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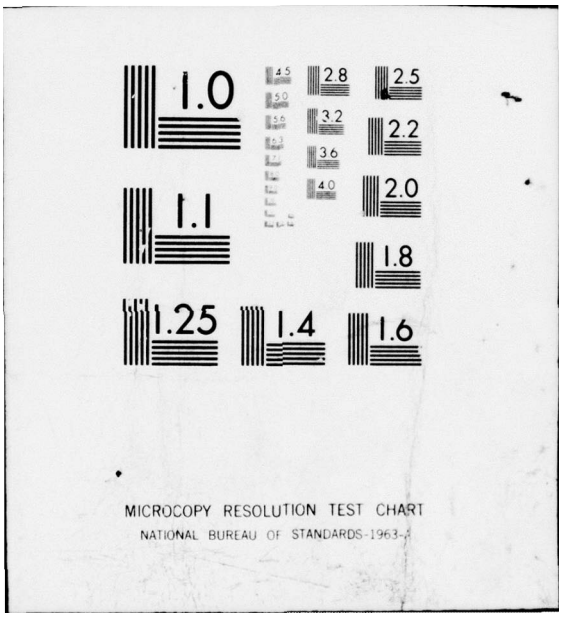
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ENERGY CONSERVATION THROUGH THE OPTIMIZATION OF HYDRAULIC POWER SUPPLIES FOR THE SIX DEGREE OF FREEDOM MOTION SYSTEM

By

Michael L. Cyrus

FLYING TRAINING DIVISION
Williams Air Force Base, Arizona 85224

March 1977

AIR FORCE SYSTEMS COMMAND
BROOKS AIR FORCE BASE, TEXAS 78235

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The objective of this project was to estimate the approximate hydraulic fluid flow requirements for the six degree of freedom motion system of the type specified in MIL-STD-1558 and determine means by which motion hydraulic supplies can be cut, combined, or made more efficient. This paper defines the approach, data collection, analysis, and results of that project.

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PREFACE

This work was conducted under project 1123, task 112303, Dr. Milton E. Wood, Project Scientist, and Mr. Michael L. Cyrus, Principal Investigator, in conjunction with Mr. Ed Martin, ASD-ENCT.

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ENERGY CONSERVATION THROUGH THE OPTIMIZATION OF HYDRAULIC POWER SUPPLIES FOR THE SIX DEGREE OF FREEDOM MOTION SYSTEM

I. INTRODUCTION

In January 1975, Aeronautical Systems Division (ENCTS) reviewed some preliminary data, taken from the Advanced Simulator for Pilot Training (ASPT), which indicated that large potential energy savings might be had through consolidation of hydraulic power supplies for platform motion systems. By 14 July 1975, the technology need (ASD-AFHRL-1307-75-49) coordination cycle was complete and time was scheduled on ASPT to develop, in detail, a mathematical model of expected hydraulic flow for the planned UPT-IFS facilities, using the ASPT motion system and aerodynamic mathematical models as a baseline. This paper describes the objective, approach, analysis and results of that study. Originally, this paper was not published in the AFHRL report system, but only as a final report for ASD/ENCT, with copies to interested parties. Recent interest in the ASPT motion system, including requests by other (non-Air Force) DOD agencies to see this data formally published has resulted in this paper.

Objective

Estimate the approximate hydraulic fluid flow requirements for the six degree of freedom motion system of the type specified in MIL-STD-1558, and determine if possible the means by which motion hydraulic supplies can be cut, combined, or made more efficient.

Approach

The completed research fell into three major efforts: preliminary data sampling and analysis, primary data collection, and analysis. Preliminary sampling and analysis provided sufficient information to specify: (a) the type of maneuvers to be flown, and (b) the necessary sampling rate. Data collection then began in earnest with approximately 300 hours of total ASPT cockpit utilization. Of this time, only 19 hours of data collected was absolutely free of any known possible system induced error. After appropriate comparisons with the overall data base (see the Analysis Section), this final data base was analyzed in terms of its time history probability distribution function and its spectral distribution.

Data Collection

Preliminary data sampling indicated that high G, "contact" maneuvers (loops, rolls, spins, etc.) provided the most extensive workout of the ASPT motion system; therefore, the only maneuver restriction placed on the pilot subjects in this study was that they continually fly one high G maneuver after another. A concerted attempt was made on the part of the author to minimize the time of straight and level flight, to thereby avoid biasing the final results towards hydraulic fluid flow estimates which would be unreasonably low. At the same time, direct comparisons in the leg cylinder time history and spectral distributions (in the preliminary analysis) demonstrated convincingly that a sampling rate of 7.5 Hz (the primary motion system software update rate for ASPT) was sufficient for data collection purposes.

Data collected, for each of the two ASPT cockpits, involved three categories of variables: demanded cylinder extensions, actual cylinder extensions, and special parameters. Special parameter recorded included discrete variables specifying which motion cue is active at a particular time (translational, rotational or gravity align). All variables were buffered from their immediate holding area in core to a large disc file (27,000 blocks of 192 words), 192 words at a time. The data transfer was controlled by three external data switches defining the activations of the data buffering routine, the initializing of the Disc File control block, and data take/hold function respectively. After a given data collection session, the appropriate contents of the disc file were dumped to tape, in blocks of 192 words. Descriptive information defining the

data, subject names, cockpits flown, and any particular system problems were recorded for use in conjunction with the later analysis.

Analysis

While the details for manipulating the recorded data are somewhat involved, there were basically only two kinds of analysis performed.

First, for each parameter whose distribution we are defining, an estimate of the parameter's mean, standard deviation, skewness, and kurtosis, minimum and maximum are obtained. Second, for each parameter, we also obtained its frequency (power) estimate and standard deviation from 0 to 3.75 Hz in increments of $\frac{3.75}{512}$ Hz. These two pieces of information we regard as uniquely describing each parameter recorded. The list of derived parameters are identical on each of ASPT's two cockpits, and are as follows:

1. Demanded flow rate. This parameter represents the demanded fluid flow required above that necessary to maintain the motion system in an erect state. The fluid flow required is computed based on a linear weighting of the actuator velocities summed across all six legs. The particular system modelled is that of the tachometer feedback system for the Instrument Flight Simulator (IFS) procurement, wherein 12.566371 cubic inches per second are required for each inch per second upstroke and 5.4977871 cubic inches per second for each inch per second downstroke. Estimates of the IFS fluid flow required, under the assumption of a motion system mathematical model similar to ASPT should be slightly high due to differences in geometry.

2. Using round numbers, with 19 inches per second for six legs as the absolute upper limit on velocity requirement, we find that fluid flow is a number which ranges from 0 to approximately 1,432 cubic inches per second or about 6.2 gallons per second.

3. Actual Flow rate. The only difference between "demanded" and "actual" flow rate is that we use the feedback positions of the cylinder instead of the commanded positions in the computations.

4. Demanded leg cylinder velocity distributions. This parameter is computed for each of six legs.

5. Actual leg cylinder velocity distributions.

6. Demanded positive velocity increments parameter. The parameter is the sum across all six legs of the velocity increments in the upward (positive) direction. Since IFS and other facilities may use servosystems whose hydraulic requirements differ as a function of direction, this information was thought critical.

7. Actual positive velocity increments parameter.

8. Demanded negative velocity increments parameter.

9. Actual negative velocity increments parameter.

Six other parameters specifying most frequent cues, and motion cueing distribution were also recorded, but are not included in this paper. As stated in the approach section, only the distilled data base need be used. Comparisons were made of the mean, and standard deviation for all parameters for both the time history and power spectral density computations from the overall data base to the distilled data base showed extremely close agreement. The reason for excluding the "error" filled runs is that false velocity data in excess of 19 inch per second is induced whenever the ASPT linkages do not service the Analog Output buffers on time, or a crash, reset, or freeze occur. While the density of these errors was low, this paper, being a first cut at specifying the actual Motion System hydraulic distribution, persuaded the author to insist on a "pure" data base. In addition to the analysis completed above, the frequency distribution in 5 cubic inch per second intervals for the overall data base was computed for graphical purposes, making it

possible for the reader to readily visualize and interpret the results presented in this paper. (Refer to the appendices for the graphs and charts associated with each variable; and for the list of formulas used to perform each analysis.)

Results

Results are presented for the combined data sets for the two ASPT cockpits, since no significant differences between the two motion platforms were observed for any time series statistical reductions (time history or spectral). All values in the tables presented are approximate. The following conventions were used:

$\bar{X}$ mean value

S standard deviation

β_1 skewness

β_2 kurtosis

max maximum value recorded

min minimum value recorded

n total number of observations in the sample

A quick glance at the time history distribution tables shows that the choice of "Demanded" values over "Actual" values gives rise to an excellent, if slightly conservative, estimate of system performance. This result enables offline estimation of the effect of a variety of motion mathematical models on hydraulic flow requirements offline, while still providing a safe, upper bound on what will really be used, a definitely preferable situation. Note the similarities in the distribution patterns of not only between "Demanded" and "Actual," but between the Leg triad sets 1, 3, 5, and 2, 4, 5. The only reason for the small dissimilarities between triad sets is the tendency for the pilot subject to practice rolls and spins primarily to the left (each subject flew from the student seat), thereby biasing the data somewhat in that direction. For the purposes of illustration, the demanded fluid flow is also presented in the summary format of a frequency distribution in intervals of 5 cubic inches per second. This, it is hoped, will help the reader more readily "picture" the distribution. The reader will recall that 1,432 cubic inches per second is the approximate excess (above that necessary for maintaining the neutral position) command. Therefore, we used 300 intervals of 5 cubic inches per second width to display the distribution. Although the software on ASPT limits leg cylinder velocity commands to less than 19 inches per second absolute, two legs recorded velocities in excess of 19 inches per second on the "actual" distribution. The reason for this anomaly is that "buffet" command requirements were not taken into account in the "Demanded" section. A combination stall, or spin, together with the associated aerodynamic buffet will, all conditions being right, cause this to happen (i.e., a command to overdrive the system will occur). An important point to remember in examining these tables is that the maneuvers flown were composed almost exclusively of the high G, Contact type. Inclusion of less active maneuvers will skew the distribution even further to the lower end of the scale. A final point regarding these distributions is the difference in the "Increase" function versus that of the "Decrease" function. This is due, we believe, to the fact that most aerobatic maneuvers, pull positive G's implying a motion platform "up" with respect to the floor, while most washout occurs "down." Spectral computations are likewise easy to interpret. The basic shape, that of a linear drop from .007 Hz to around .07 Hz, then a leveling or slight rise to around .19 Hz, and an exponential drop to 3.75 Hz thereafter is characteristics of all parameters examined in this study. The D.C. offset is not shown on the semilog paper, but is exactly the same as the mean value for each parameter in the Time History Distribution Table. The author chose to include only three parameters (fluid flow, increases, and decreases) because the shape of all distributions was similar. Also, it was felt by the author and Mr. Ed Martin, the originator of the Technology Need under which this work was accomplished, that these parameters represented the most important studied. Although units are not specified on the sides of the graphs, these units are precisely the same as the units shown in the Time History Distribution Table. A very high degree of agreement between

Table 1. **Demanded Flow Distribution Frequency Table**

(Each Interval Represents 5 Cubic Inch Per Second Flow, n = 499,428 Total Counts)

1	72060	51	665	101	61	151	7	201	3
2	64549	52	632	102	64	152	8	202	1
3	51337	53	592	103	67	153	10	203	2
4	40420	54	636	104	58	154	8	204	2
5	32976	55	615	105	43	155	7	205	1
6	27569	56	581	106	64	156	3	205	2
7	23063	57	602	107	52	157	3	207	1
8	19785	58	642	108	52	158	9	208	2
9	17126	59	570	109	42	159	6	209	2
10	14589	60	568	110	45	160	6	210	1
11	12076	61	558	111	41	161	9	211	2
12	10050	62	538	112	36	162	5	212	2
13	8472	63	515	113	22	163	9	213	1
14	7473	64	563	114	39	164	7	214	1
15	6388	65	506	115	36	165	6	215	0
16	5734	66	539	116	38	166	6	216	2
17	4941	67	481	117	33	167	7	217	0
18	4299	68	437	118	26	168	8	218	1
19	3988	69	502	119	25	169	4	219	0
20	3681	70	444	120	17	170	7	220	1
21	330	71	436	121	17	171	9	221	0
22	3043	72	390	122	19	172	7	222	0
23	2783	73	424	123	22	173	6	223	0
24	2839	74	381	124	22	174	4	224	0
25	2506	75	362	125	30	175	9	225	1
26	2463	76	346	126	14	176	3	226	0
27	2243	77	304	127	19	177	2	227	0
28	2123	78	300	128	12	178	2	228	1
29	2017	79	277	129	16	179	5	229	0
30	2006	80	293	130	16	180	4	230	0
31	1933	81	218	131	16	181	8	231	2
32	1856	82	204	132	8	182	2	232	0
33	1790	83	195	133	10	183	5	233	0
34	1796	84	198	134	19	184	5	232	1
35	1667	85	146	135	15	185	4	232	0
36	1586	86	170	136	9	186	8	236	0
37	1554	87	148	137	12	187	3	237	1
38	1446	88	132	138	11	188	1	238	0
39	1351	89	97	139	12	189	4	239	0
40	1186	90	98	140	10	190	3	240	0
41	1174	91	83	141	17	191	4	241	0
42	1135	92	97	142	11	192	4	242	0
43	916	93	87	143	10	193	4	243	0
44	930	94	95	144	13	194	3	244	1
45	842	95	71	145	11	195	3	245	0
46	821	96	75	146	15	196	6	246	0
47	729	97	72	147	13	197	3	247	0
48	728	98	84	148	3	198	1	248	1
49	726	99	73	149	11	199	3	249	1
50	673	100	66	150	7	200	1		

Table 2. **Time History Distribution Table**

Variable	Units	$\bar{X}$	S	B_1	B_2	Max	Min	N
Demanded Flow	in^3/sec	51.22	79.19	280.13	18.25	1245	0	499,428
Leg 1 Velocity (Demanded)	in/sec	0	1.85	−.645	19.76	18.63	−17.58	499,428
Leg 2 Velocity (Demanded)	in/sec	0	1.82	−1.21	20.29	18.82	−18.46	499,428
Leg 3 Velocity (Demanded)	in/sec	0	1.84	−.986	12.18	18.27	−18.55	499,428
Leg 4 Velocity (Demanded)	in/sec	0	2.06	.469	13.38	18.61	−18.52	499,428
Leg 5 Velocity (Demanded)	in/sec	0	1.98	.326	13.23	18.74	−18.93	499,428
Leg 6 Velocity (Demanded)	in/sec	0	1.83	−1.03	11.10	18.17	−18.24	499,428
Increases (Velocity, Demanded)	in/sec	2.84	5.31	25.55	33.93	99.06	0	499,428
Decreases (Velocity, Demanded)	in/sec	−2.84	6.00	−34.64	42.79	0	−94.48	499,428
Actual Flow	in^3/sec	45.69	71.18	246.15	17.00	1015	0	499,428
Leg 1 Velocity (Actual)	in/sec	0	1.67	−.48	19.27	17.02	−19.69	499,428
Leg 2 Velocity (Actual)	in/sec	0	1.63	−1.03	19.9	16.8	−17.02	499,428
Leg 3 Velocity (Actual)	in/sec	0	1.66	−.889	11.65	15.79	−16.4	499,428
Leg 4 Velocity (Actual)	in/sec	0	1.83	.296	12.76	15.99	−16.4	499,428
Leg 5 Velocity (Actual)	in/sec	0	1.79	.382	13.1	24.61	−16.4	499,428
Leg 6 Velocity (Actual)	in/sec	0	1.67	−.953	10.77	16.4	−16.4	499,428
Increases (Velocity, Actual)	in/sec	2.53	4.78	22.4	31.49	80.80	0	499,428
Decreases (Velocity, Actual)	in/sec	−2.53	5.36	−30.28	40.90	0	−82.24	499,428

the spectral characteristics of the "Demanded" time series and the "Actual" time series exists on each parameter. This important point confirms the utility of offline simulation of different motion mathematical models in estimating hydraulic flow requirements.

II. CONCLUSIONS AND RECOMMENDATIONS

As can be seen, the hydraulic flow requirements distribution is highly skewed and band limited. The vast preponderance of power lies to the left of 1 Hz. That fact, coupled with the large skewing coefficient, indicates considerable savings may be made through consolidation of hydraulic power supplies. The central limit theorem points out that sums of random variables, taken from an arbitrary distribution, eventually become "normally" distributed. In this case, the distribution would move toward "normality" from the left (i.e., from the direction of positive skewing). By using one or more large volume pumps, to handle the relatively constant demand from a combination of motion systems, and using smaller, faster reacting pumps, together with accumulators for the overflow, substantial reductions in hydraulic requirements and cost may be possible.

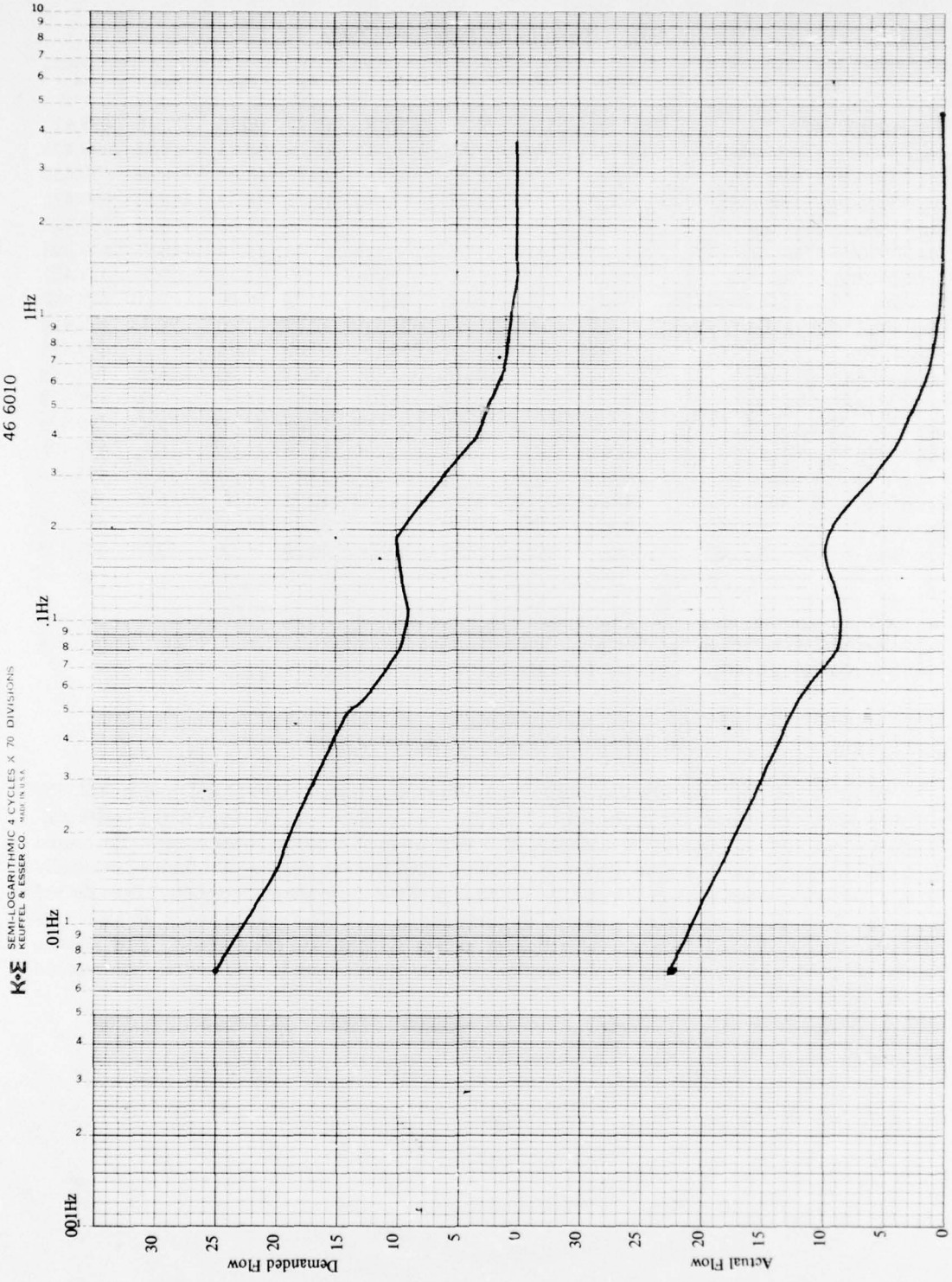

Figure 1. **Demanded actual flow fourier transform graphs.**

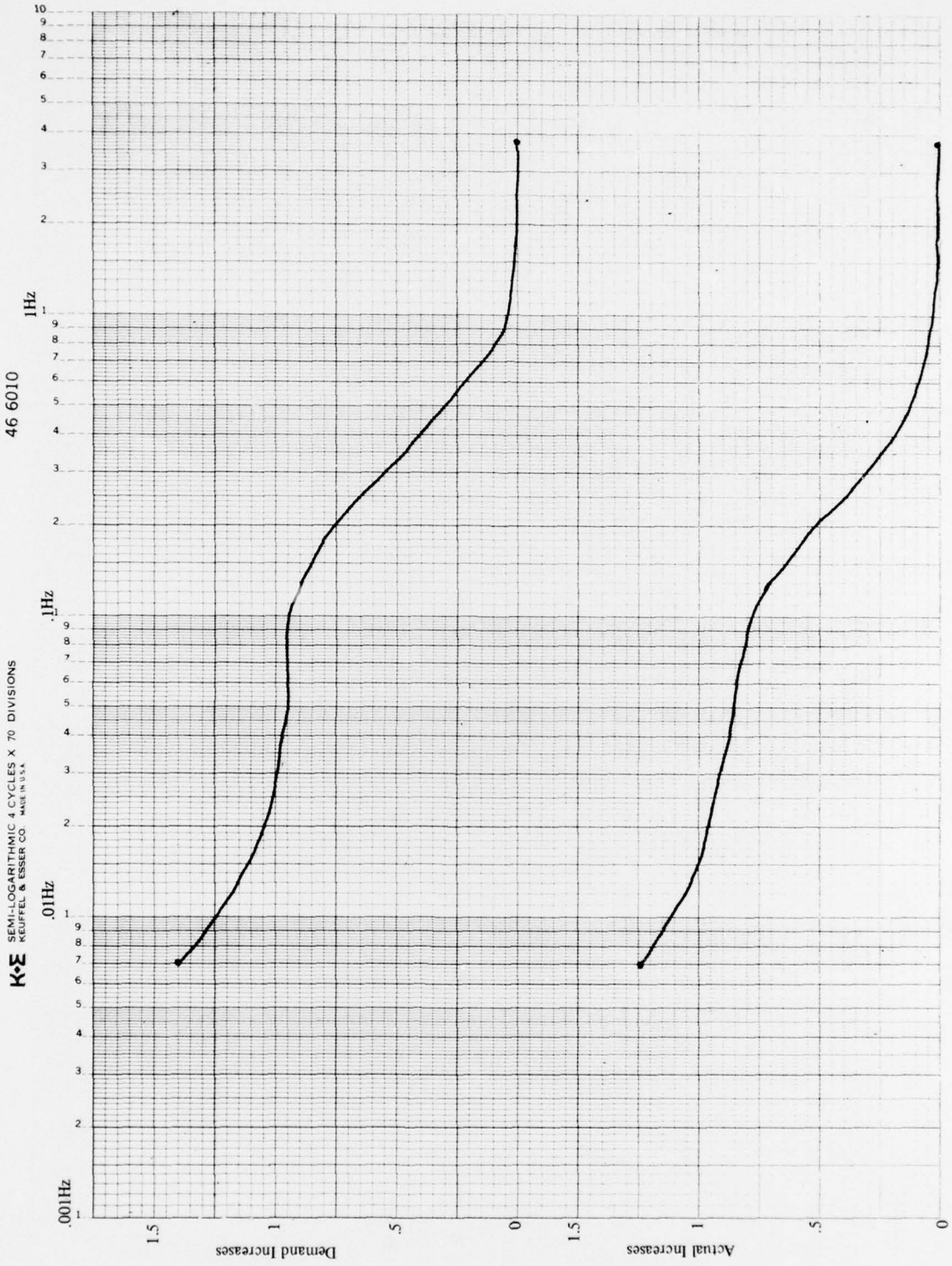

Figure 2. **Demanded and actual "increases" graphs.**

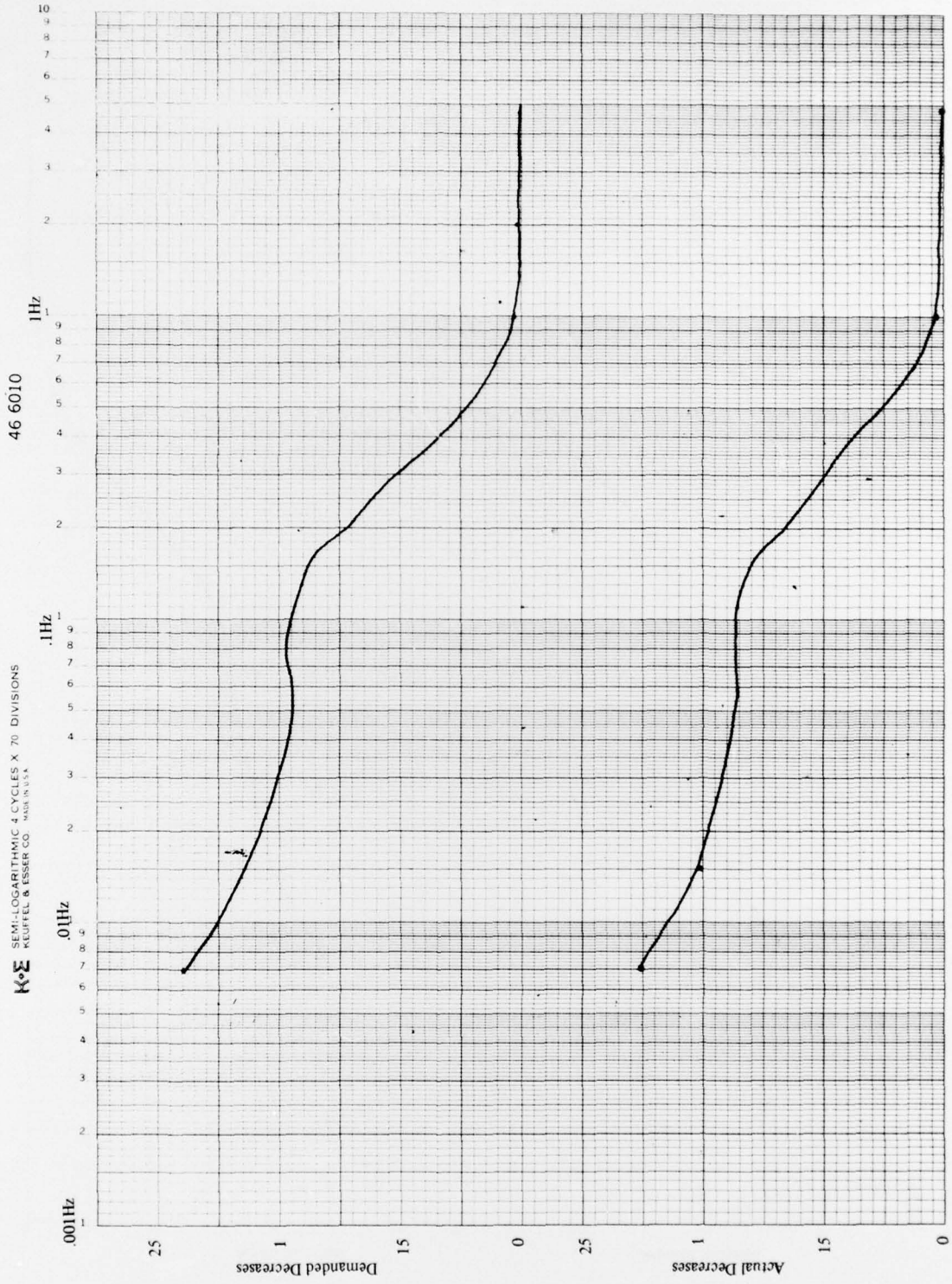

Figure 3. **Demanded and actual "decreases" graphs.**

APPENDIX A: LIST OF FORMULAS

Time History Distribution Calculations

Denoting L_{ij} as the j'th data point for the ith leg, and h as the integration interval (currently h = 2/15 seconds), then the leg cylinder velocity is given by:

$$\dot{L}_{ij} = \frac{L_{ij+1} - L_{ij}}{h}$$

a. Fluid flow, at the $_i$'th point in time, is given by

$$F_j = \sum_{L=1}^{6} \dot{L}_{ij} \cdot A_1(\dot{L}_{ij})$$

where $A_1(X)$ is given by

$$A_1(X) = \begin{cases} 12.566371 \text{ for } x \geq 0 \\ -5.4977871 \text{ for } x < 0 \end{cases}$$

To provide summary statistics, the total, number of points, n, and the sums of the first four moments about zero were saved:

$$n, \sum_{j=1}^{N} F_j, \sum_{j=1}^{N} F_j^2, \sum_{j=1}^{N} F_j^3, \sum_{j=1}^{N} F_j^4$$

During analysis phase, all separate sets of these five numbers were summed, and moments about the origin created:

$$V_1 = \frac{\sum_{j=1}^{N} F_j}{N} \qquad V_2 = \frac{\sum_{j=1}^{N} F_j^2}{N} \qquad V_3 = \frac{\sum_{j=1}^{N} F_j^3}{N} \qquad V_4 = \frac{\sum_{j=1}^{N} F_j^4}{N}$$

which were then used to create the semi-invariants of the distribution:

$$K_1 = V_1$$

$$K_2 = V_2 - V_1^2$$

$$K_3 = V_3 - 3V_1V_2 + 2V_1^3$$

$$K_4 = V_4 - 3V_2^2 - 4V_1V_3 + 12V_1^2V_2 - 6V_1^4$$

which were then turned into the mean ($\bar{x}$), standard deviation(s), skewness (β_1) and kurtosis (β_2).

$$\bar{X} = K_1$$

$$s = \sqrt{K_2}$$

$$\beta_1 = \frac{K_3}{K_2\sqrt{K_2}}$$

$$\beta_2 = \frac{K_4}{K_2^{\,2}}$$

These numbers are considered to define the essential "shape" of the distribution. The same technique is applied to either "demanded" or "actual" flow.

b. The next variables are also derived scores, known as increases "I," and decreases "D." "I" represents the sum of all the positive velocities commanded on all six legs while "D" represents the sum of all the negative velocities commanded. Thus

$$I_j = \sum_{i-1}^{6} \dot{L}_{ij} \cdot A_2(\dot{L}_{ij}) \qquad D_j = \sum_{i=1}^{6} \dot{L}_{ij} \cdot A_3(\dot{L}_{ij})$$

where $A_2(X) = \begin{cases} 1 & X \geq 0 \\ 0 & X < 0 \end{cases}$

$A_3(X) = \begin{cases} 1 & X < 0 \\ 0 & X \geq 0 \end{cases}$

These two derived time series are handled, then, identically to fluid flow, creating $\bar{X}$, S, β_1, and β_2.

c. Finally, summary statistics ($\bar{X}$, S, β_1, β_2) are created for each of the leg velocity commands separately.

Frequency Distribution Calculations

Appendix A, Part 1, described not only the summary statistics obtained, but also the generation of each time series. Using those time series, we also analyzed the frequency content of each parameter. For this purpose, a Fast Fourier Transform (FFT) was implemented. The FFT fitted the time series:

$X_o, X_h, X_{2h}, \ldots X_{1023h}$ $\qquad h = 2/15$ second

by

$$X(t) = 1/2(a_o + a_{512}\cos\pi t) + \sum_{K=1}^{511} \left(a_k \cos\frac{\pi K t}{512} + b_k \sin\frac{\pi K t}{512}\right)$$

where

$$a_k = \frac{1}{512}\sum_{t=o}^{1023h} X(t)\cdot\cos\frac{\pi K t}{512} \qquad K = 0, 1, 2, \ldots, 512$$

$$b_k = \frac{1}{512}\sum_{t=o}^{1023h} X(t)\cdot\sin\frac{\pi K t}{512} \qquad K = 1, 2, \ldots, 511$$

The sample length (approximately 136.53 seconds) was arrived at through consultation with Air Force Flight Dynamics Laboratory, examination of various time series autocorrelations and various sample lengths. Also, the original sampling rate was 15 times per second, (i.e., h = 1/15) and this proved to provide virtually no additional information about any of the parameters examined. Our "demanded" values, on ASPT, are output at 7.5 Hz and therefore constitute a band limited signal at 3.75 Hz. The followup or "actual" signal we thought could contain significantly different spectral characteristics. However, this proved not to be the case and a sampling rate of 7.5 Hz was selected.

The final output for each parameter time series was the ensemble average power, $P(f_k)$ and the ensemble standard deviation $S(f_k)$ at each frequency:

$$f = \frac{\pi k}{512} \quad k = 0, 1, 2, \ldots 512$$

where

$$P(f_o) = \frac{|a_o|}{2}$$

$$P(f_{512}) = \frac{|a_{512}|}{2}$$

$$P(f_k) = \sqrt{a_k^2 + b_k^2} \quad k = 1, 2, \ldots, 511$$

$\bar{P}(f_k)$ = Mean across all sample lengths.

$S(f_k)$ = Standard deviation of the power across all sample lengths.

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