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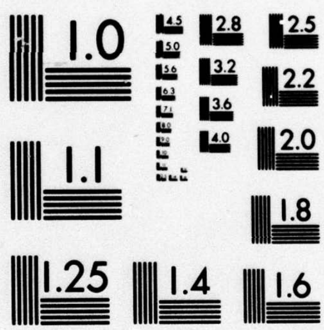
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AN OSCILLATING WING IN A PLANE-PARALLEL
FLOW OF COMPRESSIBLE FLUID

By

Yu. A. Abramov

FOREIGN TECHNOLOGY DIVISION

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AN OSCILLATING WING IN A PLANE-PARALLEL FLOW OF COMPRESSIBLE FLUID

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О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α	α	*α*	Nu	Ν	ν	
Beta	Β	β		Xi	Ξ	ξ	
Gamma	Γ	γ		Omicron	Ο	ο	
Delta	Δ	δ		Pi	Π	π	
Epsilon	Ε	ε	*ϵ*	Rho	Ρ	ρ	*ρ*
Zeta	Ζ	ζ		Sigma	Σ	σ	ς
Eta	Η	η		Tau	Τ	τ	
Theta	Θ	θ	ϑ	Upsilon	Τ	υ	
Iota	Ι	ι		Phi	Φ	φ	ϕ
Kappa	Κ	ϰ κ	*κ*	Chi	Χ	χ	
Lambda	Λ	λ		Psi	Ψ	ψ	
Mu	Μ	μ		Omega	Ω	ω	

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Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

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AN OSCILLATING WING IN A PLANE-PARALLEL FLOW OF COMPRESSIBLE FLUID

Yu. A. Abramov

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This work examines the problem of the oscillations of a wing in a plane-parallel compressible subsonic flow which was posed and solved by various authors (Birnbaum, Possio, Reisner, Sherman, Bio, Timman, Schwartz, Kyussner [as transliterated], Khaskind [as transliterated] [2, 3, 6-9] and others). This problem could not be solved in the closed form; therefore, various approximate methods for its solution have been proposed. In this regard, in the majority of methods which have been brought to numerical calculations the solution is given in the form of series in accordance with the powers of the presented frequency.

Used below is the Bergman method which permits converting the solution of the Laplace equation to solutions of linear equations of the elliptical type. It was shown that the integral equation of the problem can be reduced to the same type as for a noncompressible flow. In this connection, we can employ an algorithm for the solution which was developed by A. N. Panchenkov in work [4] for a plane problem on an oscillating wing in a noncompressible fluid.

The final formulas which were obtained for the forces on an oscillating wing in a compressible flow do not differ in type from the corresponding formulas for a noncompressible flow with the only difference that for the compressible flow the form of the oscillations profile should be obtained from the form of oscillations of an analogous profile in a noncompressible flow but with a correction for compressibility.

Let us examine the unsteady motion of a wing in a flow of compressible fluid. Let us assume that the wing is accomplishing rectilinear translational motion with average velocity v and that a simple harmonic oscillation of infinitely small amplitude is superimposed on this mean motion. Since the wing is considered plane, we will limit ourselves to linearized theory in our investigations.

Let us place the coordinate origin in the coordinate system connected with the wing in the middle of the chord and let us direct the x-axis opposite to the oncoming flow. We select the scale in such a way that the length of the wing half-chord c is equal to one; then the projection of the leading edge of the wing will be at point $x=+1$, and the projection of the trailing edge - at point $x = -1$. In these coordinates, the differential equation which the acceleration potential satisfies has the form

$$(1-M^2)\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}-\frac{2M}{c}\cdot\frac{\partial^2\theta}{\partial x\partial t}-\frac{1}{c^2}\cdot\frac{\partial^2\theta}{\partial t^2}=0, \tag{1}$$

where M - the Mach number; θ - acceleration potential; c - speed of sound in the medium.

If we now consider the periodic motion considering that all values are dimensionless in relation to the half-chord c, then for periodic motions we have $\theta=\bar{\theta}e^{i\omega t}$. Then we write equation (1) in the following manner:

$$(1-M^2)\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}-2ipM^2\frac{\partial\theta}{\partial x}+M^2p\theta=0,$$

where p - is the Strouhal number, $p=\frac{\omega c}{-v}$.

If we now write $\bar{\theta}=\theta^{*}e^{iM^2\Omega x}$, where $\Omega=\frac{p}{1-M^2}$ the differential equation takes the form

$$(1-M^2)\frac{\partial^2\theta^{*}}{\partial x^2}+\frac{\partial^2\theta^{*}}{\partial y^2}+\frac{M^2p^2}{1-M^2}\theta^{*}=0.$$

Desigrating

$$x'=x,\quad y'=y\sqrt{1-M^2},$$

we write the equation as follows:

$$\frac{\partial^2\theta^{*}}{\partial x'^2}+\frac{\partial^2\theta^{*}}{\partial y'^2}+k^2\theta^{*}=0,$$

where

$$k=M\Omega.$$

If we drop the primes and asterisks and place them where it is necessary, the differential equation will be

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+k^2\theta=0.$$

Let us formulate the boundary problem. We solve the equation

(2) $$\Delta\theta+k^2\theta=0,\quad g\in\Omega$$

with the boundary conditions

$$\theta_{y}=F',\quad g\in S;$$
$$\theta\to 0,\quad \bar{x}\to\infty.$$

When passing across the segment of axes ox (-1-+1) the function will have the discontinuity

$$\theta_+ - \theta_- = \gamma(s).$$

As we see, equation (2) pertains to the equations of an elliptical type; therefore, following Bergman's method the solution of the differential equation will be

$$\theta = \theta_0 + \sum_{n=1}^{\infty} L_n(\bar{x})\theta_n(\bar{x}, \bar{y}).$$

Taking the necessary derivatives and substituting them in equation (2), we obtain

$$k^2\theta_0 + \sum_{n=1}^{\infty}(k^2L_n\theta_n + \theta_nL_n'' + 2\theta_0L_1' + 2\theta_nL_{n+1}') = 0,$$

or finally

$$(k^2 + 2L_1')\theta_0 + \sum_{n=1}^{\infty}(k^2L_n + L_n'' + 2L_{n+1}')\theta_n = 0.$$

When satisfying the condition $L_n\theta_n \underset{n\to\infty}{\to} 0$ for the function of L_n we obtain

(3)
$$L_1 = -\frac{1}{2}\int k^2 d\bar{x};$$
$$L_{n+1} = -\frac{1}{2}\left(L_n' + \int k^2 L_n d\bar{x}\right).$$

For the function θ_n, in accordance with the Bergman method we have

(4) $$\theta_n = \int_{-1}^{x} \theta_{n-1} d\bar{x}.$$

Equations (3) and (4) permit calculating the coefficients L_n and θ_n. Being limited by three terms, we determine

$$L_1 = -\frac{1}{2} k^2 \bar{x};$$
$$L_2 = \frac{1}{4}\left(k^2 + \frac{k^4 x^2}{2}\right).$$

We write θ_0 in the form of a logarithmic potential of a double layer

$$\theta_0 = \frac{1}{2\pi} \int_{-1}^{+1} \gamma(s) \frac{\partial}{\partial \eta} \ln \frac{1}{R} ds,$$

where

$$R = \sqrt{(\bar{x} - s)^2 + (\bar{y} - \eta)^2}.$$

Then, having the connection (4), we calculate the coefficients θ_1 and θ_2. Taking the corresponding derivatives and substituting them into the boundary condition, we obtain the integral equation of the problem:

(5) $$\frac{1}{2\pi} \int_{-1}^{+1} \gamma(s) \frac{(\bar{x}-s)^2 - (\bar{y}-\eta)^2}{[(\bar{x}-s)^2 + (\bar{y}-\eta)^2]^2} ds - \frac{1}{2} k^2 \bar{x} \int_{-1}^{x} d\bar{x} \frac{1}{2\pi} \int_{-1}^{+1} \gamma(s) \times$$
$$\times \frac{(\bar{x}-s)^2 - (\bar{y}-\eta)^2}{[(\bar{x}-s)^2 + (\bar{y}-\eta)^2]^2} ds + \frac{1}{4}\left(k^2 + \frac{k^4 \bar{x}^2}{2}\right) \int_{-1}^{x} d\bar{x} \int_{-1}^{x} d\bar{x} \frac{1}{2\pi} \int_{-1}^{+1} \gamma(s) \times$$
$$\times \frac{(\bar{x}-s)^2 - (\bar{y}-\eta)^2}{[(\bar{x}-s)^2 + (\bar{y}-\eta)^2]^2} ds = F'.$$

We designate

$$\int_{-1}^{x} d\bar{x} \int_{-1}^{x} d\bar{x} \frac{1}{2\pi} \int_{-1}^{+1} \gamma(s) \frac{(\bar{x}-s)^2-(\bar{y}-\eta)^2}{[(\bar{x}-s)^2+(\bar{y}-\eta)^2]^2} ds = P,$$

then equation (5) can be written in the form

(6) $$P'' - \frac{1}{2} k^2 \bar{x} P' + \frac{1}{4}\left(k^2 + \frac{k^4 \bar{x}^2}{2}\right) P = F'.$$

With consideration of the initial coordinates, equation (6) will be

(7) $$P'' - \frac{1}{2} k^2 \bar{x} P' + \frac{1}{4}\left(k^2 + \frac{k^4 \bar{x}^2}{2}\right) P = \frac{F e^{-i\lambda \bar{x}}}{\sqrt{1-M^2}},$$

where

$$\lambda = M^2 \Omega.$$

For the solution of equation (7) we use the small parameter method. We introduce the small parameter τ in accordance with the formula

(8) $$\tau = \sqrt{\frac{1}{k^2} + 1} - \frac{1}{k}.$$

Then the solution of equation (7) takes the form

(9)
$$\dot{P} = P_0 + \tau P_1 + \tau^2 P_2 + \tau^3 P_3 + \tau^4 P_4.$$

Taking the corresponding derivatives and substituting them in equation (7), after the accomplishment of the necessary transformations we obtain

$$\begin{aligned}(1-\tau^2)^4 P'' - (2\tau^2\bar{x} - 4\tau^4\bar{x} - 2\tau^6\bar{x})\bar{P} + [\tau^2 - 2\tau^4(1-\bar{x}^2) + \tau^6]P = \\ = \frac{1}{\sqrt{1-M^2}}(F - 4\tau^2 F + 6\tau^4 F - 4\tau^6 F + \tau^8 F - iFM2\tau\bar{x} + \\ + iFM6\tau^3\bar{x} - 6iFM\tau^5\bar{x} + iFM2\tau^7\bar{x} - FM^2 2\tau^2\bar{x}^2 + \\ + FM^2 4\tau^4\bar{x}^2 - FM^2 2\tau^6\bar{x}^2 + iF\frac{4}{3}\tau^3 M^3\bar{x}^3 - \\ - iF\frac{4}{3}\tau^5 M^3\bar{x}^3 + F\frac{2}{3}\tau^4 M^4\bar{x}^4).\end{aligned}$$

Equating the equations with the same powers of τ, we have

(10)
$$\left.\begin{aligned}&\dot{P}_0 = \frac{F}{\sqrt{1-M^2}} = F_0; \quad \dot{P}_1 = -2iF_0 M\bar{x} = F_1; \\ &\dot{P}_2 = 2\bar{x}P_0' - P_0 - 2F_0 M^2\bar{x}^2 = F_2; \\ &\dot{P}_3 = 2\bar{x}\dot{P}_1 + \dot{P}_1 + 14iF_0 M\bar{x} + \frac{4}{3}iF_0 M^3\bar{x}^3 = F_3; \\ &\dot{P}_4 = 2F_z + 2\bar{x}\dot{P}_2 - 2\bar{x}^2 P_0 - P_2 + \frac{2}{3}F_0 M^4\bar{x}^4 = F_4.\end{aligned}\right\}$$

Taking into consideration expression (10), we write

(11)
$$\begin{aligned}\dot{P} = \dot{P}_0 + \tau\dot{P}_1 + \tau^2\dot{P}_2 + \tau^3\dot{P}_3 + \tau^4\dot{P}_4 = \\ = F_0 + \tau F_1 + \tau^2 F_2 + \tau^3 F_3 + \tau^4 F_4 = F_{э\phi}.\end{aligned}$$

Considering that

$$P'' = \frac{1}{2\pi}\int_{-1}^{+1}\gamma(s)\frac{(\bar{x}-s)^2-(\bar{y}-\eta)^2}{[(\bar{x}-s)^2+(\bar{y}-\eta)^2]^2}ds = F_{s\phi} \quad (12)$$

we obtain the integral equation of the problem in the form which was considered in work [4]. The algorithm of the solution was constructed there for the given type.

The kernal in equation (12) - is divergent; therefore, for the value P" we should understand

$$P'' = \lim_{(y-\eta)\to 0}\frac{1}{2\pi}\int_{-1}^{+1}\gamma(s)\frac{(\bar{x}-s)^2-(\bar{y}-\eta)^2}{[(\bar{x}-s)^2+(\bar{y}-\eta)^2]^2}ds.$$

Just as in work [1], we present the value $\gamma(s)$ in the form

$$\gamma(s) = \gamma_1(s) + \gamma_2(s) + \gamma_3(s),$$

where $\gamma_1(s)$ - the regular solution connected with turbulence; $\gamma_2(s)$ - the regular solution which determines inertial motion; $\gamma_3(s)$ - the singular solution of the problem;

$$\gamma_1(s) \in C^1;$$
$$\gamma_2(s) \in C^1$$

(here C^1 determines the class of functions which have a limited first derivative). For class C^1 equation (12) is transformed to a singular integral equation and the function $\gamma_3(s)$ determines the singular solution with a precision up to which the singular integral equation may be solved.

Next, using the algorithm for the solution of formula (12) which is presented in work [4] for the lift of an oscillating wing in a compressible flow we obtain the expression

$$\overline{P}=-2\left[C(p)\int_{-1}^{+1}\sqrt{\frac{1-\overline{x}}{1+\overline{x}}}\,v_y d\overline{x}+ip\int_{-1}^{+1}\sqrt{1-\overline{x}^2}\,v_y d\overline{x}\right], \quad (13)$$

where C(p) is the Theodorsen function

$$C(p)=\frac{H_1^{(2)}(p)}{H_1^{(2)}(p)+iH_0^{(2)}(p)}.$$

We can also obtain the expression for the moment in the known way.

Equation (13) differs from the corresponding equation in work [4] in that the shape of the oscillations v_y for a compressible flow is obtained from the form of the oscillations for a noncompressible flow with a correction for compressibility in accordance with relationships (10).

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