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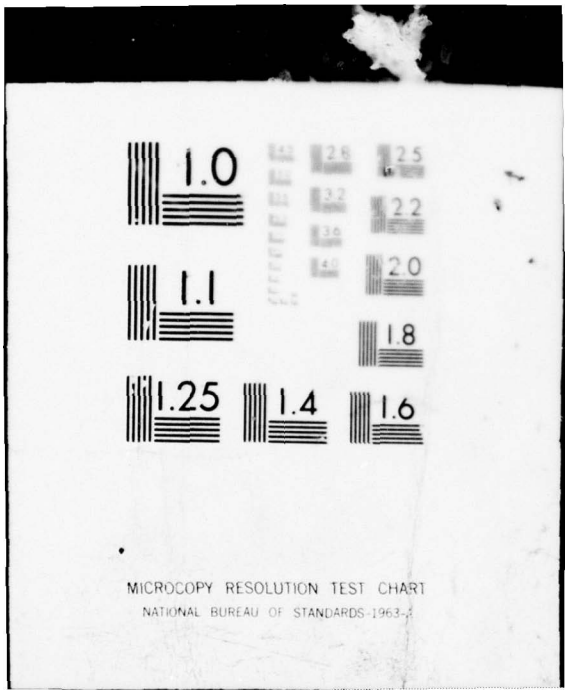
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# CRREL
## REPORT 77-12

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*Roof loads resulting from rain-on-snow*



*For conversion of SI metric units to U.S./British customary units of measurement consult ASTM Standard E380,* Metric Practice Guide, *published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.*

*Cover: Large snowdrift on the roof of a school in Buffalo, New York. (Photograph by Robert Redfield.)*

# CRREL Report 77-12

# *Roof loads resulting from rain-on-snow*

Samuel C. Colbeck

May 1977


CORPS OF ENGINEERS, U.S. ARMY
**COLD REGIONS RESEARCH AND ENGINEERING LABORATORY**
HANOVER, NEW HAMPSHIRE

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## REPORT DOCUMENTATION PAGE

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| | |
|---|---|
| 1. REPORT NUMBER: CRREL Report 77-12 | 2. GOVT ACCESSION NO. |
| 3. RECIPIENT'S CATALOG NUMBER | |
| 4. TITLE (and Subtitle): ROOF LOADS RESULTING FROM RAIN-ON-SNOW | 5. TYPE OF REPORT & PERIOD COVERED |
| | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s): Samuel C. Colbeck | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS: U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire 03755 | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: DA Project 4A161101A91D, Task 256 |
| 11. CONTROLLING OFFICE NAME AND ADDRESS: U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire 03755 | 12. REPORT DATE: May 1977<br>13. NUMBER OF PAGES: 26 |
| 14. MONITORING AGENCY NAME & ADDRESS *(If different from Controlling Office)*: 29p. | 15. SECURITY CLASS. *(of this report)*: Unclassified<br>15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Drainage
Loads (forces)
Rainfall
Roofs
Snow

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

A computer program to calculate the increased live load on a snow-covered roof due to rain-on-snow is given. For the 25-year rainstorm falling on a heavy snow load on a flat roof in Hanover, New Hampshire, an additional 98 kg/m$^2$ (20 lb/ft$^2$) of liquid water is added to the live load. The additional load due to rain-on-snow is very sensitive to the snow properties and characteristics of the roof. A wide range of live loads is possible, depending on the particular circumstances.

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## PREFACE

This report was prepared by Dr. Samuel C. Colbeck, Geophysicist, of the Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. The research was funded by DA Project 4A161101A91D, *In-House Laboratory Independent Research*, Task 256, *Roof Loads Resulting from Rain-on-Snow.*

Dr. Haldor Aamot, Thaddeus Johnson and Robert Redfield of CRREL technically reviewed the manuscript. Their contributions to this project are gratefully acknowledged.

## CONTENTS



## ILLUSTRATIONS

## SUMMARY

Rain falling on a heavy snow load can cause roof failures by adding temporary weight to a roof. This additional weight can be calculated as a function of time for any given depth, porosity, temperature and permeability of snow; size, slope, and shape of roof; and duration and intensity of rainstorm. When the 25-year rainstorm for Hanover, New Hampshire, is used as an example, it is found that a two-dimensional, flat roof could retain enough rainwater to reach about 50% of most design live loads. This weight could be partially reduced by giving the roof a slight inclination; however, the weight would be increased if the water moved radially to internal drains rather than to large gutters.

It is shown that the roof load due to rain-on-snow is very sensitive to spacing between drains, snow depth and roof inclination. The load on a flat roof is less sensitive to the duration of the 25-year rainstorm and the permeability of the snow. Roofs with radial flow to a single, central drain tend to drain more slowly than roofs with parallel flow to large gutters. The advantages of enlarged, recessed drains are shown.

In the Appendix, a computer program is given for calculating the total weight of wet snow as a function of duration for any design basis rainstorm. In addition to choosing the design basis rainstorm, the snow depth, porosity, temperature, and permeability, as well as the roof size, slope, and shape must be specified by the user.

## NOMENCLATURE

| | |
|---|---|
| $d$ | depth of saturated layer |
| $d_L$ | depth of saturated layer at drain |
| $d_0$ | maximum depth of saturated layer |
| $D$ | grain size |
| $F$ | slope correction factor |
| $g$ | gravitational constant |
| $h$ | thickness of snow |
| $i$ | rainfall intensity |
| $I$ | infiltration into the saturated layer |
| $k_s$ | intrinsic permeability of saturated layer |
| $k_u$ | intrinsic permeability of unsaturated layer |
| $L$ | roof length |
| $L_c$ | culvert length |
| $n$ | $\sqrt{\alpha k_s / I}$ |
| $q$ | discharge through the saturated layer |
| $Q_D$ | drain discharge |
| $r$ | radial coordinate |
| $r_D$ | drain radius |
| $r_e$ | roof radius |
| $R$ | $r/r_e$ |
| $R_D$ | $r_D/r_e$ |
| $R_s$ | $x/L$ |
| $S_w$ | water saturation as percent of pore volume |
| $S_{wi}$ | irreducible water saturation |
| $S^*$ | $(S_w - S_{wi})/(1 - S_{wi})$ |
| | time |
| $T$ | snow temperature (°C) |
| $u$ | volume flux, water volume per unit area per unit time |
| $V$ | water volume of saturated layer |
| $w$ | culvert width |
| $W$ | total weight per unit area |
| $W_u$ | liquid weight per unit area in the saturated layer |
| $x$ | coordinate direction along roof |
| $\frac{dz}{dt}\Big\|_{S^*}$ | speed of a value of $S^*$ |
| $\alpha$ | $5.47 \times 10^6$ m$^{-1}$ s$^{-1}$ |
| $\theta$ | roof slope |

$\rho_i$ density of ice

$\rho_s$ ice density of snow (excluding the liquid weight)

$\rho_w$ density of water

$\tau$ rainfall duration

$\phi$ snow porosity, $1 - \rho_s/\rho_i$

$\phi_e$ $\phi(1 - S_{wi})$

# ROOF LOADS RESULTING FROM RAIN-ON-SNOW

Samuel C. Colbeck

## INTRODUCTION

The cost of constructing a modern building is so great that much time and effort must be devoted to saving material and labor. Unfortunately the efficiency of the design is often limited by the many variables which must be considered. These variables – including winds, soil conditions, and snow loads – are often difficult to quantify and therefore difficult to accommodate in the design. One important aspect of many designs – roof load due to rain falling on a snow-covered roof – is described here.

Much consideration has been given to regional snow accumulations, including both depths and densities (e.g. Tobiasson and Redfield 1973). Thorburn and Schriever (1959) discussed roof failures resulting from rain-on-snow, emphasizing the significance of rain falling on an already heavy snow load. Rain has two special properties in this regard. First, rain is much more dense than snow but can be added to the snow without increasing its bulk. In fact, an observer may believe incorrectly that the roof load decreases as the snow densifies following the introduction of rain water. Second, rain falling on snow percolates downward until it reaches the roof and then spreads laterally or flows downslope. If the snow is at subfreezing temperatures, some of the percolating water will be refrozen in place, but if the snow is already wetted, all the water will pass through the snow and drain off the roof. In the former case some of the water will remain on the roof as an integral part of the snow load, whereas in the latter case all the water passes through the snow, adding only transient weight to the roof. Our purpose here is to establish techniques for calculating this weight as a function of time for any given set of conditions. The basic question is how much of the transient rainwater can be retained by the snow on the roof at any time. For a "design basis rainstorm," the roof load is considered to be at a maximum and it is this load which the roof must be designed to support.

## WATER MOVEMENT THROUGH SNOW

Rainwater falling on a snow-covered roof percolates downward until it reaches the impermeable roof surface. The water then flows along the roof surface to an internal drain or the roof edge. The two modes of flow – unsaturated vertical percolation and saturated lateral flow over the roof – are quite distinctive and require separate descriptions (see Fig. 1).

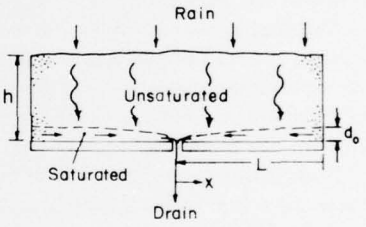



*Figure 1. Snow-covered flat roof with an internal drain where h is thickness of snow, L roof length, $d_0$ maximum depth of saturated layer, and x coordinate direction along the roof. The saturated layer of water flowing over the roof is elliptical in shape.*

### Vertical percolation

The condition of snow prior to the rainfall is very important because of the wide variety of responses that can happen in any given situation. If the snow is at a subfreezing temperature, some of the rainwater will be frozen in place in order to raise the snow to the melting point. This volume per unit area is equal to $-\rho_s Th/160$ where $\rho_s$, $T$, and $h$ are the density, temperature and depth of the snow, respectively. As shown on Figure 2 for a typical snow density of 0.5 Mg/m$^3$, the thermal

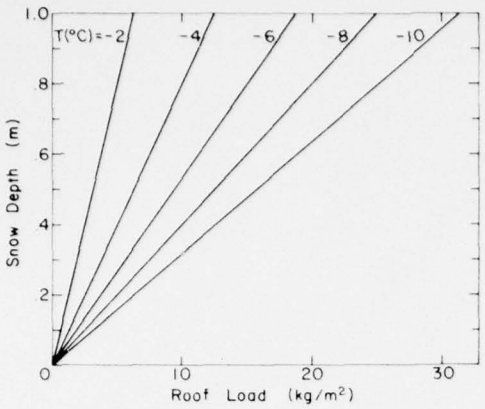



*Figure 2. Roof load due to refrozen rainwater shown as a function of snow depth for various snow temperatures. The snow density is assumed to be 0.5 Mg/m³.*

effect adds only 3.125 kg/m² of load (0.64 lb/ft²) per meter of depth and degree Celsius of subfreezing temperature. It seems unlikely that this effect could be more than a small percentage of the total roof load because heavy rains of sufficient duration are very unlikely when a thick, cold snowload occurs.

The unsaturated percolation of water through the upper portion of the snow also adds to the roof load because of the need to increase the liquid water content in order to allow water percolation. If the flow through the snow is uniform and homogeneous (i.e. does not flow in distinct drainage channels), this increase in weight due to the liquid saturation can be calculated from known principles of the unsaturated water flow in snow (e.g. Colbeck and Davidson 1973, Colbeck 1976). The intensity of the rainfall and the grain size, density and depth of the snow would have to be known to make this calculation. Assuming a constant source of rain of sufficient duration to penetrate the entire snowcover, the weight of liquid per unit area would be equal to

$$W_u = h\rho_w\phi\,[(i/\alpha k_u)^{1/3}(1 - S_{wi}) + S_{wi}] \quad (1)$$

where $\rho_w$ is density of water, $\phi$ snow porosity, $i$ intensity of the rain, $\alpha$ a constant ($5.47 \times 10^6$ m$^{-1}$ s$^{-1}$), and $S_{wi}$ the irreducible water saturation (about 0.07 depending on the degree of snow metamorphism). $k_u$ is the intrinsic permeability given by Shimizu (1970) as

$$k_u = 0.077\,D^2 \exp(-7.8\,\rho_s) \quad (2)$$

where $D$ is the grain size. For a rainstorm of $10^{-6}$ m$^3$/m$^2$ s (~ 0.14 in./h) with a snow depth of 1 m and permeability of $10^{-10}$ m$^2$, the increase in roof load due to the moving water would be 76 kg/m$^2$ (15.5 lb/ft$^2$) per meter of depth. This weight alone approaches half of most live load design limits if the rainstorm lasts long enough to deposit this much rain and if the flow is uniform throughout the snow. In fact neither of these conditions is likely. As shown later, the probability of a rainstorm of this intensity and duration falling on snow is small. Furthermore, the roof load would probably be reduced somewhat below the value calculated here because of nonuniform flow through the snow. The degree of nonuniformity would be highly variable, depending on the nature of the snow on the roof prior to the onset of the rain. Thus, while some correction could be applied to account for the channeling often observed in snow on the ground, there is no way of assessing what this correction factor might be. Since it is conservative to assume a uniform distribution of the flow, the calculations are made accordingly.

The effects of the intensity of the rainfall and permeability of the snow are reduced by their one-third power dependence. An order-of-magnitude increase in rainfall intensity only about doubles the weight due to the unsaturated snow. As shown later the more intense rainfalls are generally shorter in duration; thus, the less intense but longer lasting rainfalls can add more weight to the roof than the more spectacular (but shorter) heavy rains. Furthermore, the less intense rainstorms are more likely to cause a homogeneous soaking of the snow; hence, rapid runoff due to the formation of flow channels is less likely for rainstorms of lower intensity.

Grain growth, density increase, and a general homogenization of the snowcover occur when large quantities of rain or meltwater first infiltrate the snowcover. The properties of the snowcover change rapidly during this period of "melt metamorphism" and the permeability can increase significantly. Again because of the one-third power dependence of weight on permeability, the roof load due to the unsaturated snow would decrease very slowly with increasing permeability. Because of the general lack of observations of the physical properties of snow on roofs, it is difficult to assign values to the intrinsic permeability of the snow but the assumed value of $10^{-10}$ m$^2$ is in the correct range for the unsaturated snow.

### Lateral flow

Once the percolating water reaches the roof surface, the water moves laterally in a saturated layer. The permeability to the water in this saturated layer is orders of magnitude greater than in the unsaturated layer for two reasons. First, the permeability to the water increases as the cube of the liquid saturation (Colbeck and Davidson 1973) and the unsaturated

layer is typically 10% saturated while the saturated layer is nearly 100% saturated. Thus the saturation effect increases the permeability to the liquid in the saturated layer by a factor of about $10^3$. Second, the grain size in the saturated layer is likely to be greater because of "temperature gradient metamorphism" in the warmer basal layer prior to the introduction of liquid water and because of the rapid grain growth in saturated snow after the introduction of liquid water (Wakahama 1968). As shown by eq 2, the intrinsic permeability increases as the square of the grain size. Accordingly, an intrinsic permeability of $10^{-9}$ m$^2$ might be typical for the saturated layer $k_s$.

At the roof surface the pressure gradient available to drive the lateral flow is very small and the flow is reduced accordingly. For a house-size gable roof of 10° slope, the weight of the saturated layer could reach about 10 kg/m$^2$ (2 lb/ft$^2$) assuming no blockage at the roof edge. Since most gable roofs are steeper than this, the weight of their saturated layers would generally be negligible. At slopes of less than 10°, the flow is reduced significantly and the thickness and weight of the saturated layer are increased accordingly. Lateral flow on both flat and low-slope roofs is described below.

### Flat roofs

For the idealized, two-dimensional roof shown in Figure 1, the profile of the saturated layer is often assumed to be elliptical. For steady flow where no blockage occurs at the drain, the profile is given by

$$\left(\frac{d}{d_0}\right)^2 + \frac{(L-x)^2}{L^2} = 1. \qquad (3)$$

where $d$ is the depth of the saturated layer.

The volume of water $V$ stored in the saturated layer per unit width of roof is

$$V = 0.5\,\pi L^2 \sqrt{I/\alpha k_s \phi} \qquad (4)$$

where $I$ is the flow into the saturated layer from the overlying unsaturated layer (m$^3$/m$^2$ s), $\phi$ is snow porosity $(1 - \rho_s/\rho_i)$, and $k_s$ is intrinsic permeability of the saturated layer. The maximum thickness of the saturated layer $(d_0)$ for steady flow is

$$d_0 = \sqrt{I/\alpha k_s}\, L\,. \qquad (5)$$

For nonsteady flow where $I$ varies with time, the following equation (Colbeck 1973) describes the depth of the saturated layer as functions of $x$ and $t$:

$$\alpha k_s \frac{\partial}{\partial x}\left(d\,\frac{\partial d}{\partial x}\right) + I(t) = \phi\,\frac{\partial d}{\partial t}\,. \qquad (6)$$

This equation is solved here by assuming that transient wave effects are small and that the elliptical profile is always maintained. Then the equation can be simplified to

$$\phi\,\frac{dd_0}{dt} = I(t) - \alpha k_s\, L^{-2}\, d_0^2\,. \qquad (7)$$

This equation can be immediately solved when $I(t)$ is either a constant or zero, e.g. during the onset and then abrupt cessation of rain. The solutions for these cases are

$$d_0 = \begin{cases} d_0(0) + \sqrt{I/\alpha k_s}\; L \tanh \dfrac{\sqrt{\alpha k_s I}}{\phi L}\; t & \text{for } I > 0 \\[2ex] \dfrac{\phi L^2 d_0(0)}{\phi L^2 + \alpha k_s d_0(0) t} & \text{for } I = 0 \end{cases} \qquad (8)$$

where $d_0(0)$ is the value of $d_0$ when $I$ changes. An example of $d_0(t)$ is given on Figure 3. The thickness of the saturated layer increases rapidly following the onset of infiltration, although it is very unlikely that more than 50% of the maximum possible thickness would ever be realized because such heavy rains are not of sufficient duration. Once the infiltration ceases, the depth of the saturated layer decreases rapidly until more rain falls on the snow.

The total snow load per unit width carried by a flat roof with an internal drain is equal to the weight of the snow plus the liquid in the unsaturated layer plus $0.5\pi\phi L^2 \rho_w\, (I/\alpha k_s)^{1/2}$, the weight of the liquid in the saturated layer from eq 4. The saturated layer is somewhat more sensitive to changes in infiltration or permeability than the unsaturated layer although, because of the square root dependence, the weight of the saturated layer is less responsive to $k_s$ than might have been thought. At small values of time, the $k_s$ dependence of the saturated layer nearly vanishes as shown below. Note that flat roofs are very sensitive to the spacing of the drains since the weight per unit width of saturated layer increases as $L^2$.

### Sloping roofs

The complete equation describing flow along a sloping roof (Colbeck 1974) is

$$\alpha k_s \sin\theta\,\frac{\partial d}{\partial x} - \alpha k_s \frac{\partial}{\partial x}\left(d\,\frac{\partial d}{\partial x}\right) + \phi\,\frac{\partial d}{\partial t} = I \qquad (9)$$

where $\theta$ is the roof slope.

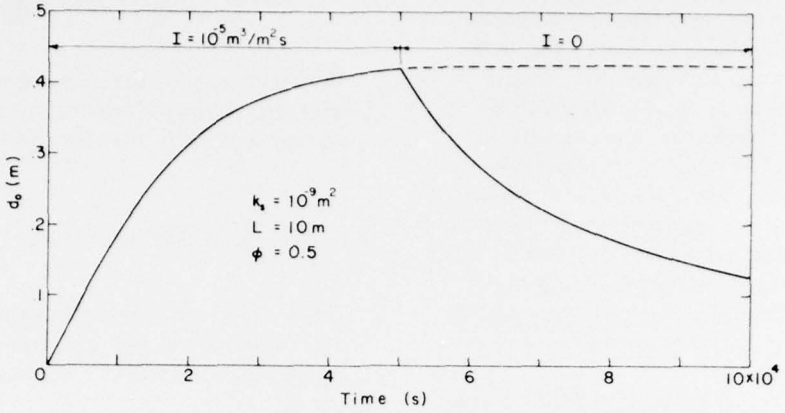



*Figure 3. Thickness of the saturated layer at the roof margin shown as a function of time for the specified conditions. The thickness increases rapidly following the onset of rain but it is unlikely that such a heavy rain could last long enough to generate more than 50% of the maximum possible value of $d_0$. Once the infiltration due to the rainfall stops, the thickness of the saturated layer decreases rapidly.*

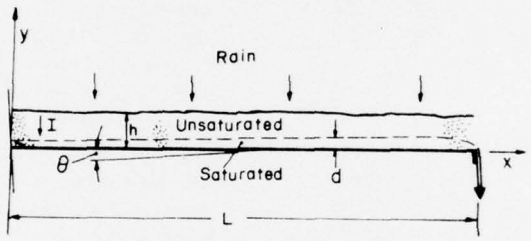



*Figure 4. Roof with a gradual slope. For a slightly dipping roof the thickness of the saturated layer is intermediate between a flat and a steeply dipping roof.*

For a steeply dipping roof this equation can be simplified by neglecting the second order derivative, whereas for a flat roof, the first term is zero. Roofs with gradual slopes are also common, and it is very important to analyze them as accurately as possible. Because of the extreme sensitivity of flow to slopes at angles of less than 10°, either of the simplifications mentioned above could cause large errors. As for flat roofs, we begin by establishing the upper limit of weight on a roof of gradual slope (see Fig. 4) during a steady rain. The flux $u$ (volume per unit area per unit time) along the roof is given by

$$u = \alpha k_s \left( \sin\theta - \frac{\partial d}{\partial x} \right). \qquad (10)$$

Hence for steady flow,

$$\alpha k_s \left( \sin\theta - \frac{dd}{dx} \right) d = Ix \qquad (11)$$

where $\theta$ is about equal to $\sin\theta$ for shallow slopes. Although eq 11 has no temporal dependence, it cannot be readily solved because of its nonlinearity. Fortunately we are not interested in solving for $d$ per se but rather we can solve for the volume of water $V$ in the saturated layer:

$$V = \phi \int_0^L d\,dx\,. \qquad (12)$$

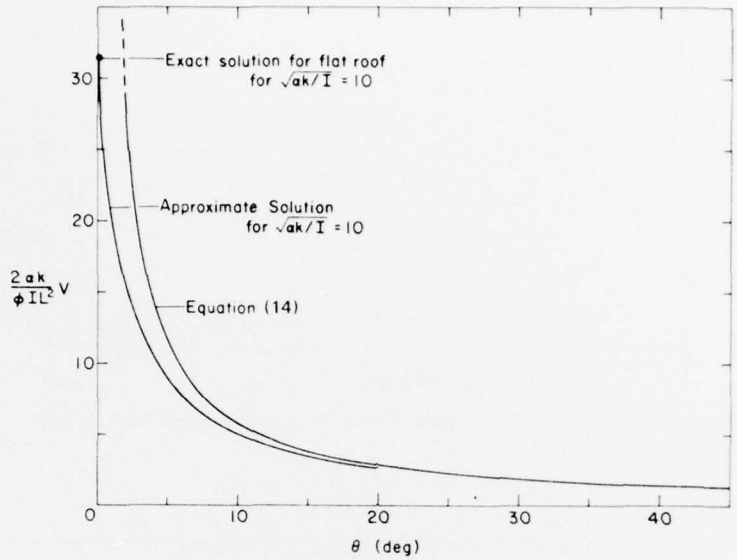



*Figure 5. Dimensionless volume of water in the saturated layer [$(2\alpha k_s/\phi I L^2)V$] shown for given parameters as a function of roof slope. The solution for V given by eq 14 is only approximate and as the roof slope decreases, the error increases. Accordingly an approximation to the solution is given for small values of θ.*

From eq 11 and 12 we get

$$V = \frac{1}{2}\phi\theta^{-1}\,(d_L^2 - d_0^2 + I\alpha^{-1}\,k_s^{-1}\,L^2) \qquad (13)$$

where $d_L$ is the depth of the saturated layer at the drain. Equation 13 is an exact expression for the volume of water in the saturated layer, but the boundary values $d_L$ and $d_0$ cannot be specified *a priori*. This equation could be used with $(d_L^2 - d_0^2)$ as a parameter, but for the practical purposes of deciding how much water can be impounded by the saturated layer, we must make the assumption that

$$I\alpha^{-1}\,k_s^{-1}\,L^2 >> |d_L^2 - d_0^2| .$$

Accordingly, eq 13 can be approximated by

$$V = \frac{1}{2}\,\phi\theta^{-1}\,I\alpha^{-1}\,k_s^{-1}\,L^2 \qquad (14)$$

which is the same as the formula for the volume of water on a steeply sloping roof, because the assumption stated above is only valid for steeper slopes.

The volume of water contained in the saturated layer as given by eq 14 is shown as a function of roof slope on Figure 5. There is a small error in the solution for slightly sloping roofs because eq 14 tends to overestimate $V$ at small slopes. This error arises because we have neglected $d_L^2 - d_0^2$, a negative number. An approximate solution for the volume of water in the saturated layer on a shallow roof is also shown on Figure 5. The approximation is made by noting that for a value of $(\alpha k/I)^{1/2}$ equal to 10, $V$ does not approach infinity as $\theta$ vanishes as suggested by eq 14, but rather $V$ has an upper limit given by eq 4 for flat roofs. Also, as the slope increases, the actual solution must approach the approximate solution of eq 14. The difference between eq 14 and the approximate solution is only significant at angles below 10° where the roof load is most sensitive to slope.

To include the effect of roof slope, we define a slope correction factor $F$ as

$$F_n(\theta) \equiv \frac{W_n(0)}{W_n(\theta)} \qquad (15)$$

where $W_n(\theta)$ is the weight of the saturated layer for any slope $\theta$ or parameter $n$, where

$$n = \sqrt{\alpha k_s/I} . \qquad (16)$$

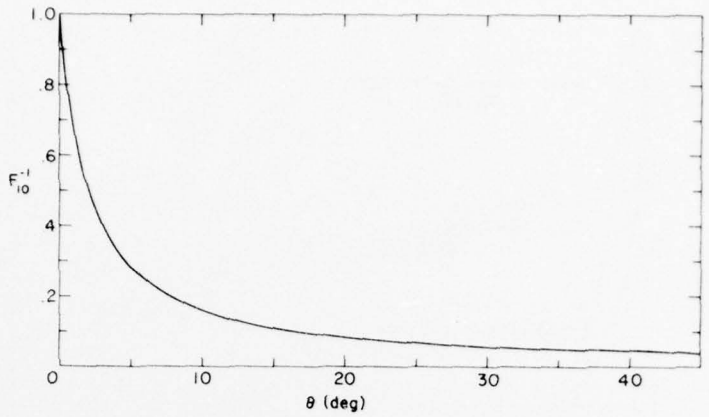



*Figure 6. Inverse of the correction factor F shown as a function of slope for n = 10.*

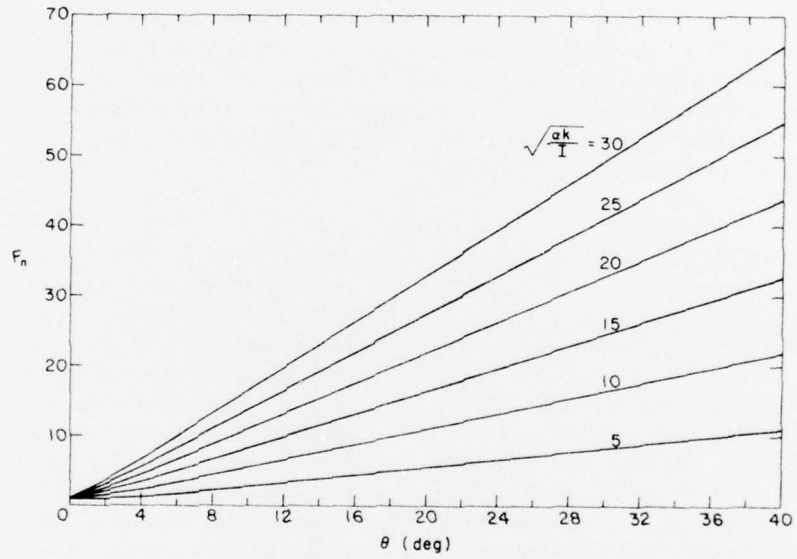



*Figure 7. Slope correction factor shown as a function of slope for various values of $(\alpha k/I)^{1/2}$.*

Using the approximate solution on Figure 5, we find $F_{10}^{-1}(\theta)$ as shown in Figure 6 and various values of $F_n(\theta)$ as shown in Figure 7. From Figure 6, $F_{10}(\theta)$ can be represented by

$$F_{10} = (1 + \theta/2.2)^{1.06} \tag{17}$$

and from Figure 7, we note that

$$[F_n(\theta) - 1] = [F_{10}(\theta) - 1]\ (n/10) \tag{18}$$

or

$$F_n(\theta) = 0.1n\ [(1 + \theta/2.2)^{1.06} - 1] + 1\ . \tag{19}$$

Now

$$W_n(\theta) = \frac{W_n(0)}{0.1n[(1 + \theta/2.2)^{1.06} - 1] + 1} \tag{20}$$

where $W_n(0)$ can be directly calculated from eq 4.

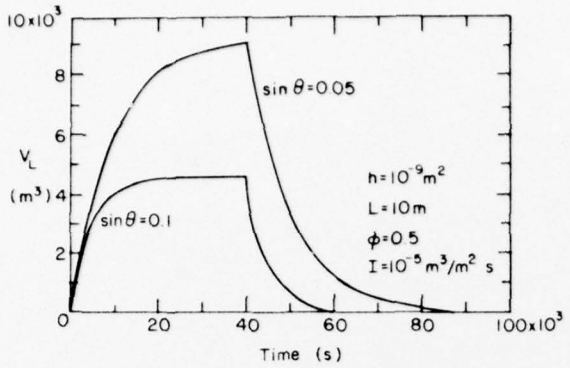



*Figure 8. Volume of water per unit width of saturated layer shown as a function of time for two roof slopes. The difference between the two slopes increases with time until $40 \times 10^3$ s when the percolating water stops reaching the roof. The more gradual roof carries about twice the weight of the steeper roof at $40 \times 10^3$ s.*

Figure 7 shows clearly that the effect of increasing the roof slope in order to provide better drainage is highly dependent on the values of $(\alpha k_s/I)^{1/2}$. The weight of the ponded layer of water decreases sharply for small increases in slope at slopes of less than 10°, but the optimum slope is dependent upon the values of rainfall intensity and snow permeability chosen for the optimization calculation. For both gently sloping and flat roofs, the weight of the saturated layer per unit width increases as the value of the spacing between the drains squared $4L^2$. This is a significant result which shows the critical value of drain spacing on shallow roofs.

Figure 8 shows the volume of water per unit width of the saturated layer as a function of time for two slopes where infiltration stops after $40 \times 10^3$ s. The more shallow slope carries a larger volume of water but it takes more time to reach its maximum value. During the first hour of infiltration into the saturated layer, the effect of slope is not as significant as suggested by Figure 5. The effect of slope increases as infiltration continues, until ultimately the more shallow slope carries more than twice as much water in its saturated layer as does the steeper slope. Also, the more shallow slope responds more slowly once the infiltration ceases by taking longer to drain. However, because it is very unlikely that such an intense rainstorm will last for this period of time, the difference due to roof slope is reduced somewhat. The decreased response time of a sloping roof is adjusted by multiplying the argument of the hyperbolic function in eq 8 by the correction factor $F$. This correction is included in the program in the Appendix.

The two-dimensional analysis given above was simplified in that the drain was assumed to be a longitudinal gutter capable of being an infinite sink. Other important factors like radial flow, drain size, basal layer permeability, and gutter slope are considered below.

## RAINFALL INTENSITY-DURATION EFFECTS

The weight added to a roof by transient water in both the unsaturated and saturated layers increases with both the intensity and duration of the rainfall. Snow-covered roofs frequently experience rainfalls which would certainly cause collapse if the rainstorms were of sufficient duration to allow complete soaking of the unsaturated layer and full development of the saturated layer. Fortunately, the duration of a rainstorm generally decreases with increasing intensity (Wisler and Brater 1965); hence some storm of intermediate intensity but longer duration may actually be the "design basis storm." The fact that the weight of the unsaturated layer only increases as $i^{1/3}$ and the weight of the saturated layer only increases as $I^{1/2}$ suggests that both duration and intensity are important parameters. Accordingly, we might suspect that long storms of less intensity may add more weight to some roofs whereas others may be loaded more by short, intense rainstorms.

The 25-year rainstorm for Hanover, New Hampshire, is used here as a design basis because we are concerned only with the coincidence of a heavy rainfall on an already heavy snow load. The probability of a 25-year rainstorm falling on a heavy snowload is not known but this example does provide an illustration. If it is decided that a larger safety margin is needed, a larger return period can be used, but because the roof weight is not too sensitive to the storm intensity, a large increase in return period would give a small increase in roof weight. For example, increasing the return period to 100 years in Hanover would only increase the weight of the saturated layer by 10%. However, if the maximum probable precipitation were used, the weight of the saturated layer would increase by about 80% above that calculated for the 25-year return period.

From Niedringhaus (1973), a $2.09 \times 10^{-5}$-m/s (3.0-in./h) rainfall lasting 1800s can be expected once every 25 years in Hanover, N.H. From the exponential formula suggested by Wisler and Brater (1965), the intensity $i$ and frequency $\tau$ of 25-year rainfalls in Hanover are found to be

$$i = 5.465\ (\tau + 360)^{-0.725}\ \text{(mm/s)} \qquad (21)$$

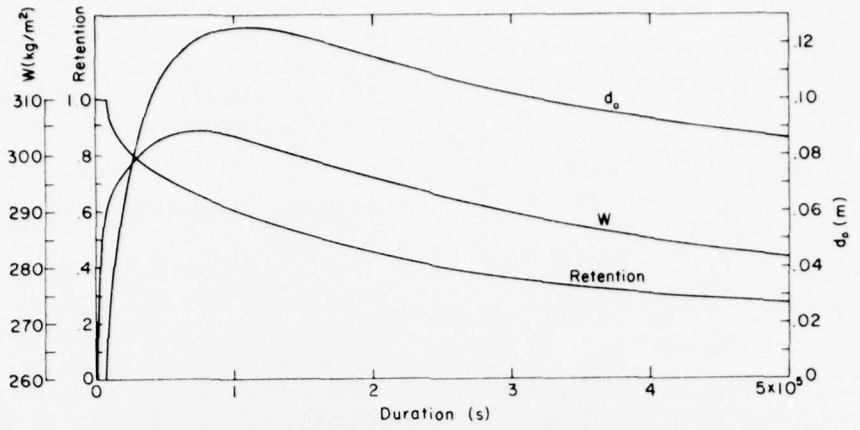



*Figure 9. Total live load W, fraction of rainfall retained, and depth of water impounded on a flat, two-dimensional roof for the 25-year rainfall at Hanover, N.H. The maximum weight occurs for a rainfall of about 75,000 s duration but is not very sensitive to duration.*

where the duration $\tau$ is expressed in seconds. The total precipitation $i\tau$ increases with duration so that lower intensity rainstorms always deposit more water on the roof than the higher intensity storms. Clearly some storm of intermediate intensity will prove to be the design basis rainstorm, since more of the water can drain from the roof during the lower intensity storms but lower intensity storms produce more total precipitation.

We make the assumption that the intensity of the rain is constant during the storm, although the worst case could arise if the intensity were unevenly distributed during the storm. Unfortunately, statistics on the intensity distribution are not available and no definitive statements about the effects of an uneven distribution can be made from the equations presented here. If there is a serious concern about distribution, it could best be accommodated by increasing the design basis to 50 or 100 years.

The intensity-duration effect is illustrated by calculating the maximum liquid retention of the saturated and unsaturated layers as separate systems and then the retention of the entire snow load. To evaluate the maximum weight of the saturated layer alone due to a 25-year rainfall, we set $d_0(0)$ equal to zero and the infiltration $I$ in eq 8 equal to the rainfall $i$. Then the maximum depth of water impounded on a flat roof is

$$d_0 = i^{1/2}\,\alpha^{-1/2}\,k_s^{-1/2}\,L\,\tanh\frac{\sqrt{\alpha k_s I}}{L\phi}\,\tau\,. \qquad (22)$$

For typical parameters for a snow with a highly permeable basal layer, Figure 9 shows that the depth of water ponded on the roof is at a maximum for an intermediate value of rainfall duration. When the rainfall in this example lasts 100,000 s (27.8 h) at an intensity of 0.00129 mm/s (0.183 in./h), the depth of the ponded layer reaches 126 mm (5 in.). Only 51% of the water reaching the saturated layer in this example is retained at the end of the period of infiltration. This liquid would add 53.4 kg/m$^2$ (11.1 lb/ft$^2$) of weight or about 30% of a normal design load. The depth of water would be significantly increased if 1) the roof were covered by fresh snow, 2) the culvert or drains were incapable of handling the water flowing to them, or as shown below, 3) the flow to the drains occurred as radial rather than parallel flow.

The maximum weight of the liquid in the unsaturated layer alone can be determined for the 25-year rainfall by combining eq 21 with the weight of the liquid in the unsaturated layer which is given by

$$W_u = h\rho_w\phi\,[(i/\alpha k_u)^{1/3}(1 - S_{wi}) + S_{wi}]\,. \qquad (23)$$

The design basis storm which gives the maximum increase in the weight of the unsaturated layer is calculated for snow which is wet but has no antecedent flow from

$$h\phi_e = 0.149\,(\alpha k_u)^{1/3}\,\tau^{0.777}\,. \qquad (24)$$

The critical duration would be longer if the snow were dry prior to the onset of rain (this option is included in the computer program) or shorter if antecedent flow existed in the snow. However, the weight of the unsaturated layer in this example would not be greatly affected. Equation 24 clearly shows that the critical duration of the rainstorm (and the weight of the unsaturated layer) increases rapidly with the depth of the snow but is much less sensitive to the permeability of the snow. For the case of Hanover with a snow of 0.5-m depth, 0.5 effective porosity and $10^{-10}$-$m^2$ permeability, the 25-year rainstorm that increases the weight of the unsaturated layer to a maximum has a duration of 8000 s (2.2 h) and an intensity of $7.83 \times 10^{-3}$ mm/s (0.23 in./h). This storm adds 62.6 kg/$m^2$ (12.8 lb/$ft^2$) of transient liquid weight to the unsaturated layer or about 35% of the usual design live load. It is very important to note that about 18.9 kg/$m^2$ (3.87 lb/$ft^2$) of additional liquid is retained as the capillary liquid. The weight of the ice would be about 211 kg/$m^2$ (43.2 lb/$ft^2$). Clearly, the additional weight of the liquid must be considered.

In these examples, the unsaturated layer can carry about the same load as the saturated layer; this has important implications for roofs of all slopes. If these weights were simply added, the total effect would require an increase of the design load of more than two-thirds to account for the total weight of liquid water on the roof. However, a more complete calculation of the combined effects of these two layers is necessary. This is illustrated here for a flat roof in Hanover.

For a flat roof the total weight of the liquid per unit area is given by

$$W = h\rho_w \phi \left[ (i/\alpha k_u)^{1/3} (1 - S_{wi}) + S_{wi} \right] + 0.5\,\pi d_0\, \phi \rho_w \qquad (25)$$

where $S_{wi}$ is the irreducible water saturation and

$$d_0 = \begin{cases} 0 & \text{for } \tau < t_u \\ (I/\alpha k_s)^{1/2}\, L \tanh \left[ \dfrac{\sqrt{\alpha k_s I}}{L\phi} (\tau - t_u) \right] & \text{for } \tau > t_u. \end{cases} \qquad (26)$$

$t_u$, the time delay before the infiltrating water reaches the roof, is given by

$$t_u = h\phi_e (\alpha k_u i^2)^{-1/3}. \qquad (27)$$

For 0.5-m depth, 0.54 porosity, 10-m drain spacing, $10^{-10}$-$m^2$ permeability for unsaturated snow, $10^{-9}$-$m^2$ permeability for saturated snow, and the 25-year rainstorm for Hanover, we find using the computer program in the Appendix that the maximum weight of liquid carried by the flat roof increases with rainfall duration until about 75,000 s (20.3 h). As shown on Figure 9, the total weight due to the liquid is not sensitive to the duration of the rainstorm as long as the duration $\tau$ is well in excess of the time $t_u$ necessary to penetrate the unsaturated layer. In fact, the weight of the transient liquid increases to about 79 kg/$m^2$ (16.4 lb/$ft^2$) before the action of drainage becomes more important than the increase of total precipitation with duration. For a rainstorm of 75,000-s duration, about 66% of the rainfall is retained on a flat roof by the end of the rainstorm.

For this situation the total liquid adds 98 kg/$m^2$ (20.4 lb/$ft^2$) of liquid to the snow on the roof of which the capillary liquid (held by capillary forces) is 19 kg/$m^2$ and the transient liquid is 79 kg/$m^2$. The design live loads on most roofs would have to be increased by about 50% to account for the total liquid weight. The total live load in this example reaches 304.3 kg/$m^2$ (62.4 lb/$ft^2$); thus it is easily seen why Thorburn and Schriever (1959) were concerned about roof failures resulting from rain falling on a heavy snowload.

Because of the large number of parameters affecting the retention of rainwater on a roof, a wide range of calculated loads might be possible. However, it has been shown that the effects of snow permeability and rainfall intensity are reduced by the square or cube root dependency, and as suggested by Figure 9, the weight of transient liquid on a flat roof is not very sensitive to duration of the 25-year rainstorm. Accordingly, a wide variety of snow conditions and rainstorms can produce about the same amount of transient roof weight.

The drain spacing will affect the saturated layer, and for this particular example, the saturated layer is very sensitive to the slope of the roof. Other important effects, especially the radial movement of water to drains and gutter overflow, are introduced in the next section.

## MISCELLANEOUS EFFECTS

### Radial flow to drains

On roofs with internal drains, radial rather than parallel flow can occur as water moves over the roof to the drains. Accordingly, the previous analysis for flow along a two-dimensional roof must be extended to allow calculation of the larger weight due to impounded water associated with radial flow to a drain.

Steady flow $q$ through the saturated layer on a flat roof ($\theta = 0$) is given by

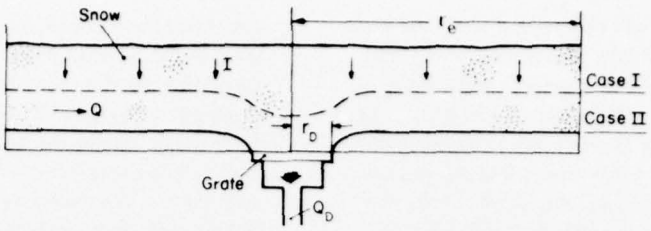



*Figure 10. Saturated layer of snow on a flat roof shown for the cases where the saturated layer does (I) and does not (II) cover the drain. Note that the effective radius of the drain ($r_D$) has been increased by providing a collector space between the drain pipe and the grate.*

$$q = -\alpha k_s 2\pi r d \frac{dd}{dr}. \tag{28}$$

The infiltrating water reaching the saturated layer increases the flux toward the drain according to

$$2\pi r I - \frac{\partial q}{\partial r} = 2\pi r \phi \frac{\partial d}{\partial t}. \tag{29}$$

Upon combining eq 28 and 29 to eliminate $q$ and integrating from the drain $r_D$ to any position $r$ for steady flow:

$$d^2 = d_L^2 - \frac{I}{2\alpha k_s}(r^2 - r_D^2) + \frac{I r_e^2}{\alpha k_s} \ln(r/r_D) \tag{30}$$

where $d_L$ is the thickness of the saturated layer at the perimeter of the drain, $r_D$ is the drain radius, and $r_e$ is the roof radius (see Fig. 10). The size and spacing of the drains are critical, since the drains must be capable of removing all the water without it ponding over them. As shown on Figure 10, the weight of the saturated layer is much greater when the drain is incapable of accepting all the water moving through the saturated layer. The vertical percolation through the saturated layer directly over the drain is independent of the thickness of that layer because the water pressures on both the upper and lower surfaces are equal. Therefore the flow through the drain $Q_D$ is given by the inequality:

$$|Q_D| \leq \alpha k_s \pi r_D^2. \tag{31}$$

The total flow reaching the saturated layer is $\pi r_e^2 I$. Thus

$$(r_e/r_D)^2 I \leq \alpha k_s \tag{32}$$

and there is no ponding over the drain as long as $(r_e/r_D)^2 I$ is less than $\alpha k_s$. It is conceivable that $(r_e/r_D)^2 I$ can be more than an order of magnitude greater than $\alpha k_s$; however, as noted before, the time required for the saturated layer to develop is very much greater than the duration of intense rainfalls. Nevertheless there are several important conclusions to be drawn at this point.

1. The rate of flow to the drain is very sensitive to the ratio of drain spacing to drain size as shown on the left hand side of eq 32. This ratio can be reduced by increasing the size of drains and/or decreasing the spacing between the drains. Since internal drains are expensive and tend to be troublesome, it is important to note that the effective diameter of a drain can be increased by simply increasing the size of the grating over the drain. The area available for flow to the drain increases as $r_D^2$ for Case I (see Fig. 10) and as $r_D$ for Case II. By increasing $r_D$ the increased weight due to ponding over the drain can be greatly reduced.

2. One severe problem with drains is the possibility of icing because of freeze-thaw cycles and/or the densification of the saturated snow overlying the drain. When the snow overlying the grating is wet it will retain by capillarity approximately 20 mm (0.8 in.) of liquid water in a saturated layer at the very base of the snow. If weather conditions change and this layer refreezes, it becomes a nearly impermeable "ice layer." If a subsequent rainstorm occurs, discharge into the drain is impeded by the reduced permeability of this layer and more of the rainwater could be retained on the roof. The "ice layer" undergoes a thermodynamic metamorphism in the presence of liquid water which eventually restores the permeability of the snow over the drain; however, it takes several days to cause a significant increase in $k_s$. As a result the weight added to a roof by a steady rainfall lasting several days could approach the total weight of the rainwater falling on the roof. One obvious approach to solving this problem is to heat the grating in order to melt away the 20-mm ice layer. This solution can be partly accomplished by

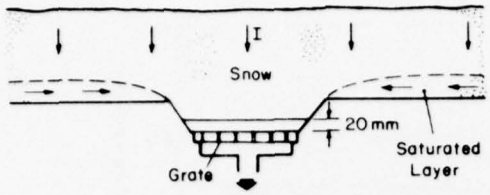



*Figure 11. Three advantages of a recessed drain over a flush drain: 1) the layer of snow saturated by capillary retention will be limited to the drain itself, thus reducing the roof weight, 2) this saturated, capillary layer will be less likely to freeze and block the drain, and 3) the lowered drain creates a "tension gradient" in the snow which slightly increases the rate of flow to the drain.*

recessing the grating as shown on Figure 11. A recess of 50 mm (1.97 in.) has several advantages over the flush grating shown in Figure 10. First, the layer of snow saturated by capillary retention is limited to the drain, thus reducing the weight that this saturated layer exerts on the entire roof. Second, internal drains form a natural heat leak from the building which should help to prevent the recessed snow from freezing. The greater the heat leak, the less the chances for ice layer formation to cause drain blockage.

3. The pressure in the water at the lowest point in the snow is approximately atmospheric and decreases with height above that point. This establishes a "tension gradient" which has been ignored in the derivation of the flow equations given here but in fact would increase the flow rate toward a recessed drain.

4. Assuming the drain is not blocked and is capable of handling the infiltrating water without forming a ponded layer directly over the drain, the profile of water in the saturated layer is given by

$$\left(\frac{2\alpha k_s}{I}\right)^{1/2} \frac{d}{r_e} = (2 \ln R/R_D - R^2 + R_D^2)^{1/2} \qquad (33)$$

where $r_e$ is the radius of the roof,

$$R_D \equiv r_D/r_e \qquad (34)$$

and

$$R \equiv r/r_e \, . \qquad (35)$$

The dimensionless ratio $(2\alpha k_s/I)^{1/2}\, d/r_e$ is shown as a function of $R$ for various values of $R_D$ in Figure 12. The effect of drain size is shown by the large increase in the thickness of the saturated layer for small values of $R_D$.

5. On a roof with radial flow, a larger volume of water is retained as a saturated layer than on a roof with parallel flow to a drain. To illustrate this point the profile of the saturated layer on a two-dimensional roof is given by

$$\left(\frac{2\alpha k_s}{I}\right)^{1/2} \frac{d}{L} = \sqrt{2}\ (1 - R_s^2)^{1/2} \qquad (36)$$

where

$$R_s = x/L. \qquad (37)$$

This is also shown in Figure 12 for a drain size of 0.1 of the roof length. The volume of ponded water on a roof with radial flow is given by

$$V = 2\pi\phi \int_0^{r_e} rd\, dr \, . \qquad (38)$$

Hence the ratio of the volume of ponded water for radial flow to that for parallel flow can be calculated as a function of $R_D$. This ratio is shown as a function of $R_D$ in Figure 13 for roofs of equal area. On the radial roof in this figure, there is a single, centrally located drain, and the two-dimensional roof has a fixed drain size. For $R_D$ equal to 0.1, the drain is $^1/_{10}$ the size of the roof and the saturated layer for radial flow has 78% more weight than for parallel flow. For $R$ equal to 0.01 which is probably a more common value, the saturated layer for radial flow contains 186% more water than for parallel flow. The advantages of increasing the relative size of the drain $R_D$ for radial flow by using an enlarged grating over the drain as suggested earlier have now been verified. Likewise, the advantage of draining a roof by parallel flow to a gutter rather than radial flow to drains has been established on a quantitative basis.

As for parallel flow, the rate of increase of the saturated layer in radial flow must be considered. The thickness of the saturated layer at the perimeter of the roof increases for a constant value of infiltration according to

$$d_0 = d_0(0) + A(I/\alpha k)^{1/2} \tanh\left[(\alpha k I)^{1/2}\, t/\phi A\right] \qquad (39)$$

where

$$A = \sqrt{r_e^2 \ln r_e/r_D - \tfrac{1}{2} r_e^2 + \tfrac{1}{2} r_D^2} \, . \qquad (40)$$

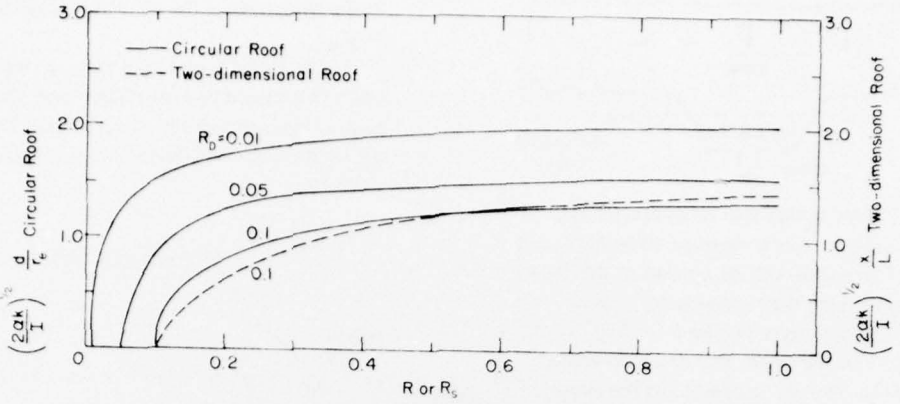



*Figure 12. Profile of the saturated layer ($d/r_e$) shown as a function of R for various drain sizes ($R_D$) for radial flow over a flat roof. The effect of decreasing the drain size is a rapid increase in the thickness of the saturated layer. For comparison, the corresponding profile is given for the two-dimensional roof considered previously (dashed line). The thickness of the saturated layer is somewhat greater for radial than for parallel flow.*

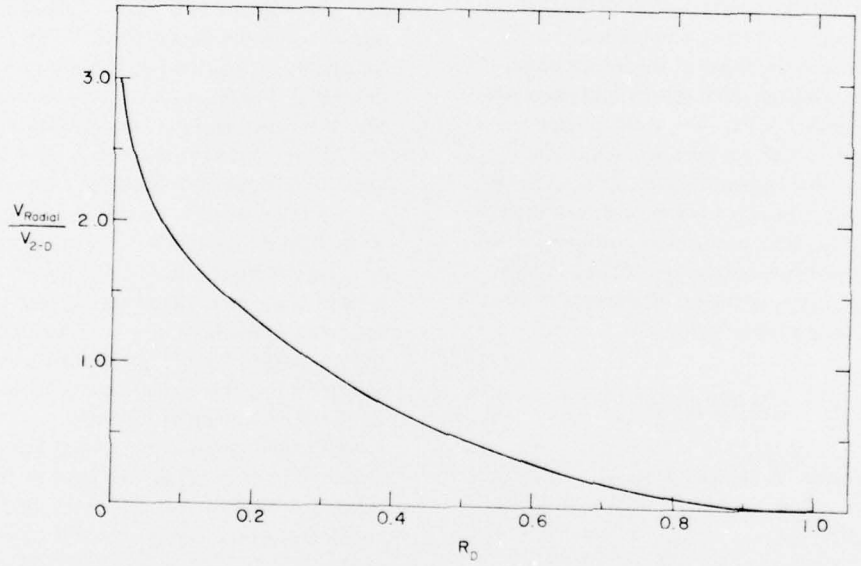
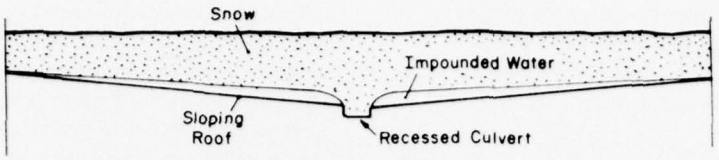
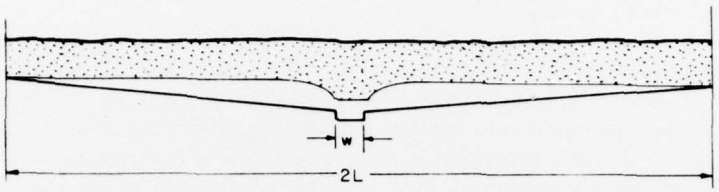



*Figure 13. Ratio of the volume of water in the saturated layers of radial and two-dimensional roofs shown as a function of the drain size on a radial roof.*




*a. Two-dimensional roof with a recessed gutter carries the minimum weight when the gutter depth exceeds the water depth.*




*b. When the gutter cannot move all of the water to the drain without overflowing, the roof slope is effectively decreased and the weight due to impounded water is greatly increased.*

*Figure 14. Flow on a flat roof for two conditions.*

From eq 38 the maximum possible volume on the roof is

$$V = \sqrt{2}\ \pi\phi\ \sqrt{l/\alpha k}\ r_e^3 \times \int_{R_D}^{R_e} \sqrt{\ln R^2 - R^2 + R_D^2 - \ln R_D^2}\ R\,dR\ . \quad (41)$$

In the formulation of the computer program it is assumed that the volume of the saturated layer increases directly as $d_0(t)$.

**Flow along gutters on snow-covered roofs**

We have shown that planar two-dimensional roofs tend to carry less weight in the saturated layer than radial roofs. The advantage of two-dimensional roofs is greatly reduced when the longitudinal gutter, which collects and moves the water to the drains, cannot move the water to the drains without overflowing. That is, when the thickness of the layer of water in a recessed gutter exceeds the depth of the gutter, the overflowing gutter effectively reduces the slope of the roof in order to increase its own slope. As shown in Figure 14, this phenomenon causes a large increase in the weight on the roof because small decreases in roof slope produce large increases in the volume of the ponded water (see Fig. 5). For level gutters, the depth of the saturated layer in the gutters assumes the elliptical profile of

$$\left(\frac{d}{d_0}\right)^2 + \frac{(L-x)^2}{L^2} = 1\ . \quad (3)$$

Hence the overflow would most likely occur near the roof edge and extend along most of the length of the gutter. For a steeply dipping gutter, the depth of the saturated layer increases linearly toward a drain and reaches a maximum just short of the drain. In this case the additional weight is concentrated near the drain so that an overflow would not cover such a large area of the roof. Gutters with gradual slopes, like roofs with gradual slopes, lie somewhere between the two extremes.

Obviously much can be gained by increasing the slope of a gutter. Unfortunately it is difficult to solve the governing equations, and unlike the case for a shallow roof where the water volume was used as the dependent variable instead of depth, we must deal with depth explicitly since the depth of the water in the gutters is the limiting criterion for design. The volume of water in the gutters per se contributes very little to the roof weight. Ideally the gutters should be able to handle all the rain intercepted by the roof without overflowing but this is very unlikely for normal roof design. Sloping the gutters reduces the weight

addition by overflow somewhat, but even so, the gutters would have to be extremely large to handle all the drainage. For a flat culvert, the maximum depth of water $d_0$ in the gutter is given by

$$d_0 = L_c \left(\frac{2L}{w}\right)^{1/2} \left(\frac{I}{\alpha k_s}\right)^{1/2} \qquad (42)$$

where $L_c$ is the length of the gutter, $2L$ is the width of the roof and $w$ is the width of the gutter. Equation 42 shows that the depth of water in the gutter increases with the spacing between the drains $L_c$ and as the square root of the ratio of roof to gutter widths $(2L/w)^{1/2}$. For reasonable values of $(2L/w)^{1/2}$ and $(I/\alpha k_s)^{1/2}$, the depth of the gutter must be excessively large unless the drain spacing is no more than a meter. Accordingly a recessed gutter similar to the suggested drain design (see Fig. 11) would be advantageous. The main advantage of this design is that the longitudinal flow along the gutter occurs in the snow-free space beneath the grating; hence, flow along the gutter would occur much more readily. For given conditions the flow rate would be greatly increased at small values of water depth. Accordingly overflow would not increase the roof weight by inhibiting flow to the gutter.

### Snow structure

In most areas of heavy snowfall, ice layers tend to form in the snowcover because of the preferential retention of liquid water at certain horizons. These ice layers interrupt the downward movement of water in the unsaturated zone causing ponded layers of water to form above them. In extreme cases these ice layers almost completely eliminate downward water movement and it would be necessary to manually penetrate such an ice layer in the event of heavy rains.

Generally ice layers are discontinuous and do not provide a complete interruption of the vertical seepage. In effect, discontinuous ice layers cause a stepwise flow of water down to the roof. There are two important effects of this mode of flow. First, more than one saturated horizon will exist since each ice layer as well as the roof tend to cause water ponding. Second, some areas of the snowcover may be bypassed and remain relatively dry while the flow is concentrated in zones of high water saturation and high flow rates. Accordingly, we expect an uneven distribution of the weight on the roof due to the liquid water.

It is difficult to generalize about the relative roof loads for homogeneous snow vs snow containing ice layers. This problem is compounded by the highly variable nature of ice layers in time and space and by the lack of studies of the particular properties of snow on roofs. The extent and nature of ice layers in the snowcover on the ground change rapidly during the warmer spring months when heavy snowloads associated with heavy rains are most likely to occur. In general it seems safe to assume that the presence of ice layers will not significantly increase the weight due to the presence of transient water. However, it must be emphasized that experimental observations of the detailed structure of snowcovers on roofs are necessary to test the validity of generalizing from snow on the ground to snow on roofs.

### Basal layer

There are two characteristics of the basal layer of snow which are important – the basal layer permeability and open channels. It was shown earlier that the permeability of the basal layer is of reduced importance because it enters the equations as a square root. Furthermore, at small values of time

$$\tanh \frac{\sqrt{\alpha k I}}{L\phi}\, t \approx \frac{\sqrt{\alpha k I}}{L\phi}\, t\,. \qquad (43)$$

Therefore, from eq 16 for small values of time

$$d_0 \simeq d_0(0) + It/\phi \qquad (44)$$

which shows that initially the thickness of the saturated layer increases linearly with time and is relatively insensitive to the permeability. This is illustrated on Figure 15 for two values of permeability. Note that the effect of permeability increases with time.

The higher permeability used in this example ($10^{-1}$ mm$^2$) could only represent a well-soaked snow with some open channels formed by melting in this basal layer. The maximum thickness of the saturated layer is 10 times larger for the snow of lower permeability and its time response is much slower. These are both important considerations regarding the weight of the basal layer. More observations of roof snow are needed to test the permeability values assumed here.

## CONCLUSION

Heavy rains falling on roofs carrying a heavy snow load are an important aspect of roof failures. The weight of the water, both capillary and transient liquid, should be considered as an additional load which must be supported by the roof. The maximum weight of liquid depends on properties of the snow, on size, slope and shape of the roof, on spacing and type of the drains, and on intensity and duration of the rainstorms.

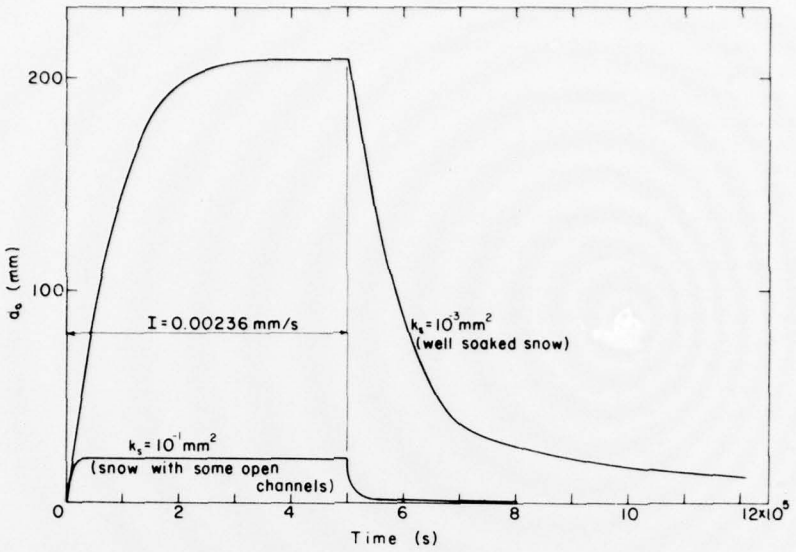



*Figure 15. Depth of the saturated layer ($d_0$) shown as a function of time for an intense rainstorm for two values of permeability of the basal layer. The maximum depth obtained differs by a factor of 10, although the permeabilities differ by a factor of 100. The more permeable basal layer has a much more rapid time response.*

While the depth of the snow is very important, the effect of permeability of the snow is reduced by its square or cube root dependence. However, the presence of open channels at the base of the snow will have an effect on reducing the weight carried by the roof. The roof size, especially for roofs with internal drains, has a very large effect, since the ponding of water over the roof increases as the square of the drain spacing. The effect of giving the roof even a small inclination is very large and must be considered in order to reduce the depth of the ponded layer. Enlarging and recessing the drains increase the rate of drainage and reduce the chances of blockage by refreezing.

Even for steeply dipping roofs, much water can be retained in the unsaturated snow. This water, both capillary and transient, must be considered since one example for a 25-year rainstorm in Hanover, New Hampshire, shows that 61 kg/m$^2$ (12.5 lb/ft$^2$) of weight could be added to the roof by the liquid in the unsaturated snow alone. The 25-year rainstorm was chosen for use as an example in order to illustrate the effect of intensity vs duration. If intense rainstorms lasted long enough they would certainly cause many failures of flat or shallow roofs covered by snow. Fortunately, more intense storms terminate before the ponded layer can be fully developed and less intense storms cause only a shallow ponded layer to develop because of concurrent drainage.

For the example given for Hanover the 25-year rainstorm falling on a heavy snow load could increase the total roof load due to liquid water to 98 kg/m$^2$ (20 lb/ft$^2$). The normal design load would have to be increased by about 50% to account for this effect. If the snow load is unevenly distributed (see cover photograph), the local load due to rain falling on the snowdrift can be as large as that produced by covering the entire roof with the drift.

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## APPENDIX A. COMPUTER PROGRAM CALCULATING ROOF LOADS FROM RAIN-ON-SNOW

This computer program in Dartmouth Basic (see Fig. A1) calculates total weight, fraction of rainfall retained, weight of the saturated layer and weight of the unsaturated layer as a function of duration for a design basis rainstorm falling on a snow-covered roof. The input required to run the program is as follows (in MKS units):

1) Specify $A, B, C$

These are the constants in a design basis rainstorm described by

$$i = A\,(\tau + B)^C \qquad \text{(A1)}$$

and must be obtained from climatological records for each area.

2) Specify $H, L, P, S, K1, K2$

$H$ is the thickness of the snow. $L$ is the distance between the parapet and drain for parallel flow to gutters or the roof edge, or the shortest distance between the parapet and drain for radial flow. $P$ is the porosity of the snow. $S$ is the irreducible water saturation of the snow. $K1$ and $K2$ are the intrinsic permeabilities of the saturated basal layer and the unsaturated upper layers, respectively.

3) Specify radial ($R$) or parallel ($P$) flow

Radial flow is flow to a central drain and parallel flow is flow to a gutter which carries the water to drains. It is assumed that a square area is drained in radial flow and that the gutter extends the entire length of the roof for parallel flow. Other configurations would require some modification of the program if they could not be represented by these assumptions.

4) Specify drain radius (meters)

The effective radius of the drain for radial flow. No drain or gutter overflow is assumed.

5) Specify temperature (°C)

The average snow temperature is required. If the snow is already at 0°C, it is assumed that only the capillary water is present in the snow such that no antecedent flow occurs prior to the onset of the rainstorm.

6) Specify roof slope (degrees)

The roof slope is input in degrees of slope.

7) Specify start time, stop time, time step

The program makes calculations for each value of rainstorm duration chosen here.

8) Another time step?

The operator can optimize the time step to minimize computer time by using a small time step at lower values of duration and increasing the time step at larger values of duration where the output changes more slowly.

The output from the program is the average weight per unit area, the maximum weight per unit area occurring anywhere on the roof, the fraction of the total rainfall retained, the average weight of the saturated layer, and the average weight of the unsaturated layer for each specified value of rainstorm duration. Other outputs such as depth of the saturated layer, weight of the capillary water, weight of refrozen water or fraction of wetting front penetration could be made with minor modifications. If the drains or gutters were not capable of removing the water without overflowing, some modifications would have to be made to the program to account for the additional weight.

```
LIST

RFDTROS   02 MAR 77  13:03

10 REM ROOF WEIGHT DUE TO RAIN ON SNOW, MKS UNITS
100 PRINT "SPECIFY A,B,C"
200 INPUT A,B,C
300 PRINT "SPECIFY H,L,P,S,K1,K2"
310 INPUT H,L,P,S,K1,K2
400 LET W1=917*H*(1-P)
405 PRINT "RADIAL(R) OR PARALLEL(P) FLOW"
410 INPUT W$
415 IF W$="P" THEN 500
417 IF W$="R" THEN 425
418 GO TO 405
425 PRINT "DRAIN RADIUS(METERS)"
430 INPUT G
435 LET J=.887*G/L
440 LET M=J/100
445 LET J1=J+M
475 FOR R=J1 TO 1 STEP M
480 LET N1=(2*LOG(R)-R^2+J^2-2*LOG(J))^(.5)*R*J
485 LET N=N+N1
490 NEXT R
500 PRINT "SPECIFY TEMP(DEG C)"
510 INPUT T9
515 LET W5=5.73*T9*H*(P-1)
525 PRINT "SPECIFY ROOF SLOPE(DEG)"
530 INPUT Q
540 LET F=(1+Q/2.2)^(1.06)
570 PRINT "SPECIFY START TIME, STOP TIME, TIME STEP"
580 INPUT T2,T3,T4
590 PRINT "DURATION", "TOT W", "RETENTION", "W(SAT)", "W(UNSAT)"
600 FOR T=T2 TO T3 STEP T4
700 LET I=A*(T+B)^C
770 IF T9<0 THEN 815
780 IF T9=0 THEN 800
790 GO TO 500
800 LET T1=(H*P*(1-S))/((5470000*K2*(I^2))^(1/3))
805 GO TO 900
815 LET T1=H*P*((I/(5470000*K2))^(1/3)*(1-S)+S)/I + 0.001*W5/I
900 LET U=((5470000*K1*I)^(1/2))*(T-T1)/(L*P)
910 LET U=U*F
1000 LET D=0
1005 IF T<T1 THEN1326
1010 IF W$="P" THEN 1200
1060 LET N2=15.95*P*L*N*(I/(5470000*K1))^(.5)
1070 LET U1=((5470000*K1*I)^.5*(T-T1))/(P*(L^2*LOG(L/G)-.5*L^2+.5*G^2)^.5)
1071 LET U1=F*U1
1080 LET W2=N2*(EXP(U1)-EXP(-U1))/(EXP(U1)+EXP(-U1))
1081 LET W2=W2/(.1*(5470000*K1/I)^(.5)*(F-1)+1)
1085 LET D=.00101*W2/P
1090 GO TO 1322
1200 LET D=((I/(5470000*K1))^(1/2))*L*((EXP(U)-EXP(-U))/(EXP(U)+EXP(-U)))
1320 LET W2=785*P*D/((F-1)*.1*(5470000*K1/I)^(.5)+1)
1322 LET E=1
1324 IF T>T1 THEN 1340
1326 LET W2=0
```

*Figure A1. Computer program.*

```
1330 LET E=T/T1
1340 LET W3=1000*H*F*((.005676*(I/K2)^.3333)*(1-S))
1350 LET W4=1000*H*F*S
1355 IF T9<0 THEN 1370
1360 LET W=W1+W2+E*(1-D/(H*F))*(W3+W4)
1364 LET V=((1-D/(H*F))*E*W3+W2)/(1000*I*T)
1366 GO TO 1495
1370 LET W=W1+W2+E*(1-D/(H*F))*(W3+W4)+E*W5
1380 LET V=(W2+E*(1-D/(H*F))*(W3+W4)+E*W5)/(1000*I*T)
1495 LET Y=(1-D/(H*F))*E*W3
1499 IF T7>0 THEN 1750
1500 PRINT T,W,V,W2,Y
1600 NEXT T
1650 PRINT "ANOTHER TIME STEP?"
1660 INPUT Z$
1670 IF Z$="YES" THEN 570
1680 IF Z$="NO" THEN 1700
1690 GO TO 1650
1700 IF F>1 THEN 2000
1710 PRINT "DURATION OF MAXIMUM TOTAL WEIGHT?"
1720 INPUT T7
1730 LET T=T7
1740 GO TO 700
1750LET W6=W1+1000*D*F+Y+W4
1800 IF T9=0 THEN 1950
1900 LET W6=W6+E*W5
1950 PRINT "MAXIMUM TOTAL WEIGHT PER UNIT AREA =",W6
2000 END
READY

BYE
```