constant delay on all four channels. In the case of tracking with a motion disturbance it is reasonable to assume that information from rates and accelerations may be processed more rapidly than position information. Since the desire of this paper is to produce a lumped representation of a human, these lags were chosen over four experimental conditions of the motion experiment. Once this model is sufficiently validated, future work can be done to investigate the lags of each individual channel and for the different experimental conditions considered here.

A description of the MATS experiment and data base used for this study is next presented.

The Multi Axis Tracking Simulator (MATS)

Figure (4) illustrates a physical diagram of the MATS. A brief description of this simulator will be presented here. A more complete description can be found in [30,31].

The MATS simulator was used only in the roll axis for this study with two independent inputs: Φ TARGET and Φ DISTURBANCE as indicated in Figure (4). Four modes of tracking were conducted:

(1) STATIC DISTURBANCE
Φ Target = 0 with Φ Disturbance ≠ 0 with no roll motion

(2) MOTION DISTURBANCE
Φ Target = 0 with Φ Disturbance ≠ 0 with roll motion

(3) TARGET STATIC
Φ Target ≠ 0 with Φ Disturbance = 0 with no roll motion

(4) TARGET MOTION
Φ Target ≠ 0 with Φ Disturbance = 0 with roll motion

The two input spectrums Φ Target and Φ Disturbance were designed based upon apriori guesses of inputs that gave rise to performance changes as indicated by the BBN optimal control pilot vehicle model. Figure (5) is a plot of the two input spectrums. The effective plant dynamics controlled by the subjects was specified by:

$$G(s) = \frac{10.0}{s(1 + s/5)\ (1 + s/20)} \quad (14)$$

The subjects involved in the experiment were six college students (male and female) 18-25 years of age. The subjects tracked each of the four experimental conditions for 165 seconds each day with the runs presented in a random sequence. The subjects were told to minimize the following score:

$$C = \text{Score} = \sigma^2_{\Phi_{error}} + 0.1\, \sigma^2_{\ddot{\Phi}_{plant}} \quad (15)$$

At the end of each run the subjects were told the score, $\sigma^2_{\Phi\ error}$, and 0.1 $\sigma^2_{\Phi\ plant}$. They were instructed to minimize the total score. When the scores reached asymptotic levels, subject training was assumed to be accomplished. The experiment was then run for an additional eight days and data was collected. The performance results are summarized in Table I for the eight days of collected data.

TABLE I *

		TARGET MOTION	TARGET STATIC	DISTURBANCE MOTION	DISTURBANCE STATIC
C (Score)	Mean	66.1	72.8	78.6	197.0
	S.D.	7.5	8.9	15.5	29.0
e^2_{RMS}	Mean	46.245	56.4558	16.248	66.6688
	S.D.	13.2547	11.45549	8.910	16.71518

One can see from Table I that in the disturbance mode of operation the effects of motion on performance were quite profound. In the target mode of operation the effects of motion were not that pronounced.

Another measure of performance is the variance of the error, error rate, and error acceleration. For the disturbance input case these variables became the plant position, rate and acceleration with just a -180° sign change in this signal. These variables were calculated and averaged across subjects. The results of these time series answers are displayed in Table II.

TABLE II

		TARGET MOTION	TARGET STATIC	DISTURBANCE MOTION	DISTURBANCE STATIC
σ_e	Mean	6.90	7.06	4.2	8.83
	S.D.	0.78	0.74	1.5	1.80
$\sigma_{\dot{e}}$	Mean	11.8	11.9	6.81	11.9
	S.D.	0.79	0.7	1.41	1.3
$\sigma_{\ddot{e}}$	Mean	40.02	39.522	24.0	33.6
	S.D.	15.161	8.33	1.9	5.0

The numerical values in Table II are also measures of performance which are an important aspect of this experiment.

Parametric Results From the Identification Algorithm

Using various values of α between 5 and 50 radians, the identification scheme was applied to the time series data $e(t)$ and $X_s(t)$ over the four conditions of motion inputs. Table III illustrates the resulting parametric values for α = 20.

TABLE III - α = 20

		TARGET MOTION	TARGET STATIC	DISTURBANCE MOTION	DISTURBANCE STATIC
a_0	Mean	.0526004	.090332	-.4435146	-.161328667
	S.D.	[illegible]	.00710845	.05885703	.0235[illegible]
a_1	Mean	.00397459	.00590767	-.1151918	-.0128[illegible]
	S.D.	.0026907	.00352577	.015809079	.0058[illegible]
a_2	Mean	[illegible]	-.00679280	-.00121933	.013[illegible]
	S.D.	.0033102	.000562707	.00130566	.00195[illegible]
a_4	Mean	-.00372[illegible]	-.000252006	.001052651	[illegible]
	S.D.	.0086512954	.000767361145	.0030386457	.001072189

In order to show that such a model has credibility it was validated two different ways. The purpose of a validation is to demonstrate that this lumped, simplified model can adequately represent the human in the tracking task. Model order tests were used to determine which parameters (or inputs) to the human had the greatest effect in reducing the output error loss functional of equation (2). In the following sections we present the validation results and parameter sensitivity tests.

Two Methods of Model Validation

The lumped model developed here was validated in the frequency domain and also in the time domain. The first method of model validation was a comparison of averaged values of spectra plots (Fast Fourier Transforms) to averaged values of the PID parameters. In this manner a spectra identification procedure is compared to a parametric identification algorithm. Using a Fast Fourier Transform program developed at AMRL, ensemble averages of the spectra of the time series $e(t)$ and $X_s(t)$ were obtained for the four tracking tasks considered here. In addition, the parametric plots of Table III were obtained for the mean values of the parameters and two additional plots of the mean values of the parameters ± 1 standard deviation of these parameter values. Since the describing functions obtained from the FFT's were also plotted as mean values ± 1 standard deviation of each spectra estimate, the two ensemble plots can be overlaid and compared.

Figure (6) illustrates the static disturbance plots of the two identification schemes overlaid. The study in [32] indicated that the two schemes match best for the case of static disturbance and motion disturbance conditions. For the static target and motion target case, the two identification schemes match with less consistency.

In all four cases the uncertainty envelope obtained from one standard deviation of the parametric plots about their mean when overlaid with the corresponding envelope obtained from the spectra plots, results in overlap of these envelopes. One interpretation of this result is that the uncertainty in the parametric estimation scheme is no worse than the uncertainty in the spectra estimates.

The second method of model validation is in the time domain by considering how well the time series $\hat{X}_s(t)$ generated from the computer model matches the experimental time history data $X_s(t)$. The ratio R given in equation (16) is a measure of this match. The ratio considered is

$$R = \left[1 - \frac{\sum_{i=1}^{N} [X_s(t_i) - \hat{X}_s(t_i)]^2}{\sum_{i=1}^{N} [X_s(t_i)]^2} \right] (100\%) \quad (16)$$

This variable is calculated and the results are displayed in Table IV.

TABLE IV

	TARGET MOTION	TARGET STATIC	DISTURBANCE MOTION	DISTURBANCE STATIC
R mean	89.98%	94.76%	95.36%	95.94%
F S.D.	1.6084	.86197	1.5274	1.1502

From the results of Table IV we can see that there is a high correlation between the model output time series and the output series from the empirical data.

It is now of interest to complete a sensitivity study on which parameters (a_o, a_1, a_2, or a_3) will

* See enlarged tables I-VIII at end of paper.

reduce the output error loss functional of equation (2). For the PID model developed here, the parameter which is most sensitive will indicate which of the possible inputs ($e(t)$, $\dot{e}(t)$, $\ddot{e}(t)$, or $\int_0^t e(\tau)d\tau$) is most important in describing the input-output characteristics of the human as a parallel processor of information. In this manner some insight can be obtained as to which input sensory variable may be used by the human when he is represented by Figure (2). This result is true only during the static mode of operation. During the motion mode of operation additional loops exist in Figure (1a) indicating rate feedback from the plant to the human. The modeling technique presented here is a lumped representation which will have only real physical meaning in the static mode of operation. Once this approach has been sufficiently validated, a more sophisticated version can be used to study the motion feedback loops.

A Study of Model Order Tests to Investigate Sensory Inputs to the Human

In an effort to investigate which sensory inputs are used by the human in the tracking task, three tests on the correct model order were considered.

Many model order tests are available to test system structure. The most familiar are, perhaps, the various max-likelihood whiteness tests on residuals as discussed in [19,20]. Some of the other methods include [21] a test on the least eigenvalue of a structural matrix constructed on the input-output time series, stochastic realization algorithms [22], entrophy estimation methods [23], and an optimization approach as considered by Bellman [7]. If the empirical data is already available then some a posteriori methods to test model order include a study of the singularity of the covariance matrix [24], Akaike's [25] final prediction error method, Astrom's F-distribution test [13,27,28] on significant reduction of the output error loss functional as parameters are increased, and an interesting index of parameter consistency measure as used in the Bioscience literature [26]. Since a parametric scheme was used here in an a posteriori manner to study empirical data, it was decided to compare Astrom's F-distribution test [13,27,28] to Akaike's [25] final prediction error method and to relate these results to the parameter consistency measures as considered in [26]. Chan et al. [29] have looked at the first two model order tests from data simulated on a computer. The approach used here differs because it utilizes real world empirical data and also Fast Fourier Transforms of the data were available to serve as a basis of comparison to the parametric methods. The third model order test considered in [26] provides an interesting check on the first two methods because it is based on the assumption that the correct model order is the one that produces the most consistent parameter estimates. In order to measure the consistency of the parameter estimates, an index of consistency was defined as the normalized standard deviation of each parameter being estimated for the particular assumed order. This normalization is computed by the ratio of the standard deviation of each parameter to its mean squared value. In order to determine standard deviations of parameters, estimates are made for different time segments of the empirical time series.

The first model order test conducted here was Astrom's F ratio test [13] using a repeated least squares approach. The term repeated least squares means that the identification process is repeated starting with a first order system and then increasing the order of the system. For this canonical model the steps of the repeated least squares approach are as follows:

(1) Assume the human can be represented by one parameter.
(2) Calculate the loss functional J_1 of equation (2) for one parameter.
(3) Now assume the human can be represented by two parameters and repeat the least squares procedure.
(4) Calculate J_2 for the two parameter case.
(5) Assume the human is represented by 3 parameters and again repeat the least squares procedure.
(6) Calculate J_3.
(7) Assume all four parameters characterize the human.
(8) Calculate J_4.

The test in [13] is based on the variable Δ defined by

$$\Delta = \frac{J_1 - J_2}{J_2} \cdot \frac{N - n_2}{n_2 - n_1} \qquad (17)$$

where J_1 and J_2 are values of the loss functional for n_1 and n_2 parameters, respectively and N is the number of input-output pairs of data points. The variable Δ is F distribution with $n_2 - n_1$, $N - n_2$ degrees of freedom. For N = 300 pairs of data points, if $\Delta > 3.09$ implies the loss functional has dropped significantly (with 95% probability). If $\Delta \leq 3.09$, no conclusions can be drawn.

In a previous study [32] it was determined that Astrom's test was sensitive to N, the number of input-output pairs. Using empirical data from one experiment in each of the four motion modes, the algorithm was applied for N=100, 200, and 300 points. Table V lists the values of the loss functional obtained here.

To extend the results of [32] for various values of N, Table VI lists the values of Astrom's model order test as applied to Table V. Plots of the loss function were also obtained. In order to examine Astrom's test, the following values of the cost function were compared.

$$J(a_0) \text{ to } J(a_0, a_1) \text{ to } J(a_0, a_1, a_2)$$
$$J(a_0) \text{ to } J(a_0, a_2) \text{ to } J(a_0, a_1, a_2)$$

In this manner we could determine if either a_1 or a_2 was the dominant factor in reducing the output error loss functional. Figures (7a-d) illustrate plots of this test for the four modes of operation and for N=300 points.

TABLE V

	TARGET MOTION			TARGET STATIC			DISTURBANCE MOTION			DISTURBANCE STATIC		
	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300
$J(a_0)$	.355	.287	.277	.492	.411	.450	.754	.583	.632	.534	.402	.547
$J(a_1)$	.277	.232	.214	.601	.505	.519	.792	.974	1.260	.683	[illegible]	[illegible]
$J(a_2)$	.356	.298	.265	.564	.435	.464	1.229	1.311	1.451	1.011	[illegible]	[illegible]
$J(a_3)$	.353	.291	.281	.571	.483	.555	1.249	1.039	1.421	1.076	[illegible]	[illegible]
$J(a_0, a_1)$	.221	.211	.225	.499	.410	.451	.517	.464	.505	.331	.423	[illegible]
$J(a_0, a_2)$	.315	.263	.215	.371	.297	.392	.425	.476	.526	.551	.603	.545
$J(a_0, a_3)$	.333	.287	.276	.437	.409	.487	.740	.568	.383	.437	.565	.572
$J(a_1, a_2)$	.247	.154	.209	.465	.350	.355	.783	.979	1.254	[illegible]	1.410	1.315
$J(a_1, a_3)$	.270	.245	.259	.569	.432	.519	.638	.594	1.075	.601	[illegible]	1.415
$J(a_2, a_3)$	.349	.288	.274	.432	.391	.462	1.217	1.206	1.412	[illegible]	1.278	[illegible]
$J(a_0, a_1, a_2)$	.243	.157	.177	.377	.235	.275	.493	.419	.433	.252	.362	.379
$J(a_0, a_1, a_3)$	.175	.199	.219	.465	.406	.443	.482	.449	.502	.204	.417	[illegible]
$J(a_1, a_2, a_3)$	.194	.196	.206	.362	.326	.355	.449	.501	1.004	.427	.533	[illegible]
$J(a_0, a_2, a_3)$	.294	.256	.250	.213	.246	.392	.403	.439	.456	.389	.506	[illegible]
$J(a_0, a_1, a_2, a_3)$	.141	.156	.177	.258	.228	.275	.362	.419	.444	.231	.331	.328

The following tests were conducted to study Figures (7a-d):

CASE I - (Target Motion):

Tests

(1) Compare $J(a_o)$ to $J(a_o, a_1)$

(2) $J(a_o)$ to $J(a_o, a_2)$

(3) $J(a_o)$ to $J(a_o, a_3)$

CASE II - (Target Static):

Tests

(1) Compare $J(a_o)$ to $J(a_o, a_1)$

(2) $J(a_o)$ to $J(a_o, a_2)$

(3) $J(a_o)$ to $J(a_o, a_3)$

CASE III - (Disturbance Motion):

Tests

(1) Compare $J(a_o)$ to $J(a_o, a_1)$

(2) $J(a_o)$ to $J(a_o, a_2)$

(3) $J(a_o)$ to $J(a_o, a_3)$

CASE IV - (Disturbance Static):

Tests

(1) Compare $J(a_o)$ to $J(a_o, a_1)$

(2) $J(a_o)$ to $J(a_o, a_2)$

(3) $J(a_o, a_1)$ to $J(a_o, a_1, a_2)$

The results of the test appear in Table VI.

TABLE VI - ASTROM'S TEST (Statistical Parameter Sensitivity)

	TARGET MOTION			TARGET STATIC			DISTURBANCE MOTION			DISTURBANCE STATIC		
	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300
(1)	59.422	71.518	68.671	.601	.483	25.769	25.772	55.047	63.141	66.024	84.255	147.37
(2)	12.444	18.068	25.769	31.962	75.959	74.50	75.864	38.722	42.254	.534	0.0	1.094
(3)	.555	0.0 (Neg)	.082	1.006	.968	1.836	1.854	1.684	25.936	30.408	33.196	33.401

Before any conclusions are drawn as to the dominant input sensory variable, the second model order test was conducted, using the index of parameter consistency developed in [26], which is defined by:

$$I_1 = \frac{1}{M} \sum_{i=1}^{M} \frac{\sigma_i}{\mu_i} \qquad (18)$$

where M = the number of parameters considered, σ_i is the standard deviation of the parameter, and μ_i is the mean value of the absolute value of this parameter. According to the test in [26], this index of consistency is smallest when the correct system order is determined. Table VII lists the calculations of this index of consistency for the parameters displayed in Table III.

From Table VII it can be seen that when the parameter a_3 is included with any other group of parameters, the index of consistency increases substantially. This can be seen, for example in the motion target case where

$$\text{index } (a_o, a_1) = .190$$

$$\text{index } (a_o, a_1, a_3) = .752$$

This result is to be expected because the integration parameter a_3 (corresponding to an input $\int_0^t e(\tau)d\tau$) was by far the most inconsistent parameter. Since this tracking task was compensatory in nature with an input that was randomly appearing to the subjects, one would not expect a memory term such as a_3 to be representative of a human's input-output characteristics.

TABLE VII - THE INDEX OF PARAMETER CONSISTENCY = I_1

PARAMETERS CONSIDERED	TARGET MOTION			TARGET STATIC			DISTURBANCE MOTION			DISTURBANCE STATIC		
	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300
a_o	.638	.767	.763	.795	.629	.646	.304	.264	.219	.176	.021	.034
a_1	.097	.074	.083	.512	.289	.160	.629	.373	.229	.232	.173	.057
a_2	.455	.351	.329	.347	.238	.153	2.115	1.778	.687	.564	.336	.183
a_3	1.371	.983	.866	1.523	.981	.768	1.961	1.382	.774	4.519	1.623	1.771
a_o, a_1	1.098	.239	.190	1.010	.667	.423	.424	.292	.415	.190	.089	.087
a_o, a_2	.965	.738	.738	7.217	1.720	11.346	.947	.972	.379	8.445	3.406	1.165
a_o, a_3	2.353	5.194	6.378	3.549	.877	.767	2.633	2.314	7.259	1.116	1.365	16.503
a_1, a_2	.139	.068	.065	.275	.183	.087	1.402	1.310	.631	.457	.373	.169
a_1, a_3	1.235	.909	.322	1.307	1.037	.744	.740	.727	.438	.812	.820	.874
a_2, a_3	.939	.728	.644	.808	.582	.433	2.440	1.932	.862	2.374	1.074	1.466
a_o, a_1, a_2	.614	.226	.157	.457	.663	.472	.455	.410	.295	.369	.253	.299
a_o, a_1, a_3	.973	.891	.752	2.840	1.131	.906	1.063	.745	.519	1.865	.413	.275
a_1, a_2, a_3	.560	.509	.215	.869	.629	.459	2.373	5.973	.720	.726	.726	.693
a_o, a_2, a_3	2.666	1.272	4.336	13.169	2.139	5.760	1.207	.927	5.691	1.415	76.106	18.619
a_o, a_1, a_2, a_3	.592	1.476	1.762	1.226	.749	.656	.970	2.042	.657	3.416	.587	.203

The last model order test conducted was based on the final prediction error test proposed by Akaike [25]. Using his procedure, the suitable model structure is the one which minimizes:

$$I_2 = \frac{N + n}{N - n} \hat{\sigma}^2 \qquad (19)$$

where again N is the number of input-output pairs of data points, n is the order of the system, and $\hat{\sigma}^2$ is the estimated variance of the (output-error) noise process. The method of implementing this approach is similar to that of the repeated least squares method previously discussed. Initially a first order system is considered and then the model order is increased one order at a time. The variable I_2 in equation (19) is calculated each time the least squares procedure is repeated. The variable I_2 should achieve a minimum for the correct system order. One can observe from equation (19) that the term N - n is strongly dominated by N for most applications of data in which a large number of samples are involved. This fact was pointed out by Chan [29] and indicates that this model order test will be very sensitive to the total number of data points considered. To study this effect, the values of I_2 were calculated for N=100, 200, and 300 points. The results are displayed in Table VIII.

TABLE VIII - I_2 VALUES = Variance of One Step Ahead Prediction

	TARGET MOTION			TARGET STATIC			DISTURBANCE MOTION			DISTURBANCE STATIC		
	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300
a_o	.268	.210	.222	.404	.325	.379	.277	.128	.104	.117	[illegible]	.082
a_1	.210	.168	.182	.305	.322	.461	.220	.575	.835	.241	1.25	.952
a_2	.315	.269	.261	.476	.304	.376	1.184	1.298	1.441	.827	1.53	1.257
a_3	.265	.231	.239	.307	.497	.392	1.228	.720	1.412	.961	1.029	.819
a_o, a_1	.296	.094	.124	.408	.327	.289	.112	.059	.065	.017	.033	.024
a_o, a_2	.222	.173	.181	.180	.121	.199	.063	.066	.066	.113	.099	.082
a_o, a_3	.267	.210	.219	.405	.322	.262	.256	.128	.093	.053	.181	.078
a_1, a_2	.076	.079	.102	.759	.173	.149	.211	.579	.884	.163	.707	.617
a_1, a_3	.192	.155	.177	.297	.324	.404	.167	.135	.651	.231	.129	.277
a_2, a_3	.231	.217	.221	.417	.259	.327	1.230	.662	1.267	.216	.943	1.326
a_o, a_1, a_2	.029	.040	.063	.075	.061	.059	.070	.033	.014	.012	.022	.018
a_o, a_1, a_3	.032	.063	.118	.547	.376	.284	.075	.054	.063	.016	.029	.024
a_1, a_2, a_3	.023	.074	.101	.169	.161	.149	.062	.127	.631	.067	.077	.302
a_o, a_2, a_3	.188	.519	.168	.156	.117	.202	.063	.044	.018	.049	.179	.079
a_o, a_1, a_2, a_3	.029	.041	.064	.063	.057	.069	.032	.039	.043	.011	.016	.017

A comparison of these model order tests indicates the following:

(1) Astrom's test is sensitive to N but provides the best method to eliminate an inconsistent parameter such as a_3 as shown in Table VI.

(2) The index of consistency I_1 suffers from small samples and outliers.

(3) The consistency index I_2 is a good sensitive measure of consistency. This was pointed out by Chan [29]. Finally, a discussion of bias in this procedure is conducted to complete a study of these identification methods.

A Discussion of Bias in This Procedure

A discussion of bias is necessary in any identification application because of the many sources which may cause inaccurate parameter estimates. Bias may occur as a result of the finite size of data [16,33], noise injection in the data [34], or to the fact that the human is not a passive control system [35] but acts in a manner to inject noise directly into the closed loop. The study of bias and algorithms for estimating bias have been obtained by Friedland [36] and Lin and Sage [37]. More recently Asher et al. [39] has studied the bias problem in the context of a state estimator which may be attempting to estimate only a portion of a known state vector. This problem of bias has been considered in both the discrete and continuous [40,41] time equations. In the approach presented here the effect of bias is removed [13] by whitening the output error. The discrete residuals from the output error are correlated (and the parameter estimates are biased) for low order systems. When the system's order is increased, the residuals reach the point where they become white and the bias is zero within the 95% whiteness test. This is achieved by using the repeated least squares approach presented here. Astrom [13] notes that this approach is one of the six possible methods of providing for bias free estimates of parameters (or whitening the residuals of the output error).

Summary and Conclusions

A study of model order tests was conducted using 3 available methods with empirical data from a motion study. The canonical model obtained here is validated in both the time domain and also in the frequency domain.

References

[1] Gustavsson, I., "Comparison of Different Methods For Identification of Industrial Processes", Automatica, Vol. 8, pp. 127-142, 1972.

[2] Gustavsson, I., "Survey of Applications of Identification in Chemical and Physical Processes", Automatica, Vol. 11, pp. 3-24, 1975.

[3] Silvey, S.D., Statistical Inference, Penguin, 1970, pp. 35-44.

[4] Tse, E., "Information Matrix and Local Identifiability of Parameters", paper 20-3, JACC, Columbus, Ohio, June, 1973.

[5] Tse, E., and H. L. Weinert, "Structure Determination and Parameter Identification For Multivariable Stochastic Linear Systems", IEEE Transactions on Automatic Control, October, 1975, pp. 603-613.

[6] Bellman, R. and K. J. Astrom, "On Structural Identifiability", Mathematical Biosciences, Vol. 7, 1970, pp. 329-339.

[7] Bellman, R., "Topics in Pharmacokinetics-IV: Approximation in Process Space and Fitting by Sums of Exponentials", Mathematical Biosciences, Vol. 14, pp. 45-47, 1972.

[8] Repperger, D. W. and A. M. Junker, "PID (Proportional Integral Derivative) Modeling Techniques Applied to Studies of Motion and Peripheral Display Effects on Human Operator Performance", The Twelfth Annual Conference on Manual Control, NASA TM X-73, 170, pp. 703-718, 1976.

[9] Repperger, D. W. and A. M. Junker, "Sensitivity Analysis of Motion and Peripheral Display Effects on Tracking Performance", The Twelfth Annual Conference on Manual Control, NASA TM X-73, 170, pp. 719-729, 1976.

[10] Junker, A. M. and C. R. Replogle, "Motion Effects on the Human Operator in a Roll Axis Tracking Task", Aviation, Space, and Enviromental Medicine, Vol. 46, pp. 819-822, June, 1975.

[11] Levison, W. H., S. Baron, and A. M. Junker, Modelling the Effects of Environmental Factors on Human Control and Information Processing", AMRL-TR-76-74, August, 1976.

[12] Box, G.E.P. and G. M. Jenkins, Time Series Analysis, Forecasting and Control, Holden Day, San Francisco, 1976 (revised edition).

[13] Astrom, K. J. and P. Eykhoff, "System Identification-A Survey", Automatica, Vol. 7, pp. 123-162, 1971.

[14] Sage, A. P. and J. L. Melsa, System Identification, Academic Press Inc., New York, 1970.

[15] Ackermann, J.E. and R.S. Bucy, "Canonical Minimal Realization of a Matrix of Impulse Response Sequences", Information and Control, Vol. 19, pp. 224-231, 1971.

[16] Soderstrom, T., "On the Uniqueness of Maximum Likelihood Identification", Automatica, Vol. 11, pp. 193-197, 1975.

[17] Bellman, R.E. and R.E. Kalaba, "Quasilinearization and Nonlinear Boundary-Value Problems", American Elsevier Pub. Co., Inc., 1965.

[18] Balakrishnan, A.V., "Communication Theory", McGraw-Hill Book Co., Inc., 1968.

[19] Mehra, R. K., "On the Identification of Variances and Adaptive Kalman Filtering", IEEE Transactions on Automatic Control, Vol. AC-15, No. 2, April, 1970, pp. 175-184.

[20] Astrom, K.J. and T. Bohlin, "Numerical Identification of Linear Dynamic Systems From Normal Operating Records", IFAC Symposium on the Theory of Self Adaptive Systems, Teddington, Sept., 1975. Also in Theory of Self-Adaptive Control Systems, P. H. Hammond (Ed.), Plenum Press, 1966.

[21] Guidorzi, R., "Canonical Structures in the Identification of Multivariable Systems", Automatica, Vol. 11, pp. 361-374, 1975.

[22] Faurre, P.L., "Stochastic Realization Algorithms", in Systems Identification Advances and Case Studies, edited by R.K. Mehra and D.G. Lainiotis, pp. 1-25, Academic Press, 1976.

[23] Rissanen, J., "Minimax Entrophy Estimation of Models for Vector Processes", in System Identification Advances and Case Studies, edited by R.K. Mehra and D.G. Lainiotis, pp. 97-119.

[24] Chow, J.C., "On Estimating the Orders of an Autoregressive Moving Average Process with Uncertain Observations", IEEE Transactions on Automatic Control, Vol. AC-17, No. 5, Oct., 1972, pp. 707-709.

[25] Akaike, H. 1971 Ann. Inst. Statist. Math., Tokyo,

Vol. 23, 163; 1972 Proc. Fifth Int. Conference Systems Science, 1973, Biometrika, 62, 255.

[26] Desai, V. K. and F. W. Fairman, "On Determining the Order of A Linear System", Mathematical Biosciences, Vol. 12, pp. 217-224, 1971.

[27] Astrom, K. J., 1970 Proc. IFAC Symposium, Identification and Process Parameters.

[28] Astrom, K. J. and B. Wittenmark, "Problems of Identification and Control", Journal of Mathematical Analysis and Applications, Vol. 34, 1971, pp. 90-113.

[29] Chan, C. W., C. J. Harris, and P. E. Wellstead, "An Order Testing Criterion for Mixed Auto-regressive Moving Average Processes", International Journal of Control, Vol. 20, No. 5, pp. 817-834, 1974.

[30] Junker, A. M. and W. H. Levison, "Recent Advances in Modeling the Effects of Roll Motion on the Human Operator", submitted for publication, Aviation, Space, and Environmental Medicine.

[31] Junker, A. M. and W. H. Levison, "Use of the Optimal Control Model in the Design of Motion Cue Experiments", The Thirteenth Annual Conference on Manual Control, MIT, Cambridge, MA., 1977.

[32] Repperger, D. W. and A. M. Junker, "Using Model Order Tests to Determine Sensory Inputs in a Motion Study", The Thirteenth Annual Conference on Manual Control, MIT, Cambridge, MA., 1977.

[33] Young, P. C., "An Instrumental Variable Method for Real-Time Identification of a Noisy Process", Automatica, Vol. 6, pp. 271-287, 1970.

[34] Eykhoff, P., System Identification-Parameter and State Estimation, John Wiley and Sons, 1974.

[35] Levison, W. H., S. Baron, and D. L. Kleinman, "A Model for Human Controller Remnant", IEEE Transactions on Man-Machine Systems, Vol. MMS-10, No. 4, December, 1969.

[36] Friedland, B., "Treatment of Bias in Recursive Filtering", IEEE Transactions on Automatic Control, Vol. AC-14, pp. 359-367, August, 1969.

[37] Lin, J. L. and A. P. Sage, "Algorithms for Discrete Sequential Maximum Likelihood Bias Estimation and Associated Error Analysis", IEEE Transactions on Systems, Man, and Cybernetics, October, 1971, pp. 314-324.

[38] Schwarz and B. Friedland, "Linear Systems", McGraw-Hill, pp. 127, 1965.

[39] Asher, R. B., K. D. Herring, and J. C. Ryles, "Bias, Variance, and Estimation Error in Reduced Order Filters", Automatica, Vol. 12, pp. 589-600, December, 1976.

[40] Brown, R. J. and A. P. Sage, "Analysis of Modeling and Bias Errors in Discrete Time State Estimation", IEEE Transactions AES, AES-7, No. 2, 1971.

[41] Brown. R. J. and A. P. Sage, "Error Analysis of Modeling and Bias Errors in Continuous Time State Estimation", Automatica, 1971.

Appendix A

The Discretization Procedure

Since a continuous differential equation is used with discrete measurements, it is necessary to discretize equation (13a) at each sample instant. Following Schwarz and Friedland [38] with the notation

$$\dot{X}(t) = A\,X(t) + B\,U(t) \qquad (A.1)$$

For equation (13a), at time t = (n + 1)T

$$X(t) = e^{At}\,X(t_o) + \int_{t_o}^{t} e^{A(t-S)}\,B\,U(s)ds \qquad (A.2)$$

let t = (n + 1) which implies

$$X[nT + T] = e^{AT}[e^{ATn}X(t_o) + \int_{t_o}^{nT} e^{A(nT-s)}B\,U(s)ds]$$

$$+ \int_{nT}^{nT + T} e^{A(nT + T - s)}\,B\,U(s)ds \qquad (A.3)$$

However, the bracket term is X(nT) and if we assume U(t) is constant between samples, then (A.3) is written

$$X(nT + T) = e^{At}\,X(nT) + [\int_{o}^{t} e^{A[nT + T - \lambda]}Bd\lambda]\ \cdot$$

$$U(nT) \qquad (A.4)$$

which is in the form

$$X_{i+1} = \bar{A}\,X_i + \bar{B}\,U_i \qquad (A.5)$$

which is the desired discretized form. To determine the gradient of X with respect to a vector parameter θ, the partial derivative of (A.1) is taken with respect to θ

$$\dot{X}_\theta = A_\theta X + AX_\theta + B_\theta\,U \qquad (A.6)$$

(This is a matrix equation)

Since X(t) can be computed from (A.5) and U(t) is known, (A.6) is expressed as

$$\frac{d}{dt}[X_\theta] = A\,X_\theta + [A_\theta\,X(t) + B_\theta\,U(t)] \qquad (A.7)$$

Using the discretized results (A.1) - (A.5), (A.7) is implemented as

$$X_{\theta\ i+1} = \bar{A}\,X_{\theta_i} + \bar{B}\,[U(t),\ X(t)] \qquad (A.8)$$

and equation (A.8) must be solved with its coupled equation (A.5) to determine X_θ.

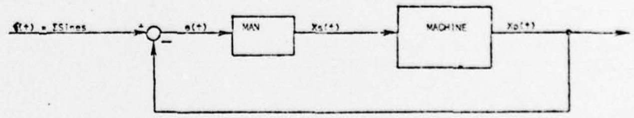


Figure (1a) - The Closed Loop Tracking Task *

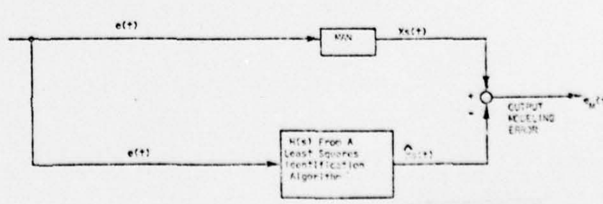


Figure (1b) - The Internal Loop Approach

*See enlarged illustrations, fig. 1-6 at end of paper.

TABLE I

		TARGET MOTION	TARGET STATIC	DISTURBANCE MOTION	DISTURBANCE STATIC
C (Score)	Mean	66.1	72.8	78.6	197.0
	S.D.	7.5	8.9	15.5	29.0
e^2RMS	Mean	46.245	56.4558	16.248	66.6688
	S.D.	13.2547	11.45549	8.910	16.71518

TABLE II

		TARGET MOTION	TARGET STATIC	DISTURBANCE MOTION	DISTURBANCE STATIC
σ_e	Mean	6.90	7.06	4.2	8.83
	S.D.	0.78	0.74	1.5	1.80
$\sigma_{\dot{e}}$	Mean	11.8	11.9	6.81	11.9
	S.D.	0.79	0.7	1.41	1.3
$\sigma_{\ddot{e}}$	Mean	40.02	39.522	24.0	33.6
	S.D.	15.161	8.33	1.9	5.0

TABLE III - $\alpha = 20$

		TARGET MOTION	TARGET STATIC	DISTURBANCE MOTION	DISTURBANCE STATIC
a_o	Mean	.0526004	.090332	-.4435146	-.161328667
	S.D.	.008456306	.00710845	.058885703	.02343810
a_1	Mean	.00397449	.00590767	-.1141918	-.01287437
	S.D.	.0028907	.00352572	.015809079	.0058299
a_2	Mean	.0002010	-.000792808	-.00121983	.0136601
	S.D.	.0003102	.000462707	.00130466	.001958
a_4	Mean	-.003723948	-.0002520066	.001052651	.0004614711
	S.D.	.0086512954	.000767361145	.0030386457	.001072189

TABLE IV

	TARGET MOTION	TARGET STATIC	DISTURBANCE MOTION	DISTURBANCE STATIC
R_{mean}	89.98%	94.76%	95.36%	95.94%
$R_{S.D.}$	1.6084	.86197	1.5274	1.1502

TABLE V

	TARGET MOTION			TARGET STATIC			DISTURBANCE MOTION			DISTURBANCE STATIC		
	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300
$J(a_o)$	.355	.287	.277	.492	.411	.490	.754	.593	.612	.554	.603	.547
$J(a_1)$	.277	.252	.254	.601	.506	.519	.790	.979	1.260	.683	1.619	1.366
$J(a_2)$	.356	.296	.285	.564	.435	.464	1.229	1.311	1.451	1.011	1.790	1.500
$J(a_3)$	.353	.291	.281	.571	.483	.555	1.249	1.039	1.421	1.078	1.319	1.481
$J(a_o, a_1)$	.221	.211	.225	.489	.410	.451	.597	.464	.505	.331	.423	.366
$J(a_o, a_2)$	.315	.263	.255	.371	.297	.392	.425	.496	.536	.551	.603	.545
$J(a_o, a_3)$	.353	.287	.276	.487	.409	.487	.740	.588	.563	.407	.565	.532
$J(a_1, a_2)$	.197	.196	.209	.465	.350	.355	.783	.979	1.254	.452	1.410	1.219
$J(a_1, a_3)$	.270	.245	.249	.569	.482	.519	.638	.594	1.075	.661	.663	1.415
$J(a_2, a_3)$	.349	.286	.274	.492	.395	.462	1.217	1.006	1.412	.998	1.278	1.383
$J(a_o, a_1, a_2)$	.143	.157	.177	.277	.235	.275	.493	.419	.453	.252	.362	.329
$J(a_o, a_1, a_2)$	.175	.199	.219	.486	.409	.443	.482	.449	.502	.269	.417	.366
$J(a_1, a_2, a_3)$	.194	.190	.206	.362	.324	.355	.449	.581	1.064	.427	.553	.989
$J(a_o, a_2, a_3)$	.294	.256	.250	.353	.286	.392	.403	.439	.456	.389	.566	.532
$J(a_o, a_1, a_2, a_3)$	.141	.156	.177	.258	.228	.275	.362	.419	.444	.233	.331	.328

TABLE VI - ASTROM'S TEST (Statistical Parameter Sensitivity)

	TARGET MOTION			TARGET STATIC			DISTURBANCE MOTION			DISTURBANCE STATIC		
	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300
TESTS (1) Δ =	59.421	71.318	68.871	.601	.483	25.769	25.772	55.047	63.141	66.024	84.255	147.37
TESTS (2) Δ =	12.444	18.068	25.709	31.962	75.999	74.50	75.864	38.722	42.254	.534	0.0	1.094
TESTS (3) Δ =	.555	0.0 (Neg)	.082	1.006	.968	1.836	1.854	1.684	25.936	30.408	33.196	33.401

TABLE VII - THE INDEX OF PARAMETER CONSISTENCY = I_1

I_1

PARAMETERS CONSIDERED	TARGET MOTION			TARGET STATIC			DISTURBANCE MOTION			DISTURBANCE STATIC		
	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300
a_o	.638	.767	.763	.795	.629	.646	.304	.264	.219	.176	.021	.034
a_1	.097	.074	.083	.512	.289	.160	.629	.370	.229	.238	.173	.087
a_2	.459	.391	.329	.347	.238	.153	2.115	1.778	.687	.564	.338	.183
a_3	1.371	.983	.866	1.523	.981	.788	1.961	1.382	.774	4.519	1.623	2.771
a_o, a_1	1.098	.239	.190	1.010	.607	.423	.424	.292	.415	.190	.089	.087
a_o, a_2	.965	.738	.738	7.257	1.720	11.346	.947	.972	.379	8.445	3.406	1.165
a_o, a_3	2.353	5.194	8.378	3.349	.877	.767	2.838	2.314	7.289	1.116	1.368	16.505
a_1, a_2	.139	.068	.065	.275	.183	.087	1.402	1.310	.631	.497	.309	.169
a_1, a_3	1.235	.909	.322	1.307	1.037	.744	.740	.727	.438	.812	.880	.874
a_2, a_3	.939	.728	.644	.808	.582	.433	2.440	1.932	.882	2.374	1.074	1.466
a_o, a_1, a_2	.614	.226	.157	.457	.668	.472	.455	.410	.295	.369	.253	.299
a_o, a_1, a_3	.973	.891	.752	2.840	1.131	.906	1.083	.745	.519	1.869	.413	.275
a_1, a_2, a_3	.560	.508	.215	.889	.629	.459	2.373	5.973	.720	.726	.726	.693
a_o, a_2, a_3	2.666	1.272	4.336	13.169	2.139	5.760	1.207	.927	5.691	1.415	76.106	19.619
a_o, a_1, a_2, a_3	.592	1.476	1.782	1.286	.749	.656	.970	2.042	.657	3.416	.587	.203

I_2

TABLE VIII - I_2 VALUES = Variance of One Step Ahead Prediction

	TARGET MOTION			TARGET STATIC			DISTURBANCE MOTION			DISTURBANCE STATIC		
	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300	N=100	N=200	N=300
a_o	.268	.210	.222	.404	.325	.379	.277	.128	.109	.117	.099	.082
a_1	.210	.166	.182	.305	.322	.461	.220	.575	.889	.261	1.25	.952
a_2	.315	.269	.261	.476	.304	.326	1.184	1.298	1.441	.827	1.554	1.257
a_3	.265	.231	.239	.307	.497	.592	1.228	.720	1.442	.961	1.029	.819
a_o, a_1	.096	.094	.124	.408	.327	.289	.112	.059	.065	.017	.033	.024
a_o, a_2	.222	.173	.181	.180	.121	.199	.043	.066	.066	.113	.099	.082
a_o, a_3	.267	.210	.219	.405	.322	.382	.256	.128	.093	.055	.083	.078
a_1, a_2	.076	.079	.102	.269	.173	.149	.211	.579	.884	.163	.797	.617
a_1, a_3	.192	.155	.177	.297	.324	.464	.167	.135	.653	.234	.129	.777
a_2, a_3	.251	.217	.221	.417	.289	.327	1.200	.662	1.292	.746	.943	1.304
a_o, a_1, a_2	.029	.040	.063	.075	.061	.069	.070	.038	.044	.012	.022	.018
a_o, a_1, a_3	.054	.083	.118	.412	.326	.289	.075	.054	.065	.016	.029	.024
a_1, a_2, a_3	.073	.074	.101	.169	.161	.149	.062	.127	.638	.067	.077	.502
a_o, a_2, a_3	.186	.519	.168	.156	.112	.202	.045	.044	.048	.049	.078	.079
a_o, a_1, a_2, a_3	.029	.041	.064	.063	.057	.069	.032	.039	.043	.011	.016	.017

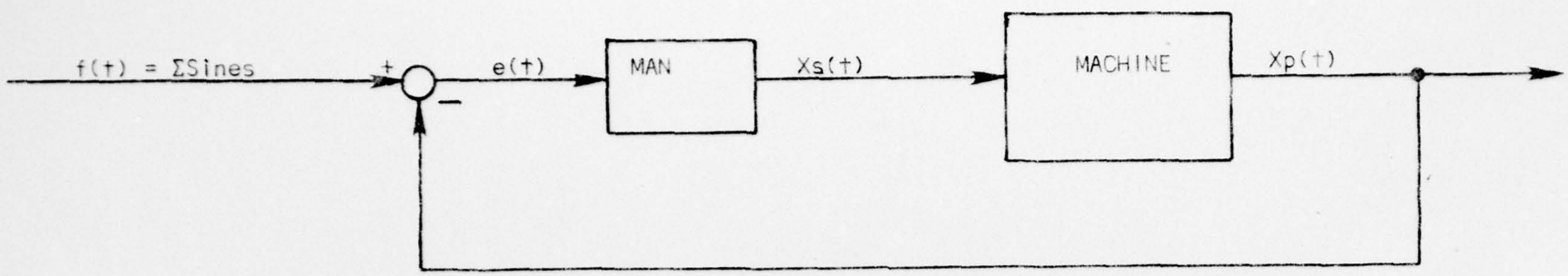


Figure (Ia) - The Closed Loop Tracking Task

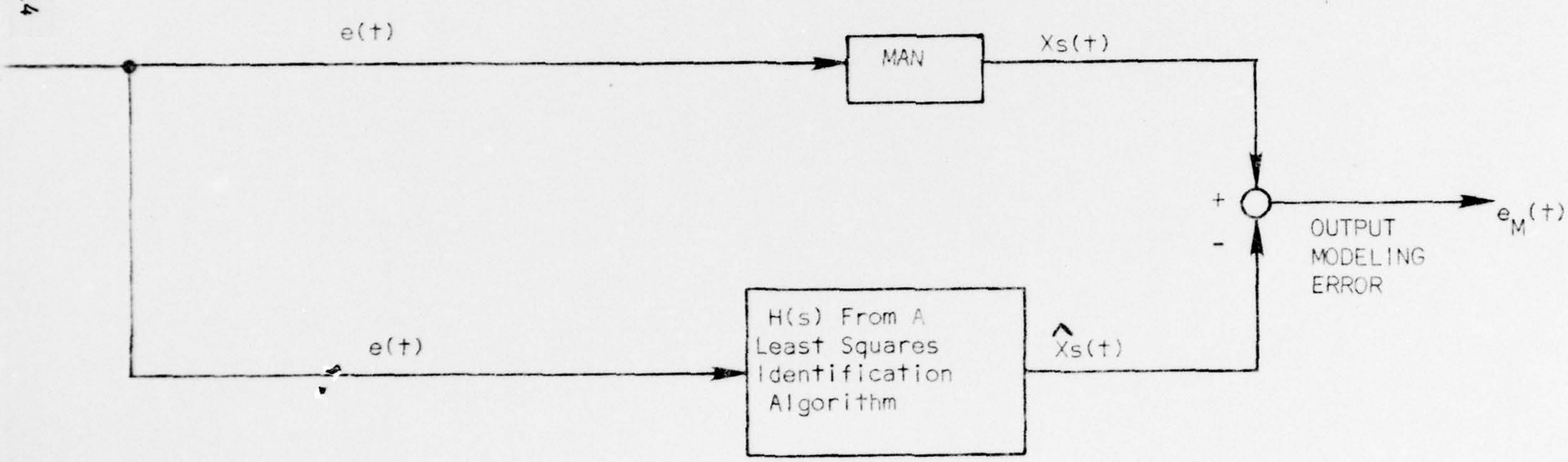


Figure (Ib) - The Internal Loop Approach

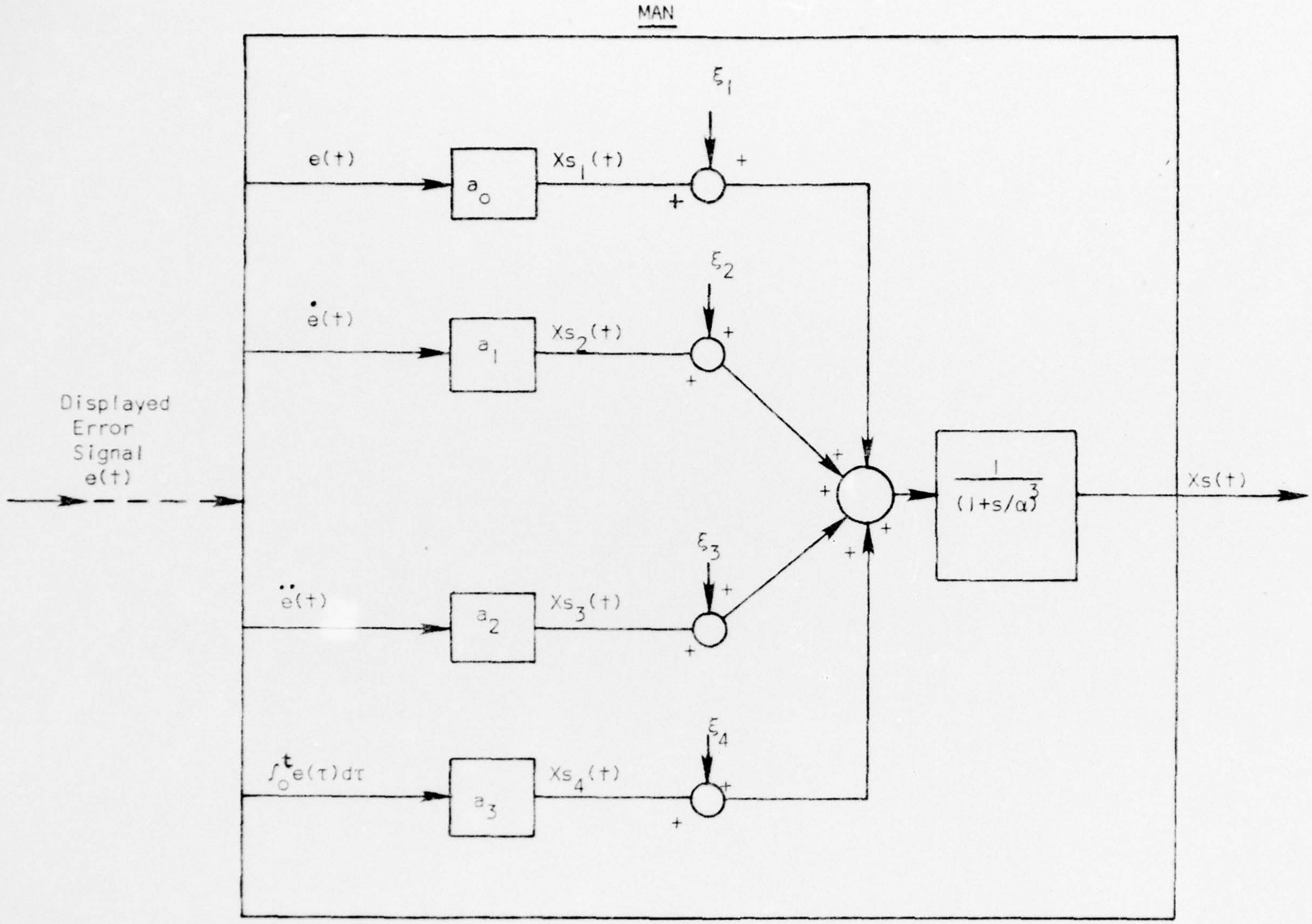


Figure (2) - Man As A Parallel Information Processor

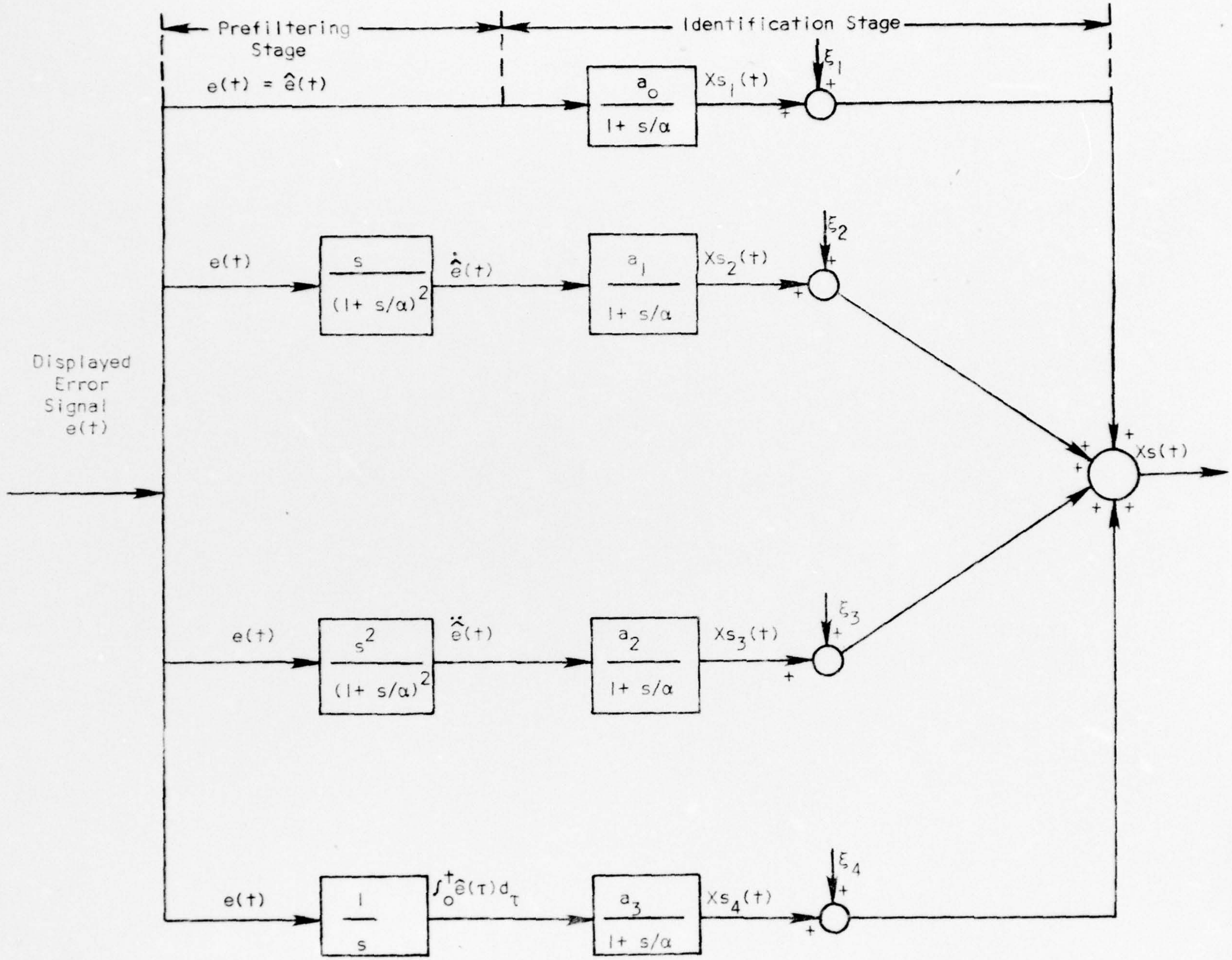


Figure (3) - Implementation of This Identification Approach

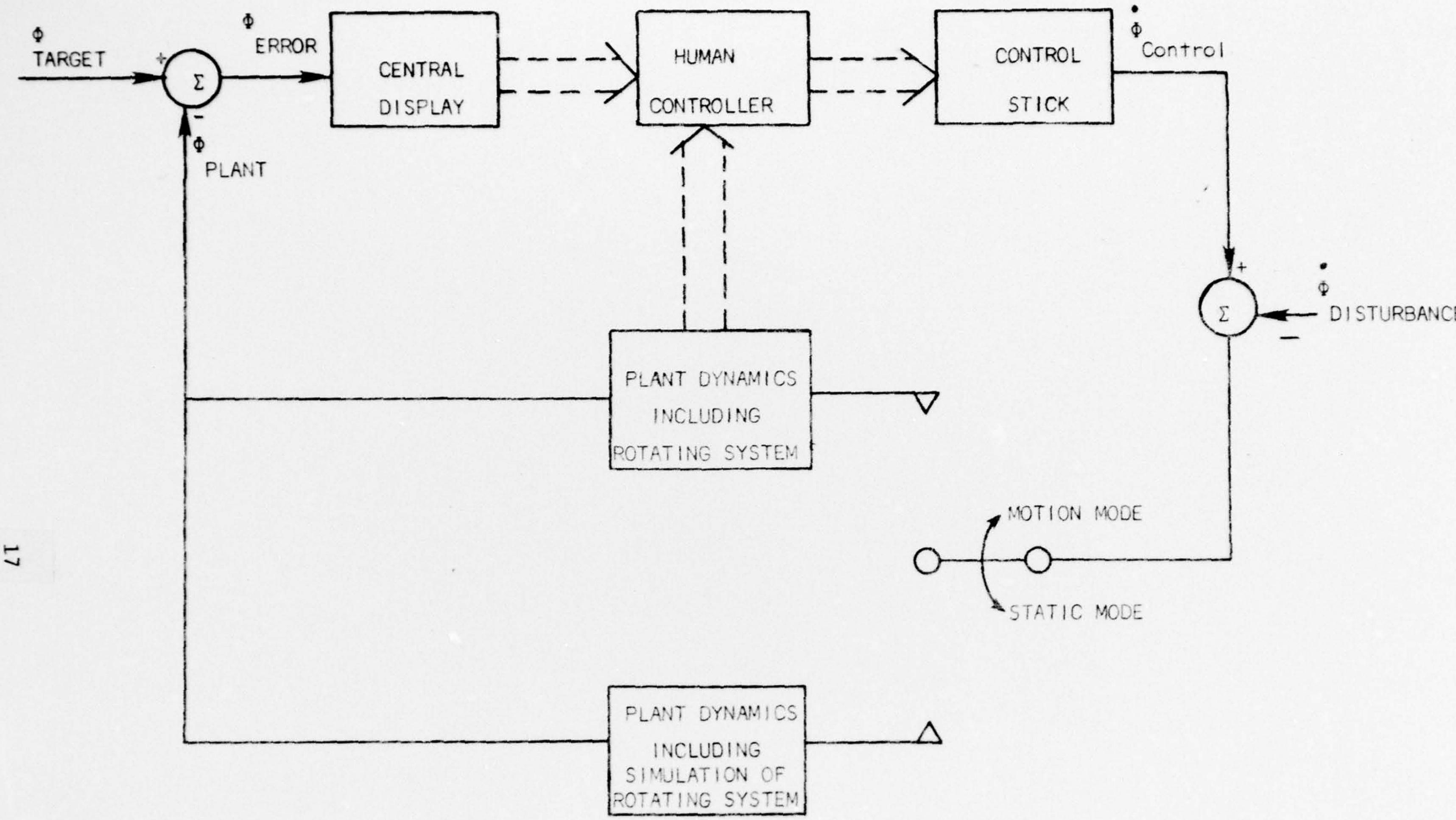


Figure (4) - The Multi Axis Tracking Simulator (MATS)

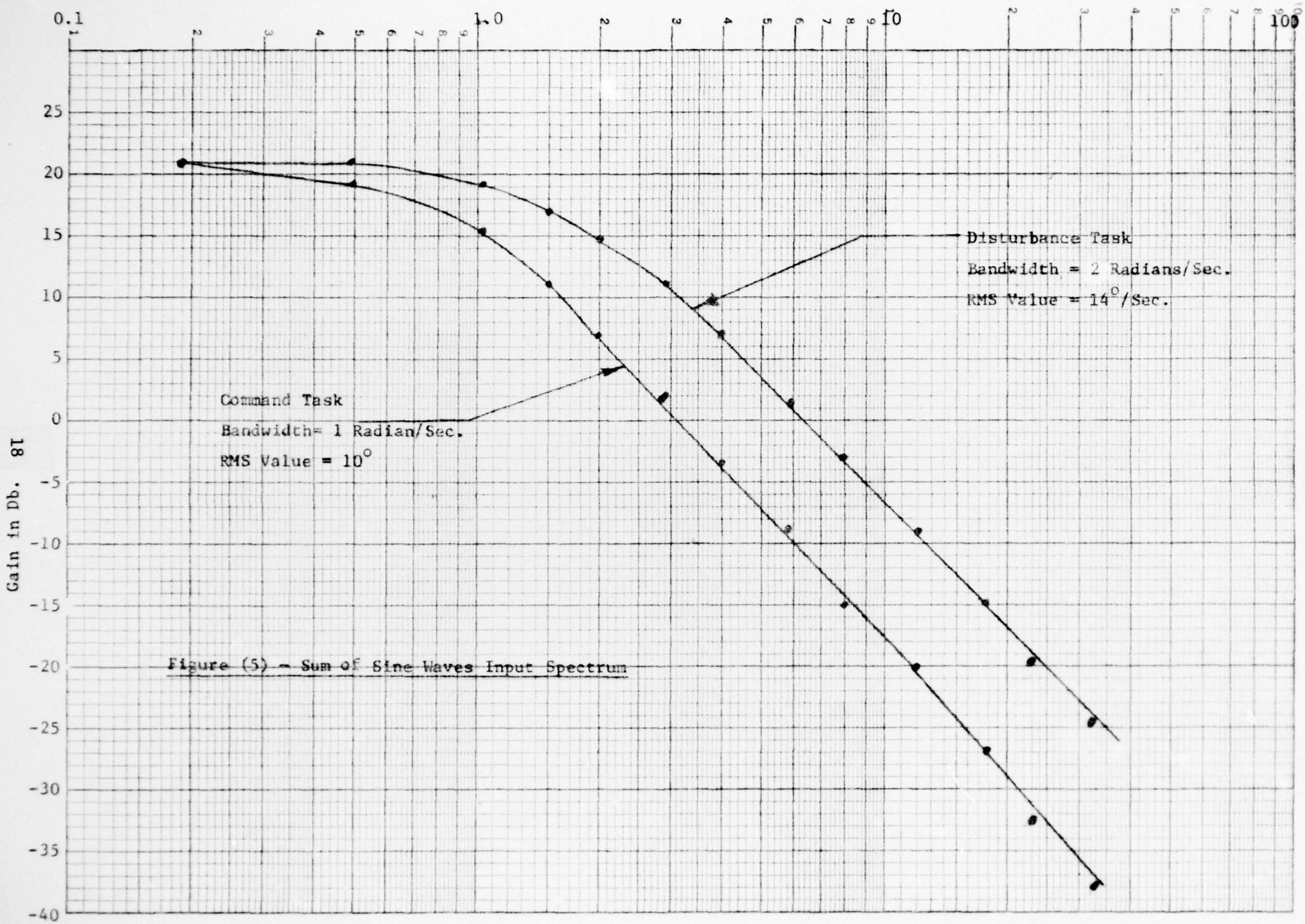
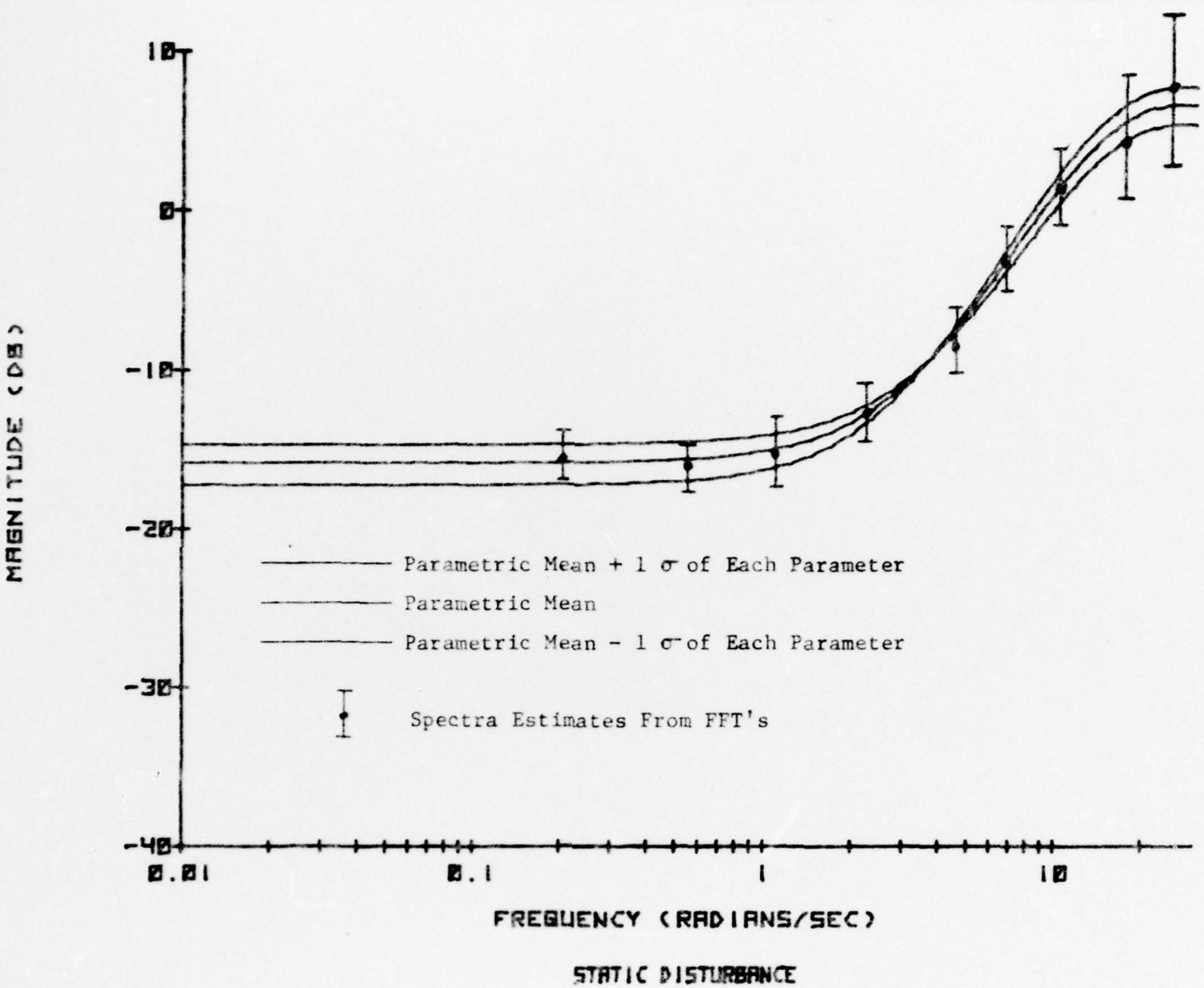


Figure (5) - Sum of Sine Waves Input Spectrum

Figure (6) Parametric Plots Versus Spectra Estimates (Static Disturbance Case)



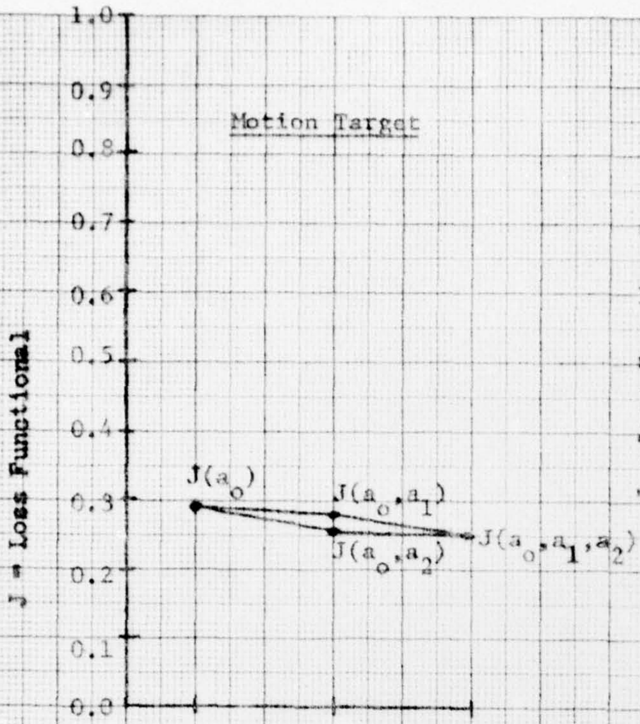


Figure (7 a)

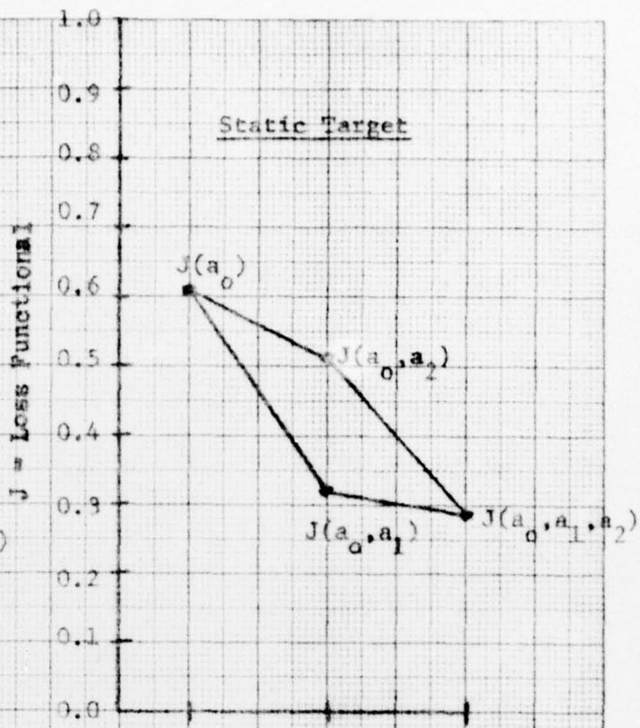


Figure (7 b)

Figures (7 a-d) Plots of The Output Error Loss Functionals

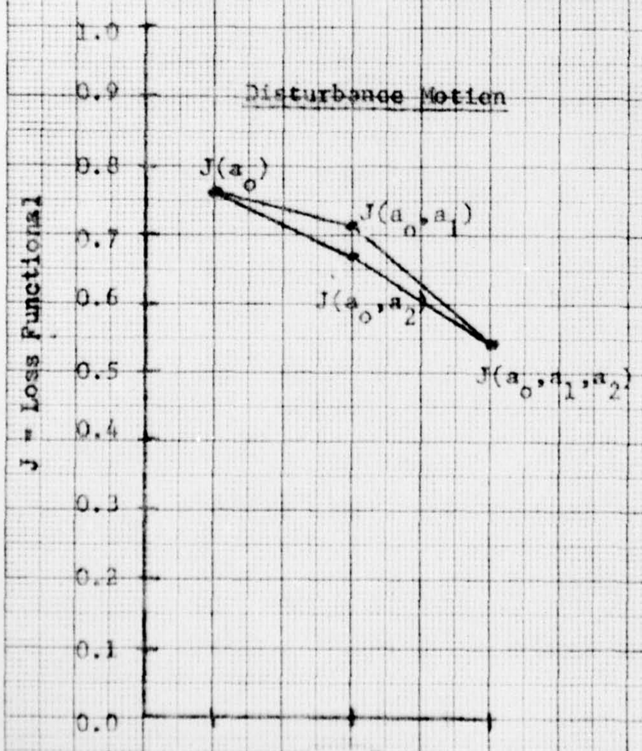


Figure (7 c)

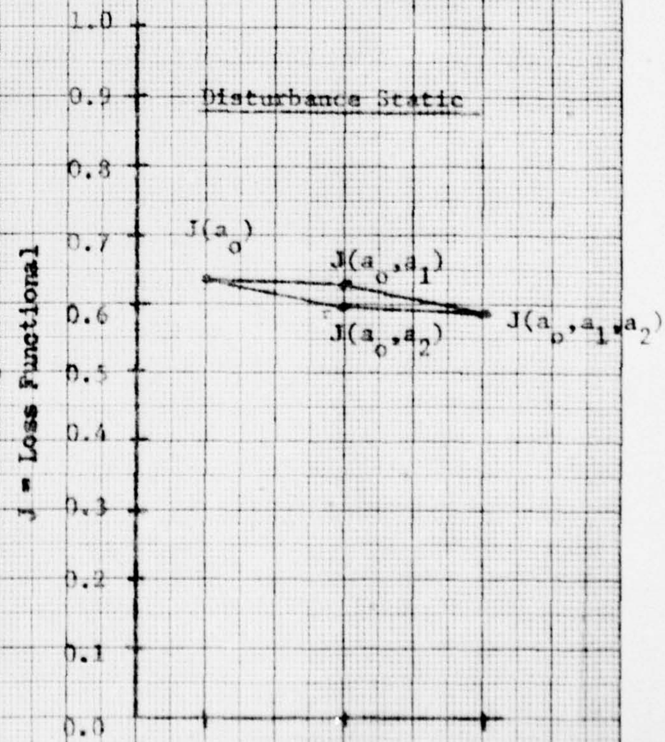


Figure (7 d)